Static Vacuum Geometry Encodes Gravitational Collapse and Cosmology

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Motivation and background

- Understanding the quantum nature of gravity is one of the central challenges in fundamental physics.
- Quantum cosmology and quantum-corrected black holes: two directions have largely developed independently, with quantum gravity modifications often introduced separately in each symmetry sector.
- Consequently, models that are individually consistent within their respective sectors need not be mutually consistent: there may be no common gravitational dynamics capable of reproducing both.
- Establishing consistency relations between these sectors is therefore important both for assessing their theoretical compatibility and for enabling observations in different gravitational settings to provide complementary constraints on quantum gravity.

Motivation and background

Cosmology: $a(t)$

$$H^2 = \frac{8\pi G}{3}\rho + \cdots$$



Black hole

Birkhoff's theorem

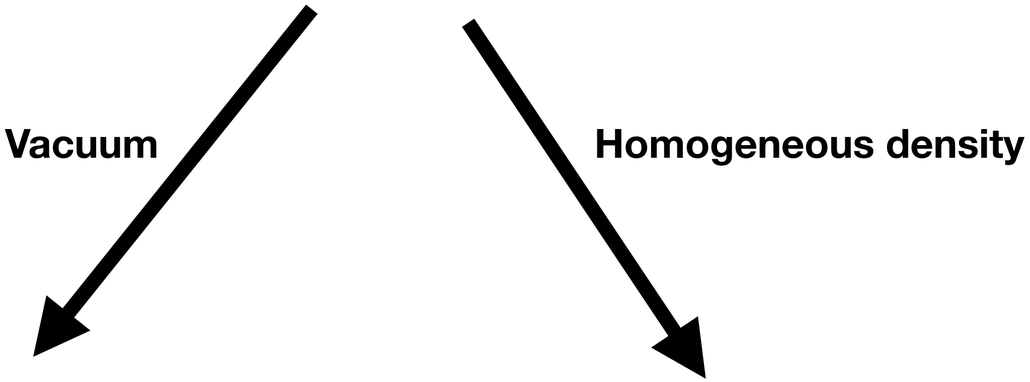


Unique family of solution in spherical symmetry

Motivation and background

In GR

LTB dynamics describes the dynamics of dust



Beyond GR

- **Quantum BH derived from LQC** [Lewadowski, Ma, Yang CZ, 23']
- **Modified FLRW dynamics derived from prescribed regular black-hole geometries within mimetic gravity** [Giesel, Liu, Singh, Weigl, 25']
- **Similar black-hole–cosmology relations have also emerged in quasi-topological gravity** [Bueno, Cano, Hennigar, and Murcia 26'; Ling, Yu 15']

Motivation and background

In our work, this issue is studied in covariant Hamiltonian gravity in spherical symmetry:

1.This framework focus on covariance issue of Hamiltonian gravity in spherical symmetry;

- **General covariance, a fundamental principle of modern physics;**
- **In Hamiltonian, the foliation of spacetime into spatial hypersurfaces makes it nontrivial to preserve general covariance;**
- **Imposing this requirement therefore places strong restrictions on the allowed gravitational dynamics**[CZ, Lewandowski, Ma, Yang, 24'&25'; CZ, Cao 25']

Concrete questions and answers

In CZ&Cao 25', the inverse problem is solved:

From one parameter family of spherically symmetric metric

$$ds^2 = -F(x;m)dt^2 + G(x,m)dx^2 + x^2 d\Omega^2$$



Find a dynamics in the Hamiltonian framework, such that its vacuum solution is the given metric

- **The reconstructed dynamics is not unique**
- **For each reconstructed dynamics, the vacuum solution is unique!**

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Coupled it to irrotational dust

Answer these two questions we established the relation

- **What is the dynamics of the dust coupled system?**
- **Dose the dust-coupled system has a cosmology sector?**

Concrete questions and answers

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- **What is the dynamics of the dust coupled system?**

For given dust profile described by $\mathfrak{m}(x)$ and $e(x)$

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \frac{[\partial_x R(t,x)]^2}{e(x)^2 \mathcal{Q}(R(t,x);\mathfrak{m}(x))}\mathrm{d}x^2 + R(t,x)^2 \mathrm{d}\Omega^2,$$

The spacetime metric in the dust comoving gauge

$$(\partial_t R)^2 = e(x)^2 \mathcal{Q}(R;\mathfrak{m}(x)) - G(R;\mathfrak{m}(x)).$$

The dynamics of the radial of each shell in the dust ball

$$\rho(t,x) = \frac{\mathfrak{m}'(x)\sqrt{\mathcal{Q}(R(t,x);\mathfrak{m}(x))}}{4\pi R(t,x)^2 \partial_x R(t,x)}.$$

How does the density of the dust evolve

with $\mathcal{Q}(x;m) = F(x;m)/G(x;m)$

The dynamics is completely encoded in the prescribed static metric, independent of the reconstructed gravitational dynamics.

[Ling, Yang, Yu and CZ, 26']

Concrete questions and answers

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- **Dose the dust-coupled system has a cosmology sector?**

The dust-coupled system has a cosmological sector iff:

$$F(r;m) = G(r;m) = 1 - r^2 g\left(\frac{\mathcal{M}(m)}{r^3}\right)$$

The sufficient and necessary condition for the model has cosmological sector.

$$H^2 + \frac{k}{a^2} = g\left(\frac{4\pi\rho}{3}\right), \quad \dot{\rho} + 3H\rho = 0,$$

The modified Friedmann equation iff the cosmological sector exist.

[Ling, Yang, Yu and CZ, 26']

Examples

Bardeen model:

$$F_B = G_B = 1 - \frac{2mr^2}{(r^2 - \ell^2)^{3/2}}$$

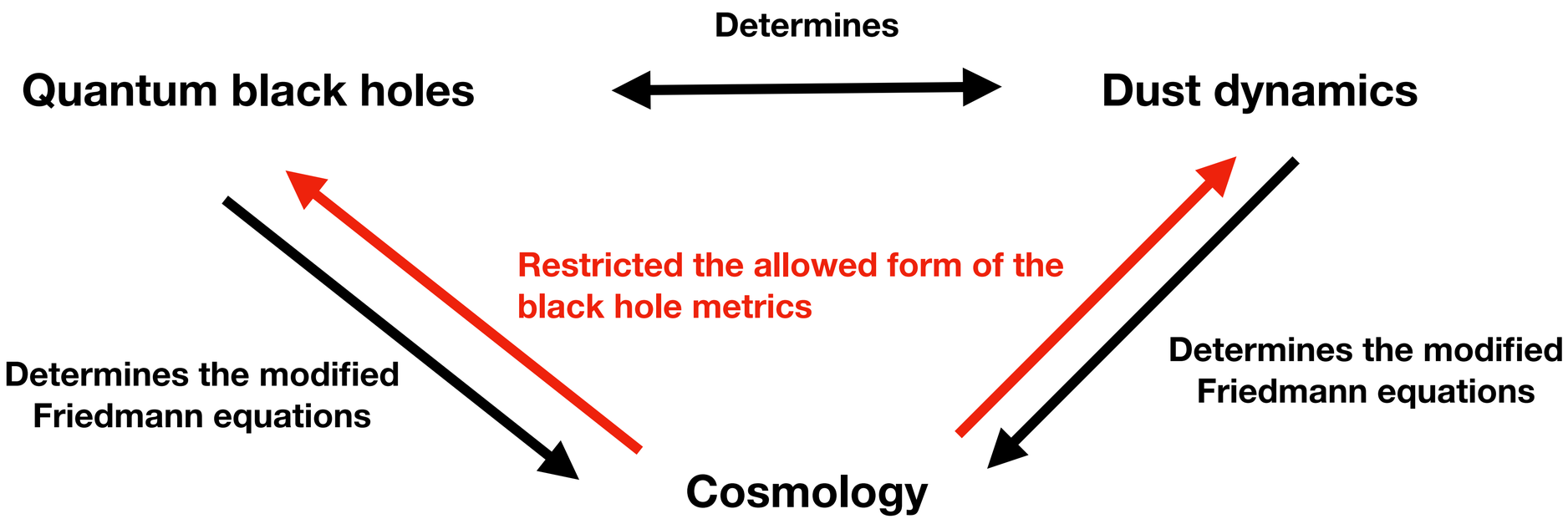
There is no cosmological sector for constant ℓ

Hayward metric:

$$F = H \equiv F_H = 1 - \frac{2mx^2}{x^3 + 2\zeta^2 m}.$$

$$H^2 + \frac{k}{a^2} = \frac{8\pi\rho/3}{1 + 8\pi\ell^2\rho/3}.$$

Conclusion



Trinity of BH, dust dynamics and cosmology

Thanks for your attention!