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S-matrix bootstrap bounds on Self-interacting dark matter

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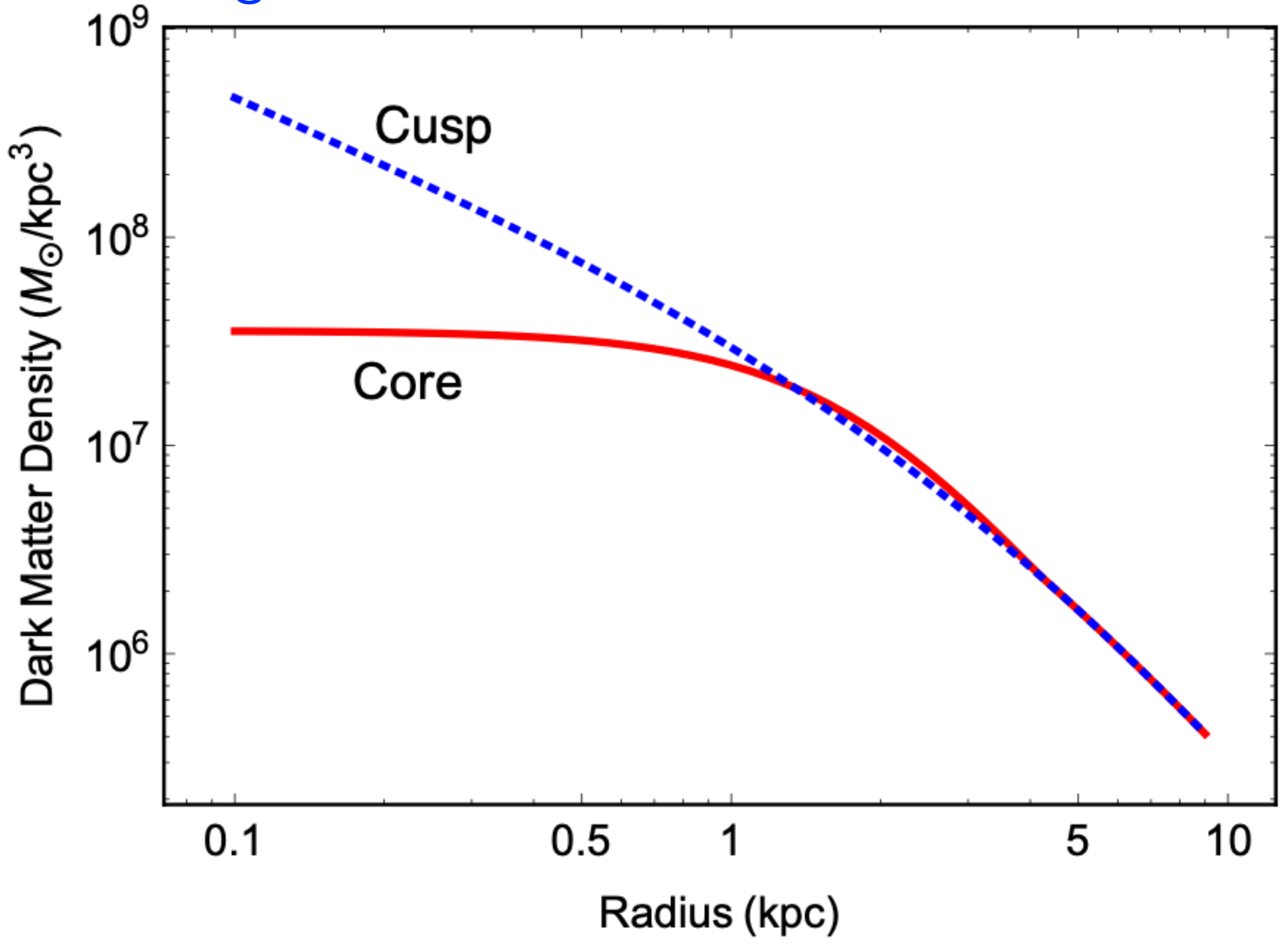
Based on arXiv:2607.13141, with Prof. Qing Chen and Prof. Shuang-Yong Zhou

Core-Cusp tension and SIDM

Figure from: Tulin, Yu, 1705.02358



- Collisionless CDM simulation: $\rho_{\rm DM} \sim r^{-1}$
- Dwarf-galaxy kinematics: $\rho_{\rm DM} \sim {\rm const}$

$\Rightarrow$ **Core-Cusp tension**

Flores, Primack, astro-ph/9402004;
Moore, astro-ph/9402009;
…

- Self-interacting dark matter (SIDM)

Scattering $\Rightarrow$ Heat transport $\Rightarrow$ Isothermal core

$$\sigma_{\rm self} \approx 10^{-24} \left(\frac{M_{\rm DM}}{\rm GeV} \right) {\rm cm}^2$$

Spergel, Steinhardt, astro-ph/9909386

Mass bounds on SIDM

- Elastic cross section

$$\sigma_{\text{self}} = \frac{|\mathscr{M}_{\text{thr}}|^2}{128\pi M_{\text{DM}}^2} + \mathcal{O}(v_{\text{rel}}^2)$$

Upper bound on $|\mathscr{M}_{\text{thr}}|$ $\oplus$ $\sigma_{\text{self}} \approx 10^{-24} \left(\frac{M_{\text{DM}}}{\text{GeV}}\right) \text{cm}^2$

- Mass bounds on dark matter

$$M_{\text{DM}} \leq 0.289\,\text{GeV} \left(\frac{\max|\mathscr{M}_{\text{thr}}|}{(4\pi)^2}\right)^{2/3}$$

Choose a model $\longrightarrow \mathscr{M}_{\text{thr}} \longrightarrow$ *DM mass*

Recent reviews: Tulin, Yu, 1705.02358;
Adhikari et al.,2207.10638; …

Bootstrap approach

$\max|\mathscr{M}_{\text{thr}}|$ *for all consistent EFT?*

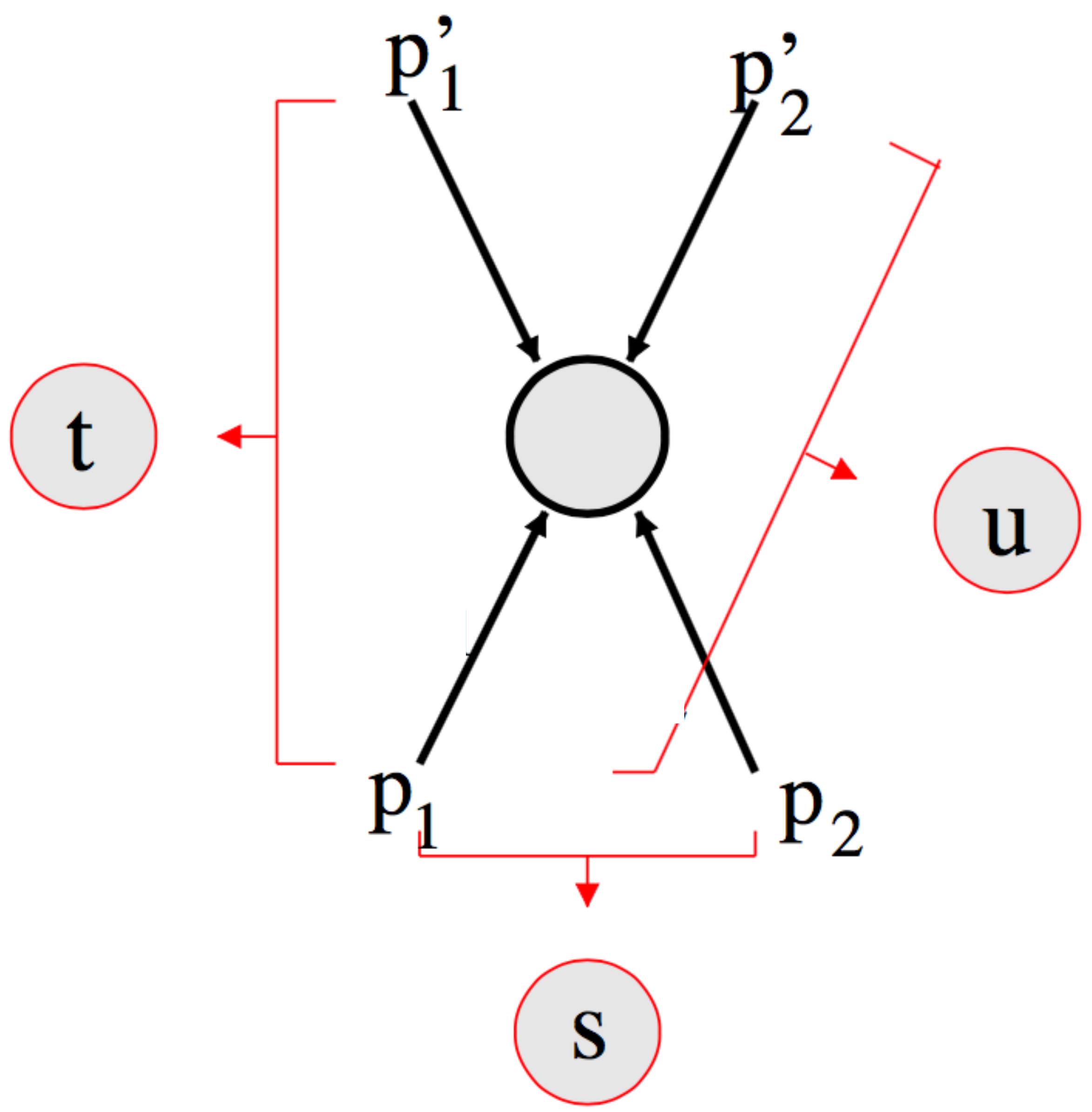
S-matrix bootstrap

Basic principles of QFTs $\Longrightarrow$ Allowed space of QFTs

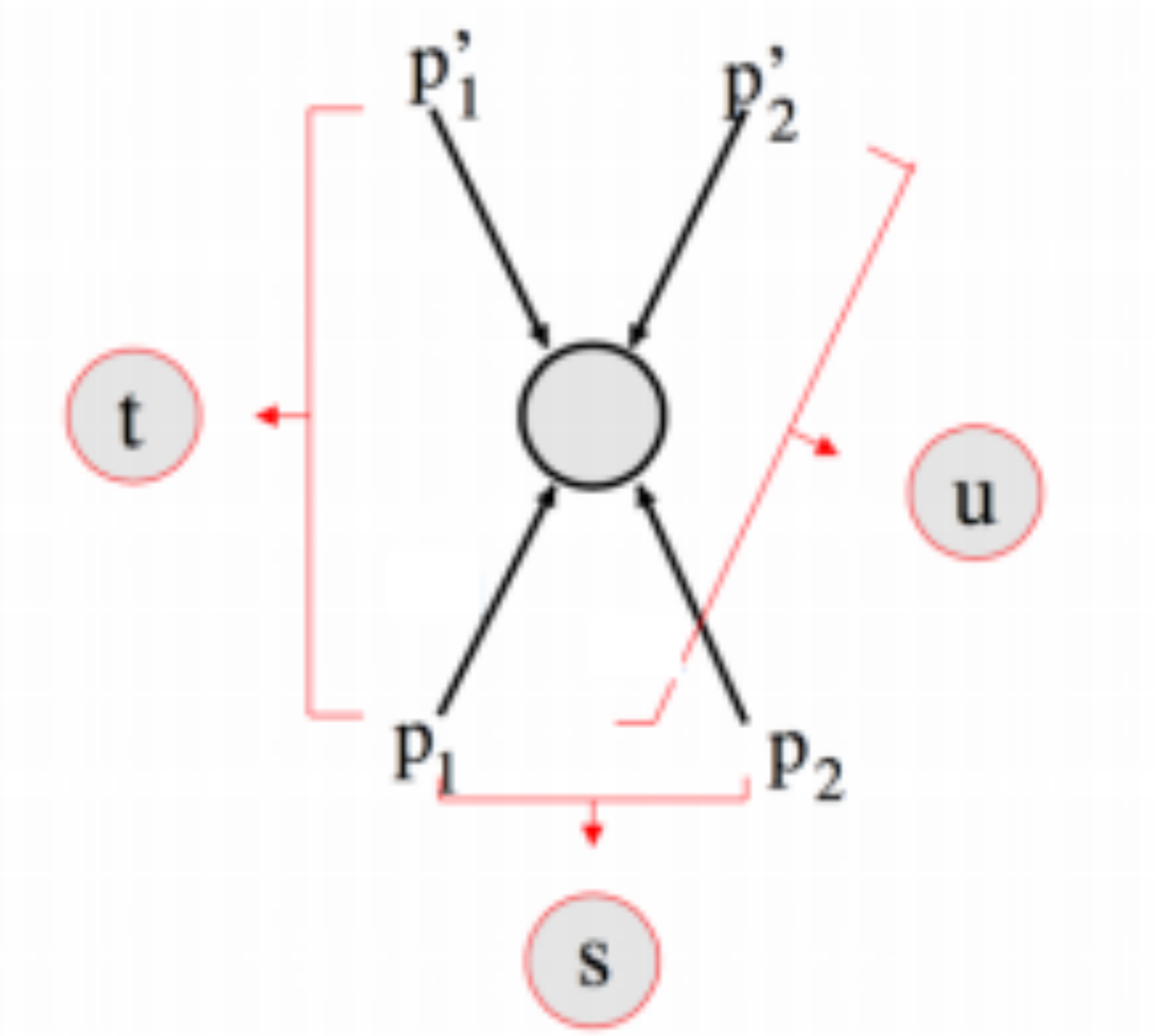
- Poincare invariance

$$\mathcal{M}(\{p\}) = \mathcal{M}(s,t), \quad s+t+u = 4m^2$$

- Unitarity: conservation of probability in quantum theory
- Causality: micro-causality $[\mathcal{O}(x), \mathcal{O}(y)] = 0, \; (x-y)^2 > 0$
- Crossing: $\mathcal{M}(s,t) = \mathcal{M}(s,u) = \mathcal{M}(u,t)$
- Locality: Froissart-Martin bound $\displaystyle \lim_{|s|\to\infty} \frac{\mathcal{M}(s,t)}{s^2} = 0, \quad t < t_* \quad \text{or} \quad |t| < t_*$



Unitarity

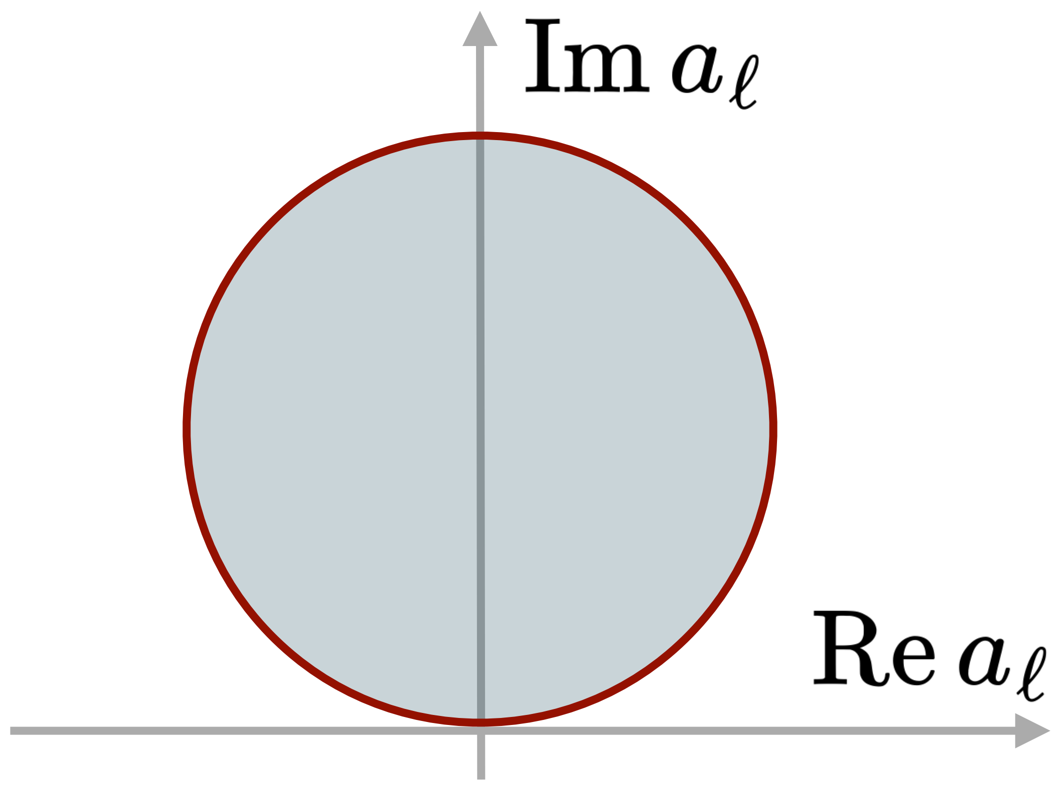


- Partial wave expansion

$$\mathcal{M} = 16\pi \sum_{\ell} (2\ell + 1) P_\ell \left(1 + \frac{2t}{s - 4M^2}\right) a_\ell(s)$$

- Conservation of probability

For $2 \to 2$ scattering: $|S_\ell| \equiv |1 + i\rho_M(s) a_\ell(s)| \leq 1$, for $s \geq 4M^2$



Phase space factor: $\rho_M(s) = \sqrt{\dfrac{s - 4M^2}{s}}$

Unitarity bounds on SIDM

Hui, astro-ph/0102349

- Short-range interactions + CDM: $\ell = 0$ wave dominance

- Unitarity allows $|a_{\ell=0}| \lesssim \dfrac{1}{v_{\rm rel}}$

Unitarity $|1 + i\rho_M a_{\ell=0}| \leq 1$

Phase space factor $\rho_M = \sqrt{(s-4M^2)/s} \sim v_{\rm rel}/2$

$$\sigma_{\rm self} \lesssim \frac{|\max \mathscr{M}_{\rm thr}|^2}{M_{\rm DM}^2} \sim \frac{1}{M_{\rm DM}^2 v_{\rm rel}^2}$$

$$\sigma_{\rm self} \approx 10^{-24}\left(\frac{M_{\rm DM}}{\rm GeV}\right)\,{\rm cm}^2 \qquad v_{\rm rel} \sim 1000\ {\rm km/s}$$

$\Rightarrow$ $M_{\rm DM} \lesssim 12\ {\rm GeV}$

Valid at any coupling strength

- Question: how much stronger is the bound for a weakly coupled IR EFT?

$\Rightarrow$ UV constrains IR

Causality and analyticity

- Micro-causality: $[\mathcal{O}(x), \mathcal{O}(y)] = 0$, when x and y are **space-like** separated
- LSZ reduction $\mathcal{M}(s,t) \propto \int dx^4 e^{-i(p_2-p_4)x/2} \langle p_3 | \theta(x_0)[J(x/2), J(-x/2)] | p_1 \rangle$

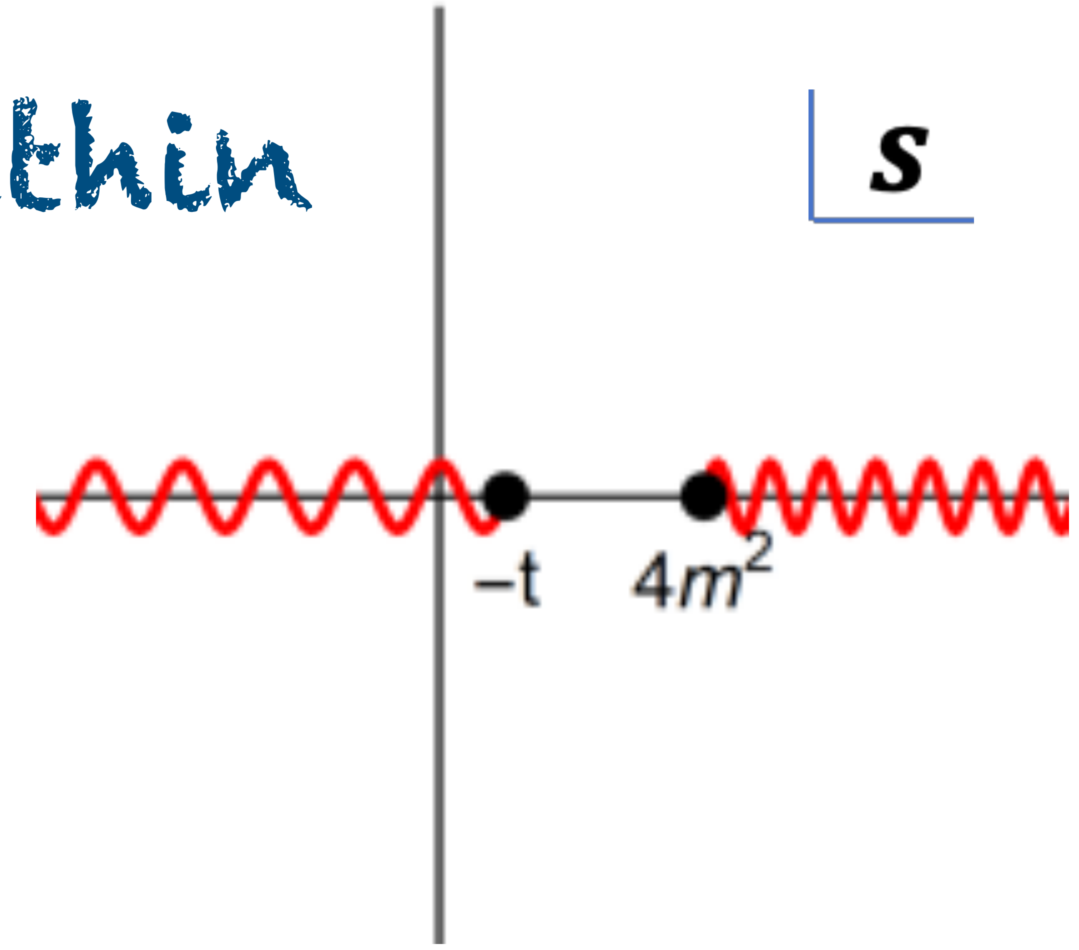
Can be continued to some complex values ⟸ Supported in forward light cone

- After much hard work …

Work from the 1950s-1970s

physical-sheet amplitude is analytic within

doubly cut s plane, $|t| \leq 4m^2$

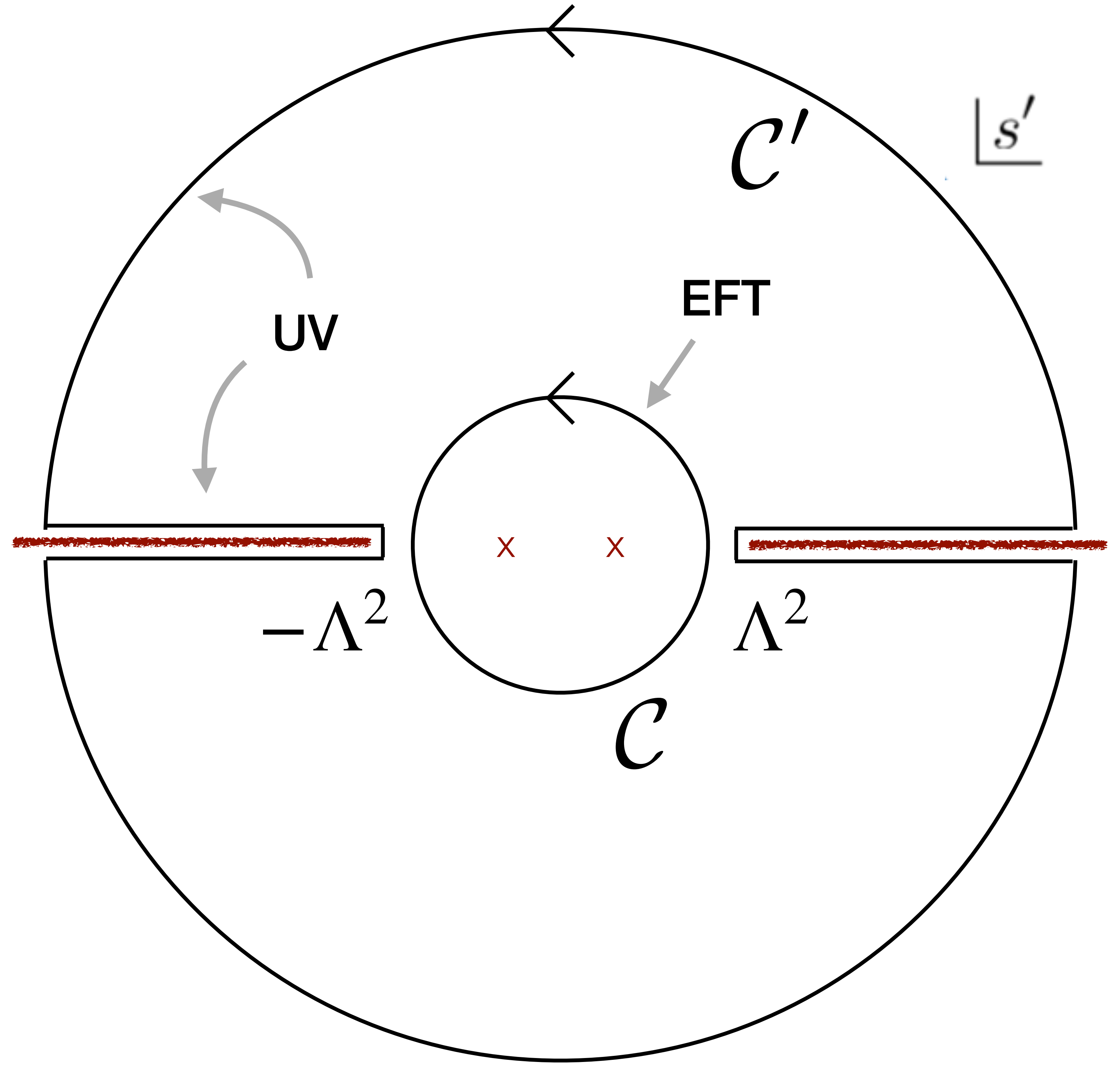


UV-IR bridge: dispersion relation

- Weakly coupled EFT: IR cut is negligible

$$\mathcal{M}(s,t) = \frac{1}{2\pi i}\int_{\mathscr{C}} \mathrm{d}s' \frac{\mathcal{M}(s',t)}{s'-s}$$

W/ locality: $\lim_{s\to\infty} \mathcal{M}(s,t) \lesssim s^2$



- EFT dispersion relation

$$\mathcal{M}(s,t) \sim \frac{1}{\pi}\int_{\Lambda^2}^{\infty} \mathrm{d}s' \left(\frac{s^2}{s'^2(s'-s)} + \frac{u^2}{s'^2(s'-u)} \right) \mathrm{Im}\mathcal{M}(s',t)$$

s': scale of UV particles

EFT amplitude

IR — UV connection

UV full amplitude

Constraints on EFT

$$\mathcal{M}(s,t) \sim \frac{1}{\pi}\int_{\Lambda^2}^{\infty} \mathrm{d}s' \left(\frac{s^2}{s'^2(s'-s)} + \frac{u^2}{s'^2(s'-u)} \right) \mathrm{Im}\mathcal{M}(s',t)$$

$$\mathcal{M}_{\mathrm{EFT}} = f_0 + f_2(s^2+t^2+u^2) + \ldots$$

Part of Unitarity

$$\mathrm{Im}\, a_\ell(s') \geq 0$$

- Expand at $s = t = 0$

Adams et al. hep-th/0602178
de Rham et al., 1702.06134
...

$$f_2 \propto \int_{\Lambda^2}^{\infty} \mathrm{d}s' \frac{1}{s'^3} \sum_\ell (2\ell+1)\mathrm{Im}\, a_\ell(s') \geq 0$$

- + full crossing symmetry: Projective bounds f_i/f_2 ; Upper bound on f_2

Tolley, Wang, Zhou, 2011.02400
Caron-Huot, Van Duong, 2011.02957
...

Recent review: de Rham, Kundu, Reece, Tolley, Zhou, Snowmass White Paper, 2203.06805

Full UV unitarity is needed

- + full crossing symmetry: Projective bounds f_i/f_2 ;   Upper bound on f_2

- $\mathscr{M}_{\rm thr} \equiv \mathscr{M}(4m^2,0)$: **no sum rule** $\leftarrow$ subtraction $\leftarrow$ $\lim_{s\to\infty} \mathscr{M}(s,t) \lesssim s^2$

- Full unitarity: constrains the real and imaginary parts

$$|S_\ell| \equiv |1 + i\rho_M(s)a_\ell(s)| \leq 1 \to \begin{pmatrix} 1 - \frac{1}{2}\rho_M {\rm Im}a_\ell & \rho_M^{1/2}{\rm Re}a_\ell \\ \rho_M^{1/2}{\rm Re}a_\ell & 2{\rm Im}a_\ell \end{pmatrix} \succeq 0$$

EFT bootstrap w/ dispersion relation

de Rham, Tolley, **ZHW**, Zhou, 2506.22546

Known function

$$\mathcal{M}(s,t) = \mathcal{M}(s_0,t_0) + \int_{\Lambda^2}^{\infty} \frac{\mathrm{d}s'}{\pi} \left(\mathrm{Im}\,\mathcal{M}(s',t)K(s',s,t;t_0) + \mathrm{Im}\,\mathcal{M}(s',t_0)K(s',t,t_0;s_0)\right)$$

Partial wave expansion: $\mathrm{Im}\mathcal{M}(s',t) = 16\pi \sum_{\ell} (2\ell+1) P_\ell\left(1 + \frac{2t}{s'-4M^2}\right) \mathrm{Im}a_\ell(s')$

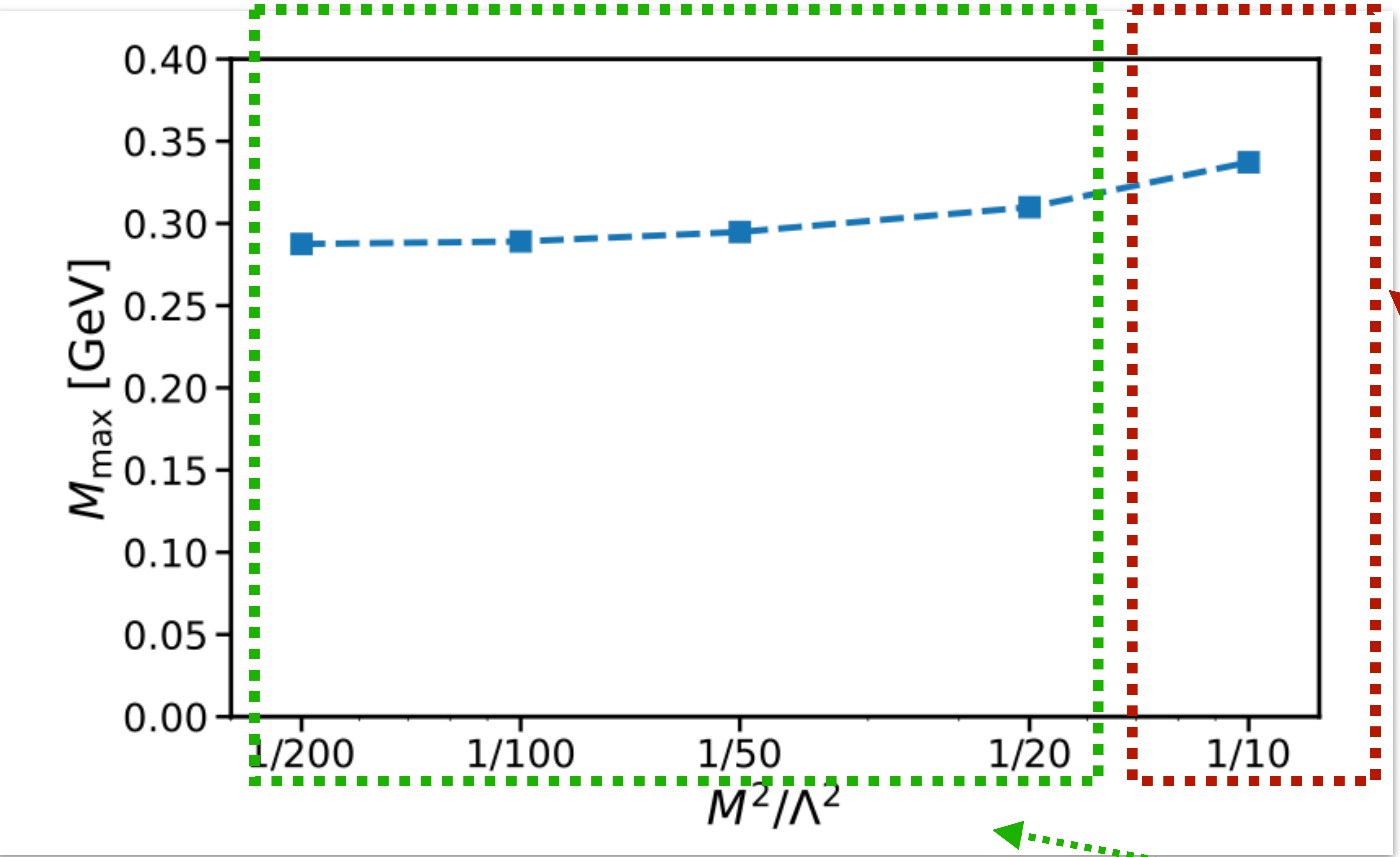
- **Amplitude** ⟸ **Imaginary part: Ansatz** $\mathrm{Im}a_\ell(s') = \sum_k c_{\ell,k} P_k\left(2\frac{\Lambda^2-4M^2}{s'-4M^2} - 1\right)$

- **Reconstruct the real part using dispersion relation**

For $s' > \Lambda^2$

$$\mathrm{Re}\mathcal{M}(s,t) = \mathcal{M}(s_0,t_0) + \mathcal{P}\int_{\Lambda^2}^{\infty} \frac{\mathrm{d}s'}{\pi} \left(\mathrm{Im}\,\mathcal{M}(s',t)K(s',s,t;t_0) + \mathrm{Im}\,\mathcal{M}(s',t_0)K(s',t,t_0;s_0)\right)$$

S-matrix bootstrap bounds on SIDM



- No explicit value of $v_{\rm rel}$ is used
- EFT loses weakly coupled regime

$2M_{\rm DM} = \Lambda$

Formally divergent, but only very slowly

- Well defined EFT: $M_{\rm DM} \ll \Lambda$

- Insensitive to EFT cutoff Λ

Weakly coupled EFT: $M_{\rm DM} \lesssim 0.3$ GeV

Approximate shift symmetry

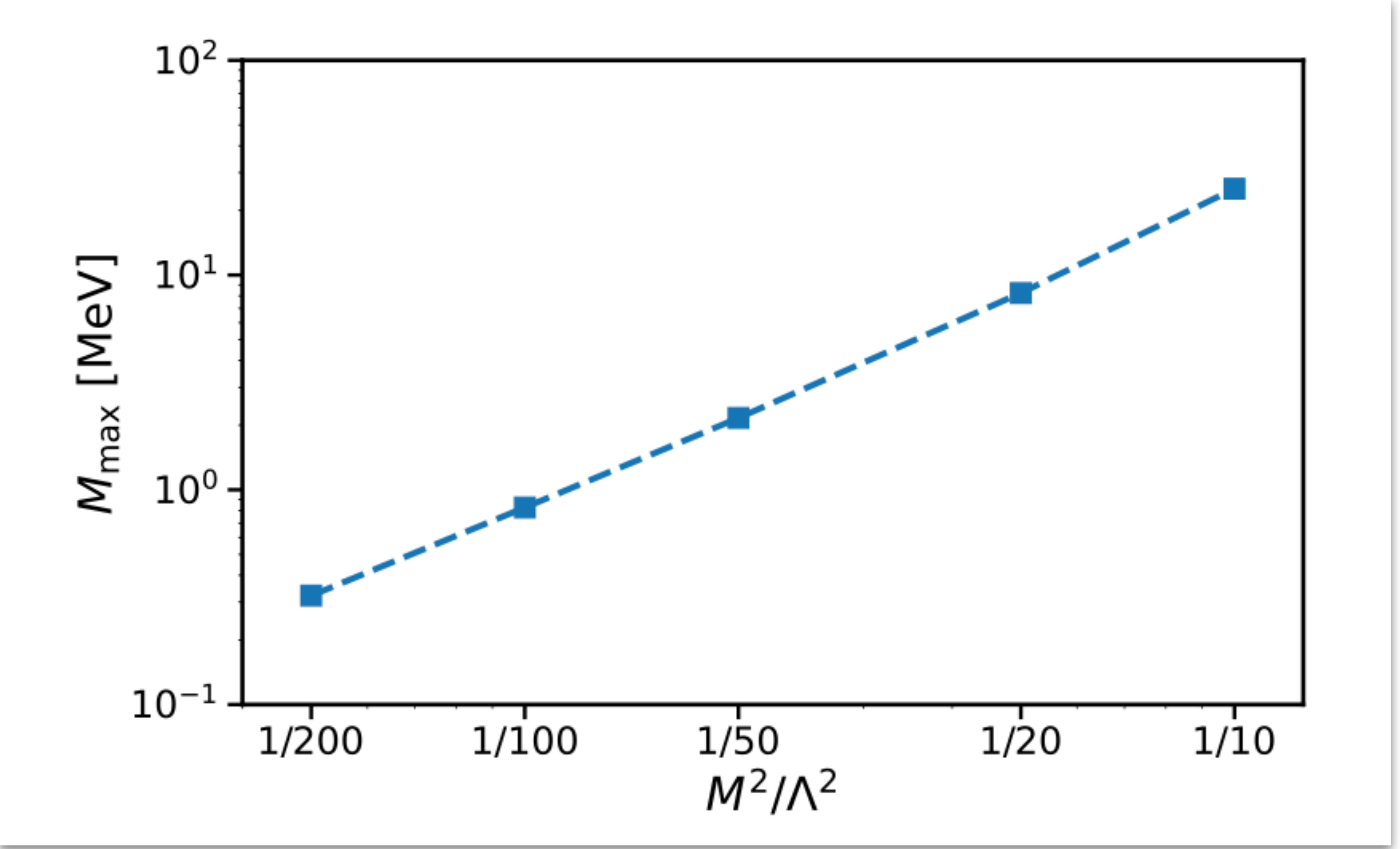
- Derivative-dominated pNGB: no φ^4

$$\mathscr{L} = -(\partial\varphi)^2/2 - m^2\varphi^2/2 - \cancel{\lambda\varphi^4} + c_4(\partial\varphi)^4 + \dots$$

- Null constraint:

$$\mathscr{M}_{\rm EFT} = f_0 + f_2 \sum_{x=s,t,u} (x - 4M_{\rm DM}^2/3)^2 + \dots$$

$$f_0 - \frac{4}{3}M_{\rm DM}^4 f_2 \approx 0 \quad \Rightarrow \quad \mathscr{M}_{\rm thr} \sim \frac{M_{\rm DM}^4}{\Lambda^4}$$



- For well defined EFT: $M_{\rm DM} \lesssim \mathcal{O}(1)$ MeV

- Similar constraints for other scenarios

Summary and outlook

- We derived bounds on the SIDM particle mass.

Hui's unitarity bound	Weakly coupled EFT	Derivative-dominated pNGB
$M_{\mathrm{DM}} \lesssim 12\ \mathrm{GeV}$	$M_{\mathrm{DM}} \lesssim 0.3\ \mathrm{GeV}$	$M_{\mathrm{DM}} \lesssim \mathrm{MeV}$

Outlook: Include controlled shift-symmetry-breaking interactions and additional light states in more general pNGB models.

Summary and outlook

- We derived bounds on the SIDM particle mass.

Hui's unitarity bound

$M_{\rm DM} \lesssim 12$ GeV

Weakly coupled EFT

$M_{\rm DM} \lesssim 0.3$ GeV

Derivative-dominated pNGB

$M_{\rm DM} \lesssim$ MeV

Thank you!