WIMP Meets ALP: Coherent Freeze-out of Dark Matter

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based on **arXiv: 2511.16731 (PRL 2026), arXiv: 2608.xxxxx**

in collaboration with Steven Ferrante and Maxim Perelstein

Outline of the talk

- Motivation
- Framework and dynamics
- Phenomenology
- Conclusions

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- **Motivation**
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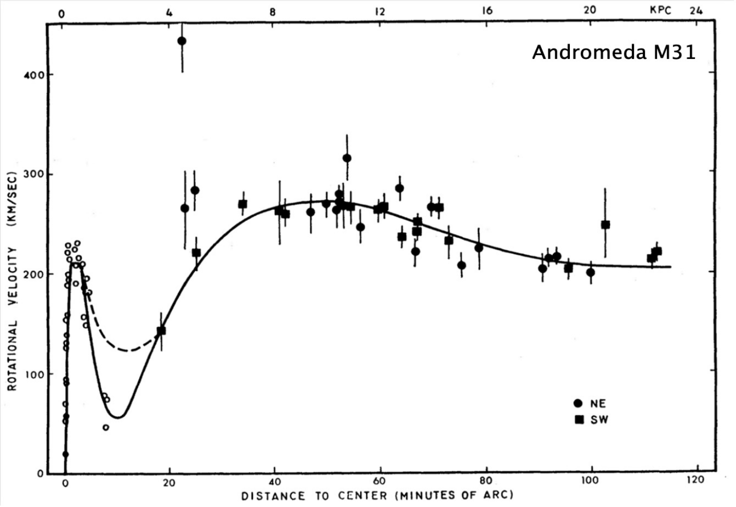
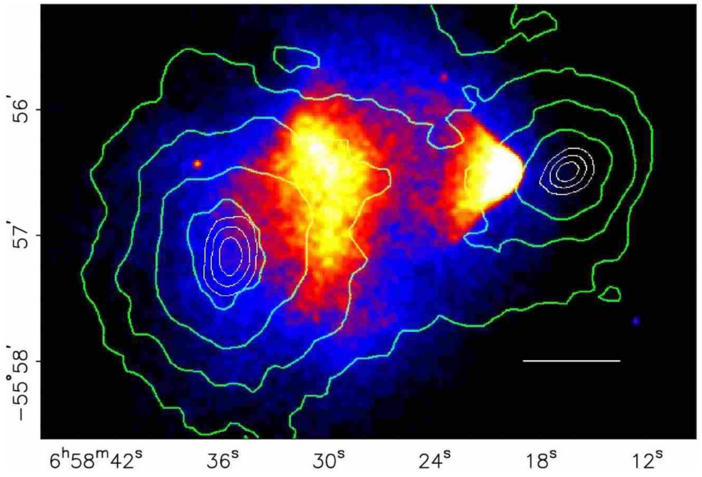
Evidence for dark matter

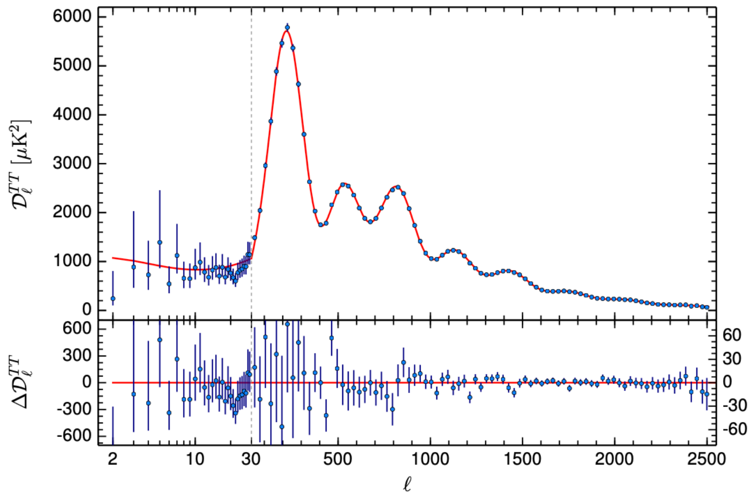
- A lot of evidence for DM from astrophysics and cosmology



Galaxy rotation curve



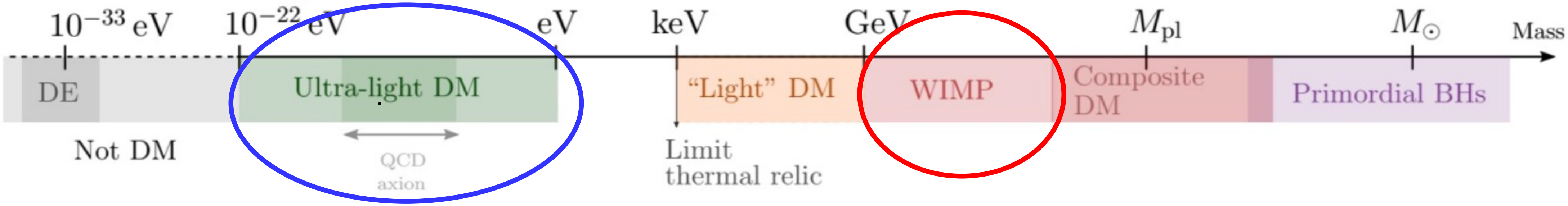
Bullet cluster



CMB anisotropies

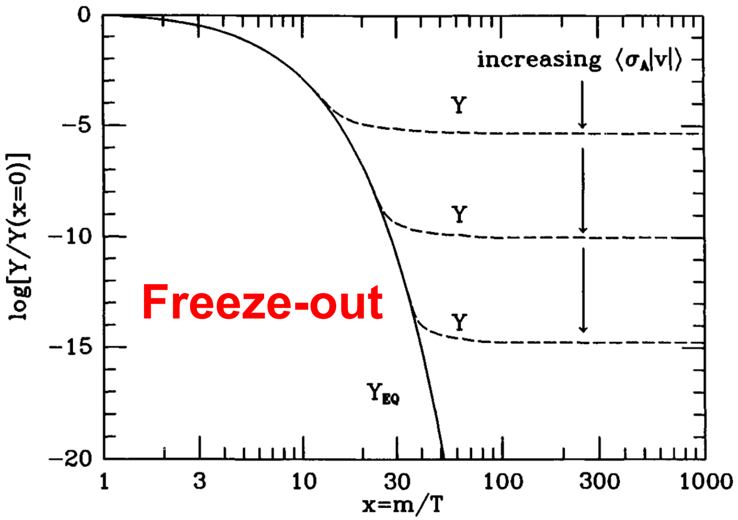
Particle nature of dark matter

- Yet, we know very little about the nature of dark matter



- For many reasons, **WIMP** and **ALP** are two leading candidates for particle dark matter -- they exhibit sharply **different** cosmological histories

Cosmic history of WIMP



- Thermalized at high temperatures, decoupled later

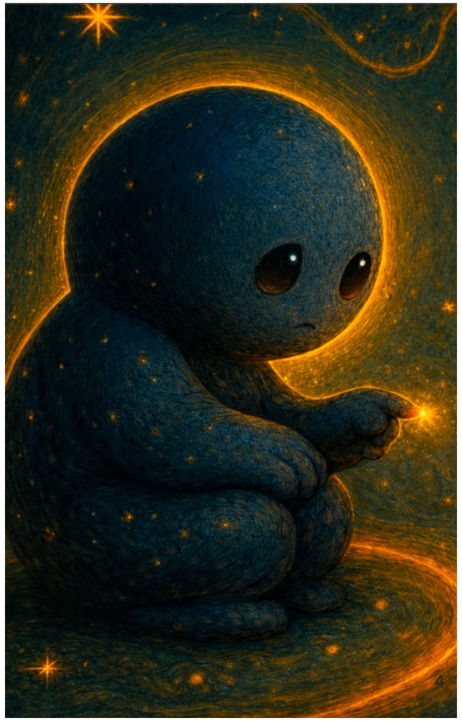
$$\frac{\mathrm{d}n_\chi}{\mathrm{d}t} + 3Hn_\chi = -\langle\sigma v\rangle\left(n_\chi^2 - n_{\chi,\mathrm{eq}}^2\right)$$

- Relic abundance insensitive to the initial condition

$$\frac{\Omega_\chi h^2}{0.12} \sim \left(\frac{x_{\mathrm{fo}}}{25}\right)\left(\frac{10^{-26}\mathrm{cm}^3/\mathrm{s}}{\langle\sigma v\rangle}\right)$$

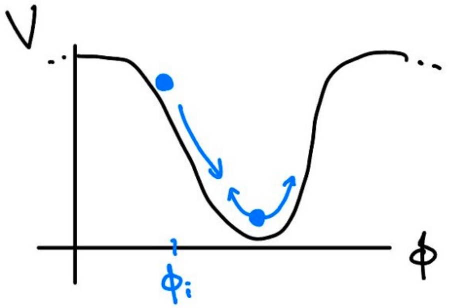
Pure weak-scale physics, “WIMP miracle”

Strong experimental constraints



Cosmic history of ALP

- Feeble coupling, never thermalized, described by a **homogeneous**, **classical** field



$$\ddot{\phi} + 3H\dot{\phi} + m_\phi^2 \phi = 0$$

- Initial field value **misaligned**, relic abundance depend on both initial field value and mass

$$\frac{\Omega_\phi h^2}{0.12} \sim \left(\frac{\phi_i}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{m_\phi}{10^{-10}\ \mathrm{eV}}\right)^{1/2}$$

Sensitive to the UV physics scale >> weak scale

What about WIMP + ALP?

$$\mathcal{L} =$$

WIMP Meets ALP: Coherent Freeze-Out

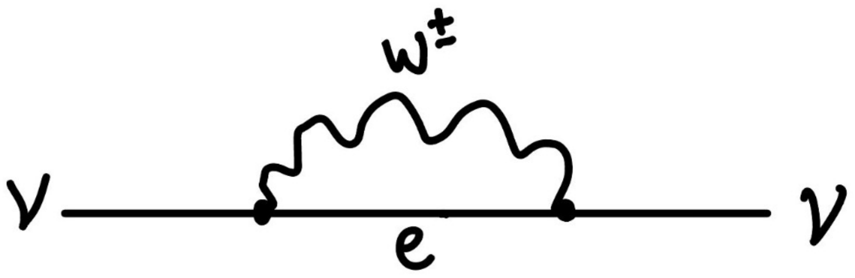
- Naively, you might think they would evolve independently, since ALP is **not** thermalized and momentum exchange is negligible

- However, physics are much richer thanks to the **coherent forward scattering** between these two sectors

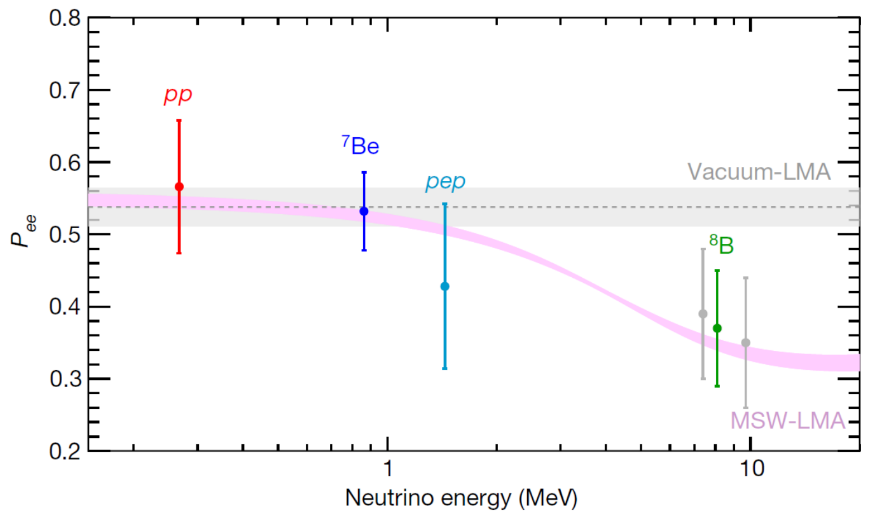
Coherent forward scattering

- No momentum exchange, but modify **dispersion relation** of scattering particles

- Example in the Standard Model: **MSW effect** of neutrino oscillation



$$E_\nu \to E_\nu + \sqrt{2} G_F n_e$$

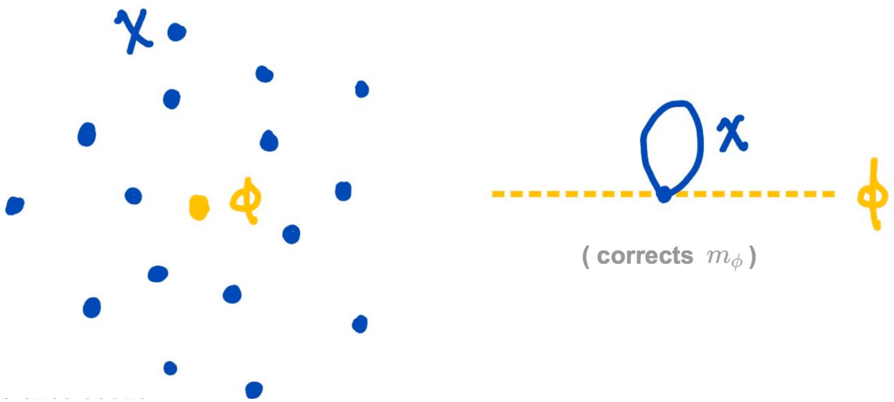


change neutrino oscillation probability by O(1),

crucial for solving the solar neutrino missing puzzle

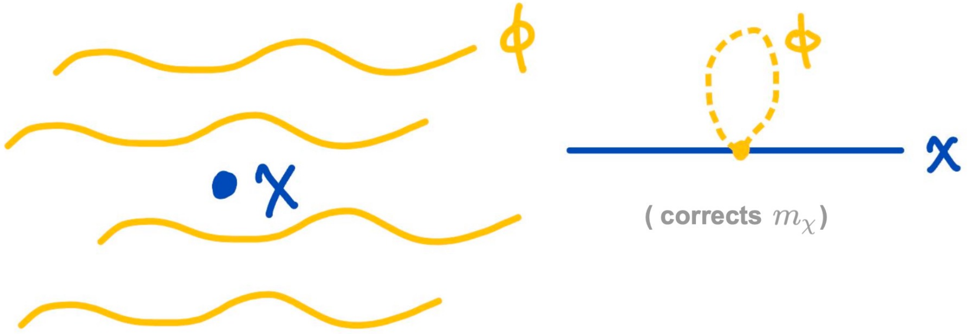
Coherent forward scattering for WIMP + ALP

- Due to coherent forward scattering, both of their dispersion relations are modified by the medium of the other one, schematically,



ALP interacts with WIMP thermal bath

$$m^2_{\phi,\mathrm{eff}} = m^2_\phi + \delta m^2_\phi$$



WIMP interacts with ALP classical background

$$m_{\chi,\mathrm{eff}} = m_\chi + \delta m_\chi$$

Coherent forward scattering for WIMP + ALP

- Due to coherent forward scattering, both of their dispersion relations are modified by the medium of the other one

- This effect is much more significant than one might naively expect. In the following, I will show it quantitatively via an explicit example

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- Motivation
- **Framework and dynamics**
- Phenomenology
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The setup

- We consider an effective **quadratic** coupling between a fermion χ (the WIMP) and a (pseudo-)scalar ϕ (the ALP):

$$\mathcal{L}_{\rm eff} \supset \frac{1}{\Lambda}\overline{\chi}\chi\frac{\phi^2}{2} \qquad\qquad (\Lambda \gg m_\chi \gg m_\phi)$$

- Motivation for this coupling: QCD axion coupling to nucleons **including the sign!**

The setup

- Non-linear sigma model contains $\mathcal{L} \sim \frac{m_q}{f_\pi^2}\,\pi^2\,\bar{N}N$
- Pion-axion mixing $\pi \to \pi + \frac{f_\pi}{f_a}\,a \quad \Rightarrow \quad \mathcal{L} \sim \frac{1}{\Lambda}\,a^2\,\bar{N}N$
- This is the **leading non-derivative** coupling of QCD axion to matter
- Our model: QCD axion -> ALP, nucleon -> WIMP (possible UV completion: dark QCD)
- Following discussions do not depend on UV completion

Dynamics

- After reheating, WIMP is in equilibrium with the SM plasma, while ALP is not populated
- Coherent forward scattering generates a "plasma mass" for ALP

$$m_{\phi,\mathrm{eff}}^2 = m_\phi^2 - \frac{\langle \bar{\chi}\chi \rangle_T}{\Lambda} \qquad \langle \bar{\chi}\chi \rangle_T = g_\chi \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \frac{m_{\chi,\mathrm{eff}}}{E_{\mathbf{k}}} f_\chi(\mathbf{k})$$

- Negative mass-squared indicates a **symmetry breaking** for ALP at high-T, generating a nonzero ALP VEV. This in turn shifts the WIMP mass

$$m_{\chi,\mathrm{eff}} = \left| m_\chi - \frac{\phi^2}{2\Lambda} \right|$$

- As the Universe expands and T decreases, the evolution of the two sectors is **coupled** via these effects

Effective ALP potential at finite T

$$x \equiv m_\chi / T$$

$$\varphi \equiv \phi / \sqrt{m_\chi \Lambda}$$

$$\gamma \equiv |1 - \varphi^2/2|$$

$$V(\phi, T) \equiv \frac{g_\chi}{2\pi^2} m_\chi^4 \mathcal{V}(\varphi, x)$$

- Coherent forward scattering on WIMP plasma modifies the ALP potential

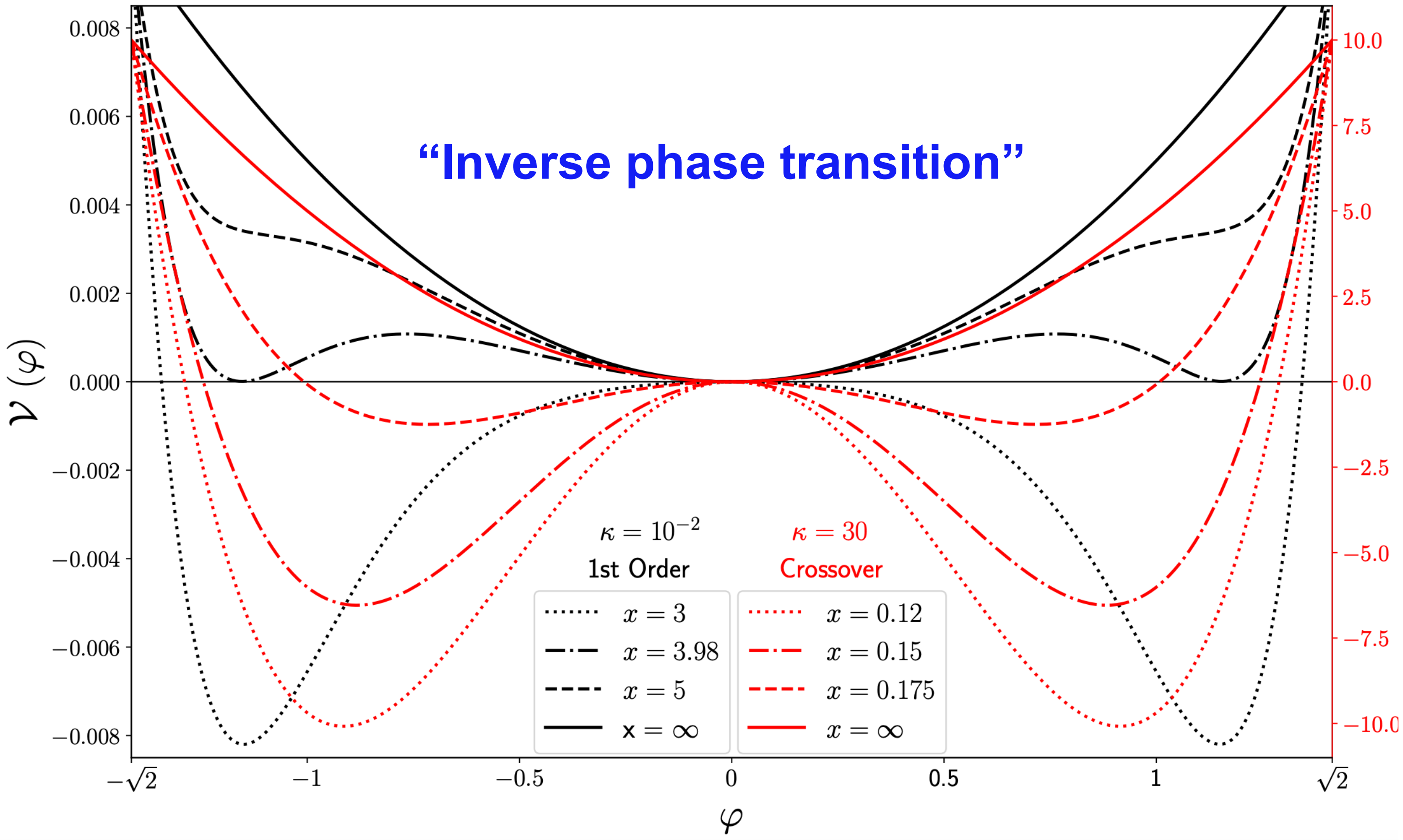
$$\mathcal{V}(\varphi, x) = \boxed{-\frac{\varphi^2}{2} \frac{\gamma^2 K_1(\gamma x)}{x}} + \boxed{\kappa \frac{\varphi^2}{2}}$$

$$\kappa \equiv \frac{2\pi^2}{g_\chi} \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

thermal mass term **bare mass term**

- **High-T:** symmetry is broken, ALP obtains a VEV, reducing the WIMP mass

  Low-T: symmetry is restored, VEV returns to zero, WIMP freezes out



- Thermodynamics picture --- system tends to minimize the free energy:

$$F = E - TS$$

- Broken-symmetry phase has higher entropy, because WIMPs are ~ massless and so relativistic! It is therefore preferred at high T

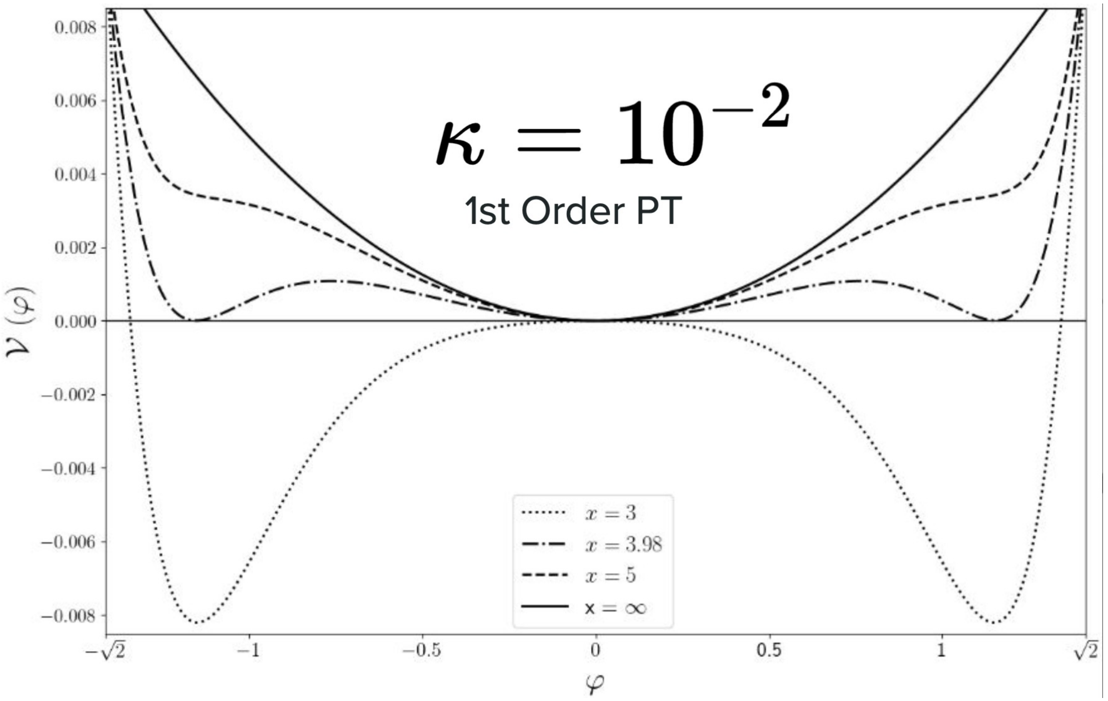
- **High-T:** symmetry is broken, ALP obtains a VEV, reducing the WIMP mass

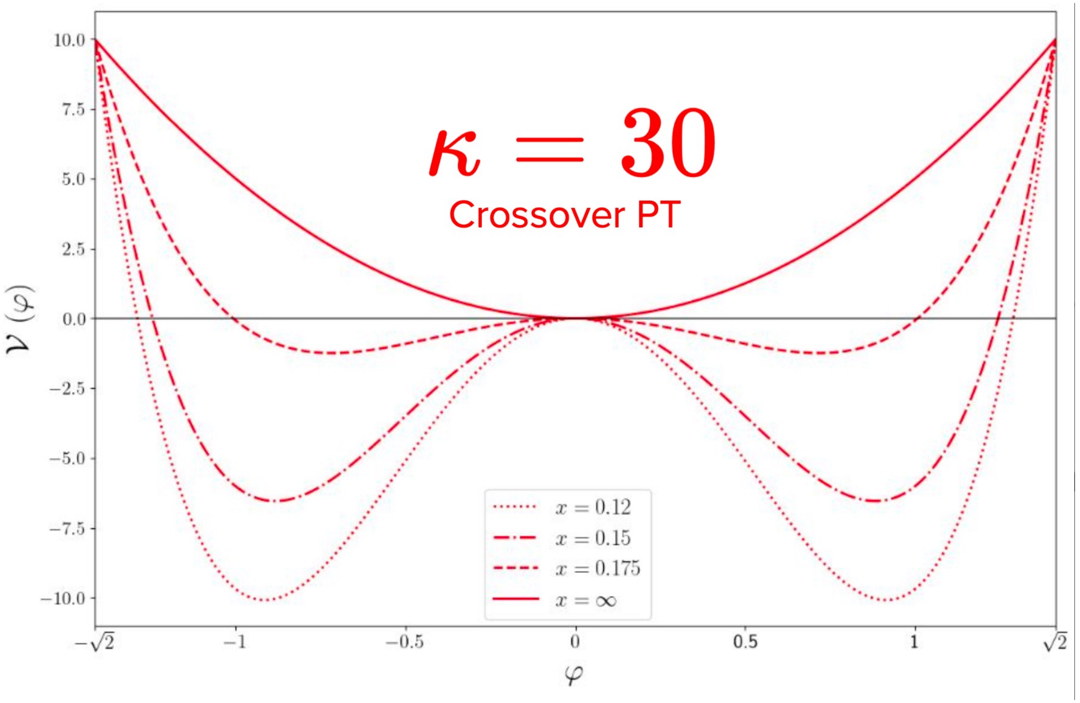
Low-T: symmetry is restored, VEV returns to zero, WIMP freezes out

Orders of the phase transition

$$\mathcal{V}(\varphi, x) = -\frac{\varphi^2}{2}\frac{\gamma^2 K_1(\gamma x)}{x} + \kappa\frac{\varphi^2}{2}$$

- Using Ginzburg-Landau theory, we find $\kappa_c \sim 0.27$ splits FOPT and crossover

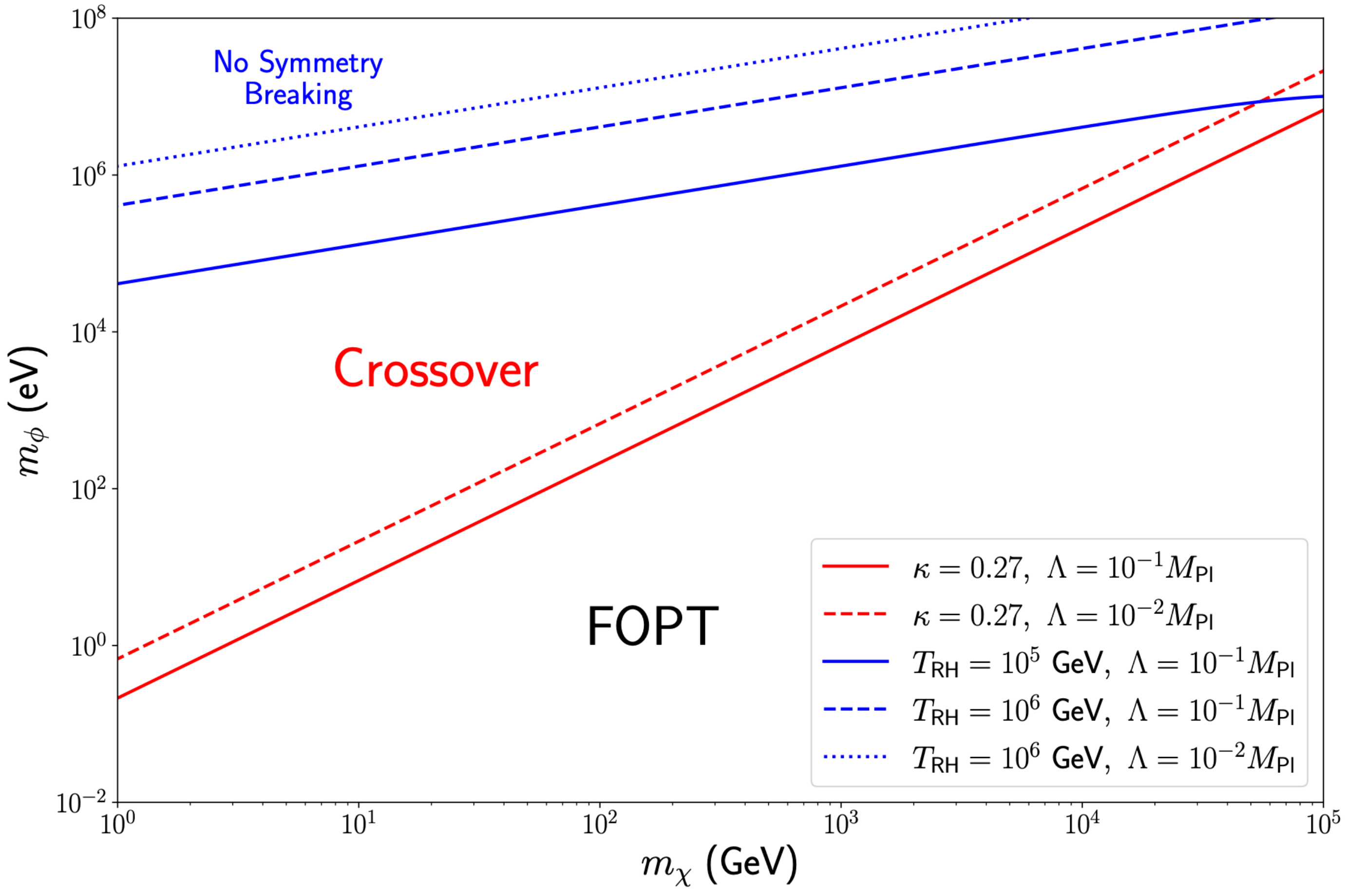




$\kappa < \kappa_c$: FOPT

$\kappa > \kappa_c$: Crossover

Classification of parameter space



- The boundary between the two is dictated by the dimensionless parameter

$$\kappa \equiv \frac{2\pi^2}{g_\chi} \frac{m_\phi^2 \Lambda}{m_\chi^3} \,.$$

- At high T it can be shown that

$$\kappa \sim \frac{m_\phi^2}{|m_T^2|\, x^2}$$

- So κ is a measure of relative importance of the thermal vs. bare ALP mass

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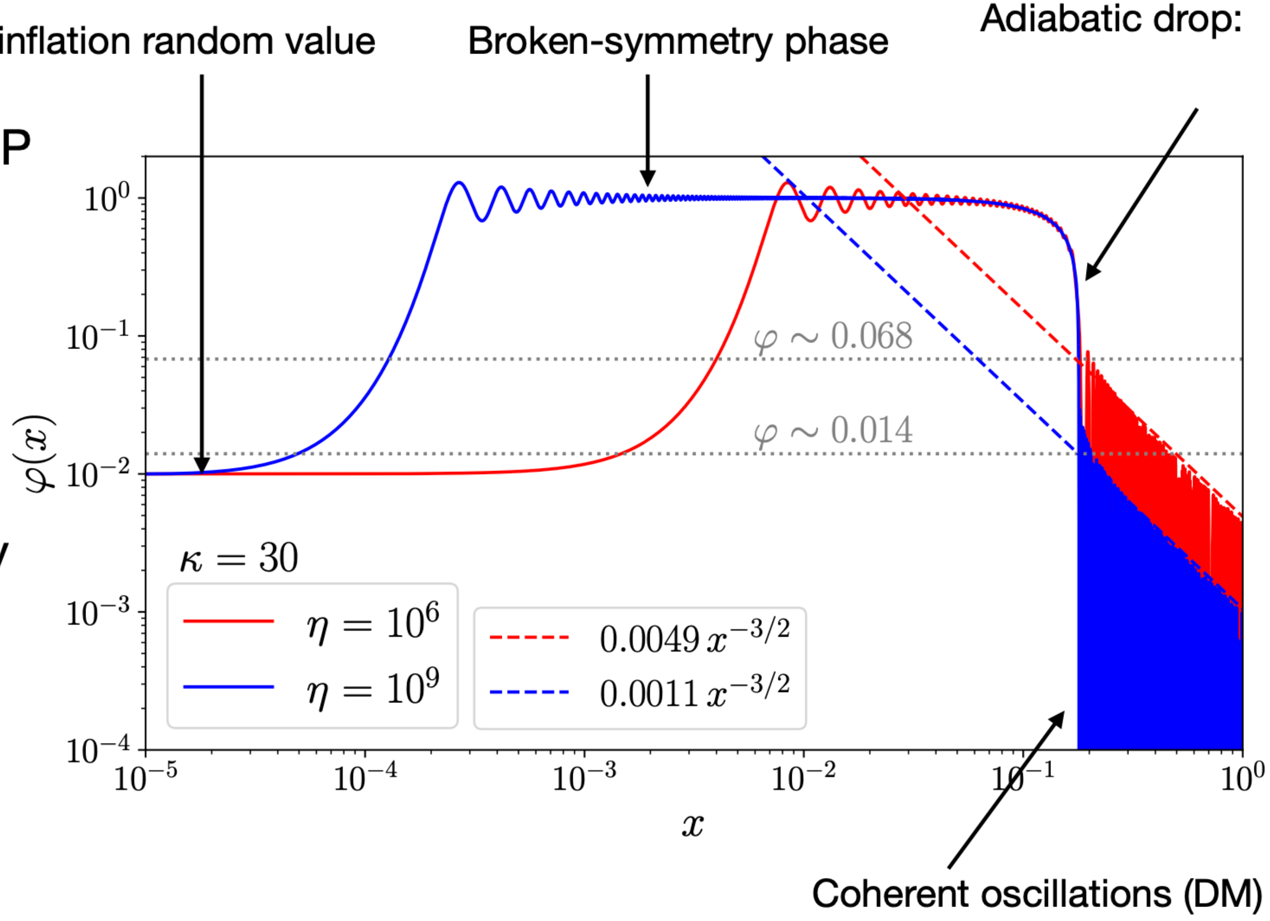
Crossover regime

(both WIMP and ALP contribute to dark matter)

Crossover Regime

- In the crossover regime, both ALP and WIMP can contribute to DM
- WIMP contribution is computed by the standard freeze-out
- ALP DM is coherent field oscillations as in misalignment, but the initial value is fixed by the v.e.v. due to thermal potential!
- More precisely:

$$\rho_\phi(x_0) \approx \underbrace{m_\phi^2 \phi^2(x_c)}_{\text{Value at PT}} \underbrace{\left(\frac{\kappa}{\eta}\right)^{\frac{1}{2}}}_{\text{Adiabatic drop}} \underbrace{\left(\frac{x_c}{x_0}\right)^3 \frac{g_{*S}(x_0)}{g_{*S}(x_c)}}_{\text{Matter-like scaling}}$$



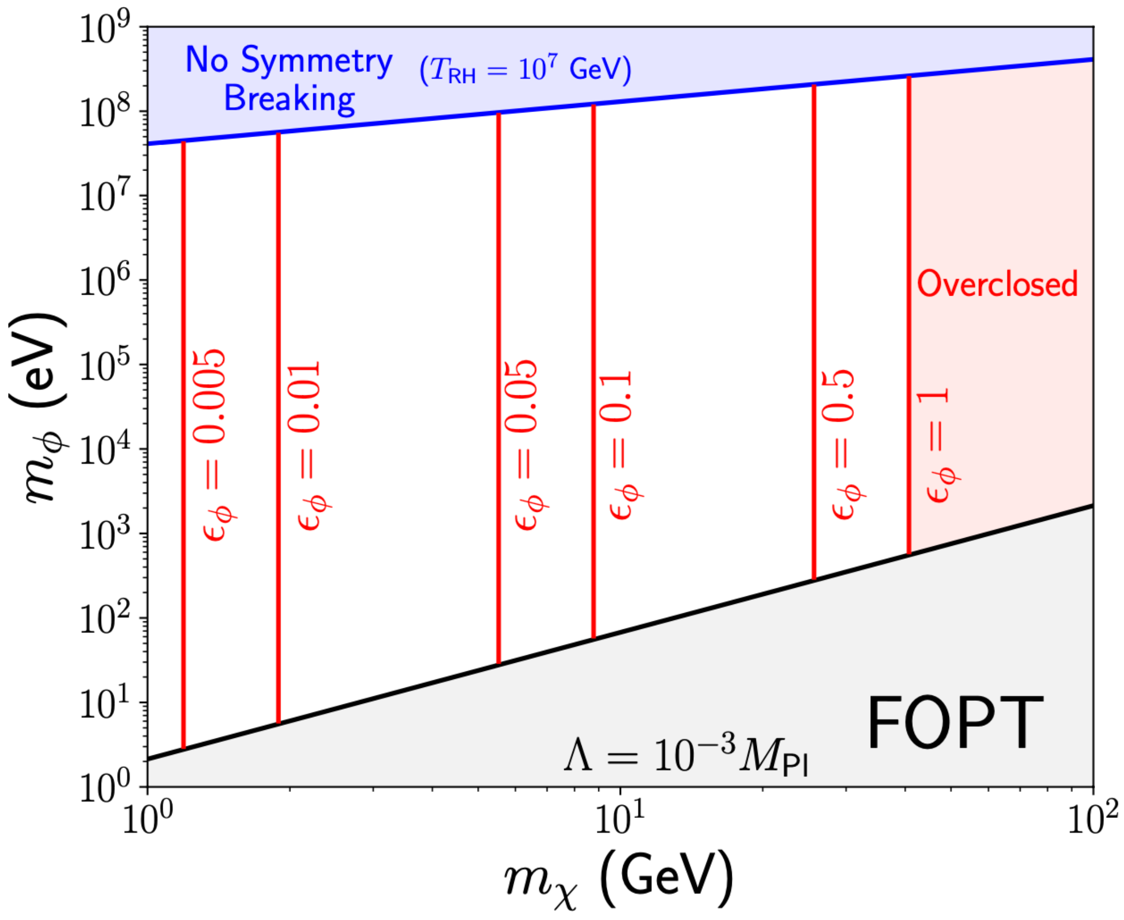
ALP Dark Matter

- Predicted ALP relic abundance:

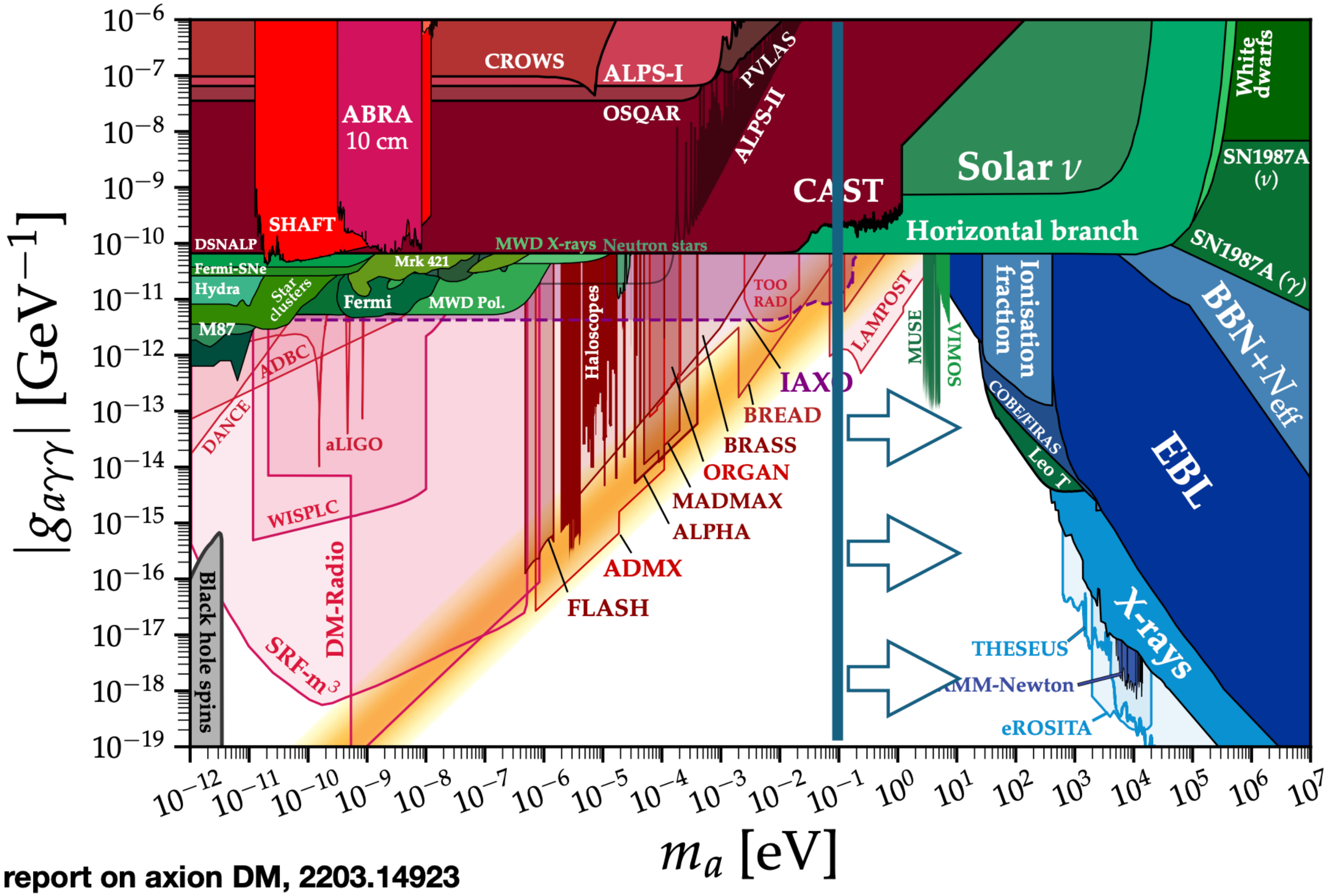
$$\Omega_\phi = \frac{1.66 \cdot 4 g_{*S}(x_0)\sqrt{2 g_\chi g_*(x_c)}}{3 g_{*S}(x_c)} \frac{m_\chi^{3/2}\Lambda^{1/2} T_0^3}{H_0^2 M_{\rm Pl}^3}$$
$$\approx 0.3 \left(\frac{m_\chi}{10\ \rm GeV}\right)^{3/2} \left(\frac{\Lambda}{0.1 M_{\rm Pl}}\right)^{1/2},$$

- Correct relic density for reasonable values of WIMP mass and coupling! (“ALP miracle”)
- No dependence on initial conditions, as opposed to the usual misalignment
- The dependence on the ALP mass has dropped out (as long as it’s consistent with the cross-over regime)
- Works for ALP mass ~ eV - MeV

$\epsilon_\phi \equiv \Omega_\phi/\Omega_{\rm DM}$



Experimental Searches for ALPs



Source: Snowmass report on axion DM, 2203.14923

First-order phase transition regime

(only WIMP contributes to dark matter)

First-Order Phase Transition

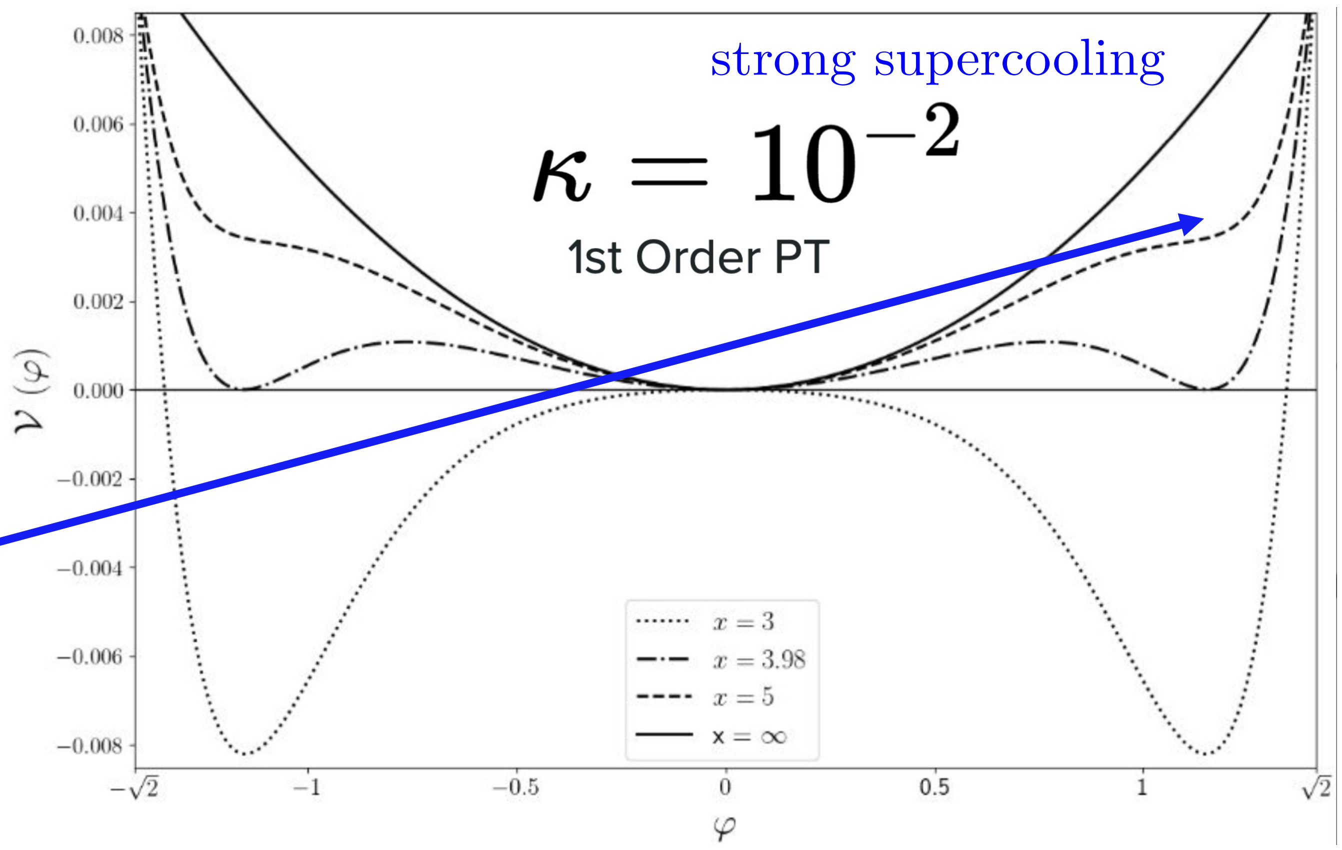
- Critical temperature is given by

$$T_c \approx \frac{m_\chi}{\sqrt[4]{2}}\kappa^{1/4}$$

- However, the ALP field remains stuck in the false vacuum long after that (tunneling rate is tiny). The PT occurs at

$$x_{\rm cfo} = (c_2/\kappa)^{1/3} \approx 0.836\,\kappa^{-1/3}$$

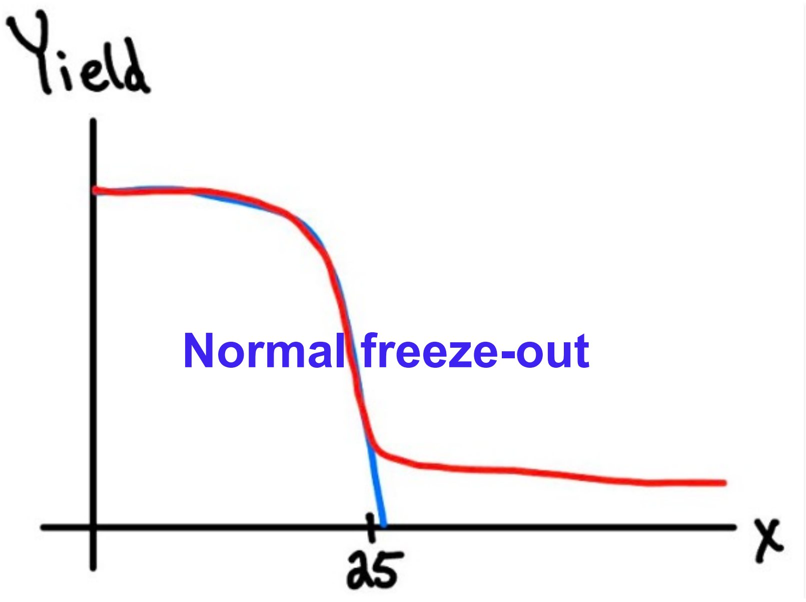
- As long as ALP has a vev, WIMP's effective mass is close to zero, and it does not freeze out!

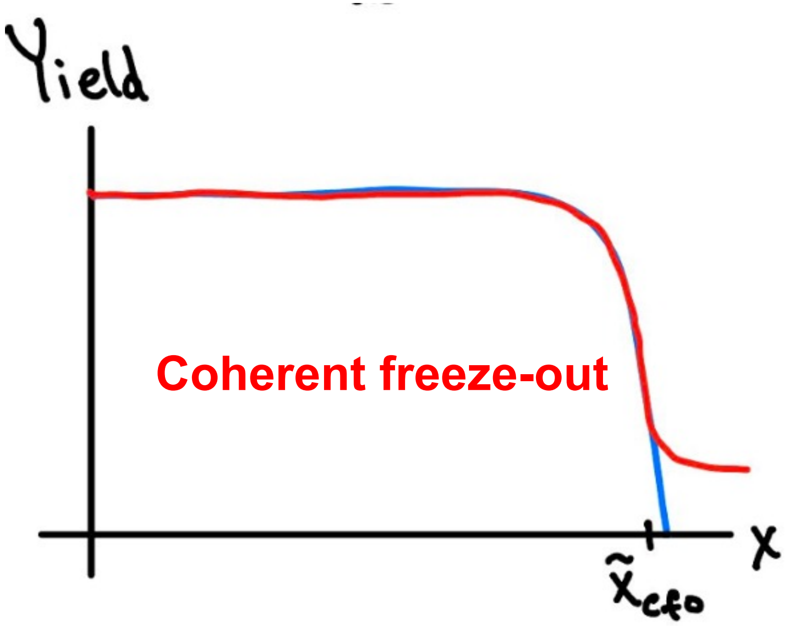


$$x \equiv m_\chi/T, \quad \varphi \equiv \phi/\sqrt{m_\chi \Lambda}, \quad \kappa \equiv \frac{2\pi^2}{g_\chi}\frac{m_\phi^2 \Lambda}{m_\chi^3}.$$

Normal Vs. Coherent Freeze-Out

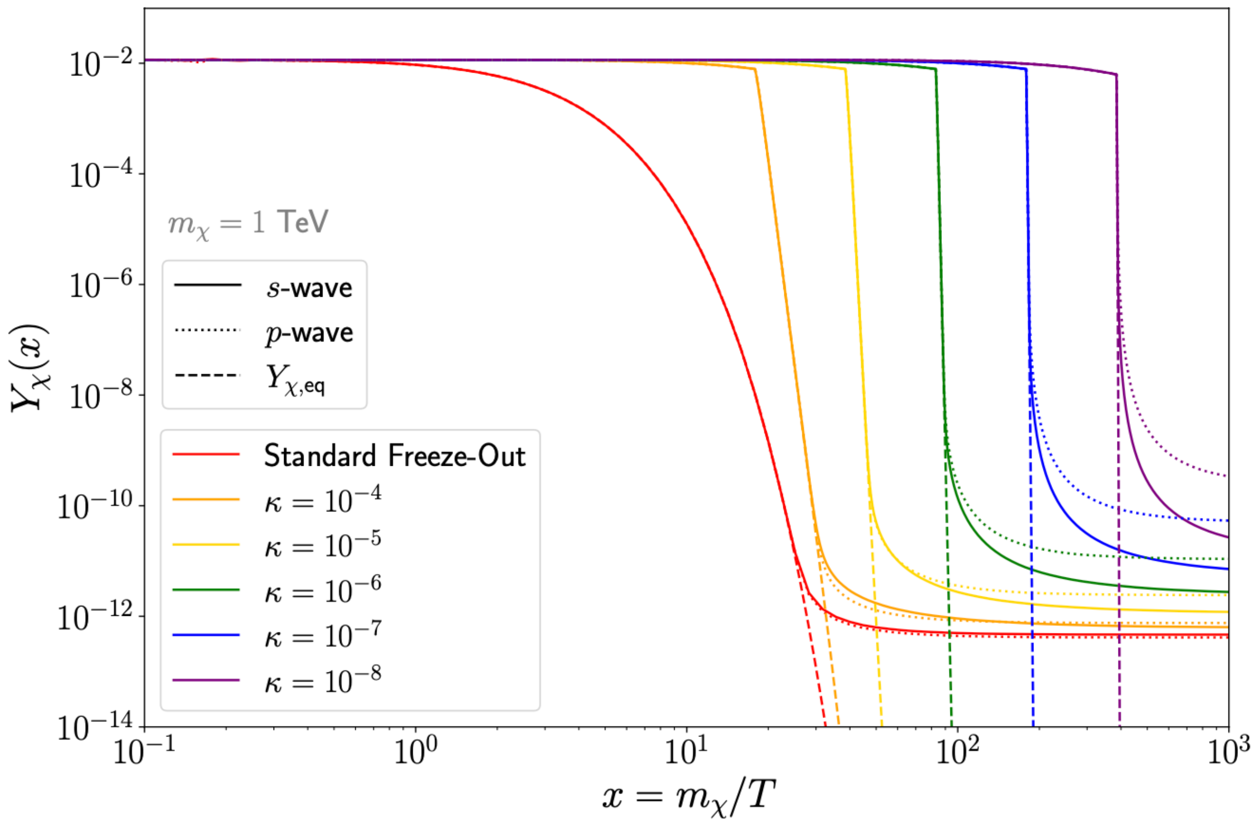
- Normal freeze-out: triggered by universe expansion, typically occurs at $x_{\mathrm{fo}} \sim 25$
- Coherent freeze-out: triggered by phase transition, can occur at $x_{\mathrm{cfo}} \gg 25$





Coherent Freeze-Out

- The WIMP freeze-out occurs suddenly as the ALP v.e.v. disappears. i.e. at x_{cfo}
- It can be delayed by 1-2 orders of magnitude compared to the usual thermal freeze-out value, $x_{\text{fo}} \approx 25$
- If delay is too much, ALP vacuum energy starts to dominate and there is a burst of inflation To avoid inflation, $x_{\text{cfo}} \lesssim 500$
- There may be a way to have consistent cosmology with a short inflationary phase, at the expense of some fine-tuning (work in progress)



Relic abundance of coherent freeze-out

- Because freeze-out is delayed, the relic abundance is enhanced thanks to less redshift between freeze-out and present day

- Quantitively, we find $$\frac{Y^{\rm cfo}_{\chi,\infty}}{Y^{\rm fo}_{\chi,\infty}} = \frac{14}{9}\frac{x_{\rm cfo}}{x_{\rm fo}}\frac{\sigma_0^{\rm fo} + 3\sigma_1^{\rm fo}/x_{\rm fo}}{\sigma_0^{\rm cfo} + 21\sigma_1^{\rm cfo}/(10x_{\rm cfo})}$$

- To get the observed yield*, we have

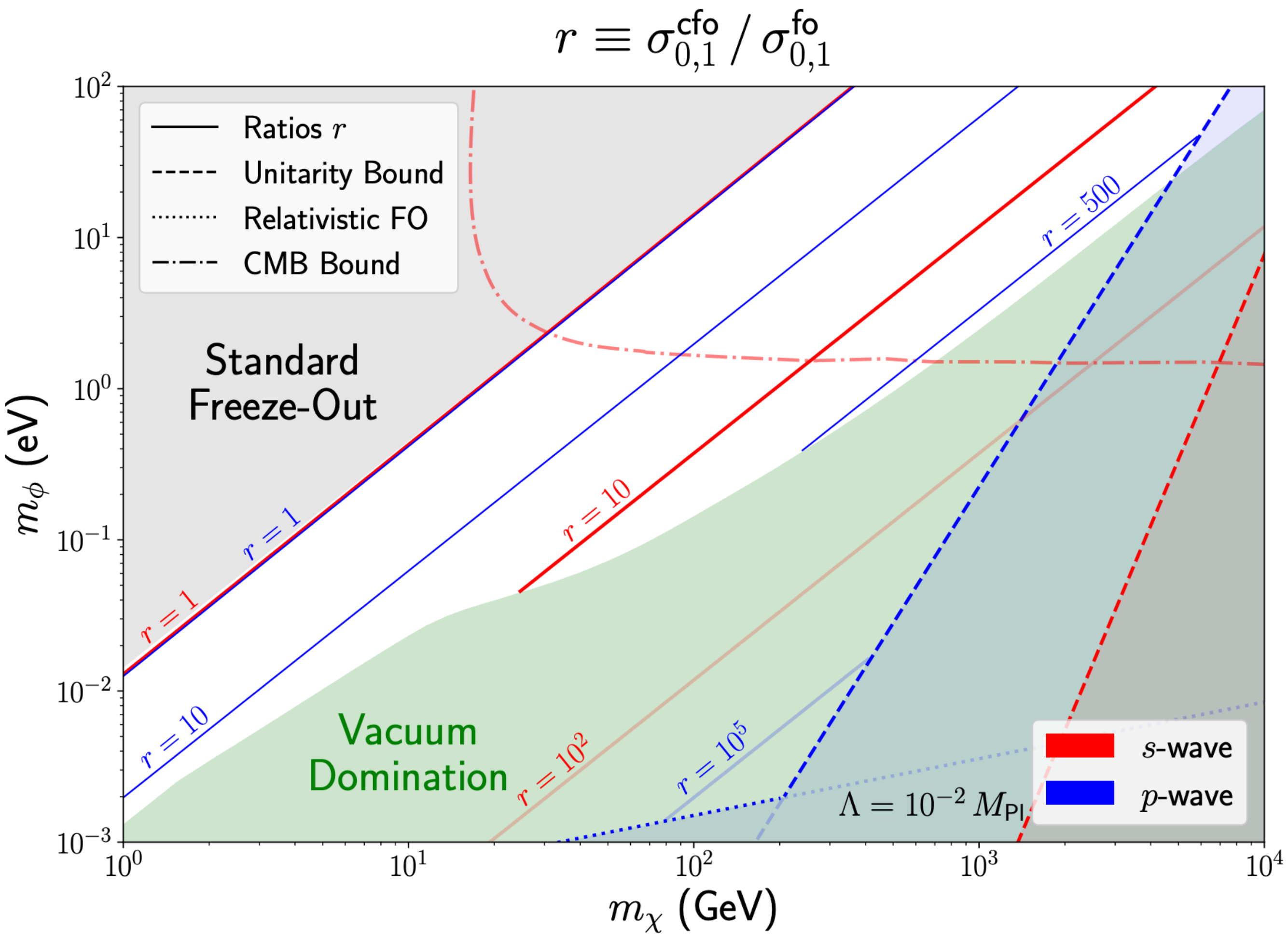
  s-wave: $\sigma_0^{\rm cfo}/\sigma_0^{\rm fo} \sim x_{\rm cfo}/x_{\rm fo}$

  p-wave: $\sigma_1^{\rm cfo}/\sigma_1^{\rm fo} \sim (x_{\rm cfo}/x_{\rm fo})^2$

- So, coherent freeze-out allows for an **enhanced** WIMP cross section!

* In the FOPT regime, ALP coherent oscillations must decay to avoid over-closure, so generically WIMP is 100% of DM

Enhancement of annihilation cross section



s-wave: enhanced up to 30

p-wave: enhanced up to 10^3

This may modify the existing bounds from indirect detection and collider searches of dark matter

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Conclusions

- Considered a coupling between WIMP and ALP motivated by QCD axion-nucleon coupling
- Even a tiny (Planck-suppressed) coupling can substantially modify cosmological histories of both WIMP and ALP through coherent forward scattering
- This may qualitatively change the predicted properties of WIMP and ALP dark matter particles, such as their masses and couplings
- This in turn leads to important consequences for targeted experimental searches for particle dark matter

The story of "WIMP meets ALP"

Outlook

- UV completion of the WIMP-ALP interaction
- Crossover regime: identify the targeted experiments searching for the “ALP miracle” window
- FOPT regime: compute possible gravitational-wave signals during WIMP coherent freeze-out
- For strong enough phase transition (small enough κ), vacuum domination may occur (a new regime with interesting dynamics and signals)
- Axion-WIMP interaction to realize phantom crossing for axion dark energy arXiv: 2607.28721

Thank you for your attention!

Backup slides

Separation of scales

$$\mathcal{L} = \frac{1}{\Lambda}\bar{\chi}\chi\frac{\phi^2}{2}$$

- We assume the hierarchy among three relevant scales

$$\Lambda \gg m_\chi \gg m_\phi \qquad\qquad m_\chi \sim \text{electroweak scale}$$

- The coupling is assumed to be small enough to prevent the ALP from ever being thermalized via its scattering with the WIMP

$$\Gamma \sim \frac{T^3}{\Lambda^2} \ll H \sim \frac{T^2}{M_{\rm Pl}} \quad \Rightarrow \quad \Lambda \gg \sqrt{T_{\rm RH} M_{\rm Pl}}$$

Modification of dispersion relation

- Both masses are shifted due to the coherent forward scattering inside the other medium

$$m^2_{\phi,\mathrm{eff}} = m^2_\phi - \frac{\langle\bar{\chi}\chi\rangle_T}{\Lambda}$$

$$m_{\chi,\mathrm{eff}} = \left| m_\chi - \frac{\phi^2}{2\Lambda} \right|$$

$$\langle\bar{\chi}\chi\rangle_T = g_\chi \int \frac{\mathrm{d}^3\mathbf{k}}{(2\pi)^3} \frac{m_{\chi,\mathrm{eff}}}{E_\mathbf{k}} f_\chi(\mathbf{k})$$

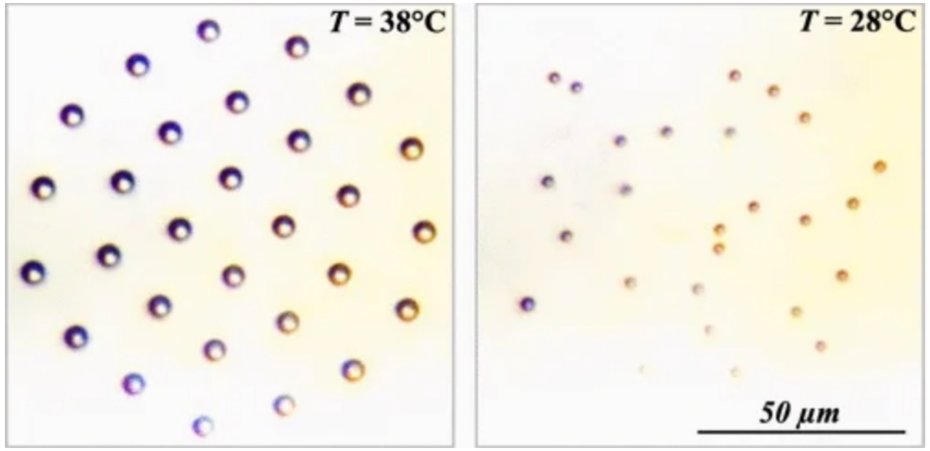
$g_\chi = 4$ (or 2) for Dirac (or Majorana)

f_χ is the phase-space distribution

- The dynamics of the two sectors are therefore **coupled**

Why inverse phase transition

- Thermodynamics tend to minimize the free energy
- Normal phase transition: the symmetric (disordered) phase has a larger entropy than the broken (ordered) phase
- Inverse phase transition (our case): the broken phase has a larger entropy due to the suppression of the WIMP mass by the ALP VEV



inverse melting

$$F = E - TS$$

The full one-loop thermal potential

$$V_{\rm full}(\phi, T) = -\frac{g_\chi}{2\pi^2} T^4 \int_0^\infty {\rm d}y\, y^2 \log\left[1 + e^{-\sqrt{y^2 + m_{\chi,{\rm eff}}^2/T^2}}\right] + \frac{1}{2} m_\phi^2 \phi^2 \equiv \frac{g_\chi}{2\pi^2} m_\chi^4 \mathcal{V}_{\rm full}(\varphi, x)$$

$$m_{\chi,{\rm eff}} = \left| m_\chi - \frac{\phi^2}{2\Lambda} \right|$$

$$\mathcal{V}_{\rm full}(\varphi, x) = -\frac{1}{x^4} \int_0^\infty {\rm d}y\, y^2 \log\left[1 + e^{-\sqrt{y^2 + \gamma^2 x^2}}\right] + \kappa \frac{\varphi^2}{2}$$

$$\kappa \equiv \frac{2\pi^2}{g_\chi} \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

Equation of motion for ALP

$$x \equiv m_\chi / T$$

$$\varphi \equiv \phi / \sqrt{m_\chi \Lambda}$$

$$\gamma \equiv |1 - \varphi^2/2|$$

- The EOM is non-linear due to the backreaction

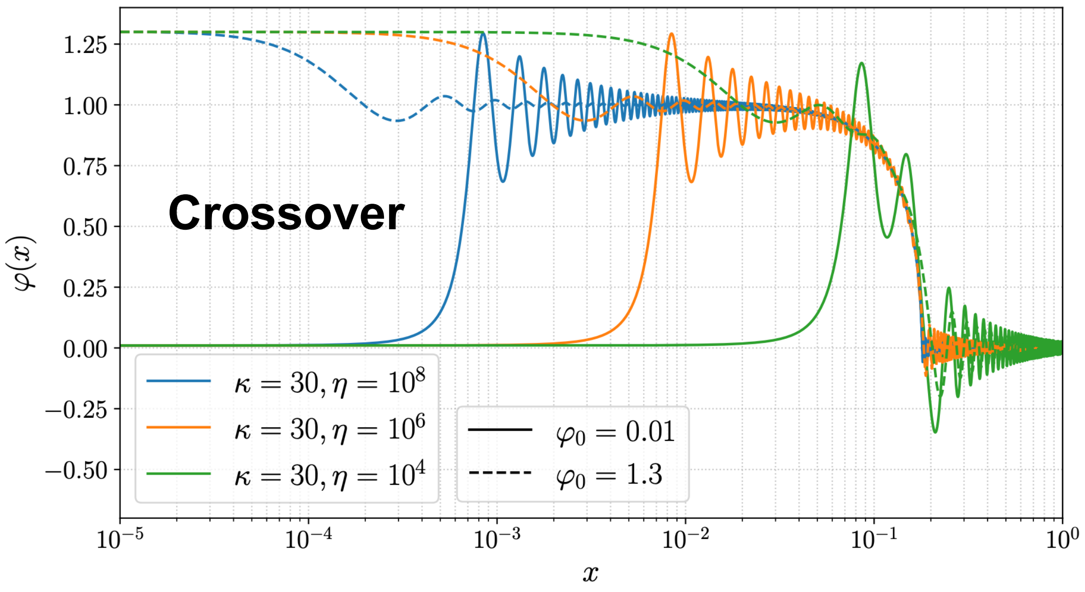
$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi, T) = 0$$

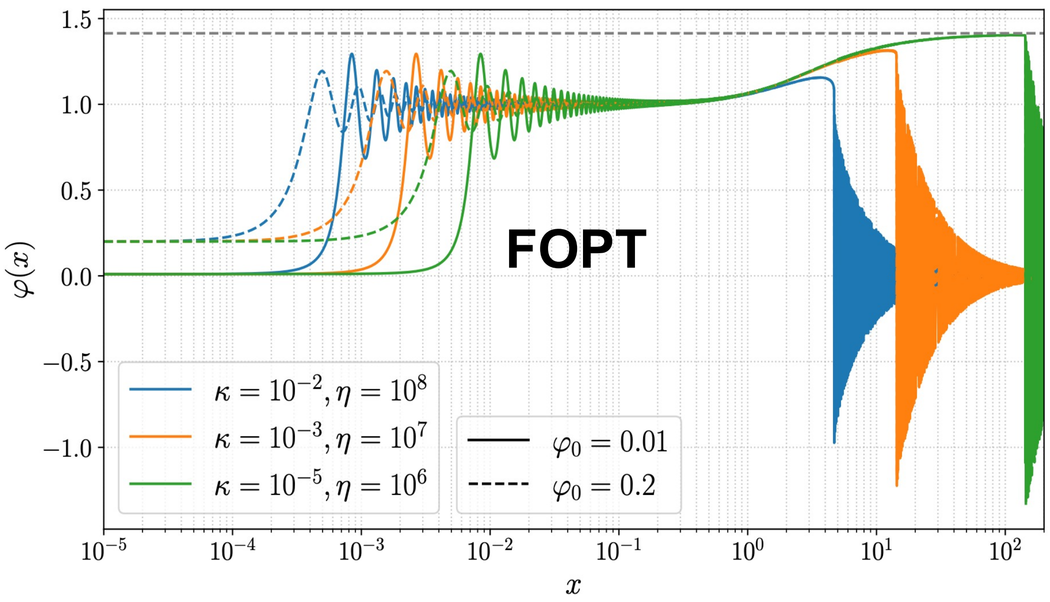
$$\varphi'' + \frac{2}{x}\varphi' - \eta x \left(1 - \frac{\varphi^2}{2}\right)\left[\left(1 - \varphi^2\right) K_1(\gamma x) + \frac{\varphi^2}{2}\gamma x K_0(\gamma x)\right]\varphi + \eta\kappa x^2 \varphi = 0$$

- The EOM depends only on two dimensionless parameters:

$$\kappa \equiv \frac{2\pi^2}{g_\chi}\frac{m_\phi^2 \Lambda}{m_\chi^3} \qquad \eta \equiv \frac{g_\chi}{2\pi^2}\frac{1}{1.66^2 g_*}\frac{M_{\rm Pl}^2}{m_\chi \Lambda}$$

Solutions of the EOM

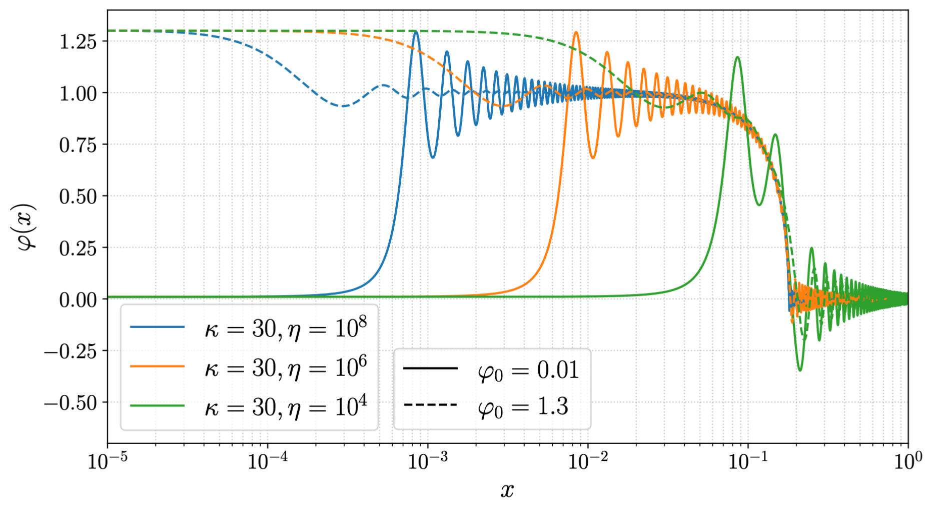




- Crossover regime: ALP evolution is **nearly adiabatic**, decouples earlier, does not affect freeze-out
- FOPT regime: ALP evolution is **non-adiabatic**, decouples later, affects WIMP freeze-out

More details in the crossover regime

Evolution of the ALP field



- In this regime, the symmetry is restored earlier at $x \ll 25$, so freeze-out is not affected

- The EOM in this regime is given by

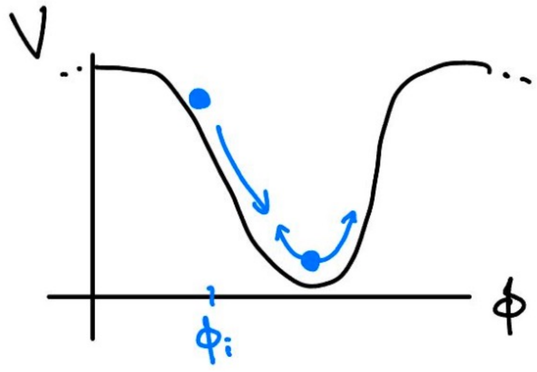
$$\varphi'' + \frac{2}{x}\varphi' - \eta\left(1 - \kappa x^2\right)\varphi + \eta\,\varphi^3 = 0$$

$$\kappa \sim \frac{m_\phi^2 \Lambda}{m_\chi^3}$$

$$\eta \sim \frac{M_{\rm Pl}^2}{m_\chi \Lambda}$$

- for $x \ll 1/\sqrt{\eta}$: the ALP field is frozen by Hubble friction
- for $1/\sqrt{\eta} \lesssim x \lesssim 1/\sqrt{\kappa}$: it evolves toward the vacuum at $\varphi_* = 1$ **wash out the initial dependence**
- for $x \gtrsim 1/\sqrt{\kappa}$: symmetry is restored and the field relaxes back toward 0

Relic abundance of ALP



- Usual misalignment: φ_i ~ UV dependent

$$\frac{\Omega_\phi h^2}{0.12} \sim \left(\frac{\phi_i}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{m_\phi}{10^{-10}\ \mathrm{eV}}\right)^{1/2}$$

Relic abundance of ALP

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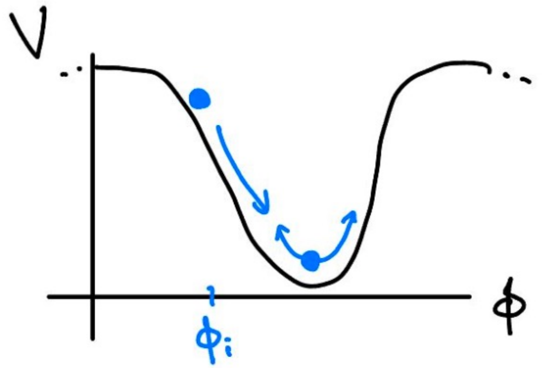
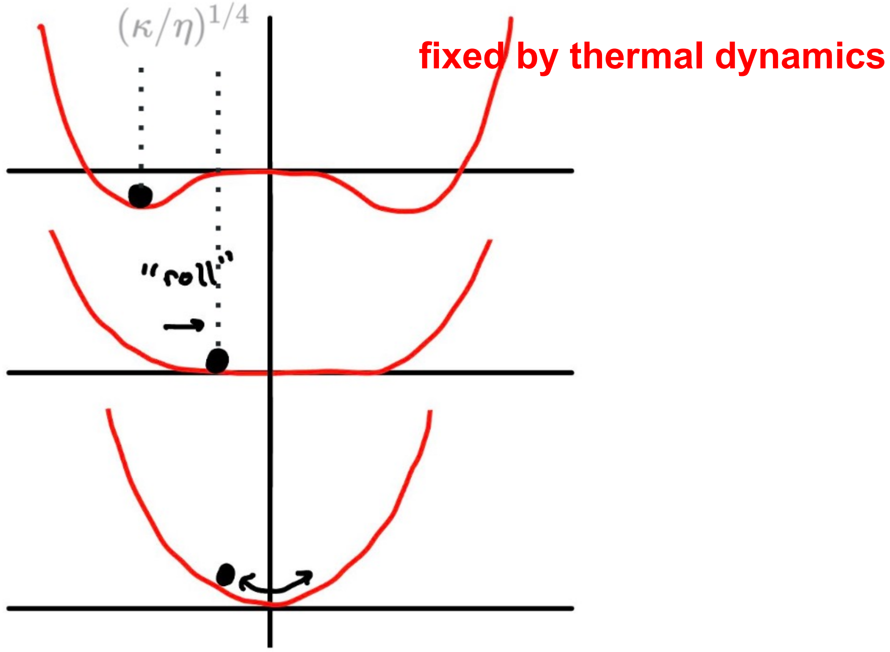
$$\frac{\Omega_\phi h^2}{0.12} \sim \left(\frac{\phi_i}{10^{14}\ \text{GeV}}\right)^2 \left(\frac{m_\phi}{10^{-10}\ \text{eV}}\right)^{1/2}$$



- Our case: $\varphi_i \sim \varphi_* (\kappa/\eta)^{1/4} \sim \varphi_* \sqrt{H(x_c)/m_\phi}$



Relic abundance of ALP

- Usual misalignment: φ_i ~ UV dependent

$$\frac{\Omega_\phi h^2}{0.12} \sim \left(\frac{\phi_i}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{m_\phi}{10^{-10}\ \mathrm{eV}}\right)^{1/2}$$

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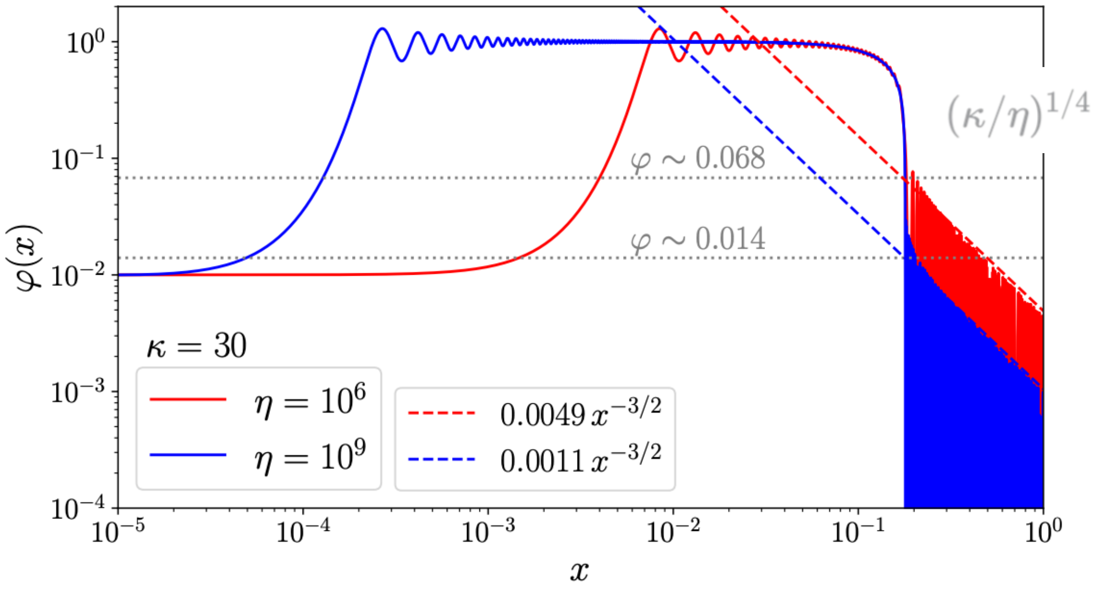

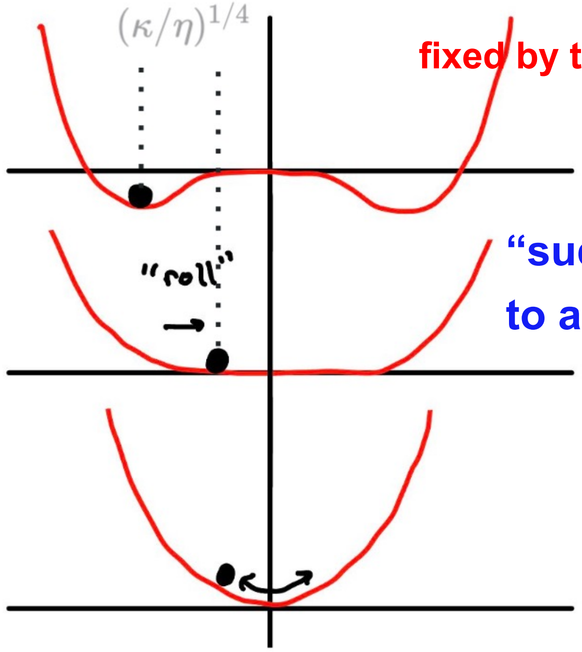

fixed by thermal dynamics

"sudden drop" due to adiabatic invariant

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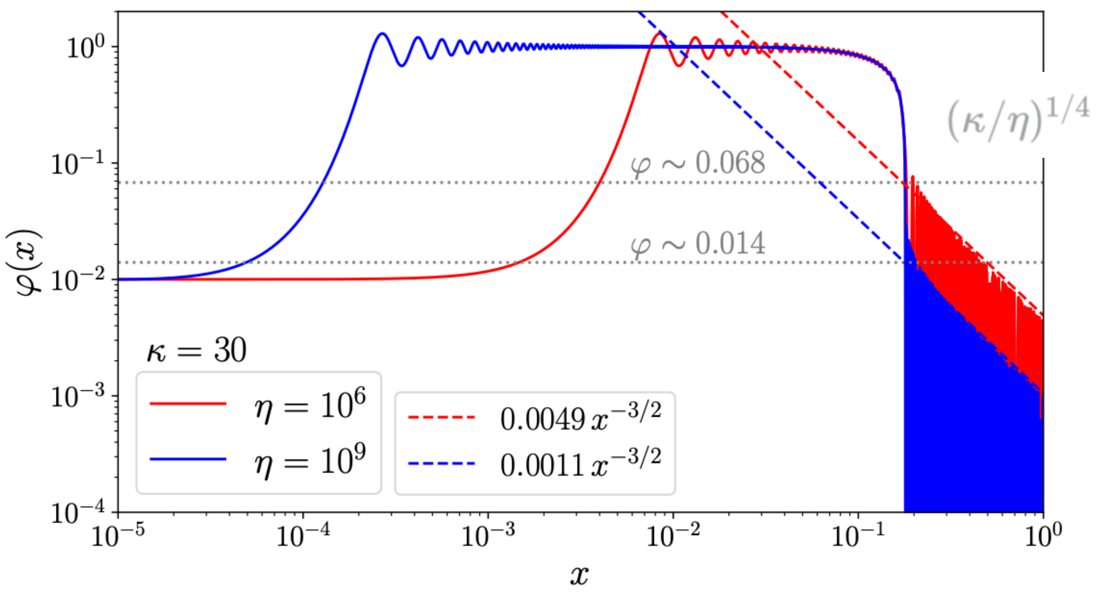

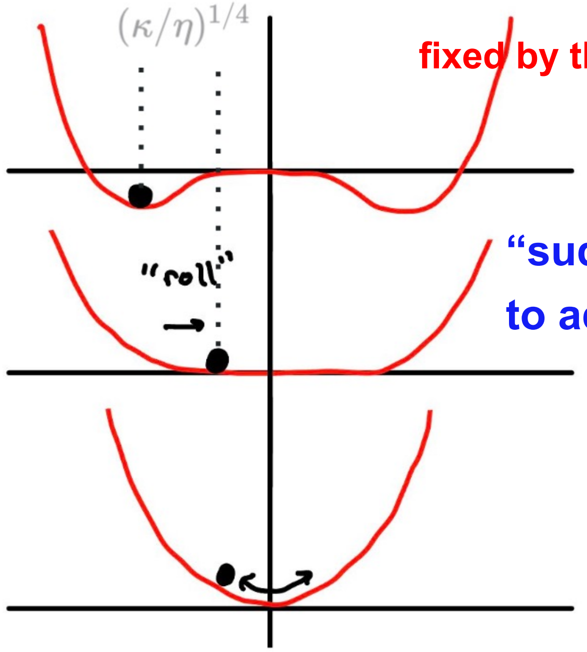

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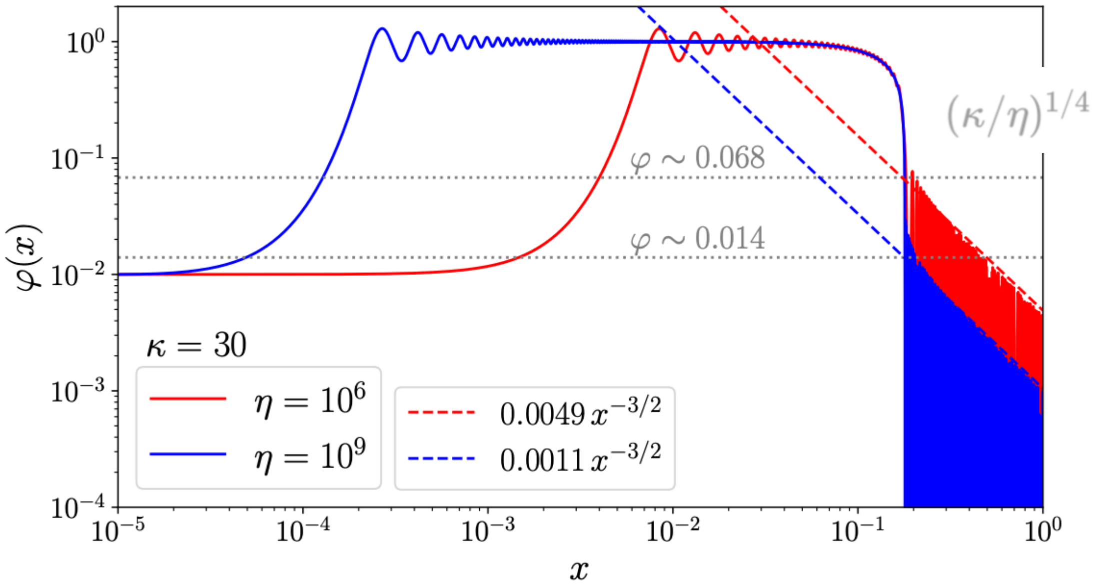

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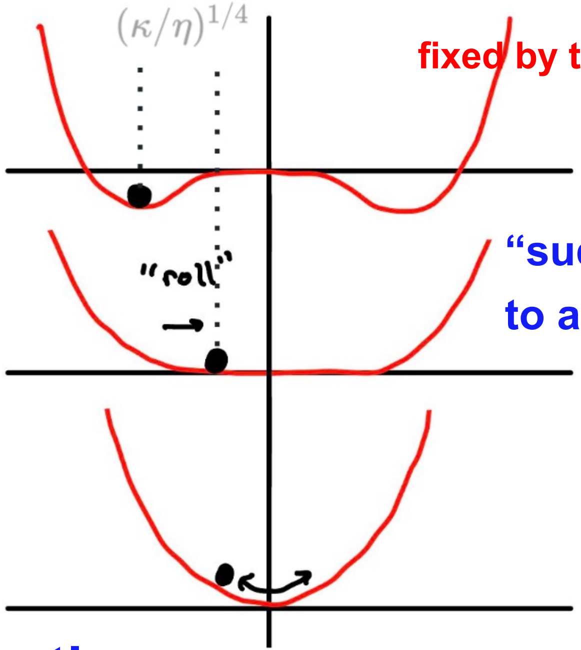


fixed by thermal dynamics

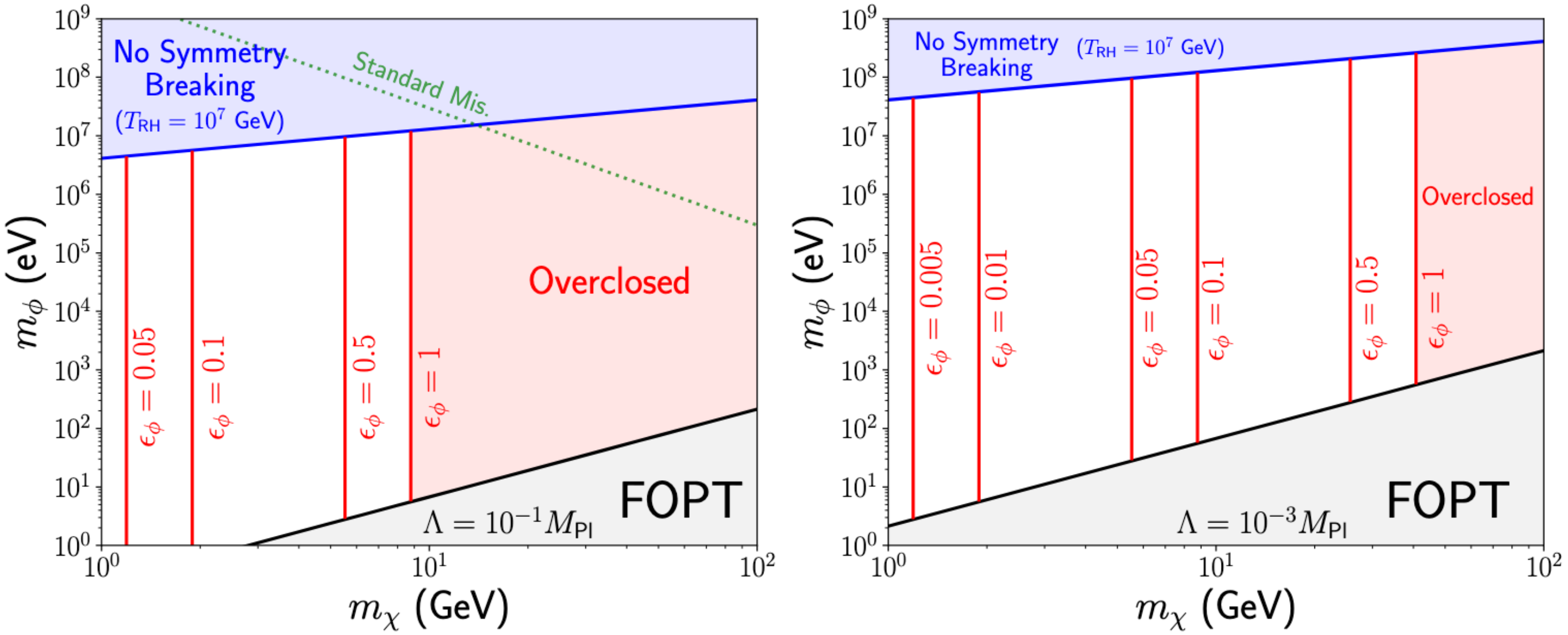
"sudden drop" due to adiabatic invariant

Surprisingly, the ALP mass also drops out!

The "ALP miracle"

$$\Omega_\phi \approx 0.3 \left(\frac{m_\chi}{10\ \mathrm{GeV}}\right)^{3/2} \left(\frac{\Lambda}{0.1 M_{\mathrm{Pl}}}\right)^{1/2}$$

- For m_χ ~ weak scale, Λ ~ Planck scale, the ALP obtains correct relic abundance
- This is largely insensitive to both the initial ALP field value and the ALP mass



$\epsilon_\phi \equiv \Omega_\phi / \Omega_{\mathrm{DM}}$

More detailed calculations of relic abundance

- Symmetry is restored at $x_c \equiv 1/\sqrt{\kappa}$

$$\varphi'' + \frac{2}{x}\varphi' - \eta\left(1 - \kappa x^2\right)\varphi + \eta\,\varphi^3 = 0$$

- The adiabatic approximation holds when $\eta\left(1 - \kappa x^2\right) \gg 1/x^2$

- We define $x_1 = x_c + \delta x$ the time when adiabaticity is restored, i.e., when effective mass = Hubble friction

$$\left|\eta\left(1 - \kappa x_1^2\right)\right| \;\equiv\; 1/x_1^2$$

- For $\eta \gg \kappa$ (which always holds), the loss of adiabaticity is extremely brief $\delta x/x_c \approx \kappa/(2\eta)$

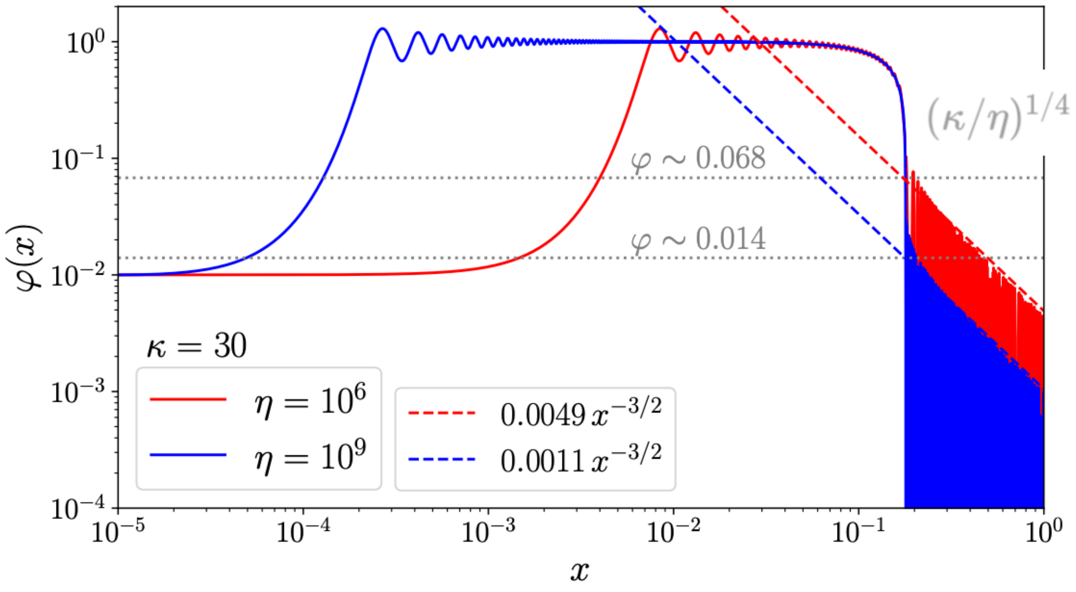
- For $x > x_1$, we have adiabatic invariant

$$m_{\phi,\mathrm{eff}}(x)\phi^2(x)\,a^3(x) = m_{\phi,\mathrm{eff}}(x_1)\phi^2(x_1)\,a^3(x_1)$$

More detailed calculations of relic abundance

- Near $x_c \approx x_1$, the field undergoes a sudden drop due to the adiabatic condition

$$\frac{\phi^2(x_0)\,a^3(x_0)}{\phi^2(x_1)\,a^3(x_1)} = \frac{H(x_1)}{m_\phi} \approx \frac{H(x_c)}{m_\phi} = \left(\frac{\kappa}{\eta}\right)^{\frac{1}{2}} \ll 1$$



- So, the present-day energy density is given by

$$\rho_\phi(x_0) \approx m_\phi^2 \phi^2(x_c) \left(\frac{\kappa}{\eta}\right)^{\frac{1}{2}} \left(\frac{x_c}{x_0}\right)^3 \frac{g_{*S}(x_0)}{g_{*S}(x_c)}$$

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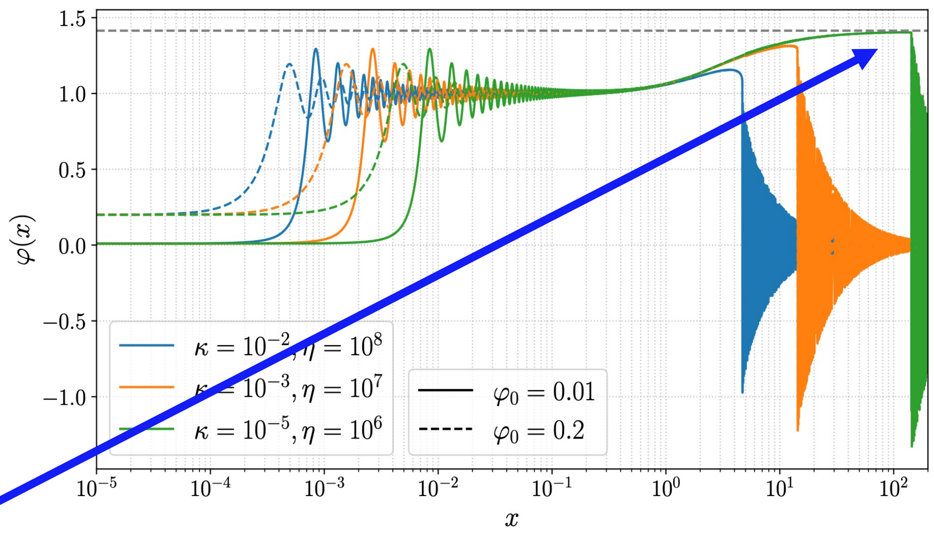
$$\begin{aligned}
\rho_\phi(x_0) &= m_\phi^2 \times 2m_\chi\Lambda \times \frac{H(x_c)}{m_\phi} \times \frac{x_c^3}{x_0^3} \times \frac{g_{*S}(x_0)}{g_{*S}(x_c)} \\
&= m_\phi^2 \times 2m_\chi\Lambda \times \frac{1.66\sqrt{g_*(x_c)}\, m_\chi^2}{M_{\rm Pl}\, m_\phi} \times \frac{x_c}{x_0^3} \times \frac{g_{*S}(x_0)}{g_{*S}(x_c)} \\
&= \cancel{m_\phi^2} \times 2m_\chi\Lambda \times \frac{1.66\sqrt{g_*(x_c)}\, m_\chi^2}{M_{\rm Pl}\, \cancel{m_\phi}} \times \frac{\pi}{2\sqrt{3}} \left(\frac{g_\chi}{2\pi^2}\right)^{1/2} \frac{m_\chi^{3/2}}{\Lambda^{1/2} \cancel{m_\phi} x_0^3} \times \frac{g_{*S}(x_0)}{g_{*S}(x_c)} \\
&= 1.66\sqrt{\frac{g_\chi g_*(x_c)}{6}}\, \frac{g_{*S}(x_0)}{g_{*S}(x_c)}\, \frac{m_\chi^{3/2}\Lambda^{1/2} T_0^3}{M_{\rm Pl}} \,.
\end{aligned}$$

ALP mass precisely drops out in the relic abundance!

More details in the FOPT regime

Evolution of the vacuum

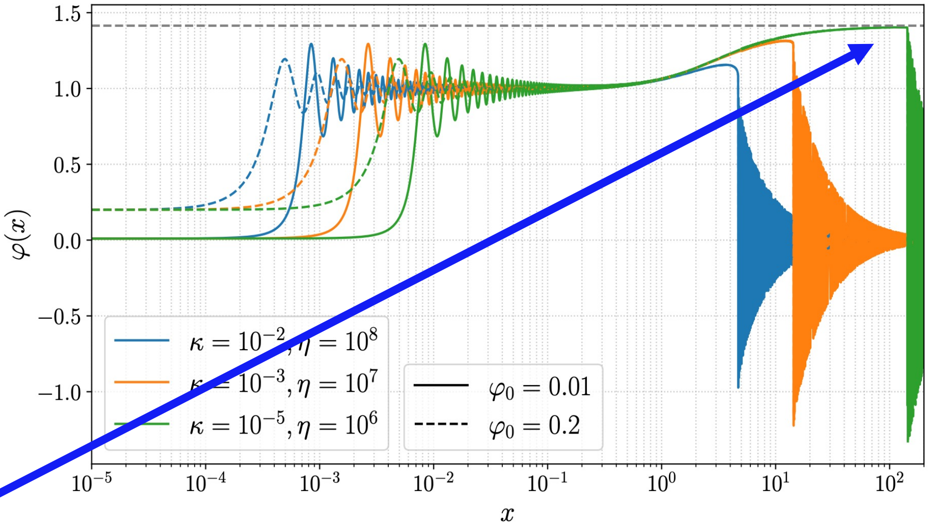


- Given the potential, for $\kappa \ll 1$ and $x \gg 1$, the symmetry-breaking vacuum evolves as

$$\varphi_* = \sqrt{2\left(1 - \frac{c_1}{x}\right)}$$

$c_1 \approx 1.33$ is the root of $c_1 K_0(c_1) = K_1(c_1)$

Evolution of the vacuum



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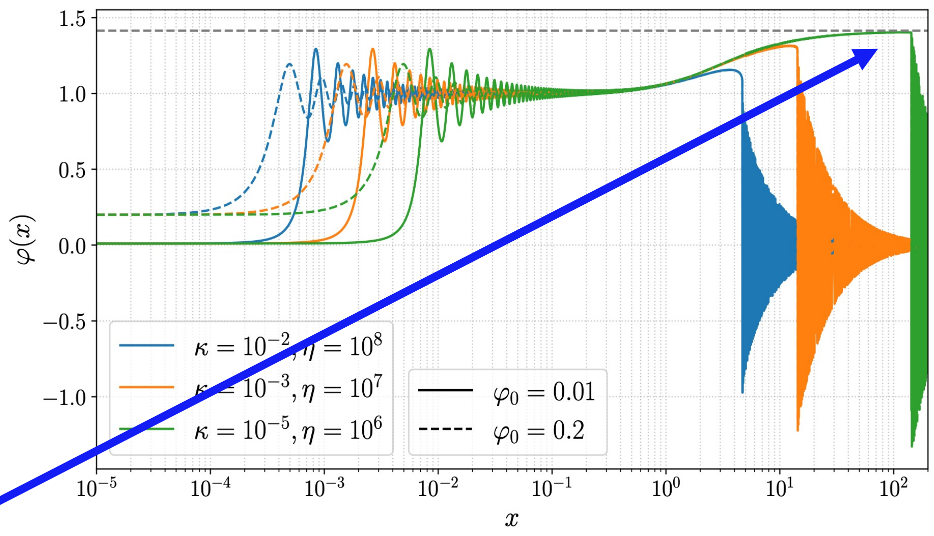
- The evolving vacuum reduces the effective WIMP mass as temperature decreases

$$\boxed{m_{\chi,\mathrm{eff}} = \left(1 - \varphi_*^2/2\right) m_\chi \sim T}$$

Evolution of the vacuum



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- The evolving vacuum reduces the effective WIMP mass as temperature decreases

$$\boxed{m_{\chi,\text{eff}} = \left(1 - \varphi_*^2/2\right) m_\chi \sim T}$$

- This makes the WIMP stay in equilibrium even at $x \equiv \frac{m_\chi}{T} \gg O(25)$

$$n_{\chi,\text{eq}} \sim m_{\chi,\text{eff}}^2 T K_2(m_{\chi,\text{eff}}/T) \sim T^3$$

No Boltzmann suppression at large x!

WIMP Mass Evolution in FOPT

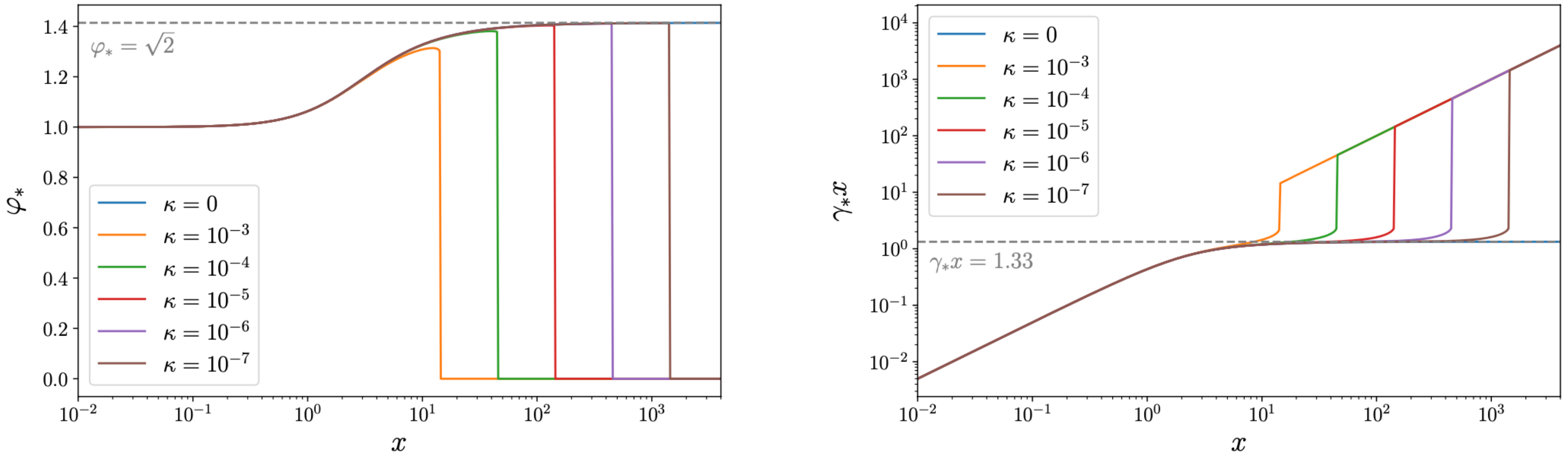
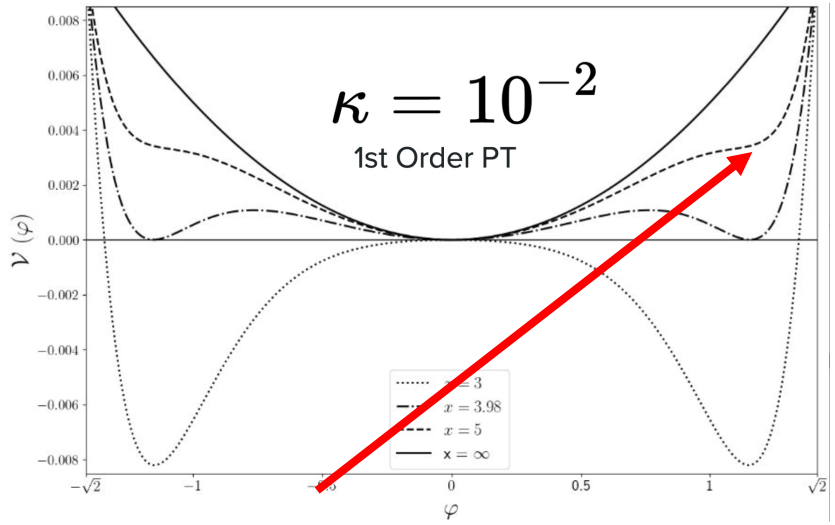


FIG. 7. The evolution of the symmetry-breaking vacuum φ_* (left panel) and the boost factor $\gamma_* \equiv |1 - \varphi_*^2/2|$ (right panel) as a function of the inverse temperature $x \equiv m_\chi/T$. The ratio $m_{\chi,\text{eff}}/T = \gamma_* x$ remains smaller than $\mathcal{O}(1)$ until x reaches $\tilde{x}_{\text{cfo}}$ (corresponding to the sudden drop of φ_* and the sharp increase of $\gamma_* x$ in the plots), after which the coherent freeze-out begins.

Coherent freeze-out



$x_{\mathrm{cfo}} \sim \kappa^{-1/3}$

- WIMP freeze-out is controlled by ALP dynamics
- We define x_{cfo} as the time when the local minimum disappears
- For $x < x_{\mathrm{cfo}}$, WIMP stays in thermal equilibrium, regardless how large x is
- For $x > x_{\mathrm{cfo}}$, ALP rolls back to the origin, $m_{\chi,\mathrm{eff}}$ rapidly tends to m_χ, driving WIMP out of equilibrium --- the beginning of **coherent freeze-out**

Dark matter in the FOPT regime

- Freeze-out of WIMP is delayed in a *dynamical* way, allowing for orders of magnitude enhancement of its annihilation cross section while still yielding the correct relic abundance

- In this regime, however, ALP only behaves as a spectator field. In order not to overclose the Universe, it must decay to radiation after WIMP freeze-out. Therefore, dark matter consists solely of the WIMP in the FOPT regime

Making ALP Decay After FOPT

- ALP Coherent oscillations in FOPT regime would over-close the universe
- OK if they decay into SM radiation before BBN, but SM-ALP coupling must be small enough to not thermalize the ALP particles
- Solution: introduce dark photon with ALP-DP coupling $\mathcal{L}_{\rm dark} \supset -\frac{\phi}{4f_\phi} F_{\mu\nu}\tilde{F}^{\mu\nu}$
- Preheating quickly transfers energy from ALP to DP
- Dark photon then decays to SM e+e- before BBN
- See paper for details, consistency checks etc.