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Bias with a Timer: Axion Domain Wall Decay and Dark Matter

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We study a cosmological scenario in which the post-inflationary QCD axion interacts with a light scalar field, resulting in a dynamically controlled mechanism for the axion domain wall decay and DM production.

1. Introduction

The Standard Model is a very successful model of physics, however, strong CP problem remains one of the most important puzzles in it.

QCD allows a CP-violating term in its Lagrangian.

But the EDM results require $\bar{\theta}$ to be unnaturally small.

$$|\bar{\theta}| \lesssim 10^{-10}$$

C. A. Baker *et al.*, Phys. Rev. Lett. **97**, 131801 (2006)

A particularly elegant solution to the strong CP problem is provided by the Peccei-Quinn (PQ) mechanism.

It promotes the parameter θ to a dynamical field by introducing a new global $U(1)$ symmetry, spontaneously broken at some high energy scale.

Peccei and Quinn (1977)

$$\mathscr{L} = \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu a} \Longrightarrow \mathscr{L} = \left(\theta + \frac{a}{f_a}\right) \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \tilde{G}^{\mu\nu a}$$

The associated pseudo-Nambu-Goldstone boson, the axion dynamically relaxes θ to zero.

$\underline{T > v_{PQ}}$ $v_{\mathrm{PQ}} \equiv N_{\mathrm{DW}} f_a$

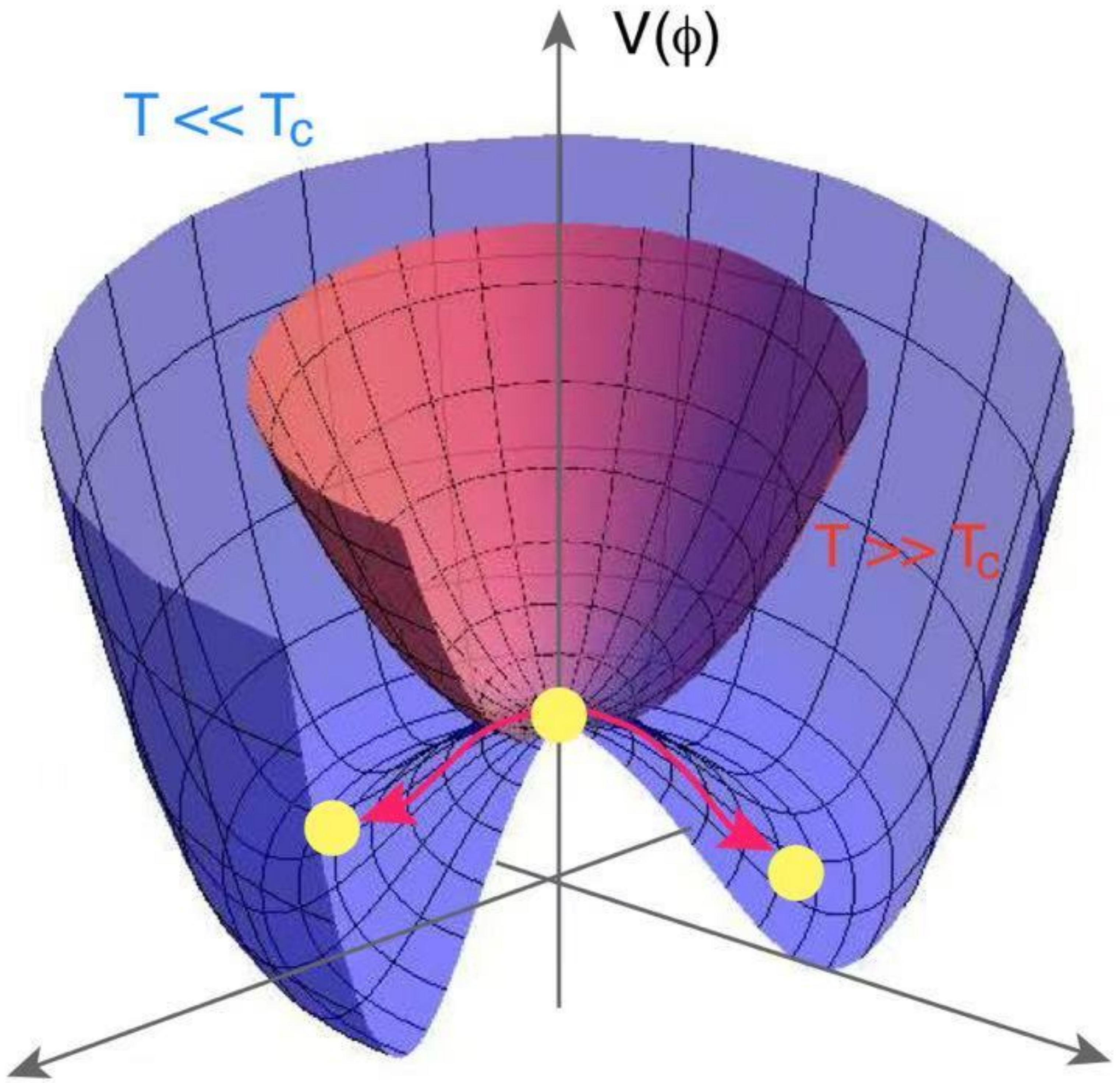
- $U(1)_{PQ}$ symmetry holds.

$\underline{T \sim v_{\mathrm{PQ}}}$

- $U(1)_{PQ}$ is broken, cosmic string appears.

Axion (a) manifests as phase of PQ scalar $|\phi| e^{i\theta} = |\phi| e^{ia/v_{PQ}}$.

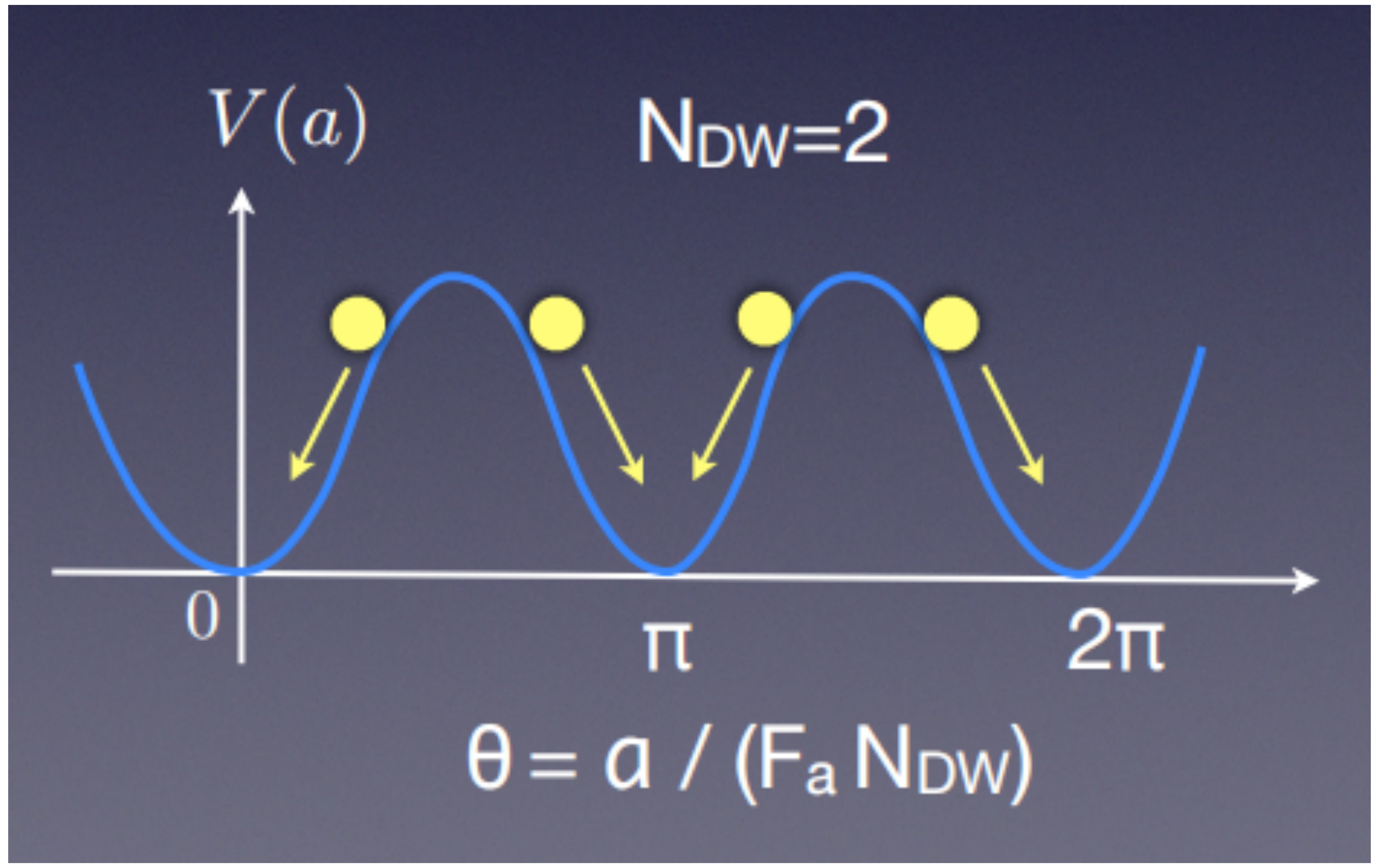
Kibble (1976), Vilenkin (1981)



from Cosmology(Masahiro Kawasaki)The University of Tokyo)

$\underline{T \sim \Lambda_{\mathrm{QCD}}}$ $U(1)_{PQ}$ ⟶ discrete Z_N

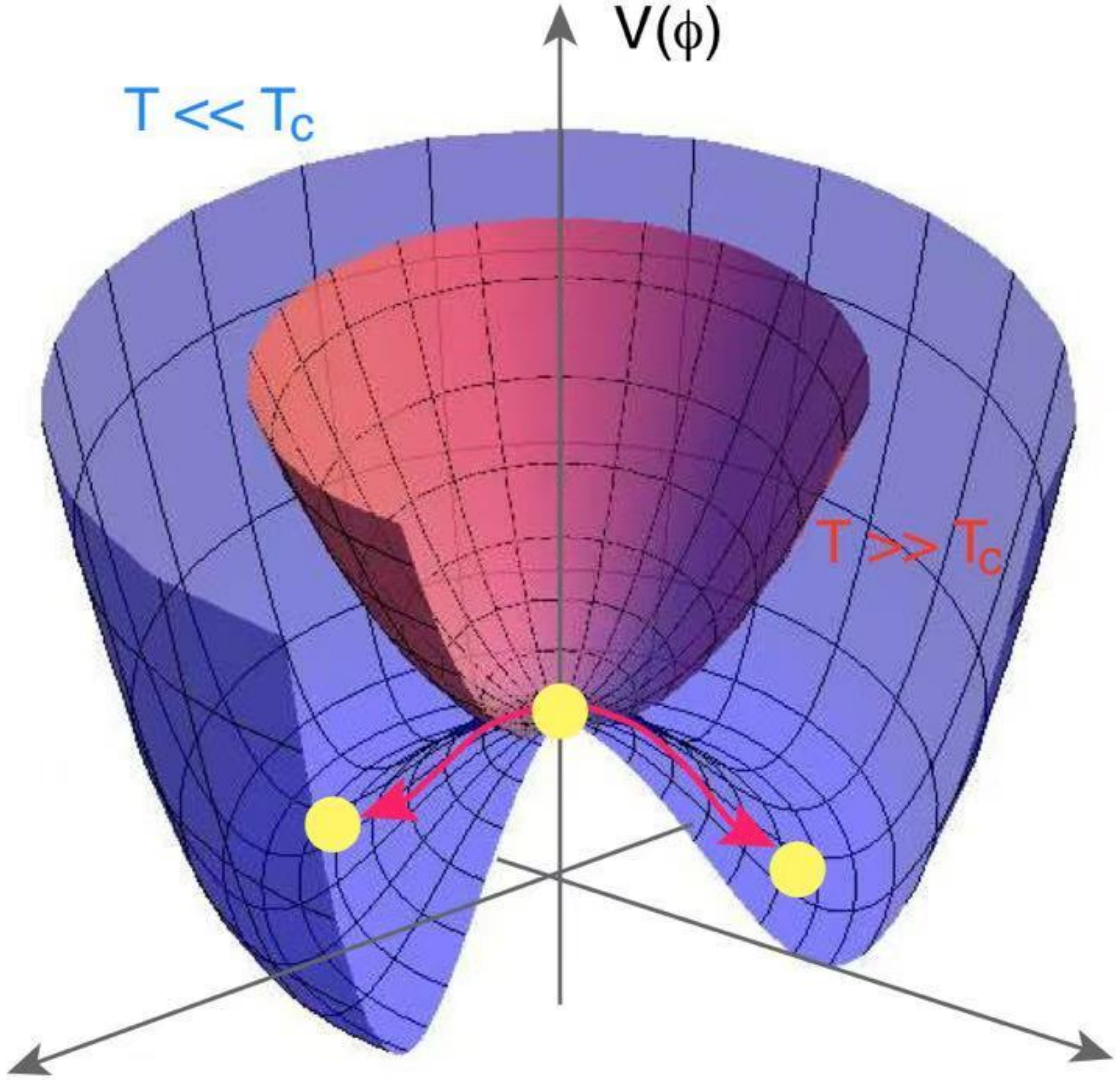
- QCD axion obtains mass, domain wall (DW) appears. Vilenkin (1981)



- For $N_{\rm DW} > 1$,
- it is stable and dominates the Universe.

Domain wall .

Zeldovich, Kobzarev (1976), Vilenkin (1985)



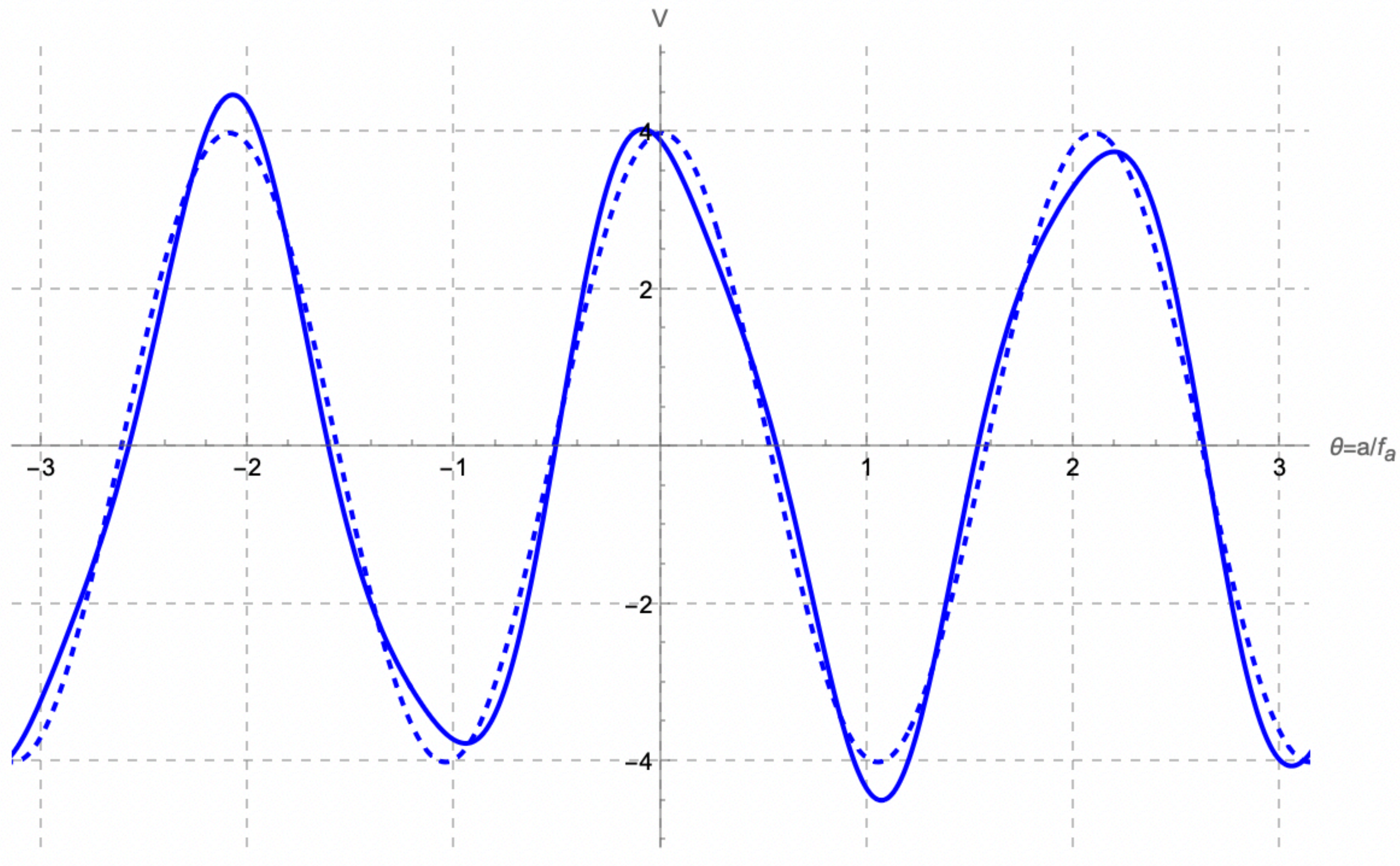
from Cosmology(Masahiro Kawasaki)The University of Tokyo)

- One possible way to aviod the domain wall problem is to introduce a "bias" term which explicitly breaks the PQ symmetry.

Ringwald, Saikawa (2016)

- However, such explicit breaking potentials misalign the potential minimum from the CP conserving one.

Eg: Large bias can drag the θ away from 0.



Bias term lifts degenerated vacua

What kind of bias term can safely solve the problem?

→ **Bias with a timer**

Above introductions address several problems:

1. How to determine this changing bias term?
2. What is the evolution of string-wall system?
3. What's the DM production in this model? Does it satisfy the DM abundance?
4. Do we have any viable parameter spaces in effective Lagrangian?

Contents

2. PQ mechanism with a light scalar

The Lagrangian can be divided into 3 parts:

- Evolution of S $\longrightarrow$ Background $|S|$
- Evolution of PQ field $\longrightarrow$ V_{QCD}
- Evolution of mixing term (bias) $\longrightarrow$ $V_{\cancel{PQ}}$

$$V_{\rm PQ}(P,S) \supset \lambda_P \left(|P|^2 - \frac{v_{\rm PQ}^2}{2}\right)^2 + \frac{1}{(n!)^2}\frac{\lambda_S^2}{M_{\rm Pl}^{2n-4}}|S|^{2n}$$

$$+\ m_S^2|S|^2 + \left(\frac{\lambda}{m!\ell!M_{\rm Pl}^{m+\ell-4}}S^m P^\ell + {\rm h.c.}\right)$$

PQ scalar P

a light complex scalar field S

Hao, Nakagawa, Nakai, Suzuki (2025)

Evolution of S

Throughout the history in the Universe until the oscillation of S starts, the spectator field has a large field value.

$$|\ddot{S}| + 3H|\dot{S}| + \frac{n\lambda_S^2}{(n!)^2 M_{\rm Pl}^{2n-4}}|S|^{2n-1} = 0$$

$$\longrightarrow \quad \langle|S|\rangle \simeq \left[\frac{2(n-3)(n!)^2}{n(n-1)^2\lambda_S^2}\right]^{\frac{1}{2(n-1)}} \left(\frac{H}{M_{\rm Pl}}\right)^{\frac{1}{n-1}} M_{\rm Pl}$$

for $n \geq 6$

Evolution of PQ field & mixing term

At $T \sim v_{PQ}$, P acquires a nonzero VEV and the PQ symmetry is spontaneously broken. We can parameterize the PQ field and S field by:

$$P = \frac{v_{\rm PQ}}{\sqrt{2}} e^{ia/v_{\rm PQ}}$$

$$S = \frac{\chi}{\sqrt{2}} e^{ib/\chi}$$

Obtain the potential $\longrightarrow$

$$V_{\rm QCD}(a) = m_a^2(T) f_a^2 \left(1 - \cos\frac{a}{f_a}\right),$$

$$V_{\cancel{\rm PQ}} \simeq -\frac{1}{\ell^2} m_{\cancel{\rm PQ}}^2 v_{\rm PQ}^2 \cos\left(\ell \frac{a}{v_{\rm PQ}} + m\frac{b}{\chi} + \delta\right)$$

Our focus is on a scenario that the QCD axion starts to oscillate earlier than the conventional onset of oscillation,

$T_{osc} \gg T_{osc}^{conv}$, which is estimated by $H \sim m_a$.

Now we've obtained three important constraints:

- Mixing term shouldn't backreact the evolution.

Limited parameter sets

$$\cancel{\left(\frac{\lambda}{m!\ell! M_{\mathrm{Pl}}^{m+\ell-4}} S^m P^\ell + \mathrm{h.c.}\right)} \longrightarrow \frac{1}{(n!)^2}\frac{\lambda_S^2}{M_{\mathrm{Pl}}^{2n-4}}|S|^{2n} > \frac{|\lambda|}{m!\ell! M_{\mathrm{Pl}}^{m+\ell-4}}|S|^m v_{\mathrm{PQ}}^\ell$$

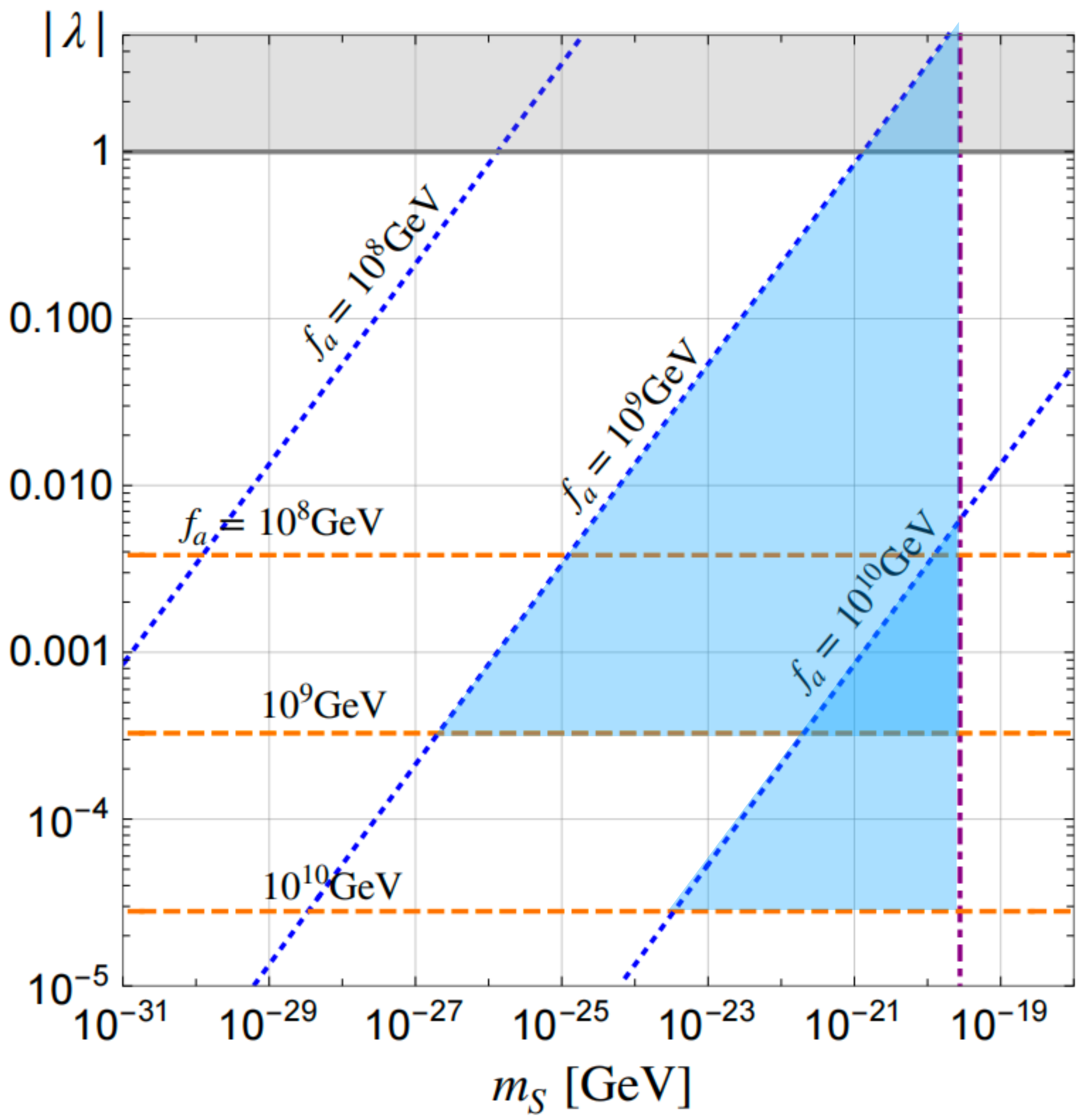
- S keeps the large field value until the oscillation $m_S \sim H$.

- $T_{\mathrm{osc}} > T_{\mathrm{osc}}^{(\mathrm{conv})}$

$$T_{\mathrm{osc}}^{(\mathrm{conv})} \simeq 2.6 \text{ GeV} \left(\frac{g_*}{80}\right)^{-0.084} \left(\frac{f_a}{10^{10} \text{ GeV}}\right)^{-0.17}$$

$$m_S \lesssim \sqrt{\frac{\pi^2 g_*}{90}} \frac{\Lambda_{\mathrm{QCD}}^2}{M_{\mathrm{Pl}}} \simeq 3 \times 10^{-11} \mathrm{eV}$$

Hao, Nakagawa, Nakai, Suzuki (2025)



$(N_{\rm DW}, l, m, n) = (2,3,9,6)$

Fix this set during this talk.

We can obtain the maximum value of λ for each parameter set (N_{DW}, l, m, n, f_a) .

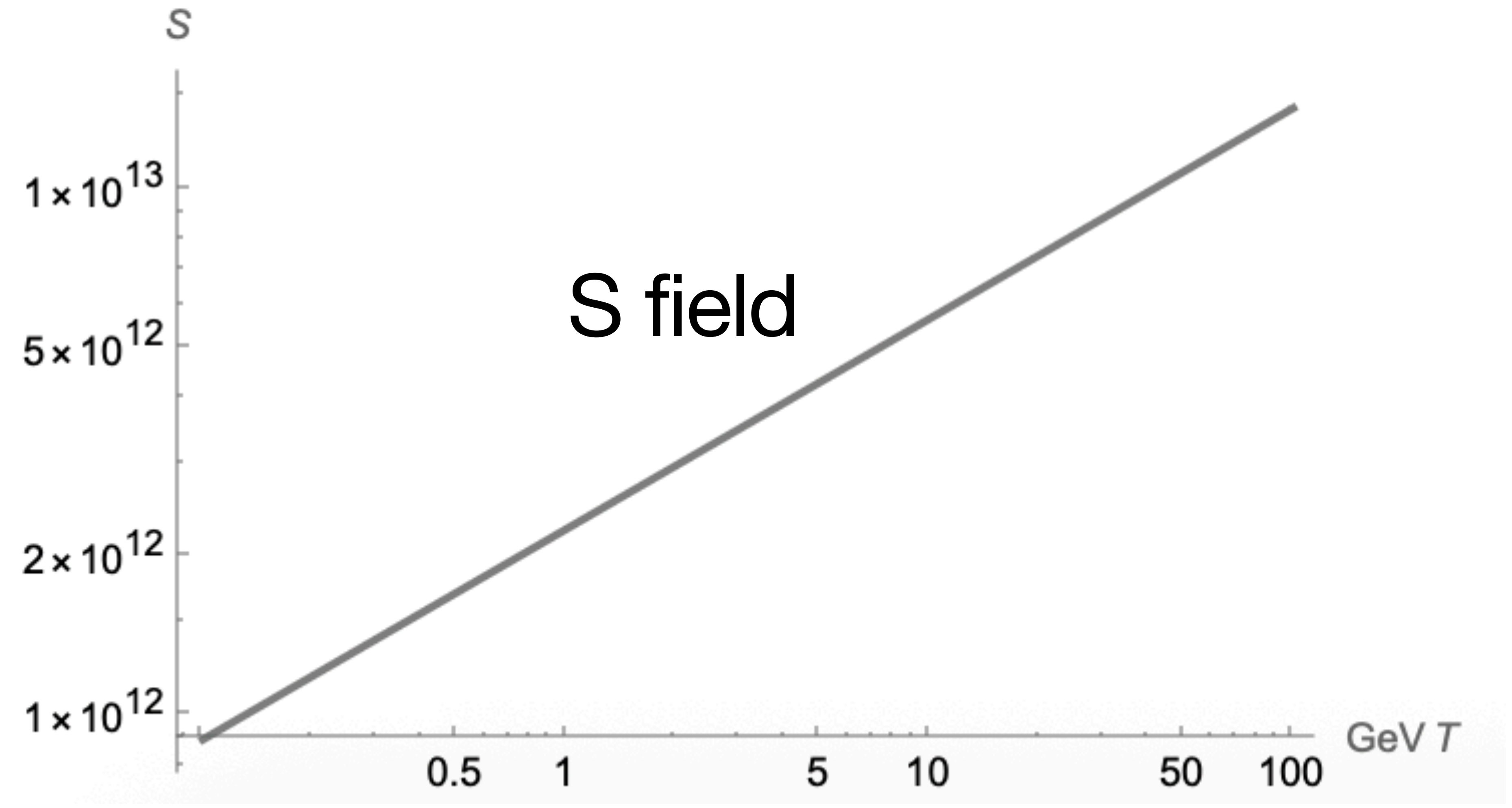
Possible sets are as follow:

$$
\begin{aligned}
(\ell, m, n) \ni\ & (1,10,6),(2,10,6) \\
& (1,9,6),(2,9,6),(3,9,6) \\
& (1,8,6),(2,8,6),(3,8,6),(4,8,6) \\
& (1,7,6),(2,7,6),(3,7,6),(4,7,6),(5,7,6).
\end{aligned}
$$

N_{DW} should be prime with l.

Hao, Nakagawa, Nakai, Suzuki (2025)

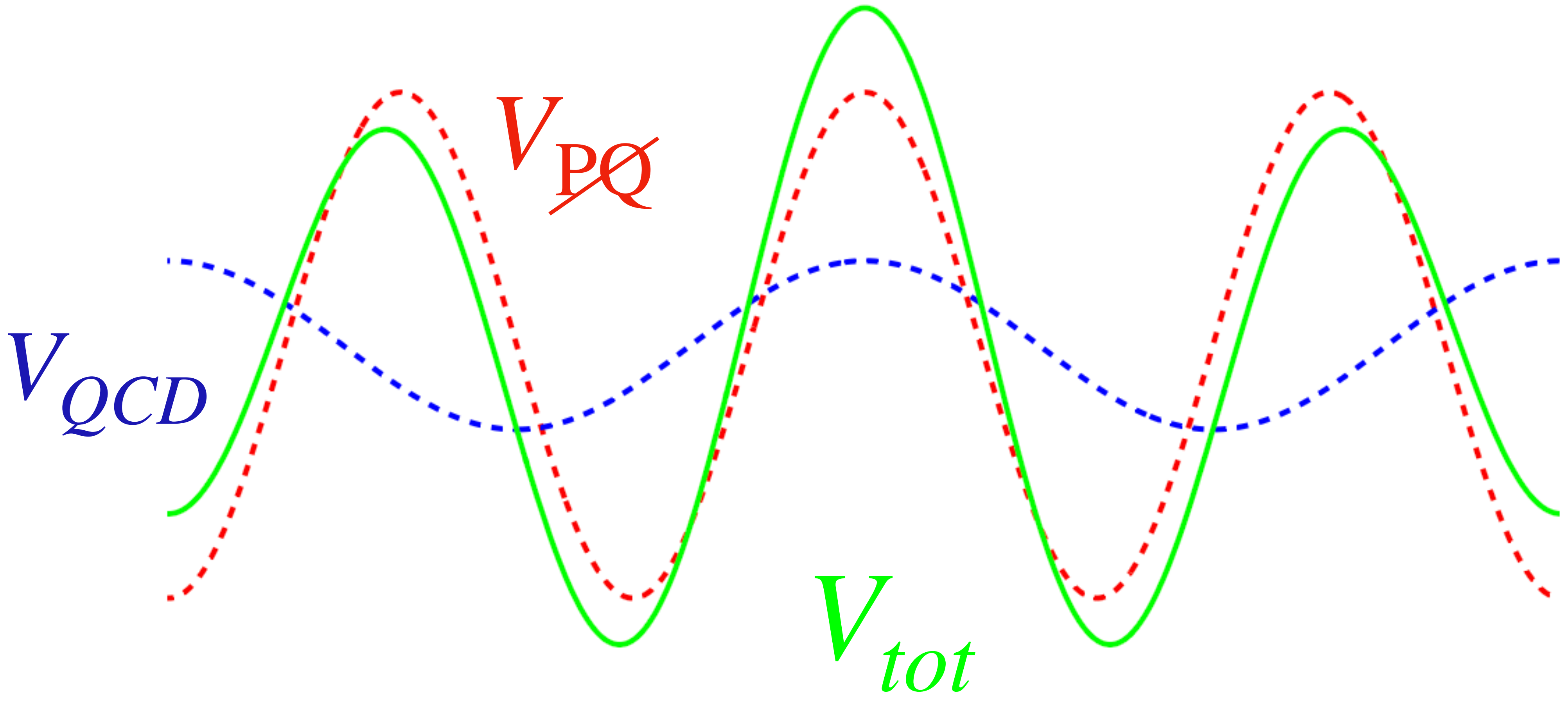
$T \sim v_{PQ}$, P obtains VEV. S keeps large field value.



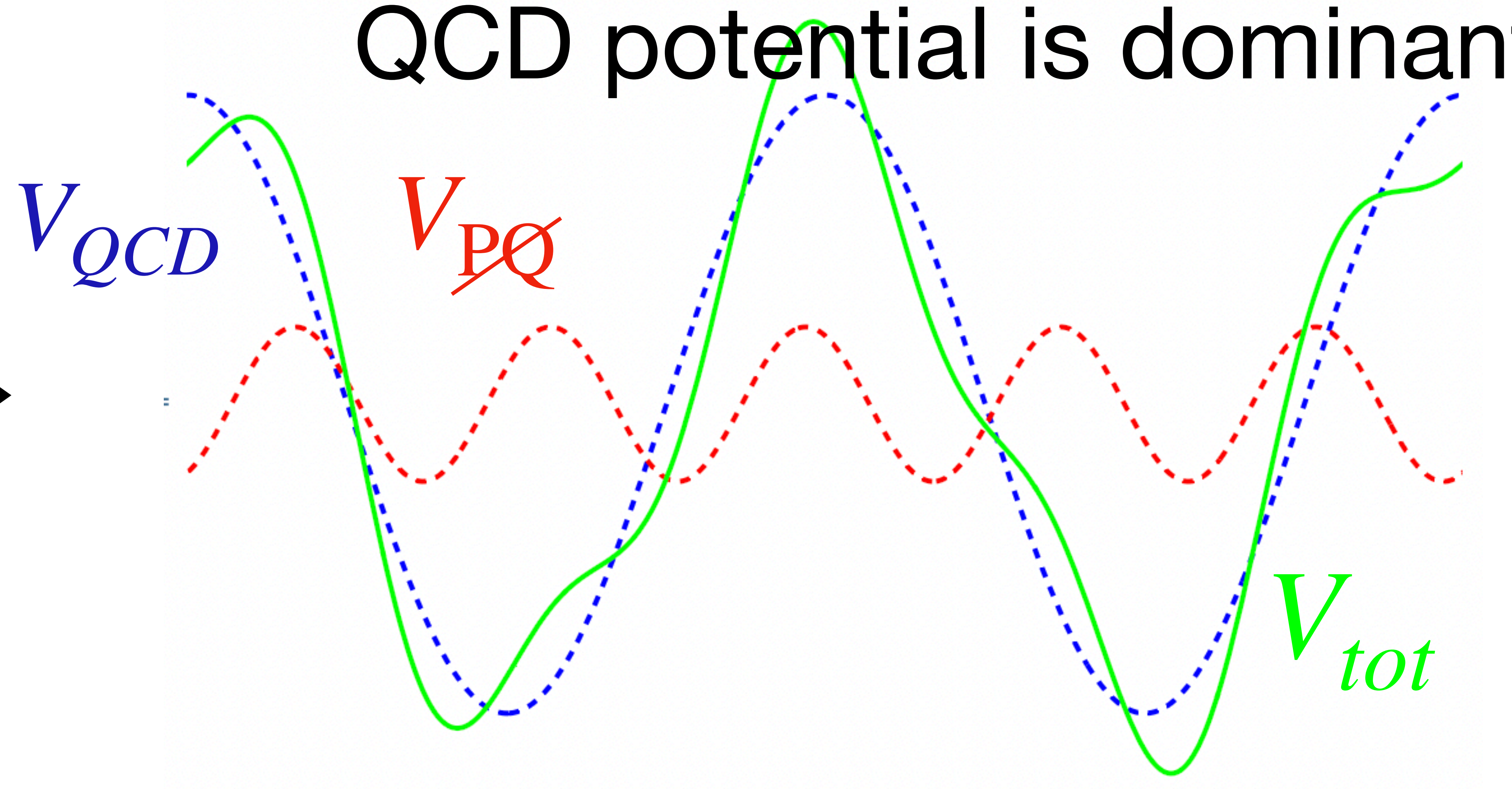
$H \sim m_{PQ}$, axion oscillates and $V_{\cancel{PQ}}$ is dominant.

$T > \Lambda_{\rm QCD}$ $\quad \langle |S_{\rm RD}| \rangle \propto \left(\frac{H}{M_{Pl}}\right)^{\frac{1}{n-1}} M_{Pl}$

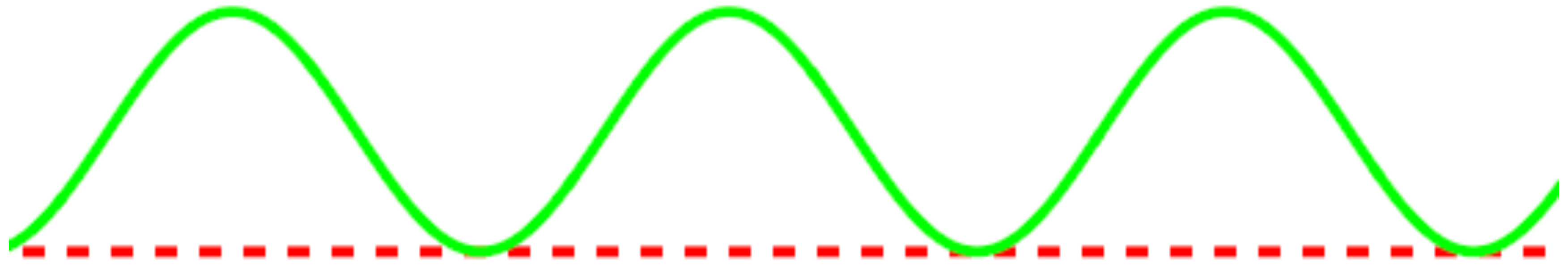
$T \sim \Lambda_{\rm QCD}$, QCD potential appears.



QCD potential is dominant.



$H \sim m_S$,S field oscillates to zero.

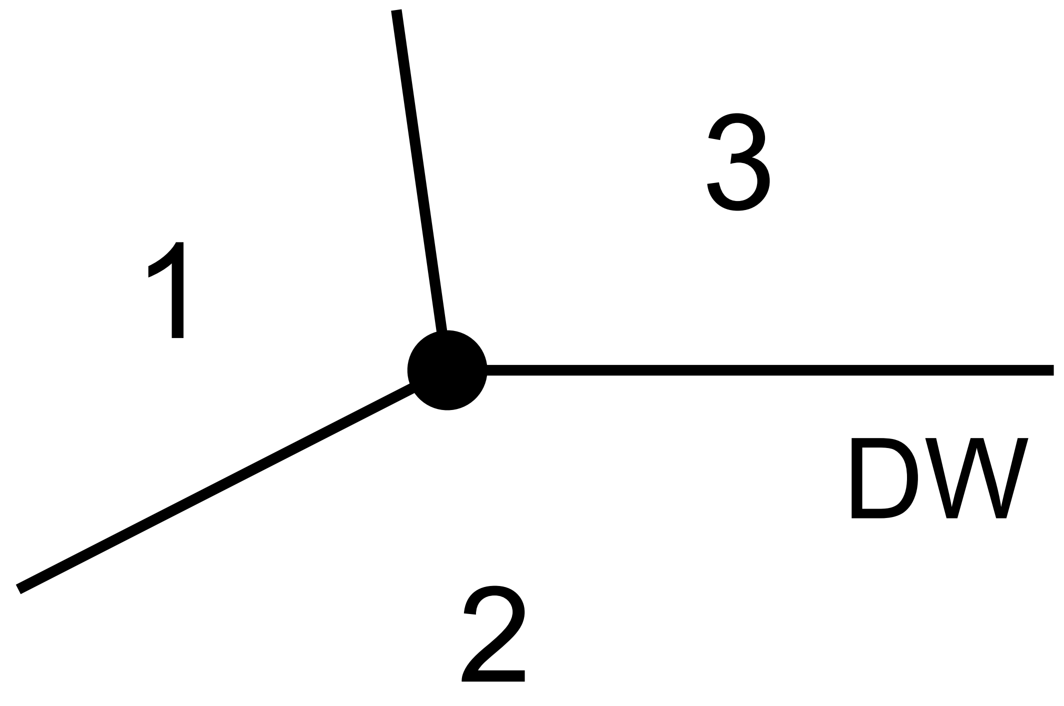


$V_{\cancel{PQ}}$ dynamically vanishes to zero.

3. Evolution of string-wall system

After inflation, $T \sim v_{PQ}$ then P acquires a VEV, cosmic string appears.
In our scenario, the axion starts to oscillate before the QCD crossover, DWs are produced through the mixing potential and connected with the cosmic strings.

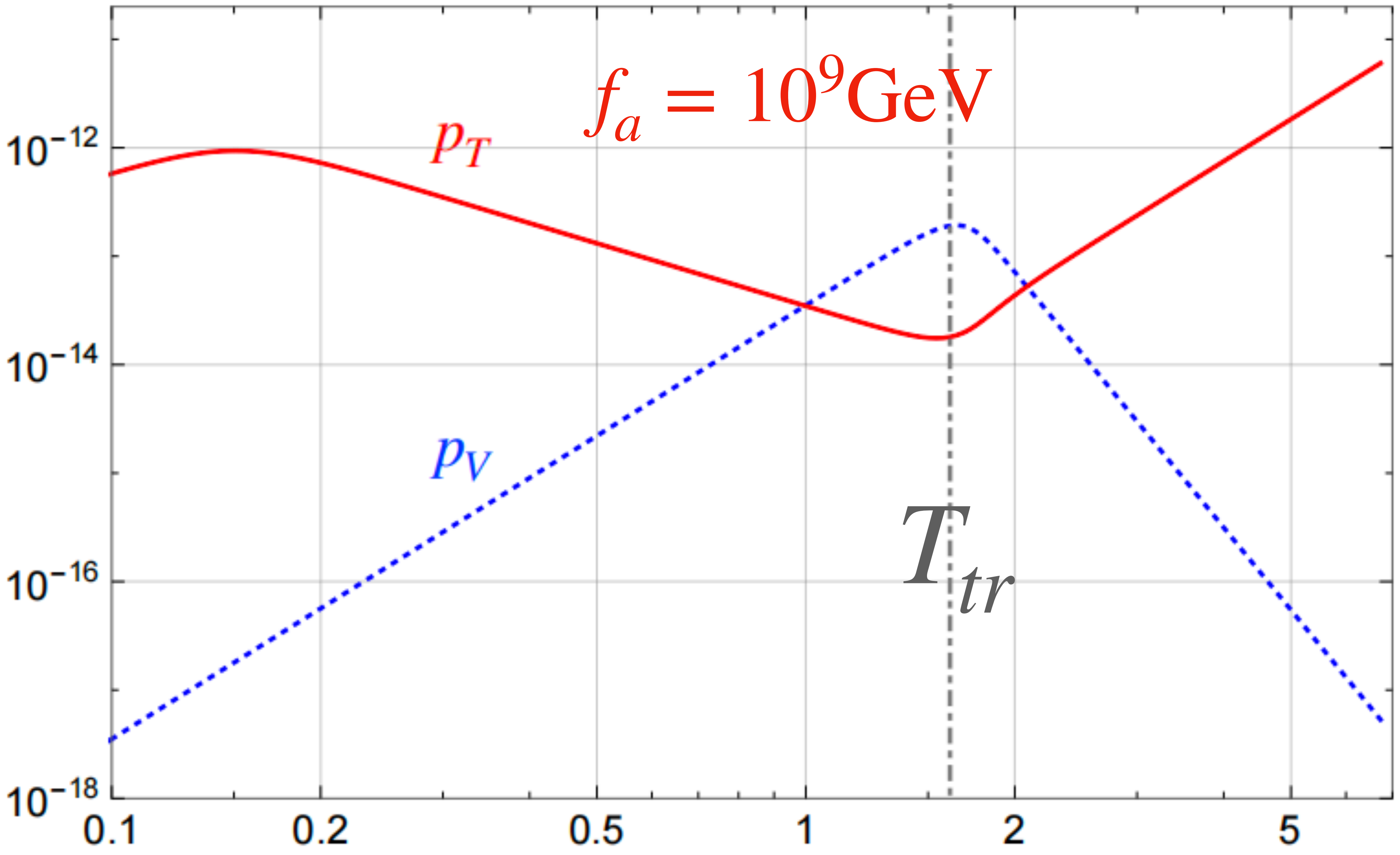
$l = 3$ # DW for $V_{\cancel{PQ}}$



For $l > 1$ long-lived system, the string-wall system must be broken due to some destabilization effect.

- Volume pressure
- Structural instability

Volume pressure



Hao,Nakagawa, Nakai, Suzuki (2025)

A bias term against the string-wall systems generates the volume pressure

$$p_V \simeq \Delta V_{bias}$$

When the volume pressure is comparable to the tension force, the system starts to destabilized.

$$p_T \sim \sigma_{\rm wall} H$$

$f_a \lesssim 10^9$GeV is required for the system collapse due to p_V.

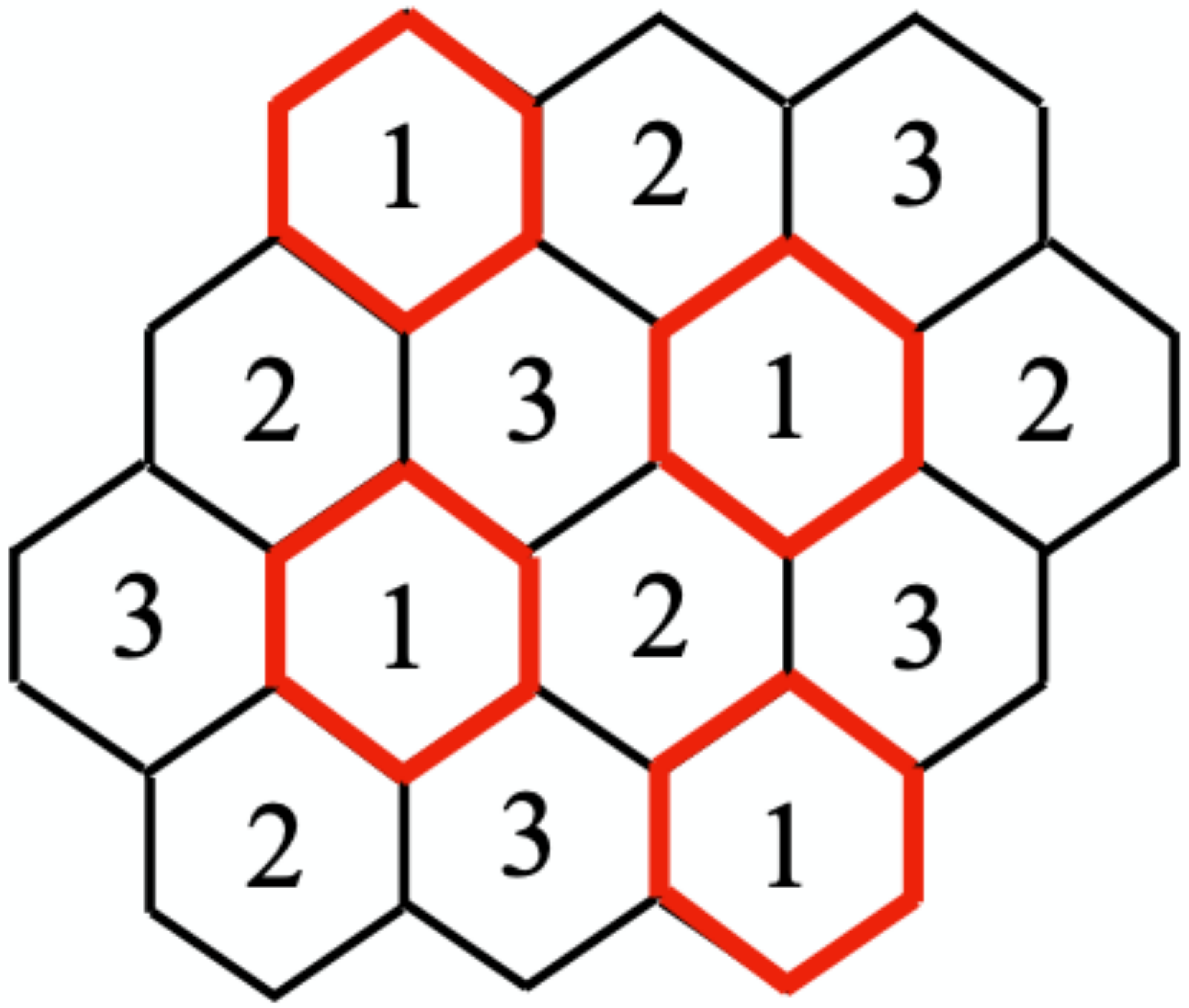
When $|V_{\cancel{PQ}}| \sim |V_{\rm QCD}|$ $\longrightarrow$ $T_{\rm tr} \simeq 1.6\ {\rm GeV} \left(\frac{|\lambda|}{0.01}\right)^{-\alpha} \left(\frac{v_{PQ}}{2\times 10^9\ {\rm GeV}}\right)^{-\ell\alpha}$

We assume that the annihilation occurs at $T_{\rm ann} = \kappa T_{\rm tr}$. $(\kappa < 1)$

$$\Omega_{a,\rm dec}h^2 \simeq 0.12 \frac{1}{\sqrt{1+\epsilon_a^2}} \left(\frac{\kappa}{0.1}\right)^{-1} \left(\frac{|\lambda|}{2\times 10^{-4}}\right)^{\alpha} \left(\frac{N_{\rm DW}}{2}\right)^{\ell\alpha} \left(\frac{f_a}{2.4\times 10^{10}\ {\rm GeV}}\right)^{1+\ell\alpha}$$

Structural instability

The first time be proposed!



$(N_{\mathrm{DW}}, l) = (2,3)$

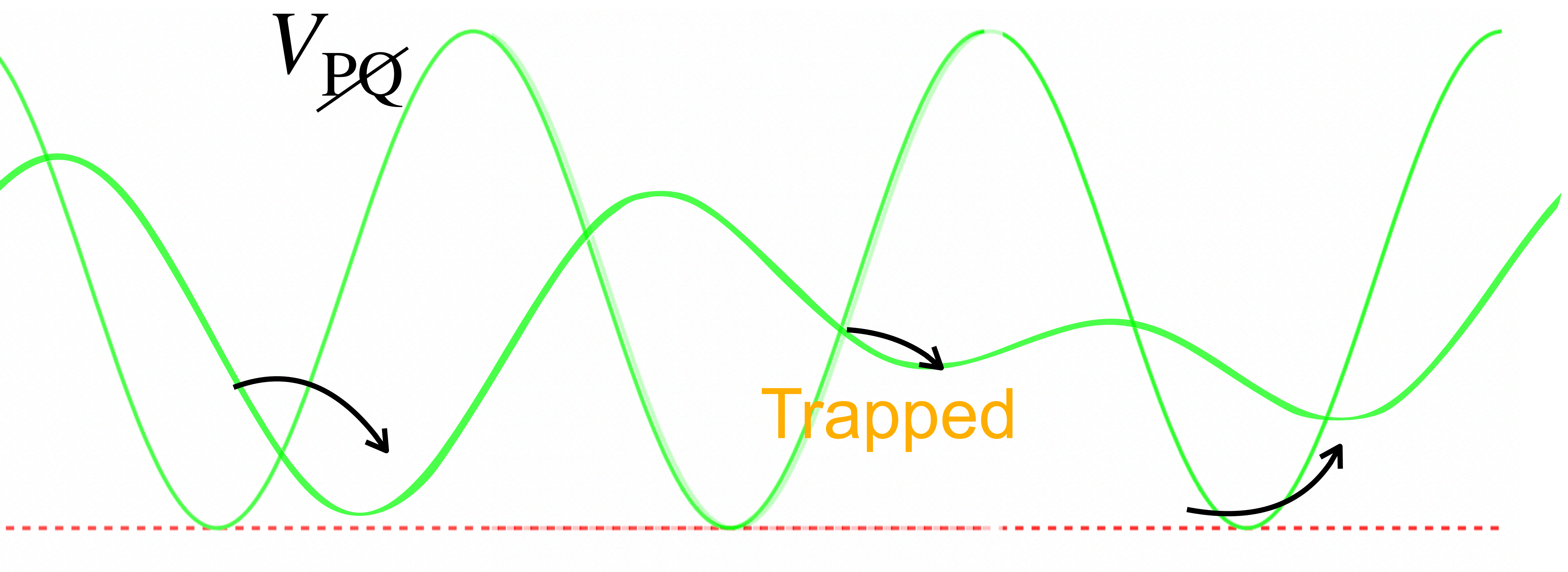
Hao,Nakagawa, Nakai, Suzuki (2025)

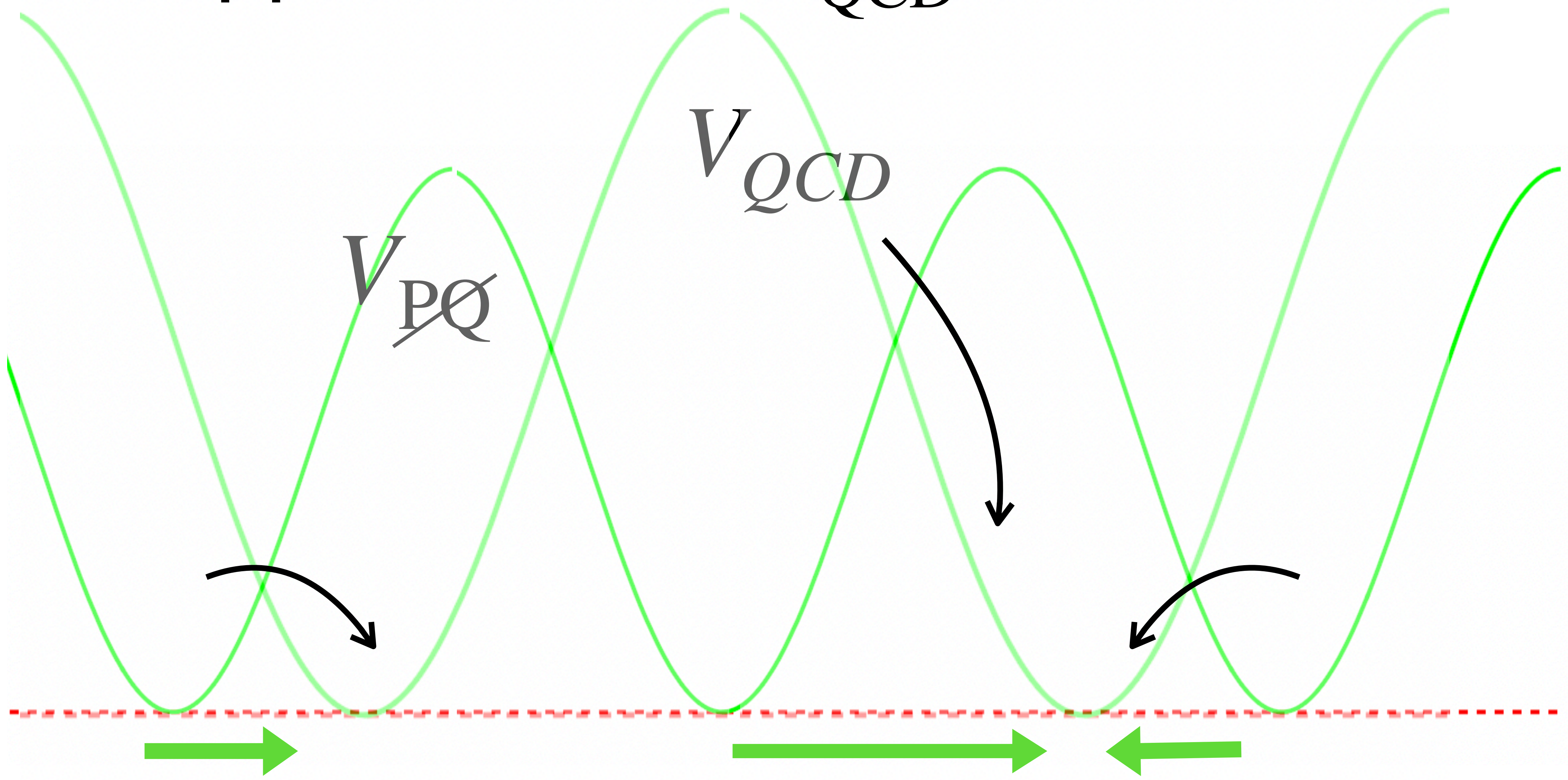
After V_{QCD} becomes dominant, the vacua numbered by 2 and 3 are integrated, and one vacuum is isolated. The red lines detnote the final domain walls.

Even for $f_a > 10^9\mathrm{GeV}$, the system is able to be destabilized by the structural instability.

4. Misalignment contribution

- In the smooth shift regime, the minimum of the PQ breaking term where the axion first starts to oscillate is continuously connected to the minimum of $V_{\cancel{PQ}}$.
- In the trapping regime, the axion first starts to oscillate in a wrong vacuum, and gets trapped there for a while until the false vacuum disappears when $V_{\rm QCD}$ becomes dominant.

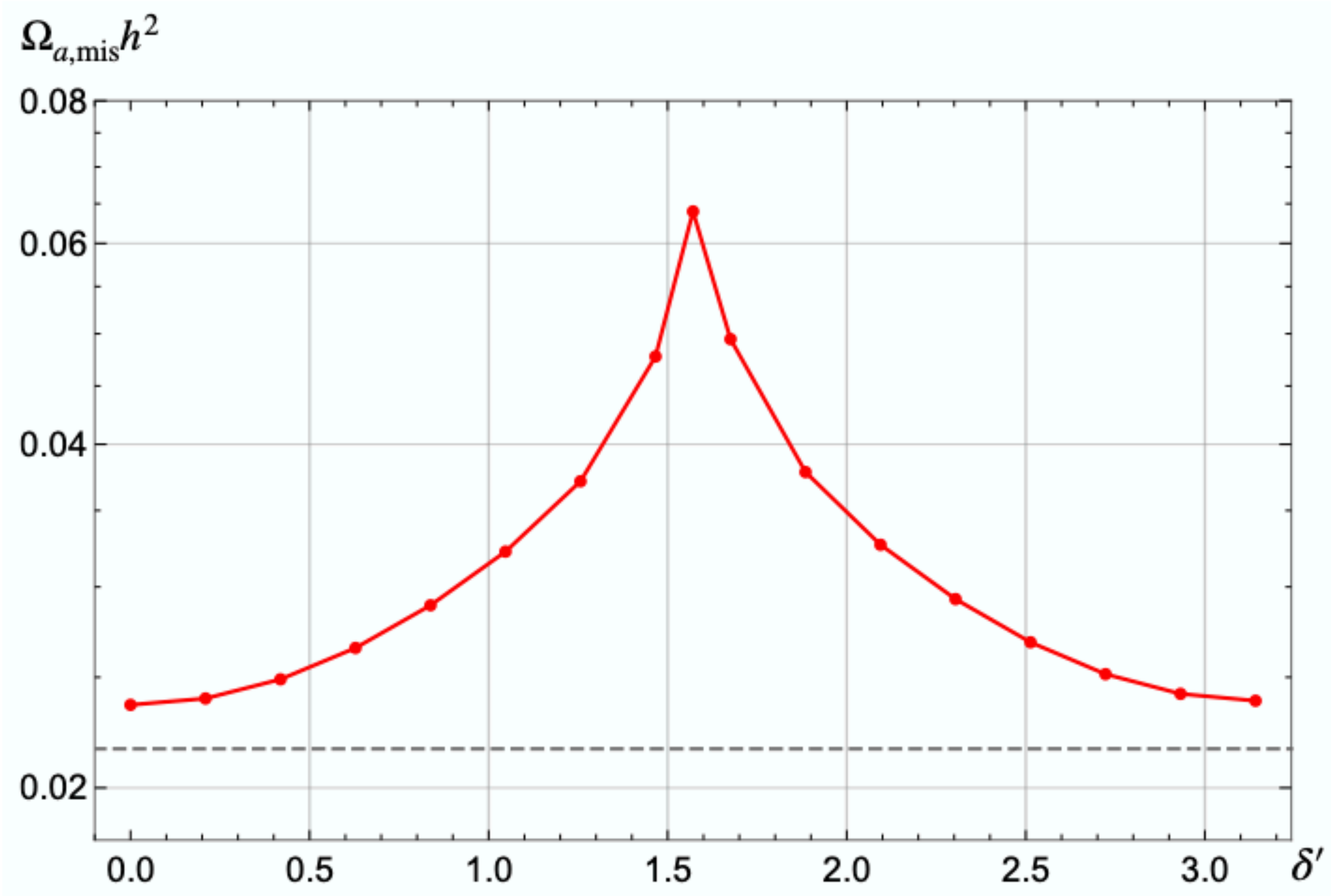




- The number density can be estimated as the average,

$$n_a \simeq \frac{1}{2\pi}\int_{-\pi}^{\pi} d\theta_{\rm ini} n_a(\theta_{\rm ini}) \simeq \frac{1}{l}\sum_{r=1}^{l} n_a^{(r)}$$

cf) Trapped misalignment,Shota Nakagawa, Takahashi, Yamada (2020). Di Luzio, Gavela, Quilez, and Ringwald (2021).
Jeong, Matsukawa,Shota Nakagawa, Takahashi (2022).

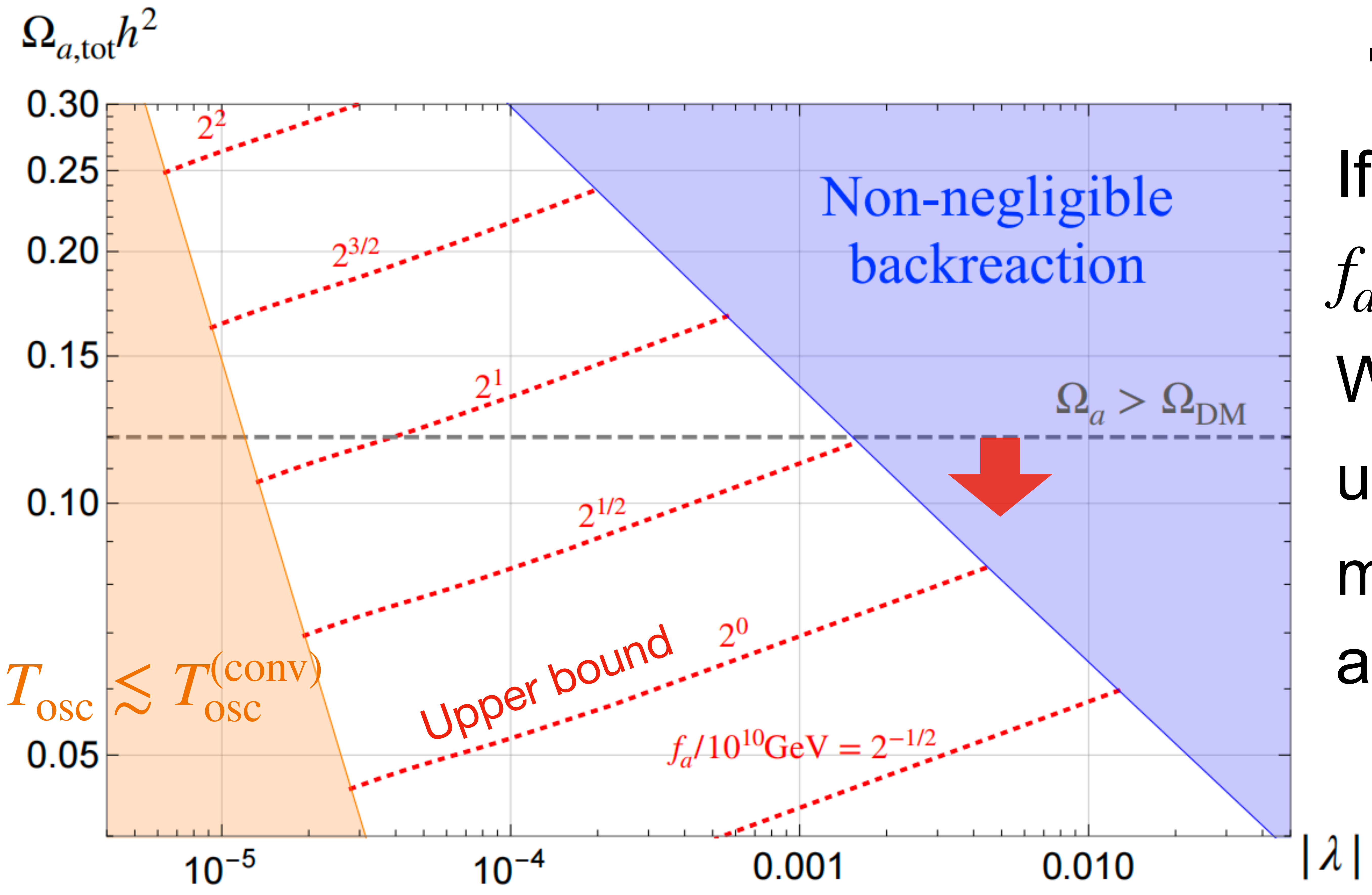


The abundance can be enhanced compared to the conventional one due to the trapping effect.

$$f_a = 10^{10}\mathrm{GeV}$$

$$\delta' \equiv m\frac{b}{\chi} + \delta$$

5. Viable parameter spaces



Hao,Nakagawa, Nakai, Suzuki (2025)

$(N_{\mathrm{DW}}, l, m, n) = (2,3,9,6) \quad \kappa = 0.1$

$$\Omega_{a,\mathrm{tot}} = \Omega_{a,\mathrm{dec}} + \Omega_{a,\mathrm{mis}}$$

If κ is very small, the upper bound on f_a would be severer.

When κ is relatively large and the upper bound on f_a is large, the misalignment axion production gives a relevant contribution.

DW problem can be solved.

$f_a \lesssim 2 \times 10^{10}\mathrm{GeV}$

Summary

- In order to solve the strong CP problem, we consider axion as an elegant solution, which leads to the domain wall problem.
- To solve it, we consider the mixing coupling that induces a time-dependent bias potential, which makes the string-DW system unstable.
- DM provides a natural constraints of abundance so that we can research on appropriate parameter sets.
- The abundance is contributed by the misalignment mechanics and the decay of domain walls.

Thanks you for listening !

Back up

We neglect the phase direction of S and assume that the mixing term is negligibly small compared to portion of S field.

$(m \gg l)$

$$\cancel{V_{\rm PQ}} \simeq -\frac{1}{\ell^2} m^2_{\cancel{\rm PQ}} v^2_{\rm PQ} \cos\left(\ell \frac{a}{v_{\rm PQ}} + m\frac{b}{\chi} + \delta\right)$$

For simplicity, b is a constant.

The energy density is estimated as the time average of the kinetic energy, $\rho_a = \frac{\dot{a}^2}{2} + \cancel{V_{\rm PQ}} + V_{\rm QCD} = 2/\delta t \int_{\delta t} \frac{\dot{a}^2}{2} dt$

2 Axions: a & b

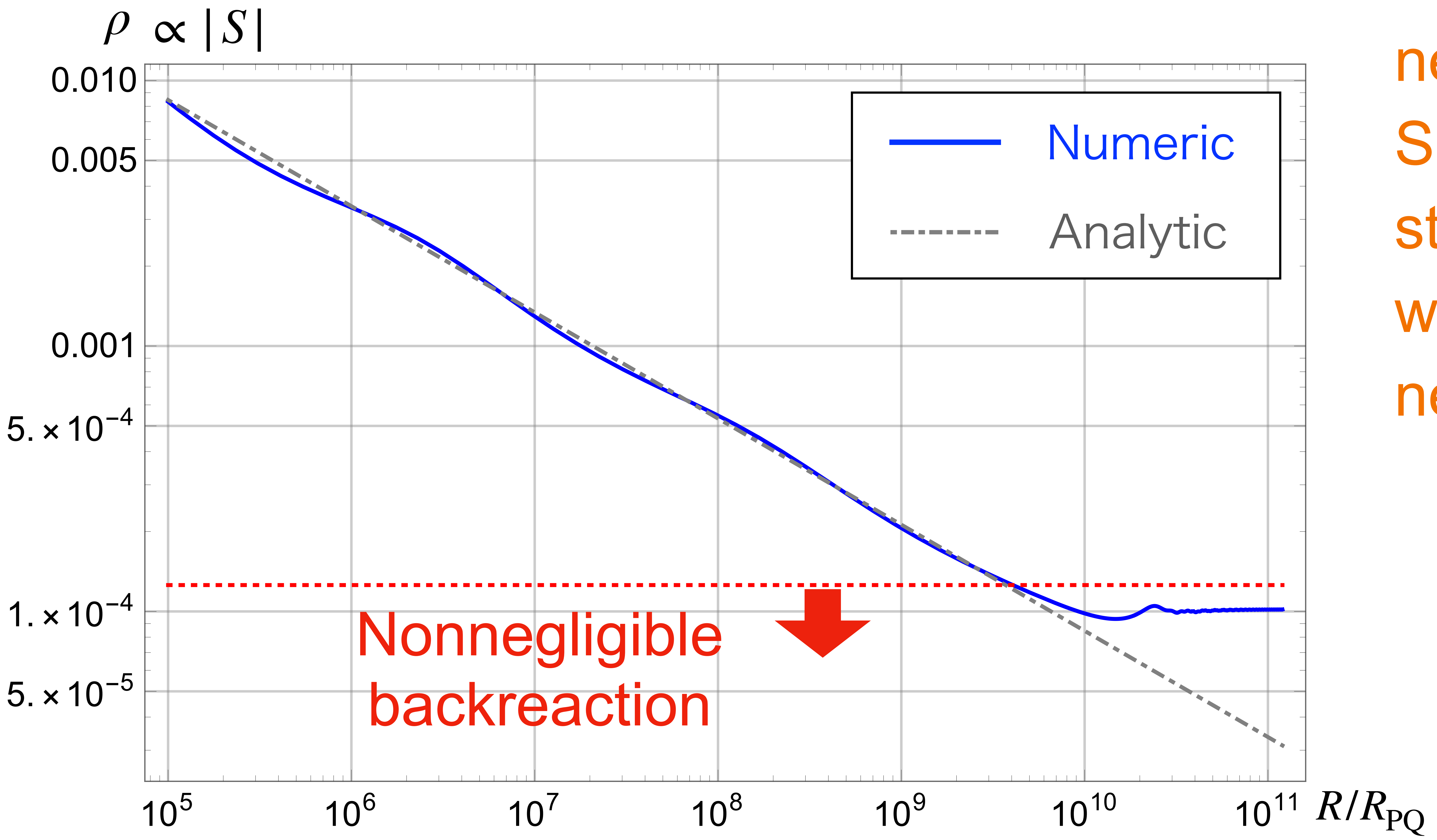
$$\left(\frac{\lambda}{m!\ell!M_{\rm Pl}^{m+\ell-4}}S^m P^\ell + \text{h.c.}\right) \quad \propto \cos(l\theta + m\theta_b + \delta)$$

$\rightarrow$ $T \sim m_b$ b slightly changes the potential given by a, but a still oscillates around the local minimum. DWs exists

$\rightarrow$ $T \sim \Lambda_{\rm QCD}$ DWs decay due to the bias term.

2 Axions: a & b

In this case, backreaction is not excluded.



Energy is minimized, so that the sign is flipped to negative.
S doesn't disappear but is stuck at a fixed point, which introduces the negative dark energy.