# Meson Structure Using the Basis Light-Front Quantization (BLFQ)


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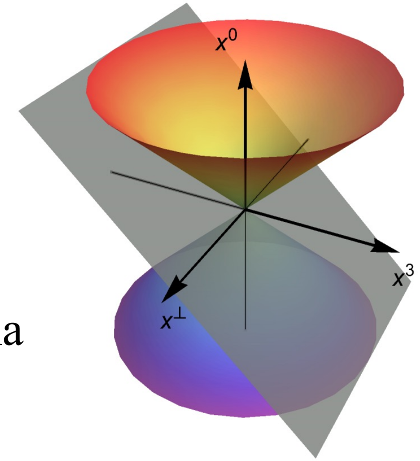



5 Feb 2026, ITP, Beijing

# Outline

- Basis Light-front Quantization
- Application to Light Mesons
  - Light mesons with $q\bar{q}$
  - Light mesons with $q\bar{q} + q\bar{q}g$
  - Light mesons with $q\bar{q} + q\bar{q}g$ BLFQ .vs. BSE
- Conclusion & Outlook

*LFWF, EMFF,*

*PDA, PDF, …*

# Light-front Hamiltonian Framework

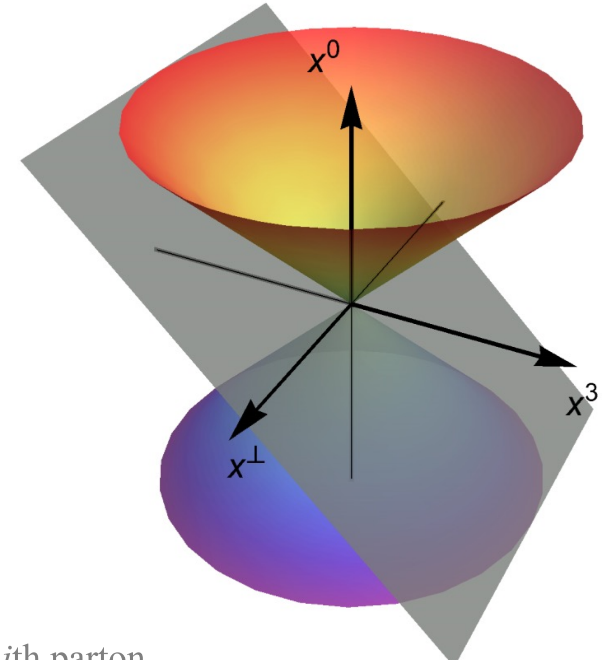
Quantization on the light-front : $x^+ = x^0 + x^3$

LF momenta : $P^+ = P^0 + P^3; \vec{P}_\perp$

Choose transverse rest frame: $\vec{P}_\perp = 0$

LF Hamiltonian: $H_{LF} \equiv P^+ P^-$

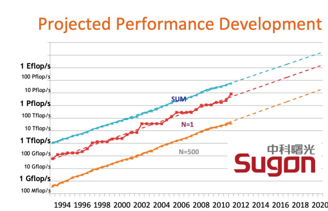
Eigenvalue problem : $H_{LF}|P, \Lambda\rangle = M^2|P, \Lambda\rangle$




$$H_{LF} = \sum_i \frac{m_i^2 + \vec{p}_{i\perp}^{\,2}}{x_i} + \mathrm{H}_{int}$$

$x_i$ LF momentum fraction carried by $i$th parton

**DLCQ** → **BLFQ**




Both are constructed in Minkowski space-time

Longitudinal direction (plane wave)

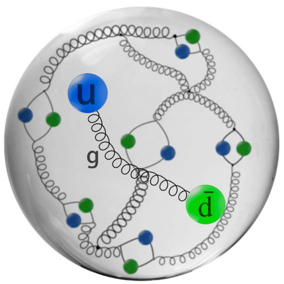
-Brodsky, Pauli, Pinsky Phys. Reports. 301, 299 (1998)
-…

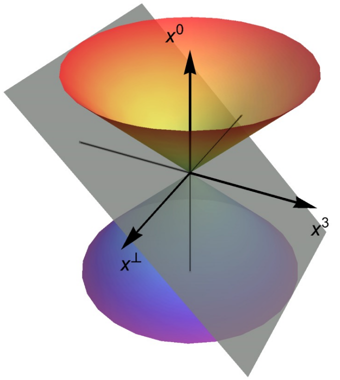
Longitudinal direction (plane wave)
Transverse direction (harmonic oscillator)

-Vary, et al, PRC 81, 035205 (2010)
-…

# Basis Light-front Quantization

[Vary et al, PRC 81, 035205 (2010)]




- Nonperturbative eigenvalue problem
$$P^-|\beta\rangle = P^-_\beta|\beta\rangle$$
  - $P^-$: light-front Hamiltonian
  - $|\beta\rangle$: mass eigenstate
  - $P^-_\beta$: eigenvalue for $|\beta\rangle$
- Evaluate observables for eigenstate
$$O \equiv \langle\beta|\hat{O}|\beta\rangle$$
- Fock sector expansion
  - Eg. $|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + d|q\bar{q}gg\rangle + \cdots$

| **Discretized basis** | **Truncation** |
|---|---|
| Transverse: 2D harmonic oscillator basis: $\Phi^b_{n,m}(\vec{p}_\perp)$. | $\sum_i(2n_i + \lvert m_i\rvert + 1) \le N_{max}$ |
| Longitudinal: plane-wave basis, labeled by $k$. | $\sum_i k_i = K, \quad x_i = k_i/K$ |
| Light-front helicity state for spin d.o.f. | $\sum_i(m_i + \lambda_i) = M_J$ |
| $\alpha_i = (k_i, n_i, m_i, \lambda_i)$<br>$\lvert\alpha\rangle = \otimes_i \lvert\alpha_i\rangle$ | Fock sector truncation |

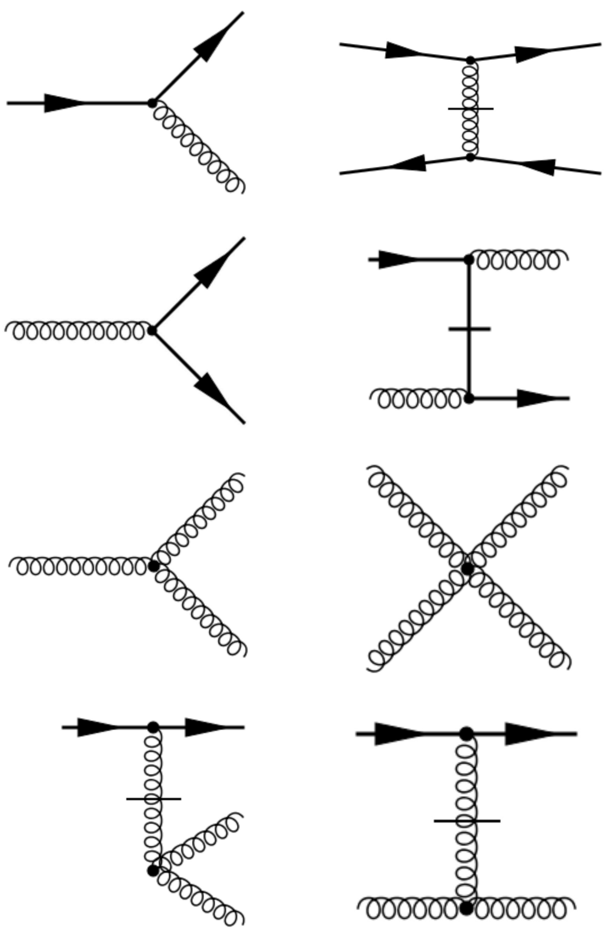
$\Lambda_{IR} \sim \frac{b}{\sqrt{N_{max}}}$ and $\Lambda_{UV} \sim b\sqrt{N_{max}}$. $b$ is HO energy scale

Large $N_{max}$ and $K$ → High UV cutoff & low IR cutoff

# Light-front Hamiltonian

QCD light-front Hamiltonian can be derived from QCD Lagrangian:

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4}G^{\alpha}_{\mu\nu}G^{\mu\nu}_{\alpha} \Rightarrow \boldsymbol{P^{-}_{QCD} = H_K + H_{int}} \qquad A^{+} = 0$$



$$\boldsymbol{H_K} = \frac{1}{2}\int d^3x\, \bar{\psi}\gamma^{+}\frac{(i\partial^{\perp})^2 + m^2}{i\partial^{+}}\psi + \frac{1}{2}\int d^3x\, A^{i}_{a}(i\partial^{\perp})^2 A^{i}_{a}$$

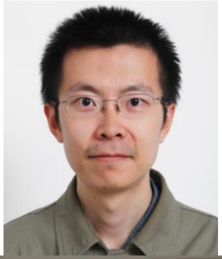
$$\begin{aligned}
\boldsymbol{H_{int}} &= g\int d^3x\, \bar{\psi}\gamma_{\mu}A^{\mu}\psi \\
&\quad + \frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma_{\mu}A^{\mu}\frac{\gamma^{+}}{i\partial^{+}}\gamma_{\nu}A^{\nu}\psi \\
&\quad - ig^2\int d^3x\, f^{abc}\bar{\psi}\gamma^{+}T^{c}\psi\frac{1}{(i\partial^{+})^2}\left(i\partial^{+}A^{\mu}_{a}A_{\mu b}\right) \\
&\quad + \frac{1}{2}g^2\int d^3x\, \bar{\psi}\gamma^{+}T^{a}\psi\frac{1}{(i\partial^{+})^2}\bar{\psi}\gamma^{+}T^{a}\psi \\
&\quad + ig\int d^3x\, f^{abc}i\partial^{\mu}A^{\nu a}A^{b}_{\mu}A^{c}_{\nu} \\
&\quad - \frac{1}{2}g^2\int d^3x\, f^{abc}f^{ade}\, i\partial^{+}A^{\mu}_{b}A_{\mu c}\frac{1}{(i\partial^{+})^2}(i\partial^{+}A^{+}_{d}A_{\nu e}) \\
&\quad + \frac{1}{4}g^2\int d^3x\, f^{abc}f^{ade}A^{\mu}_{b}A^{\nu}_{c}A_{\mu d}A_{\nu e}.
\end{aligned}$$

$\psi$: quark field operator

$A^{a}_{\mu}$: gluon field operator

[Brodsky, Pauli, Pinsky, 1998]

# Basis Light-front Quantization Group



**Staff**: 4 (1 Prof. Xingbo Zhao, 2 Assoc. Res., 1 Asst. Res.)

**Postdocs**: 2 **Students**: 11 (9 Ph.D., 2 M.S.)

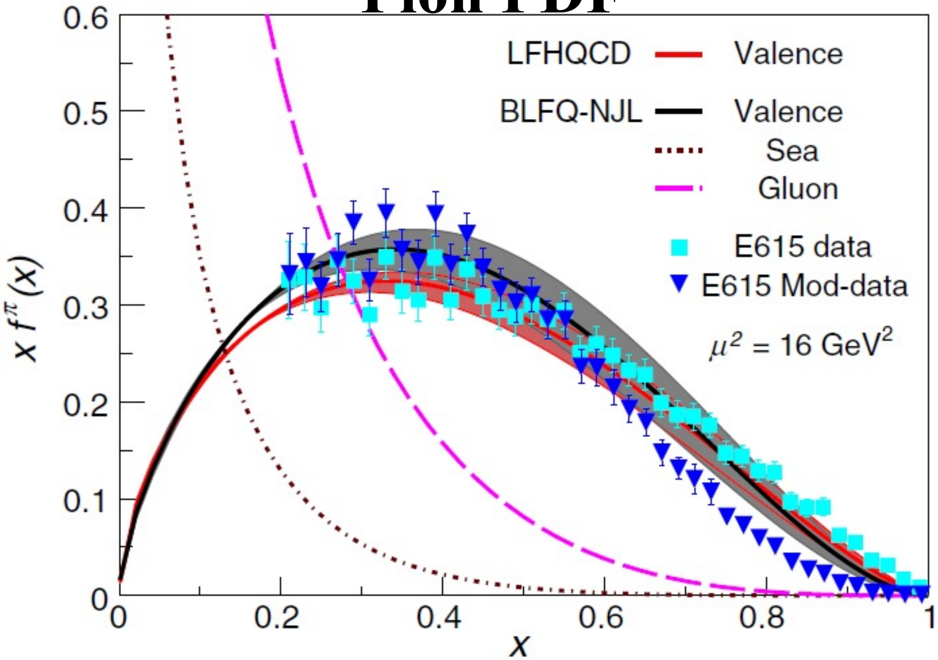
# PDFs for Light Mesons

$|\text{meson}\rangle = |q\bar{q}\rangle + \cdots$

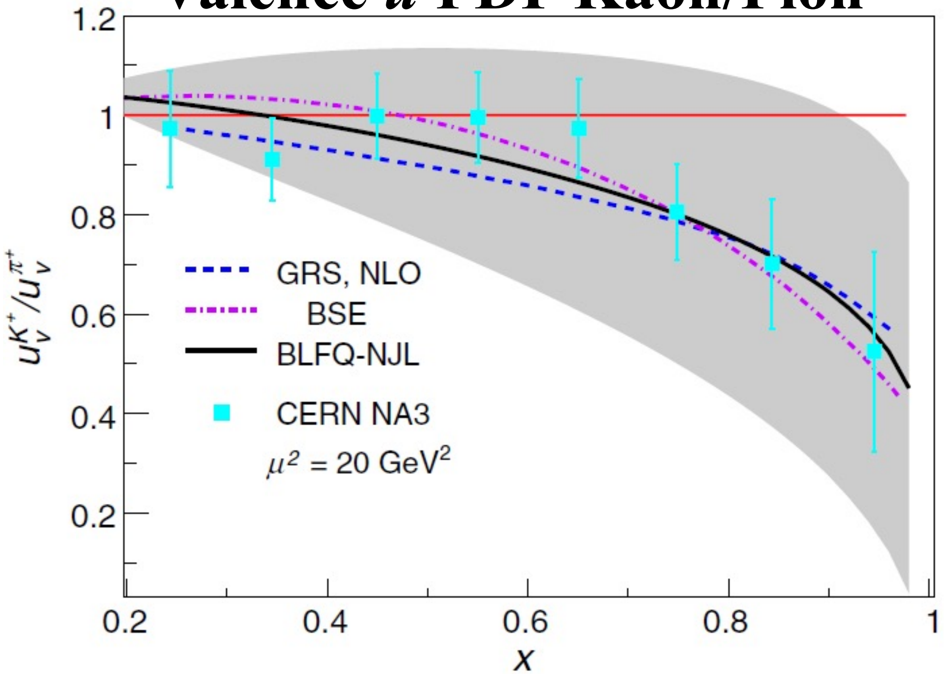
$$H_{\text{eff}} = \frac{\vec{k}_\perp^2 + m_q^2}{x} + \frac{\vec{k}_\perp^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x)\vec{r}_\perp^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2}\partial_x(x(1-x)\partial_x) + H_{\text{eff}}^{\text{NJL}}$$

Diagonalizing $H_{\text{eff}}$ → LF wave function → Initial PDFs → Scale evolution

**Pion PDF**




**Valence $u$ PDF Kaon/Pion**

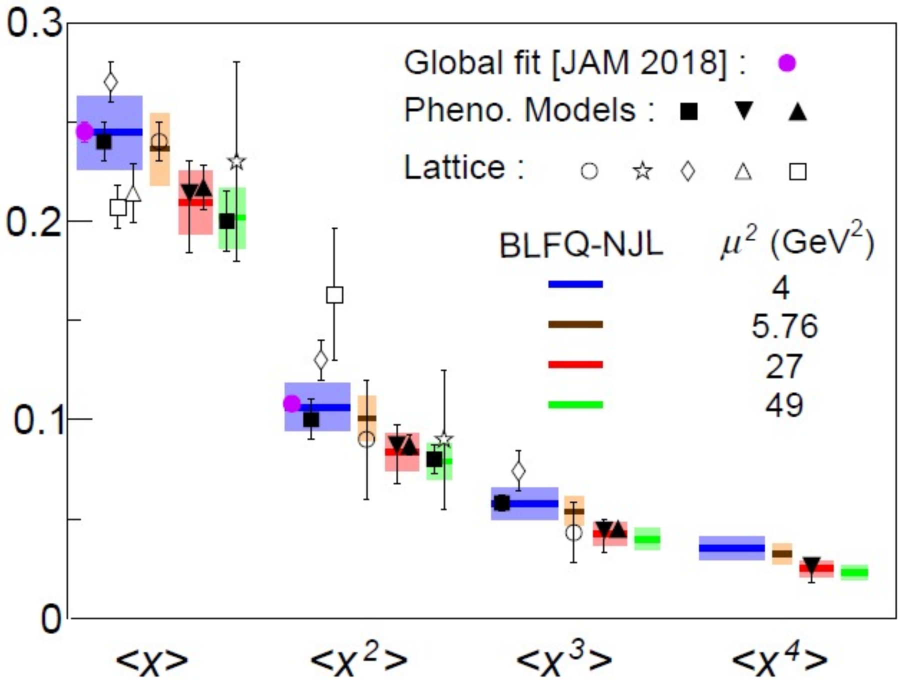


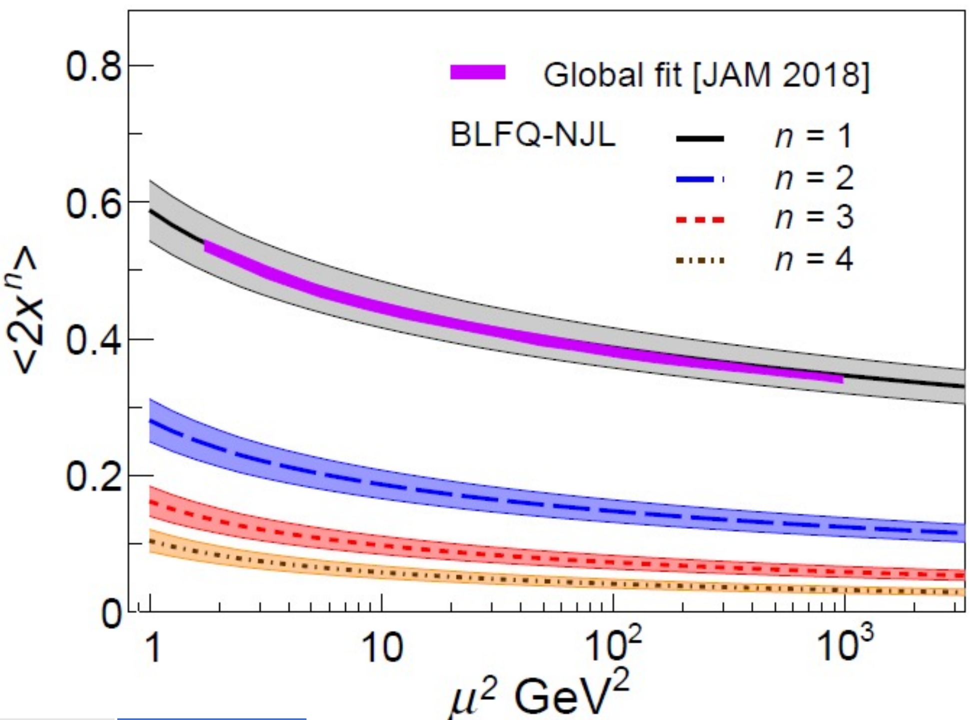

[Lan, Mondal, Jia, Zhao, Vary, PRL122(2019)172001]

**Agree with experimental results**

# The Moments of Pion Valence Quark PDF

$$\langle x^n \rangle = \int_0^1 dx\ x^n f_v^{\pi/K}(x, \mu^2),\ \ n = 1, 2, 3, 4.$$




| $\langle x \rangle$ @ 4 GeV² | Valence | Gluon | Sea |
|---|---|---|---|
| **BLFQ-NJL** | **0.489** | **0.398** | **0.113** |
| [Ding *et. al.*, BSE model 2019'] | 0.48(3) | 0.41(2) | 0.11(2) |

**Agree with other results**

[Lan, Mondal, Jia, Zhao, Vary, PRD101(2020)034024]

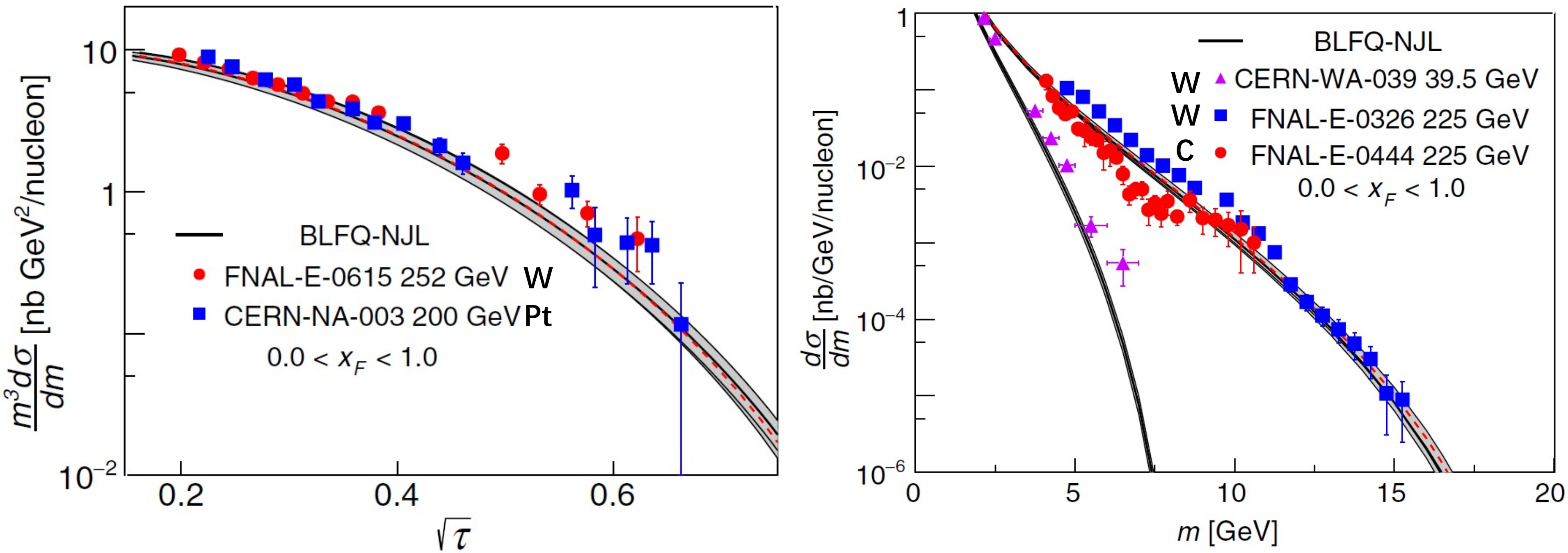
# Drell-Yan Cross Section

$$\pi^- \ \text{Nucleus} \rightarrow \mu^+ \ \mu^- \ X$$

[S. D. Drell and T.-M. Yan, PRL (1970)]
[T. Becher et al, JKEP07(2008)030]; [P. C. Barry et al, PRL121(2018)152001]
[C. Anastasiou et al, PRL91(2003)182002]

[nCTEQ 2015]

$$\frac{m^3 d^2\sigma}{dmdY} = \frac{8\pi\alpha^2}{9}\frac{m^2}{s}\sum_{ij} dx_1 dx_2 \tilde{C}_{ij}(x_1, x_2, s, m, \mu_f) f_{i/\pi}(x_1, \mu_f) f_{j/N}(x_2, \mu_f)$$




Agree with experimental data (**FNAL E615**, **326**, **444**, & **CERN NA3**, **WA-039**).

[Lan, Mondal, Jia, Zhao, Vary, PRD101(2020)034024]

# Light mesons up to $|q\bar{q}g\rangle$

$$|\text{meson}\rangle = |q\bar{q}\rangle + \cdots$$

[J. Lan et al, PRL122, 172001(2019)]
[J. Lan et al, PRD 101 (2020) 034024]

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

[J. Lan et al, PLB 825 (2022) 136890]
[Z. Zhu et al, PLB 839(2023) 137808]
[J. Lan et al, PLB 868 (2025) 139654]
[J. Lan et al, PRD 111 (2025) 11, L111903]
[J. Lan et al, arXiv: 2509.22016 [hep-ph]]

# Light mesons up to $|q\bar{q}g\rangle$

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle$$

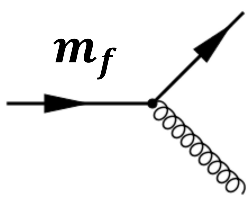
$$\boldsymbol{P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}}$$

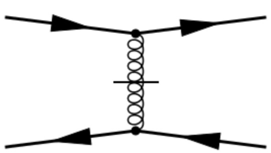
$$\boldsymbol{H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}}$$

$\boldsymbol{H_{trans} \sim \kappa_T^4 r^2}$ ---Brodsky, Teramond arXiv: 1203.4025

$\boldsymbol{H_{longi} \sim -\sum_{ij} \kappa_L^4 \partial_{x_i}\left(x_i x_j \partial_{x_j}\right)}$ ---Y Li, X Zhao , P Maris , J Vary, PLB 758(2016)

$$\boldsymbol{H_{Interact} = H_{Vertex} + H_{inst} = g\bar{\psi}\,\gamma^\mu T^a\,\psi\,A_\mu^a + \frac{g^2 C_F}{2}\,j^+\frac{1}{(i\partial^+)^2}j^+}$$



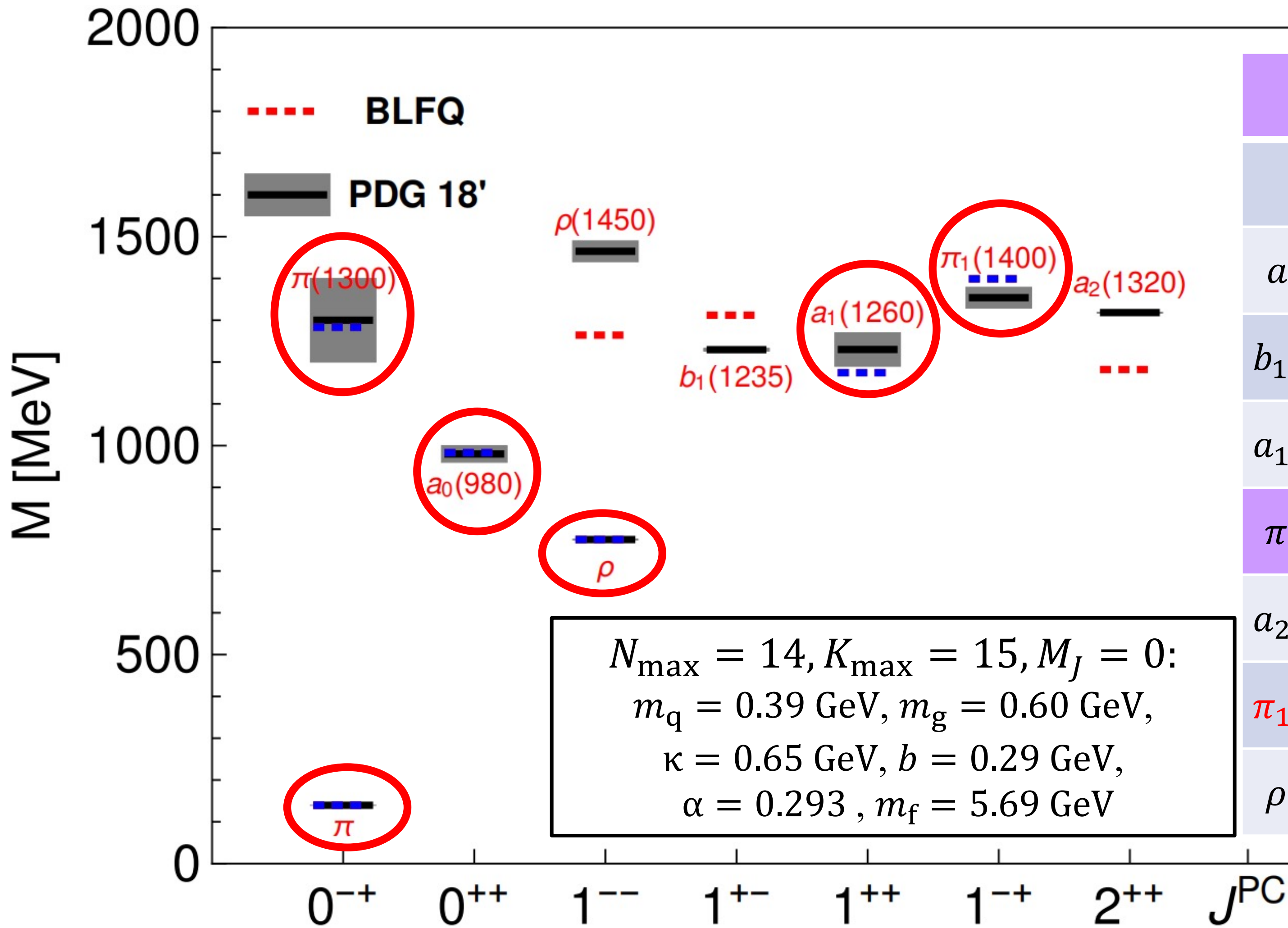



We allow an independent quark mass $\boldsymbol{m_f}$ in the vertex interaction.

[PRD 45 (1992) 3740–3754]

[J. Lan et al, PLB 825 (2022) 136890]

# Light meson mass spectrum




| | Norm$q\bar{q}$ | DC [MeV] |
|---|---|---|
| $\pi$ | 0.492 | 151 |
| $\rho$ | 0.486 | 141 |
| $a_0(980)$ | 0.370 | 0 |
| $b_1$ (1235) | 0.30 | 0 |
| $a_1$ (1260) | 0.324 | 13 |
| $\pi(1300)$ | 0.284 | 59 |
| $a_2$ (1320) | 0.320 | 0 |
| $\pi_1$ (1400) | 0.002 | 0 |
| $\rho(1450)$ | 0.312 | 50 |

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

[J. Lan et al, PLB 825 (2022) 136890]

Fix the parameters by fitting six blue states

- $\pi_1(1400)$ : $|q\bar{q}g\rangle$ dominates
- $\pi(1300)$: the DC is smaller than the DC of pion

# The Wave Function in Leading Fock Sector

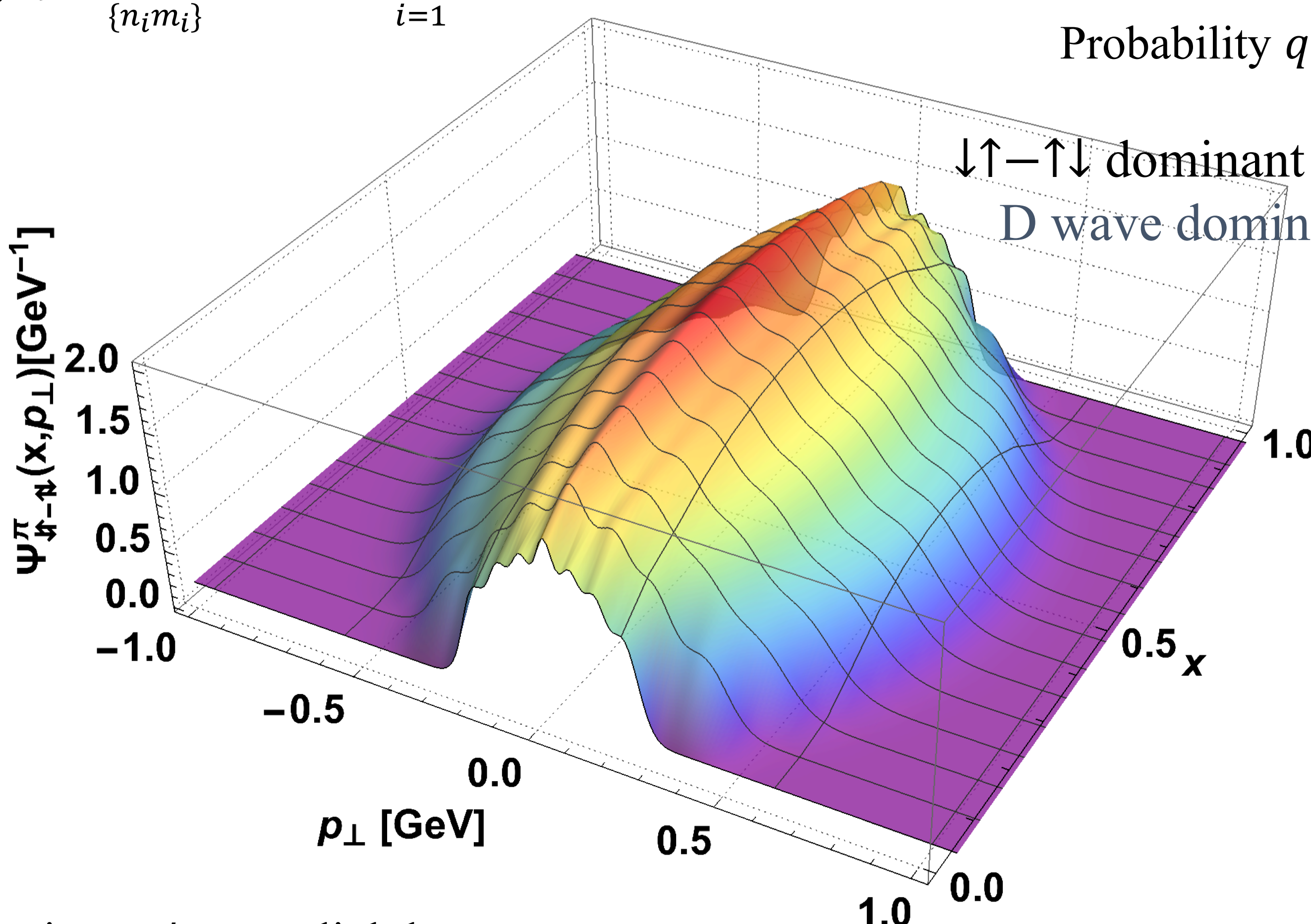
$$\Psi^{\mathcal{N},M_J}_{\{x_i,\vec{p}^{\,2}_{\perp i},\lambda_i\}} = \sum_{\{n_i m_i\}} \psi^{\mathcal{N}}(\{\bar{\alpha}_i\}) \prod_{i=1}^{\mathcal{N}} \phi_{n_i m_i}(\vec{p}_{\perp i}, b)$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

Probability $q\bar{q}$ =0.486

$\downarrow\uparrow - \uparrow\downarrow$ dominant in $|q\bar{q}\rangle$ 99.7%

D wave dominant in $|q\bar{q}g\rangle$



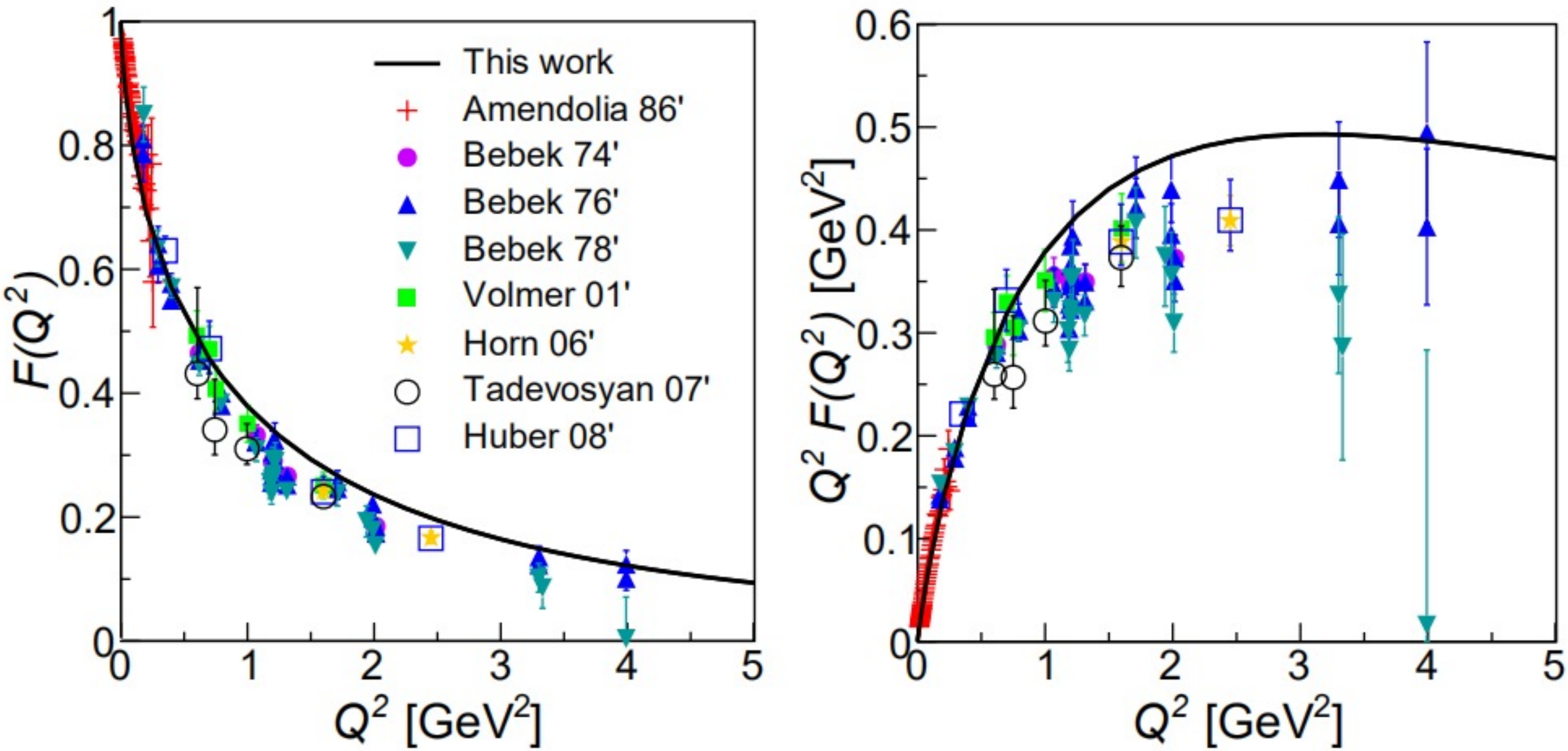

- At endpoint x, $\psi \sim p_\perp$: lightly narrow
- At middle x, $\psi \sim p_\perp$: a little bit wide

# Pion Electromagnetic Form Factor

[Brodsky & de Teramond, PRD 77(2008)056007]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

$$\langle\Psi(p')|J^{+}_{EM}(0)|\Psi(p)\rangle = (\mathrm{p}+\mathrm{p}')^{+}\,\mathrm{F}(\mathrm{Q}^2)$$




- FF is in reasonable agreement with experimental data
- $F(Q^2) \propto 1/Q^2$ for large $Q^2$



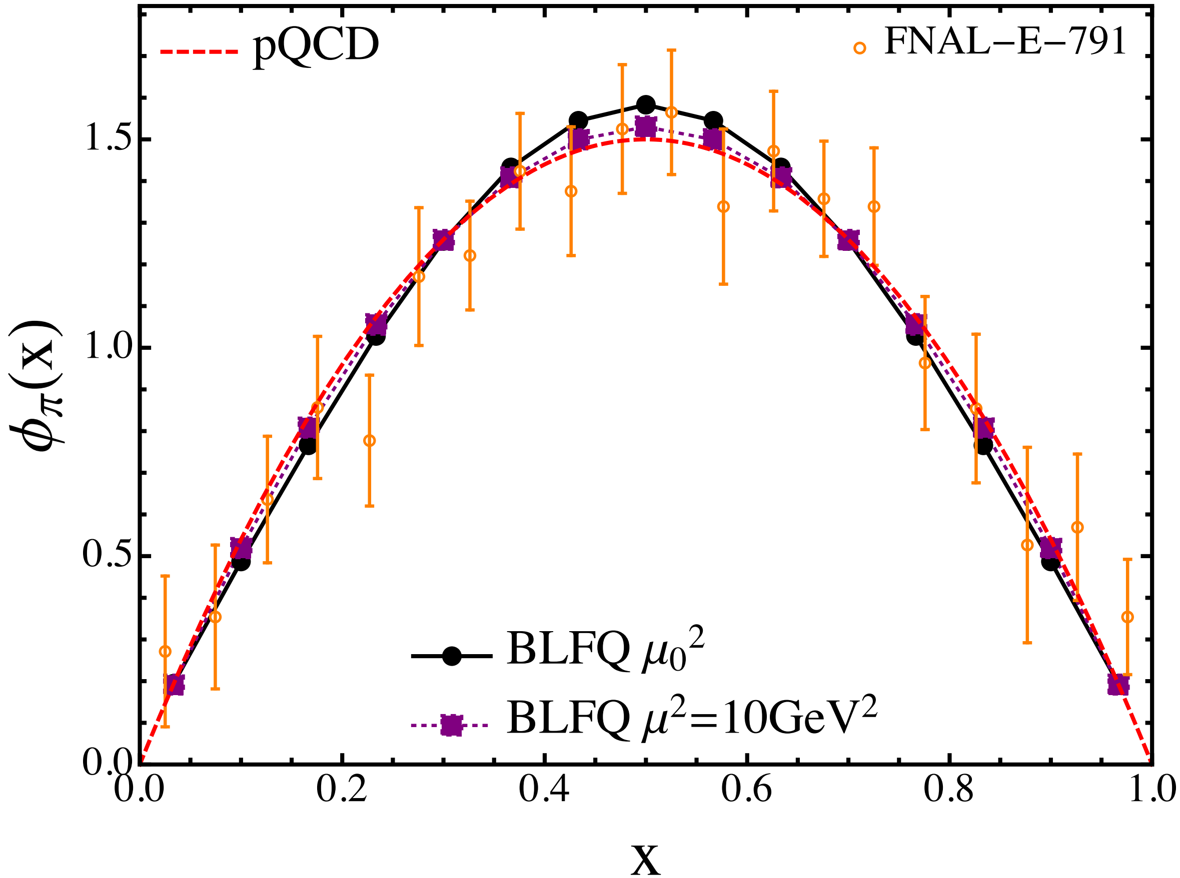
[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]

# Pion PDA

$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$

[Ruiz Arriola & Broniowski, PRD 66(2002)094016]

[PRL86. 4768]

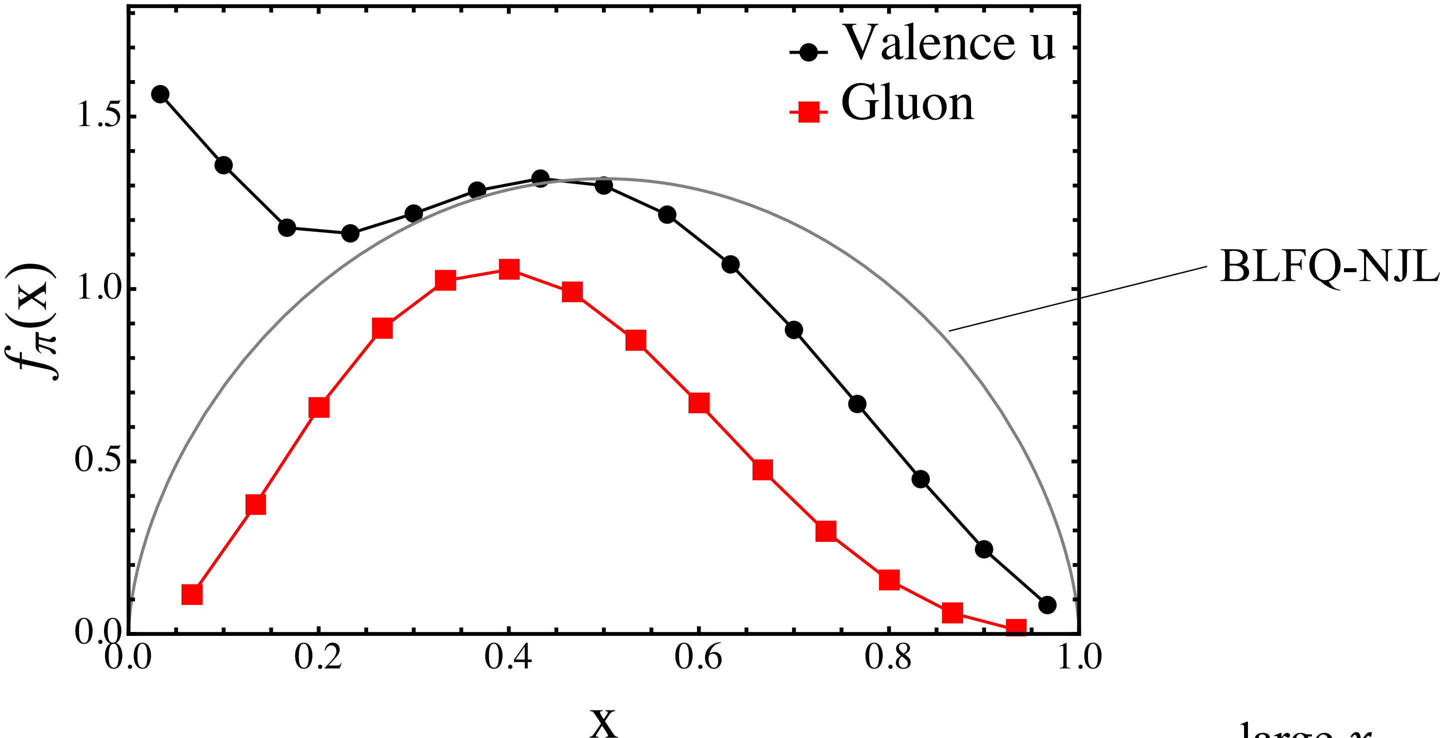



- Endpoint behavior almost agrees with pQCD
- Consistent with FNAL-E-791 experiment on Drell-Yan (pion-platinum) process

# Pion PDF at Model Scale

$$f_i(x) = \sum_{\mathcal{N},\lambda_i} \int [d\mathcal{X} d\mathcal{P}^\perp]_{\mathcal{N}} \left| \psi^{\mathcal{N}, M_j=0}_{\{x_i, \vec{p}^2_{\perp i}, \lambda_i\}} \right|^2 \delta(x - x_i)$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$



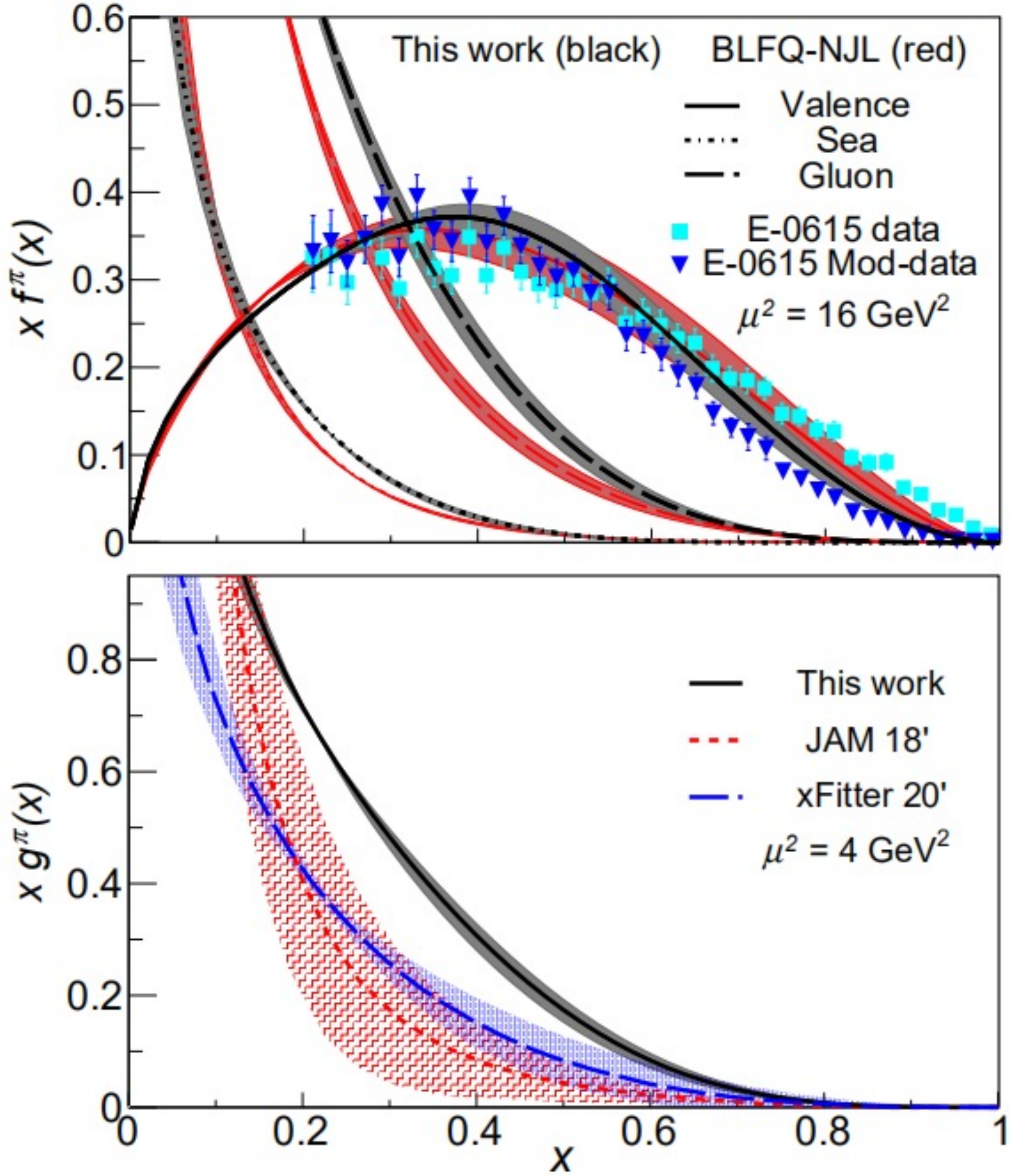

large $x$

| | | | |
|---|---|---|---|
| $\mu^2_{0\text{BLFQ-NJL}} = 0.240\ \text{GeV}^2$ | $\langle x \rangle_{\text{gluon}} = \mathbf{0}$; | $\langle x \rangle_{\text{valence } u} = \mathbf{0.5}$ | $(1-x)^{0.596}$ |
| $\mu^2_{0\text{BLFQ}} = 0.43\ \text{GeV}^2$ | $\langle x \rangle_{\text{gluon}} = \mathbf{0.216}$; | $\langle x \rangle_{\text{valence } u} = \mathbf{0.392}$ | $(1-x)^{1.4}$ |



[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]

# Pion PDF with QCD Evolution




$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

- Large-x behavior $(1-x)^{1.77}$ closer to pQCD
- The gluon distribution significantly increases

| $\langle x \rangle$ @ 4 GeV$^2$ | Valence | Gluon | Sea |
|---|---|---|---|
| **BLFQ** | **0.483** | **0.421** | **0.096** |
| BLFQ-NJL | 0.489 | 0.398 | 0.113 |
| [BSE 2019'] | 0.48(3) | 0.41(2) | 0.11(2) |



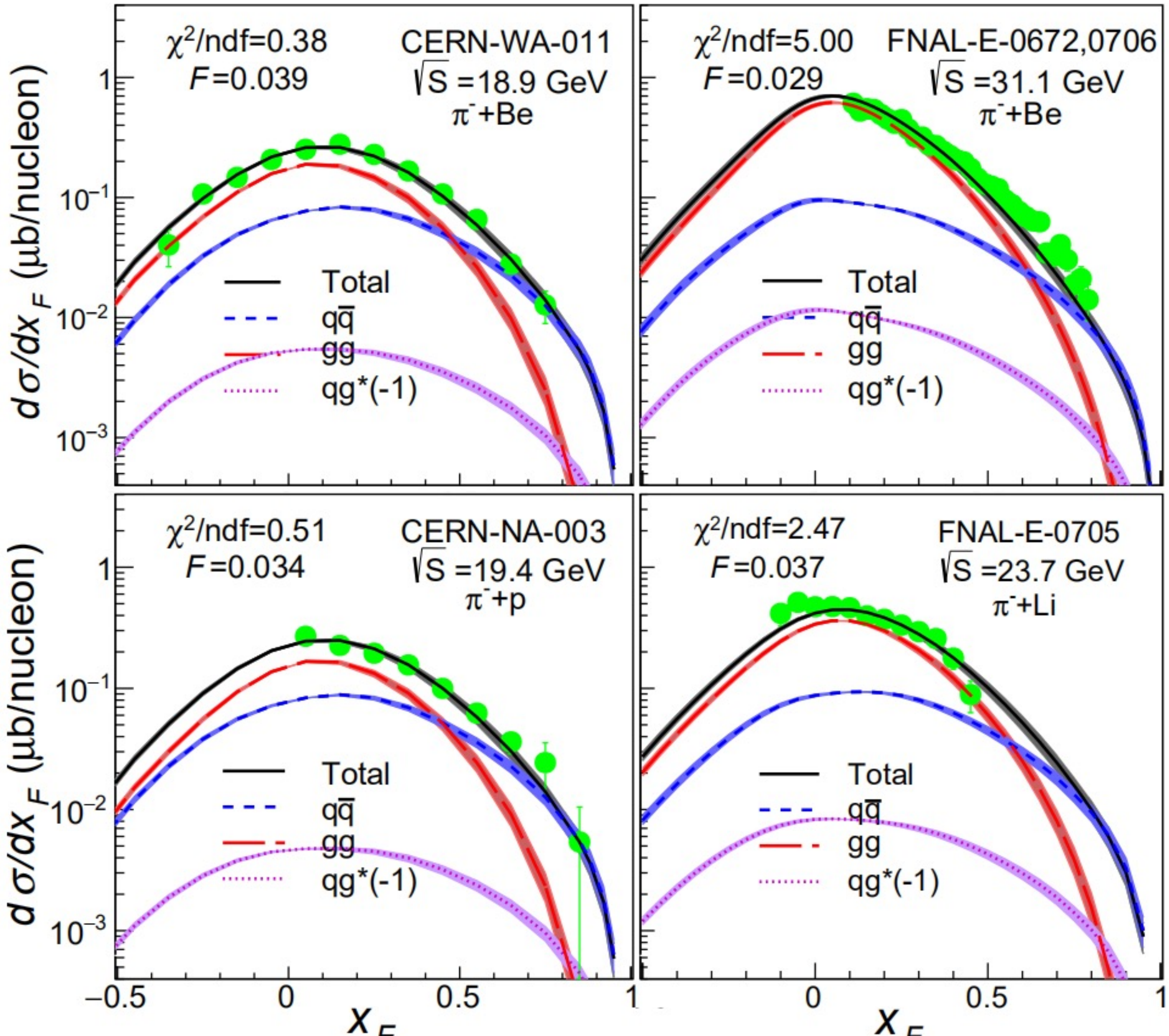
[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]

# $J/\psi$ production cross section

$\pi^{\pm} N \to J/\psi\ X$

$$\frac{d\sigma}{dx_F}|J/\psi = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{S\sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{S}}} \hat{\sigma}_{ij}(s, m_c^2, \mu_R^2, \mu_F^2) f_i^{\pi^{\pm}}(x_1, \mu_F^2) f_j^N(x_2, \mu_F^2)$$

[nCTEQ 2015]




CEM

[Chang, et al, PRD 102 (2020) 054024];
[Nason, et al, NPB 303 (1988) 607];
[Mangano, et al, NPB 405 (1993) 507]

- significantly gg contribution
- various energies of pions
- different target

Agree with experimental data (FNAL E672, E706, E705, CERN NA3,WA11).

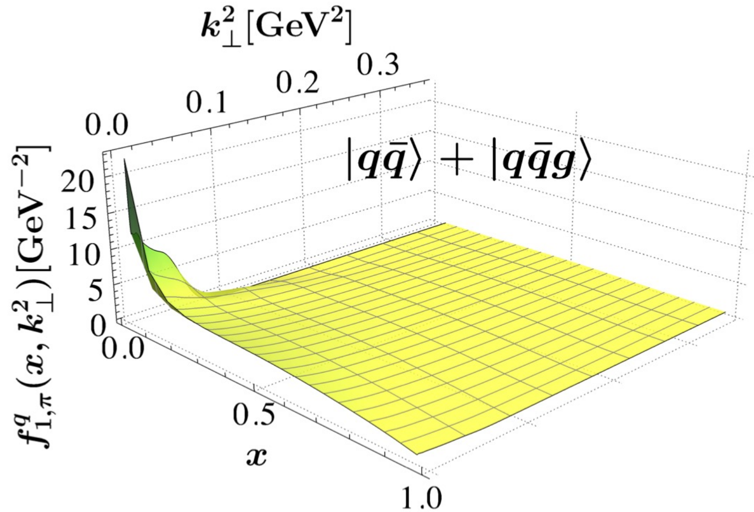
[Lan, Fu, Mondal, Zhao, Vary, Phys. Lett. B 825 (2022) 136890]

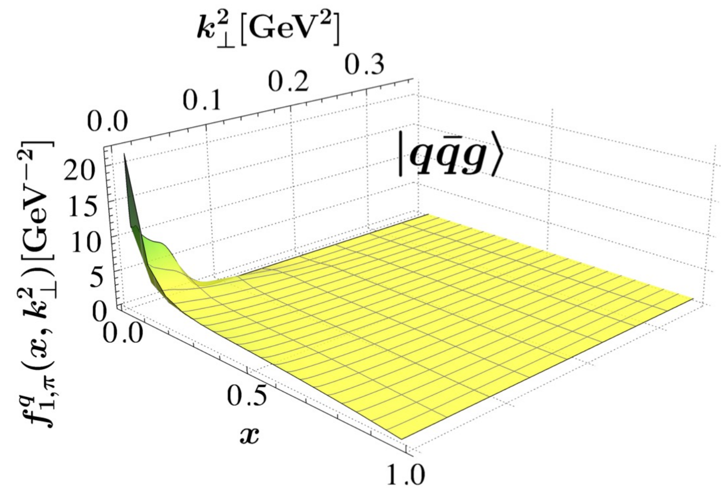
# Pion TMD

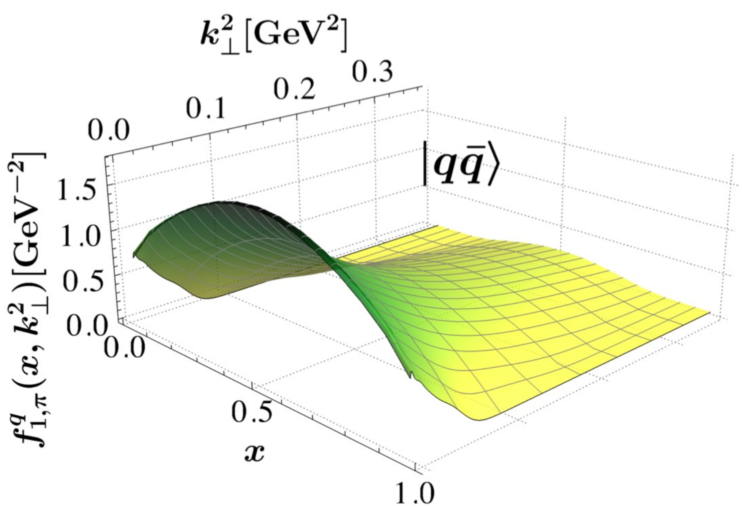
$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

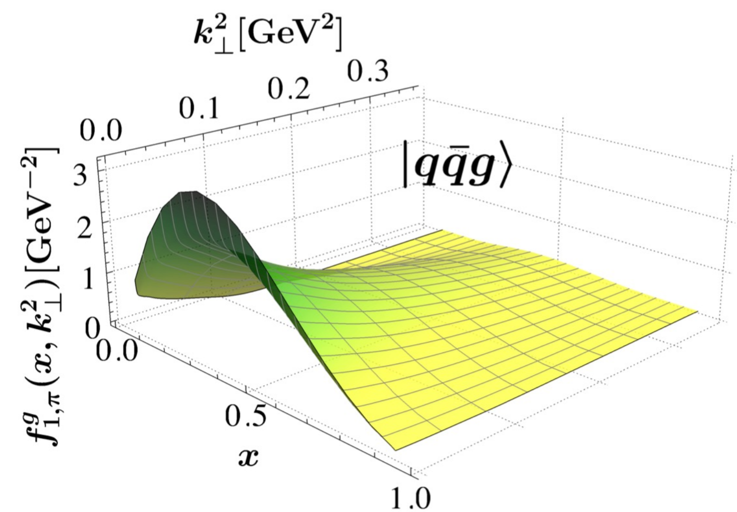
$$f^q_{1,\pi}(x,k_\perp^2) = \sum_{\mathcal{N},\lambda_i}\int_{\mathcal{N}}\left|\Psi^{\mathcal{N},M_J=0}_{\{x_i,\vec{k}_{\perp i},\lambda_i\}}\right|^2 \delta(x-x_q)\delta^2(\vec{k}_\perp-\vec{k}_{\perp q}),$$

$$f^g_{1,\pi}(x,k_\perp^2) = \sum_{\mathcal{N},\lambda_i}\int_{\mathcal{N}}\left|\Psi^{\mathcal{N},M_J=0}_{\{x_i,\vec{k}_{\perp i},\lambda_i\}}\right|^2 \delta(x-x_g)\delta^2(\vec{k}_\perp-\vec{k}_{\perp g}),$$




- The TMD decreases with $k_\perp$
- Vanishes after $k_\perp \sim 0.6$ GeV

[J. Lan et al, arXiv: 2509.22016 [hep-ph]]

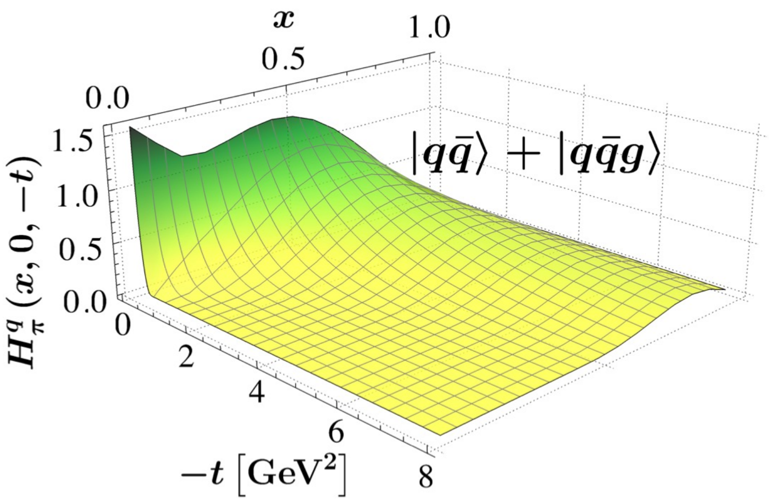
# Pion GPD

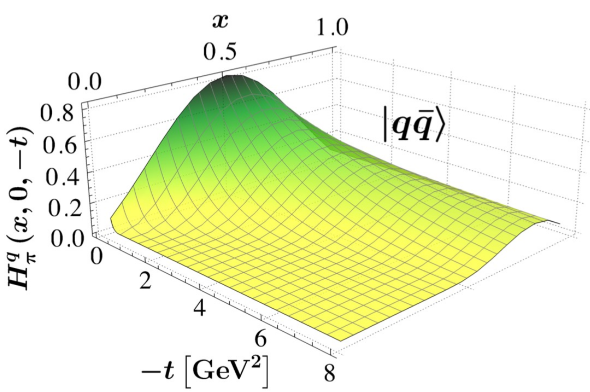
$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

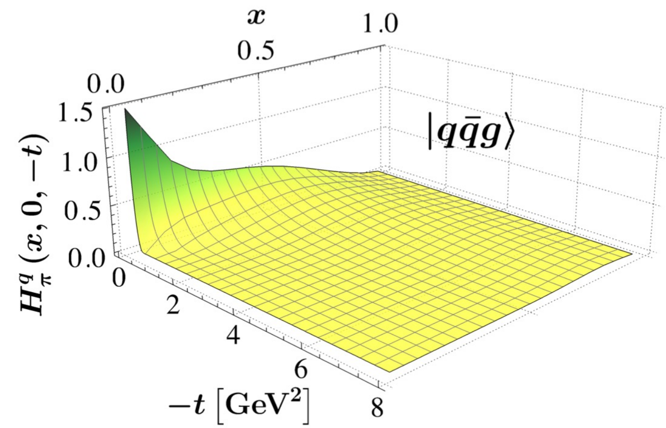
$$H_\pi^q(x,0,-t) = \sum_{\mathcal{N},\lambda_i} \int_{\mathcal{N}} \Psi^{\mathcal{N},M_J=0*}_{\{x_i,\vec{k}'_{\perp i},\lambda_i\}} \Psi^{\mathcal{N},M_J=0}_{\{x_i,\vec{k}_{\perp i},\lambda_i\}} \delta(x - x_q),$$

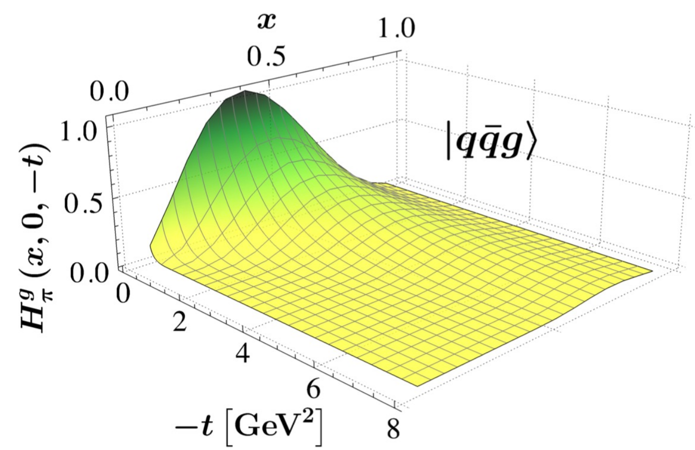
$$H_\pi^g(x,0,-t) = \sum_{\mathcal{N},\lambda_i} \int_{\mathcal{N}} \Psi^{\mathcal{N},M_J=0*}_{\{x_i,\vec{k}'_{\perp i},\lambda_i\}} \Psi^{\mathcal{N},M_J=0}_{\{x_i,\vec{k}_{\perp i},\lambda_i\}} \delta(x - x_g),$$

$$\vec{k}'_{\perp i} = \begin{cases} \vec{k}_{\perp i} + (1 - x_i)\vec{\Delta}_\perp, & \text{for the struck parton,} \\ \vec{k}_{\perp i} - x_i\vec{\Delta}_\perp, & \text{for spectator partons.} \end{cases}$$



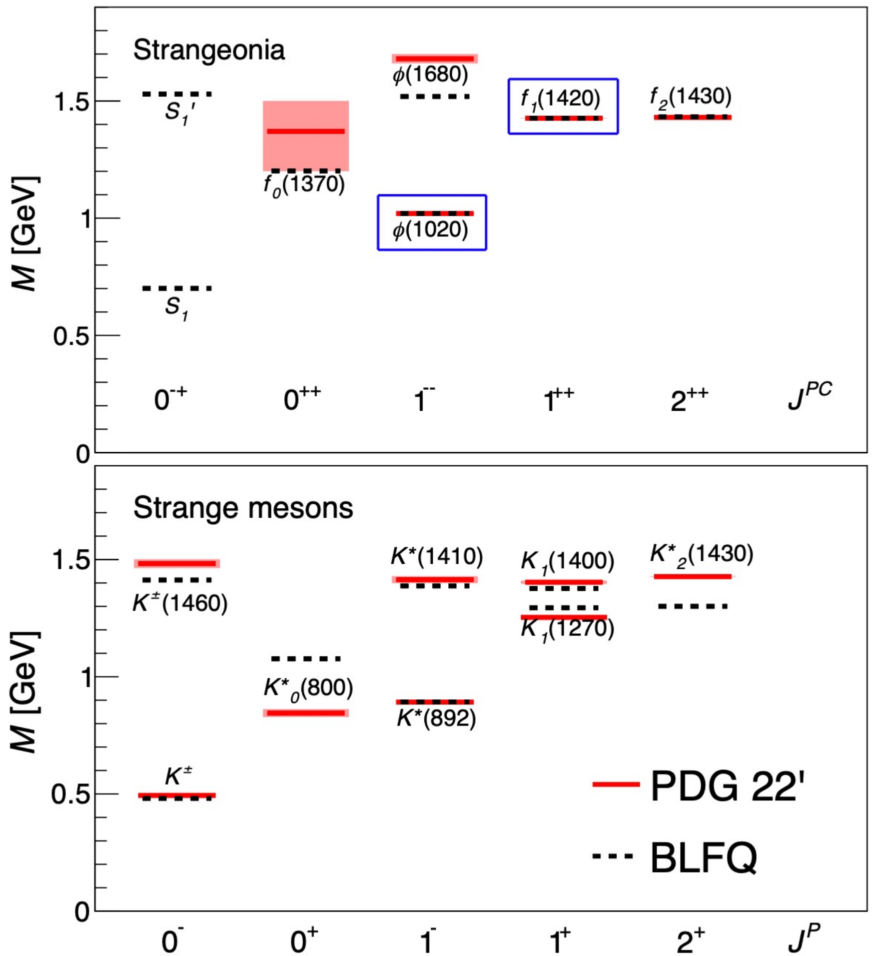



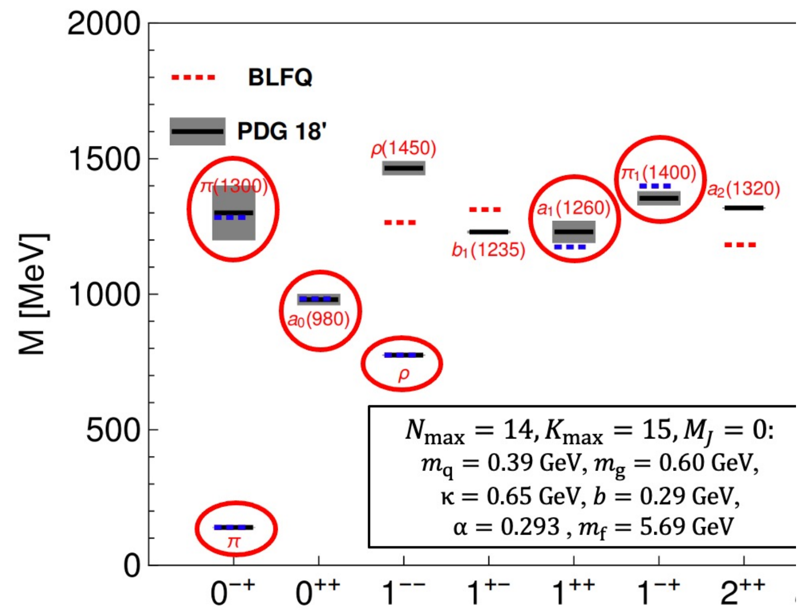

- Quark content enhanced at small $x$ with $|q\bar{q}g\rangle$
- Falls slowly at larger $x$
- Emerge at larger $x$ range for larger $-t$



[J. Lan et al, arXiv: 2509.22016 [hep-ph]]

# Strange meson mass spectrum




| $m_q$ | $m_{f_q}$ | $m_g$ | $b$ | $\kappa$ | $g_s$ | $m_s$ | $m_{f_s}$ |
|---|---|---|---|---|---|---|---|
| 0.39 | 5.69 | 0.60 | 0.29 | 0.65 | 1.92 | 0.55 | 7.13 |

- Agree with experimental data $(K, K^*)$
- The DC of kaon: 156.9 MeV (Exp. 155.6 $\pm$ 0.4 MeV)

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

[J. Lan et al, Phys.Lett.B 868 (2025) 139654]

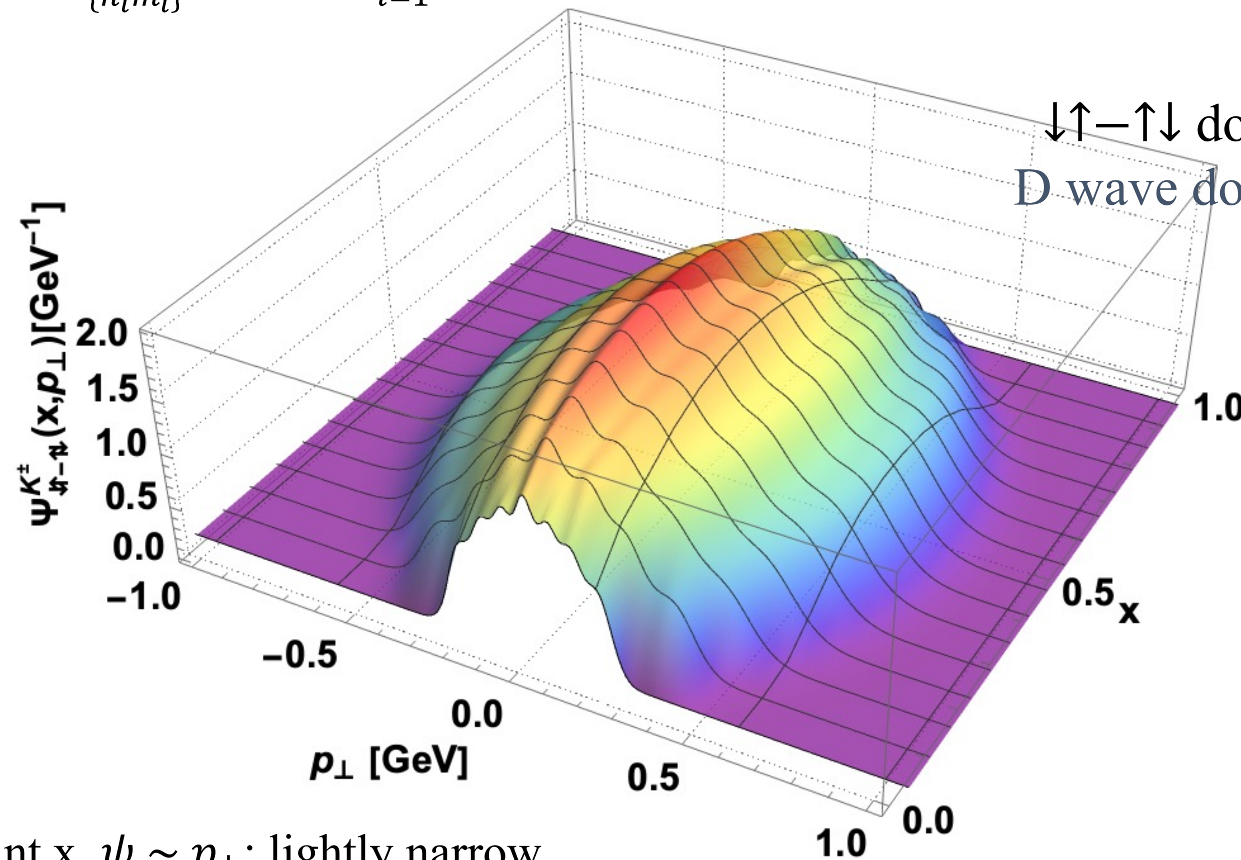
# The Wave Function in Leading Fock Sector

$$\Psi^{\mathcal{N},M_J}_{\{x_i,\vec{p}^{\,2}_{\perp i},\lambda_i\}} = \sum_{\{n_i m_i\}} \psi^{\mathcal{N}}(\{\bar{\alpha}_i\}) \prod_{i=1}^{\mathcal{N}} \phi_{n_i m_i}(\vec{p}_{\perp i}, b)$$

$$|K\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \cdots$$

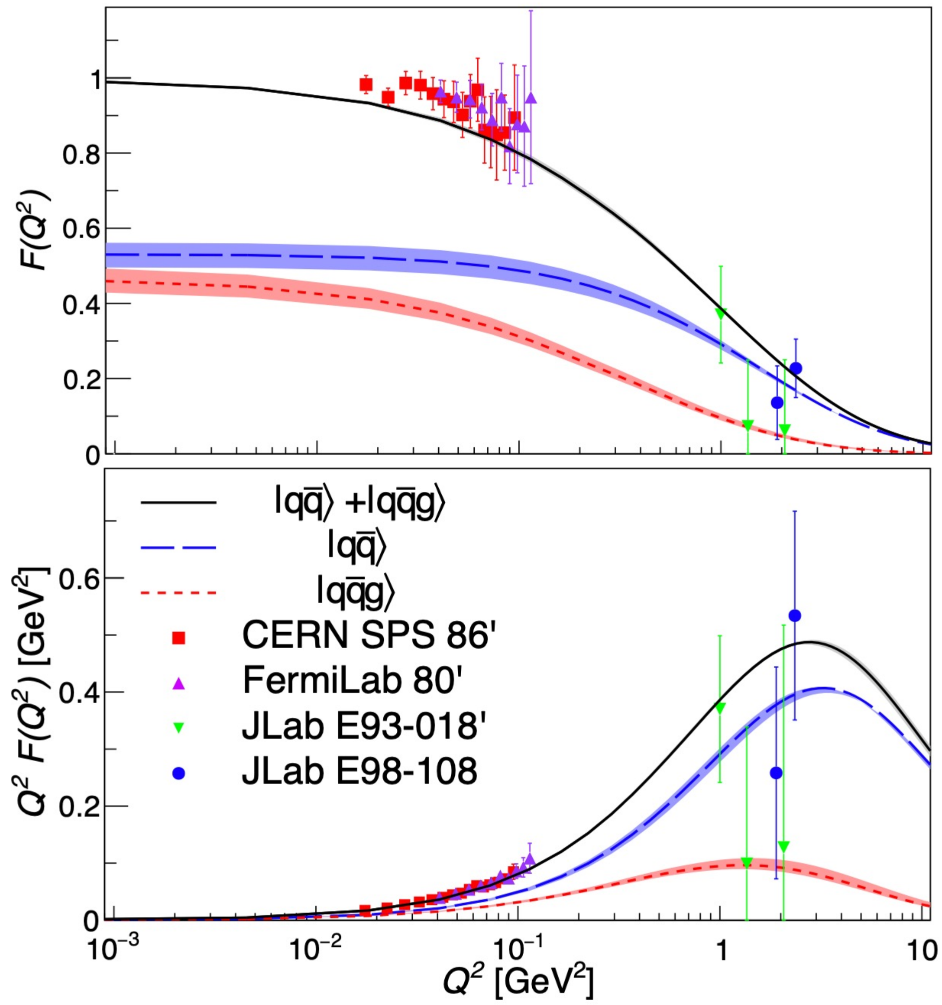
$\downarrow\uparrow - \uparrow\downarrow$ dominant in $|q\bar{s}\rangle$

D wave dominant in $|q\bar{s}g\rangle$




- At endpoint x, $\psi \sim p_\perp$: lightly narrow
- At middle x, $\psi \sim p_\perp$: a little bit wide
- The peak slightly less than x=1/2

# Kaon form factor

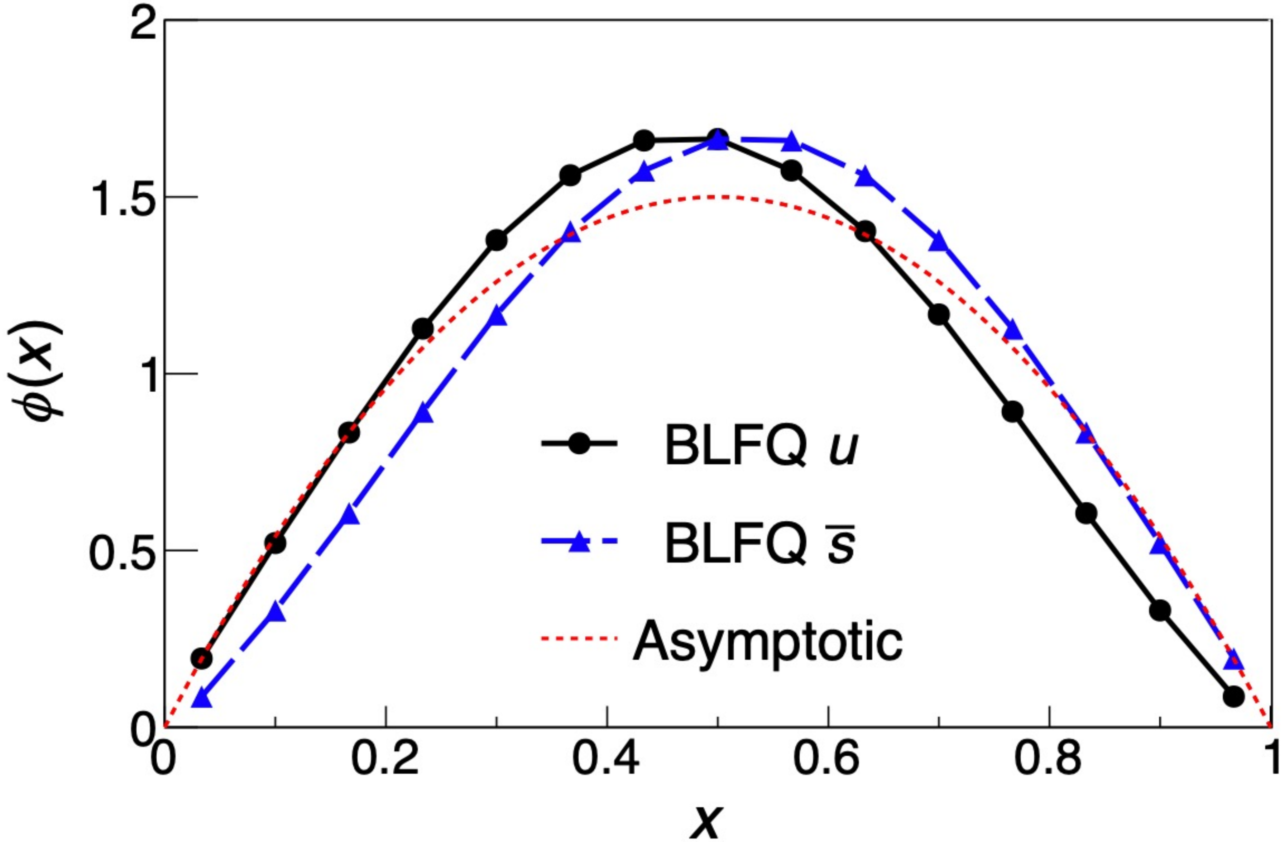



[J. Lan et al, Phys.Lett.B 868 (2025) 139654]

- FF is in reasonable agreement with experimental data
- $F(Q^2) \propto 1/\ Q^2$ for large $Q^2$

# Kaon PDA

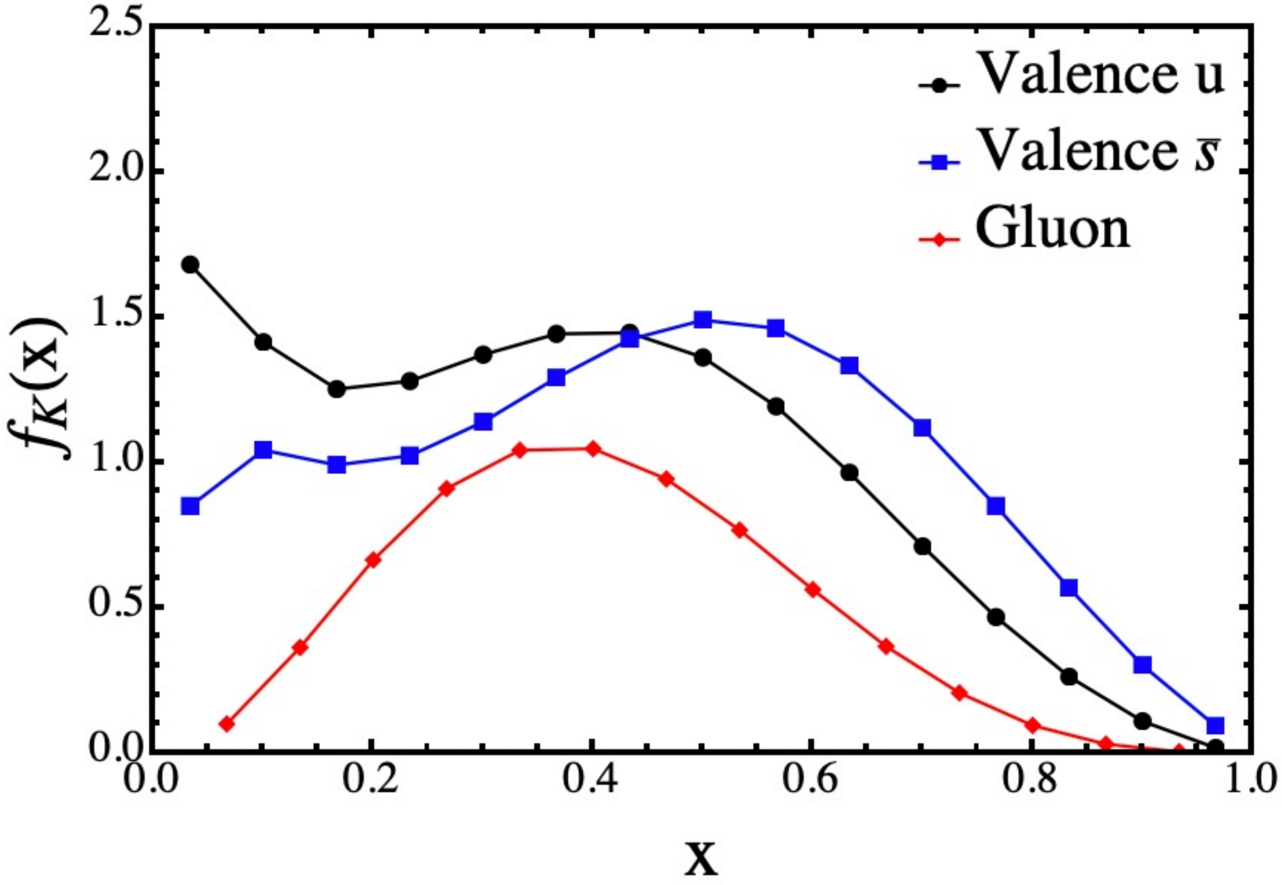
$$|K\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \cdots$$




- Endpoint behavior almost agrees with pQCD

[J. Lan et al, Phys.Lett.B 868 (2025) 139654]

# Kaon PDF at model scale

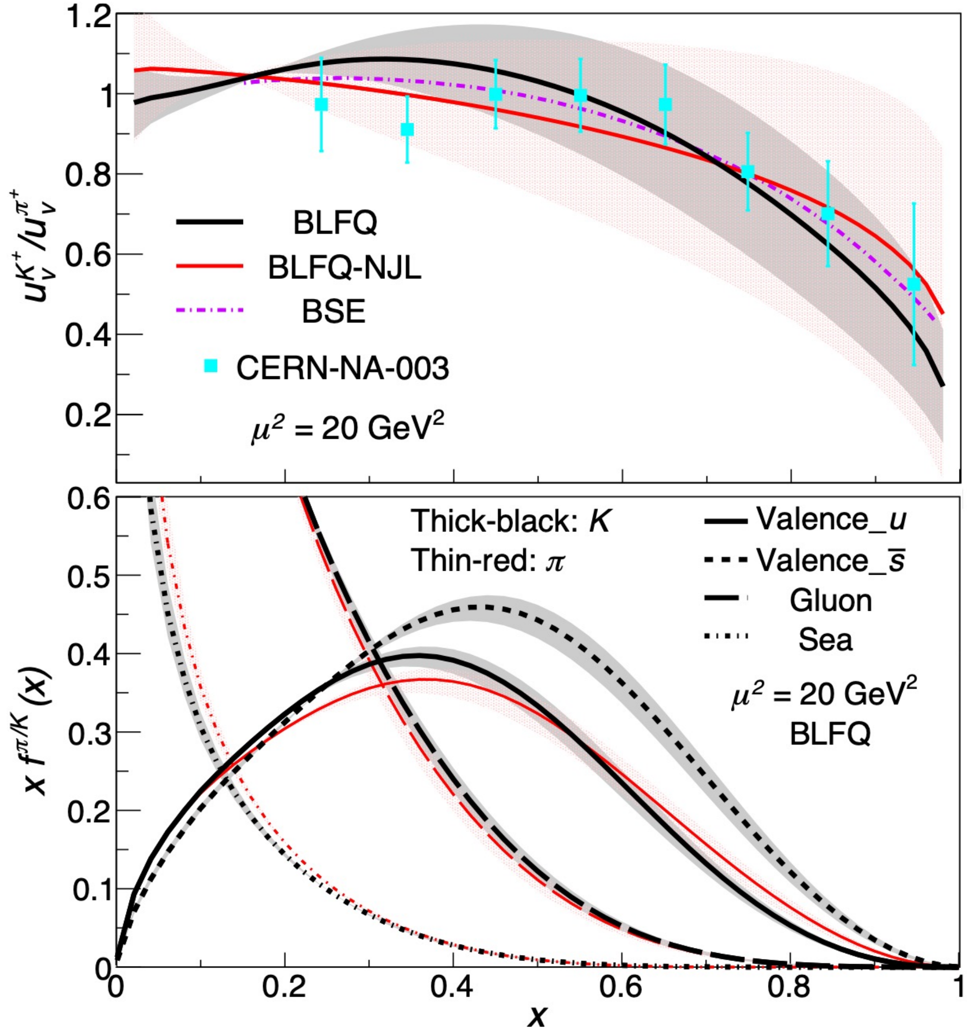



$\pi$: $\mu^2_{0\mathrm{BLFQ}} = 0.43\ \mathrm{GeV}^2$ $\langle x \rangle_{\mathrm{gluon}} = 0.216$; $\langle x \rangle_{\mathrm{val.}\,u} = 0.392$

$K$: $\mu^2_{0\mathrm{BLFQ}} = 0.52\ \mathrm{GeV}^2$ $\langle x \rangle_{\mathrm{gluon}} = 0.193$; $\langle x \rangle_{\mathrm{val.}\,u} = 0.363$; $\langle x \rangle_{\mathrm{val.}\,\bar{s}} = 0.444$

[J. Lan et al, Phys.Lett.B 868 (2025) 139654]

# Kaon PDF with QCD evolution



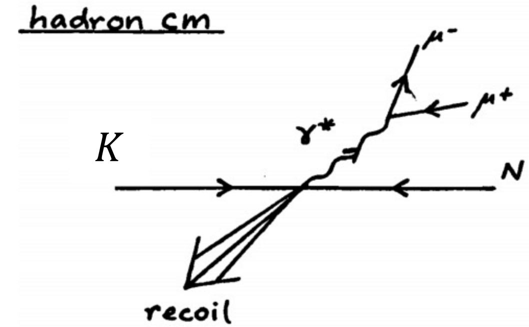

$$|\boldsymbol{K}\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \cdots$$

- Large-x behavior
  u: $(1-x)^{2.24}$
  s: $(1-x)^{1.72}$

| $\langle x \rangle$ @ 20 GeV$^2$ | BLFQ |
|---|---|
| Valence u | 0.21 |
| Valence s | 0.26 |
| Gluon | 0.42 |
| Sea | 0.11 |

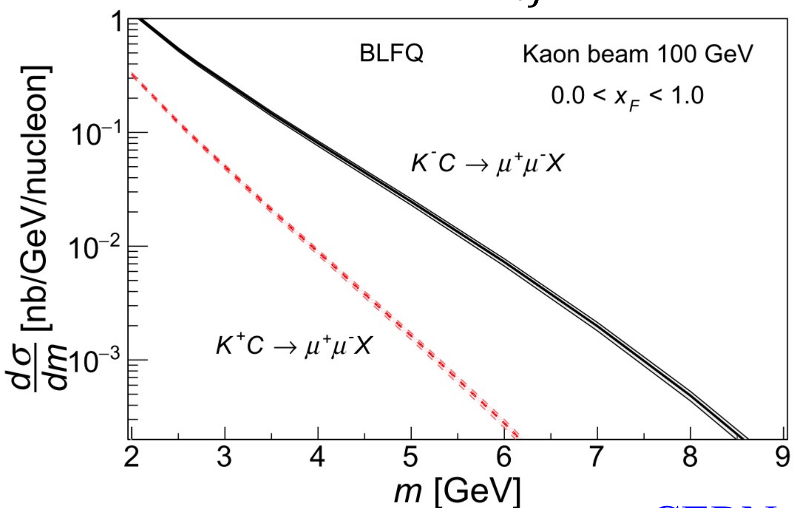
[J. Lan et al, Phys.Lett.B 868 (2025) 139654]

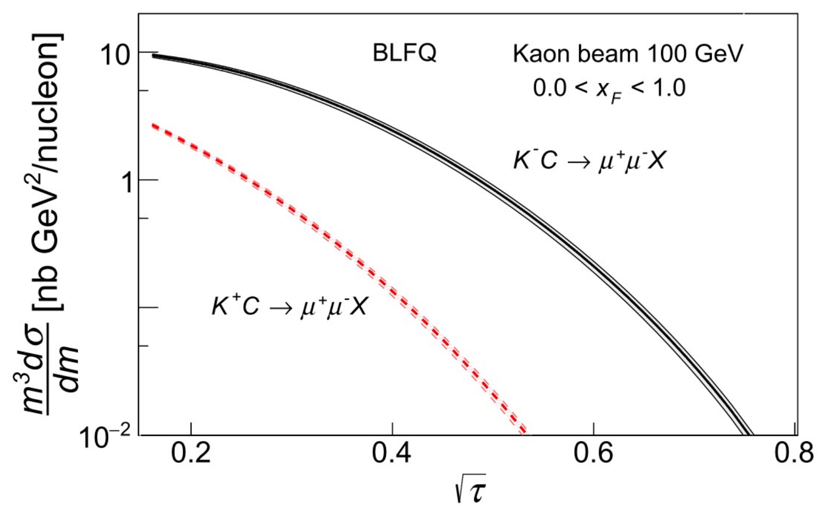
# Drell-Yan Cross Section




[S. D. Drell and T.-M. Yan, PRL (1970)]
[T. Becher et al, JKEP07(2008)030]; [P. C. Barry et al, PRL121(2018)152001]
[C. Anastasiou et al, PRL91(2003)182002]

$$\frac{m^3 d^2\sigma}{dm dY} = \frac{8\pi\alpha^2}{9}\frac{m^2}{s}\sum_{ij} dx_1 dx_2 \tilde{C}_{ij}(x_1, x_2, s, m, \mu_f) f_{i/K}(x_1, \mu_f) f_{j/N}(x_2, \mu_f)$$




CERN-AMBER [CERN-SPSC-2019–022 /SPSC-P-360]

| Program | Physics Goals | Beam Energy [GeV] | Beam Intensity [$s^{-1}$] | Trigger Rate [kHz] | Beam Type | Target | Earliest start time, duration | Hardware additions |
|---|---|---|---|---|---|---|---|---|
| Drell-Yan (RF) | Kaon PDFs & Nucleon TMDs | $\sim$100 | $10^8$ | 25-50 | $K^\pm, \bar{p}$ | $NH_3^\uparrow$, C/W | 2026 2-3 years | "active absorber", vertex detector |
| Primakoff (RF) | Kaon polarisa-bility & pion life time | $\sim$100 | $5 \cdot 10^6$ | $> 10$ | $K^-$ | Ni | non-exclusive 2026 1 year | |
| Prompt Photons (RF) | Meson gluon PDFs | $\geq 100$ | $5 \cdot 10^6$ | 10-100 | $K^\pm$ $\pi^\pm$ | LH2, Ni | non-exclusive 2026 1-2 years | hodoscope |
| $K$-induced Spectroscopy (RF) | High-precision strange-meson spectrum | 50-100 | $5 \cdot 10^6$ | 25 | $K^-$ | LH2 | 2026 1 year | recoil TOF, forward PID |
| Vector mesons (RF) | Spin Density Matrix Elements | 50-100 | $5 \cdot 10^6$ | 10-100 | $K^\pm, \pi^\pm$ | from H to Pb | 2026 1 year | |

[J. Lan et al, Phys.Lett.B 868 (2025) 139654]

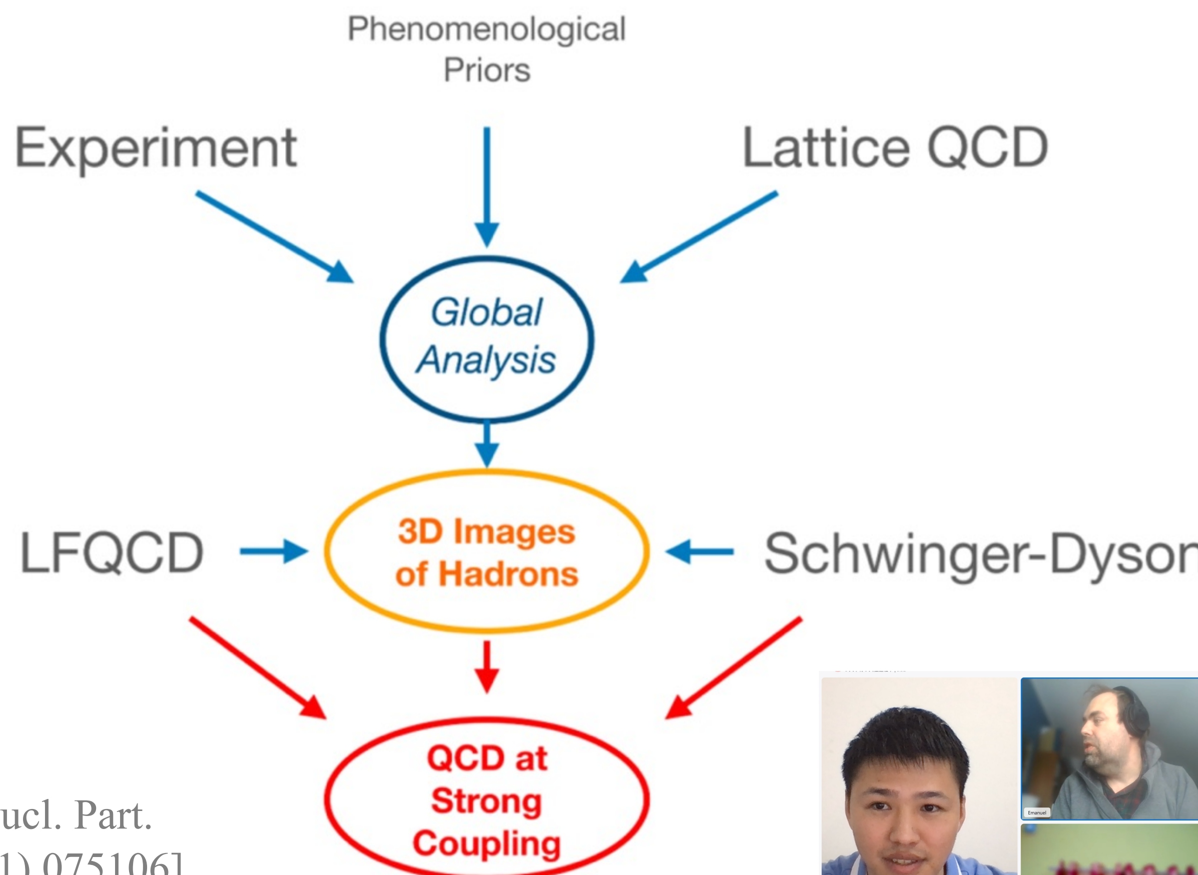
| Experiment | Target type | Beam type | Beam intensity (part/sec) | Beam energy (GeV) | DY mass (GeV/$c^2$) | DY events $\mu^+\mu^-$ | DY events $e^+e^-$ |
|---|---|---|---|---|---|---|---|
| NA3 | 6 cm Pt | $K^-$ | | 200 | 4.2 – 8.5 | 700 | 0 |
| This exp. | 100 cm C | $K^-$ | $2.1 \times 10^7$ | 80 | 4.0 – 8.5 | 25,000 | 13,700 |
| | | | | 100 | 4.0 – 8.5 | 40,000 | 17,700 |
| | | | | 120 | 4.0 – 8.5 | 54,000 | 20,700 |
| | | $K^+$ | $2.1 \times 10^7$ | 80 | 4.0 – 8.5 | 2,800 | 1,300 |
| | | | | 100 | 4.0 – 8.5 | 5,200 | 2,000 |
| | | | | 120 | 4.0 – 8.5 | 8,000 | 2,400 |
| This exp. | 100 cm C | $\pi^-$ | $4.8 \times 10^7$ | 80 | 4.0 – 8.5 | 65,500 | 29,700 |
| | | | | 100 | 4.0 – 8.5 | 95,500 | 36,000 |
| | | | | 120 | 4.0 – 8.5 | 123,600 | 39,800 |

# The pion $q\bar{q}g$ BLFQ .vs. BSE

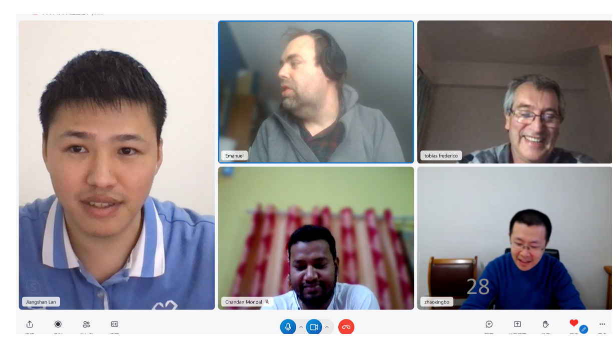
BLFQ
Phys. Lett. B 825 (2022) 136890

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \cdots$$

BSE in Minkowski space
Phys. Rev. D 103 (2021) 014002
Phys. Rev. D 105 (2022) L071505
Phys. Lett. B 820 (2021) 136494



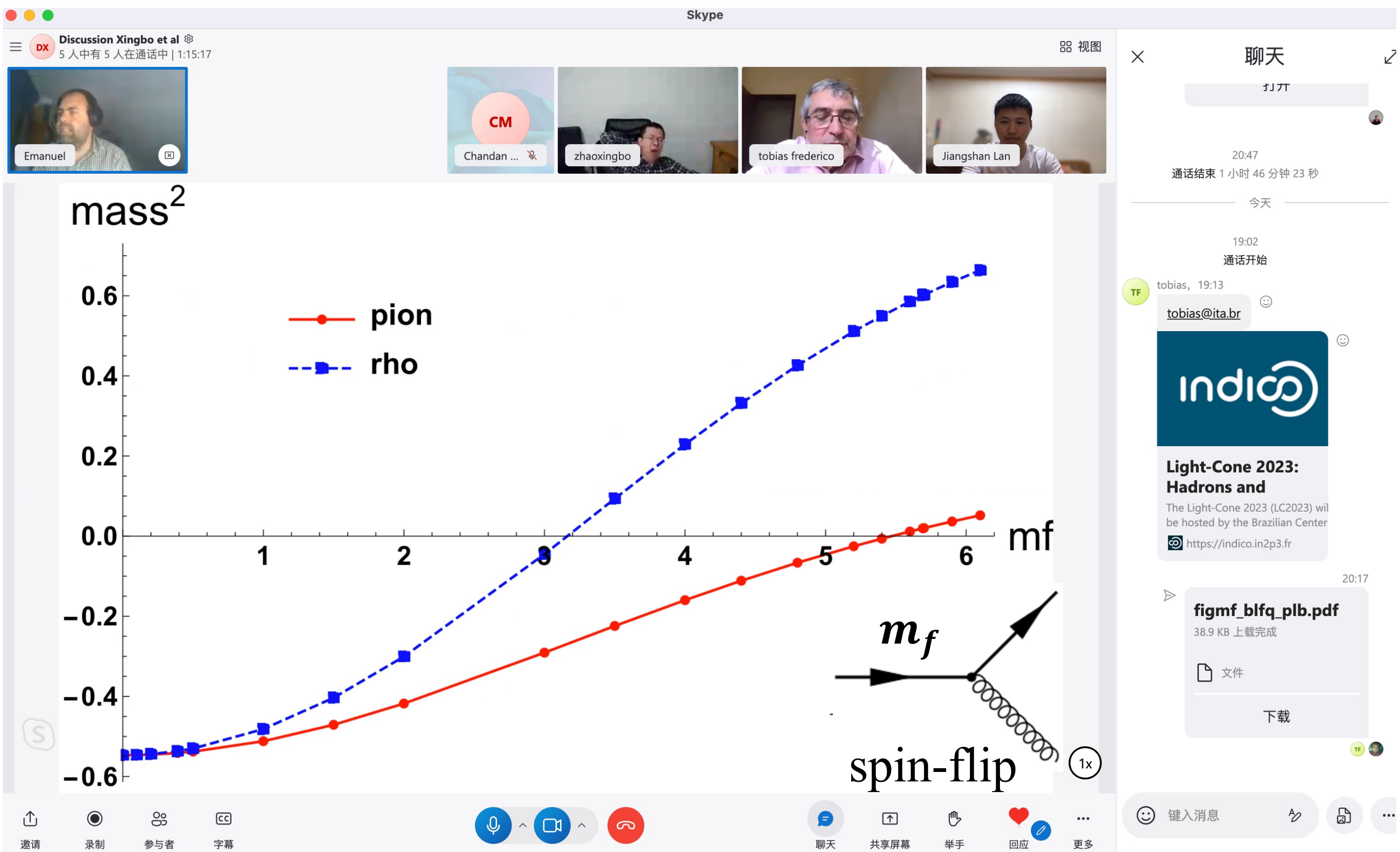

[J. Phys. G: Nucl. Part.
Phys. 48 (2021) 075106]



# The pion $q\bar{q}g$ BLFQ .vs. BSE

We explicitly show an enhancement of the low-x contribution in the quark PDF is directly associated with the large spin-flip matrix element, necessary to provide the $\pi - \rho$ mass splitting. [M. Burkardt, *Dynamical vertex mass generation and chiral symmetry breaking on the light front*, Phys. Rev. D 58 (1998) 096015].

*It was proposed to enhance the spin flip matrix element of the effective quark-gluon coupling QCD LF-Hamiltonian by introducing a large effective quark vertex mass (mf).*



# Pion $q\bar{q}g$ BLFQ .vs. BSE

We introduce a LF model:

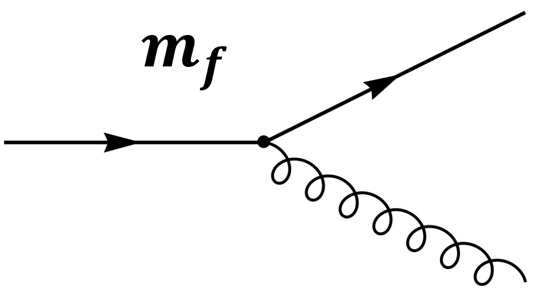
$$|\text{Meson}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle$$

$$\psi^{(s_q,s_{\bar{q}},\lambda)}_{q\bar{q}g;(i_q,i_{\bar{q}},a)} = \frac{1}{M_\pi^2 - M_{0,q\bar{q}g}^2}\left[V\psi^{(s_q,s_{\bar{q}})}_{q\bar{q};(i_q,i_{\bar{q}})}\right],$$

$$W_\lambda^{(+,-)}(p_q, p_g) = m_f(x_1 - x_q)/\sqrt{x_1 x_q}\delta_{\lambda,-},$$
$$W_\lambda^{(-,+)}(p_q, p_g) = -m_f(x_1 - x_q)/\sqrt{x_1 x_q}\delta_{\lambda,+},$$
$$\bar{W}_\lambda^{(+,-)}(p_{\bar{q}}, p_g) = \lambda m_f(x_1 - x_{\bar{q}})/\sqrt{x_1 x_{\bar{q}}},$$
$$\bar{W}_\lambda^{(-,+)}(p_{\bar{q}}, p_g) = 0,$$

$V$: interaction connecting $q\bar{q}$ and $q\bar{q}g$ sector

$$\left[V\psi^{(s_q,s_{\bar{q}})}_{q\bar{q};(i_q,i_{\bar{q}})}\right] = \frac{g_s\sqrt{2}}{x_q + x_g}\sum_{i_1 s_1} T^a_{i_q i_1} W_\lambda^{(s_q,s_1)}(p_q, p_g)$$
$$\times \psi^{(s_1,s_{\bar{q}})}_{q\bar{q};(i_1,i_{\bar{q}})}(x_{\bar{q}}, \vec{p}_{\bar{q}\perp}) - \frac{g_s\sqrt{2}}{x_{\bar{q}} + x_g}$$
$$\times \sum_{i_1 s_1} T^a_{i_1 i_{\bar{q}}} \bar{W}_\lambda^{(s_1,s_{\bar{q}})}(p_{\bar{q}}, p_g)\psi^{(s_q,s_1)}_{q\bar{q};(i_q,i_1)}(x_q, \vec{p}_{q\perp}).$$

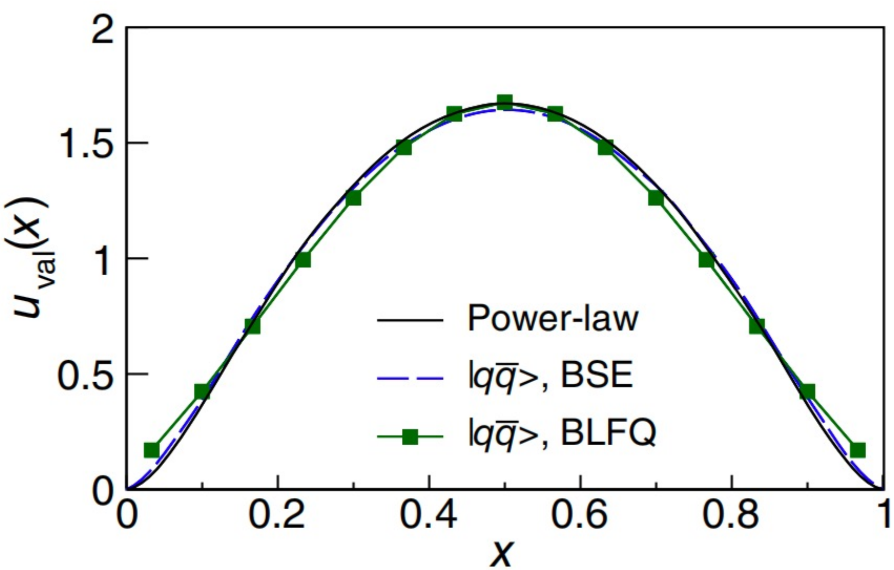



- To get the LFWF for $|q\bar{q}g\rangle$ based on the LFWF for $|q\bar{q}\rangle$ with the interaction $V$.

[J. Lan et al, Phys.Rev.D 111 (2025) 11, L111903]

# Pion $q\bar{q}g$ BLFQ .vs. BSE

To get LFWF for $|q\bar{q}g\rangle$




BLFQ: [PLB 825 (2022) 136890]
BSE: [PRD 103 (2021) 014002]

[J. Lan et al, Phys.Rev.D 111 (2025) 11, L111903]

[Phys. Rev. D 50 (1994) 6895]

Take a power-law form for simplicity:

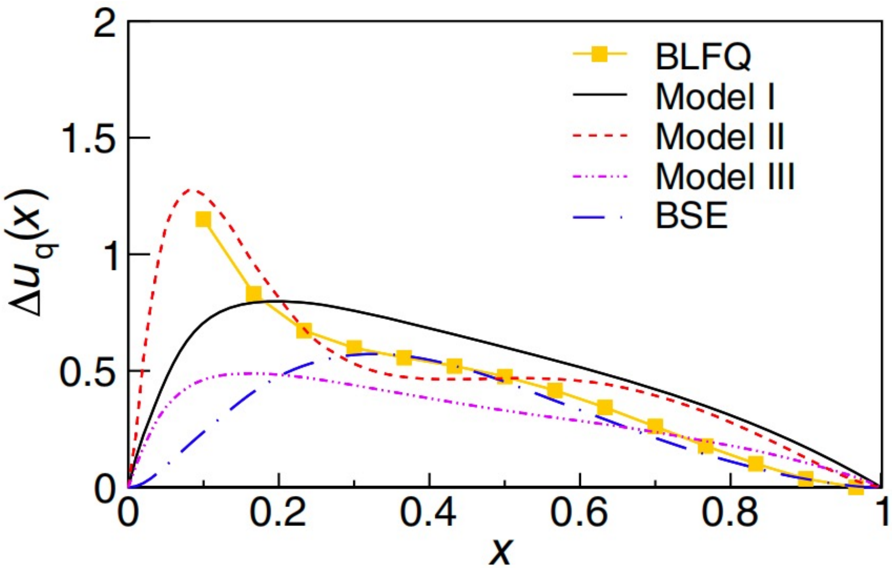
$$\psi_{pl}(x_q, \boldsymbol{p}_{q\perp}) = N\left[1 + \frac{A_{0,eff}(x_q, \boldsymbol{p}_{q\perp})/4 - m_q^2}{\beta^2}\right]^{-s}$$

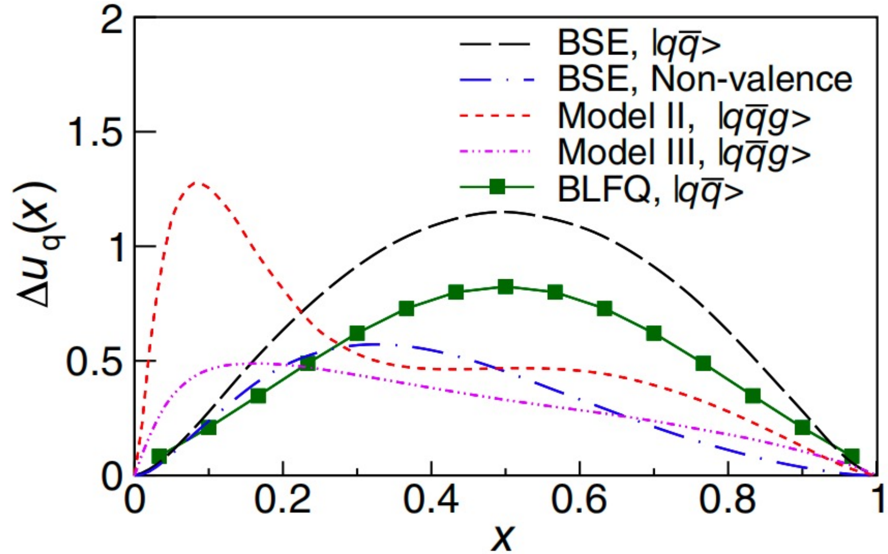
$$A_{0,eff}(x_q, \boldsymbol{p}_{q\perp}) = \frac{\boldsymbol{p}_{q\perp}^2 + m_q^2}{x_q} + \frac{\boldsymbol{p}_{\bar{q}\perp}^2 + m_{\bar{q}}^2}{x_{\bar{q}}}$$

Fitting the valence PDF of both the BLFQ and the BSE:

$$s = 1.4$$
$$\beta/m_q = 1.16$$

# Pion $q\bar{q}g$ BLFQ .vs. BSE



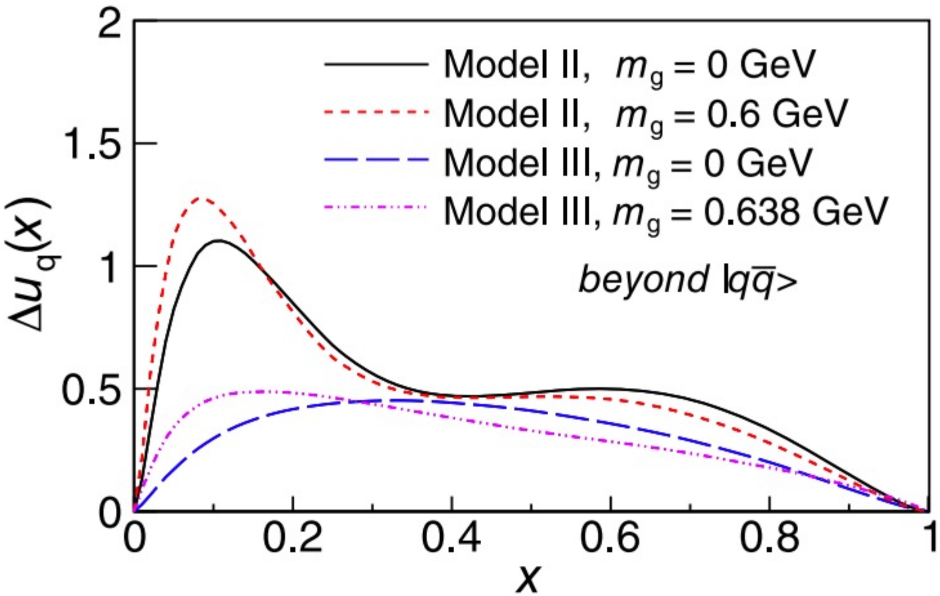



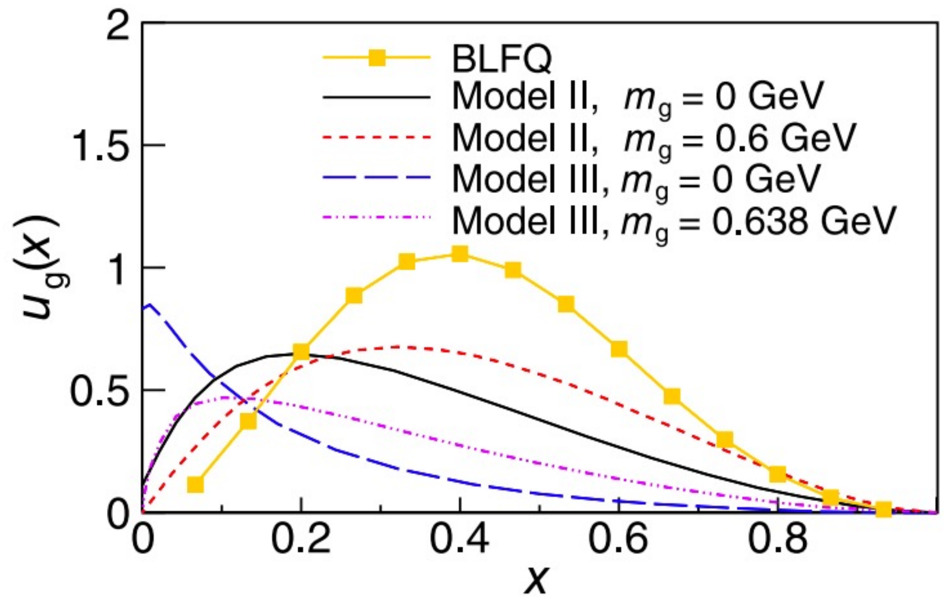

| Model | $m_q$[GeV] | $m_f$[GeV] | $m_g$[GeV] | 1-$P_{val}$ |
|---|---|---|---|---|
| I | 0.390 | 0.390 | 0.600 | 0.508 |
| II | 0.390 | 5.69 | 0.600 | 0.508 |
| III | 0.255 | 0.255 | 0.638 | 0.300 |

- Model II: input from BLFQ PLB 22'
- Model III: input from BSE PRD 21'
- Model I .vs. II: bump is related to large value of $m_f$
- Model II agrees with BLFQ: a large bump at low-$x$ is reproduced
- BSE result differs from Model III: BSE result contains contributions from $q\bar{q}ng$, where $n = 1,2,\ldots,\infty$
- $q\bar{q}g$ Fock sector is mostly important at small- $x$

[J. Lan et al, Phys.Rev.D 111 (2025) 11, L111903]

# Pion $q\bar{q}g$ BLFQ .vs. BSE




Impact of $m_g$ on pion PDFs:

- Model II: input from BLFQ PLB 22'
- Model III: input from BSE PRD 21'

| Model | $m_q$[GeV] | $m_f$[GeV] | $m_g$[GeV] | 1-$P_{val}$ |
|---|---|---|---|---|
| I | 0.390 | 0.390 | 0.600 | 0.508 |
| II | 0.390 | 5.69 | 0.600 | 0.508 |
| III | 0.255 | 0.255 | 0.638 | 0.300 |

- An increase of $m_g$ leads to a shift of quark PDF to lower values of $x$
- A larger $m_g$ gives a gluon PDF shifted towards larger- $x$
- Perturbative results agree qualitatively with the BLFQ

[J. Lan et al, Phys.Rev.D 111 (2025) 11, L111903]

# Conclusion & Outlook

- BLFQ : A non-perturbative approach based on light-front QCD Hamiltonian
- LF QCD Hamiltonian ⇒ Wave functions ⇒ Observables
- Solved for light mesons structure based on $q\bar{q}$ + $q\bar{q}$g Fock components, EMFF, PDF, TMD, GPD…
- $\pi - \rho$ mass splitting ~ a large spin-flip matrix element
  ~ a peak at low-$x$ for quark PDF

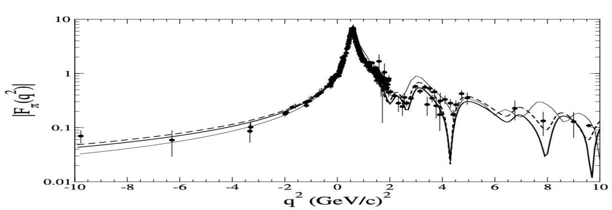
---

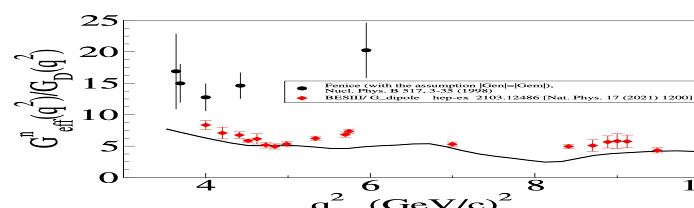
**Plan 1**

$$|\text{Meson}\rangle = |q\bar{q}\rangle + |q\bar{q}\,g\rangle + |q\bar{q}\,q\bar{q}\rangle + |q\bar{q}\,gg\rangle + |q\bar{q}\,q\bar{q}\,g\rangle + |q\bar{q}\,ggg\rangle + |gg\rangle + \cdots$$

**Plan 2**

Combine different perturbative methods ⇒ Timelike Physics, ...




*with Tobias, Pacheco, Xiaorong, ..., in future*


***Good health!***

***Smooth work!***

***Happy New Year!***

***Thank you!***