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A unified framework to study f-family of modified gravity theories

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Cosmographic reconstruction and the past limit of general relativity

A common framework for two phenomenological questions

I. Cosmographic reconstruction

What local measures of departure from GR can be extracted from $(q_0, j_0, s_0, \ldots)$ alone?

cosmography $\longrightarrow$ local shape of f.

II. Theory-space evolution

For a prescribed cosmic history, do nearby theories approach GR in the past?

$j = j(q) \longrightarrow$ flow of $(r, m, \ldots)$.

Comparison today: metric $f(R)$ and the trivial spatially flat FLRW branch of $f(Q)$.

$$R = 6H^2(1-q), \qquad Q = -6H^2, \qquad R = Q + B.$$

Linear GR actions differ by a boundary term; nonlinear extensions generally do not.

Two recursive hierarchies

Expansion kinematics

$$c_n = \frac{a^{(n+2)}}{aH^{n+2}}, \qquad (c_0, c_1, c_2) = (-q, j, s).$$

$$\frac{a(t)}{a_0} = 1 + H_0\Delta t - \frac{q_0}{2!}(H_0\Delta t)^2 + \frac{j_0}{3!}(H_0\Delta t)^3 + \cdots .$$

$$c_i' = \big[(i+2)(1+q) - 1\big]c_i + c_{i+1}.$$

$$q' = 2q^2 + q - j, \quad j' = s + (2+3q)j,$$

Theory shape

$$m_i = \frac{Xf^{(i+1)}}{f^{(i)}}, \qquad m_0 = r, \quad m_1 = m.$$

$$\frac{f(X)}{f(X_0)} = 1 + r_{(0)}\xi + \frac{r_{(0)}m_{(0)}}{2!}\xi^2 + \cdots ,$$

$\xi = X/X_0 - 1$; $r_{(0)}, m_{(0)}$ are evaluated at X_0.

$$m_i' = m_i(1 - m_i + m_{i+1})\frac{X'}{X}.$$

$$\frac{R'}{R} = \frac{j - q - 2}{1 - q}, \quad \frac{Q'}{Q} = -2(1+q).$$

The field equations couple cosmography to theory shape.

Prime is $\mathrm{d}/\mathrm{d}N$, $N = \ln a$. The theory-series coefficients are evaluated at X_0.

The background equations in a common notation

Flat FLRW, conserved dust: $\dot{\rho}_m + 3H\rho_m = 0$, $\Omega_m = \kappa\rho_m/(3H^2)$.

Metric $f(R)$, $F = f_R$

$$3FH^2 = \kappa\rho_m + \tfrac{1}{2}(RF - f) - 3H\dot{F},$$
$$-2F\dot{H} = \kappa\rho_m + \ddot{F} - H\dot{F}.$$

Fourth order in the metric.

$$\boxed{\frac{\Omega_m}{F} = q + \frac{1-q}{r} + m\frac{j-q-2}{1-q}}.$$

Trivial-branch $f(Q)$, $F = f_Q$

$$3FH^2 = \kappa\rho_m + \tfrac{1}{2}(QF - f),$$
$$-2(F + 2Qf_{QQ})\dot{H} = \kappa\rho_m.$$

Second order in $a(t)$ on this branch.

$$\boxed{\frac{\Omega_m}{F} = 2 - \frac{1}{r} = \frac{2}{3}(1+2m)(1+q)}.$$

Spatially flat symmetric teleparallel FLRW admits three connection branches. The results here refer to the trivial branch.

Standard reconstruction retains normalization inputs

The usual local reconstruction is

$$\frac{f(X)}{f(X_0)} = 1 + r(X_0)\xi + \frac{r(X_0)m(X_0)}{2}\xi^2 + \cdots .$$

For $f(Q)$,

$$r(X_0) = \frac{1}{2 - \Omega_{m0}/F_0}, \qquad m(X_0) = \frac{1}{2}\left[\frac{3\Omega_{m0}}{2F_0(1+q_0)} - 1\right].$$

For $f(R)$,

$$r(X_0) = \frac{6H_0^2(1-q_0)F_0}{f_0}, \qquad m(X_0) = \frac{1-q_0}{j_0 - q_0 - 2}\left[\frac{\Omega_{m0}}{F_0} - q_0 - \frac{1-q_0}{r(X_0)}\right].$$

Cosmography alone does not specify a unique normalized $f(X)$.

Theory space and the GR reference point

$$\boxed{r = \frac{Xf_X}{f}, \qquad m = \frac{Xf_{XX}}{f_X}}.$$

A specified $f(X)$ traces a curve $(r(X), m(X))$. Locally, a curve $m = m(r)$ determines a family of functions through

$$\frac{\mathrm{d}\ln|X|}{\mathrm{d}r} = \frac{1}{r(1-r+m)}, \qquad \frac{\mathrm{d}\ln|f|}{\mathrm{d}r} = \frac{1}{1-r+m}.$$

GR, $f = X - 2\Lambda$, lies on $m = 0$. Its reference location on a dust–Λ background is

$$r^{Q}_{\mathrm{GR}}(q) = \frac{3}{2(2-q)}, \qquad r^{R}_{\mathrm{GR}}(q) = \frac{3(q-1)}{q-2}.$$

Reconstruct local theory shape relative to a specified GR anchor.

Local reconstruction around the GR anchor

At a chosen epoch define $\Delta r = r - r_{\rm GR}(q_0)$ and impose $m(r_{\rm GR}) = 0$:

$$m(r) = a_1 \Delta r + \frac{a_2}{2!}\Delta r^2 + \frac{a_3}{3!}\Delta r^3 + \cdots .$$

The coefficients follow by differentiation along the cosmological solution,

$$\mathcal{D}_r := \frac{d}{dr} = \frac{1}{r'}\frac{d}{dN}, \qquad \boxed{a_n = \mathcal{D}_r^{\,n} m\big|_{r=r_{\rm GR},\, m=0}}.$$

$$\frac{\mathrm{d}}{\mathrm{d}N} = r'\partial_r + m'\partial_m + (2q^2 + q - j)\partial_q + [s + (2+3q)j]\partial_j + \cdots .$$

Differentiate first; evaluate at the GR point afterwards.

The resulting coefficients, when evaluated at the GR point, depend only on cosmography, with no explicit Ω_m or F.

The leading response to cosmographic departure from GR

$f(Q)$: a linear response

$$\boxed{a_1^Q = \frac{(j-1)(q-2)^2}{3(q+1)^2(2q-1)}}.$$

$$m_Q(r) = a_1^Q \Delta r + \tfrac{1}{2} a_2^Q \Delta r^2 + \cdots .$$

(q, j) fix the slope; adding s fixes the quadratic term.

$f(R)$: a quadratic response

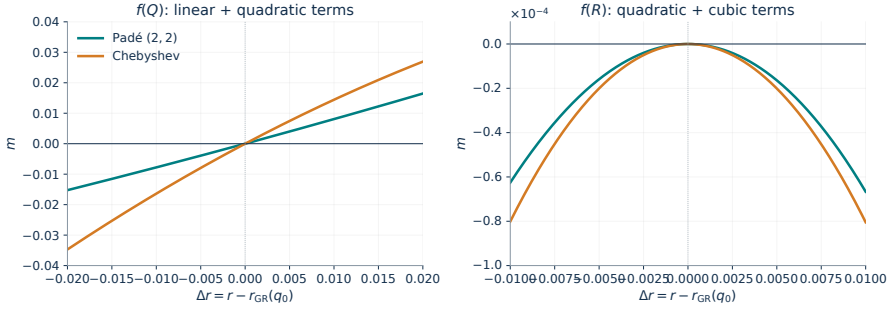
$$a_1^R = 0, \qquad \boxed{a_2^R = \frac{2(j-1)(q-2)^4(q-1)}{9(1-2q)^2(j-q-2)^3}}.$$

$$m_R(r) = \tfrac{1}{2} a_2^R \Delta r^2 + \tfrac{1}{6} a_3^R \Delta r^3 + \cdots .$$

(q, j) fix the curvature; adding s fixes the cubic term.

$j - 1$ controls the first nonzero local coefficient in both frameworks. Within the framework where there is no additional dark energy component in the matter sector, deviation of j_0 from 1 is a definitive sign of modified gravity.

Reconstruction using DESI DR2 cosmographic central values



Cosmography	q_0	j_0	s_0
Padé $(2,2)$	-0.397	0.73	0.55
Chebyshev	-0.472	0.59	-0.48

Recomputed local curves using the published central values in Mishra et al., MNRAS 546, stag197 (2026), doi:10.1093/mnras/stag197, Table 1. TAxes have different scales. Curves show Taylor truncations, not confidence bands.

Physical interpretation of the reconstructed curves

For the displayed central values, $a_2^R < 0$. Near the anchor, $m_R < 0$ for nonzero Δr.

$$R > 0, \quad f_R > 0, \quad m < 0 \quad \Longrightarrow \quad f_{RR} < 0.$$

This fails the usual $f_{RR} \geq 0$ stability condition of the viable near-GR metric sector.

Statistical scope: the published marginal jerk intervals are

$$j_0^{\mathrm{Pade}} = 0.73^{+0.28}_{-0.44}, \qquad j_0^{\mathrm{Cheb}} = 0.59^{+0.16}_{-0.24} \qquad (68\%).$$

The inference depends on cosmographic parametrization, the GR anchor and joint uncertainties.

$$G_* := \frac{G_N}{f_X}, \qquad \frac{\mathrm{d} \ln G_*}{\mathrm{d}N} = -m\frac{X'}{X}.$$

Local shape diagnostics motivate viability tests and perturbation predictions.

A prescribed history with a matter-like past

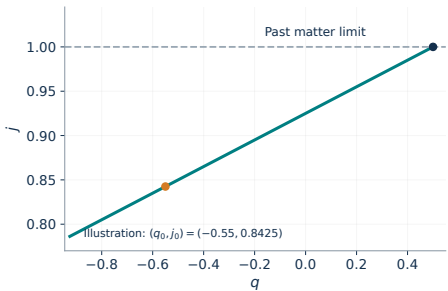
$$\boxed{j(q) = 1 + 3\epsilon\left(q - \frac{1}{2}\right)}, \quad q_* = -1 + \frac{3}{2}\epsilon.$$

$$q' = 2\left(q - \frac{1}{2}\right)(q - q_*).$$

$$H^2 = C_m a^{-3} + C_e a^{-3\epsilon}, \quad C_m, C_e > 0.$$

For $\epsilon < 2/3$, the branch $q_* < q < 1/2$ kinematically approaches ΛCDM $(q, j) = (1/2, 1)$ in the past, while having an accelerated future attractor.

$$\epsilon = \frac{j_0 - 1}{3(q_0 - 1/2)}.$$



Use $\epsilon = 0.05$ for the flow figures. This is an illustrative choice, not a fit to the preceding data.

Does kinematic past asymptoticity to ΛCDM also imply past asymptoticity to GR?

Closed reconstruction systems for the prescribed $j(q)$

Set $\mathcal{K} = 2q^2 + q - j(q)$ and $\mathcal{A} = [j(q) - q - 2]/(1 - q)$.

$f(R)$: (r, m, q)

$$
\begin{aligned}
q' &= \mathcal{K}, \\
r' &= r(1 - r + m)\mathcal{A}, \\
m' &= \frac{2 - q - 3(1 - q)/r}{\mathcal{A}} + (q - 1)m \\
&\quad - m^2\mathcal{A} - m\frac{\mathcal{A}_{,q}}{\mathcal{A}}\mathcal{K}.
\end{aligned}
$$

$$
\mathcal{C}_R := \frac{\Omega_m}{f_R} = q + \frac{1 - q}{r} + m\mathcal{A}.
$$

$f(Q)$: (m, q)

$$
\begin{aligned}
q' &= \mathcal{K}, \\
m' &= \frac{(1 + 2m)[j - 1 + 2m(1 + q)^2]}{2(1 + q)}.
\end{aligned}
$$

$$
r = \frac{3}{2[2(1 - m) - q(1 + 2m)]},
$$

$$
\mathcal{C}_Q := \frac{\Omega_m}{f_Q} = \frac{2}{3}(1 + 2m)(1 + q).
$$

The GR matter point has different past stability

Framework	GR matter point	Eigenvalues in $N = \ln a$
$f(R)$	$(r, m, q) = (1, 0, \frac{1}{2})$	$\frac{5+\sqrt{73}}{4}, \quad \frac{5-\sqrt{73}}{4}, \quad 3(1-\epsilon)$
$f(Q)$	$(m, q) = (0, \frac{1}{2})$	$\frac{3}{2}, \quad 3(1-\epsilon)$

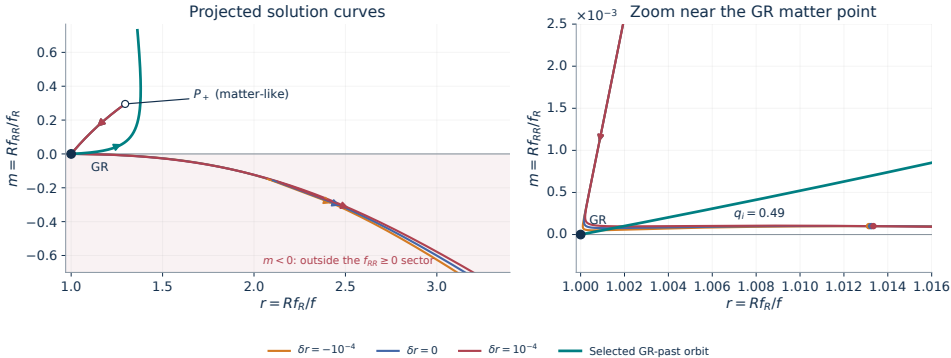
$f(R)$: a saddle for every $\epsilon < 1$

One stable and two unstable directions in forward time. A GR past requires initial data on the unstable manifold.

$f(Q)$: a local past attractor for $\epsilon < 1$

Both eigenvalues are positive, so nearby reduced-system trajectories contract toward GR as $N \to -\infty$.

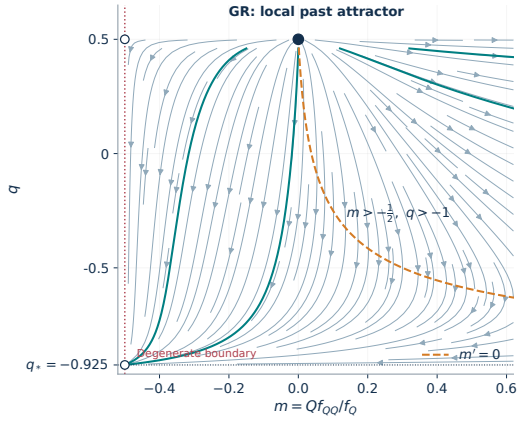
$f(R)$: proximity to GR does not imply a GR past



$\epsilon = 0.05$: generic nearby seeds turn away from GR toward the past. The selected orbit is seeded close to the forward unstable manifold.

Arrows: increasing N. Projection of the 3D (r, m, q) flow. Generic seeds: $q_i = 0.49$, $m_i = 10^{-4}$, $r_i = 3(q_i - 1)/(q_i - 2) + \delta r$, $\delta r = \{-10^{-4}, 0, 10^{-4}\}$. Forward endpoint $q = -0.55$. Green curve is a numerically converged asymptotic seed, not an exact manifold construction.

$f(Q)$: the GR point attracts nearby histories in the past



$$\epsilon = 0.05, \quad q_* = -0.925.$$

Near GR

$$\lambda_1 = 1.5, \qquad \lambda_2 = 2.85.$$

Both directions contract toward the past.

$$\mathcal{C}_Q = \tfrac{2}{3}(1+2m)(1+q) > 0.$$

For $q > -1$, positive dust and $f_Q > 0$ require $m > -1/2$. $m = -1/2$ is a degenerate boundary.

Arrows: increasing N. Dashed curve: nontrivial $m' = 0$ nullcline. The remote saddle $(m, q) = (19, -0.925)$ is outside the displayed window. Local past attraction does not establish global future viability or stability of inhomogeneous perturbations.

Summary: Local reconstruction and past selection

	Local reconstruction	Prescribed-history evolution
Input	$(q_0, j_0, s_0, \ldots)$ and a GR-reference point	$j(q) = 1 + 3\epsilon(q - 1/2)$
$f(R)$	Leading quadratic response in Δr	GR matter point is a saddle
$f(Q)$	Leading linear response in Δr	GR matter point is a local past attractor

Outlook: from background histories to perturbative signatures

1. Embed a prescribed $j(q)$, $w' = g(w)$ or $w = w(\Omega_m)$ in different frameworks; impose physical and normalization constraints.

2. Propagate cosmographic posteriors into local theory-shape diagnostics and present-day observable planes.

3. Append perturbation variables and test whether background-degenerate theories have distinguishable growth.

4. Extend the hierarchy to additional connection branches and scalar-field models, including their extra degrees of freedom.

$$f_g := \frac{\mathrm{d}\ln\delta_m}{\mathrm{d}N}, \qquad f_g' + f_g^2 + (1-q)f_g = \frac{3}{2}\mu(k,N)\Omega_m.$$

In viable-model subhorizon quasi-static metric $f(R)$, $\mu \simeq f_R^{-1}(1+4\chi)/(1+3\chi)$, $\chi = (k/a)^2 f_{RR}/f_R$; the usual trivial-branch $f(Q)$ dust-growth reduction gives $\mu \simeq f_Q^{-1}$. De Felice & Tsujikawa (2010); Beltrán Jiménez et al. (2020). Full perturbative consistency remains essential.

Thank you

I would be happy to discuss collaborative projects in this direction.

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Theory-space reconstruction Cosmographic closures Perturbative signatures