Revisiting PBH Accretion, Evaporation and Their Cosmological Consequences

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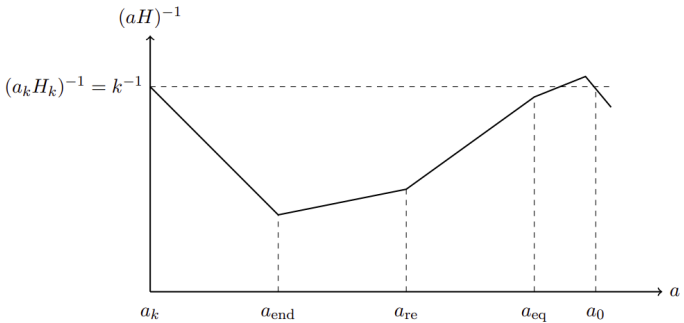
Collaborator: Prof. Debaprasad Maity

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Hangzhou, China | November 8

Introduction and Motivation: Cosmic Epochs

❖ After the Big Bang, the Universe evolves through distinct cosmic epochs:



Horizon Evolution:

- $a_k \rightarrow a_{\mathbf{end}}$
  Cosmic Inflation
- $a_{\mathbf{end}} \rightarrow a_{\mathbf{re}}$
  Reheating Epoch
- $a_{\mathbf{re}} \rightarrow a_{\mathbf{eq}}$
  Radiation Domination

Introduction and Motivation: Primordial Black Holes

Definition and Formation Mechanism

- Primordial Black Holes (PBHs) are black holes formed in the early Universe, prior to any stellar collapse.
- At the time of horizon entry, if the density perturbation exceeds a critical threshold (δ_c)

$$\frac{\rho - \bar{\rho}}{\bar{\rho}} = \delta \gtrsim \delta_c \simeq 0.4$$

- These overdense regions directly collapse to form PBHs.

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Initial PBH Mass

$$M_{\mathrm{BH}}^{\mathrm{in}} = \gamma\, M_H \sim 10^{15}\mathrm{g}\left(\frac{t}{10^{-23}\mathrm{s}}\right)$$

- Collapse fraction: $\gamma \approx \omega^{3/2} = 1/3^{3/2} \approx 0.2$ (in the radiation-dominated era)[1].

[1] Carr, B. J. et al., *Primordial black holes as dark matter candidates*, 2022.

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Mass Scales vs. Epoch

Initial Mass	Time (t)	Temp (T)
10^8g	10^{-30}s	10^{12} GeV
$1M_\odot$	10^{-5}s	100 MeV
$10^5 M_\odot$	1s	1 MeV

Introduction and Motivation: Relativistic accretion and evaporation

Accretion and Hawking Evaporation

- PBHs in the early Universe are surrounded by hot thermal plasma and accrete in a relativistic manner.

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- PBHs in the early Universe are surrounded by hot thermal plasma and accrete in a relativistic manner.
- Simultaneously, PBHs evaporate via Hawking radiation

$$n_\omega = \frac{\Gamma_\omega}{e^{2\pi\omega/\kappa} \pm 1}$$

- The evaporation time scales with mass as $t_{\rm ev} \propto M_{\rm BH}^3$.

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$$\text{PBH Accretion} \xrightarrow{\text{changes}} M_{\rm BH} \xrightarrow{\text{changes}} \text{PBH Lifetime}$$

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Cosmological Consequences

- Big Bang Nucleosynthesis (BBN) bounds
- Shifts in Dark Matter parameter space
- Gravitational Wave (GW) production
- PBH dark matter fraction ($f_{\rm PBH}$)

General Relativistic Accretion by PBHs

- We want to obtain the Navier-Stokes equation in general axisymmetric spacetime

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu = g_{tt}dt^2 + 2g_{t\phi}dt\,d\phi + g_{rr}dr^2 + g_{\theta\theta}d\theta^2 + g_{\phi\phi}d\phi^2$$

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Conservation Laws

$$\nabla_\nu T^{\mu\nu} = 0$$
$$\nabla_\mu j^\mu = 0$$

$$T^{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^{\mu\nu}$$
$$j^\mu = \tilde{\rho}u^\mu$$

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Navier-Stokes Projection

Define a projection operator $h^i_\mu = \delta^i_\mu + u^i u_\mu$, with $i = 1, 2, 3$ and $h^i_\mu u^\mu = 0$.

- $h^i_\mu(\nabla_\nu T^{\mu\nu}) = 0 \longrightarrow \gamma^2 v\partial_r v + \frac{1}{h\tilde{\rho}}(\partial_r p) + \partial_r \Phi^{\text{eff}} = 0$
- $u_\mu(\nabla_\nu T^{\mu\nu}) = 0 \longrightarrow h\frac{d\tilde{\rho}}{dr} - \frac{d\rho}{dr} = 0$

$$\Phi^{\text{eff}} = \frac{1}{2}\ln\left(\frac{g_{t\phi}^2}{g_{\phi\phi}} - g_{tt}\right), \quad \gamma = \frac{1}{\sqrt{1-v^2}}, \quad h = \frac{\rho + p}{\tilde{\rho}}$$

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$$\nabla_\mu j^\mu = 0 \longrightarrow \dot{M} = -\oint \sqrt{-g}\, j^r\, d\theta d\phi = 2\pi \int_0^\pi \tilde{\rho}\,\gamma v \sqrt{g^{rr}}\sqrt{-g}\, d\theta$$

General Relativistic Accretion in Schwarzschild Background

Schwarzschild Metric

$$ds^2 = -f(r)dt^2 + f(r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

- where the metric function is defined as $f(r) = 1 - 2GM/r$.

Fluid Equations

- Effective potential

$$\Phi^{\text{eff}} = \frac{1}{2}\ln f(r)$$

- Accretion rate

$$\frac{dM}{dt} = 4\pi r^2 \tilde{\rho}\gamma v\sqrt{f(r)}$$

- Radial momentum equation transforms to

$$v\gamma^2\frac{dv}{dr} + \frac{1}{h\tilde{\rho}}\frac{dp}{dr} + \frac{1}{2}\frac{f'(r)}{f(r)} = 0$$

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Equation of State: $p = \omega\rho$ ($\omega = 1/3$ in a radiation-dominated Universe)

Accretion in the Expanding Background

Background Expansion Kinematics

- Physical distance: $r = ax$
- Velocity: $v = \frac{dr}{dt} = Hax + v_p$

where v_p $(= a\dot{x})$ is defined as the peculiar velocity.

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Integrated Momentum Equation

$$\ln\left(1 - (Hax + v_p)^2\right) + \frac{2\omega}{(1+\omega)} \ln\left(\frac{\rho_\infty}{\rho}\right) - \ln f(ax) = 0$$

Boundary Condition: Far away from the BH ($x \to \infty$), $v \to 0$, and $\rho \to \rho_\infty \propto a^{-3(1+\omega)}$, which corresponds to the background energy density of the Universe.

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Modified Accretion Rate

$$\frac{dM}{dt} = 4\pi(ax)^2 \rho_\infty^{\frac{\omega}{1+\omega}} \rho^{\frac{1}{1+\omega}} \frac{(Hax + v_p)}{\sqrt{1 - (Hax + v_p)^2}} \sqrt{f(ax)}$$

Critical Accretion Rate

Flow Dynamics & Critical Point

- Conserved mass accretion rate $d\dot{M}/dx = 0$.
- Radial variation of comoving fluid velocity

$$\frac{dv_p}{dx} = \frac{\frac{2\omega}{(ax)} + \frac{\omega f'(ax)}{2f(ax)} - \frac{f'(ax)}{2f(ax)} - H\frac{v^2-\omega}{v(1-v^2)}}{\frac{v^2-\omega}{v(1-v^2)}\frac{1}{a}} \equiv \frac{N}{D}$$

- The inflowing matter passes through a critical point where $N = D = 0$
  - $x_c = GM(1+3\omega)/(2a\omega)$
  - $v_p^c = \sqrt{\omega} - Hax_c$

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Rate at Critical Point

$$\frac{dM}{dt} = 4\pi\,\lambda_c(\omega)\,G^2\,M^2\rho_\infty$$

- $\lambda_c(\omega) = (1/4)\omega^{-3/2}(1+3\omega)^{(1+3\omega)/2\omega}$

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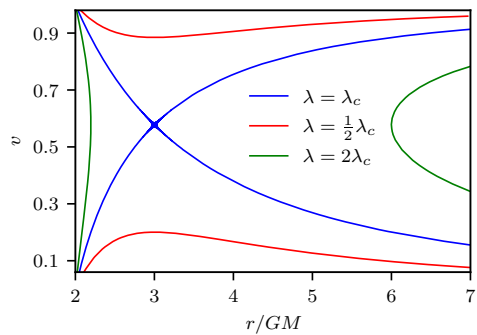
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Accretion Rate in Kerr Spacetime

Kerr Metric Components

$$g_{tt} = -\left(1 - \frac{r_g r}{\Sigma}\right), \quad g_{rr} = \frac{\Sigma}{\Delta}, \quad g_{\theta\theta} = \Sigma,$$
$$g_{\phi\phi} = \frac{A \sin^2\theta}{\Sigma}, \quad g_{t\phi} = -\frac{a_k r_g r \sin^2\theta}{\Sigma}$$

Accretion Rate

$$\frac{dM}{dt} = 4\pi\lambda_{\text{Kerr}}(a_*, \omega) G^2 M^2 \rho_\infty$$

» $a_* = J/GM^2$ *is the dimensionless Kerr parameter.*

Accretion Rate in Kerr Spacetime

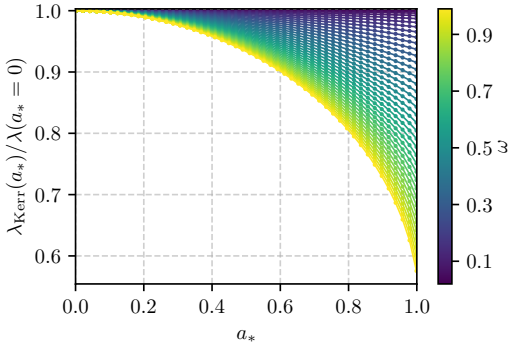
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Efficiency Factor (λ_{Kerr})

$$\lambda_{\mathrm{Kerr}}(a_*, \omega) = \frac{1}{2}(1-\omega)^{\frac{1-\omega}{2\omega}} \sqrt{\omega} \int_0^\pi [\mathcal{F}(y_c, \theta, a_*)]^{-\frac{1}{2\omega}} \left\{y_c^2 - 2y_c + a_*^2\right\}^{\frac{1}{2}} \left\{y_c^2 + a_*^2 \cos^2\theta\right\}^{\frac{1}{2}} \sin\theta \, d\theta$$

Accretion Rate in Kerr Spacetime: Velocity profile

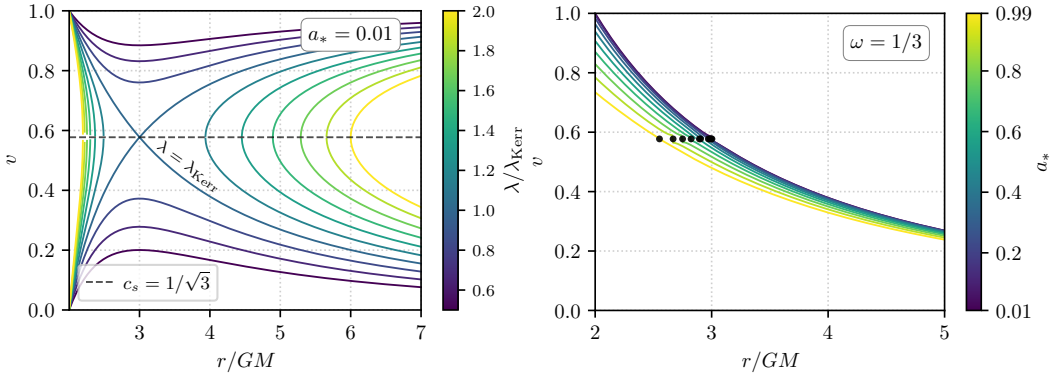


Figure: The radial velocity $v(r)$ as a function of dimensionless radius r/GM.

Accretion in a Cosmological Background

Background Evolution

- Energy density: $\rho_\infty(t) = 3M_p^2 H(t)^2$
- Hubble parameter: $H(t) = \frac{2}{3(1+\omega)t}$

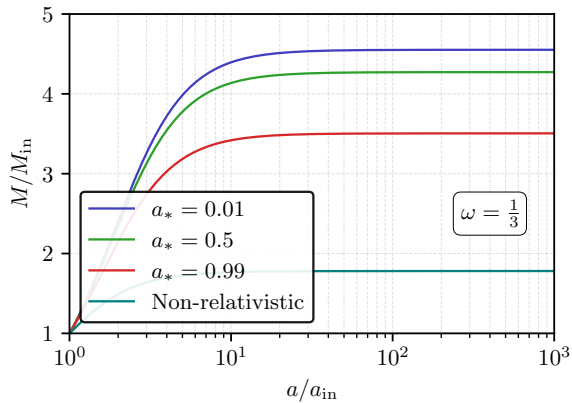
Integrated Mass Evolution

$$\frac{M(t)}{M_{\rm in}} = \left[1 - \frac{\lambda_{\rm Kerr}(a_*)\gamma}{2(1+\omega)}\left(\frac{\frac{3}{2}H_{\rm in}(1+\omega)(t-t_{\rm in})}{\frac{3}{2}H_{\rm in}(1+\omega)(t-t_{\rm in})+1}\right)\right]^{-1}$$

Non-Relativistic Limit

- NR accretion efficiency[2]

$$\lambda_c^{\rm NR} = (1/4)\exp(3/2)\omega^{-3/2}$$



[2] Y. Ali-Haïmoud and M. Kamionkowski, "Cosmic microwave background limits on accreting primordial black holes," 2017.

PBH Mass and Spin Evolution

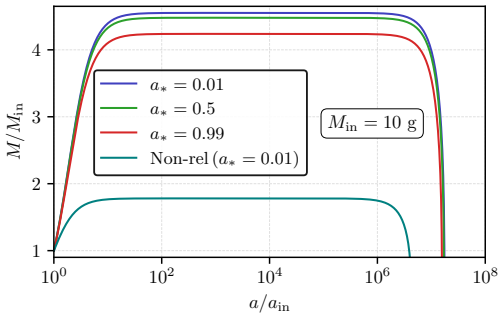
Total Mass Evolution

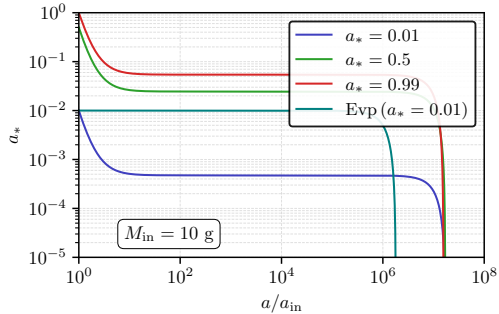
$$\frac{dM}{dt} = \left(\frac{dM}{dt}\right)_{\text{Acc}} + \left(\frac{dM}{dt}\right)_{\text{Evap}}$$

$$= 4\pi\lambda_{\text{Kerr}}(a_*)G^2M^2\rho_\infty(t) - \epsilon(M, a_*)\frac{M_p^4}{M^2}$$

Spin Evolution

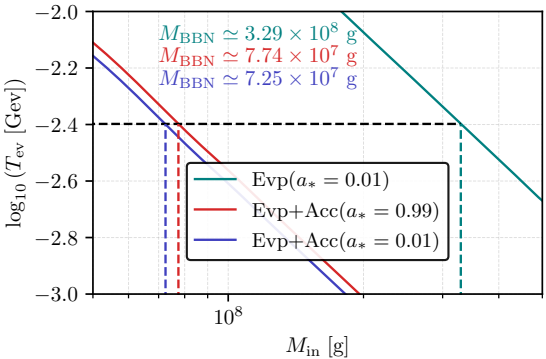
$$\frac{da_*}{dt} = -\frac{2a_*}{M}\left(\frac{dM}{dt}\right) - a_*\left(8\pi\gamma(M, a_*)\right)\frac{M_p^4}{M^3}$$





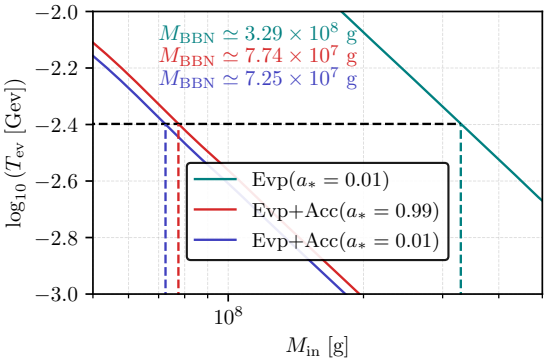
BBN Constraint

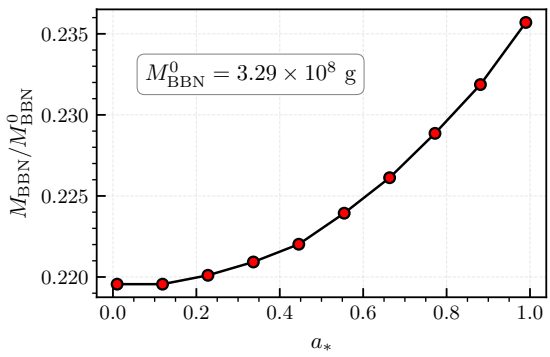
- PBH must evaporate before the onset of BBN (at ≈ 4Mev background temperature). ⇒ BBN constraint



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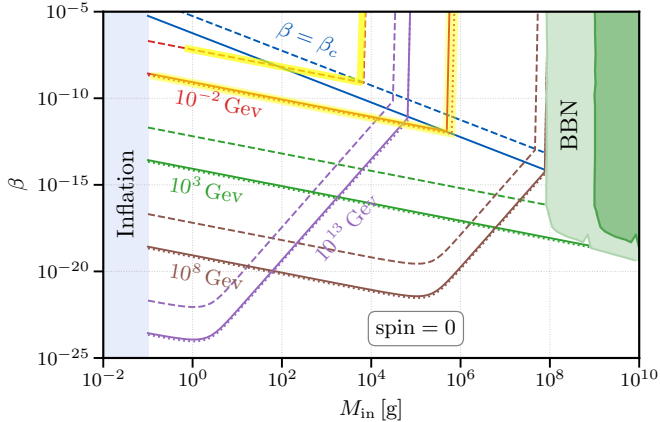


Dark Matter from Evaporating PBHs

- Let PBH is evaporating SM radiations and DM of mass m_{DM}.
- PBH abundance: $\beta(M) = \rho^{in}_{PBH}/\rho^{in}_{total}$
- Present DM abundance can be written as

$$\Omega_{DM,0}h^2 \simeq 1.724 \left(\frac{1\text{g}}{M^{in}_{BH}}\right)^{\frac{3}{2}} \left(\frac{m_{DM}}{1\text{Gev}}\right) \beta N_{DM}$$

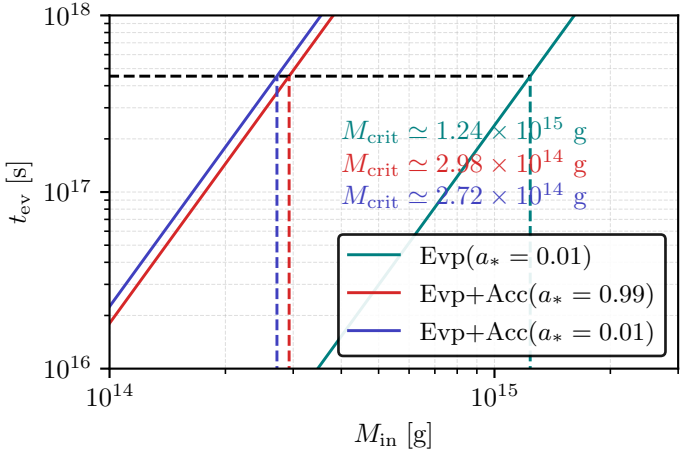
where, N_{DM} total DM produced by a PBH in its lifetime[3].



[3] Cheek, Andrew et al., "Primordial black hole evaporation and dark matter production", 2022.

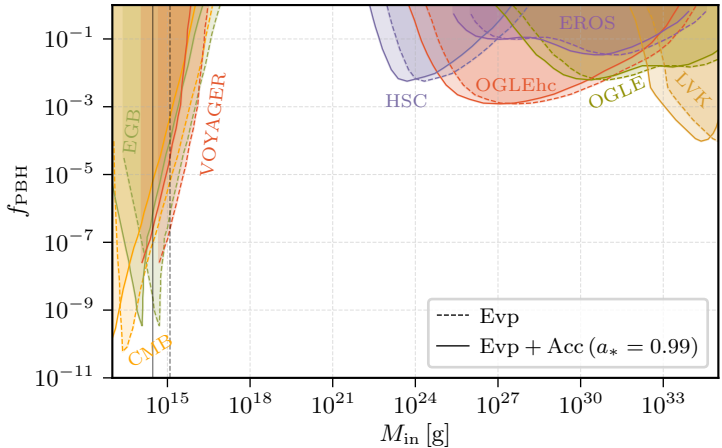
Constraints on PBHs as Dark Matter: New Evaporation Threshold

- **Standard scenario:** A PBH must have an initial mass $M_{\rm in} \gtrsim 10^{15}$ g to survive for the entire age of the universe.



Constraints on PBHs as Dark Matter: Observational Constraints

- Fraction of PBHs as DM, $f_{\rm PBH}(M) = \Omega_{\rm PBH}/\Omega_{\rm DM}$.



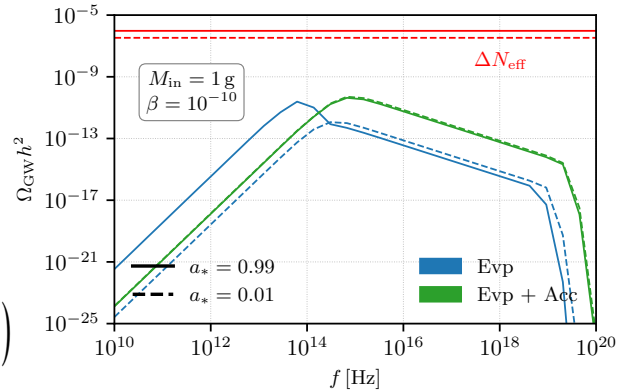
Present-Day GW Abundance

- The instantaneous emission rate of gravitons (spin $s = 2$) from a Kerr BH, per unit frequency interval, is

$$\frac{d^2 N_{\rm grav}}{dt d\omega} = \sum_{l,m} \frac{\Gamma^{lm}_{(s=2)}(\omega)}{e^{(\omega - m\Omega)/T_{\rm BH}} - 1} \equiv Q_{\rm GW}(t, \omega)$$

- The present-day GW energy density spectrum

$$\Omega_{\rm GW}(f_0) = \frac{n^{\rm in}_{\rm BH}}{\rho_{\rm crit,0}} \frac{\omega_0^2}{2\pi} \frac{a_{\rm in}^3}{a_0^2} \int_{t_{\rm in}}^{t_{\rm ev}} \frac{dt}{a(t)} Q_{\rm GW}\left(t, \frac{\omega_0 a_0}{a(t)}\right)$$



Summary and Conclusions

- PBHs in the early Universe undergo simultaneous relativistic accretion and Hawking evaporation, which fundamentally alters their mass and spin evolution.
- This modified evolution shifts key cosmological boundaries, notably altering BBN constraints and the critical mass threshold required for PBHs to survive to the present day.
- Accretion effects significantly modify the DM parameter space, also impacting the allowable fraction of PBHs as DM ($f_{\rm PBH}$).
- Accretion-induced spin-down makes Kerr PBHs evaporate as Schwarzschild black holes, yielding a smooth, single-peaked GW spectrum without the high-frequency bump.

Thank You!

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Questions & Discussion

Backup Slides

Gravitational Wave Detector Sensitivities

Ground-Based (Audio Band: 1 Hz – 10 kHz)

- **Present:** LIGO, Virgo, KAGRA.
- **Next-Gen:** Einstein Telescope (ET), Cosmic Explorer (CE).

❑ Can strictly probe the extreme low-frequency tail of the PBH evaporation spectrum.

Space-Based & PTAs (nHz – dHz)

- **mHz to dHz:** LISA, DECIGO, BBO.
- **nHz:** Pulsar Timing Arrays (NANOGrav, SKA).

❑ Ideal for massive PBH inspirals, but completely blind to the high-frequency evaporation peak.

Ultra-High-Frequency (MHz – GHz)

The PBH evaporation peak requires novel, non-interferometric UHF-GW detection proposals:

- **Microwave Cavities:** Detect GWs via photon conversion in strong magnetic fields.
- **Bulk Acoustic Wave (BAW):** Detecting MHz-frequency GWs with high quality factor resonators.
- **Magnons:** Detecting GHz-frequency GWs via spin excitations.
- **Levitated Optomechanics:** Trapped nanoparticles responding to very high-frequency spacetime metric perturbations.