Discovering the Axiverse via Fifth Forces

[2606.06606]

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- fifth force experiments try to measure small deviations from the four known forces
  - ➤ torsion-balance experiments measure small twists in a fibre caused by forces acting on test masses
- many theories predict multiple axions (e.g. String Theory)
  - ➤ may enhance the expected force

Basics

- Axions and Axion-Like Particles (ALPs) can couple to fermions like

$$\mathcal{L}_{\text{int}} = \sum_{i=1}^{N_a} \frac{\partial_\mu a_i}{2f_i} \bar{N} g_{N,i} \gamma^\mu \gamma^5 N + \sum_{i,j=1}^{N_a} \frac{a_i a_j}{f_i f_j} \bar{N} c_{N,ij} N$$

- At low energies, these axions generate a potential between two fermions
  - ➤ Fifth Forces

$$V(\mathbf{r}) = \frac{1}{4M_{\psi_1} M_{\psi_2}} \int \frac{\mathrm{d}^3 q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{r}} \mathcal{M}\left(t = -\mathbf{q}^2\right)$$

Tree-Level Axion Exchange

- When only one axion is exchanged between two fermions, the potential is

$$V_1^{(N_a)}(\mathbf{r}) \approx -\boxed{\sum_{i=1}^{N_a}} \frac{g_{N_1,i}g_{N_2,i}}{16\pi f_i^2}\frac{1}{r^3}e^{-m_i r}(1+m_i r)$$
$$\times\left[\vec{S}_1\cdot\vec{S}_2 - 3\left(\vec{S}_1\cdot\hat{r}\right)\left(\vec{S}_2\cdot\hat{r}\right)\right]$$

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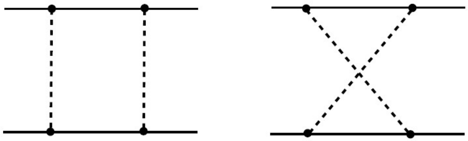
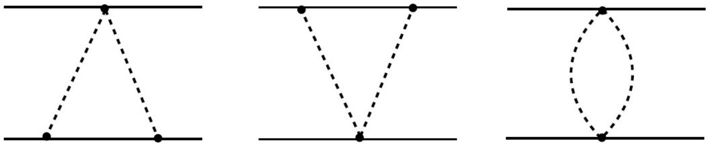
- Relatively long range, with $V_1 \propto r^{-3}$
- **BUT!** V_1 is spin-dependent
  - ➤ vanishes for unpolarised objects

Double Axion Exchange

- Exchange of two axions results in loop diagrams
- linear coupling $(\partial_\mu a_i)\bar{\psi}\gamma^\mu\gamma^5\psi$ leads to



- quadratic coupling $a_i a_j \bar{\psi}\psi$ leads to further diagrams



Simplifying the Loops

- Remember that we need to take the Fourier transform of the loop

$$V(\mathbf{r}) \sim \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, e^{-i\mathbf{q}\cdot\mathbf{r}} \mathcal{M}\left(-\mathbf{q}^2\right)$$

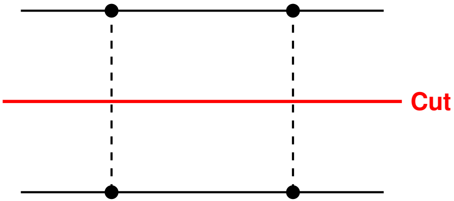
- if we know the discontinuities of the amplitude, we get a Laplace transform instead

$$V(r) \sim \frac{1}{r} \int_{t_0}^{\infty} \mathrm{d}t\, \mathrm{Disc}(\mathcal{M}(t))\, e^{-\sqrt{t}r}$$

➤ discontinuity is easy to calculate!

Cutkosky Rules

- Discontinuities of Feynman diagrams can be found by “cutting” them into two pieces



- one then sets the cut propagators to be on-shell by replacing

$$\frac{1}{p^2 - m^2 + i\epsilon} \to -2\pi i \delta(p^2 - m^2)\theta(p^0)$$

Potential

- Resulting potential is generally not analytic
- if all axions had the same masses, then

$$V_2^{(N_a)}(r) = \frac{1}{128\pi^3} \sum_{i,j=1}^{N_a} \frac{1}{f_i^2 f_j^2} \Bigg\{ \frac{3}{r^5} g_i^\alpha g_i^\beta g_j^\alpha g_j^\beta \mathcal{K}(x_a) + \frac{12}{r^5} \left[\frac{g_i^\alpha g_j^\alpha c_{ij}^\beta}{M_\alpha} + (\alpha \leftrightarrow \beta) \right] \left[\mathcal{K}(x_a) + \frac{x_a^4}{48} K_0(x_a) \right] - \frac{8}{r^3} c_{ij}^\alpha c_{ij}^\beta x_a K_1(x_a) \Bigg\}, \quad x_a = 2 m_a r$$

$$\mathcal{K}(x_a) = \frac{x_a^2}{2} K_0(x_a) + \left(x_a + \frac{x_a^3}{6} \right) K_1(x_a)$$

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$$+\frac{12}{r^5}\left[\frac{g_i^\alpha g_j^\alpha c_{ij}^\beta}{M_\alpha} + (\alpha \leftrightarrow \beta)\right]\left[\mathcal{K}(x_a) + \frac{x_a^4}{48}K_0(x_a)\right]$$

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Kaluza-Klein ALP

- Kaluza-Klein Axions arise in theories of compactified extra dimensions
- KK ALPs are characterised by the mass and f_a distributions

$$m_n = \sqrt{m_0^2 + (n\mu_1)^2}$$
$$f_n = f_a = \text{const.}$$

- as a benchmark, we assign the following parameter values

$$m_0 = 10^{-4}\,\text{eV}$$
$$\mu_1 = 5 \times 10^{-5}\,\text{eV}$$

Kaluza-Klein Maxion

- KK Maxions are maximally far from the QCD axion band

$$m_n = \left(n - \frac{1}{2}\right)\mu_1$$
$$f_n^{(N_a)} = f_a \times \frac{2N_a - 1}{2n - 1}$$

- as a benchmark, we assign the parameter value

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Type IIB String Theory

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- we can approximate the distribution of τ_i by sampling from an **empirical distribution**
- number of axions is equal to the Hodge number $h^{1,1}$

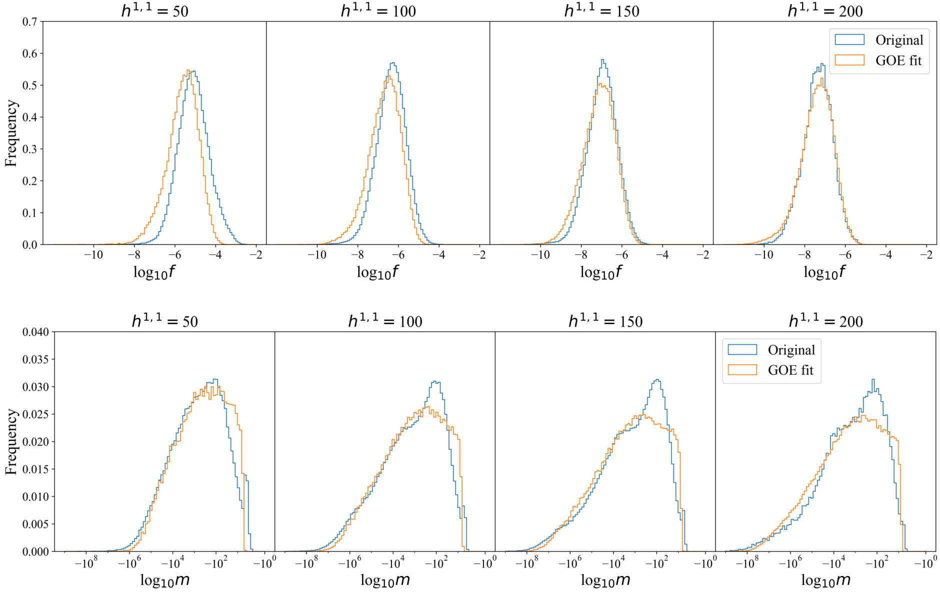

Figure: Distributions of axion decay constants in units of M_{pl} computed using a database of compactifications (blue) and the approximated distribution (orange) [2507.12516].

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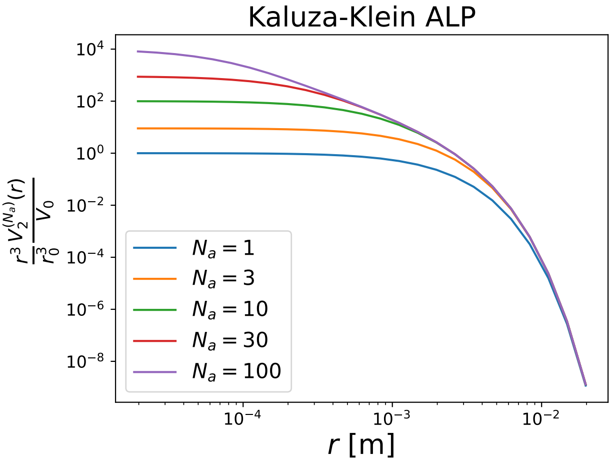
- we nevertheless assume a shift symmetry conserving coupling of order $g_i \sim 1$
- for the shift symmetry breaking coupling, we assume $c_{ij} \lesssim M_{\mathrm{neutron}}$

Kaluza-Klein ALP Spin-Independent Potential



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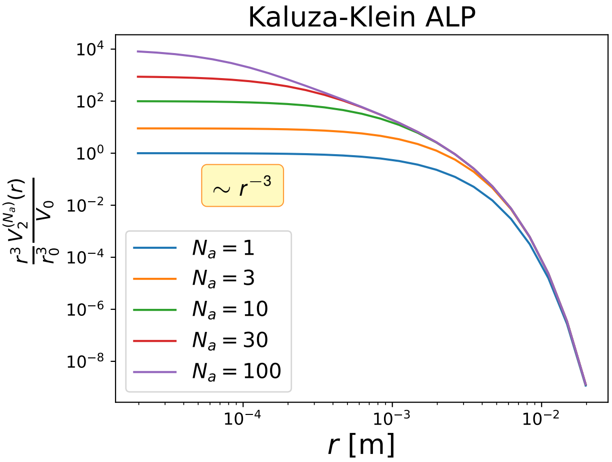

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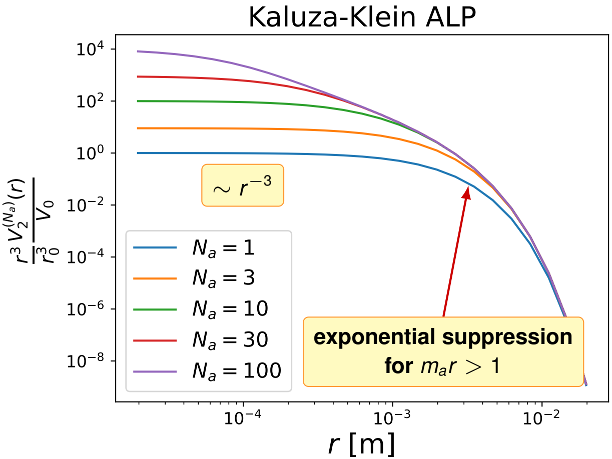

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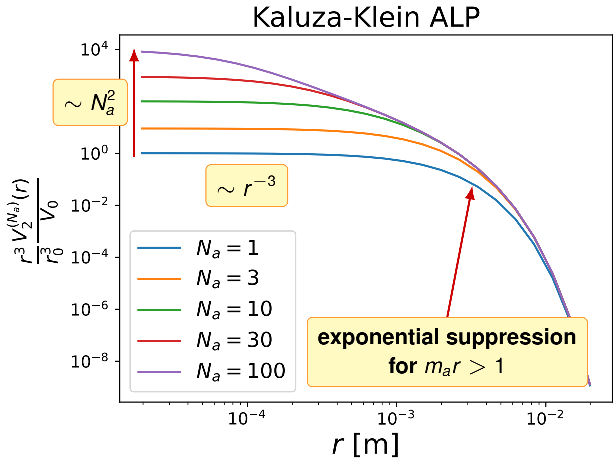

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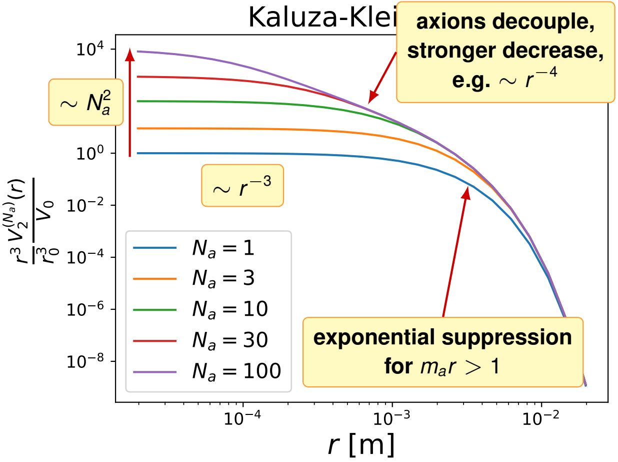
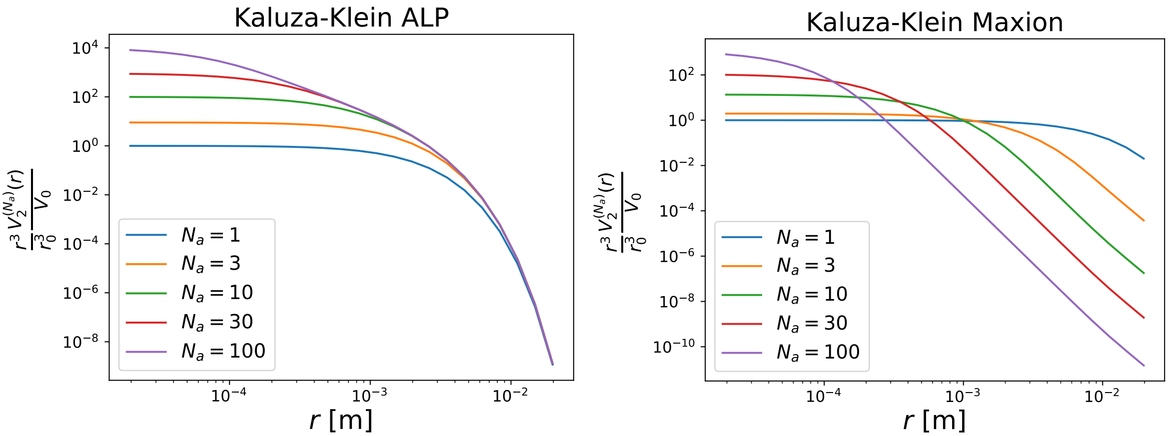


Kaluza-Klein Models



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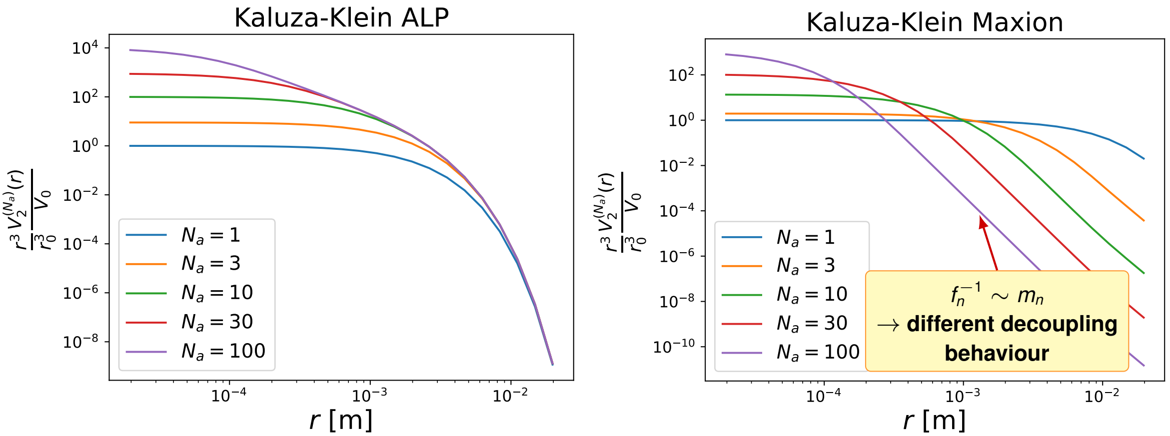
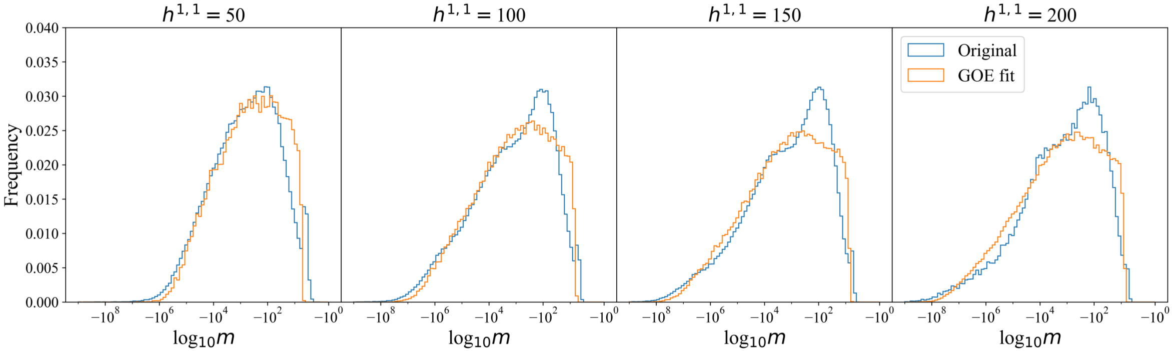


Type IIB String Theory



- Most of the axions have ultralight masses
- potential will have a behaviour very close to $V \propto r^{-3}$ for most distances with only slight deviations

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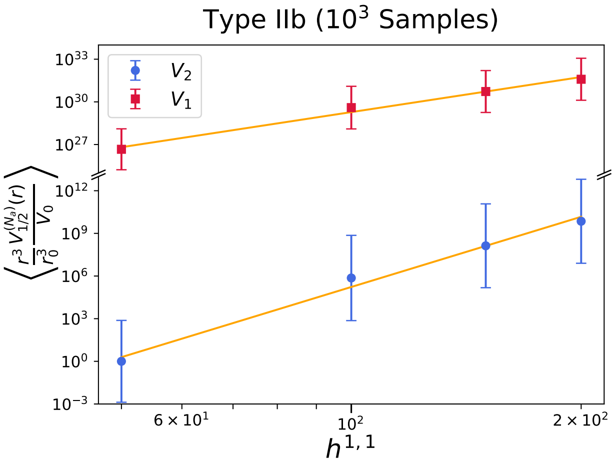
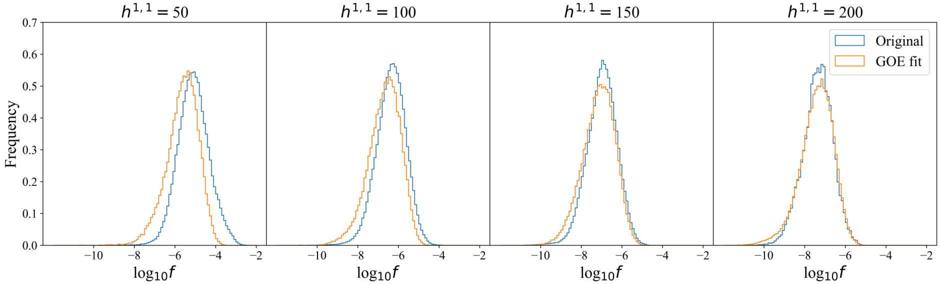


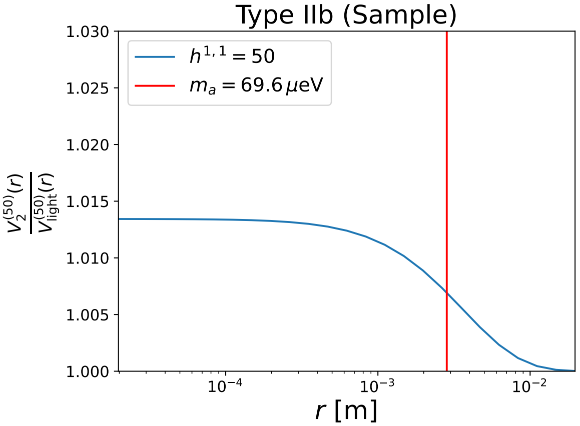
Type IIB String Theory: Potential Scaling

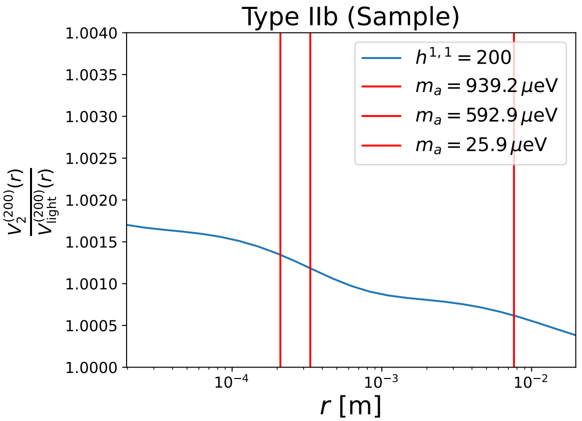


- Why is $V_2 \sim N_a^{16}$?
- on average, the f_a scale like $N_a^{-7/2}$

$$V_2 \sim \sum_{i,j=1}^{N_a} \frac{1}{f_i^2} \frac{1}{f_j^2} \sim N_a^{16}$$

Type IIB String Theory: Potential Steps





Conclusions

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- N_a axions can generally lead to an N_a^2 enhancement of the spin-independent potential
- potential can be further enhanced by changes to the f_a distribution
- "steps" in the potential may lead to potentials that cannot be fit by r^{-n}
  - ➤ can be used to distinguish models with one and multiple new light degrees of freedom

Type IIB String Theory: Sampling

- Define normalised decay volumes $\hat{\tau}^i$

$$\hat{\tau}^i \equiv \frac{\log_{10}(\tau^i)}{\langle \log_{10}(\vec{\tau}) \rangle}$$

$$\langle \log_{10}(\vec{\tau}) \rangle = 3.433 - 5.404 \times (\log_{10} h^{1,1})^{-4.050}$$

- sample $h^{1,1} + 4$ decay volumes from the distribution

$$p(\hat{\tau}) = \frac{\pi}{2} \hat{\tau} e^{-\frac{\pi}{4}\hat{\tau}^2}$$

- normalise all volumes such that $\hat{\tau}_{\mathrm{min}} = 1$
- discard the largest four volumes

Type IIB String Theory: Sampling

- use the remaining $\hat{\tau}^i$ to calculate τ^i and

$$m_i = \frac{M_{pl}\left(\tau^i\right)^{3/4}}{\left(\tau^{\max}\right)^{3/4}} e^{-\pi\tau^i}$$

$$f_i = \frac{M_{pl}}{\left(\tau^{\max}\right)^{3/4}\left(\tau^i\right)^{1/4}}$$