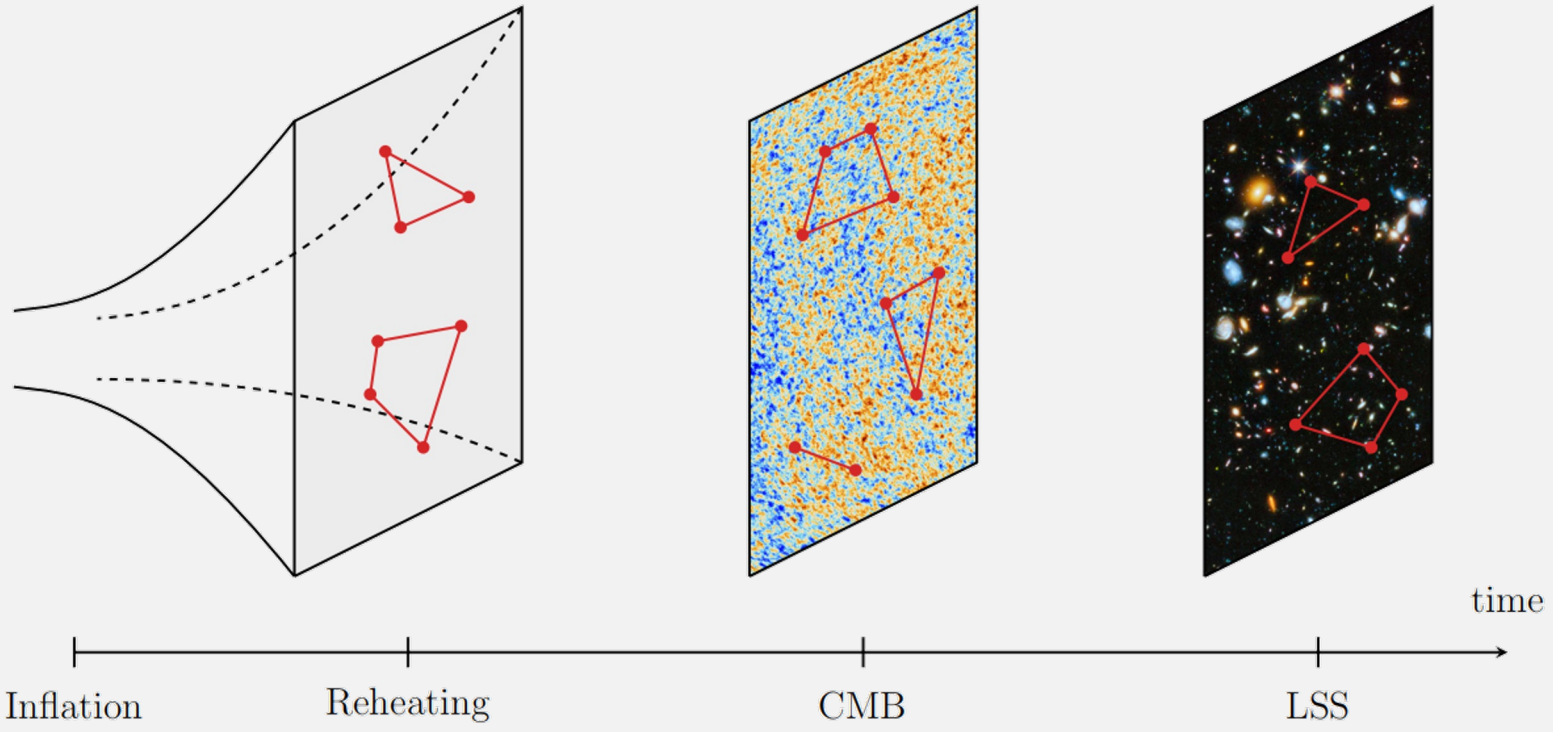
Tracing Out Massive Fields in Cosmology

Guanhao Sun
Chongqing University

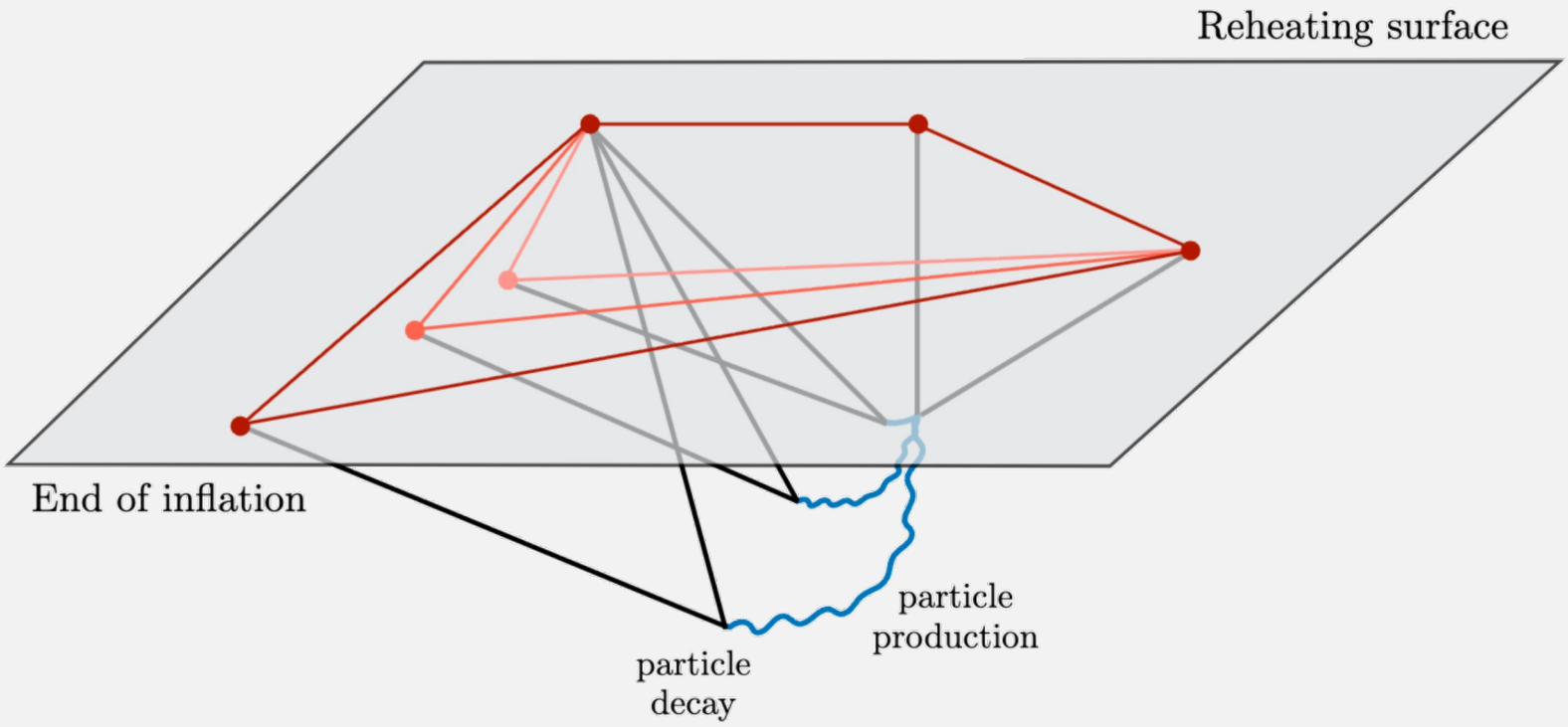
Based mainly on 2609.30370 with Sam S.C. Wong (CityU HK)
and 2412.02739 with Daniel Green (UCSD)

Correlation Functions



C. Duaso Pueyo, A Bootstrap Approach to Cosmology. PhD thesis, University of Amsterdam, 2022

Correlation Functions



Correlation Functions

- EFT of inflation – most general action for SSB of time translation:

$$S_\pi = \int d^4x \sqrt{-g} \left[M_{\rm Pl}^2 \dot{H} \left(\partial_\mu \pi\right)^2 + 2M_2^4 \left(\dot{\pi}^2 + \dot{\pi}^3 - \dot{\pi} \frac{1}{a^2} (\partial_i \pi)^2 \right) - \frac{4}{3} M_3^4 \dot{\pi}^3 - \frac{\bar{M}^2}{2} \frac{1}{a^4} (\partial_i^2 \pi)^2 + \dots \right]$$

Cheung et. al. 0709.0293 and many more

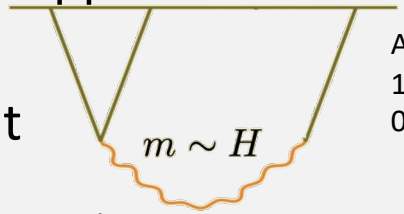
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  - Oscillatory signals in squeezed limit
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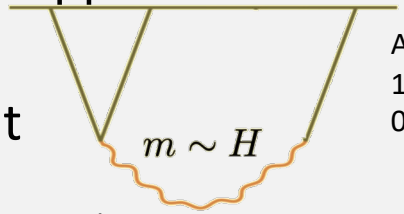
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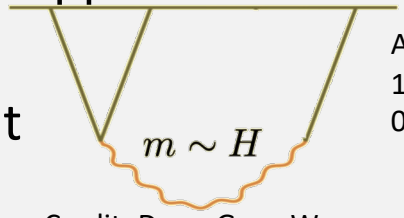
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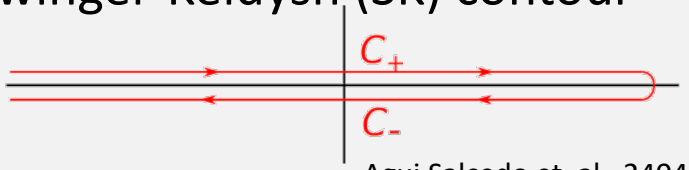
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- Open EFT of inflation – EFT on a Schwinger-Keldysh (SK) contour
  - Bottom-up construction
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Agui Salcedo et. al. 2404.15416 and many more

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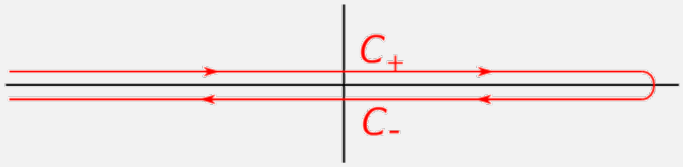
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- Explicit calculations for branch-crossing and boundary terms
- Basis for understanding UV completion and more

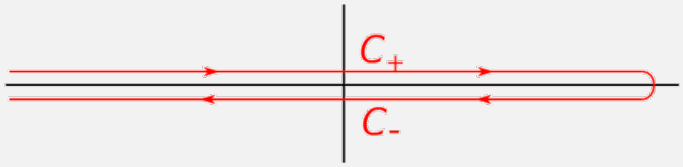
In-Out vs. In-In



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$$\langle \mathcal{O}(t) \rangle = \left\langle \bar{\mathrm{T}} \left\{ e^{i \int_{-\infty(1+i\epsilon)}^{t} dt\, H_{\mathrm{int}}(t)} \right\} \mathcal{O}^{\mathrm{int}}(t) \mathrm{T} \left\{ e^{-i \int_{-\infty(1-i\epsilon)}^{t} dt\, H_{\mathrm{int}}(t)} \right\} \right\rangle$$

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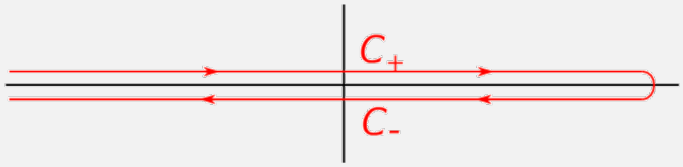
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- Expand in interacting Hamiltonian

$$\langle \mathcal{O}(t_0) \rangle = \sum_{N=0}^{\infty} i^N \int_{-\infty}^{t_0} dt_N \int_{-\infty}^{t_N} \cdots \int_{-\infty}^{t_2} dt_1 \langle [H_I(t_1), [H_I(t_2), \cdots [H_I(t_N), \mathcal{O}(t_0)] \cdots]] \rangle$$

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- Alternatively,

$$\begin{aligned} \langle \mathcal{O}(t=0) \rangle &= \int \mathcal{D}\bar{\phi}\mathcal{D}\bar{\sigma}\ \langle 0|\bar{\phi},\bar{\sigma}\rangle\langle\bar{\phi},\bar{\sigma}|\mathcal{O}|0\rangle \\ &= \int \mathcal{D}\bar{\phi}\mathcal{D}\bar{\sigma} \left(\int^{\bar{\phi},\bar{\sigma}} \mathcal{D}\phi'\mathcal{D}\sigma'\ e^{iS[\phi',\sigma']} \right)^* \mathcal{O}[\bar{\phi},\bar{\sigma}] \left(\int^{\bar{\phi},\bar{\sigma}} \mathcal{D}\phi\mathcal{D}\sigma\ e^{iS[\phi,\sigma]} \right) \\ &= \int \mathcal{D}\bar{\phi}\mathcal{D}\bar{\sigma}\ |\Psi[\bar{\phi},\bar{\sigma}]|^2\ \mathcal{O}[\bar{\phi},\bar{\sigma}]\ . \end{aligned}$$

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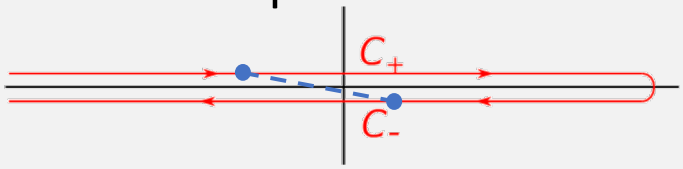
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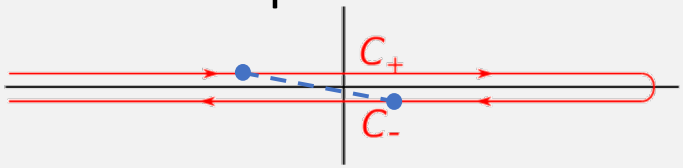


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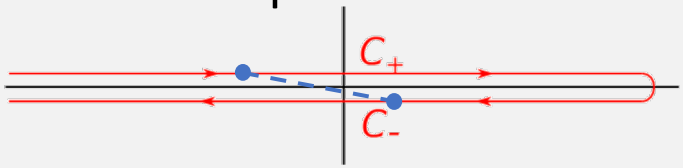
- Their existence admits a non-perturbative proof through exact RG (see Green and GS 2412.02739 for details)

Integrating Out Heavy Fields

- The action follows as

$$S \supset \frac{1}{2}\int d^4x\, \lambda^2\phi^2 \frac{1}{M^2-\Box}\phi^2 + \frac{i}{2}\int d^3x \lambda^2 \left(\frac{1}{M^2-\Box}+\frac{1}{(i\partial_t - E)E}\right)\bar{\phi}^2 \frac{i\partial_t + E}{M^2-\Box}\bar{\phi}^2$$

- Using this, we recover the correct result
- The existence of these corrections is a result of possible branch-mixing terms that cannot be reproduced within single-branch



- Their existence admits a non-perturbative proof through exact RG (see Green and GS 2412.02739 for details)
- The seemingly single-branch action is the result after integrating over time evolutions

General Procedure

- Start with the formal solution

$$\sigma_{k,+}(\tau) = \sigma^*_{k,-}(\tau) = \bar{\sigma}_k K_k(\tau) + \int_{\mathcal{C}_+} d\tau'\, a^4(\tau') G_k(\tau, \tau') J_{+,k}(\tau')$$

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  or explicitly integrating out $\bar{\sigma}$ in the action.

Quasi-Single Field Example

- Work in de Sitter background

$$ds^2 = a^2(\tau)\left(-d\tau^2 + d\mathbf{x}^2\right) \ , \quad a(\tau) = -\frac{1}{H\tau}$$

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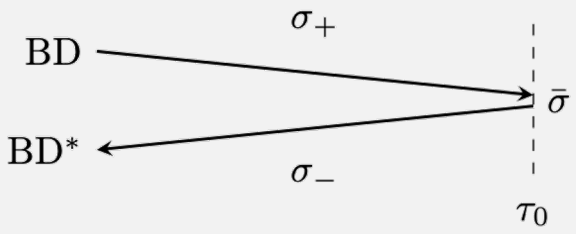
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- Here F is a generic function with zero constant term
- We write the formal solution to the heavy field

$$\sigma^{(0)}_{\pm,\mathbf{k}}(\tau) = \bar{\sigma}_{\mathbf{k}} K_{\pm,k}(\tau) + g\int_{\mathcal{C}_\pm} d\tau'\, a^2(\tau') G_{D,\pm,k}(\tau,\tau') F_{\pm,\mathbf{k}}(\tau') \ .$$

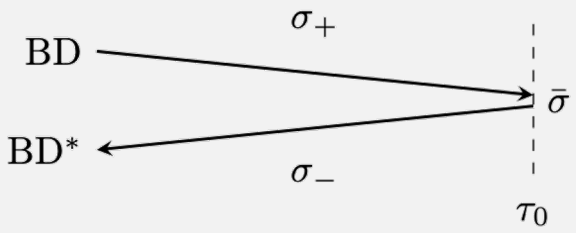
$$K_{+,k}(\tau) = \frac{u_k(\tau)}{u_k(\tau_0)} \ , \quad G_{D,\pm,k}(\tau_0,\tau') = G_{D,\pm,k}(\tau,\tau_0) = 0$$

The Boundary Value



- Now plug in σ and integrate over $\bar{\sigma}$

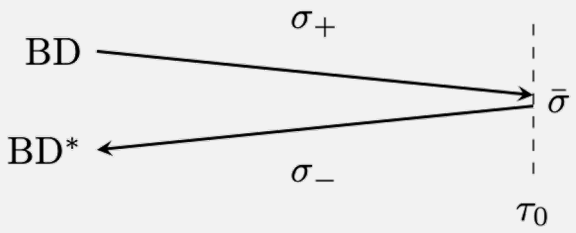
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$$S^{(0),\mathrm{os}}_{\sigma,\pm} = \frac{1}{2}\int_{\mathbf{k}} \Omega_{\pm,k}\bar{\sigma}_{\mathbf{k}}\bar{\sigma}_{-\mathbf{k}} + g\int_{\mathbf{k}} \bar{\sigma}_{-\mathbf{k}} I_{\pm,\mathbf{k}}$$
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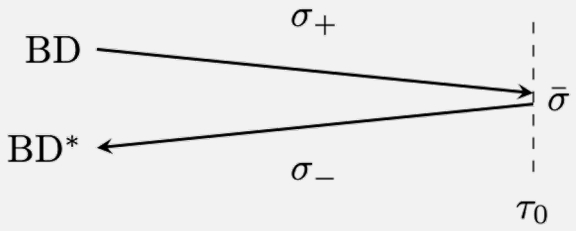
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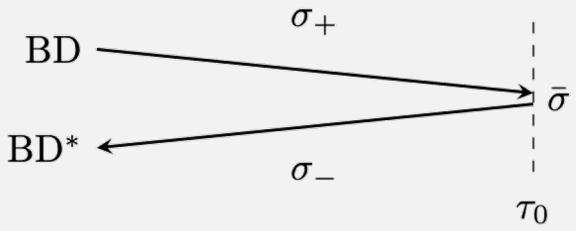
$$I_{\pm,\mathbf{k}}[F_\pm] \equiv \int_{\mathcal{C}_\pm} d\tau\, a^2(\tau)K_{\pm,k}(\tau)F_{\pm,\mathbf{k}}(\tau) \qquad \Omega_{+,k} = a_0^2 K'_{+,k}(\tau_0)$$

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- Then calculate $\int \mathcal{D}\,\bar{\sigma}\; e^{iS_+ - iS_-} \;=\; \int \mathcal{D}\,\bar{\sigma}\; e^{iS_{\mathrm{SK}}}$

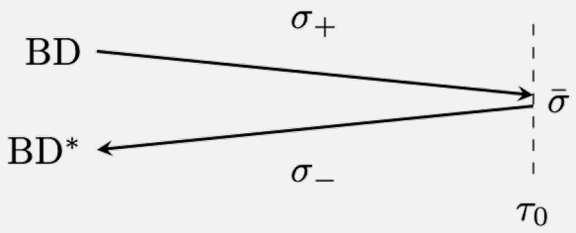
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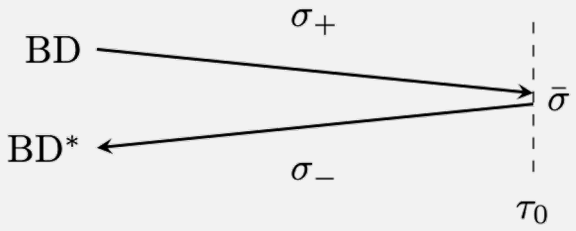
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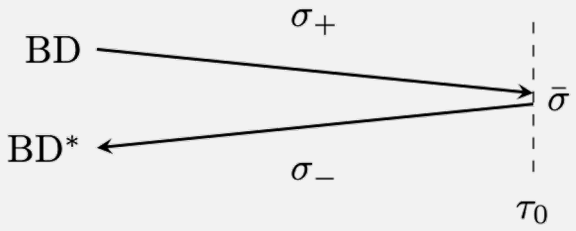
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- We can then complete the square for a Gaussian integral, with mean value given by

$$\bar{\sigma}_{*,\mathbf{k}} = \; -\, g\frac{\Delta I_{\mathbf{k}}}{\Delta\Omega_k} = igP_k\Delta I_{\mathbf{k}}$$

The Boundary Value

- Completing the square leaves us with an extra term

$$iS_{\mathrm{SK,EFT}} \supset -\frac{g^2}{2}\int_{\mathbf{k}} P_k \Delta I_{\mathbf{k}} \Delta I_{-\mathbf{k}}$$

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- The boundary value admits a bulk and a boundary view

Branch-Mixing terms

- To clearly separate out the branch-mixing terms, change G_D to G_F, check the full Schwinger-Keldysh action,

$$
\begin{aligned}
iS^{(2)}_{\mathrm{IF}} = \frac{ig^2}{2}\int_{\mathbf{k}}\Bigg(&\int_{\mathcal{C}_+} d\tau \int_{\mathcal{C}_+} d\tau'\, a^2(\tau)a^2(\tau')F_{+,\mathbf{k}}(\tau)G_{F,k}(\tau,\tau')F_{+,-\mathbf{k}}(\tau') \\
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- The last line contains all the branch-mixing terms

The Reduced Density Matrix

- The proper object to study is then a reduced density matrix

$$\rho_{\rm red}[\bar{\zeta}_+,\bar{\zeta}_-] \propto \int_{\zeta_\pm(\tau_0)=\bar{\zeta}_\pm} \mathcal{D}\zeta_+\mathcal{D}\zeta_- \, \exp\Bigg\{ iS_\zeta[\zeta_+] - iS_\zeta[\zeta_-]$$
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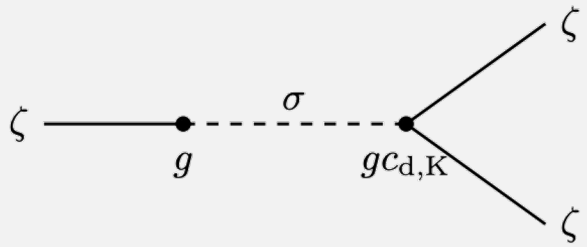
- Note the branch-mixing terms that cannot be reproduced from a pure state
- The density matrix cannot be written as product of some pure state, or, $\rho_{\rm red} \neq \Psi[\bar{\zeta}_+]\Psi^*[\bar{\zeta}_-]$

Cubic Results

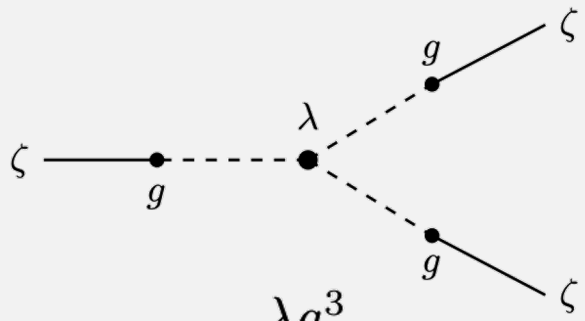
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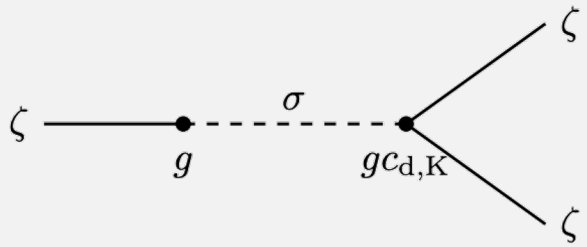
$g^2 c_{\rm d,K}$



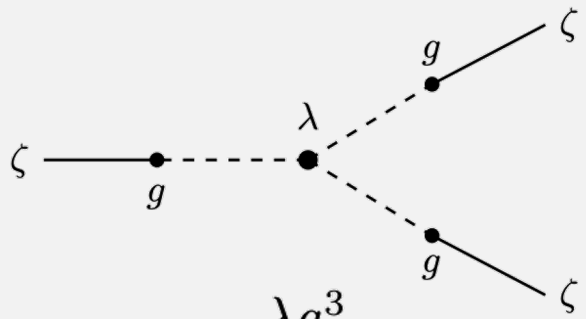
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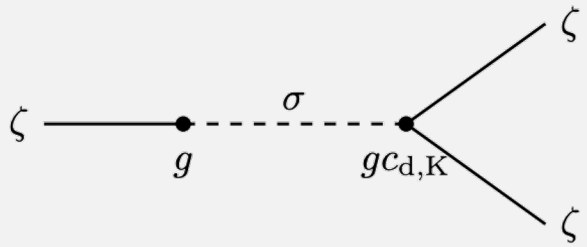
λg^3

- Represent cubic interactions in the reduced density matrix using

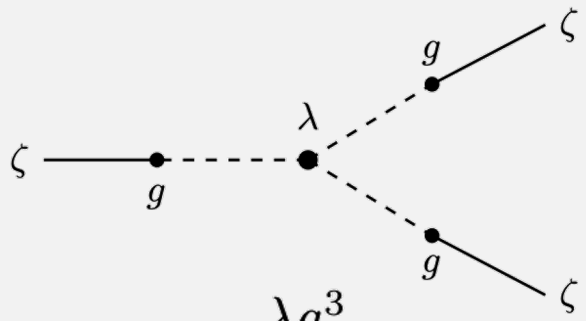
$$\ln \rho_{\rm red}|_{\bar{\zeta}^3} = -\frac{1}{3!}\int_{\mathbf{k}_1\mathbf{k}_2\mathbf{k}_3}(2\pi)^3\delta^3\left(\sum_{a=1}^{3}\mathbf{k}_a\right)\sum_{s_1,s_2,s_3=\pm}\Gamma^{(3)}_{s_1s_2s_3}(1,2,3)\prod_{a=1}^{3}\bar{\zeta}_{s_a,\mathbf{k}_a}$$

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- The Γ's can be calculated through convolutions of propagators

Cubic results

- We can check several results. For example, when the heavy field is conformally coupled, $m^2 = 2H^2$, we can do the integrals explicitly, leading to

$$\left\langle \bar{\zeta}_{\mathbf{k}_1} \bar{\zeta}_{\mathbf{k}_2} \bar{\zeta}_{\mathbf{k}_3} \right\rangle_c' = \left\{ \frac{6c_{\mathrm{d}} g^2}{H^2} \left[k_t - \frac{k_1 k_2 + k_2 k_3 + k_3 k_1}{k_t} - \frac{k_1 k_2 k_3}{k_t^2} \right] + \frac{\pi \lambda g^3}{8H^4} \right\} \prod_{a=1}^{3} P_\zeta(k_a) + \cdots$$

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- Even though the mode function decays at $\tau_0 \to 0$, we still obtain non-zero contributions from branch-mixing terms

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- We can check several results. For example, when the heavy field is conformally coupled, $m^2 = 2H^2$, we can do the integrals explicitly, leading to

$$\left\langle \bar{\zeta}_{\mathbf{k}_1} \bar{\zeta}_{\mathbf{k}_2} \bar{\zeta}_{\mathbf{k}_3} \right\rangle_c' = \left\{ \frac{6c_{\mathrm{d}} g^2}{H^2} \left[k_t - \frac{k_1 k_2 + k_2 k_3 + k_3 k_1}{k_t} - \frac{k_1 k_2 k_3}{k_t^2} \right] + \frac{\pi \lambda g^3}{8H^4} \right\} \prod_{a=1}^{3} P_\zeta(k_a) + \cdots$$

- Even though the mode function decays at $\tau_0 \to 0$, we still obtain non-zero contributions from branch-mixing terms
- The behavior in the squeezed limit for a generic mass (of order Hubble) recovers previous cosmological collider calculations

Cubic Results

- For very heavy mass, under heavy mass expansion, with adiabatic approximation, the branch-mixing terms are

$$\Delta \ln \rho_{\partial} \simeq \frac{g^2}{4a_0} \int_{\mathbf{k}} \frac{(F_+ + F_-)_{\mathbf{k}} (F_+ + F_-)_{-\mathbf{k}}}{\omega_k^3} \Big|_{\tau_0}$$

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- Perturbatively, we could obtain a local EFT in heavy mass expansion in de Sitter, given adiabatic evolution

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- These are standard for open system EFT

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with constraints

$$\mathcal{D}_R(x,y)^* = \mathcal{D}_R(x,y) \ , \qquad \mathcal{D}_R(x,y) = 0 \quad \text{for} \quad \tau_x < \tau_y \ ,$$
$$\mathcal{N}(x,y)^* = \mathcal{N}(x,y) = \mathcal{N}(y,x) \ ,$$
$$\int_{x,y} f(x) \mathcal{N}(x,y) f(y) \geq 0 \qquad \text{for every real test function } f \ .$$

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- For a generic kernel $\mathcal{K}$, the dilatation identity becomes

$$\left(\tau\partial_\tau + \tau'\partial_{\tau'} - k\partial_k - 3\right) \mathcal{K}_k(\tau,\tau') = 0$$

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- This leads to

$$\left[\sum_{a=1}^{N-1}\boldsymbol{k}_a\cdot\frac{\partial}{\partial\boldsymbol{k}_a} - 3 + N\Delta_\zeta\right]\Gamma^{(N)}_{s_1\cdots s_N} = 0$$

$$\sum_{a=1}^{N-1}\left[-2\Delta_\zeta\frac{\partial}{\partial k_a^i} + k_a^i\frac{\partial^2}{\partial k_a^j\partial k_a^j} - 2k_a^j\frac{\partial^2}{\partial k_a^j\partial k_a^i}\right]\Gamma^{(N)}_{s_1\cdots s_N} = 0$$

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- Our example $F_{\mathrm{d}}[\zeta] \equiv \zeta' + 3c_{\mathrm{d}}\zeta\zeta' = \partial_\tau \left(\zeta + \frac{3}{2}c_{\mathrm{d}}\zeta^2\right)$ breaks dS symmetry by a choice of a specific clock

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- One can derive Ward identities for theories with explicit choices of a clock, or with a specific symmetry breaking pattern, depending on the exact model

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- Future directions:
  - Realistic inflationary scenarios
  - Loops and non-trivial states/environments
  - Massive/massless spin fields
  - Beyond weak mixing
  - Other systems in later universe…
  - …

Use of AI

- We used ChatGPT to assist with calculations and manuscript preparation
- Calculations and texts are checked and reviewed
- Everything was done through work mode in Codex

This is the final version at the moment. Please check. Let me know of any inconsistencies and quick fixes. Make it a list so that I can quickly find and fix them.

A few optional cleanups:

- **Title page:** names and affiliations are absent; restore them unless intentionally anonymous.

Thank you!

Questions?