

The Fifth International Conference on Axion Physics and Experiment (Axion 2026)

A Study on the $K \rightarrow \pi a$ decay within the large- N_c chiral perturbation theory

Based on work to appear on arXiv

Speaker: Jin-bao Wang (王金葆)

Co-authors: Zhi-hui Guo (郭志辉) Hai-qing Zhou (周海清)



河北师范大学
HEBEI NORMAL UNIVERSITY

物理学院
College of Physics

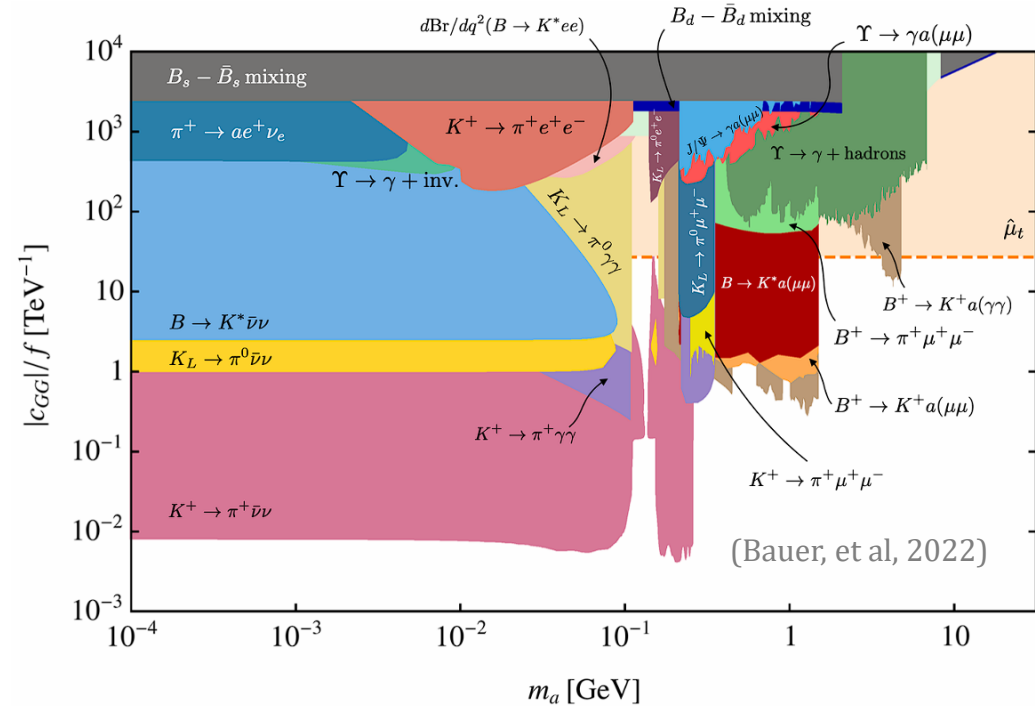
□ Axion-like particles (ALPs)

- Motivated by the QCD axion and many BSM scenarios.
- Pseudo-Nambu-Goldstone boson (pNGB) from spontaneous $U(1)_{PQ}$ breaking at $\Lambda \equiv 4\pi f \gg v_{EW}$.
- Effective interactions: dim-5 (or higher) operators suppressed by Λ^{-1} .
- Precision probes: intensity-frontier experiments (BESIII, Belle II, JLab, STCF, ...).
- Flavor physics: rare meson decays strongly constrain ALPs with $m_a \sim \text{MeV--GeV}$.

□ $K^+ \rightarrow \pi^+ a$ decay provides most stringent particle-physics constraint.

- SM: $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is theoretically very clean.
- Experiment: Precisely measured by NA62.
- Long-lived ALPs: $K^+ \rightarrow \pi^+ a$ has an identical missing-energy signature,

$$f \gtrsim 5 \times 10^8 \text{ TeV assuming } C_{ds}^v = O(1).$$



□ Status of $K \rightarrow \pi a$ calculations

- Pioneering EFT calculation (Georgi, et al, 1986)
 - ✓ First analysis of $K \rightarrow \pi a$ in a general EFT framework.
 - ✗ Incomplete treatment of ALP terms in the weak effective Lagrangian.
 - ✗ Resulting amplitudes depended on the chiral rotation scheme.
- Resolution (Bauer, et al, 2021)
 - ✓ Implemented the correct flavor-changing quark currents in SU(3) χ PT.
 - ✓ Obtained scheme-independent decay amplitudes.
 - ✓ Predicted $\mathcal{B}(K^+ \rightarrow \pi^+ a)$ nearly 40 times larger than previous estimates.
- NLO calculation (Cornella, et al, 2024)
 - ✓ Complete NLO analysis in SU(3) χ PT.
 - ✓ Included novel ALP-induced operators.
 - ✓ NLO corrections can be sizable; uncertainties are dominated by the relevant LECs.

□ Our motivation

- Previous studies
 - ✓ Based on SU(3) χ PT, where η_0 is integrated out.
- Why go beyond SU(3)?
 - ✓ η - η' mixing is numerically important for processes involving η .
 - ✓ The ALP belongs to the same singlet sector as the η_0 .
 - ✓ Large- N_c QCD matching requires the explicit inclusion of the η_0 .
- We study $K \rightarrow \pi a$ decay at LO in U(3) χ PT (large- N_c χ PT), including the explicit η_0 and the associated new weak operators.

□ Key question: How important are the explicit η_0 contributions to $K \rightarrow \pi a$?

□ Effective Lagrangian at quark level

□ ALP effective Lagrangian up to dim-5 at $\mu_0 \simeq 1.2$ GeV (kaon decay scale) (Georgi, et al, 1986)

$$\mathcal{L}_{\text{ALP}}^{D \leq 5}(\mu_0) = \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2 + \mathcal{L}_{\text{ALP-lepton}} + \frac{\partial_\mu a}{f} (\bar{q}_L k_Q \gamma^\mu q_L + \bar{q}_R k_q \gamma^\mu q_R) - c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a} - c_{\gamma\gamma} \frac{\alpha}{4\pi} \frac{a}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$k_Q = \begin{pmatrix} [k_U]_{11} & & \\ & [k_D]_{11} & [k_D]_{12} \\ & [k_D]_{21} & [k_D]_{22} \end{pmatrix}, \quad k_q = \begin{pmatrix} [k_u]_{11} & & \\ & [k_d]_{11} & [k_d]_{12} \\ & [k_d]_{21} & [k_d]_{22} \end{pmatrix}$$

$|\Delta S| = 1$ FCNC ALP-quark couplings induce $K \rightarrow \pi a$ directly

□ $|\Delta S| = 1$ transition can also be induced by the SM nonleptonic weak interaction

$$\mathcal{L}_{\text{eff}}^{|\Delta S|=1} = -\frac{G_F}{\sqrt{2}} V_{ud}^* V_{us} \sum_{i=1}^6 c_i(\mu_0) \hat{Q}_i + \text{h.c.}, \quad \text{ignoring higher-order electroweak corrections}$$

$$\hat{Q}_2 = 4 (\bar{d}_L \gamma^\mu u_L) (\bar{u}_L \gamma_\mu s_L) \quad \text{additional operators induced by non-factorizable gluon exchanges}$$

➤ We match quark and gluon d.o.f. onto mesonic fields using Chiral Perturbation Theory

□ QCD chiral effective Lagrangian in the U(3) framework

□ SU(3) framework

□ In the χ -limit ($m_{u,d,s} \rightarrow 0$), $\mathcal{L}_{\text{QCD}}^{n_f=3}$ exhibits a global chiral symmetry $U(1)_V \otimes SU(3)_L \otimes SU(3)_R$ in flavor space

□ $U(1)_A$ is broken by the QCD anomaly $\partial_\mu (\bar{q} \gamma^\mu \gamma_5 q) = 6\omega(x) = \frac{3\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a}$

□ U(3) framework

□ In the χ -limit and the **large- N_C limit** (QCD anomaly disappears), chiral symmetry group becomes $G_\chi = U(3)_L \otimes U(3)_R$

□ It breaks down to $H_\chi = U(3)_V$ by vacuum, giving rise to **9 massless Goldstone bosons**

$$U(x) = \exp\left(\frac{i\phi(x)}{F}\right), \quad \phi(x) = \sum_{i=0}^8 \lambda_i \phi_i(x) = \begin{pmatrix} \pi^3 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^3 + \frac{1}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}\bar{K}^0 & -\frac{2}{\sqrt{3}}\eta_8 + \sqrt{\frac{2}{3}}\eta_0 \end{pmatrix}$$

- Under a chiral transformation $(V_L, V_R) \in G_\chi$, $U(x)$ transforms as $U \rightarrow V_R U V_L^\dagger$

□ To reproduce the correct large- N_C behavior of QCD, the U(3) framework is more suitable:

- a) In the large- N_C limit, there are nine pNGBs, not eight.
- b) SU(3) χ PT can be obtained by integrating out the η_0 . However, $m_{\eta_0}^2 \sim m_\pi^2$ in the large- N_C limit.

- G_χ is explicitly broken by the light-quark masses, $U(1)_A$ anomaly, and non-QCD couplings

External source method $\mathcal{L}_{\text{QCD}}^{\text{ext}} = \mathcal{L}_{\text{QCD}}^0 + \bar{q}_R \gamma^\mu r_\mu q_R + \bar{q}_L \gamma^\mu \ell_\mu q_L - \bar{q}_R (s + ip) q_L - \bar{q}_L (s - ip) q_R - \boxed{\theta \omega},$

QCD chiral Lagrangian is constructed guided by local G_χ symmetry, discrete P and C symmetries, and Lorentz invariance

- Power counting rule: δ expansion $\partial_\mu \sim p^\mu = O(\delta^{1/2}), \quad m_{u,d,s} = O(\delta), \quad N_C^{-1} = O(\delta).$

Symmetry breaking effects from $m_{u,d,s}$ and $U(1)_A$ anomaly are introduced order by order

- Leading-order QCD chiral Lagrangian

$$\mathcal{L}_{\text{eff}}^{(0)} = \frac{F^2}{4} \langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle - \frac{F^2 M_0^2}{12} X^2 \quad M_0^2 \text{ can be identified as the } U(1)_A \text{ anomaly contribution to the } \eta_0 \text{ mass}$$

$$D_\mu U = \partial_\mu U - i r_\mu U + i U \ell_\mu, \quad \chi = 2B_0(s + ip), \quad X = \psi + \boxed{\theta}, \quad \psi = \frac{\sqrt{6}\eta_0}{F}.$$

Fix external fields to their “physical values”, one introduce symmetry breaking and non-QCD couplings

Chiral low-energy constants have their N_C scalings: $F = O(N_C^{1/2}), B_0 = O(N_C^0), M_0^2 = O(N_C^{-1})$

- ✓ With the external field $\theta(x)$ coupled to ω , the $aG\tilde{G}$ term **does not need to be removed** before matching onto χ PT

□ Chiral rotation on quark field

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2 + \frac{\partial_\mu a}{f} (\bar{q}_L k_Q \gamma^\mu q_L + \bar{q}_R k_q \gamma^\mu q_R) - c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a} - c_{\gamma\gamma} \frac{\alpha}{4\pi} \frac{a}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}$$



$$q(x) \rightarrow \exp \left(-i \kappa_q \gamma_5 c_{GG} \frac{a(x)}{f} \right) q(x), \quad \kappa_q = \text{diag}(\kappa_u, \kappa_d, \kappa_s)$$

$$\begin{aligned} \mathcal{L}_{\text{eff}}(\kappa_q) = & \mathcal{L}_{\text{QCD}}^0 + \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2 + \frac{\partial_\mu a}{f} \bar{q}_R \gamma^\mu \hat{k}_q(a) q_R + \frac{\partial_\mu a}{f} \bar{q}_L \gamma^\mu \hat{k}_Q(a) q_L \\ & - \bar{q}_R \hat{M}(a) q_L - \bar{q}_L \hat{M}^\dagger(a) q_R - 2 \hat{c}_{GG} \frac{a}{f} \omega - \hat{c}_{\gamma\gamma} \frac{\alpha}{4\pi} \frac{a}{f} F_{\mu\nu} \tilde{F}^{\mu\nu} \end{aligned}$$

$$\hat{k}_q(a) = k_q + c_{GG} \kappa_q + \mathcal{O}(a/f), \quad \hat{k}_Q(a) = k_Q - c_{GG} \kappa_q + \mathcal{O}(a/f),$$

$$\hat{M}(a) = M e^{2i \kappa_q c_{GG} \frac{a}{f}}, \quad \boxed{\hat{c}_{GG} = [1 - \text{Tr}(\kappa_q)] c_{GG}},$$

$$\hat{c}_{\gamma\gamma} = c_{\gamma\gamma} - 2N_C \text{Tr}(\kappa_q Q_q^2) c_{GG}. \quad \text{Tr}(\kappa_q) \text{ is not necessary to be unity!}$$

$$r_\mu = \frac{\partial_\mu a}{f} \hat{k}_q(a),$$

$$\ell_\mu = \frac{\partial_\mu a}{f} \hat{k}_Q(a),$$

$$s + ip = \hat{M}(a),$$

$$\theta = 2 \hat{c}_{GG} \frac{a}{f}.$$



Match onto chiral EFT

✓ Physical amplitudes must be independent of κ_q

$$\mathcal{L}_{\text{eff}}^\chi(\kappa_q) = \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2 - \hat{c}_{\gamma\gamma} \frac{\alpha}{4\pi} \frac{a}{f} F_{\mu\nu} \tilde{F}^{\mu\nu} + \mathcal{L}_{\text{eff}}^{(0)}(\kappa_q)$$

□ U(3) chiral Lagrangian for nonleptonic weak interaction

- Four-fermion operators $\hat{Q}_{i=1-6}$ are decomposed into $(8_L, 1_R)$ and $(27_L, 1_R)$ representations under G_χ
- Construct $|\Delta S| = 1$ chiral operators that have the same chiral structures

$$\mathcal{L}_{\text{weak}}^{(-1)}(\kappa_q) = \underbrace{e^{i(\kappa_s - \kappa_d) c_{GG} \frac{a}{f}}}_{\text{induced by chiral rotation}} \left(\boxed{G_8 O_8 + G_8^\psi O_8^\psi} + \boxed{G_{27}^{1/2} O_{27}^{1/2} + G_{27}^{3/2} O_{27}^{3/2}} \right) + \text{h.c.},$$

induced by

chiral rotation

$$O_8 = (L_\mu L^\mu)_{32}, \quad O_8^\psi = D_\mu \psi (L^\mu)_{32},$$

$$L_\mu \equiv iU^\dagger D_\mu U$$

$$O_{27}^{1/2} = (L_\mu)_{32} (L^\mu)_{11} + (L_\mu)_{31} (L^\mu)_{12} + 2 (L_\mu)_{32} (L^\mu)_{22} - 3 (L_\mu)_{32} (L^\mu)_{33},$$

$$D_\mu \psi \equiv -\langle L_\mu \rangle$$

$$O_{27}^{3/2} = (L_\mu)_{32} (L^\mu)_{11} + (L_\mu)_{31} (L^\mu)_{12} - (L_\mu)_{32} (L^\mu)_{22}.$$

- The weak LECs start at $\mathcal{O}(N_C^2)$ in the large- N_C expansion $\{G_8, G_8^\psi, G_{27}^{1/2}, G_{27}^{3/2}\} = \mathcal{O}(N_C^2)$

$$G_i = G_W g_i, \quad G_W \equiv -\frac{F^4 G_F}{\sqrt{2}} V_{ud}^* V_{us}, \quad \checkmark \text{ Values of } g_i \text{ are determined from kaon weak decays (Cirigliano, et al, 2012)(Gerard, et al, 2005)}$$

	p^2	p^4	p^6	$\dots$
N_C^2	δ^{-1}	δ^0	δ	$\dots$
N_C	δ^0	δ	δ^2	$\dots$
N_C^0	δ	δ^2	δ^3	$\dots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$

- In $N_C \rightarrow \infty$, non-factorizable contributions vanish, and the two color-singlet currents can be matched independently onto mesonic currents

$$\mathcal{L}_{\text{eff}}^{|\Delta S|=1}|_{N_C \rightarrow \infty} = -\frac{4G_F}{\sqrt{2}} V_{ud}^* V_{us} (\bar{d}_L \gamma^\mu u_L) (\bar{u}_L \gamma_\mu s_L) + \text{h.c.} \rightarrow G_W (L_\mu)_{31} (L^\mu)_{12} + \text{h.c.} + \mathcal{O}(p^4)$$

$$\boxed{g_8|_{N_C \rightarrow \infty} = \frac{3}{5}, \quad g_{27}^{1/2}|_{N_C \rightarrow \infty} = \frac{1}{15}, \quad g_{27}^{3/2}|_{N_C \rightarrow \infty} = \frac{1}{3}, \quad g_8^\psi|_{N_C \rightarrow \infty} = \frac{2}{5}.}$$

- ✓ consistent with the SU(3) values (Pich & de Rafael, 1991)

□ Mixings between fields $A = (a, \pi^3, \eta_8, \eta_0)^T$

$$\mathcal{L}_{\text{eff}}^\chi|_{\text{quadratic}} = \frac{1}{2} \partial_\mu A^T \begin{pmatrix} 1 & -\frac{F}{f} k_{a\pi^3} & -\frac{F}{f} k_{a\eta_8} & -\frac{F}{f} k_{a\eta_0} \\ -\frac{F}{f} k_{a\pi^3} & 1 & 0 & 0 \\ -\frac{F}{f} k_{a\eta_8} & 0 & 1 & 0 \\ -\frac{F}{f} k_{a\eta_0} & 0 & 0 & 1 \end{pmatrix} \partial^\mu A - \frac{1}{2} A^T \begin{pmatrix} m_a^2 & -\frac{F}{f} m_{a\pi^3}^2 & -\frac{F}{f} m_{a\eta_8}^2 & -\frac{F}{f} m_{a\eta_0}^2 \\ -\frac{F}{f} m_{a\pi^3}^2 & m_{0\pi}^2 & -\frac{1}{\sqrt{3}} \delta_I & -\sqrt{\frac{2}{3}} \delta_I \\ -\frac{F}{f} m_{a\eta_8}^2 & -\frac{1}{\sqrt{3}} \delta_I & m_{0\eta_8}^2 & -\frac{2\sqrt{2}}{3} \Delta_8 \\ -\frac{F}{f} m_{a\eta_0}^2 & -\sqrt{\frac{2}{3}} \delta_I & -\frac{2\sqrt{2}}{3} \Delta_8 & m_{0\eta_0}^2 \end{pmatrix} A$$

$$+ \partial_\mu \pi^+ \partial^\mu \pi^- - m_{0\pi}^2 \pi^+ \pi^- + \partial_\mu K^+ \partial^\mu K^- - m_{0K^\pm}^2 K^+ K^- + \partial_\mu K^0 \partial^\mu \bar{K}^0 - m_{0K^0}^2 K^0 \bar{K}^0 - \frac{F(k_d - k_D)_{21}}{\sqrt{2}f} \partial_\mu a \partial^\mu K^0 - \frac{F(k_d - k_D)_{12}}{\sqrt{2}f} \partial_\mu a \partial^\mu \bar{K}^0$$

- ALP-meson mixing coefficients depend on κ_q

$$\begin{aligned} k_{a\pi^3} &= \frac{1}{2} [(c_{uu}^a - c_{dd}^a) + 2c_{GG} (\kappa_u - \kappa_d)] , & m_{a\pi^3}^2 &= 2c_{GG} B_0 (\kappa_u m_u - \kappa_d m_d) , \\ k_{a\eta_8} &= \frac{1}{2\sqrt{3}} [(c_{uu}^a + c_{dd}^a - 2c_{ss}^a) + 2c_{GG} (\kappa_u + \kappa_d - 2\kappa_s)] , & m_{a\eta_8}^2 &= \frac{2}{\sqrt{3}} c_{GG} B_0 (\kappa_u m_u + \kappa_d m_d - 2\kappa_s m_s) , \\ k_{a\eta_0} &= \frac{1}{\sqrt{6}} [(c_{uu}^a + c_{dd}^a + c_{ss}^a) + 2c_{GG} (\kappa_u + \kappa_d + \kappa_s)] , & m_{a\eta_0}^2 &= 2\sqrt{\frac{2}{3}} c_{GG} B_0 (\kappa_u m_u + \kappa_d m_d + \kappa_s m_s) - \frac{\sqrt{6}}{3} \hat{c}_{GG} M_0^2 , \end{aligned}$$

- η - η' mixing at leading order

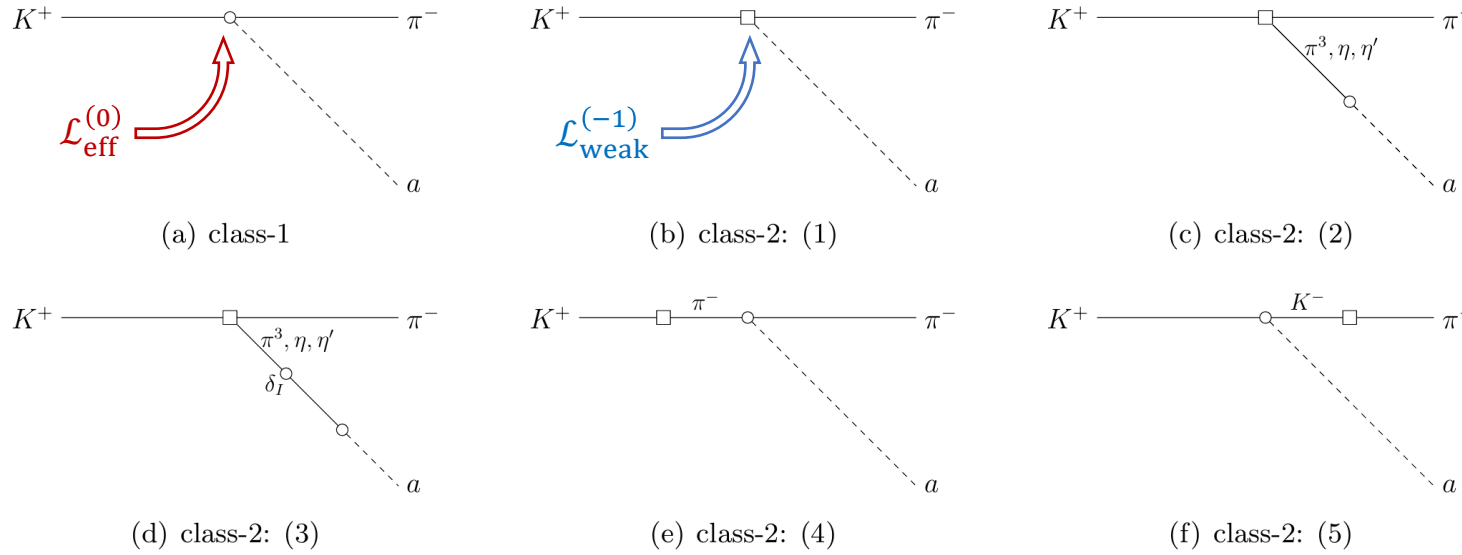
$$\begin{pmatrix} \eta_8 & \eta_0 \end{pmatrix} \begin{pmatrix} m_{0\eta_8}^2 & -\frac{2\sqrt{2}}{3} \Delta_8 \\ -\frac{2\sqrt{2}}{3} \Delta_8 & m_{0\eta_0}^2 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} = \begin{pmatrix} \eta & \eta' \end{pmatrix} \begin{pmatrix} m_\eta^2 & \\ & m_{\eta'}^2 \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix}$$

$$\begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix} \longrightarrow \begin{pmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{pmatrix} \begin{pmatrix} \eta \\ \eta' \end{pmatrix}$$

Introduce higher-order corrections in η - η' mixing

✓ General η - η' mixing matrix

□ Leading order $\bar{K} \rightarrow \pi a$ decay amplitude in U(3) χ PT



$$A_{\bar{K} \rightarrow \pi a} = A_{\bar{K} \rightarrow \pi a}^{\text{AFC}} + A_{\bar{K} \rightarrow \pi a}^{\text{WFC}}$$

$$A_{K^- \rightarrow \pi^- a}^{\text{AFC}} = -\frac{(k_d + k_D)_{12}}{2f} (m_{K^\pm}^2 - m_\pi^2)$$

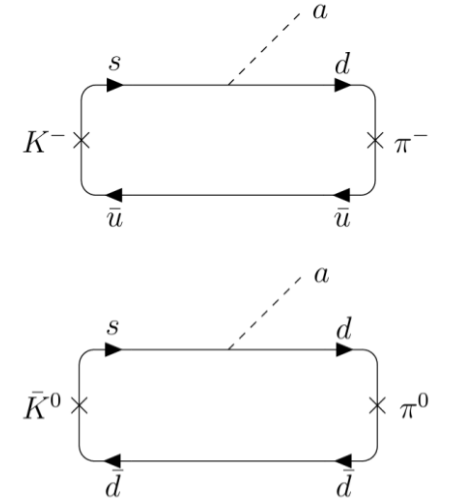
$$A_{\bar{K}^0 \rightarrow \pi^0 a}^{\text{AFC}} = \frac{(k_d + k_D)_{12}}{2\sqrt{2}f} (m_{K^0}^2 - m_\pi^2) \left\{ 1 + \delta_I \left[\frac{\beta_{11}(\beta_{11} + \sqrt{2}\beta_{21})}{m_\pi^2 - m_\eta^2} + \frac{\beta_{12}(\beta_{12} + \sqrt{2}\beta_{22})}{m_\pi^2 - m_{\eta'}^2} \right] \right\}$$

$$A_{\bar{K} \rightarrow \pi a}^{\text{WFC}} = G_W \sum_{\text{cALP}} \frac{c_{\text{ALP}}}{f} F_{\bar{K} \rightarrow \pi a}^{\text{cALP}} \quad c_{\text{ALP}} \in \{c_{GG}, c_{uu}^a, c_{dd}^a, c_{ss}^a, c_{dd}^v - c_{ss}^v\}$$

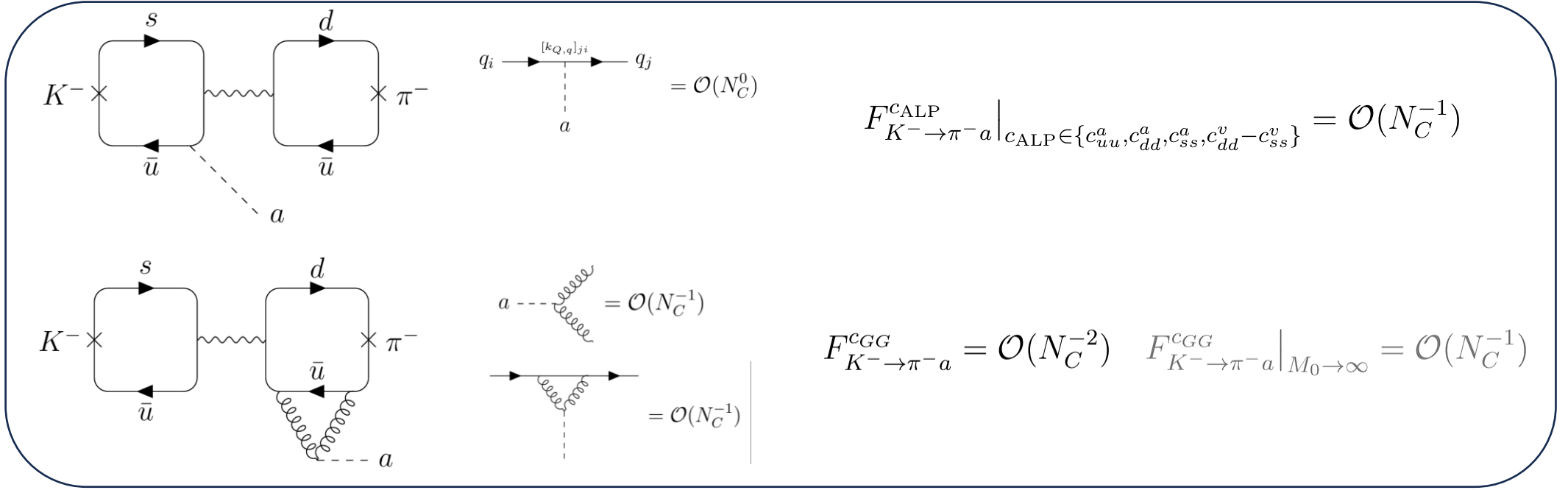
$$c_{uu}^{v/a} = [k_u]_{11} \pm [k_U]_{11}, \quad c_{dd}^{v/a} = [k_d]_{11} \pm [k_D]_{11}, \quad c_{ss}^{v/a} = [k_d]_{22} \pm [k_D]_{22}$$

□ Three self-consistency checks:

- ✓ Complete cancellation of the κ_q dependence
- ✓ Recovery of the SU(3) results in the $M_0 \rightarrow \infty$ limit
- ✓ Correct large- N_C behavior of the decay amplitudes



□ Large- N_C behavior of the $K \rightarrow \pi a$ decay amplitudes



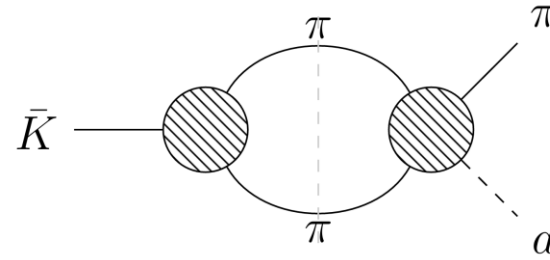
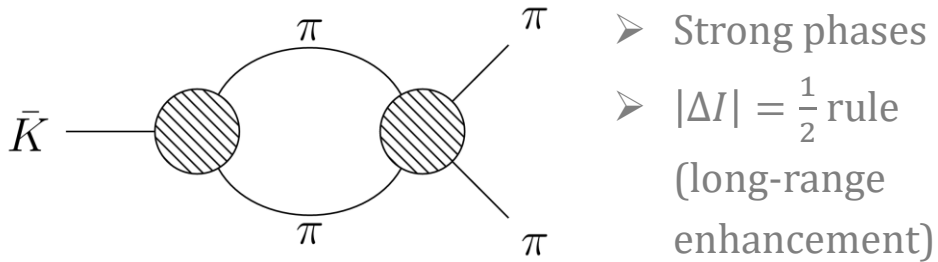
$$\mathcal{L} = \mathcal{L}_{\text{QCD}} + \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2 - c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a} \quad \xrightarrow{\quad} \quad q \rightarrow e^{-i\gamma_5 c_{GG} \frac{a}{3f}} q$$

$$\mathcal{L} = \mathcal{L}_{\text{QCD}} + \frac{1}{2} \partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2 - \frac{a}{3f} c_{GG} [\partial_\mu (\bar{q} \gamma^\mu \gamma_5 q) - 2i \bar{q} M \gamma_5 q] + \mathcal{O}\left(\frac{a^2}{f^2}\right)$$

- The following PCAC relation can not be fully reproduced in SU(3) theory

$$\partial_\mu (\bar{q} \gamma^\mu \gamma_5 q) - 2i \bar{q} M \gamma_5 q = \frac{3\alpha_s}{4\pi} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a} = \mathcal{O}(N_C^{-1})$$

■ A first look at higher-order effects: $\pi\pi$ FSI



- How important is $\pi\pi$ FSI?

$$\text{Im}A_{\bar{K} \rightarrow \pi a}^{\text{WFC}} = \sum_I \rho_{\pi\pi}(m_K^2) T_{(\pi\pi, I) \rightarrow \pi a}^{J=0}(m_K^2) a_{\bar{K} \rightarrow (\pi\pi, I)}^*$$

$$A_{\bar{K} \rightarrow \pi a}^{\text{WFC, uni}} = A_{\bar{K} \rightarrow \pi a}^{\text{WFC, tree}} + \sum_I \frac{a_{\bar{K} \rightarrow (\pi\pi, I)}^{\text{tree}} G_{\pi\pi}(m_K^2) T_{(\pi\pi, I) \rightarrow \pi a}^{J=0, \text{LO}}(m_K^2)}{1 - T_{\pi\pi \rightarrow \pi\pi}^{I0, \text{LO}}(m_K^2) G_{\pi\pi}(m_K^2)}$$

■ K^- : $I = 2$ only, definitely small by the $|\Delta I| = 1/2$ rule

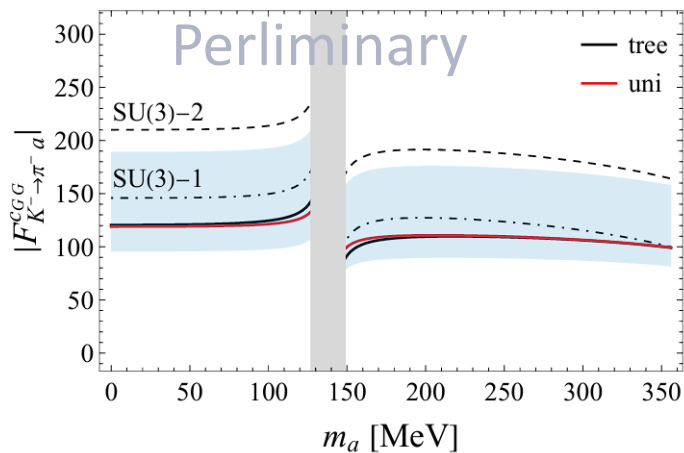
■ $\bar{K}^0$: both $I = 0$ and 2, potentially significant

$$G_{\pi\pi}(s) = -i \int \frac{d^d q}{(2\pi)^d} \frac{1}{((p-q)^2 - m_\pi^2)(q^2 - m_\pi^2)} \Big|_{\left(\frac{2}{d-4} + \dots\right) \rightarrow a_{\text{sc}}}$$

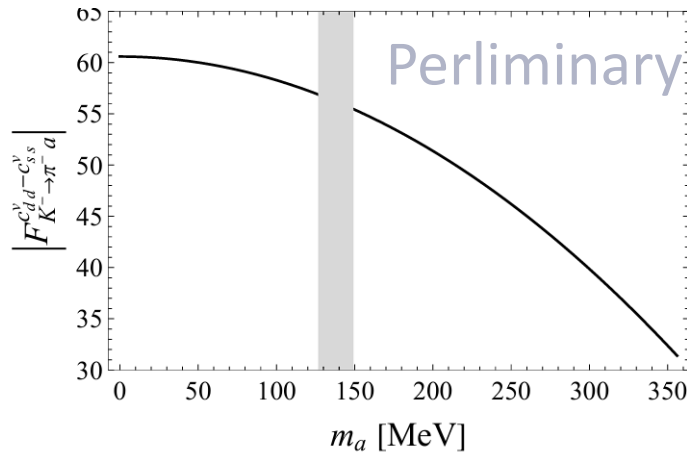
Determined by S-wave $\pi\pi$ phase shifts

$$\text{Im}G_{\pi\pi}(s + i\varepsilon) = \theta(s - 4m_\pi^2) \rho_{\pi\pi}(s)$$

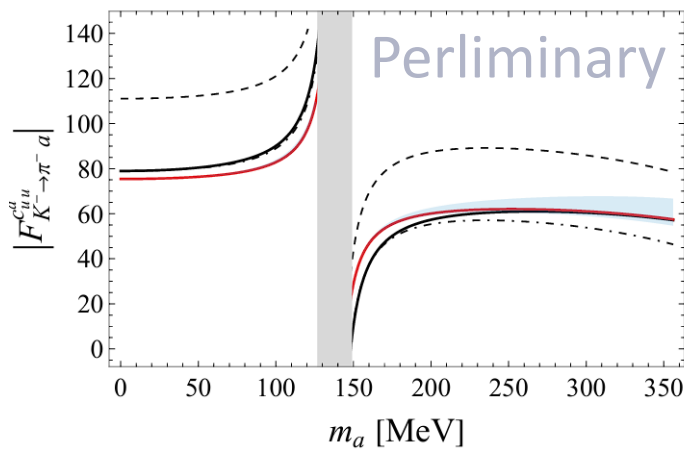
□ Numeric discussion



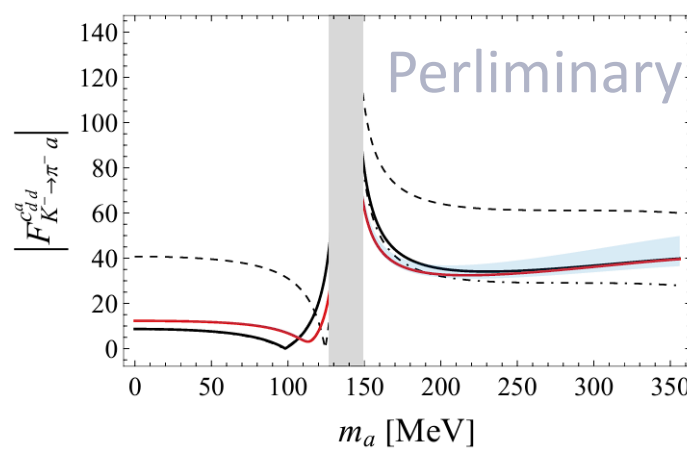
(a)



(b)

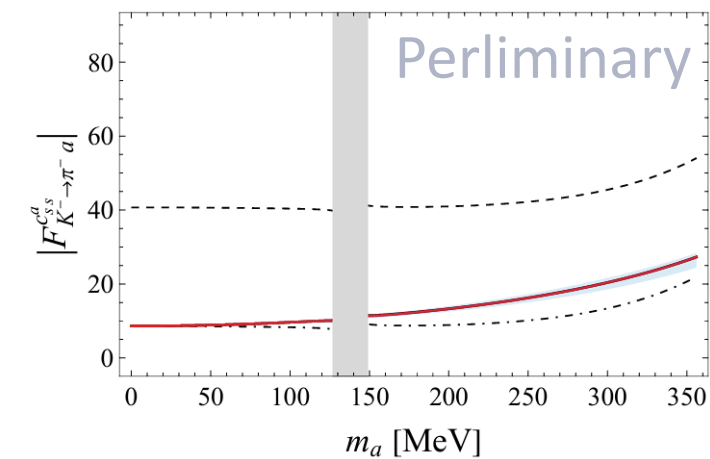


(c)



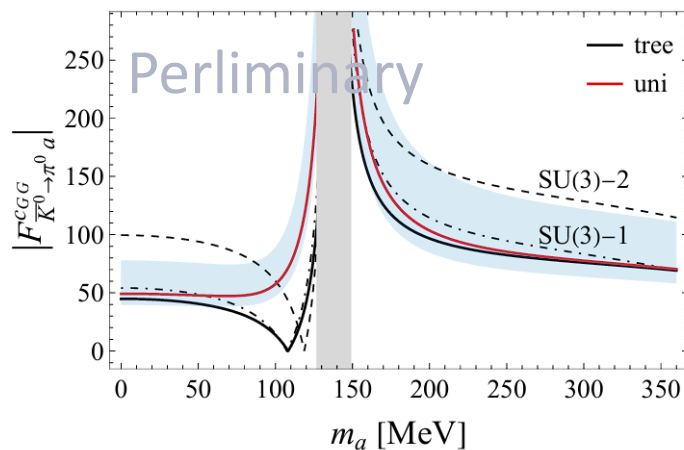
(d)

- $SU(3)-1 = U(3)|_{M_0 \rightarrow \infty}$:
explicit η_0 contribution is small
- $SU(3)-2 = SU(3)-1|_{g_8^{\psi \rightarrow 0}}$:
effect of new weak operator is pronounced

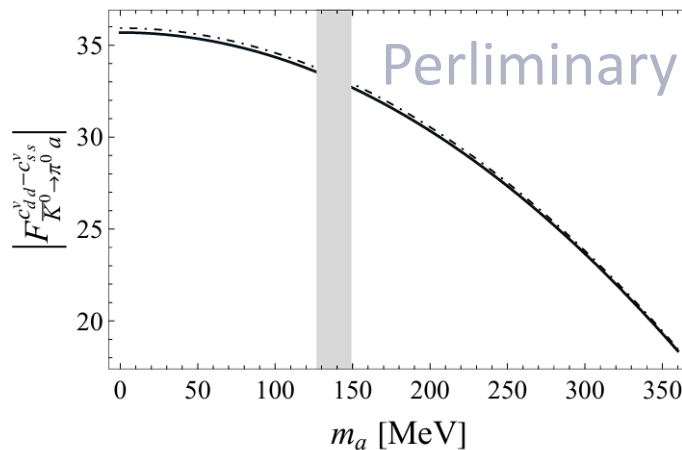


(e)

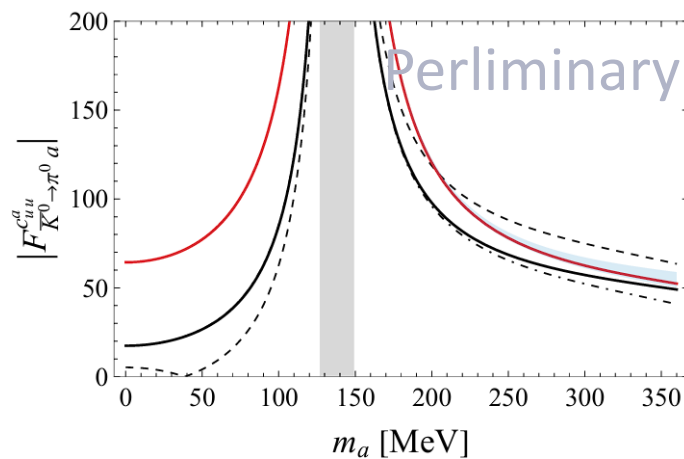
□ Numeric discussion



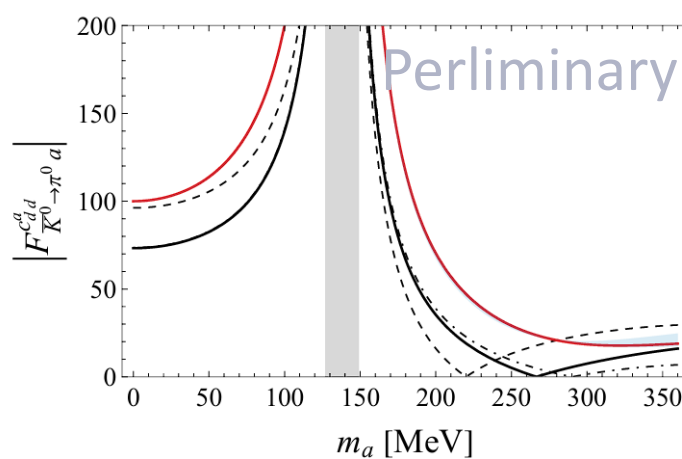
(a)



(b)

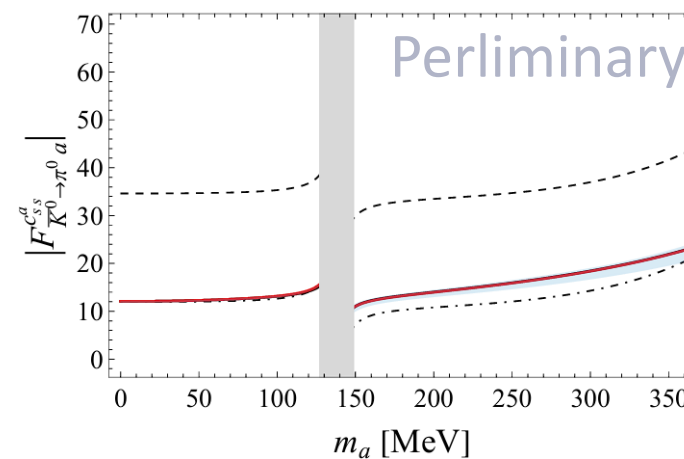


(c)



(d)

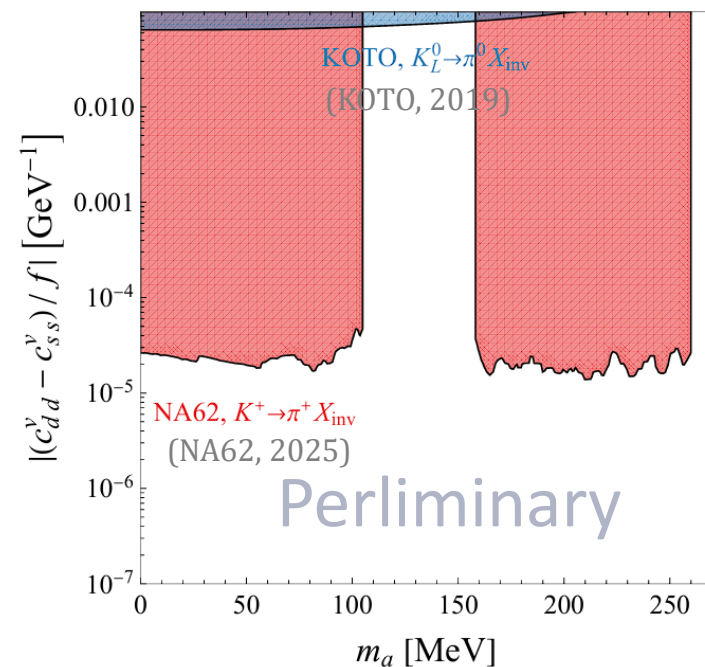
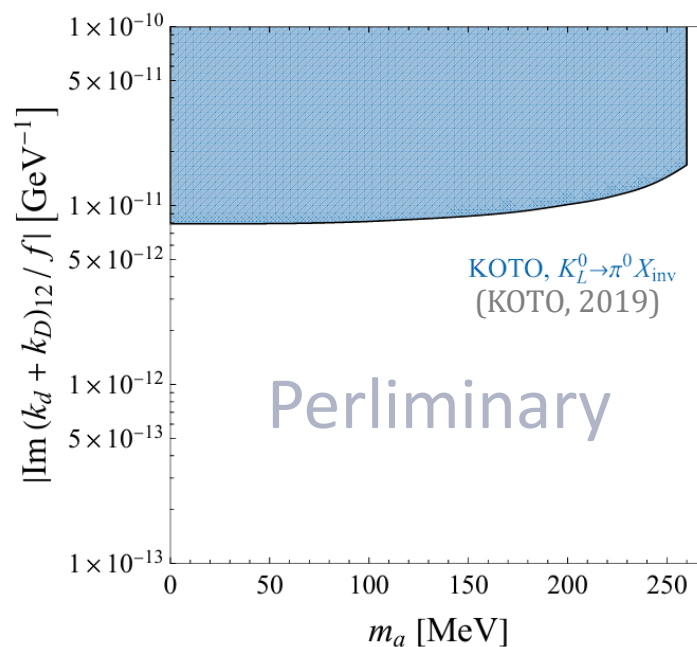
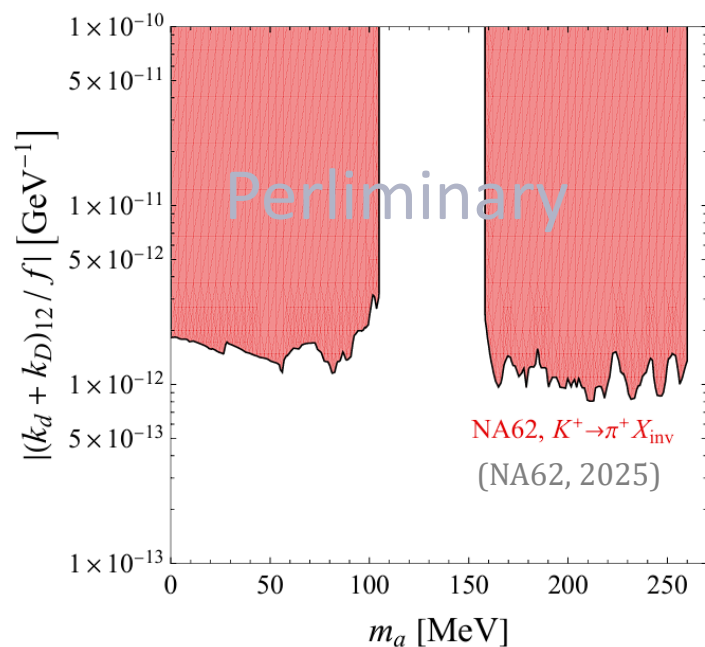
- SU(3)-1 = U(3)| $M_0 \rightarrow \infty$:
explicit η_0 contribution is small
- SU(3)-2 = SU(3)-1| $g_8 \psi \rightarrow 0$:
effect of new weak operator is pronounced



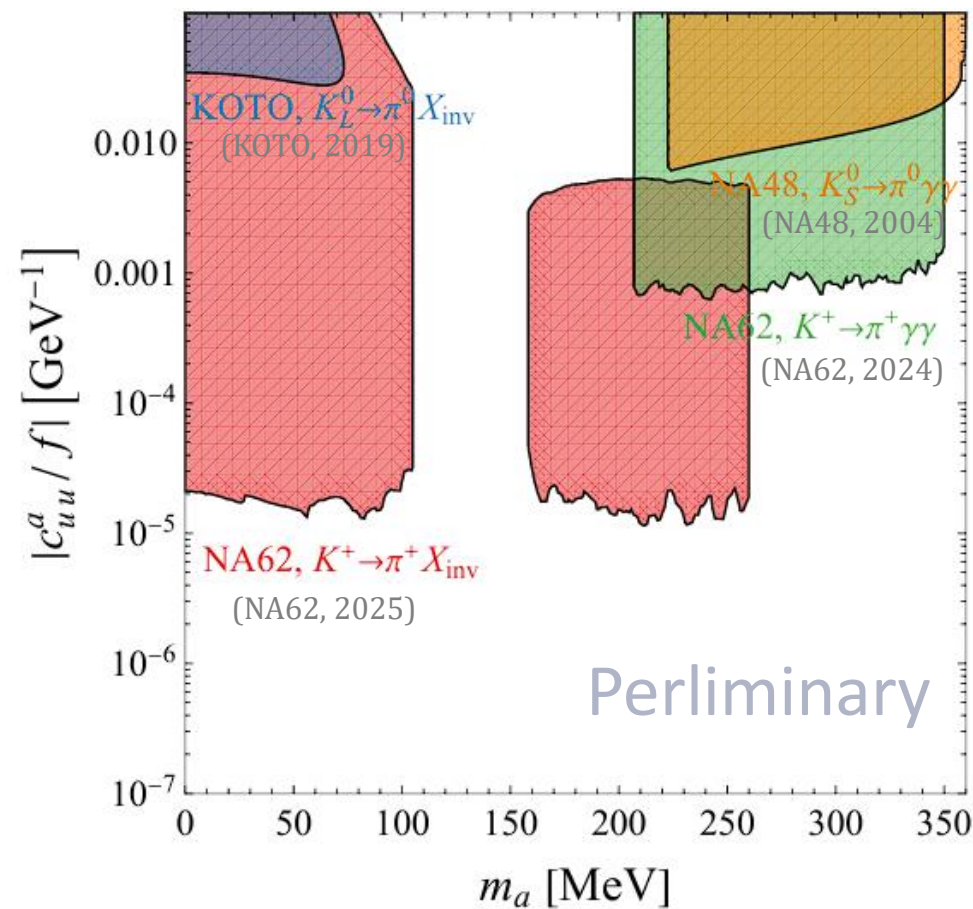
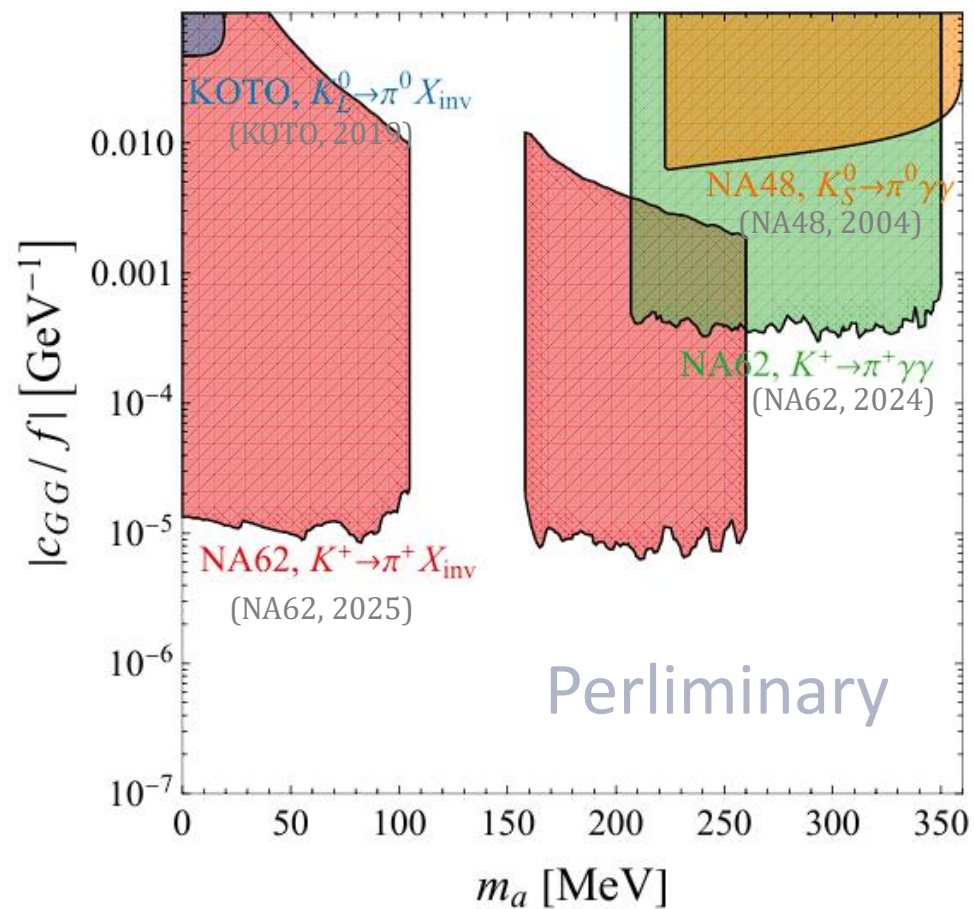
(e)

□ ALP constraints from kaon decays

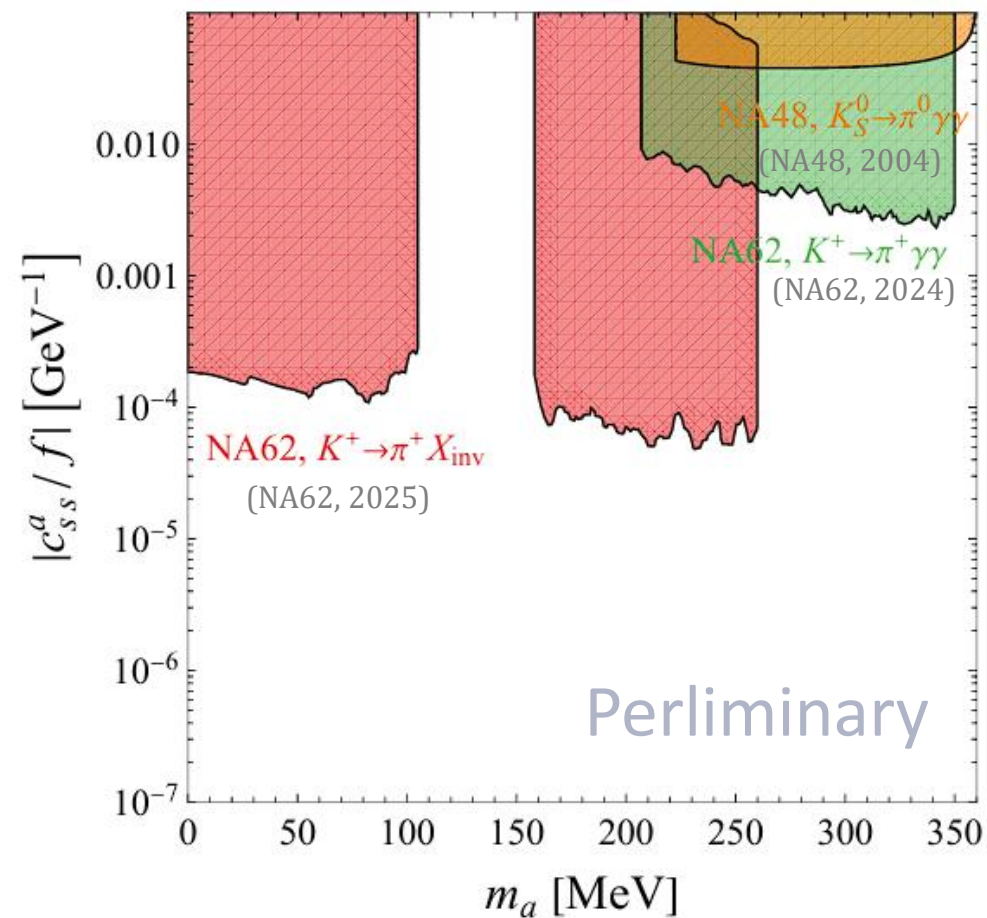
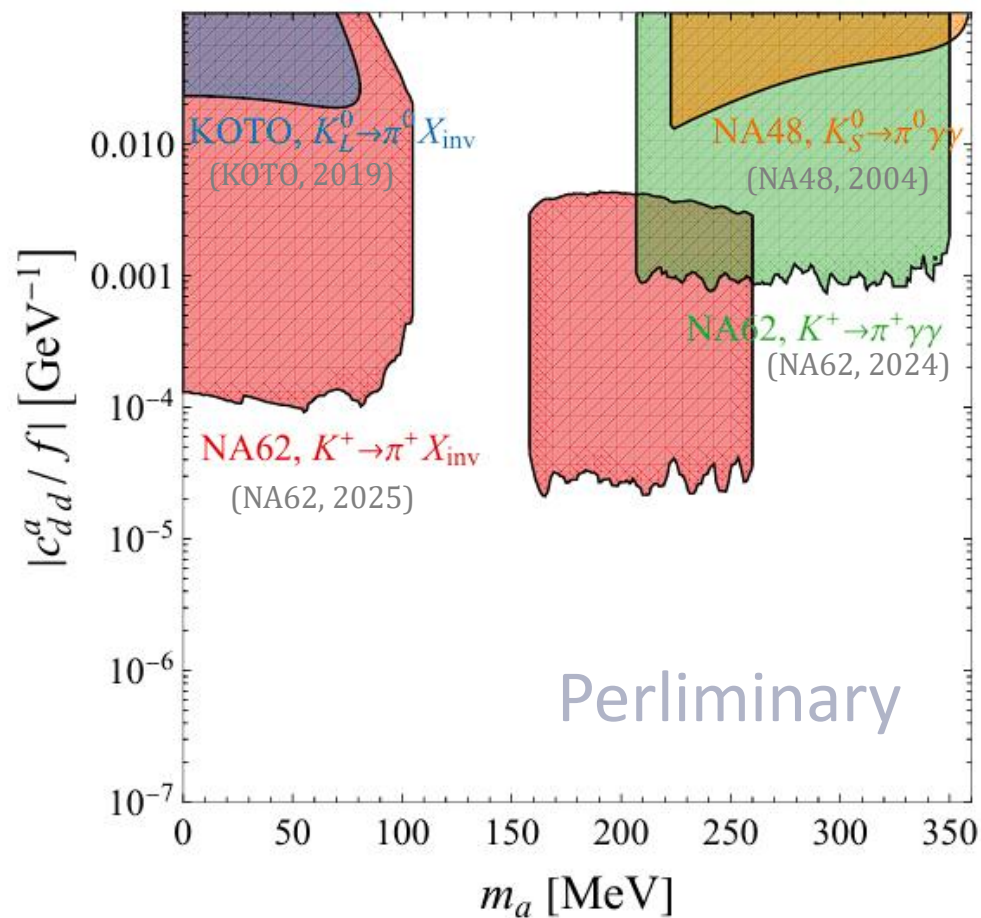
- Assume ALPs decay only into $\gamma\gamma$
- Switch on one ALP coupling (kaon scale) at a time



ALP constraints from kaon decays



□ ALP constraints from kaon decays



Summary

- ❑ In this talk, we have studied the $K \rightarrow \pi a$ decay at leading order in $U(3)$ χ PT.
- ❑ We verified that our treatment of ALP interactions in $U(3)$ χ PT is self-consistent, as the resulting decay amplitudes are independent of the reparameterization parameters.
- ❑ $U(3)$ χ PT fully reproduces the large- N_C behavior of QCD. We show that our amplitudes exhibit the correct large- N_C scaling, especially for the c_{GG} contribution.
- ❑ Numerically, we found that the **explicit η_0 contribution is small** in the $K \rightarrow \pi a$ decay, whereas the **new weak operator unique to $U(3)$ χ PT gives a sizable contribution**. This demonstrates the importance of studying the $K \rightarrow \pi a$ decay within the $U(3)$ framework.

Outlook

- ❑ Extending the present work to the complete one-loop calculation in $U(3)$ χ PT would be an important next step toward a precision prediction of the $K \rightarrow \pi a$ decay. The main challenge is our limited knowledge of the many weak low-energy constants that enter at higher orders. We hope that this difficulty can be overcome in the future with the help of nonperturbative approaches, such as the large- N_C expansion and lattice QCD.

Thank you for your attention

□ Backup:

□ The fraction of ALPs escape the detector:

$$F_{esc}^{exp} = F(m_a, \tau_0, \beta_K^{exp}, L_{max}^{exp}) = \int_0^{\frac{\pi}{2}} d\theta \sin\theta \exp\left(-\frac{L_{max}^{exp}}{\Delta x_L}\right), \quad \Delta x_L = \frac{p_{a,L} \tau_0}{m_a}, \quad \tau_0 = \Gamma_{a \rightarrow \gamma\gamma}^{-1}$$
$$p_{a,L} = \gamma_K^{exp} (\beta_K^{exp} E_a^* + p_a^* \cos\theta)$$

- Long-lived: $F_{esc}^{exp} \mathcal{B}(K \rightarrow \pi a) \leq \mathcal{B}(K \rightarrow \pi X_{inv})_{UL}^{exp}$
- $\gamma\gamma$: $(1 - F_{esc}^{exp}) \mathcal{B}(K \rightarrow \pi a) \mathcal{B}(a \rightarrow \gamma\gamma) \leq [\mathcal{B}(K \rightarrow \pi a) \mathcal{B}(a \rightarrow \gamma\gamma)]_{UL}^{exp}$