New type of SVT theory via disformal transformation.

partly based on papers with Volkova, Shtennikova, Valencia-Villegas and Rubakov

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ST and SVT

- Scalar-Tensor theories (ST)
  - Local covariant theory of metric and real scalar field
  - Serves as a simplest, yet quite general modification of gravity
  - Spectrum contains (massless tensor = graviton) and (scalar mode)

- Scalar-Vector-Tensor theories (SVT)
  - Local covariant theory of metric, U(1) gauge vector field and real scalar field
  - Contains all ST modifications of gravity, but additionally presents a non-trivial interaction with (modified) Maxwell field
  - Spectrum contains (massless tensor = graviton), (massless vector =? photon) and (scalar mode)

Motivation

- Motivation and applications: Why and where to use ST and SVT?
  - early Universe cosmology
  - compact objects
  - modern Universe cosmology

Early Universe (pre-Big Bang)

- Mostly Scalar-Tensor theories:

- Different "successful" models of the early Universe:
  - different new models of Inflations
  - bouncing cosmologies
  - genesis models
  - various combinations

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- By "successful" we mean not only stable (healthy), but also satisfying experimental data
  - amplitude and tilt of the scalar spectum
  - amplitude of tensor spectrum (r-ratio)
  - anisotropies and non-gaussianities

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- SVT theories: primordial magnetic fields?

Compact objects

- Stable wormhole solutions in ST teories
- Many different "successful" black hole solutions (with or without hair)

- Mostly Scalar-Tensor theories, vectors (if any) are usually non-modified

Modern Universe cosmology

- Dark matter
- "mimetic"dark matter
- heavy vector or scalar dark matter
- or even light scalar dark matter

- Dark energy
- quintessence and other models
- phantom crossing
  (DESI BAO data)
- selftuning models
  (Cosmological Constant Problem, Weinberg (1989))

- Modifications to hot stage

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  - isotropy
  - simplicity

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  - vector particles
  - (electro) magnetic fields
  - modified photon

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- Why Vectors then?
  - vector particles
  - (electro) magnetic fields
  - modified photon why?

    A. Shtennikova explained on Saturday

Why modified photon?

- There is an additional phenomenological restriction in the late Universe (models of dark energy and dark matter)

$$\boxed{GW170817}$$

- Speed of gravitational waves is very close to the speed of light

$$c_{\mathcal{T}} \approx c$$

$$\left|\frac{c_{\mathcal{T}}}{c} - 1\right| < 10^{-15}$$

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- Speed of gravitational waves is very close to the speed of light

$$c_{\mathcal{T}} \approx c$$

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The idea is to allow $c_{\mathcal{T}} \neq 1$ while keeping $c_{\mathcal{T}} = c$

Plan: Scalar-Tensor theories

- ST and SVT theories, properties and perturbations:
  - 2nd order equations (Horndeski type)
  - Degenerate theories (DHOST type)
  - Fully degenerate theories (MMG)
  - Perturbations in ST theories

2nd order equations

Toy example:

$$\mathcal{L} = K(\pi, X)\Box\pi \qquad X = g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$$

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$$= ... - \underline{2K_X\partial^\mu\Box\pi\partial_\mu\pi\delta\pi} + \partial_\mu(K_\pi\partial^\mu\pi + \underline{2K_X\partial^\mu\partial_\nu\pi\partial^\nu\pi})\delta\pi$$

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$$... - 2K_X\partial^\mu\partial_\nu\partial^\nu\pi\partial_\mu\pi\delta\pi + 2K_X\partial_\mu\partial^\mu\partial_\nu\pi\partial^\nu\pi\delta\pi$$

$$= ...\text{only second derivatives}$$

Horndeski theory

- Most general scalar-tensor theory with 2nd order EoM

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = -G_4(\pi, X)R + 2G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

$$\mathcal{L}_5 = G_5(\pi, X)G^{\mu\nu}\pi_{;\mu\nu} + \frac{1}{3}G_{5X}\left[(\Box\pi)^3 - 3\Box\pi\pi_{;\mu\nu}\pi^{;\mu\nu} + 2\pi_{;\mu\nu}\pi^{;\mu\rho}\pi_{;\rho}{}^{\nu}\right],$$

where π is the Galileon field, $X = g^{\mu\nu}\pi_{,\mu}\pi_{,\nu}$, $\pi_{,\mu} = \partial_\mu\pi$, $\pi_{;\mu\nu} = \nabla_\nu\nabla_\mu\pi$, $\Box\pi = g^{\mu\nu}\nabla_\nu\nabla_\mu\pi$, $G_{4X} = \partial G_4/\partial X$

Properties

- Theory of a very general form under several assumptions:
  - general covariance
  - locality
  - $S = \int d^4x\sqrt{-g}\,\mathcal{L}(g_{\mu\nu}, \phi, \nabla\phi, \nabla\nabla\phi)$
    (1 additional degree of freedom)
  - 2nd order equations of motion.

- Have sufficiently much freedom to modify gravity and scalar dynamics in different ways

  Encodes wide class of modified gravity models

Examples

- Minimally coupled scalar (k-essence)
  - $F = F(\phi, X), \quad K = 0, \quad G_4 = \frac{M_{\rm Pl}^2}{2}, \quad G_5 = 0$
  - Canonical case: $G_2 = X - V(\phi)$

- Brans–Dicke theory
  - $G_4 = \phi, \quad F = -\frac{\omega}{\phi} X - V(\phi), \quad K = G_5 = 0$
  - $f(R)$ gravity equivalent to Brans–Dicke with $\omega = 0$

$$S = \int d^4x \sqrt{-g}\, [f'(\chi)(R - \chi) + f(\chi)]$$

  - $G_4 = \frac{\phi}{2}, \quad F = -V(\phi), \quad K = G_5 = 0$

Examples

- Kinetic Gravity Braiding

  $F = F(\phi, X), \quad K = K(\phi, X), \quad G_4 = \frac{M_{\rm Pl}^2}{2}, \quad G_5 = 0$

  Qubic Galileon: $F = X, \quad K = \frac{X}{\Lambda^3}, \quad G_4 = \frac{M_{\rm Pl}^2}{2}, \quad G_5 = 0$

- Coupling to Gauss-Bonnet term
  $\xi(\phi)\left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\rho}R^{\mu\nu\lambda\rho}\right)$

  $F = 8\xi_{\phi\phi\phi\phi}X^2(3 - \ln X) \qquad K = 4\xi_{\phi\phi\phi}X(7 - 3\ln X)$

  $G_4 = 4\xi_{\phi\phi}X(2 - \ln X) \qquad G_5 = -4\xi_{\phi}\ln X$

Degenerate theories

Toy example:

Kinetic matrix

$$\mathcal{L} = \frac{A}{2}\,\ddot{\phi}^2 + B\,\ddot{\phi}\dot{g} + \frac{C}{2}\,\dot{g}^2 + \frac{1}{2}\dot{\phi}^2 - \frac{1}{2}\phi^2 - \frac{1}{2}g^2 ...$$

Equations of motion

$$A\,\dddot{\phi} + B\,\dddot{g} - \ddot{\phi} - \phi = 0$$

$$B\,\dddot{\phi} + C\,\ddot{g} + g = 0$$

if $AC - B^2 = 0$ system is equivalent to

$$\ddot{\phi} + \frac{B}{C}\,\dot{g} + \phi = 0$$

$$\left(1 - \frac{B^2}{C^2}\right)\,\ddot{g} - \frac{B}{C}\,\dot{\phi} + \frac{1}{C}\,g = 0$$

Main idea:

Hessian

$$\det \begin{pmatrix} \partial^2\mathcal{L}/\partial\ddot{\phi}^2 & \partial^2\mathcal{L}/\partial\ddot{\phi}\partial\dot{g} \\ \cdots & \partial^2\mathcal{L}/\partial\dot{g}^2 \end{pmatrix} = 0$$

Primary constraint $\Rightarrow$ no extra DOF

beyond Horndeski

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5 + \mathcal{L}_{\mathcal{BH}}\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

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$$\mathcal{L}_{\mathcal{BH}} = F_4(\pi, X)\epsilon^{\mu\nu\rho}{}_{\sigma}\epsilon^{\mu'\nu'\rho'\sigma}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'} +$$
$$+F_5(\pi, X)\epsilon^{\mu\nu\rho\sigma}\epsilon^{\mu'\nu'\rho'\sigma'}\pi_{,\mu}\pi_{,\mu'}\pi_{;\nu\nu'}\pi_{;\rho\rho'}\pi_{;\sigma\sigma'}$$

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$$F_4\ G_{5X}X = -3F_5\ \left[G_4 - 2XG_{4X} + \frac{1}{2}G_{5\pi}X\right],$$

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$$F_4\ G_{5X}X = -3F_5\left[G_4 - 2XG_{4X} + \frac{1}{2}G_{5\pi}X\right],$$

H $\to$ BH $\qquad$ $g_{\mu\nu} \to g_{\mu\nu} + \Gamma(\pi, X)\partial_\mu\pi\partial_\nu\pi.$

DHOST theory

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DHOST theory

$$\mathcal{L}_{\text{Quad}} = \sum_{i=1}^{5} A_i(\pi, X)\, L_i^{(2)} \,,$$

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$$L_1^{(2)} = (\pi_{\mu\nu})^2\,, \quad L_2^{(2)} = (\Box\pi)^2\,, \quad L_3^{(2)} = \Box\pi\,(\pi_{\mu\nu}\pi^\mu\pi^\nu)\,,$$

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$$L_4^{(2)} = (\pi_{\mu\rho}\pi^\mu)^2\,, \; L_5^{(2)} = (\pi_{\mu\nu}\pi^\mu\pi^\nu)^2\,,$$

$$A_2 = -A_1\,,$$

$$\begin{aligned} A_4 = \frac{1}{8(F_2 - XA_1)^2}\,\big[&-16XA_1^3 + 4(3F_2 + 16XF_{2X})A_1^2 \\ &-(16X^2F_{2X} - 12XF_2)A_3A_1 - X^2F_2A_3^2 \\ &-16F_{2X}(3F_2 + 4XF_{2X})A_1 + 8F_2(XF_{2X} - F_2)A_3 + 48F_2F_{2X}^2\big]\,, \end{aligned}$$

$$A_5 = \frac{(4F_{2X} - 2A_1 + XA_3)\left(-2A_1^2 - 3XA_1A_3 + 4F_{2X}A_1 + 4F_2A_3\right)}{8(F_2 - XA_1)^2}\,.$$

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$$\mathcal{L}_{\text{Cubic}} = \sum_{j=1}^{10} B_j(\pi, X)\, L_j^{(3)}$$

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$$L_6^{(3)} = (\pi_{\rho\sigma})^2\,(\pi_{\mu\nu}\pi^\mu\pi^\nu)\,, \qquad L_7^{(3)} = \pi^{\mu\nu}\pi_{\nu\rho}\pi^{\rho\sigma}\pi_\mu\pi_\sigma\,,$$

$$L_8^{(3)} = (\pi^{\mu\nu}\pi_\mu)^2(\pi^{\rho\sigma}\pi_\rho\pi_\sigma)\,, \qquad L_9^{(3)} = \Box\pi(\pi^{\rho\sigma}\pi_\rho\pi_\sigma)^2\,,$$

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+ Relations between F_3 and B_j

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+ Relations

H $\rightarrow$ DHOST $\qquad g_{\mu\nu} \rightarrow \Omega^2(\pi, X)\, g_{\mu\nu} + \Gamma(\pi, X)\partial_\mu\pi\partial_\nu\pi.$

Fully degenerate theories

- Additional constraint on the kinetic matrix $\Rightarrow$ only two DOF remains.

  Scalar is a non-dynamical auxiliary field

- Minimally Modified Gravity theories:
- Type-I MMG
- Have an Einstein frame
- Without matter are equivalent to GR
- "Veiled gravity"

- Type-II MMG

  = VCDM theory
- Does not have an Einstein frame
- "Cuscuton"

Perturbations above FLRW background: DHOST

$$S = \int \mathrm{d}t\mathrm{d}^3x a^3 \left[\frac{\mathcal{G}_\mathcal{T}}{8} \left(\dot{h}^T_{ik} \right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2} \left(\partial_i h^T_{kl} \right)^2 + \mathcal{G}_\mathcal{S} \dot{\zeta}^2 - \mathcal{F}_\mathcal{S} \frac{(\nabla\zeta)^2}{a^2} \right]$$

The speeds of sound for tensor and scalar perturbations are, respectively,

$$c^2_\mathcal{T} = \frac{\mathcal{F}_\mathcal{T}}{\mathcal{G}_\mathcal{T}}, \qquad c^2_\mathcal{S} = \frac{\mathcal{F}_\mathcal{S}}{\mathcal{G}_\mathcal{S}}$$

These coefficients are combinations of Lagrangian functions and have non-trivial relations

$$\begin{aligned} \mathcal{G}_\mathcal{S} &= \frac{\Sigma {\mathcal{G}_\mathcal{T}}^2}{\Theta^2} + 3\mathcal{G}_\mathcal{T}, \\ \mathcal{F}_\mathcal{S} &= \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T}, \\ \xi &= \frac{a{\mathcal{G}_\mathcal{T}}^2}{\Theta}. \end{aligned} \qquad \Rightarrow \qquad \begin{aligned} \mathcal{G}_\mathcal{S} &= \frac{\Sigma {\mathcal{G}_\mathcal{T}}^2}{\Theta^2} + 3\mathcal{G}_\mathcal{T}, \\ \mathcal{F}_\mathcal{S} &= \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T}, \\ \xi &= \frac{a\left(\mathcal{G}_\mathcal{T} + \mathcal{D}\dot{\pi} + \mathcal{F}_\mathcal{T}\Delta\right)\mathcal{G}_\mathcal{T}}{\Theta}. \end{aligned}$$

Perturbations above FLRW background: simplest MMG - generalized cuscuton

$$S = \int \mathrm{d}t\mathrm{d}^3x a^3 \left[\frac{\mathcal{G}_\mathcal{T}}{8} \left(\dot{h}^T_{ik} \right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2} \left(\partial_i h^T_{kl} \right)^2 + \mathcal{G}_\mathcal{S} \dot{\zeta}^2 - \mathcal{F}_\mathcal{S} \frac{(\nabla\zeta)^2}{a^2} \right]$$

The speeds of sound for tensor and scalar perturbations are, respectively,

$$c_\mathcal{T}^2 = \frac{\mathcal{F}_\mathcal{T}}{\mathcal{G}_\mathcal{T}}, \qquad c_\mathcal{S}^2 = \frac{\mathcal{F}_\mathcal{S}}{\mathcal{G}_\mathcal{S}}$$

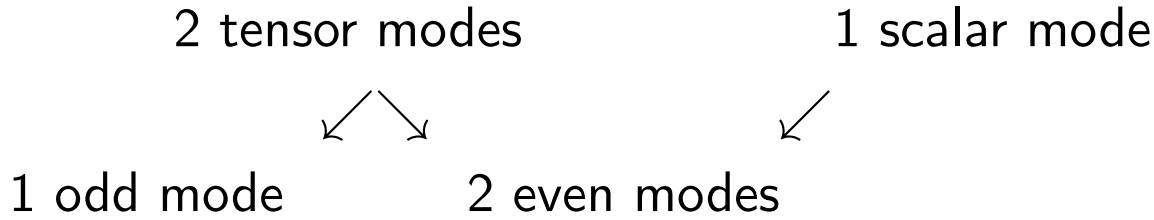
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$$\mathcal{G}_\mathcal{S} = \frac{\Sigma {\mathcal{G}_\mathcal{T}}^2}{\Theta^2} + 3\mathcal{G}_\mathcal{T},$$
$$\mathcal{F}_\mathcal{S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T},$$
$$\xi = \frac{a{\mathcal{G}_\mathcal{T}}^2}{\Theta}.$$

$$\Rightarrow$$

$$\mathcal{G}_\mathcal{S} = 0$$
$$\mathcal{F}_\mathcal{S} = \frac{1}{a}\frac{\mathrm{d}\xi}{\mathrm{d}t} - \mathcal{F}_\mathcal{T},$$
$$\xi = \frac{a{\mathcal{G}_\mathcal{T}}^2}{\Theta}.$$

Perturbations above spherically symmetric background



$$S^{(2)}_{odd} = \int \mathrm{d}t\,\mathrm{d}r\,\left[\mathcal{A}\dot{q}^2 - \mathcal{B}(q')^2 + \mathcal{C}\dot{q}(q') - \frac{l(l+1)}{J^2}\cdot\mathcal{H}q^2 - V(r)q^2\right]$$

$$S^{(2)}_{even} = \int \mathrm{d}t\mathrm{d}r\,\left[\mathcal{K}_{ij}\dot{v}^i\dot{v}^j - \mathcal{G}_{ij}v^{i\prime}v^{j\prime} + Q_{ij}\dot{v}^i v^{j\prime} + \text{...less derivative terms...}\right]$$

$i,\ j = 1..2$

Scalar-Vector-Tensor theories

- Scalar-tensor theory + Vector interactions

  U(1) gauge vector-ST vertexes look like:

$$F_{\alpha\beta}F^{\alpha\beta}\,,\qquad F_{\alpha\beta}F^{\alpha\beta}(\nabla_\gamma\nabla^\gamma\pi)\,,\qquad F_{\alpha\beta}F_{\gamma\delta}(\nabla_\gamma\nabla^\gamma\pi)\nabla^\beta\pi\,\nabla^\gamma\nabla^\alpha\pi\,\nabla^\delta\pi\,,$$
$$F^{\alpha}{}_{\epsilon}\nabla_\nu\pi\,\nabla^\nu\nabla_\mu\pi\,\nabla_\alpha F^{\epsilon\mu}\,,\qquad F_{\delta\epsilon}\nabla_\nu\pi\,\nabla^\nu\nabla^\alpha\pi\,\nabla_\alpha F^{\delta\epsilon}\,,$$
$$F_{\alpha\beta}F_{\gamma\delta}\nabla_\nu\nabla^\alpha\pi\,\nabla^\nu\nabla^\gamma\pi\,\nabla^\beta\pi\,\nabla^\delta\pi\,,\qquad F_{\alpha\beta}F^{\alpha}{}_{\delta}\nabla^\beta\pi\,\nabla^\delta\pi\,,\qquad F_{\alpha\beta}F^{\alpha}{}_{\delta}\nabla^\delta\nabla^\beta\pi\,,$$
$$F_{\alpha\beta}F^{\alpha\beta}\nabla_\nu\nabla^\mu\pi\,\nabla^\nu\pi\,\nabla_\mu\pi\,,\qquad F_{\alpha\beta}F^{\alpha\beta}R\,,\qquad F_{\alpha\beta}F^{\alpha\beta}(\nabla_\gamma\nabla^\gamma\pi)^2\,,$$
$$F_{\alpha\beta}F^{\alpha}{}_{\delta}R\nabla^\beta\pi\,\nabla^\delta\pi\,,\qquad F_{\alpha\beta}F^{\alpha\beta}\nabla_\eta\nabla_\mu\pi\,\nabla^\eta\nabla^\mu\pi\,,\qquad F_{\alpha\beta}F^{\alpha}{}_{\delta}\nabla_\nu\nabla^\beta\pi\,\nabla^\nu\pi\,\nabla^\delta\pi\,,$$
$$F_{\alpha\beta}F^{\alpha}{}_{\delta}(\nabla_\gamma\nabla^\gamma\pi)\nabla^\delta\nabla^\beta\pi\,,\qquad F_{\delta\epsilon}(\nabla_\gamma\nabla^\gamma\pi)\nabla_\alpha F^{\delta\alpha}\nabla^\epsilon\pi\,,$$
$$F_{\alpha\beta}F^{\alpha}{}_{\delta}\nabla_\nu\nabla^\beta\pi\,\nabla^\nu\nabla^\delta\pi\,,\qquad F_{\alpha\beta}F_{\gamma\delta}R^{\alpha\gamma}\nabla^\beta\pi\,\nabla^\delta\pi\,,\qquad F_{\delta\epsilon}\nabla_\alpha F^{\delta\alpha}\nabla^\epsilon\pi\,,$$
$$F_{\delta\epsilon}\nabla_\alpha F^{\alpha\mu}\nabla^\delta\pi\,\nabla^\epsilon\nabla_\mu\pi\,,\qquad F_{\alpha\beta}F_{\gamma\delta}\nabla^\gamma\nabla^\alpha\pi\,\nabla^\delta\nabla^\beta\pi\,,\qquad F_{\delta\epsilon}\nabla_\alpha F^{\delta\eta}\nabla^\epsilon\pi\,\nabla_\eta\nabla^\alpha\pi\,,$$
$$F_{\delta\epsilon}\nabla_\nu\pi\,\nabla^\nu\nabla^\epsilon\pi\,\nabla_\alpha F^{\delta\alpha}\,,\qquad F_{\alpha\beta}F^{\alpha}{}_{\delta}(\nabla_\gamma\nabla^\gamma\pi)\nabla^\beta\pi\,\nabla^\delta\pi\,,$$
$$F_{\alpha\beta}F_{\gamma\delta}\nabla^\beta\pi\,\nabla^\gamma\nabla^\alpha\pi\,\nabla^\delta\pi\,,\qquad F_{\alpha\beta}F^{\alpha}{}_{\delta}R^{\beta\delta}\,,\qquad F_{\alpha\beta}F_{\gamma\delta}R^{\alpha\gamma\beta\delta}\,,$$
$$F_{\alpha\beta}F^{\alpha}{}_{\delta}(\nabla_\gamma\nabla^\gamma\pi)^2\nabla^\beta\pi\,\nabla^\delta\pi\,,\qquad F_{\alpha\beta}F^{\alpha}{}_{\delta}\nabla^\beta\pi\,\nabla^\delta\pi\,\nabla_\eta\nabla_\mu\pi\,\nabla^\eta\nabla^\mu\pi\,,$$

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  $F_{\delta\epsilon}\nabla_\alpha F^{\alpha\mu}\nabla^\delta\pi\,\nabla^\epsilon\nabla_\mu\pi$, $F_{\alpha\beta}F_{\gamma\delta}\nabla^\gamma\nabla^\alpha\pi\,\nabla^\delta\nabla^\beta\pi$, $F_{\delta\epsilon}\nabla_\alpha F^{\delta\eta}\nabla^\epsilon\pi\,\nabla_\eta\nabla^\alpha\pi$,
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  degeneracy conditions/disformal transformations $\Rightarrow$ degenerate SVT

  full degeneracy, additional constraint $\Rightarrow$???

Scalar-Vector-Tensor theories: another approach

KK reduction of Horndeski/DHOST/MMG theory

$$g_{mn} = \begin{pmatrix} g_{\mu\nu} - \phi^2 A_\mu A_\nu & \phi^2 A_\mu \\ \phi^2 A_\nu & -\phi^2 \end{pmatrix}$$

- $R^5 \longrightarrow R^4 \times S^1$

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- $R^5 \longrightarrow R^4 \times S^1$

- Generalized Galileons $\longrightarrow$ Generalized Galileons

  2nd derivatives in the action $\longrightarrow$ 2nd derivatives in the action

  no higher derivatives in EOMs $\longrightarrow$ no higher derivatives in EOMs

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  degenerate theory $\longrightarrow$ degenerate theory

- Metric + scalar $\longrightarrow$ Metric + vector + scalar + scalar

  [U(1) gauge]

Scalar-Vector-Tensor theories: another approach

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  [U(1) gauge]
- Metric $\longrightarrow$ Metric + vector + scalar

Scalar-Vector-Tensor theories: another approach

KK reduction of Horndeski/DHOST/MMG theory

$$g_{mn} = \begin{pmatrix} g_{\mu\nu} & A_\mu \\ A_\nu & -1 \end{pmatrix}$$

- $R^5 \longrightarrow R^4 \times S^1$

- Generalized Galileons $\longrightarrow$ Generalized Galileons

  2nd derivatives in the action $\longrightarrow$ 2nd derivatives in the action

  no higher derivatives in EOMs $\longrightarrow$ no higher derivatives in EOMs

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  [U(1) gauge]
- Metric $\longrightarrow$ Metric + vector + ~~scalar~~

2nd order SVT theories

- The largest SVT theory (U(1) gauge invariant vector), of order $\mathcal{O}\left((\nabla\nabla\phi)^2\right)$, with second order equations of motion can be written as

$$L_{\rm SVT} = L_0 + L_1 + L_2 \ ,$$

with $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$, g_i and w_i functions of π and X

$$L_0 = -\frac{1}{4}g_0\ F_{\mu\nu}F^{\mu\nu} + g_2\ F^{\mu}{}_{\rho}F^{\nu\rho}\pi_{;\mu}\pi_{;\nu}$$

$$L_1 = w_1\ \tilde{F}^{\mu}{}_{\rho}\tilde{F}^{\nu\rho}\ \pi_{;\mu\nu} + w_2\ \tilde{F}^{\mu\rho}\tilde{F}^{\nu\sigma}\ \pi_{;\rho}\ \pi_{;\sigma}\ \pi_{;\mu\nu}$$

$$L_2 = \left(w_3\ R_{\mu\rho\nu\sigma} + w_{3,X}\ \pi_{;\mu\rho}\pi_{;\nu\sigma}\right)\tilde{F}^{\mu\nu}\tilde{F}^{\rho\sigma}\ .$$

L. Heisenberg (2018)

Ali Gorji, Mukohyama, Petrov, Yamaguchi (2025)

SM, A. Shtennikova and M. Valencia-Villegas (2025)

2nd order SVT theories

- The pure SVT part of Kaluza-Klein reductions of 5D Horndeski theory can be written in the latter basis.
- Larger sets of SVT theories also fit in the latter basis of SVT's (SM, A. Shtennikova and M. Valencia-Villegas (2025)), at least in 4D (Using Dimensional dependent identities).
- Is L_{SVT} everything that there can be in U(1) gauge invariant SVT's? No!

Disformal transformations

Beyond Horndeski (BH) theory can be obtained from Horndeski (H) theory via disformal transformation

$$\bar{g}_{\mu\nu} = C(\pi)\, g_{\mu\nu} + D(\pi,\, X)\, \pi_{;\mu}\pi_{;\nu}\ .$$

Jacob Bekenstein, 1993

- H and BH related by a transformation of the metric (e.g. with $C = 1$),

$$F_4 = \frac{1}{2}\frac{-D_{,X}}{1 + 2\, D_{,X}\, X^2}\left(G_4 - 2\, X\, G_{4,X} + G_{5,\pi}\, X\right) \tag{6}$$

$$F_5 = \frac{D_{,X}\, G_{5,X}\, X}{3\,(1 + 2 D_{,X}\, X^2)} \tag{7}$$

where the degeneracy condition is automatically satisfied

$$F_4\, G_{5,X}\, X = -3\, F_5\, \left(G_4 - 2\, X\, G_{4,X} + G_{5,\phi}\, X\right)\ . \tag{8}$$

Conformal/ Disformal transformations in ST theories

H and bH can be seen in many cases as the same gravity theory, just two different geometries

However equivalence H$\leftrightarrow$BH breaks if:

- The relation between $\bar{g}$ and g is not invertible (No-Go theorem)

  SM, V.Rubakov, V.Volkova (2018)

- If the same matter Lagrangian is coupled to $\bar{g}$ and g.

All in all, ST theories behave as follows under these transformations:

- BH is closed under $C(\pi)$/ $D(\pi, X)$ transformations
- DHOST is closed under $C(\pi, X)$/ $D(\pi, X)$ transformations

Disformal transformations in SVT theories

As a tool, Disformal (or Conformal-Disformal) transformations are useful to uncover new theories. For instance, H $\rightarrow$ BH.

What happens in the L_{SVT} theory? the theory is not closed under Disformal transformations.

$$L_{\rm SVT} \xrightarrow{D} L_{\rm bSVT} = L_{\rm SVT} + L_{d\,1} + L_{d\,2} \tag{9}$$

where

$$L_{d\,1} = w_0(\pi, X)\, \tilde{F}^{\alpha\zeta} \tilde{F}_{\zeta\beta} \pi_{;\alpha} \pi^{;\gamma} (2\, X \pi^{;\beta}_{;\gamma} + \tfrac{1}{2} \pi^{;\beta} \pi_{;\delta\gamma} \pi^{;\delta}) \tag{10}$$

$$\begin{aligned} L_{d\,2} = {} & w4(\pi, X) \big(\tilde{F}^{\alpha\delta} \tilde{F}^{\gamma\zeta}{}_{;\delta}\, \pi_{;\alpha} \pi^{;\beta} \pi_{;\gamma} \pi_{;\zeta\beta} \\ & \qquad + \tilde{F}^{\alpha\beta} \tilde{F}^{\gamma\delta} \pi_{;\delta\beta} (X \pi_{;\gamma\alpha} + \pi_{;\alpha} \pi_{;\zeta\gamma} \pi^{;\zeta}) \big) \\ & + w5(\pi, X)\, \tilde{F}^{\alpha\beta} \tilde{F}^{\gamma\delta} \Big(2X^2 \pi_{;\gamma\alpha} \pi_{;\delta\beta} \\ & \qquad + \pi_{;\alpha} \pi_{;\zeta} \big(4X \pi^{;\zeta}_{;\gamma} \pi_{;\delta\beta} + \pi_{;\gamma} (-\pi^{;\zeta}_{;\beta} \pi_{;\nu\delta} + \pi_{;\delta\beta} \pi_{;\nu\zeta}) \pi^{;\nu} \big) \Big) \,. \end{aligned} \tag{11}$$

Disformal transformations in SVT theories

Where w_0, w_4 and w_5 are related as follows

$$w_0 = -\frac{w_1\ w_4}{w_3}, \qquad w_5 = \frac{w_4\ (X\ w_4 + w_{3,X})}{w_3 - 2X^2\ w_4} \tag{12}$$

Alternatively, in terms of the C/D factors,

$$w_4 = -w_3\ D_{,X}\ , \qquad w_5 = \frac{D_{,X}\ (X\ w_3\ D_{,X} - w_{3,X})}{1 + 2\ X^2\ D_{,X}}\ . \tag{13}$$

- $L_{d\,1} + L_{d\,2}$ lead to higher order EOMs, but they are free of ghosts.

- The equivalence $L_{SVT} \leftrightarrow L_{bSVT}$ is broken if the same matter Lagrangian for χ is coupled to $\bar{g}$ and g. The degeneracy is not spoiled, as long as they do not alter the off–diagonal terms in the kinetic matrix,

$$\frac{\partial^2 L}{\partial \dot{g}\ \partial \dot{\chi}}\ , \qquad \frac{\partial^2 L}{\partial \dot{\pi}\ \partial \dot{\chi}}\ , \qquad \frac{\partial^2 L}{\partial \dot{A}\ \partial \dot{\chi}}\ , \tag{14}$$

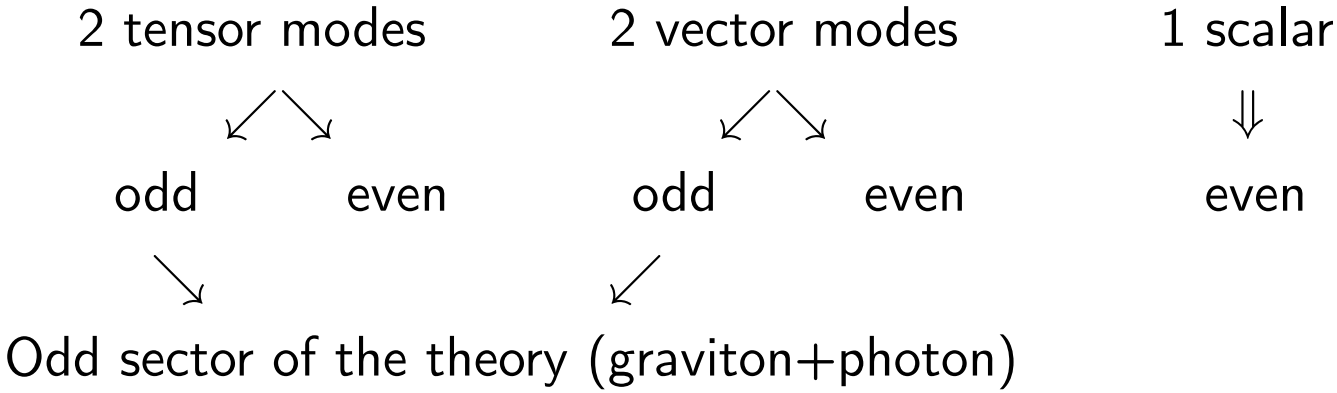
Degenerate SVT: Perturbations above FLRW background

$$S = \int \mathrm{d}t\mathrm{d}^3x a^3 \Big[\frac{\mathcal{G}_\mathcal{T}}{8} \left(\dot{h}^T_{ik} \right)^2 - \frac{\mathcal{F}_\mathcal{T}}{8a^2} \left(\partial_i h^T_{kl} \right)^2 + \\ + \mathcal{G}_\mathcal{S} \dot{\zeta}^2 - \mathcal{F}_\mathcal{S} \frac{(\nabla\zeta)^2}{a^2} + \mathcal{G}_\mathcal{V} \dot{A}_i^2 - \mathcal{F}_\mathcal{V} \frac{(\partial_j A_i)^2}{a^2} \Big]$$

The speeds of sound for tensor, scalar and vector perturbations are, respectively,

$$c^2_\mathcal{T} = \frac{\mathcal{F}_\mathcal{T}}{\mathcal{G}_\mathcal{T}}, \qquad c^2_\mathcal{S} = \frac{\mathcal{F}_\mathcal{S}}{\mathcal{G}_\mathcal{S}}, \qquad c^2 = c^2_\mathcal{V} = \frac{\mathcal{F}_\mathcal{V}}{\mathcal{G}_\mathcal{V}}$$

Perturbations above spherically symmetric background



Applications of ST and SVT theories

Motivation to study them, solutions

- early Universe cosmology
- compact objects
- modern Universe cosmology

Early Universe (pre-Big Bang)

$\pm$ Horndeski

$+$ beyond Horndeski and DHOST

$+$ MMG

$-$ SVT

Early Universe

- Different "successful" models of the early Universe
- Including those with NEC violation
- But it took some efforts

Early Universe: first challenge

Null Energy Condition $\Longleftrightarrow$ Null Convergence Condition

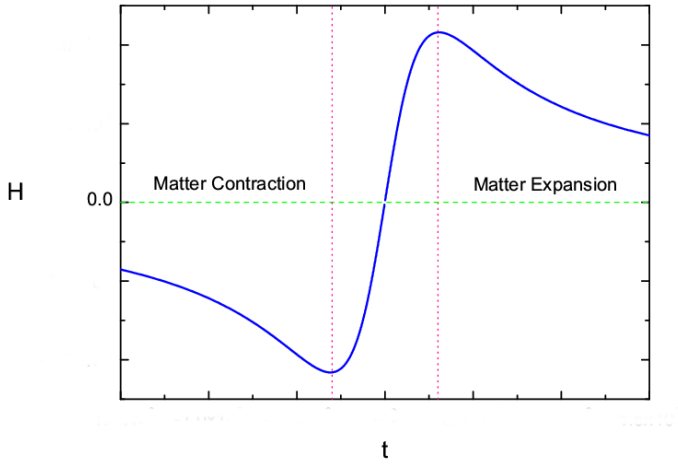
$T_{\mu\nu}k^\mu k^\nu \geq 0$ $\qquad$ $R_{\mu\nu}k^\mu k^\nu \geq 0$

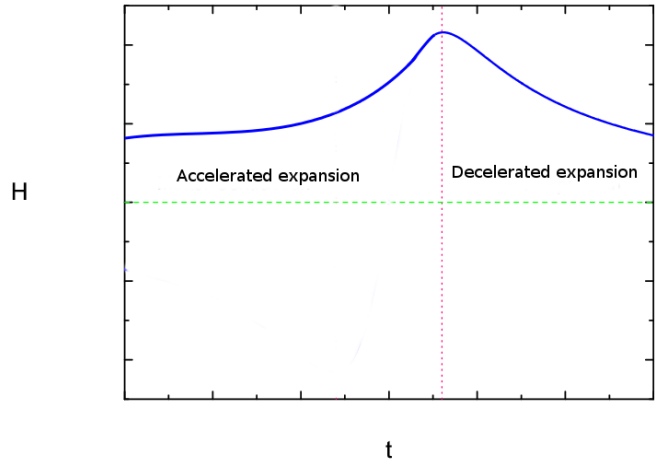
Friedmann equations

$\dot{H} = -4\pi G(p+\rho) \quad \leq 0$

Penrose theorem

no singularity $\Rightarrow$ NEC-violation





Cosmological bounce and genesis solutions require NEC-violation

Early Universe: first challenge

Null Energy Condition $\Longleftrightarrow$ Null Convergence Condition

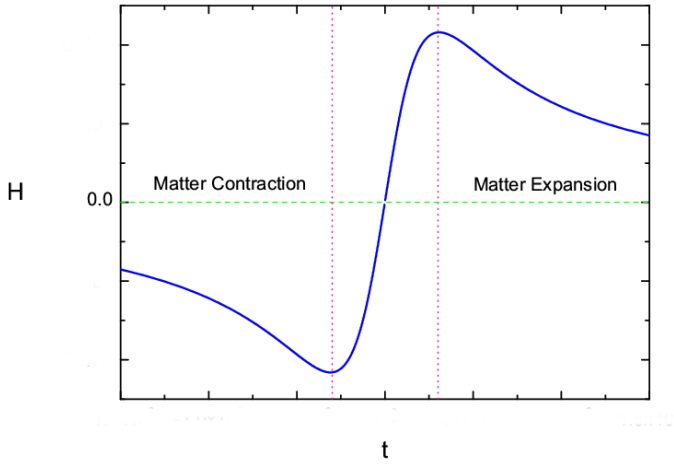
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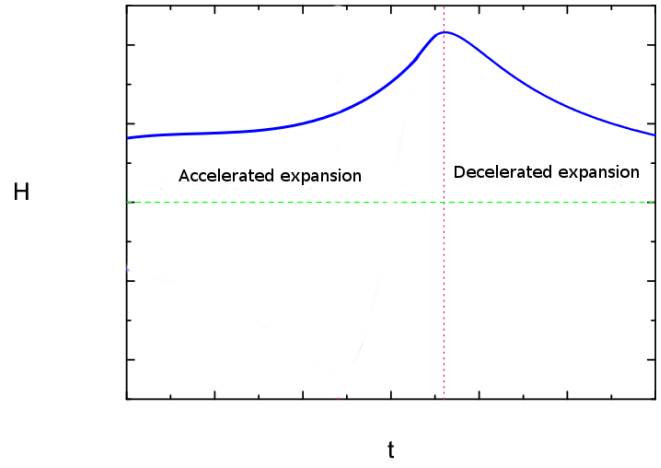
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Penrose theorem

no singularity $\Rightarrow$ NEC-violation





Cosmological bounce and genesis solutions require NEC-violation

As well as wormhole-like solutions

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- No-go for wormhole solutions in Horndeski theory
- "almost" resolved in beyond Hondeski

Modern Universe cosmology

+ Horndeski

+ beyond Horndeski and DHOST

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++ SVT

Modern Universe cosmology

- Different "successful" models of Dark Matter and Dark Energy
- Including those with phantom crossing (NEC violation) and those solving cosmological constant problem
- And theories with modified Maxwell field

Modern Universe challenges

- There is are additional phenomenological restrictions, when we study modern Universe (models of dark energy and dark matter)

$$\boxed{GW170817}$$

- Speed of gravitational waves is very close to the speed of light

$$c_{\mathcal{T}} \approx c$$

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Horndeski cosmology Speed of graviton

$$S = \int \mathrm{d}^4x\sqrt{-g}\left(\mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right),$$

$$\mathcal{L}_2 = F(\pi, X),$$

$$\mathcal{L}_3 = K(\pi, X)\Box\pi,$$

$$\mathcal{L}_4 = G_4(\pi, X)R + G_{4X}(\pi, X)\left[(\Box\pi)^2 - \pi_{;\mu\nu}\pi^{;\mu\nu}\right],$$

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$$\mathcal{S}_T = \int \mathrm{d}\eta\,\mathrm{d}^3x\,a^4\left[\frac{1}{2\,a^2}\left(\mathcal{G}_\tau\left(\dot{h}_{ij}\right)^2 - \mathcal{F}_\tau(\partial_k\,h_{ij})^2\right)\right]$$

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Viable ST models after GW 170817

- General relativity
- quintessence/k-essence
- Brans-Dicke/f(R)
- Kinetic Gravity Braiding
- Derivative Conformal
- Disformal Tuning
- quartic Galileons
- Fab Four
- quadratic DHOST
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- $f(\phi)$ Gauss-Bonnet

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Subclass of Fab Four allows self tuning with simultaneous Vainshtein screening of Dark Energy

No decay for gravitational waves

$$\mathcal{L}_{ssh} = a^3\left[\frac{c_1}{a^2}h_{ij}\zeta_{,i}\zeta_{,j} + \frac{c_2}{a^2}\dot{h}_{ij}\zeta_{,i}\zeta_{,j} + c_3\dot{h}_{ij}\zeta_{,i}\psi_{,j} + \frac{c_4}{a^2}\partial^2 h_{ij}\zeta_{,i}\psi_{,j} + \frac{c_5}{a^4}\partial^2 h_{ij}\zeta_{,i}\zeta_{,j} + c_6\partial^2 h_{ij}\psi_{,i}\psi_{,j} + \frac{c_7}{a^2}\dot{h}_{ij}\dot{\zeta}_{,i}\zeta_{,j} + c_8\dot{h}_{ij}\dot{\zeta}_{,i}\psi_{,j}\right],$$

where $\partial^2 h_{ij} \equiv h_{ij,kk}$, $\psi \equiv \partial^{-2}\dot{\zeta}$ and the coefficients read

$$c_1 = \mathcal{F}_S, \qquad c_5 = \frac{\mathcal{G}_T}{4\tilde{\Theta}^2}(\mathcal{G}_T + \mathcal{D} + \mathcal{F}_T\Delta_1)^2,$$

$$c_2 = -\frac{\mathcal{G}_T}{2\tilde{\Theta}}(\mathcal{G}_T + \mathcal{D} + \mathcal{F}_T\Delta_1), \qquad c_6 = \frac{\mathcal{G}_S^2}{4\mathcal{G}_T},$$

$$c_3 = \frac{3}{2}\mathcal{G}_S, \qquad c_7 = \frac{\mathcal{G}_T(\Gamma - 3\mathcal{G}_T\Delta_1)}{2\tilde{\Theta}^2}(\mathcal{G}_T + \mathcal{D} + \mathcal{F}_T\Delta_1),$$

$$c_4 = -\frac{\mathcal{G}_S}{2\tilde{\Theta}}(\mathcal{G}_T + \mathcal{D} + \mathcal{F}_T\Delta_1), \qquad c_8 = -\frac{\mathcal{G}_S(\Gamma - 3\mathcal{G}_T\Delta_1)}{2\tilde{\Theta}}.$$

No decay constraint solved in SVT trivial gravity in ST

Disformal SVT theories

- The speed of the vector modes about the cosmological background acquires new terms after the C/D transformation

$$c_A^2 = \frac{F_A}{G_A} \tag{18}$$

$$F_A = g_0 + 4\,H^2\,w_3 + 4\,\dot{H}\,w_3 - 2\,\ddot{\pi}\,(w_1 + 2\,X^2\,w_0) + H\,(4\,\dot{\pi}\,\ddot{\pi}\,w_{3,X} - 2\,\dot{\pi}\,(w_1 - 2\,w_2\,X)) \tag{19}$$

$$G_A = g_0 + 4\,g_2\,X - 4\,H\,\dot{\pi}\,w_1 + 4\,H^2\,(w_3 + 2\,X\,(w_{3,X} + X\,(w_4 + 2\,w_5\,X))) \tag{20}$$

- Recall that the light cones are modified by the Disformal transformation.

– Nevertheless $c_g^2 = c_A^2$ remains impossible if $G_{5,X} \neq 0$.

\+ Allows healthy coupling to beyond Horndeski theory

Conclusion and outlook

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