Can higher derivatives resolve the black-hole singularity?

Breno L. Giacchini

Institute of Theoretical Physics, Charles University, Prague, Czech Republic

Based on:

Borissova, Giacchini & Held, *JCAP* **07** [2512.14148]
Giacchini & Kolář, *PRD* **113** [2509.07317]
Giacchini & Kolář, *PRD* **112** [2503.17318]
Giacchini & Kolář, *PRD* **110** [2406.00997]



The 6th BRICS-AGAC Symposium, Hangzhou, 8/XI/2026

Can higher derivatives resolve the black-hole singularity?

- No.
  - It depends – what is meant by higher derivatives and by singularity?

Outline

- Higher derivatives in gravity

- Exact solutions in sixth-derivative gravity
- Exact solutions in quasi-local Einstein-Weyl gravity
- More complicated solutions in quadratic gravity

Higher derivatives & gravity

- General Relativity (GR)
  - Physical singularities for high curvatures.
  - Quantum gravity based on general relativity is not perturbatively renormalizable.

$$d = 2p + 2$$

d: number of derivatives in the counterterms
p: order in the loop expansion

- Way out: GR is not valid at all scales.
- The "true" gravity theory is expected to resolve the singularities and coincide with GR in the low energy domain.

- Natural extension with higher-derivatives:
  - Higher-derivative terms appear already in QFT in curved spacetimes.

$$R^2, \quad R_{\mu\nu}^2, \quad R_{\mu\nu\alpha\beta}^2, \quad \Box R$$

  - Fourth-order gravity is renormalizable [Stelle, 1977]

$$S_4 = \int \mathrm{d}^4x \sqrt{-g} \left[\Lambda + \alpha R + \beta_1 R^2 + \beta_2 R_{\mu\nu}^2 + \beta_3 R_{\mu\nu\alpha\beta}^2 + \beta_4 \Box R \right]$$

  - Models with more than 4 derivatives are super-renormalizable [Asorey, López, Shapiro, 1997]

$$\begin{aligned} S_{2N+4} = \int \mathrm{d}^4x \sqrt{-g} \Big[& \Lambda + \alpha R + \beta_1 R^2 + \beta_2 R_{\mu\nu}^2 + \beta_3 R_{\mu\nu\alpha\beta}^2 + \beta_4 \Box R \\ & + \gamma_1 R \Box R + \gamma_2 R_{\mu\nu} \Box R^{\mu\nu} + \gamma_3 R_{\mu\nu\alpha\beta} \Box R^{\mu\nu\alpha\beta} + \gamma_4 R^3 + \dots \\ & + \rho_1 R \Box^N R + \rho_2 R_{\mu\nu} \Box^N R^{\mu\nu} + \rho_3 R_{\mu\nu\alpha\beta} \Box^N R^{\mu\nu\alpha\beta} + \rho_{\dots} R_{\dots}^{N+2} \Big] \end{aligned}$$

- Problem of ghosts: negative energy, violation of unitarity.

$$G_{\text{spin}-2}(k) \sim \left(\frac{1}{k^2} - \frac{1}{k^2 + m^2}\right)$$

- Higher-derivative and non-local terms appear as perturbative corrections to GR, in the low-energy regime of string theory and of most quantum gravity models.

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\,\sqrt{-g}\left[R + R_{\mu\nu}F_1(\Box)R^{\mu\nu} + RF_2(\Box)R\right]$$

- Modern insights to avoid, tame or understand ghosts:
  - Complex ghosts (Lee-Wick gravity) [Modesto & Shapiro, 2016; Asorey, Krein, Shapiro, 2024]
  - Quantization via *fakeons* [Anselmi & Piva, 2017]
  - Unstable ghosts [Donoghue & Menezes, 2019]
  - Stable and meta-stable scenarios with ghosts [Salvio & Strumia, 2016; Smilga, 2017; Reis, Chapiro & Shapiro, 2019; Donoghue & Menezes, 2021; Deffayet, Mukohyama & Vikman, 2022; Held & Lim, 2025]
  - Nonlocal ghost-free theories [Krasnikov, 1987; Kuz'min, 1989; Tomboulis, 1997; Modesto, 2012; Biswas, Gerwick, Koivisto & Mazumdar, 2012]
- Systematic study of classical solutions of higher-derivative theories.

We shall assume that higher derivatives exist and study their consequences

Higher derivatives & singularities

- Most of the solutions available are in the linear approximation and static, spherically symmetric.
- Linearized higher-derivative gravity:
  - Four-derivative gravity: finite Newtonian potential, but curvature invariants diverge [Stelle, 1978]
  - 6+ derivatives: finite potential and bounded curvature invariants [BLG & Netto, 2019; Burzillà, BLG, Netto & Modesto, 2021]

Exact solutions in higher-derivative gravity

- Four-derivative gravity:
  - Several families of solutions are known [Stelle, 1978; Lü, Perkins, Pope & Stelle, 2015; Bonanno; Holdom; Podolský; Pravda; Pravdová; Ren; Silveravalle; Švarc;…]
  - It seems it cannot resolve the singularity: asymptotically flat solutions that couple to normal matter have a naked singularity at $r = 0$ [Lü, Perkins, Pope & Stelle, 2015]

$$\mathrm{d}s^2 = -B(r)\mathrm{d}t^2 + A(r)\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)$$

Families of solutions: (1,-1): A(r)∼r, B(r)∼1/r

(2,2): A,B∼r^2

(0,0): A,B=const.+O(r)

- General higher-derivative models: Holdom (2002) – models with 4, 6, 8 and 10 derivatives.
  - Reported only finding regular solutions for general models with 6, 8 and 10 derivatives. Suggests that models admitting singular solutions form a zero-measure set.

• Solutions are (almost) always obtained using the Frobenius method (expansion in power-series) and numerical methods.

- Metric ansatz with unknown functions $F_i(r)$ of a certain coordinate r.
- Expansion in Frobenius series around a point r_0:

$$F_i(r) = (r - r_0)^{p_i} \sum_{n=0}^{\infty} f_{i,n}(r - r_0)^n, \qquad f_0 \neq 0.$$

- Solve the indicial equations (field equations at lowest order) for the leading exponents p_i.
- Solve field equations order by order (recurrent relations).
- Try to obtain a global solution based on this local solution (often requires numerical treatment).

Example 1: Six-derivative gravity

[BLG & Kolář, arXiv:2406.00997; 2503.17318]

- General six-derivative gravity

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\Big[\alpha R + \beta_1 R^2 + \beta_2 R_{\mu\nu}^2 + \gamma_1 R\Box R + \gamma_2 R_{\mu\nu}\Box R^{\mu\nu}$$
$$+ \gamma_3 R^3 + \gamma_4 R R_{\mu\nu}R^{\mu\nu} + \gamma_5 R_{\mu\nu}R^{\mu}{}_{\rho}R^{\rho\nu} + \gamma_6 R_{\mu\nu}R_{\rho\sigma}R^{\mu\rho\nu\sigma}$$
$$+ \gamma_7 R R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \gamma_8 R_{\mu\nu\rho\sigma}R^{\mu\nu\tau\upsilon}R^{\rho\sigma}{}_{\tau\upsilon}\Big],$$

- Static spherically symmetric solutions

$$\mathrm{d}s^2 = -B(r)\mathrm{d}t^2 + A(r)\mathrm{d}r^2 + r^2\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)$$

- Field equations

$$H_{\mu\nu} \equiv \frac{1}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = 0 \qquad \nabla^\nu H_{\mu\nu} = 0$$

$$H_{tt} = 0\,, \qquad H_{rr} = 0$$

- Frobenius series solutions around $r = 0$

$$A(r) = r^s \sum_{n=0}^{\infty} a_n r^n, \qquad \frac{B(r)}{b_0} = r^t \left(1 + \sum_{n=1}^{\infty} b_n r^n\right), \qquad s, t \in \mathbb{R}, \quad a_0, b_0 \neq 0$$

- Solution families are identified by the labels $(s, t)_0$, which are assumed to be independent of the couplings.
- System of indicial equations (lowest-order term of the field equations)

$$\frac{H_{tt}(r)}{B(r)} = r^{p(s)} \sum_{i=1}^{8} \gamma_i \, g_i(s, t, a_0) + \ldots = 0, \qquad H_{rr}(r) = r^{q(s)} \sum_{i=1}^{8} \gamma_i \, h_i(s, t, a_0) + \ldots = 0$$

$$g_i(s, t, a_0) = h_i(s, t, a_0) = 0, \quad \forall \, i = 1, \ldots, 8.$$

$$
\begin{aligned}
g_1(s,t) &= 4(s+2)[s(t+4)-t(t+2)-4][(6s-t)(10s^2-st-t^2)+2(118s^2-34st-t^2)+4(69s-14t+22)],\\
h_1(s,t) &= 4(s+2)[s(t+4)-t(t+2)-4][t(10s^2-st-t^2)+2(20s^2+6st-3t^2)+4(31s-6t+14)],\\
g_2(s,t) &= 2[st(60s^4-76s^3t+11s^2t^2+6st^3-t^4)+2(60s^5+146s^4t-208s^3t^2+35s^2t^3+8st^4-t^5+328s^4+152s^3t\\
&\quad -263s^2t^2+96st^3-3t^4)+8(124s^3-69s^2t-15st^2+21t^3-5s^2-110st+2t^2)-96(9s+2t+3)],\\
h_2(s,t) &= 2[st^2(10s^3-11s^2t+t^3)+2t(20s^4+6s^3t-21s^2t^2+2st^3+t^4)+2(60s^4+56s^3t-71s^2t^2-4st^3+21t^4)\\
&\quad +8(64s^3-9s^2t-33st^2-7t^3+47s^2-26st+6t^2)+96(2t-5s-3)],\\
g_3(s,t) &= 4[s(t+4)-t(t+2)-4]^2[30s^2+st-t^2+2(47s-t+37)],\\
h_3(s,t) &= 4[s(t+4)-t(t+2)-4]^2[t(5s+t)+2(10s+4t+23)],\\
g_4(s,t) &= 2[t^2(s-t)^2(30s^2+st-t^2)+2t(80s^4-55s^3t-71s^2t^2+47st^3-t^4)+2(140s^4+146s^3t-193s^2t^2+22st^3\\
&\quad +25t^4)+8(59s^3-20s^2t+23st^2+12t^3-39s^2+28st+56t^2)-32(8s-21t-19)],\\
h_4(s,t) &= 2[t^3(s-t)^2(5s+t)+8t^2(5s^3-6s^2t+t^3)+2t(70s^3-35s^2t-34st^2+27t^3)+8(30s^3+5s^2t-23st^2+14t^3\\
&\quad +19s^2-28st+48t^2)-32(21s-16t-17)],\\
g_5(s,t) &= t^2(s-t)^2(30s^2+st-t^2)+12st(10s^3-3s^2t-14st^2+7t^3)+6(40s^4+84s^3t-23s^2t^2-8st^3+7t^4)\\
&\quad +8(71s^3+69s^2t+24st^2+8t^3)+24(6s^2+16st+11t^2)-32(3s-9t-10),\\
h_5(s,t) &= t^3(s-t)^2(5s+t)+6t^2(5s^3-5s^2t-st^2+t^3)+6t(20s^3+3s^2t-12st^2+9t^3)+8(25s^3+27s^2t-3st^2\\
&\quad -5t^3)+24(10s^2+8st+9t^2)-32(9s-15t-14),\\
g_6(s,t) &= t^2(s-t)^2(30s^2+st-t^2)+2t(40s^4+19s^3t-97s^2t^2+37st^3+t^4)+4(20s^4+24s^3t-22s^2t^2-43st^3\\
&\quad +11t^4)+8(10s^3-5s^2t+23st^2-10t^3-25s^2+54st+39t^2)-32(5s-18t-7),\\
h_6(s,t) &= t^3(s-t)^2(5s+t)+4t^2(5s^3-3s^2t-3st^2+t^3)+4st(10s^2-9st-3t^2)+8(20s^3-7s^2t-14st^2+19t^3\\
&\quad +11s^2-14st+21t^2)-32(15s-4t-11),\\
g_7(s,t) &= 4[t^2(s-t)^2(30s^2+st-t^2)+2t(40s^4+19s^3t-97s^2t^2+37st^3+t^4)+2(40s^4+128s^3t+43s^2t^2-92st^3\\
&\quad +21t^4)+16(15s^3-3s^2t+27st^2-4t^3)+8(46s^2-36st+45t^2)+32(15s+13)],\\
h_7(s,t) &= 4[t^3(s-t)^2(5s+t)+4t^2(5s^3-3s^2t-3st^2+t^3)+2t(20s^3+17s^2t-20st^2+11t^3)+8(20s^3-2s^2t\\
&\quad +5st^2-t^3+26s^2-8st+31t^2)-32(6s-3t-11)],\\
g_8(s,t) &= 4[t^2(s-t)^2(30s^2+st-t^2)+6t^2(31s^3-41s^2t+9st^2+t^3+70s^2-54st+4t^2)-8(s^3-66st^2+19t^3)\\
&\quad +32(9t^2+1)],\\
h_8(s,t) &= 4[t^3(s-t)^2(5s+t)+24st^3(s-t)+12t^3(3s-t)+8(10s^3-3s^2t+7t^3)+144s^2+160].
\end{aligned}
$$

- Frobenius series solutions around $r = 0$

$$A(r) = r^s \sum_{n=0}^{\infty} a_n r^n, \qquad \frac{B(r)}{b_0} = r^t \left(1 + \sum_{n=1}^{\infty} b_n r^n\right), \qquad s, t \in \mathbb{R}, \quad a_0, b_0 \neq 0$$

- Solution families are identified by the labels $(s, t)_0$, which are assumed to be independent of the couplings.
- System of indicial equations (lowest-order term of the field equations)

$$\frac{H_{tt}(r)}{B(r)} = r^{p(s)} \sum_{i=1}^{8} \gamma_i \, g_i(s, t, a_0) + \ldots = 0, \qquad H_{rr}(r) = r^{q(s)} \sum_{i=1}^{8} \gamma_i \, h_i(s, t, a_0) + \ldots = 0$$

$$g_i(s, t, a_0) = h_i(s, t, a_0) = 0, \quad \forall \, i = 1, \ldots, 8.$$

- Only one family of solutions: $s = t = 0, \qquad a_0 = 1.$

- Only solution family: $(0,0)$, characterized by 6 parameters (5 physical)

$$A(r) = 1 + a_2 r^2 + a_3 r^3 + a_4 r^4 + \frac{(a_3 + b_3)\bar{a}_5}{\gamma_2(3\gamma_1 + \gamma_2)} r^5 + O(r^6),$$

$$\frac{B(r)}{b_0} = 1 + b_2 r^2 + b_3 r^3 + b_4 r^4 + \frac{(a_3 + b_3)\bar{b}_5}{\gamma_2(3\gamma_1 + \gamma_2)} r^5 + O(r^6)$$

with $(8\gamma_1 + 3\gamma_2)a_3 = 3(4\gamma_1 + \gamma_2)b_3$

- Solutions in this family are regular at $r = 0$: $R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta} \underset{r\to 0}{=} 12\left(a_2^2 + b_2^2\right) + O(r)$
- No non-trivial solutions such that $R = 0$ and/or $g_{tt}g_{rr} = -1$.
- Other solutions expanded around a finite $r = r_0 \neq 0$:
  - Expansions around a regular point.
  - Expansions around Schwarzschild- and non-Schwarzschild-like horizons.
  - Expansions around a double (extremal) horizon.
  - Expansions around wormhole throats.

[BLG & Kolář, arXiv:2503.17318]

❖ Solutions of incomplete (but renormalizable) sixth-derivative models

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\Big[\alpha R + \beta_1 R^2 + \beta_2 R_{\mu\nu}^2 + \gamma_1 R\Box R + \gamma_2 R_{\mu\nu}\Box R^{\mu\nu} + \gamma_3 R^3 + \gamma_4 R R_{\mu\nu}R^{\mu\nu} + \gamma_5 R_{\mu\nu}R^{\mu}{}_{\rho}R^{\rho\nu} + \gamma_6 R_{\mu\nu}R_{\rho\sigma}R^{\mu\rho\nu\sigma} + \gamma_7 R R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} + \gamma_8 R_{\mu\nu\rho\sigma}R^{\mu\nu\tau\upsilon}R^{\rho\sigma}{}_{\tau\upsilon}\Big],$$

Sixth-derivative terms in the action	(s,t) solution family	Number of free parameters
$R\Box R,\ R_{\mu\nu}\Box R^{\mu\nu},\ R_{\mu\nu\alpha\beta}\Box R^{\mu\nu\alpha\beta}$	$(0,0)$	$6 \to 5$
$R\Box R,\ R_{\mu\nu}\Box R^{\mu\nu}$	$(0,0)$ $(1,-1)$	$6 \to 5$ $3 \to 2$
$R\Box R,\ R_{\mu\nu\alpha\beta}\Box R^{\mu\nu\alpha\beta}$	$(0,0)$	$6 \to 5$
$R\Box R,\ C_{\mu\nu\alpha\beta}\Box C^{\mu\nu\alpha\beta}$	$(0,0)$ $(2,2)$	$6 \to 5$ $8 \to 7$

[BLG & Kolář, arXiv:2406.00997]

Example 2: Quasi-local Einstein-Weyl gravity

$$S[g] = \frac{m_{\rm Pl}^2}{2}\int \mathrm{d}^4x\,\sqrt{-g}\Big[R + \mu C_{\mu\nu\rho\sigma}\big(\eta\Box - m^2\big)^{-1}C^{\mu\nu\rho\sigma}\Big]$$

- Localization procedure with auxiliary field, results in

$$S[g,\psi] \equiv \frac{m_{\rm Pl}^2}{2}\int \mathrm{d}^4x\,\sqrt{-g}\big[R - \mu\big(2C^{\mu\nu\rho\sigma}\psi_{\mu\nu\rho\sigma} + \psi^{\mu\nu\rho\sigma}\big(\eta\Box - m^2\big)\psi_{\mu\nu\rho\sigma}\big)\big]$$

- Conformal-to-Kundt metric ansatz

$$\mathrm{d}s^2 = \Omega^2(\rho)\big[\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2 - 2\,\mathrm{d}u\,\mathrm{d}\rho + \mathcal{H}(\rho)\,\mathrm{d}u^2\big]$$

$$\Omega(\rho) = \Delta^n\sum_{i=0}^{\infty}A_{n+i}\Delta^i\,,\qquad \mathcal{H}(\rho) = \Delta^p\sum_{i=0}^{\infty}B_{p+i}\Delta^i\,,\qquad \Psi(\rho) = \Delta^q\sum_{i=0}^{\infty}C_{q+i}\Delta^i\,,\qquad \Delta \equiv \rho - \rho_0\,.$$

(and similarly for asymptotic expansions in powers of 1/r)

- Solutions labelled by [n,p,q]

- Comparison to the classes of solutions of Einstein-Weyl gravity and quadratic gravity – with coupling-independent exponents:

$[n,p]$ or $[n,p]^\infty$	interpretation	quasi-local EW	EW	quadratic gravity	GR
$[-1,3]^\infty$	Schwarzschild-like core		✓	✓	✓
$[-1,2]^\infty$	regular core	✓	✓	✓	✓
$[1,0]$	2-2 hole core		✓	✓	
$[0,1]_{A_1\neq 0}$	Schwarzschild-like horizon	✓	✓	✓	✓
$[0,1]_{A_1=0}$	horizon	✓	✓	✓	
$[0,1]_{A_{1,2}=0}$	horizon	✓		✓	
$[0,0]_{A_1\neq 0}$	generic point	✓	✓	✓	✓
$[0,0]_{A_1=0,A_2\neq 0}$	non-symmetric throat	✓	✓	✓	
$[0,0]_{A_1=0,A_2\neq 0,\text{ even}}$	symmetric throat	✓	✓	✓	
$[0,0]_{A_{1,2}=0,A_3\neq 0}$	non-symmetric throat	✓	✓	✓	
$[0,0]_{A_{1,2,3}=0,A_4\neq 0}$	non-symmetric throat	✓		✓	
$[0,0]_{A_{1,2,3}=0,A_4\neq 0,\text{ even}}$	symmetric throat			✓	
$[-1,2]$	asymptotic corrections to Schwarzschild	✓	✓	✓	✓

• The only solution expandable in Frobenius series describing the radial core are of type [0,0] – i.e., a regular core. Back to "standard coordinates":

$$\mathrm{d}s^2 = -f(r)\,\mathrm{d}t^2 + h(r)\,\mathrm{d}r^2 + r^2\,\mathrm{d}\Omega^2$$

$$\begin{aligned}
f(r) &= a_0\left[1 + b_2 r^2 + \frac{(240\mu^2 + 76\mu\eta - 9\eta^2)b_2 + 10\mu m^2}{200\mu(\eta + 2\mu)} b_2 r^4\right] + \mathcal{O}(r^6)\,, \\
h(r) &= 1 + b_2 r^2 + \frac{(120\mu^2 + 28\mu\eta - 3\eta^2)b_2 + 10\mu m^2}{100\mu(\eta + 2\mu)} b_2 r^4 + \mathcal{O}(r^6)\,, \\
\psi(r) &= \frac{1}{20\mu} b_2 r^2 + \frac{(320\mu + 147\eta)b_2 + 5m^2}{1400\mu(\eta + 2\mu)} b_2 r^4 + \mathcal{O}(r^6)\,.
\end{aligned}$$

• EW gravity's solutions satisfy $R = 0$. Quasi-local ones generally have $R \neq 0$.

• Solutions in the quasi-local model seem to be more constrained than their equivalents in the local model, as the parameter space has "forbidden regions". (Could also indicate "bad" choice of parameters.)

• Regular black holes might not exist: absence of solutions of type [0,2].

[Borissova, BLG & Held (2026), arXiv:2512.14148]

Example 3: Neglected solutions of quadratic gravity

- General quadratic (fourth-derivative) gravity

$$S = \int \mathrm{d}^4x\,\sqrt{-g}\left(\gamma R + \beta R^2 - \alpha\, C_{\mu\nu\alpha\beta}\, C^{\mu\nu\alpha\beta}\right)$$

- Static spherically symmetric solutions

$$\mathrm{d}s^2 = -B(\bar{r})\mathrm{d}t^2 + \frac{\mathrm{d}\bar{r}^2}{A(\bar{r})} + \bar{r}^2\left(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2\right)$$

$$\mathrm{d}s^2 = -H(r)\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{H(r)} + F^2(r)\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right)$$

$$\mathrm{d}s^2 = \Omega^2(r)\left[\,\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2 - 2\,\mathrm{d}u\,\mathrm{d}r + \mathcal{H}(r)\,\mathrm{d}u^2\,\right]$$

- Field equations are independent of r.
- Existence of recursive relations.
- Precise counting of parameters.

$$ds^2 = -H(r)dt^2 + \frac{dr^2}{H(r)} + F^2(r)\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

- Solutions around $r = r_0$:

$$F(r) = \Delta^\sigma \sum_{n=0}^{\infty} f_n \Delta^n, \qquad H(r) = \Delta^\tau \sum_{n=0}^{\infty} h_n \Delta^n, \quad \text{with} \quad \Delta \equiv r - r_0 \quad \text{and} \quad f_0, h_0 \neq 0.$$

- Solutions around $r = \infty$: $F(r) = r^\sigma \sum_{n=0}^{\infty} f_n r^{-n}, \qquad H(r) = r^\tau \sum_{n=0}^{\infty} h_n r^{-n}$

- Solution families are identified by the labels $\{\sigma, \tau\}$ and $\{\sigma, \tau\}^\infty$

$$ds^2 = -B(\bar{r})dt^2 + \frac{d\bar{r}^2}{A(\bar{r})} + \bar{r}^2\left(d\theta^2 + \sin^2\theta\, d\phi^2\right)$$

$$A(\bar{r}) = (\bar{r} - \bar{r}_0)^s \sum_{n=0}^{\infty} a_n (\bar{r} - \bar{r}_0)^n,$$

$$B(\bar{r}) = b_0(\bar{r} - \bar{r}_0)^t\left[1 + \sum_{n=1}^{\infty} b_n (\bar{r} - \bar{r}_0)^n\right], \qquad a_0, b_0 \neq 0,$$

- Solution families are identified by the labels $(s,t)_0$, $(s,t)_{r_0}$ and $(s,t)\infty$

- Summary of solutions previously known, valid for generic values of the couplings:
  [Stelle, Holdom, Lü, Perkins, Pope]

Solution family (s,t)	Solution family $\{\sigma,\tau\}$	Parameters	Number of free parameters	Interpretation
$(0,0)_0$	$\{1,0\}$	f_0, f_2, h_2, r_0	$4 \to 2$	regular core
$(-1,-1)_0$	$\{1,-1\}$	f_0, f_3, h_0, h_3, r_0	$5 \to 3$	Schwarzschild-like core
$(-2,2)_0$	$\left\{\frac{1}{3},\frac{2}{3}\right\}$ (nF)	$f_0, f_1, f_2, f_3, h_0, h_3, r_0$	$7 \to 5$	2-2-hole core

Footnote of [Lü, Perkins, Pope & Stelle, 2015]: extra family for $\frac{\alpha}{3\beta} = \frac{(s^2-2s+2)^2}{s^4}, \quad s \in \mathbb{Z}^-$

$(0,0)_{\bar{r}_0}$	$\{0,0\}$	$(f_0, f_1, f_2, f_3, h_0, h_1, h_2, h_3), r_0$	$8 \to 6$	generic point
$(1,0)_{\bar{r}_0,1/2}$ (nF)	$\{0,0\}_{f_1=0,\, f_2\neq 0}$	$(f_0, f_2, f_3, h_0, h_1, h_2, h_3), r_0$	$7 \to 5$	non-symmetric throat
$(1,0)_{\bar{r}_0}$	$\{0,0\}_{f_{1,3}=h_{1,3}=0,\, f_2\neq 0}$	$(f_0, f_2, h_0, h_2), r_0$	$4 \to 2$	symmetric throat
$\left(\frac{4}{3},0\right)_{\bar{r}_0,1/3}$ (nF)	$\{0,0\}_{f_{1,2}=0,\, f_3\neq 0}$	$(f_0, f_3, h_0, h_1, h_2, h_3), r_0$	$6 \to 4$	non-symmetric throat

$(1,1)_{\bar{r}_0}$	$\{0,1\}$	f_0, f_1, h_0, h_1, r_0	$5 \to 3$	Schwarzschild-like horizon
$\left(\frac{3}{2},\frac{1}{2}\right)_{\bar{r}_0,1/2}$ (nF)	$\{0,1\}_{f_1=0,\, f_2\neq 0}$	f_0, h_0, h_1, r_0	$4 \to 2$	horizon

$(0,0)_\infty$	$\{1,0\}^\infty$	f_0, f_1, h_1	$3 \to 1$	Schwarzschild

nF = non-Frobenius series (Puiseux series, with fractional increments)

Footnote of [Lü, Perkins, Pope & Stelle, 2015]: extra family for $\frac{\alpha}{3\beta} = \frac{(s^2-2s+2)^2}{s^4}, \quad s \in \mathbb{Z}^-$

- In modified coordinates, they are characterized by

$$\sigma = \frac{3}{2} - \tau, \qquad \omega = \frac{(2\tau^2 - 6\tau + 5)^2}{4(\tau-1)^4}, \qquad 1 < \tau < \frac{3}{2}.$$

These relations uniquely define τ as a function of ω,

$$\tau_+(\omega) = \frac{1}{2}\left(2 - \frac{1}{\sqrt{\omega}-1} + \sqrt{\frac{2\sqrt{\omega}-1}{(\sqrt{\omega}-1)^2}}\right), \qquad \omega > 1,$$

- Assuming a rational τ, $\tau = \frac{p}{q}$.

it follows:

$$\sigma = \frac{3}{2} - \frac{p}{q}, \qquad \omega = \frac{(2p^2 - 6pq + 5q^2)^2}{4(p-q)^4}, \qquad \frac{2p}{3} < q < p.$$

We proved that there exist solutions in the form of Puiseux series:

$$F(r) = \Delta^\sigma \sum_{n=0}^{\infty} f_n \Delta^{\frac{n}{q}}, \qquad H(r) = \Delta^\tau \sum_{n=0}^{\infty} h_n \Delta^{\frac{n}{q}},$$

$$f_0, h_0 \neq 0, \qquad \Delta \equiv r - r_0.$$

[BLG & Kolář, arXiv:2509.07317]

Solution family (s,t)	Solution family $\{\sigma,\tau\}$	Parameters	Number of free parameters	Interpretation	Remarks
$(0,0)_0$	$\{1,0\}$	f_0, f_2, h_2, r_0	$4 \to 2$	regular core	
$(-1,-1)_0$	$\{1,-1\}$	f_0, f_3, h_0, h_3, r_0	$5 \to 3$	Schwarzschild-like core	
$(-2,2)_0$	$\left\{\frac{1}{3},\frac{2}{3}\right\}$ (nF)	$f_0, f_1, f_2, f_3, h_0, h_3, r_0$	$7 \to 5$	2-2-hole core	
$(s_*, 2-3s_*)_0$ (nF)	$\left\{\frac{3}{2}-\tau_+, \tau_+\right\}$ (nF)	f_0, h_0, r_0	$3 \to 1$	singular core	$\omega > 1$, $\tau_+ \in \mathbb{Q}$, τ_+ in Eq. (11)
$(0,0)_{\bar{r}_0}$	$\{0,0\}$	$(f_0, f_1, f_2, f_3, h_0, h_1, h_2, h_3), r_0$	$8 \to 6$	generic point	
$(1,0)_{\bar{r}_0,1/2}$ (nF)	$\{0,0\}_{f_1=0,\, f_2\neq 0}$	$(f_0, f_2, f_3, h_0, h_1, h_2, h_3), r_0$	$7 \to 5$	non-symmetric throat	
$(1,0)_{\bar{r}_0}$	$\{0,0\}_{f_{1,3}=h_{1,3}=0,\, f_2\neq 0}$	$(f_0, f_2, h_0, h_2), r_0$	$4 \to 2$	symmetric throat	
$\left(\frac{4}{3},0\right)_{\bar{r}_0,1/3}$ (nF)	$\{0,0\}_{f_{1,2}=0,\, f_3\neq 0}$	$(f_0, f_3, h_0, h_1, h_2, h_3), r_0$	$6 \to 4$	non-symmetric throat	
$\left(\frac{3}{2},0\right)_{\bar{r}_0,1/4}$ (nF)	$\{0,0\}_{f_{1,2,3}=0,\, f_4\neq 0}$	$(f_0, h_0, h_1, h_2, h_3), r_0$	$5 \to 3$	non-symmetric throat	$\omega \neq 1$
$\left(\frac{3}{2},0\right)_{\bar{r}_0,1/2}$ (nF)	$\{0,0\}_{f_{1,2,3}=h_{1,3}=0,\, f_4\neq 0}$	$(f_0, h_0, h_2), r_0$	$3 \to 1$	symmetric throat	$\omega \neq 1$
$\left(\frac{8}{5},0\right)_{\bar{r}_0,1/5}$ (nF)	$\{0,0\}_{f_{1,2,3,4}=0,\, f_5\neq 0}$	$(f_0, h_0, h_1, h_2), r_0$	$4 \to 2$	non-symmetric throat	$\omega \neq 1$ and Eq. (24)
$\left(\frac{5}{3},0\right)_{\bar{r}_0,1/6}$ (nF)	$\{0,0\}_{f_{1,2,3,4,5}=0,\, f_6\neq 0}$	h_0, h_1, r_0	$3 \to 1$	non-symmetric throat	$\omega \neq 1$, see Eq. (25)
$(1,1)_{\bar{r}_0}$	$\{0,1\}$	f_0, f_1, h_0, h_1, r_0	$5 \to 3$	Schwarzschild-like horizon	
$\left(\frac{3}{2},\frac{1}{2}\right)_{\bar{r}_0,1/2}$ (nF)	$\{0,1\}_{f_1=0,\, f_2\neq 0}$	f_0, h_0, h_1, r_0	$4 \to 2$	horizon	
$\left(\frac{5}{3},\frac{1}{3}\right)_{\bar{r}_0,1/3}$ (nF)	$\{0,1\}_{f_{1,2}=0,\, f_3\neq 0}$	f_0, h_0, r_0	$3 \to 1$	horizon	$\omega \neq 1$ and Eq. (24)
$\left(\frac{7}{4},\frac{1}{4}\right)_{\bar{r}_0,1/4}$ (nF)	$\{0,1\}_{f_{1,2,3}=0,\, f_4\neq 0}$	h_0, r_0	$2 \to 0$	horizon	$\omega \neq 1$ and Eq. (25)
$(s_*, 2-3s_*)_\infty$ (nF)	$\left\{\frac{3}{2}-\tau_-, \tau_-\right\}$ (nF)	f_0, h_0, r_0	$3 \to 1$	singular boundary	$1/4 < \omega < 1$, $\tau_- \in \mathbb{Q}$, τ_- in Eq. (22)
$(0,0)_\infty$	$\{1,0\}^\infty$	f_0, f_1, h_1	$3 \to 1$	Schwarzschild	

$$\frac{\alpha}{3\beta} \equiv \omega, \qquad \tau_\pm(\omega) = \frac{1}{2}\left(2 - \frac{1}{\sqrt{\omega}-1} \pm \sqrt{\frac{2\sqrt{\omega}-1}{\left(\sqrt{\omega}-1\right)^2}}\right) \qquad \omega = \frac{(s_*^2 - 2s_* + 2)^2}{s_*^4}$$

Solution family (s,t)	$\{\sigma,\tau\}$	Parameters	Number of free parameters	Interpretation	Remarks
$(0,0)_0$	$\{1,0\}$	f_0, f_2, h_2, r_0	$4 \to 2$	regular core	
$(-1,-1)_0$	$\{1,-1\}$	f_0, f_3, h_0, h_3, r_0	$5 \to 3$	Schwarzschild-like core	
$(-2,2)_0$	$\left\{\frac{1}{3},\frac{2}{3}\right\}$ (nF)	$f_0, f_1, f_2, f_3, h_0, h_3, r_0$	$7 \to 5$	2-2-hole core	
$(s_*, 2-3s_*)_0$ (nF)	$\left\{\frac{3}{2}-\tau_+, \tau_+\right\}$ (nF)	f_0, h_0, r_0	$3 \to 1$	singular core	$\omega > 1$, $\tau_+ \in \mathbb{Q}$, τ_+ in Eq. (11)
$(0,0)_{\bar{r}_0}$	$\{0,0\}$	$(f_0, f_1, f_2, f_3, h_0, h_1, h_2, h_3), r_0$	$8 \to 6$	generic point	
$(1,0)_{\bar{r}_0,1/2}$ (nF)	$\{0,0\}_{f_1=0,\, f_2\neq 0}$	$(f_0, f_2, f_3, h_0, h_1, h_2, h_3), r_0$	$7 \to 5$	non-symmetric throat	
$(1,0)_{\bar{r}_0}$	$\{0,0\}_{f_{1,3}=h_{1,3}=0,\, f_2\neq 0}$	$(f_0, f_2, h_0, h_2), r_0$	$4 \to 2$	symmetric throat	
$\left(\frac{4}{3},0\right)_{\bar{r}_0,1/3}$ (nF)	$\{0,0\}_{f_{1,2}=0,\, f_3\neq 0}$	$(f_0, f_3, h_0, h_1, h_2, h_3), r_0$	$6 \to 4$	non-symmetric throat	
$\left(\frac{3}{2},0\right)_{\bar{r}_0,1/4}$ (nF)	$\{0,0\}_{f_{1,2,3}=0,\, f_4\neq 0}$	$(f_0, h_0, h_1, h_2, h_3), r_0$	$5 \to 3$	non-symmetric throat	$\omega \neq 1$
$\left(\frac{3}{2},0\right)_{\bar{r}_0,1/2}$ (nF)	$\{0,0\}_{f_{1,2,3}=h_{1,3}=0,\, f_4\neq 0}$	$(f_0, h_0, h_2), r_0$	$3 \to 1$	symmetric throat	$\omega \neq 1$
$\left(\frac{8}{5},0\right)_{\bar{r}_0,1/5}$ (nF)	$\{0,0\}_{f_{1,2,3,4}=0,\, f_5\neq 0}$	$(f_0, h_0, h_1, h_2), r_0$	$4 \to 2$	non-symmetric throat	$\omega \neq 1$ and Eq. (24)
$\left(\frac{5}{3},0\right)_{\bar{r}_0,1/6}$ (nF)	$\{0,0\}_{f_{1,2,3,4,5}=0,\, f_6\neq 0}$	h_0, h_1, r_0	$3 \to 1$	non-symmetric throat	$\omega \neq 1$, see Eq. (25)
$(1,1)_{\bar{r}_0}$	$\{0,1\}$	f_0, f_1, h_0, h_1, r_0	$5 \to 3$	Schwarzschild-like horizon	
$\left(\frac{3}{2},\frac{1}{2}\right)_{\bar{r}_0,1/2}$ (nF)	$\{0,1\}_{f_1=0,\, f_2\neq 0}$	f_0, h_0, h_1, r_0	$4 \to 2$	horizon	
$\left(\frac{5}{3},\frac{1}{3}\right)_{\bar{r}_0,1/3}$ (nF)	$\{0,1\}_{f_{1,2}=0,\, f_3\neq 0}$	f_0, h_0, r_0	$3 \to 1$	horizon	$\omega \neq 1$ and Eq. (24)
$\left(\frac{7}{4},\frac{1}{4}\right)_{\bar{r}_0,1/4}$ (nF)	$\{0,1\}_{f_{1,2,3}=0,\, f_4\neq 0}$	h_0, r_0	$2 \to 0$	horizon	$\omega \neq 1$ and Eq. (25)
$(s_*, 2-3s_*)_\infty$ (nF)	$\left\{\frac{3}{2}-\tau_-, \tau_-\right\}$ (nF)	f_0, h_0, r_0	$3 \to 1$	singular boundary	$1/4 < \omega < 1$, $\tau_- \in \mathbb{Q}$, τ_- in Eq. (22)
$(0,0)_\infty$	$\{1,0\}^\infty$	f_0, f_1, h_1	$3 \to 1$	Schwarzschild	

These new type of solutions looks exotic, but it can be important in more complicated models.

[BLG, Kolář, Pravda & Pravdová, arXiv:2605.14658]

Conclusions

- Higher-derivative gravity models have many applications in quantum gravity, but the study of their classical solutions are still at early stages.

At linear approximation:

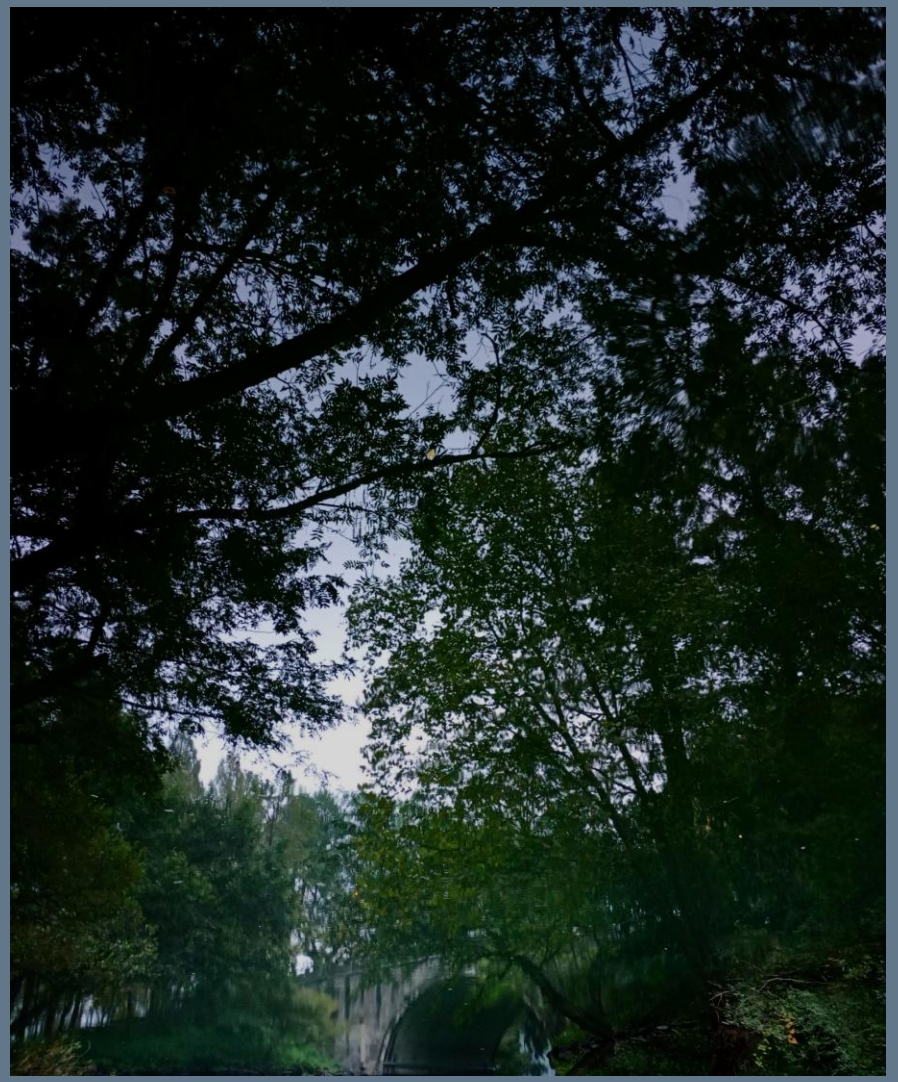
- Theories with *more* than four derivatives in both spin-2 and spin-0 sectors have not only a finite Newtonian potential, but also a regular non-relativistic limit.
- Divergences are softened when going from 2 to 4, and to 6+ derivatives:
  - Non-renormalizable ---> renormalizable ---> super-renormalizable
  - Singular potential and curvature ---> finite potential ---> finite potential and regular curvature ---> regular curvature-derivative scalars

Exact static spherically symmetric solutions of six-derivative gravity:

- Solutions that possess a Frobenius expansion around $r = 0$ are necessarily regular.
- This is in contrast to fourth-derivative gravity.
- We have identified solutions with black hole horizons and double-degenerate horizons.
- Conditions $R = 0$ and/or $g_{tt}g_{rr} = -1$ are too restrictive.
- There exist singular solutions with coupling-dependent leading exponents.

Quasilocal Einstein-Weyl gravity:

- Similar situation, but without extremal horizon.



West Lake, Hangzhou, 6/XI/2026

Thank you!