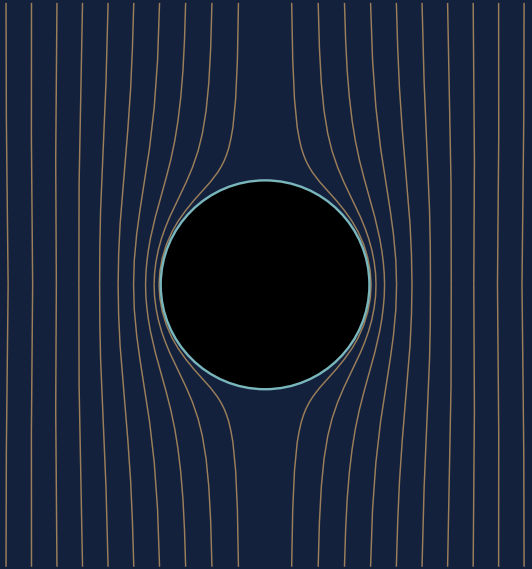
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Meissner Effect and Kerr/CFT Holography of Extremal Kerr–Bertotti–Robinson Black Holes

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The talk in one slide

1 One extremal throat

An exact rotating black hole in a Bertotti–Robinson field. At extremality all is set by

$$I_2 = 1 - B^2 a^2 > 0,$$

$$k = \frac{2\sqrt{I_2}}{1 + I_2} \quad \text{(twist)}.$$

2 Kerr/CFT entropy

The rotational Kerr/CFT construction gives

$$c_L = 12\, k\, \mathcal{J} = 12 a^2 \sqrt{I_2},$$

$$T_L = \frac{1}{2\pi k},$$

and Cardy returns $A_{\text{ext}}/4$ for every $|Ba| < 1$.

3 Meissner effect

Two exact identities at extremality force the horizon potential to become x-independent as $k \to 0$:

$$\Phi_N = \mathcal{O}\big(\sqrt{I_2}\big) \to 0$$

as $Ba \to 1^-$.

The same twist $k(a, B)$ controls both. Its zero, the static boundary $Ba \to 1^-$, is where $c_L \to 0$, $T_L \to \infty$ and the magnetic flux is expelled.

Conventions: $G = c = \hbar = 1$, $a > 0$, $B \geq 0$, $x = \cos\theta$; φ has period 2π at extremality. NPB 1028, 117514; EPJC 86, 1285 (2026).

Black holes in external magnetic fields

- Black holes in magnetised environments motivate studying rotation, fields and horizon flux
- Strong-field backreaction calls for exact Einstein–Maxwell solutions
  - **1974** Wald: uniform test field on Kerr
  - **1976** Ernst: Kerr–Melvin via Harrison transformation
  - **2025** Podolský & Ovcharenko: Kerr black hole in a Bertotti–Robinson universe
- Two questions at extremality:
  - Is there a Kerr/CFT description, and how does B enter?
  - Is the magnetic flux expelled from the horizon?

Bertotti–Robinson universe

- Direct product $\mathrm{AdS}_2 \times S^2$ supported by a homogeneous Maxwell field
- Conformally flat; the near-horizon geometry of extremal Reissner–Nordström
- $M \to 0$ limit of Kerr–BR: $R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma} = 8B^4(1 - B^2a^2)^2$ [NPB, Eq. (2.38)]

Kerr–BR interpolates between **Kerr** ($B \to 0$) and the **BR universe** ($M \to 0$).

Wald, PRD 10, 1680 (1974); Ernst, J. Math. Phys. 17, 54 (1976); Podolský & Ovcharenko, PRL 135, 181401 (2025).

The Kerr–BR metric

$$\mathrm{d}s^2 = \frac{1}{\Omega^2}\left[-\frac{Q}{\rho^2}\left(\mathrm{d}t - a\Delta_x\,\mathrm{d}\varphi\right)^2 + \frac{\rho^2}{Q}\,\mathrm{d}r^2 + \frac{\rho^2}{P\,\Delta_x}\,\mathrm{d}x^2 + \frac{P\,\Delta_x}{\rho^2}\left(a\,\mathrm{d}t - (r^2+a^2)\,\mathrm{d}\varphi\right)^2\right]$$

$$\rho^2 = r^2 + a^2x^2, \qquad \Delta_x = 1 - x^2, \qquad x = \cos\theta,$$

$$Q = (1 + B^2r^2)\,\Delta, \qquad \Omega^2 = (1 + B^2r^2) - B^2\Delta\,x^2,$$

$$\Delta = \left(1 - B^2M^2\frac{I_2}{I_1^2}\right)r^2 - 2M\frac{I_2}{I_1}\,r + a^2,$$

$$P = 1 + B^2\left(M^2\frac{I_2}{I_1^2} - a^2\right)x^2,$$

$$I_1 = 1 - \tfrac{1}{2}B^2a^2, \qquad I_2 = 1 - B^2a^2.$$

- Three parameters: mass M, rotation a, field B
- $B \to 0$: Kerr in Boyer–Lindquist form
- $M \to 0$: the (conformal) BR universe
- $a \to 0$: Schwarzschild–BR, $r_h = \dfrac{2M}{1 - B^2M^2}$
- Ω is a genuine conformal factor: not asymptotically flat
- The metric functions do not depend on the duality angle σ of the Maxwell field

Podolský & Ovcharenko, PRL 135, 181401 (2025); Ovcharenko & Podolský, PRD 112, 064076 (2025).

Horizons and extremality

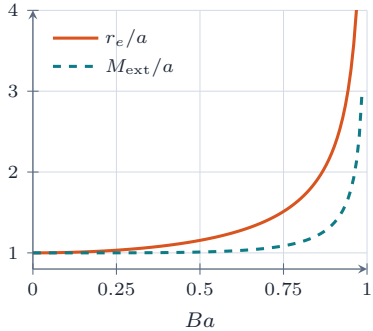
Horizons: $Q = 0 \;\Rightarrow\; \Delta(r_\pm) = 0$

$$r_\pm = I_1 \,\frac{M I_2 \pm \sqrt{M^2 I_2 - a^2 I_1^2}}{I_1^2 - B^2 M^2 I_2}$$

Extremality (mass parameter M): $M^2 I_2 = a^2 I_1^2$

$$M_{\rm ext} = \frac{a\, I_1}{\sqrt{I_2}}, \qquad r_e = \frac{M_{\rm ext}}{I_1} = \frac{a}{\sqrt{I_2}}, \qquad |Ba| < 1$$

- Double root: $\Delta = I_2\,(r - r_e)^2$; a two-parameter family (a, B)
- Equivalently $a = k\, M_{\rm ext}$ with $k = \dfrac{2\sqrt{I_2}}{1 + I_2} \le 1$; Kerr–Melvin keeps $a = M$



Both diverge as $Ba \to 1^-$.

Schwarzschild–BR ($a = 0$) has a single horizon: no extremal member.

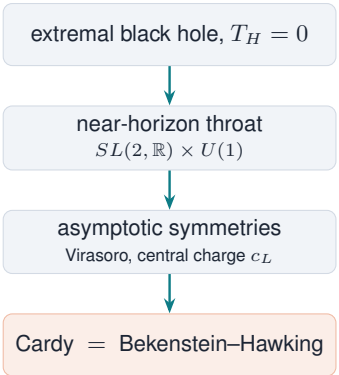
Kerr/CFT in one slide

- Extremal Kerr has a NHEK throat with $SL(2,\mathbb{R}) \times U(1)$ symmetry.
- Suitable boundary conditions extend the rotational symmetry to a Virasoro algebra.
- The Frolov–Thorne thermal weight supplies a left temperature.

For extremal Kerr

$$c_L = 12J, \qquad T_L = \frac{1}{2\pi},$$

$$S_{\text{Cardy}} = \frac{\pi^2}{3} c_L T_L = 2\pi J = \frac{A_H}{4}.$$



Next: deform the throat with the BR field.

Bardeen & Horowitz (1999); Guica, Hartman, Song & Strominger (2009); Hartman et al. (2009); Compere (2012).

Zooming into the extremal throat

Bardeen–Horowitz scaling

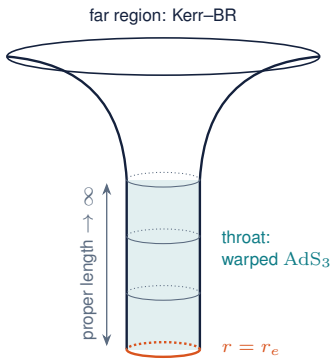
$$r = r_e + \lambda\, y, \quad t = \frac{r_e^2 + a^2}{\lambda}\,\tau, \quad \varphi = \phi + \Omega_H t, \quad \lambda \to 0$$

- Double root: $Q \;\to\; (1 + B^2 r_e^2)\, I_2\, \lambda^2 y^2$
- Key identity, from $r_e^2 = a^2/I_2$:

$$(1 + B^2 r_e^2)\, I_2 = 1$$

The (τ, y) sector is canonical AdS_2, with no further rescaling

- Horizon values: $\rho_0^2 = r_e^2 + a^2 x^2, \quad \Omega_0^2 = 1 + B^2 r_e^2 = \dfrac{1}{I_2}$



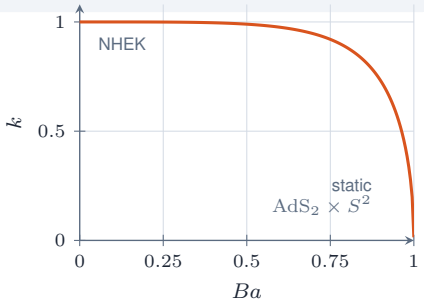
Near-horizon geometry: NHEK–BR

$$\mathrm{d}s^2_{\mathrm{nh}} = \Gamma(x)\Big[-y^2\mathrm{d}\tau^2 + \frac{\mathrm{d}y^2}{y^2} + \frac{\mathrm{d}x^2}{1-x^2}\Big] + \gamma(x)^2\big(\mathrm{d}\phi + k\,y\,\mathrm{d}\tau\big)^2$$

$$\Gamma = \frac{\rho_0^2}{\Omega_0^2} = a^2\big(1+I_2x^2\big), \qquad \gamma^2 = \frac{a^2(1+I_2)^2(1-x^2)}{1+I_2x^2}$$

twist parameter $$k = \frac{2ar_e}{r_e^2+a^2} = \frac{2\sqrt{1-B^2a^2}}{2-B^2a^2}$$

- Isometry $SL(2,\mathbb{R}) \times U(1)$: a warped AdS_3 at each x
- $B \to 0$: $k \to 1$, the standard NHEK geometry
- $Ba \to 1^-$: $k \to 0$, the fibre untwists



The near-horizon Maxwell sector

Near-horizon decomposition [EPJC, Eq. (2.16)]
Under the Bardeen–Horowitz scaling a smooth axisymmetric potential becomes

$$A_\mu \mathrm{d}x^\mu \;\to\; L(x)\, y\, \mathrm{d}\tau + A_\varphi\big|_{r_e}(x)\, \mathrm{d}\phi + \mathrm{d}\Lambda,$$

with $L(x)$ fixed by the bulk solution and $\mathrm{d}\Lambda$ a regular gauge transformation.

- $A_\varphi|_{r_e} \equiv A_\varphi(r_e, x)$: the horizon value of the azimuthal potential in the co-rotating angle ϕ
- Only differences of $A_\varphi|_{r_e}$, i.e. $\partial_x A_\varphi|_{r_e}$, enter the magnetic flux (Part III)

Compact fibre-aligned form [NPB, Eq. (3.15)]

$$A_{\mathrm{nh}} = Z\,(\mathrm{d}\phi + k\, y\, \mathrm{d}\tau),$$
$$Z = \frac{a^2 B\,(2 - B^2 a^2)}{2\sqrt{1 - B^2 a^2}} = \frac{B a^2}{k}$$

In a gauge regular on the future horizon; the constant $\mathrm{d}\phi$ term is locally pure gauge, the physical content is in $F_{\mathrm{nh}} = \mathrm{d}A_{\mathrm{nh}}$.

- c_L, T_L and S_{CFT} below use only the metric data Γ, γ, k

NPB 1028, 117514 (2026), Sec. 3; EPJC 86, 1285 (2026), Sec. 2.2 and Appendix A.

Asymptotic symmetries and the central charge

- Impose the standard Kerr/CFT boundary conditions on perturbations of the metric and gauge field.
- The diffeomorphisms

$$\zeta_n = -e^{-in\phi}\partial_\phi - in\, e^{-in\phi} y\partial_y + \cdots$$

span one copy of the Virasoro algebra of the $U(1)$ rotational symmetry.

- Covariant phase-space charges give the central term

$$c_L = 3k\int_{-1}^{1}\sqrt{\Gamma\,\gamma^2\alpha^2}\,\mathrm{d}x = \frac{3k}{2\pi}\,A_{\mathrm{ext}}$$

with $\alpha^2 = 1/(1-x^2)$.

$$A_{\mathrm{ext}} = 4\pi\,\frac{r_e^2+a^2}{1+B^2r_e^2} = 4\pi a^2(1+I_2),$$

$$c_L = 12\,k\,\mathcal{J} = 12a^2\sqrt{1-B^2a^2}.$$

- $\mathcal{J} \equiv \dfrac{A_{\mathrm{ext}}}{8\pi} = \dfrac{a^2(1+I_2)}{2}$: the paper's effective angular momentum; $\mathcal{J} \to aM$ as $B \to 0$
- The external field enters only through k and $\mathcal{J}$
- $B \to 0$: $c_L = 12J$, the Kerr result

NPB 1028, 117514 (2026), Eqs. (3.19)–(3.23); Guica et al. (2009); Hartman, Song & Strominger (2010); Compère (2012).

Temperature and Cardy entropy

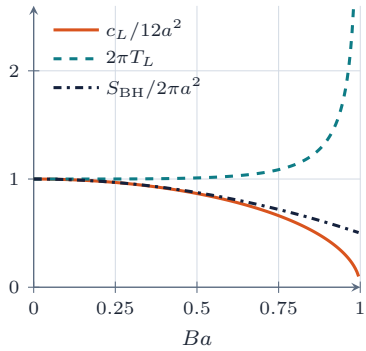
Frolov–Thorne temperature (extremal limit of the Hartle–Hawking vacuum, $T_R = 0$):

$$T_L = \frac{1}{2\pi k} = \frac{r_e^2 + a^2}{4\pi a r_e} = \frac{2 - B^2 a^2}{4\pi\sqrt{1 - B^2 a^2}}$$

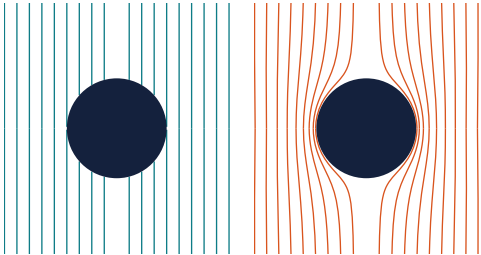
Cardy formula:

$$S_{\text{Cardy}} = \frac{\pi^2}{3}\,\frac{3kA_{\text{ext}}}{2\pi}\,\frac{1}{2\pi k} = \frac{A_{\text{ext}}}{4}$$

- The twist cancels: $S_{\text{CFT}} = S_{\text{BH}}$ for every $|Ba| < 1$, for any member of the duality-rotated family
- Explicitly $S = \pi(r_e^2 + a^2) I_2 = \pi a^2 (2 - B^2 a^2)$
- $k \to 0$: $c_L \to 0$, $T_L \to \infty$, their product stays finite



The black-hole Meissner effect



$a = 0$: flux threads $\qquad$ $a = M$: flux expelled

Wald's uniform test field on Kerr (Boyer–Lindquist, units of r_+).
Hemisphere flux $\Phi = \pi B r_+^2\,(1 - a^4/r_+^4)$.

- **1974–85** Wald; Bičák & Janiš: expulsion of external axisymmetric fields from extremal horizons, test-field limit
- **1991** Karas & Vokrouhlický: exact magnetised Kerr–Newman
- **2015** Bičák & Hejda: near-horizon framework; in the static limit $k \to 0$ the flux density $F_{x\varphi}|_{r_e}$ vanishes
- **2023** Ghezelbash & Siahaan: a NUT charge destroys the effect (Melvin–Kerr–Newman–Taub–NUT)
- **2025** Ovcharenko & Podolský: numerically, the field is weakened in the equatorial plane of Kerr–BR

Question. Does the effect survive in Kerr–BR: non-aligned Maxwell and Weyl directions, $\mathrm{AdS}_2 \times S^2$ asymptotics, deformed extremality?

EPJC 86, 1285: an **analytic proof**, valid for all external fields compatible with extremality.

References as cited in EPJC 86, 1285 (2026), Sec. 1; full list on the References backup.

How to diagnose flux expulsion

Gauge-robust quantities on the extremal horizon

- flux density: $F_{x\varphi}\big|_{r_e} = \partial_x A_\varphi\big|_{r_e}$
- flux through the northern hemisphere (Stokes):

$$\Phi_N \equiv \int_N F = \oint_{\partial N} A = 2\pi\Big[A_\varphi\big|_{r_e}(0) - A_\varphi\big|_{r_e}(1)\Big]$$

- $A_\varphi|_{r_e}$ is defined up to a constant, $A_\varphi \to A_\varphi + \partial_\varphi \Lambda$; the flux depends only on differences

For a smooth, monopole-free configuration the constant is fixed by regularity on the axis, $A_\varphi(x=1) = 0$.

Bičák–Hejda criterion (2015)

In the static near-horizon limit $k \to 0$ the horizon potential becomes x-independent (pure gauge),

$$\partial_x A_\varphi\big|_{r_e} \to 0,$$

so the horizon-threading magnetic flux vanishes.

Task. Evaluate $A_\varphi|_{r_e}(x)$ exactly for Kerr–BR and follow it to $Ba \to 1^-$.

EPJC 86, 1285 (2026), Sec. 2.2 and Eq. (3.7); Bičák & Hejda, PRD 92, 104006 (2015).

Two exact identities at extremality

The double root $\Delta = I_2(r - r_e)^2$ gives two identities for every $0 \le Ba < 1$.

Identity 1

$$\Omega_{,x}\big|_{r_e} = -\frac{B^2 x}{\Omega_0}\,\Delta(r_e) = 0$$

- The horizon condition removes the angular term of A_φ

Identity 2

$$\Omega_{,r}\big|_{r_e} = \frac{B^2}{\Omega_0}\cdot\frac{a}{\sqrt{I_2}} = B^2 a$$

- Independent of x: the radial gradient of the conformal factor is uniform over the horizon

Both identities in the horizon value of the parent potential, magnetic sector ($w = \cos\sigma = 1$):

$$A_\varphi\big|_{r_e} = \frac{\Omega_0 - 1}{B} - \frac{Ba\, r_e(r_e^2 + a^2)}{\rho_0^2}, \qquad \rho_0^2 = r_e^2 + a^2 x^2, \quad \Omega_0 = \frac{1}{\sqrt{I_2}}\ :$$

all x-dependence sits in ρ_0^2; the constant axis value is removed by the regular gauge choice.

EPJC 86, 1285 (2026), Eqs. (3.1)–(3.3).

Exact horizon potential

Purely magnetic external field ($w = 1$), gauge regular on the axis:

$$A_\varphi\big|_{r_e}(x) = -\,Ba^2\sqrt{1-B^2a^2}\;\frac{1-x^2}{1+(1-B^2a^2)\,x^2}\,, \qquad F_{x\varphi}\big|_{r_e} = \frac{2Ba^2\sqrt{I_2}\,(1+I_2)\,x}{(1+I_2x^2)^2}$$

- Vanishes at both poles: smooth and monopole-free on S^2; $F_{x\varphi}|_{r_e}$ is odd in x
- A closed form for the whole extremal family
- Amplitude $\propto \sqrt{I_2}$: near the boundary

$$A_\varphi\big|_{r_e} \simeq -a\sqrt{I_2}\,(1-x^2) \;\to\; 0$$

uniformly in x

In the parent gauge (series in $\sqrt{I_2}$)

$$A_\varphi\big|_{r_e} = -a + a\,x^2\sqrt{I_2} + \mathcal{O}(I_2),$$
$$\partial_x A_\varphi\big|_{r_e} = \mathcal{O}\big(\sqrt{I_2}\big).$$

The potential approaches the x-independent constant $-a$: a pure-gauge configuration on the horizon S^2.

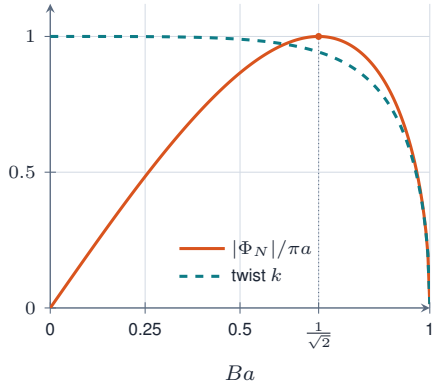
EPJC 86, 1285 (2026), Eqs. (3.4)–(3.6) and (4.1) [(3.11)–(3.12) in the published version]: the closed form is Eq. (3.4) at $w = 1$ in the gauge $A_\varphi(x = 1) = 0$.

The hemisphere flux

$$\Phi_N = \int_N F = 2\pi\Big[A_\varphi\big|_{r_e}(0) - A_\varphi\big|_{r_e}(1)\Big]$$

$$\boxed{\Phi_N = -\,2\pi\, B\, a^2\sqrt{1 - B^2a^2}}$$

- Non-zero for $0 < Ba < 1$: extremality alone does not expel the field
- At fixed a, largest at $Ba = 1/\sqrt{2}$, where $|\Phi_N| = \pi a$
- $\Phi_N = \mathcal{O}(\sqrt{I_2}) \to 0$ as $Ba \to 1^-$, like k: Meissner effect in the static near-horizon limit



EPJC 86, 1285 (2026), Eqs. (3.7)–(3.8): $\Phi_N = \mathcal{O}(\sqrt{I_2}) \to 0$; the closed form is the exact evaluation of Eq. (3.7) with the regular horizon potential.

Where is the static limit? At the edge of parameter space



As $Ba \to 1^-$ (at fixed a)

- k decreases monotonically from 1 to 0^+: the static limit is approached continuously
- diverge: $r_e = \dfrac{a}{\sqrt{I_2}},\ \Omega_0 = \dfrac{1}{\sqrt{I_2}},\ M_{\mathrm{ext}} = \dfrac{aI_1}{\sqrt{I_2}}$
- stay finite: $S_{\mathrm{BH}} = \pi(r_e^2 + a^2)I_2 \to \pi a^2,\ \mathcal{J} \to \dfrac{a^2}{2}$

The fibre trivialises: the throat becomes a direct product $\mathrm{AdS}_2 \times S^2$ of radius a.

Contrast

- **Kerr–Melvin:** $k = 0$ is reached at a *finite, interior* field strength [Bičák–Hejda 2015]
- **Kerr–BR:** only asymptotically, at the *boundary* $Ba = 1$ of the allowed range
- $a \to 0$ **at fixed** B**:** $I_1 = I_2 = 1, k \to 1$: the static limit is *not* reached without rotation

> Order of limits: construct the extremal throat first, then send $Ba \to 1^-$ at fixed $a > 0$.

Limiting cases and duality

$B \to 0$: extremal Kerr

$k \to 1$, $r_e \to M = a$, $\Omega_0 \to 1$: NHEK. The horizon potential reduces to the NHEK result and $\Phi_N \to 0$, the classic Meissner effect of extremal Kerr. $\mathcal{J} \to a^2$, $S_{\rm BH} \to 2\pi a^2$.

$a \to 0$: Schwarzschild–BR

$I_1 = I_2 = 1$ and $k \to 1$ for all B; the near-horizon metric takes the Robinson–Bertotti form. $k = 0$ would need $I_2 = 0$, impossible at finite B: the static limit is not reached by switching off rotation.

Duality rotation ($w = \cos\sigma$; the metric is σ-independent)

- purely magnetic, $w = 1$: $A_\varphi|_{r_e} \to -a$, $\partial_x A_\varphi|_{r_e} = \mathcal{O}(\sqrt{I_2}) \to 0$
- purely electric, $w = 0$: $A_\varphi|_{r_e} \to -a\,x$, $\partial_x A_\varphi|_{r_e} \to -a \neq 0$

Not a failure of expulsion: $F(\sigma) = \cos\sigma\, F + \sin\sigma \star F$, so $w = 0$ interchanges the electric and magnetic sectors. The Meissner statement concerns the *magnetic* flux in the magnetic sector; for $w = 0$ the natural quantity is the duality-rotated (electric) flux.

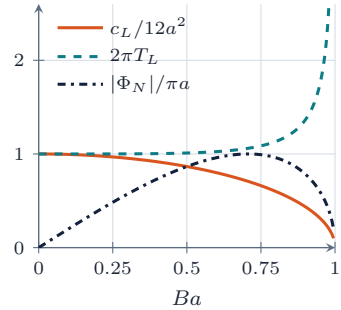
Area, temperature and near-horizon geometry are the same for every σ: the difference is entirely electromagnetic.

EPJC 86, 1285 (2026), Sec. 4.

The static boundary is a degenerate point of the dual CFT

$$c_L = 12\,k\,\mathcal{J}, \qquad T_L = \frac{1}{2\pi k}, \qquad S_{\mathrm{CFT}} = \frac{A_{\mathrm{ext}}}{4}$$

- c_L and T_L depend on the field only through $k(a, B)$
- $k \to 0$: $T_L \to \infty$, $c_L \to 0$, the breakdown of the chiral CFT description at $Ba = 1$; $S_{\mathrm{CFT}} = S_{\mathrm{BH}}$ stays finite
- The Meissner condition $k = 0$ is a degenerate point in the moduli space of the dual CFT: the holographic description degenerates together with the expulsion
- Penna's picture: disentanglement across the extremal horizon; without the $U(1)$ fibration the two sides decouple and no gauge flux is threaded



Along the extremal family at fixed a.

EPJC 86, 1285 (2026), Sec. 5; NPB 1028, 117514 (2026), Sec. 3; Penna, PRD 90, 043003 (2014).

The infinite throat and Blandford–Znajek jets

Proper length of the throat. At extremality $Q = (1 + B^2 r^2)\, I_2\, (r - r_e)^2$, with ρ^2, Ω^2 finite at r_e:

$$\sqrt{g_{rr}} \;\simeq\; \frac{\rho_0}{\Omega_0 \sqrt{(1 + B^2 r_e^2) I_2}}\, \frac{1}{|r - r_e|} \quad \Rightarrow \quad L_{\text{throat}} = \int_{r_e}^{r_e + \epsilon} \sqrt{g_{rr}}\, \mathrm{d}r \;\sim\; \frac{\rho_0}{\Omega_0}\, \ln \frac{\epsilon}{r_e} \;\to\; \infty$$

- The prefactor is finite and positive for all $Ba \in [0, 1)$: the logarithmic divergence is universal, a property of the double root of Q alone, independent of Ω
- The paper reads it, through Penna's cylinder argument, as the geometric counterpart of the flux suppression

Blandford–Znajek estimate

$$P_{\text{jet}} = \frac{1}{8\pi}\, \Omega_H^2\, F_H^2, \qquad \Omega_H = \frac{a}{r_e^2 + a^2}, \quad F_H \equiv \Phi_N$$

- $\Phi_N \to 0$ as $Ba \to 1^-$: jets driven by smooth, axisymmetric multipole fields are suppressed in this limit
- Exception: split-monopole fields, $A_\varphi \sim \cos\theta$, singular at the poles, lie outside the smooth multipole basis; whether they are realised here is open

EPJC 86, 1285 (2026), Sec. 5, Eqs. (5.1)–(5.3); Penna, PRD 89, 104057 (2014); PRD 90, 043003 (2014).

Comparison across magnetised extremal black holes

Spacetime	Asymptotics	Maxwell alignment	Meissner effect	Static limit $k = 0$
Kerr–Melvin (MKN)	Melvin	aligned	Yes [B–H 2015]	finite interior field
MKN–Taub–NUT	non-standard	aligned	No [G–S 2023]	reached, flux stays
Kerr–BR	$\mathrm{AdS}_2 \times S^2$	non-aligned	Yes (EPJC 2026)	boundary $Ba \to 1^-$

- Neither the alignment of the Maxwell field with the Weyl directions nor the asymptotics is essential for the mechanism
- What matters is the double root of Δ and its two identities: $\Omega_{,x}|_{r_e} = 0$ and the x-independent $\Omega_{,r}|_{r_e} = B^2 a$
- NUT charge breaks the x-independence of $\Omega_{,r}|_{r_e}$ and leaves flux on the horizon

The mechanism of magnetised Kerr–Newman, driven here by the approach to the boundary $Ba = 1$ rather than by a finite interior critical field. [EPJC 86, 1285, Table 1 and Sec. 5]

Summary and outlook

1. **Kerr/CFT survives the BR embedding.** The extremal throat is a warped AdS_3 with twist $k = \dfrac{2\sqrt{1 - B^2a^2}}{2 - B^2a^2}$; $c_L = 12k\mathcal{J} = 12a^2\sqrt{I_2}$, $T_L = (2\pi k)^{-1}$, and Cardy reproduces $A_{\mathrm{ext}}/4$ for every admissible field.

2. **The Meissner effect holds, analytically.** Two exact identities, $\Omega_{,x}|_{r_e} = 0$ and $\Omega_{,r}|_{r_e} = B^2a$, make $A_\varphi|_{r_e}$ x-independent as $k \to 0$: $\Phi_N = \mathcal{O}(\sqrt{I_2}) \to 0$ at the boundary $Ba \to 1^-$, not at a finite interior field as in Kerr–Melvin.

3. **One parameter controls both.** At $k = 0$ the chiral CFT degenerates ($c_L \to 0$, $T_L \to \infty$) exactly where the flux is expelled; the throat length diverges for every extremal member and the BZ estimate vanishes in the limit.

Outlook (from the papers): Kerr–Newman–BR and Kerr–BR–Taub–NUT; the Bertotti–Robinson–Bonnor–Melvin class; quasi-normal modes, stability and low-frequency scattering of test fields; split-monopole fields and jets near $Ba = 1$.

Thank you. Haryanto M. Siahaan • haryanto.siahaan@unpar.ac.id

NPB 1028, 117514 (2026), arXiv:2512.12533 | EPJC 86, 1285 (2026), arXiv:2603.00653. Supported by LPPM-UNPAR.

Backup slides

Derivations, limits and consistency checks behind the main talk.

Two exact embeddings: Melvin versus Bertotti–Robinson

	Kerr–Melvin	Kerr–Bertotti–Robinson
Background	Melvin universe: flux tube focused on the axis	Bertotti–Robinson $\mathrm{AdS}_2 \times S^2$: homogeneous field
Construction	Harrison transformation of Kerr [Ernst 1976]	new type-D family [Podolský–Ovcharenko 2025]
Algebraic type	not type D	type D; Maxwell and Weyl null directions non-aligned
Horizons	Δ and the bound $a \leq M$ untouched by B	B deforms $r_\pm$ and the extremal bound
Axis	conical defects in general	regular with one conicity C; $C = 1$ at extremality
Far region	curvature set by the Melvin field	non-asymptotically flat; large-r curvature depends on direction

Same parameters (M, a, B), very different global structure $\Rightarrow$ a new arena for extremal-horizon physics.

The Maxwell field

$$\mathcal{A} = \frac{e^{i\sigma}}{2B}\left[\Omega_{,r}\,\frac{a\,\mathrm{d}t - (r^2+a^2)\,\mathrm{d}\varphi}{r+iax} - i\,\Omega_{,x}\,\frac{\mathrm{d}t - a\Delta_x\,\mathrm{d}\varphi}{r+iax} + (\Omega-1)\,\mathrm{d}\varphi\right], \qquad A = 2\,\mathrm{Re}\,\mathcal{A}$$

$$\Omega_{,r} = \frac{B^2}{\Omega}\left[r(1-x^2) + x^2 I_2\left(\frac{M}{I_1} + \frac{B^2M^2r}{I_1^2}\right)\right]$$

$$\Omega_{,x} = -\frac{B^2x}{\Omega}\,\Delta(r)$$

- σ is a duality angle: $\sigma = 0$ purely magnetic, $\sigma = \frac{\pi}{2}$ purely electric; $w \equiv \cos\sigma$
- The metric does not depend on σ

Purely magnetic case ($w = 1$)

$$A_\varphi = \frac{1}{B}\left[\Omega - 1 - \Omega_{,r}\frac{r(r^2+a^2)}{\rho^2} + \Omega_{,x}\frac{a^2x\Delta_x}{\rho^2}\right]$$

- $A_\varphi \simeq -\frac{1}{2}Br^2\sin^2\theta$ for $M \ll r \ll 1/B$: a uniform field along the axis

The physical potential is $2\,\mathrm{Re}\,\mathcal{A}$; the Einstein–Maxwell equations hold with a source-free field $F = \mathrm{d}A$. The horizon structure is determined by (M, a, B) alone.

NPB 1028, 117514 (2026), Eqs. (2.8)–(2.13); Podolský & Ovcharenko, PRL 135, 181401 (2025).

Axis regularity and thermodynamics

Regular axis. With $\varphi \in [0, 2\pi C)$ the ratio circumference/radius is $2\pi C\, P(\pm 1)$, and P is even:

$$C = \frac{1}{P(1)} = \left[1 + B^2\left(M^2 \frac{I_2}{I_1^2} - a^2\right)\right]^{-1}$$

removes the conical defect on both poles at once.

Horizon area and temperature.

$$A = 4\pi C\, \frac{r_+^2 + a^2}{1 + B^2 r_+^2}, \qquad S = \frac{A}{4}, \qquad \Omega_H = \frac{a}{r_+^2 + a^2}$$

$$T_H = \frac{\kappa}{2\pi} = \frac{1 + B^2 r_+^2}{2\pi\,(r_+^2 + a^2)}\left(M \frac{I_2}{I_1} - \frac{a^2}{r_+}\right)$$

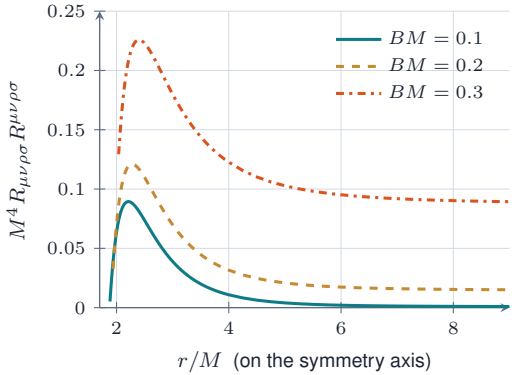
At extremality

- $P(x) \equiv 1 \;\Rightarrow\; C = 1$: standard 2π period, no defect
- $T_H = 0$
- $\Omega_H = \dfrac{a}{r_e^2 + a^2} = \dfrac{I_2}{a(1 + I_2)}$
- Entropy stays finite for all $|Ba| < 1$:

$$S_{\mathrm{BH}} = \frac{A_{\mathrm{ext}}}{4} = \pi a^2\,(2 - B^2 a^2)$$

NPB 1028, 117514 (2026), Eqs. (2.22)–(2.33) and (3.5)–(3.7).

Curvature: a black hole that is not asymptotically flat



$a = 0.5M$; each curve starts at its own horizon $r_+(B)$.

- For these regular exterior examples, the axis curvature approaches a nonzero plateau
- On the axis, for large r:

$$K_\infty^{\mathrm{KBR}} = 8B^4 + (32M^2 - 16a^2)B^6 + \mathcal{O}(B^8)$$
$$K_\infty^{\mathrm{MK}} = 320B^4 - 23552\, M^2a^2B^8 + \mathcal{O}(B^{10})$$

- $M \to 0$: $K^{\mathrm{KBR}} = 8B^4(1 - B^2a^2)^2$ still remembers a; $K^{\mathrm{MK}} = 320B^4$ does not
- Not asymptotically flat: the invariant saturates to a nonzero constant controlled by B

NPB 1028, 117514 (2026), Sec. 2 and Eqs. (2.34)–(2.39); the curves are evaluated along the symmetry axis, where they reproduce those closed forms. MK = Melvin–Kerr in the paper's convention.

Limits of the Kerr–BR family

$B \to 0$: Kerr

$\Omega = 1$, $P = 1$,
$Q = \Delta = r^2 - 2Mr + a^2$
$r_\pm = M \pm \sqrt{M^2 - a^2}$
Extremal: $M = a = r_e$, $k = 1$: the NHEK throat.

$M \to 0$: Bertotti–Robinson

$\Delta = r^2 + a^2$: no horizon.
Homogeneous: at every point
$R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = 8B^4(1-B^2a^2)^2$

Eq. (2.38) of NPB: it keeps a memory of a.

$a \to 0$: Schwarzschild–BR

$I_1 = I_2 = 1$,
$\Delta = (1 - B^2M^2)r^2 - 2Mr$
$P = 1 + B^2M^2x^2$,
$C = \dfrac{1}{1 + B^2M^2}$
$r_h = \dfrac{2M}{1 - B^2M^2}$: enlarged horizon, never extremal.

Kerr–BR interpolates between a rotating black hole and a homogeneous $\mathrm{AdS}_2 \times S^2$ electromagnetic background; unlike Kerr–Melvin, B enters $\Delta(r)$ and the extremality condition.

NPB 1028, 117514 (2026), Sec. 2; EPJC 86, 1285 (2026), Sec. 4.

Conicity, area and surface gravity

Axis. Small circle around $x = \pm 1$ at fixed t, r:

$$\mathcal{C} = \int_0^{2\pi C} \sqrt{g_{\varphi\varphi}}\, \mathrm{d}\varphi, \qquad \mathcal{R} = \int \sqrt{g_{xx}}\, \mathrm{d}x$$

$$\lim_{x \to \pm 1} \frac{\mathcal{C}}{\mathcal{R}} = 2\pi C\, P(\pm 1) \overset{!}{=} 2\pi$$

P is even, so one value $C = 1/P(1)$ regularises both poles.

- $B = 0$: $C = 1$; $a = 0$: $C = 1/(1 + B^2 M^2)$
- extremal: $P \equiv 1$, $C = 1$

Area. On the horizon $Q = 0$, $\Omega^2 = 1 + B^2 r_+^2$:

$$A = \int_0^{2\pi C} \mathrm{d}\varphi \int_{-1}^{1} \mathrm{d}x\, \sqrt{g_{xx} g_{\varphi\varphi}}\Big|_{r_+} = 4\pi C\, \frac{r_+^2 + a^2}{1 + B^2 r_+^2}$$

Surface gravity. With $\Delta(r_+) = 0$:

$$\kappa = \frac{Q'(r_+)}{2(r_+^2 + a^2)} = \frac{1 + B^2 r_+^2}{r_+^2 + a^2} \left(M \frac{I_2}{I_1} - \frac{a^2}{r_+} \right)$$

Curvature invariants: closed forms

Large-r limit, NPB Eqs. (2.34)–(2.39) ($\zeta \equiv B^2a^2$); evaluated here along the symmetry axis:

$$K_\infty^{\mathrm{KBR}} = \frac{8B^4(\zeta-1)^2}{(\zeta-2)^8}\Big[(\zeta-2)^8 - 8B^2(\zeta-2)^7M^2 + 16B^4(\zeta-2)^4(3\zeta^2-8\zeta+6)M^4 - 256B^6(\zeta-1)^2(\zeta-2)^2(2\zeta-1)M^6 + 1792B^8(\zeta-1)^4M^8\Big]$$

$$K_\infty^{\mathrm{KBR}} = 8B^4 + (32M^2-16a^2)B^6 + (48M^4-48M^2a^2+8a^4)B^8 + \mathcal{O}(B^{10})$$

$$K_\infty^{\mathrm{MK}} = -\frac{64B^4\,(48B^4M^2a^2-5)}{(16B^4M^2a^2+1)^4} = 320B^4 - 23552\,M^2a^2B^8 + \mathcal{O}(B^{10})$$

- $M \to 0$: Kerr–BR gives $8B^4(1-B^2a^2)^2$ (keeps a memory of a); Melvin–Kerr gives $320B^4$ (forgets a)
- Leading term $\propto B^4$, independent of M and a; the black-hole parameters enter only through corrections

NPB 1028, 117514 (2026), Sec. 2.

Near-horizon limit, step by step

With $r = r_e + \lambda y,\ t = (r_e^2 + a^2)\tau/\lambda,\ \varphi = \phi + \Omega_H t$:

- $\mathrm{d}t - a\Delta_x \mathrm{d}\varphi \ \to\ \dfrac{\rho_0^2}{r_e^2 + a^2}\,\mathrm{d}t - a\Delta_x\,\mathrm{d}\phi$
- $a - (r^2 + a^2)\,\Omega_H = -\dfrac{2ar_e}{r_e^2 + a^2}\,\lambda y + \mathcal{O}(\lambda^2)$, so

$$a\,\mathrm{d}t - (r^2 + a^2)\mathrm{d}\varphi \to -(r_e^2 + a^2)\big[\mathrm{d}\phi + k\,y\,\mathrm{d}\tau\big]$$

- $-\dfrac{Q}{\rho^2}\big(\mathrm{d}t - a\Delta_x \mathrm{d}\varphi\big)^2 \to -\rho_0^2\,(1 + B^2 r_e^2) I_2\, y^2 \mathrm{d}\tau^2 = -\rho_0^2\, y^2 \mathrm{d}\tau^2$
- $\dfrac{\rho^2}{Q}\,\mathrm{d}r^2 \to \rho_0^2\,\dfrac{\mathrm{d}y^2}{y^2}$

$$k = \frac{2ar_e}{r_e^2 + a^2} = \frac{2\sqrt{I_2}}{1 + I_2}$$

$$\Gamma = \frac{\rho_0^2}{\Omega_0^2} = a^2(1 + I_2 x^2)$$

$$\gamma^2 = \frac{a^2(1 + I_2)^2(1 - x^2)}{1 + I_2 x^2}$$

- $r_e^2 + a^2 = \dfrac{a^2(1 + I_2)}{I_2}$,
  $\rho_0^2 = \dfrac{a^2(1 + I_2 x^2)}{I_2}$
- $\sqrt{-g_{\mathrm{nh}}} = a^2(1 + I_2)\,\Gamma(x)$

Frolov–Thorne temperature from the near-extremal limit

The formal Frolov–Thorne weight for a mode $e^{-i\omega t+im\varphi}$ is

$$\exp\left[-\frac{\omega-m\,\Omega_H}{T_H}\right]=\exp\left[-\frac{n_R}{T_R}-\frac{n_L}{T_L}\right],\qquad n_L=m.$$

At extremality $T_R\to 0$ and

$$T_L=-\lim_{r_+\to r_e}\frac{T_H}{\Omega_H-\Omega_H^{\rm ext}}\,.$$

Near extremality (fixed a, B):

$$T_H\simeq\frac{r_+-r_-}{4\pi(r_e^2+a^2)},\qquad \Omega_H-\Omega_H^{\rm ext}\simeq-\frac{a r_e\,(r_+-r_-)}{(r_e^2+a^2)^2}$$

$$T_L=\frac{r_e^2+a^2}{4\pi a r_e}=\frac{1}{2\pi k}$$

- Uses $(1+B^2r_e^2)I_2=1$ once more
- Agrees with the general rule $T_L=1/2\pi k$ for a throat with fibre $\mathrm{d}\phi+k\,y\,\mathrm{d}\tau$
- Kerr: $T_L=1/2\pi$

Frolov & Thorne, PRD 39, 2125 (1989); Guica et al., PRD 80, 124008 (2009).

What the external field does to the dual CFT

	Kerr $(B = 0)$	**Kerr–BR** $(0 < Ba < 1)$	$Ba \to 1^-$
Extremal mass $M_{\rm ext} = aI_1/\sqrt{I_2}$	a	a/k	∞
Twist k	1	$\dfrac{2\sqrt{I_2}}{1+I_2}$	0
Central charge c_L	$12a^2$	$12a^2\sqrt{I_2}$	0
Temperature T_L	$\dfrac{1}{2\pi}$	$\dfrac{1+I_2}{4\pi\sqrt{I_2}}$	∞
Entropy $S_{\rm CFT} = S_{\rm BH}$	$2\pi a^2$	$\pi a^2(1+I_2)$	πa^2

The product $c_L T_L$ is fixed by the area, but each factor degenerates at the boundary $Ba = 1$.
This boundary is where the magnetic flux is expelled (Part III).

Proof of $\Omega_{,r}\big|_{r_e} = B^2 a$

$$\Omega_{,r} = \frac{B^2}{\Omega}\Big[r(1-x^2) + x^2\, I_2\Big(\frac{M}{I_1} + \frac{B^2 M^2 r}{I_1^2}\Big)\Big]$$

Insert the extremal values $r = r_e = \dfrac{a}{\sqrt{I_2}},\ M = \dfrac{a I_1}{\sqrt{I_2}}$:

$$\frac{M}{I_1} + \frac{B^2 M^2 r_e}{I_1^2} = \frac{a}{\sqrt{I_2}} + \frac{B^2 a^3}{I_2^{3/2}} = \frac{a}{I_2^{3/2}}\big(I_2 + B^2 a^2\big) = \frac{a}{I_2^{3/2}}$$

$$\Rightarrow \quad \Big[\ \cdots\ \Big] = \frac{a}{\sqrt{I_2}}\,(1-x^2) + x^2\,\frac{a}{\sqrt{I_2}} = \frac{a}{\sqrt{I_2}} \qquad \text{for all } x.$$

With $\Omega_0 = 1/\sqrt{I_2}$ (no x-dependence, since $\Delta(r_e) = 0$):

$$\Omega_{,r}\big|_{r_e} = B^2\sqrt{I_2}\cdot\frac{a}{\sqrt{I_2}} = B^2 a.$$

Horizon potential: the algebra

1. With the two identities, EPJC Eqs. (3.1)–(3.3) ($w = 1$):

$$A_\varphi\big|_{r_e} = \frac{\Omega_0 - 1}{B} - \frac{B\,a\,r_e\,(r_e^2 + a^2)}{\rho_0^2}$$

2. Insert the extremal values $\Omega_0 = \frac{1}{\sqrt{I_2}}$, $r_e = \frac{a}{\sqrt{I_2}}$, $r_e^2 + a^2 = \frac{a^2(1+I_2)}{I_2}$, $\rho_0^2 = \frac{a^2(1+I_2x^2)}{I_2}$:

$$A_\varphi\big|_{r_e} = \frac{1 - \sqrt{I_2}}{B\sqrt{I_2}} - \frac{Ba^2\,(1 + I_2)}{\sqrt{I_2}\,(1 + I_2x^2)}$$

3. On the axis, for any r, $\Omega = \sqrt{I_2}\left(1 + B^2 M r / I_1\right)$:

$$A_\varphi(x = \pm 1) = \frac{\Omega - 1 - r\,\Omega_{,r}}{B} = \frac{\sqrt{I_2} - 1}{B}$$

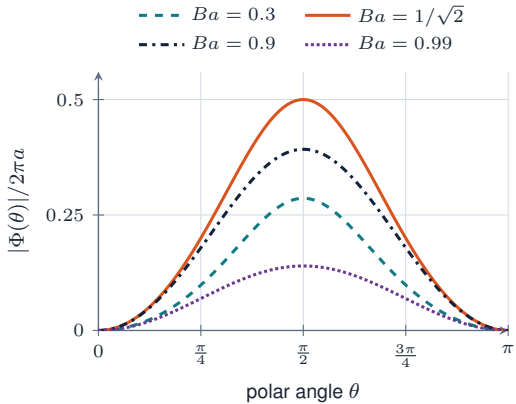
4. Regular representative: subtract the polar value by adding $\frac{1-\sqrt{I_2}}{B}$ and use $1 - I_2 = B^2 a^2$:

$$A_\varphi\big|_{r_e} = -\,Ba^2\sqrt{I_2}\,\frac{1 - x^2}{1 + I_2 x^2}$$

5. Stokes' theorem on the northern hemisphere:

$$\Phi_N = 2\pi\Big[A_\varphi\big|_{r_e}\Big]_{x=1}^{x=0} = -2\pi Ba^2\sqrt{I_2}$$

Flux distribution over the horizon



Flux through the polar cap $0 \le \theta' \le \theta$

$$\Phi(\theta) = -\,2\pi\, Ba^2 \sqrt{I_2}\, \frac{\sin^2\theta}{1 + I_2 \cos^2\theta}$$

- Equal and opposite contributions from the two hemispheres: the total flux through the horizon is zero
- Shape: between $\sin^2\theta/(1+\cos^2\theta)$ at weak field and $\sin^2\theta$ near $Ba = 1$
- Amplitude: rises with B, then is switched off by the factor $\sqrt{1 - B^2a^2}$
- Every cap, not only the hemisphere, loses its flux as $Ba \to 1^-$

Sample values along the extremal family

Ba	r_e/a	M_{ext}/a	k	c_L/a^2	T_L	S_{BH}/a^2	$\lvert\Phi_N\rvert/a$
0	1.000	1.000	1.0000	12.000	0.1592	6.283	0
0.25	1.033	1.001	0.9995	11.619	0.1592	6.087	1.521
0.5	1.155	1.010	0.9897	10.392	0.1608	5.498	2.721
$1/\sqrt{2}$	1.414	1.061	0.9428	8.485	0.1688	4.712	3.142
0.9	2.294	1.365	0.7326	5.231	0.2173	3.739	2.465
0.99	7.089	3.615	0.2766	1.693	0.5753	3.204	0.878
0.999	22.37	11.21	0.0892	0.537	1.783	3.148	0.281

In every row $\frac{\pi^2}{3} c_L T_L = S_{\mathrm{BH}}$. The twist stays close to 1 until $Ba \gtrsim 0.7$, then falls toward the static boundary. All quantities are evaluated at fixed a.

Wald's test field and the classic Meissner effect

Killing vectors of Kerr generate test Maxwell fields:

$$A = \tfrac{1}{2} B \left(\psi^{\flat} + 2a\, \xi^{\flat} \right) \qquad \text{(Wald, uncharged)}$$

$$A_{\varphi} = \frac{B \sin^2 \theta}{2\rho^2} \Big[(r^2 + a^2)^2 - \Delta\, a^2 \sin^2 \theta - 4Ma^2 r \Big]$$

Flux through a horizon hemisphere:

$$\Phi = 2\pi A_{\varphi}\big(r_+, \tfrac{\pi}{2}\big) = \pi B r_+^2 \left(1 - \frac{a^4}{r_+^4} \right) \xrightarrow{a \to M} 0$$

- Wald's field is the test-field analogue of a uniform magnetic field on Kerr
- Bičák & Janiš: the hemisphere flux of stationary axisymmetric test fields vanishes at extremality
- Karas & Vokrouhlický: the same for the exact magnetised Kerr–Newman solution
- Bičák & Hejda: the general near-horizon criterion used in Part III

This is the classic effect that Part III establishes analytically for the Kerr–BR family.

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