

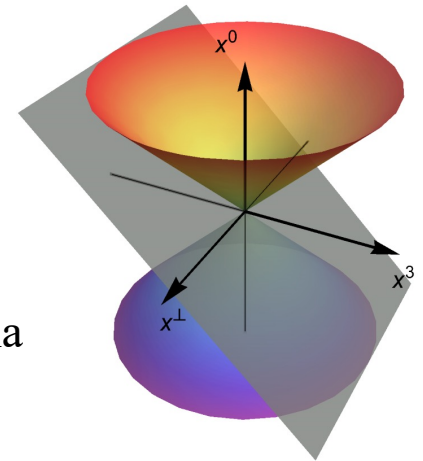


Meson Structure Using the Basis Light-Front Quantization (BLFQ)



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Outline

- Basis Light-front Quantization
- Application to Light Mesons
 - Light mesons with $q\bar{q}$ *LFWF, EMFF,*
 - Light mesons with $q\bar{q} + q\bar{q}g$ *PDA, PDF, ...*
 - Light mesons with $q\bar{q} + q\bar{q}g$ **BLFQ .vs. BSE**
- Conclusion & Outlook

Light-front Hamiltonian Framework

Quantization on the light-front : $x^+ = x^0 + x^3$

LF momenta : $P^+ = P^0 + P^3; \vec{P}_\perp$

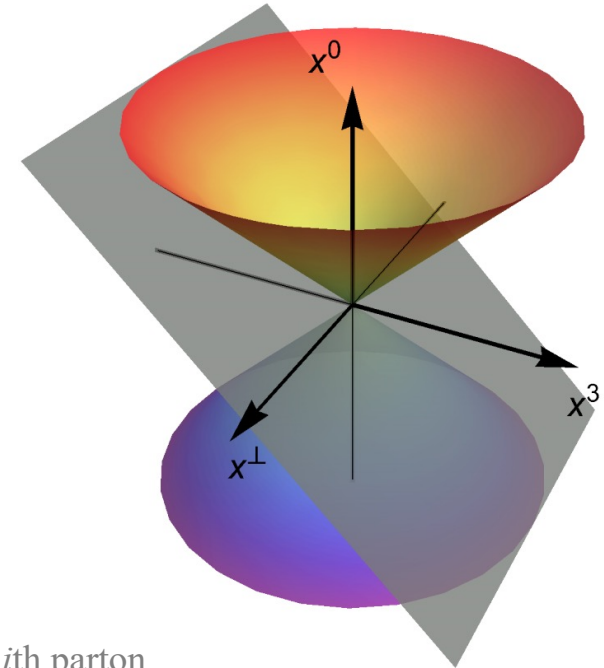
Choose transverse rest frame: $\vec{P}_\perp = 0$

LF Hamiltonian: $H_{LF} \equiv P^+ P^-$

Eigenvalue problem : $H_{LF} |P, \Lambda\rangle = M^2 |P, \Lambda\rangle$

$$H_{LF} = \sum_i \frac{m_i^2 + \vec{p}_{i\perp}^2}{x_i} + H_{int}$$

x_i LF momentum fraction carried by i th parton



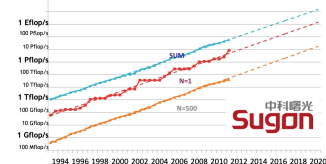
DLCQ



BLFQ

Both are constructed in Minkowski space-time

Projected Performance Development



Longitudinal direction (plane wave)

-Brodsky, Pauli, Pinsky Phys. Reports. 301, 299 (1998)

...

Longitudinal direction (plane wave)

Transverse direction (harmonic oscillator)

-Vary, et al, PRC 81, 035205 (2010)

...

Basis Light-front Quantization

- Nonperturbative eigenvalue problem

[Vary et al, PRC 81, 035205 (2010)]

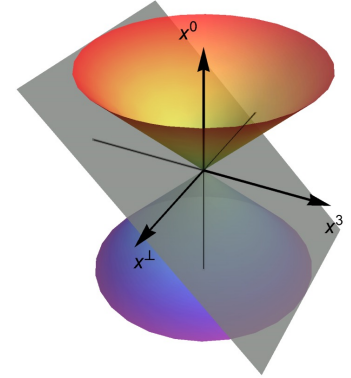
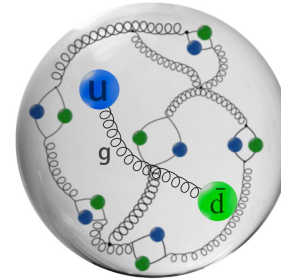
$$P^- |\beta\rangle = P_\beta^- |\beta\rangle$$

- P^- : light-front Hamiltonian
 - $|\beta\rangle$: mass eigenstate
 - P_β^- : eigenvalue for $|\beta\rangle$
- Evaluate observables for eigenstate

$$O \equiv \langle \beta | \hat{O} | \beta \rangle$$

- Fock sector expansion

- Eg. $|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + c|q\bar{q}q\bar{q}\rangle + d|q\bar{q}gg\rangle + \dots$



Discretized basis	Truncation
Transverse: 2D harmonic oscillator basis: $\Phi_{n,m}^b(\vec{p}_\perp)$.	$\sum_i (2n_i + m_i + 1) \leq N_{max}$
Longitudinal: plane-wave basis, labeled by k .	$\sum_i k_i = K, \quad x_i = k_i/K$
Light-front helicity state for spin d.o.f.	$\sum_i (m_i + \lambda_i) = M_J$
$\alpha_i = (k_i, n_i, m_i, \lambda_i)$ $ \alpha\rangle = \otimes_i \alpha_i\rangle$	Fock sector truncation

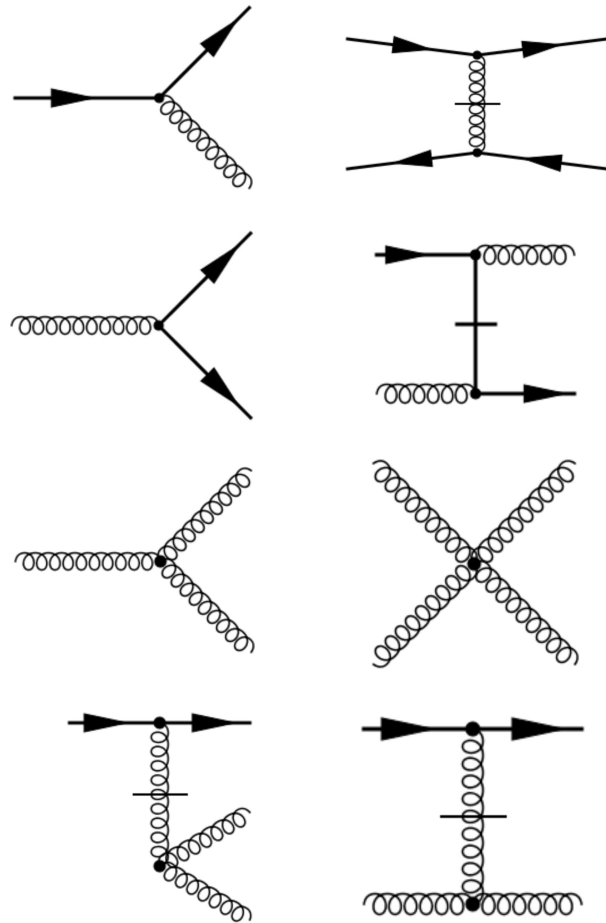
$$\Lambda_{IR} \sim \frac{b}{\sqrt{N_{max}}} \quad \text{and} \quad \Lambda_{UV} \sim b \sqrt{N_{max}}. \quad b \text{ is HO energy scale}$$

Large N_{max} and $K \rightarrow$ High UV cutoff & low IR cutoff

Light-front Hamiltonian

QCD light-front Hamiltonian can be derived from QCD Lagrangian:

$$\mathcal{L}_{QCD} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} G_{\mu\nu}^\alpha G_\alpha^{\mu\nu} \quad \Rightarrow \quad P_{QCD}^- = H_K + H_{int} \quad A^+ = 0$$



$$H_K = \frac{1}{2} \int d^3x \bar{\psi} \gamma^+ \frac{(i\partial^\perp)^2 + m^2}{i\partial^+} \psi + \frac{1}{2} \int d^3x A_a^i (i\partial^\perp)^2 A_a^i$$

$$H_{int} = g \int d^3x \bar{\psi} \gamma_\mu A^\mu \psi$$

$$+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \gamma_\mu A^\mu \frac{\gamma^+}{i\partial^+} \gamma_\nu A^\nu \psi$$

$$- i g^2 \int d^3x f^{abc} \bar{\psi} \gamma^+ T^c \psi \frac{1}{(i\partial^+)^2} (i\partial^+ A_a^\mu A_{\mu b})$$

$$+ \frac{1}{2} g^2 \int d^3x \bar{\psi} \gamma^+ T^a \psi \frac{1}{(i\partial^+)^2} \bar{\psi} \gamma^+ T^a \psi$$

$$+ i g \int d^3x f^{abc} i\partial^\mu A^{\nu a} A_\mu^b A_\nu^c$$

$$- \frac{1}{2} g^2 \int d^3x f^{abc} f^{ade} i\partial^+ A_b^\mu A_{\mu c} \frac{1}{(i\partial^+)^2} (i\partial^+ A_d^+ A_{ve})$$

$$+ \frac{1}{4} g^2 \int d^3x f^{abc} f^{ade} A_b^\mu A_c^\nu A_{\mu d} A_{ve}.$$

ψ : quark field operator

A_μ^a : gluon field operator

PDFs for Light Mesons $|\text{meson}\rangle = |q\bar{q}\rangle + \dots$

$$H_{\text{eff}} = \frac{\vec{k}_{\perp}^2 + m_q^2}{x} + \frac{\vec{k}_{\perp}^2 + m_{\bar{q}}^2}{1-x} + \kappa^4 x(1-x) \vec{r}_{\perp}^2 - \frac{\kappa^4}{(m_q + m_{\bar{q}})^2} \partial_x (x(1-x) \partial_x) + H_{\text{eff}}^{\text{NJL}}$$

Diagonalizing H_{eff}



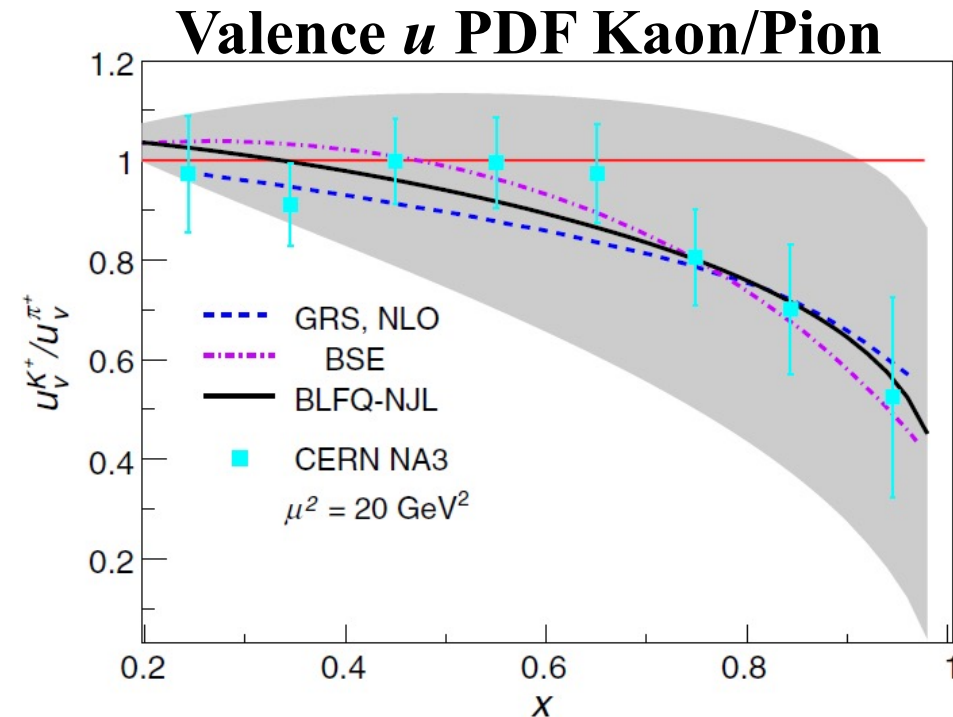
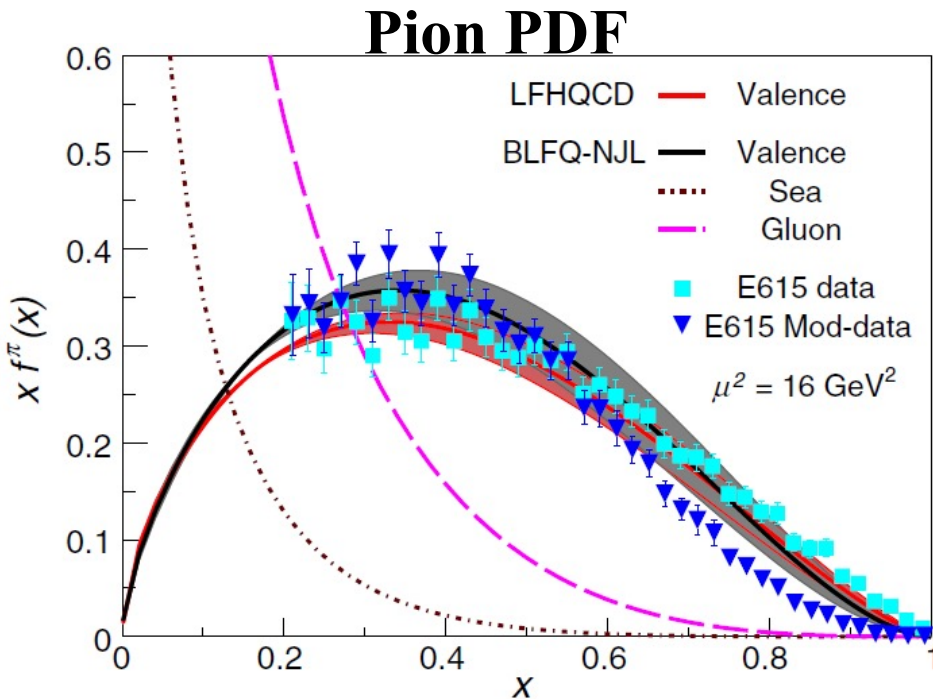
LF wave function



Initial PDFs

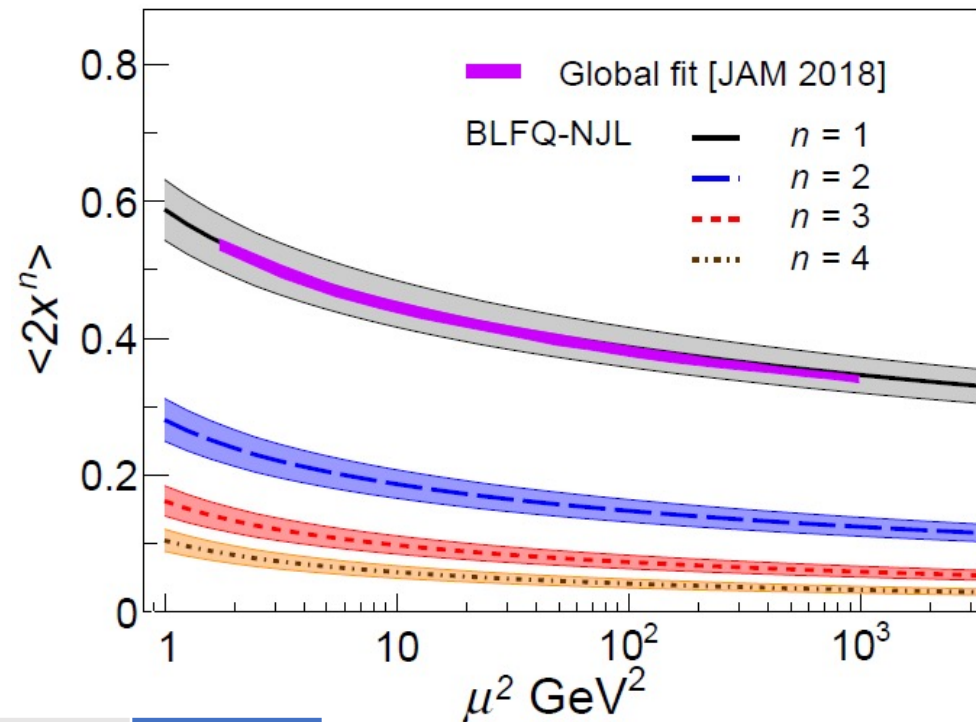
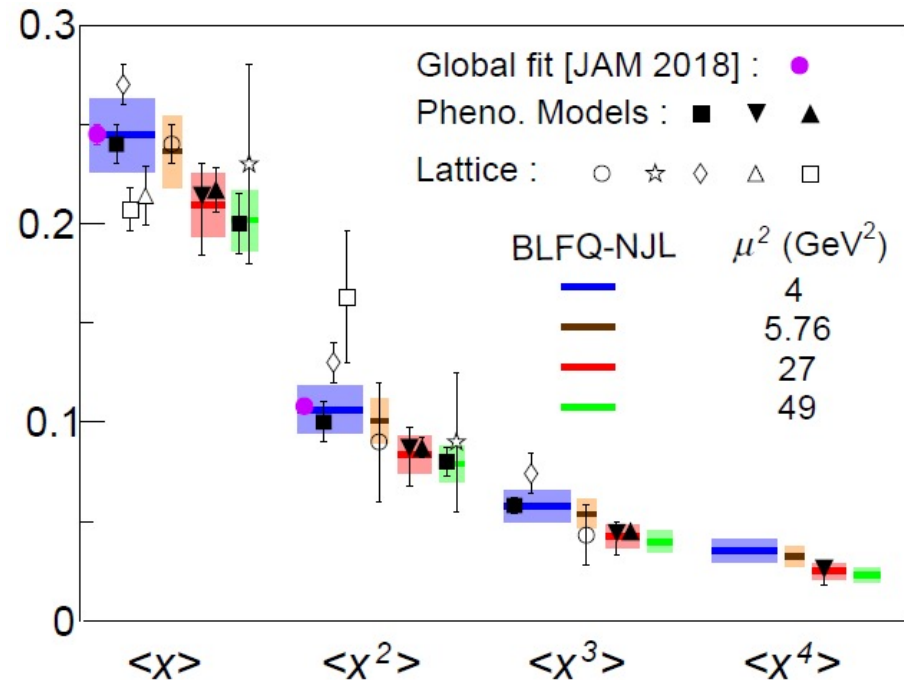


Scale evolution



The Moments of Pion Valence Quark PDF

$$\langle x^n \rangle = \int_0^1 dx x^n f_v^{\pi/K}(x, \mu^2), \quad n = 1, 2, 3, 4.$$



$\langle x \rangle @ 4 \text{ GeV}^2$	Valence	Gluon	Sea
BLFQ-NJL	0.489	0.398	0.113
[Ding <i>et. al.</i> , BSE model 2019']	0.48(3)	0.41(2)	0.11(2)

Agree with other results

Drell-Yan Cross Section

[S. D. Drell and T.-M. Yan, PRL (1970)]

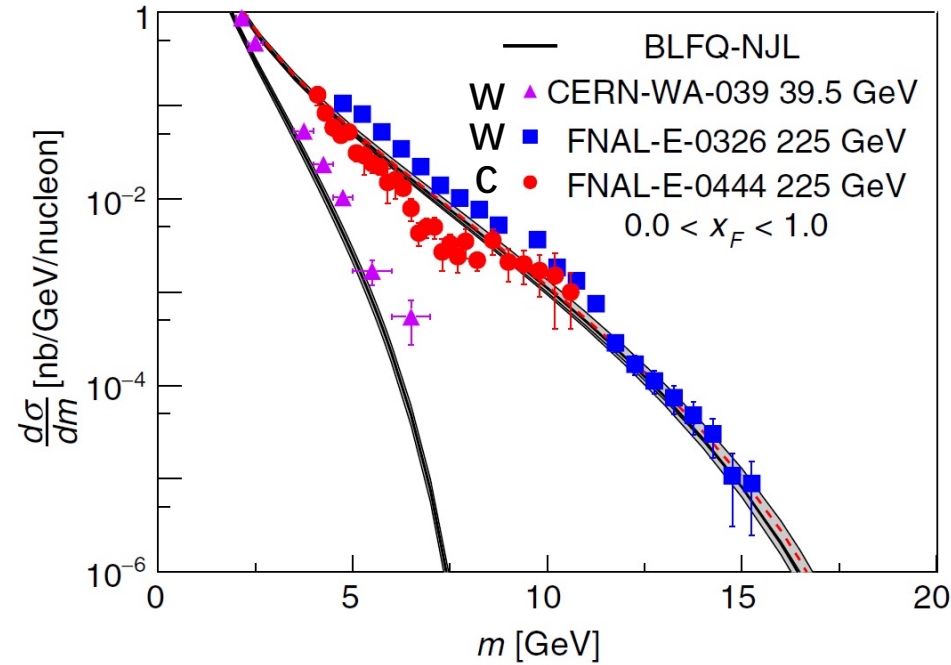
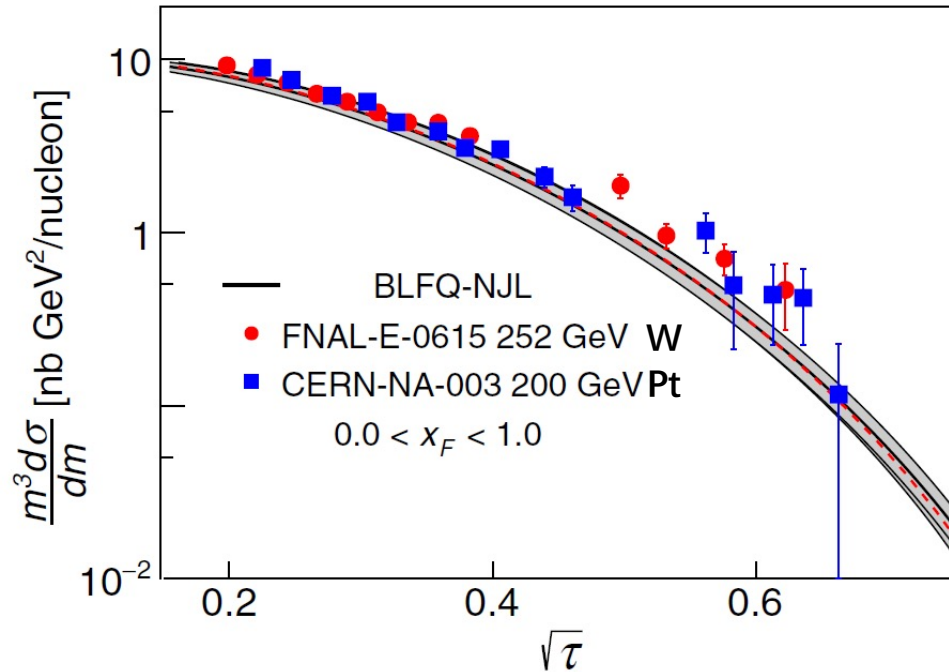
[T. Becher et al, JKEP07(2008)030]; [P. C. Barry et al, PRL121(2018)152001]

[C. Anastasiou et al, PRL91(2003)182002]

$$\pi^- \text{ Nucleus} \rightarrow \mu^+ \mu^- X$$

[nCTEQ 2015]

$$\frac{m^3 d^2 \sigma}{dm dY} = \frac{8\pi\alpha^2 m^2}{9 s} \sum_{ij} dx_1 dx_2 \tilde{C}_{ij}(x_1, x_2, s, m, \mu_f) f_{i/\pi}(x_1, \mu_f) f_{j/N}(x_2, \mu_f)$$



Agree with experimental data (FNAL E615, 326, 444, & CERN NA3, WA-039).

[Lan, Mondal, Jia, Zhao, Vary, PRD101(2020)034024]

Light mesons up to $|q\bar{q}g\rangle$

$$|\text{meson}\rangle = |q\bar{q}\rangle + \dots$$

[J. Lan et al, PRL122, 172001(2019)]

[J. Lan et al, PRD 101 (2020) 034024]



$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

[J. Lan et al, PLB 825 (2022) 136890]

[Z. Zhu et al, PLB 839(2023) 137808]

[J. Lan et al, PLB 868 (2025) 139654]

[J. Lan et al, PRD 111 (2025) 11, L111903]

[J. Lan et al, arXiv: 2509.22016 [hep-ph]]

Light mesons up to $|q\bar{q}g\rangle$

$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle$$

$$P^- = H_{K.E.} + H_{trans} + H_{longi} + H_{Interact}$$

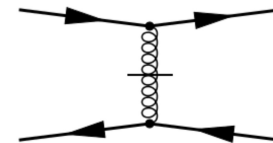
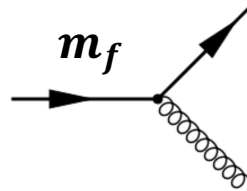
$$H_{K.E.} = \sum_i \frac{p_i^2 + m_q^2}{p_i^+}$$

$$H_{trans} \sim \kappa_T^4 r^2$$

---Brodsky, Teramond arXiv: 1203.4025

$$H_{longi} \sim - \sum_{ij} \kappa_L^4 \partial_{x_i} (x_i x_j \partial_{x_j}) \quad \text{---Y Li, X Zhao, P Maris, J Vary, PLB 758(2016)}$$

$$H_{Interact} = H_{Vertex} + H_{inst} = g\bar{\psi} \gamma^\mu T^a \psi A_\mu^a + \frac{g^2 C_F}{2} j^+ \frac{1}{(i\partial^+)^2} j^+$$

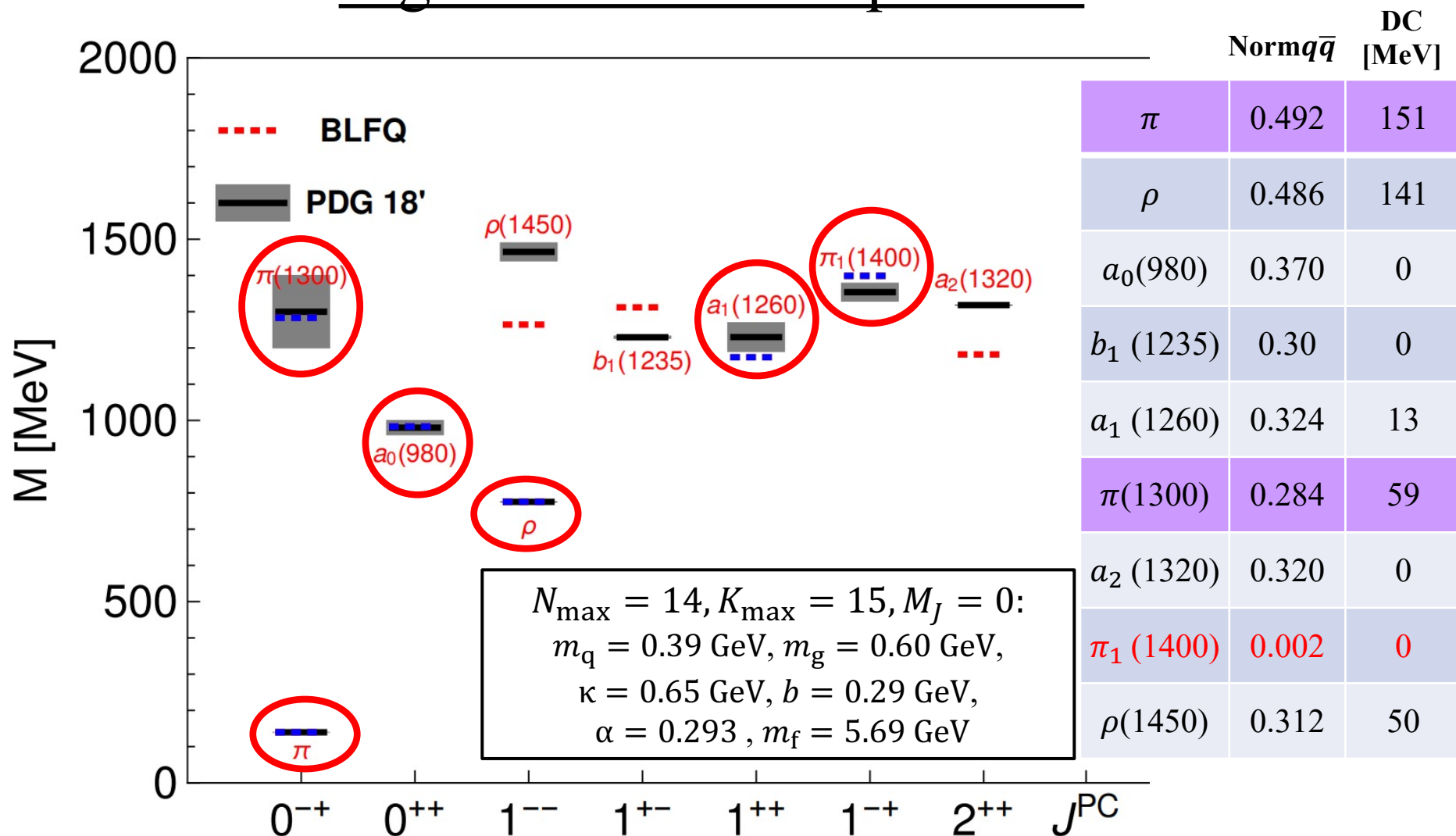


We allow an independent quark mass m_f in the vertex interaction.

[PRD 45 (1992) 3740–3754]

[J. Lan et al, PLB 825 (2022) 136890]

Light meson mass spectrum



$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

Fix the parameters by fitting six blue states

- $\pi_1(1400)$: $|q\bar{q}g\rangle$ dominates
- $\pi(1300)$: the DC is smaller than the DC of pion

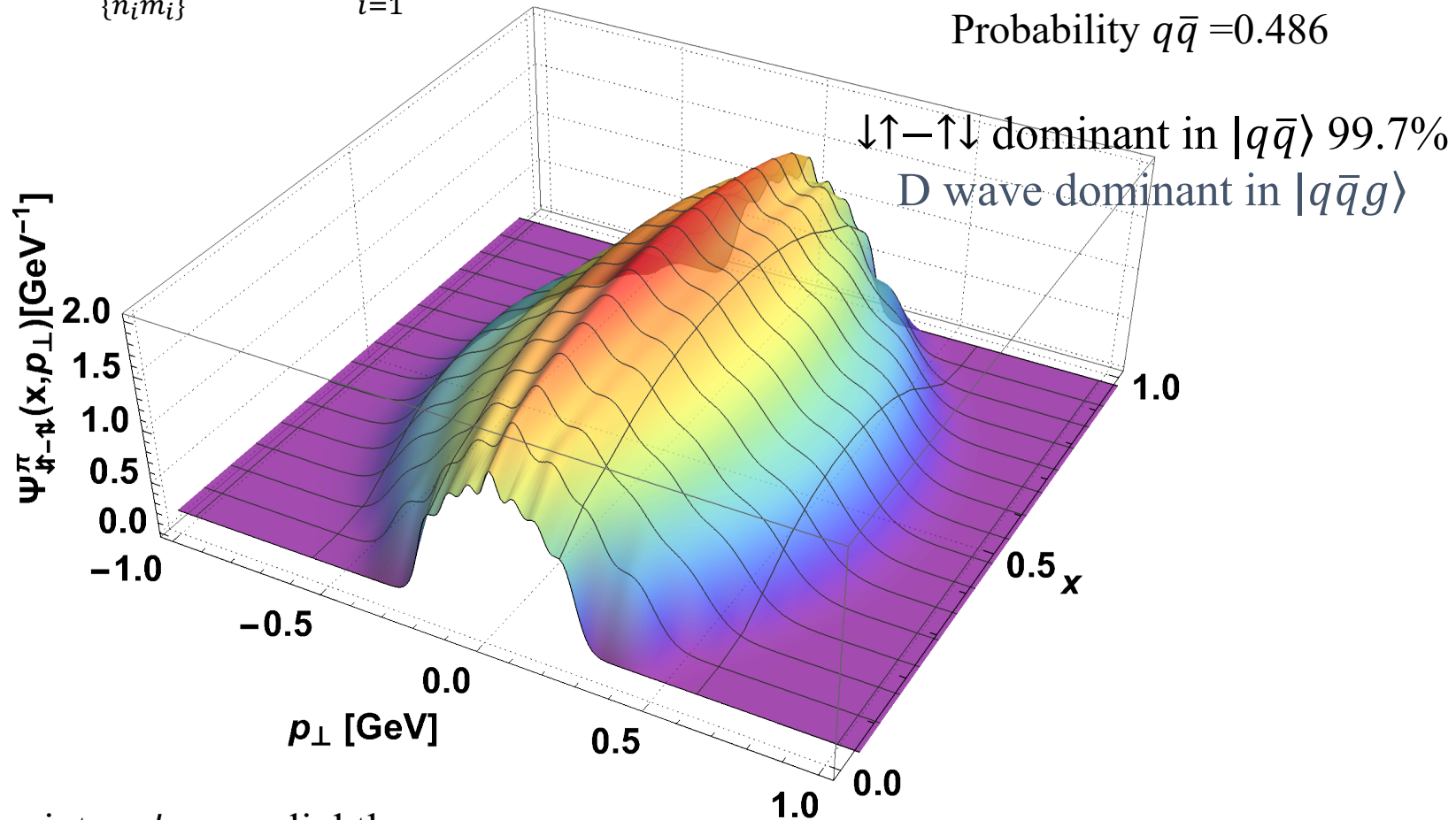
[J. Lan et al, PLB 825 (2022) 136890]

The Wave Function in Leading Fock Sector

$$\Psi_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_J} = \sum_{\{n_i m_i\}} \psi^{\mathcal{N}}(\{\vec{\alpha}_i\}) \prod_{i=1}^{\mathcal{N}} \phi_{n_i m_i}(\vec{p}_{\perp i}, b)$$

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

Probability $q\bar{q} = 0.486$



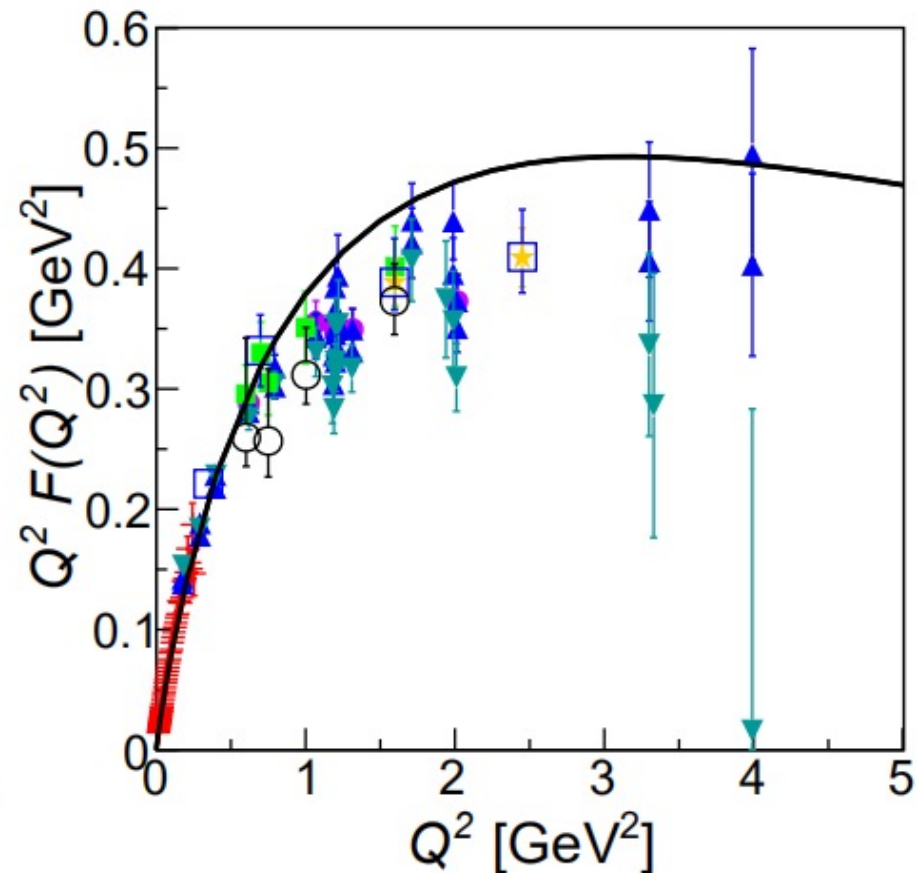
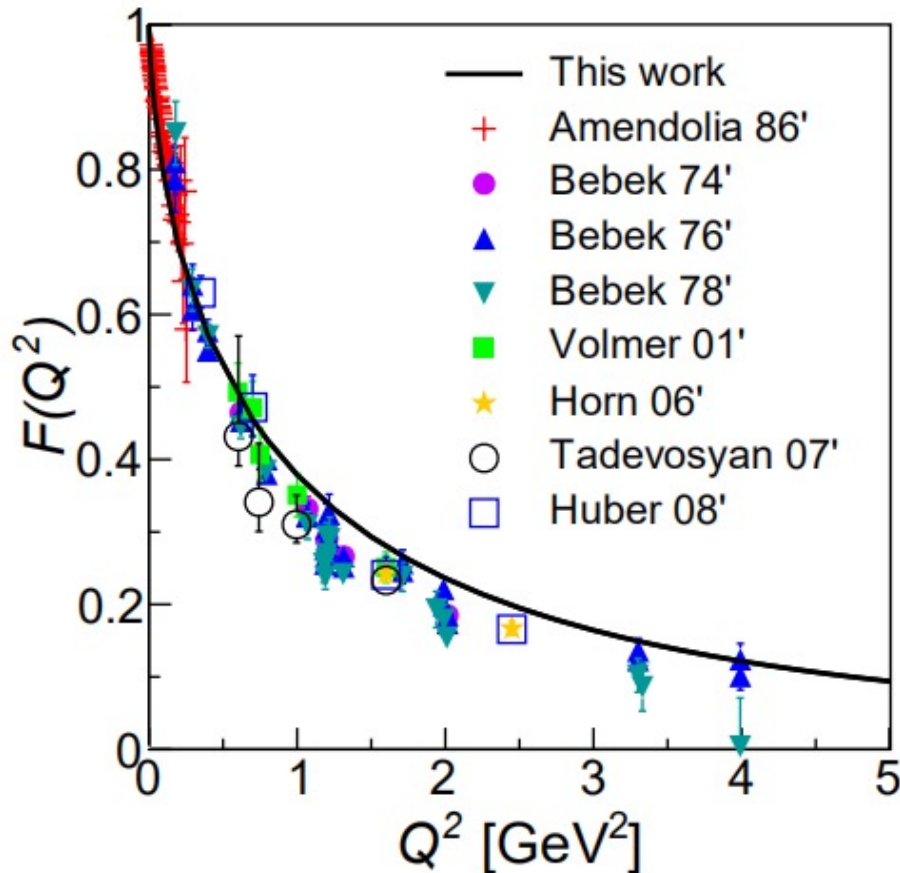
- At endpoint x , $\psi \sim p_{\perp}$: lightly narrow
- At middle x , $\psi \sim p_{\perp}$: a little bit wide

Pion Electromagnetic Form Factor

[Brodsky & de Teramond, PRD 77(2008)056007]

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

$$\langle\Psi(p')|J_{EM}^+(0)|\Psi(p)\rangle = (p+p')^+ F(Q^2)$$

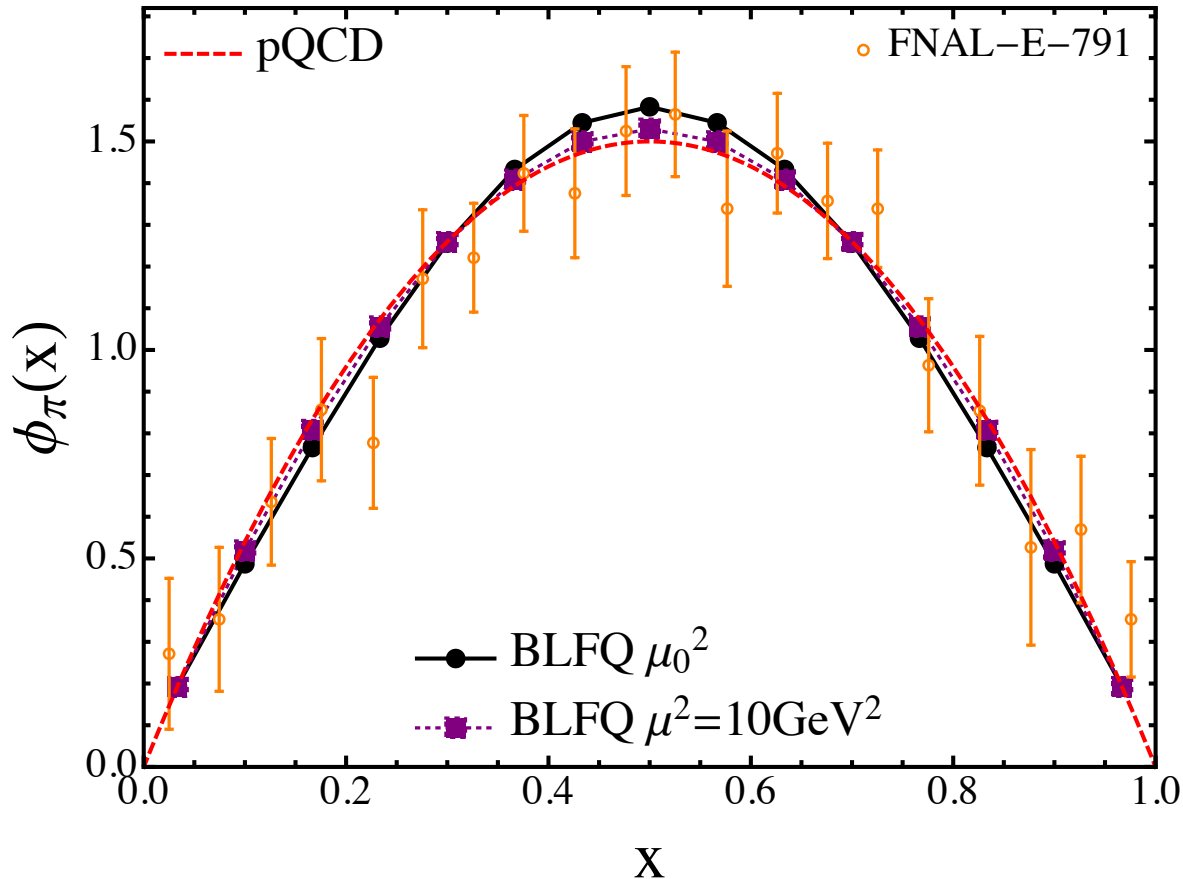


- FF is in reasonable agreement with experimental data
- $F(Q^2) \propto 1/Q^2$ for large Q^2

Pion PDA

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

[Ruiz Arriola & Broniowski, PRD 66(2002)094016]

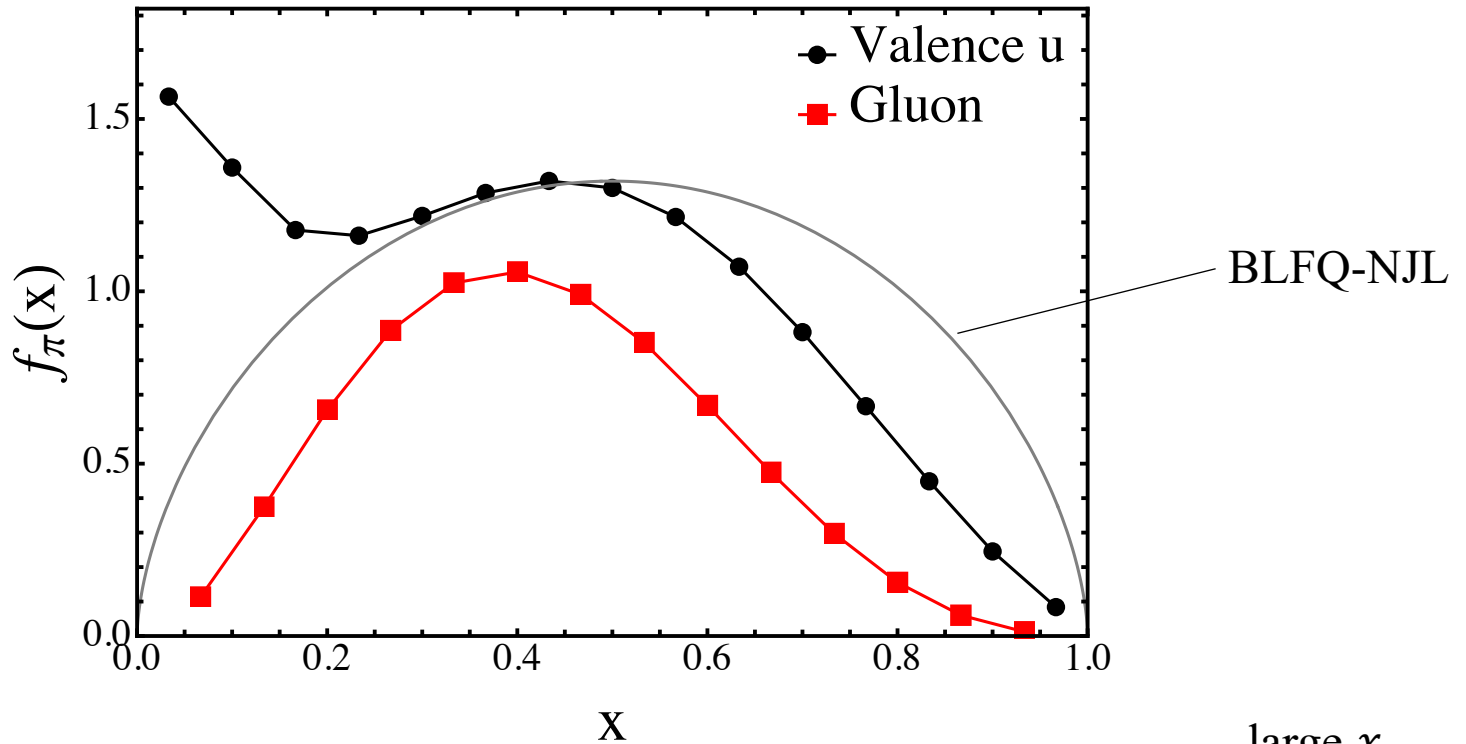


[PRL86. 4768]

- Endpoint behavior almost agrees with pQCD
- Consistent with FNAL-E-791 experiment on Drell-Yan (pion-platinum) process

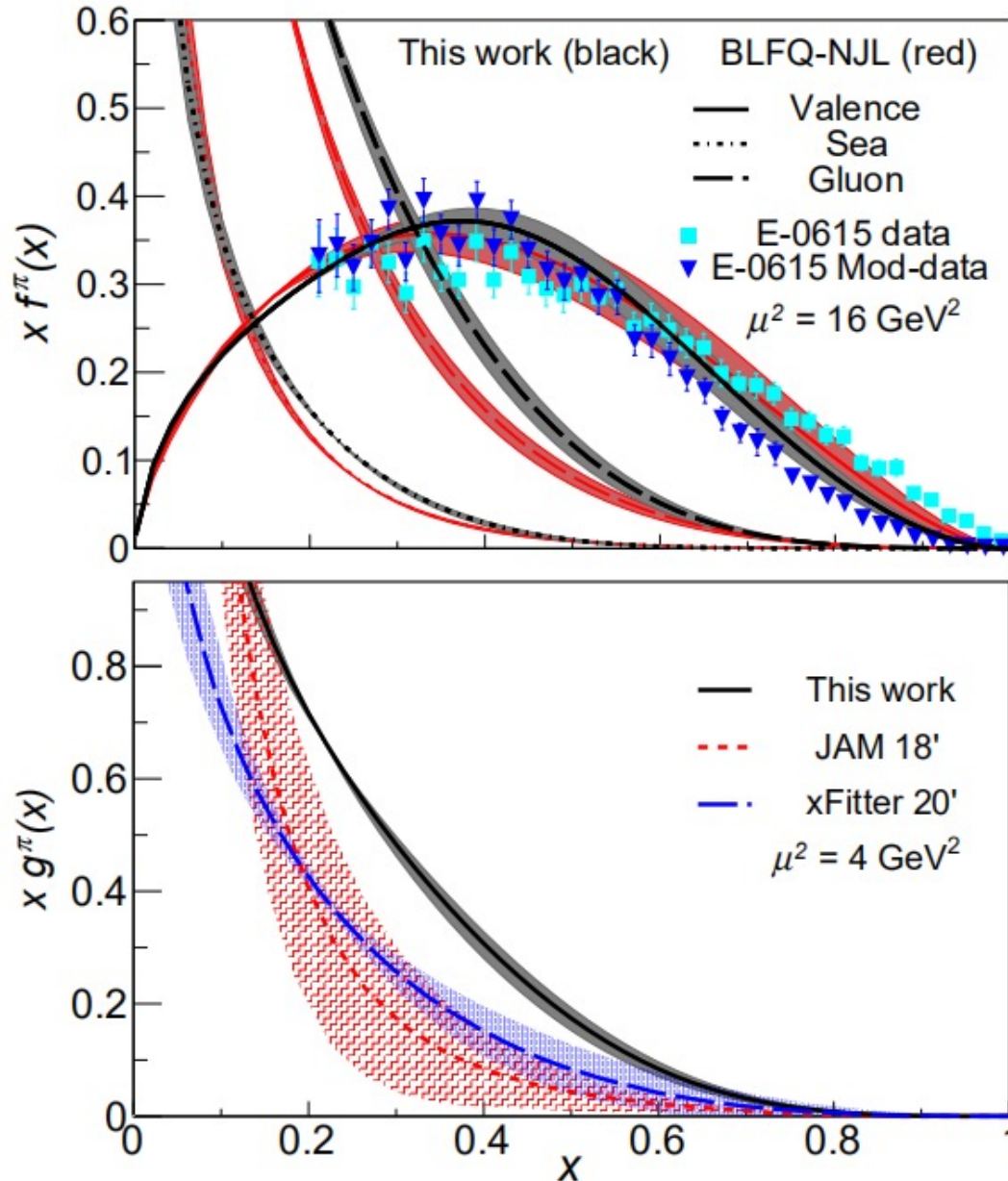
Pion PDF at Model Scale

$$f_i(x) = \sum_{\mathcal{N}, \lambda_i} \int [d\mathcal{X} d\mathcal{P}^\perp]_{\mathcal{N}} \left| \psi_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_j=0} \right|^2 \delta(x - x_i) \quad |\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$



$\mu_{0\text{BLFQ-NJL}}^2 = 0.240 \text{ GeV}^2$	$\langle x \rangle_{\text{gluon}} = 0$;	$\langle x \rangle_{\text{valence } u} = 0.5$	$(1-x)^{0.596}$
$\mu_{0\text{BLFQ}}^2 = 0.43 \text{ GeV}^2$	$\langle x \rangle_{\text{gluon}} = 0.216$;	$\langle x \rangle_{\text{valence } u} = 0.392$	$(1-x)^{1.4}$

Pion PDF with QCD Evolution



$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

- Large- x behavior $(1 - x)^{1.77}$ closer to pQCD
- The gluon distribution significantly increases

$\langle x \rangle @ 4 \text{ GeV}^2$	Valence	Gluon	Sea
BLFQ	0.483	0.421	0.096
BLFQ-NJL	0.489	0.398	0.113
[BSE 2019']	0.48(3)	0.41(2)	0.11(2)

J/ψ production cross section

$$\pi^\pm N \rightarrow J/\psi X$$

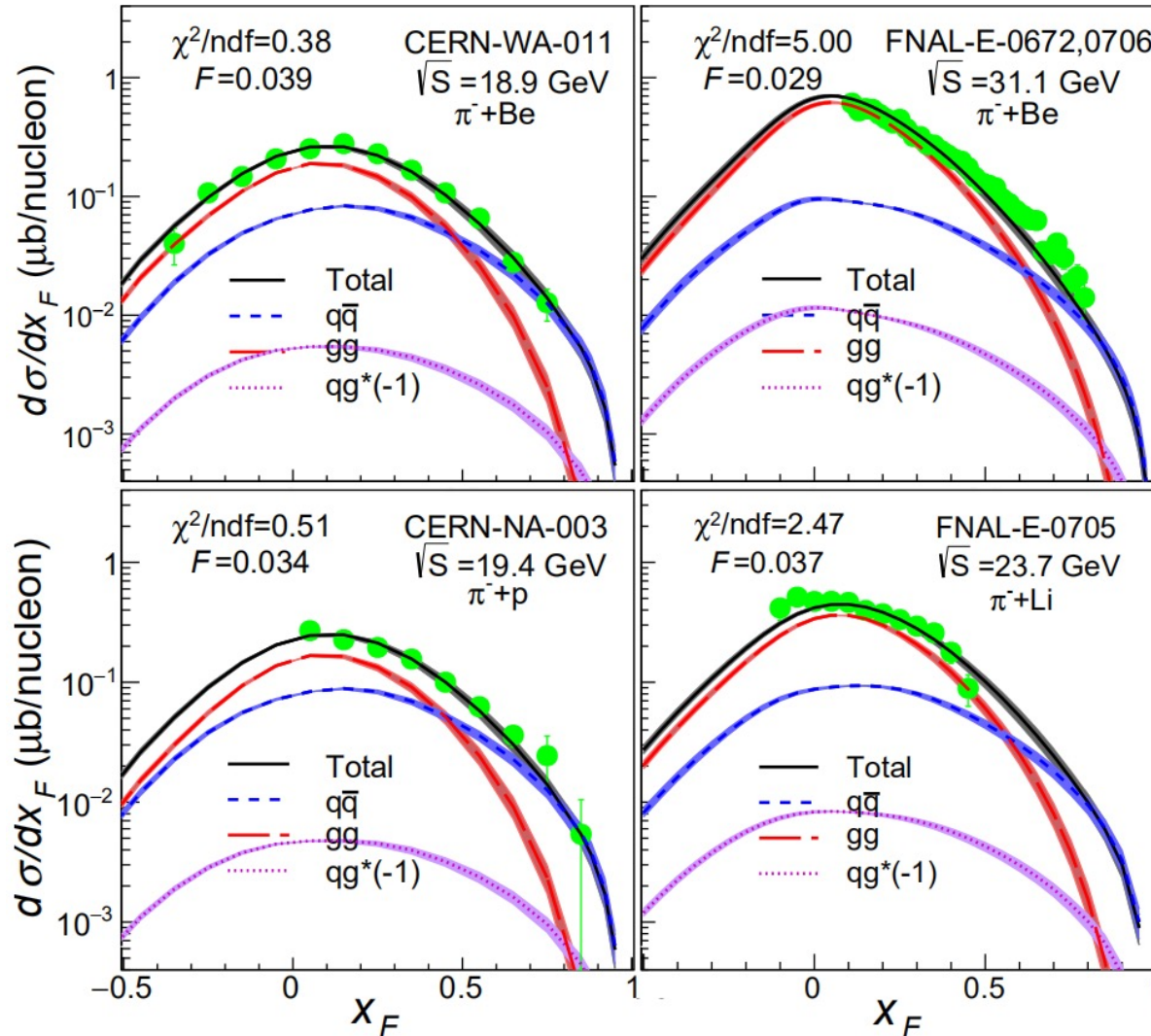
$$\frac{d\sigma}{dx_F} |J/\psi = F \sum_{i,j=q,\bar{q},g} \int_{2m_c}^{2m_D} dM_{c\bar{c}} \frac{2M_{c\bar{c}}}{S \sqrt{x_F^2 + \frac{4M_{c\bar{c}}^2}{S}}} \hat{\sigma}_{ij}(s, m_c^2, \mu_R^2, \mu_F^2) f_i^{\pi^\pm}(x_1, \mu_F^2) f_j^N(x_2, \mu_F^2)$$

[nCTEQ 2015]

CEM

[Chang, et al, PRD 102 (2020) 054024];
[Nason, et al, NPB 303 (1988) 607];
[Mangano, et al, NPB 405 (1993) 507]

- significantly gg contribution
- various energies of pions
- different target



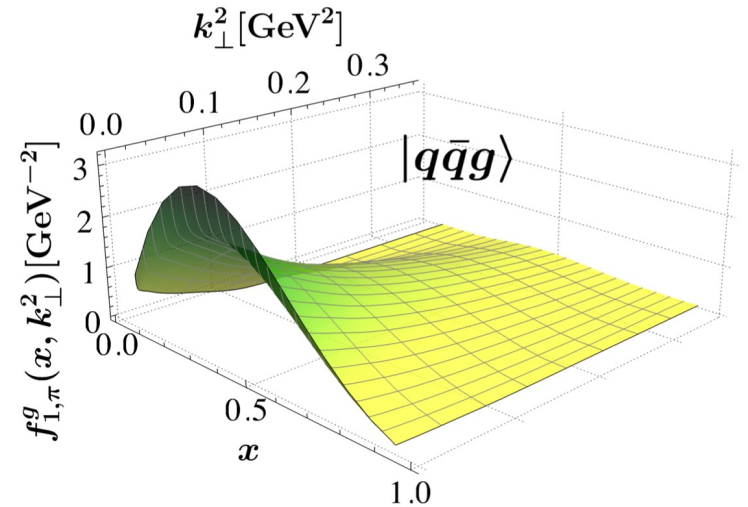
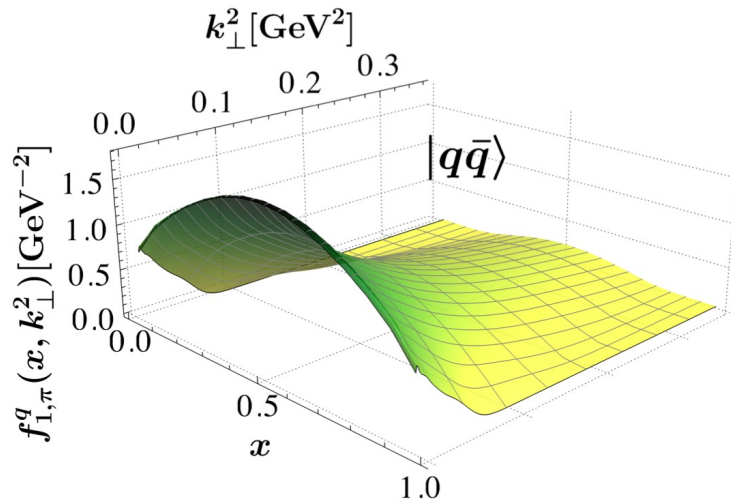
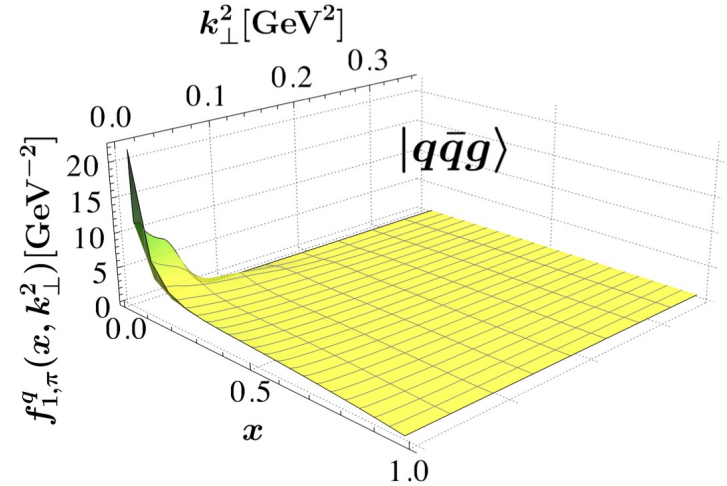
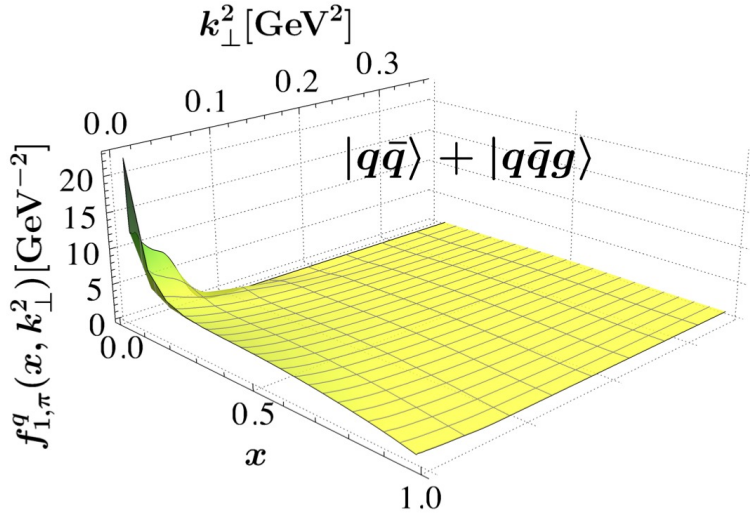
Agree with experimental data (FNAL E672, E706, E705, CERN NA3, WA11).

Pion TMD

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

$$f_{1,\pi}^q(x, k_\perp^2) = \sum_{\mathcal{N}, \lambda_i} \int_{\mathcal{N}} |\Psi_{\{x_i, \vec{k}_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_J=0}|^2 \delta(x - x_q) \delta^2(\vec{k}_\perp - \vec{k}_{\perp q}),$$

$$f_{1,\pi}^g(x, k_\perp^2) = \sum_{\mathcal{N}, \lambda_i} \int_{\mathcal{N}} |\Psi_{\{x_i, \vec{k}_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_J=0}|^2 \delta(x - x_g) \delta^2(\vec{k}_\perp - \vec{k}_{\perp g}),$$



- The TMD decreases with $k_\perp$
- Vanishes after $k_\perp \sim 0.6$ GeV

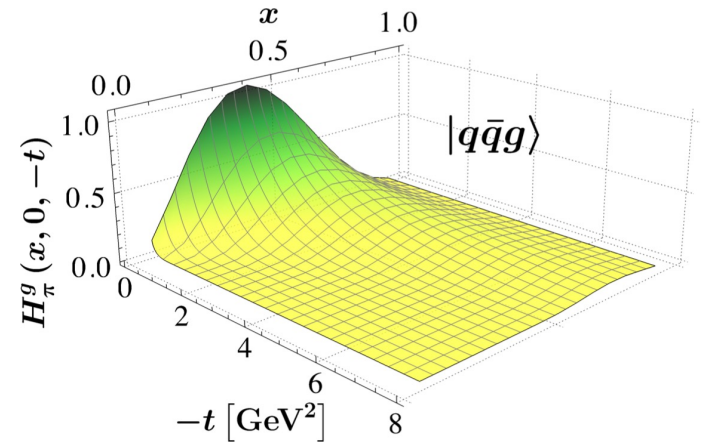
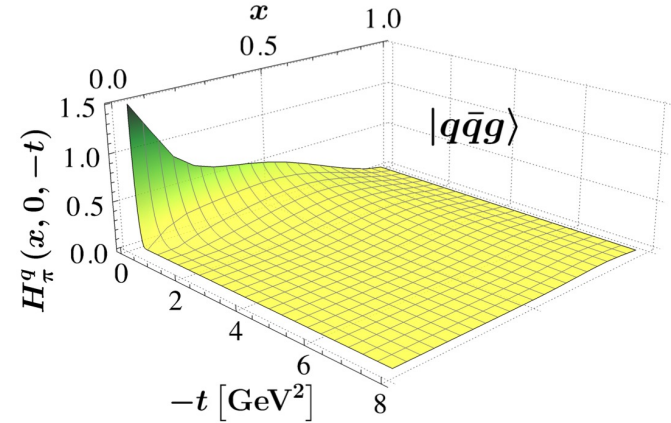
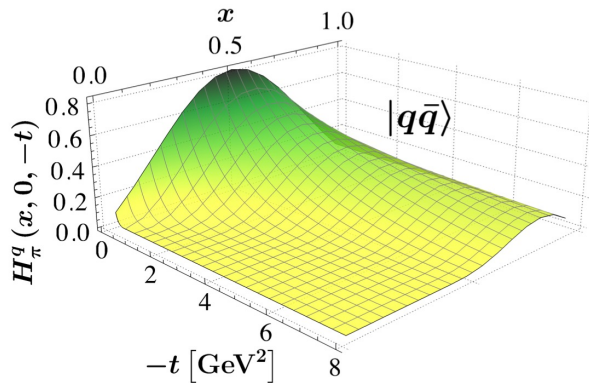
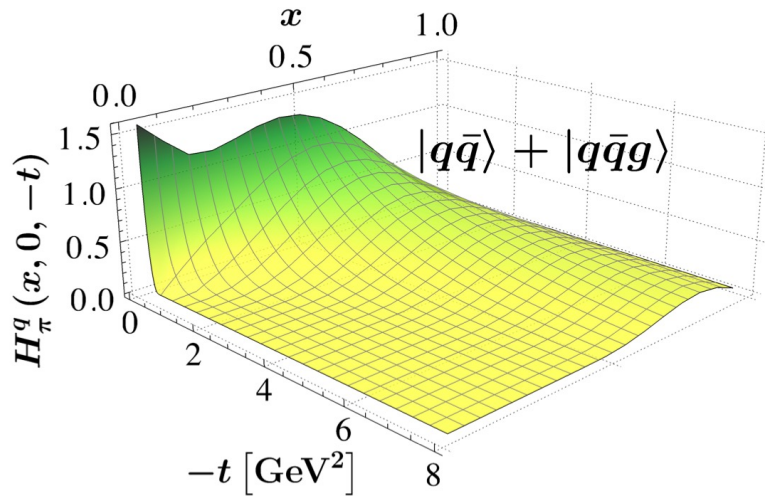
Pion GPD

$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

$$H_\pi^q(x, 0, -t) = \sum_{\mathcal{N}, \lambda_i} \int_{\mathcal{N}} \Psi_{\{x_i, \vec{k}'_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_J=0*} \Psi_{\{x_i, \vec{k}_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_J=0} \delta(x - x_q),$$

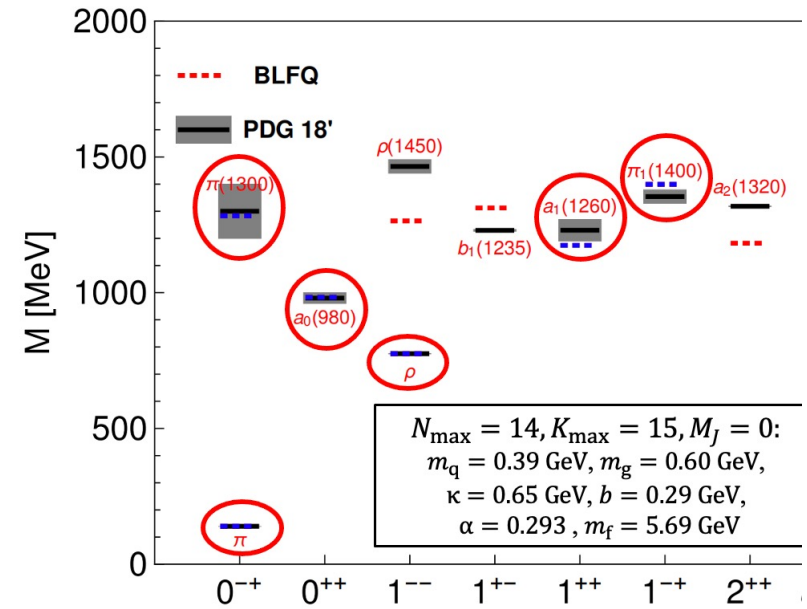
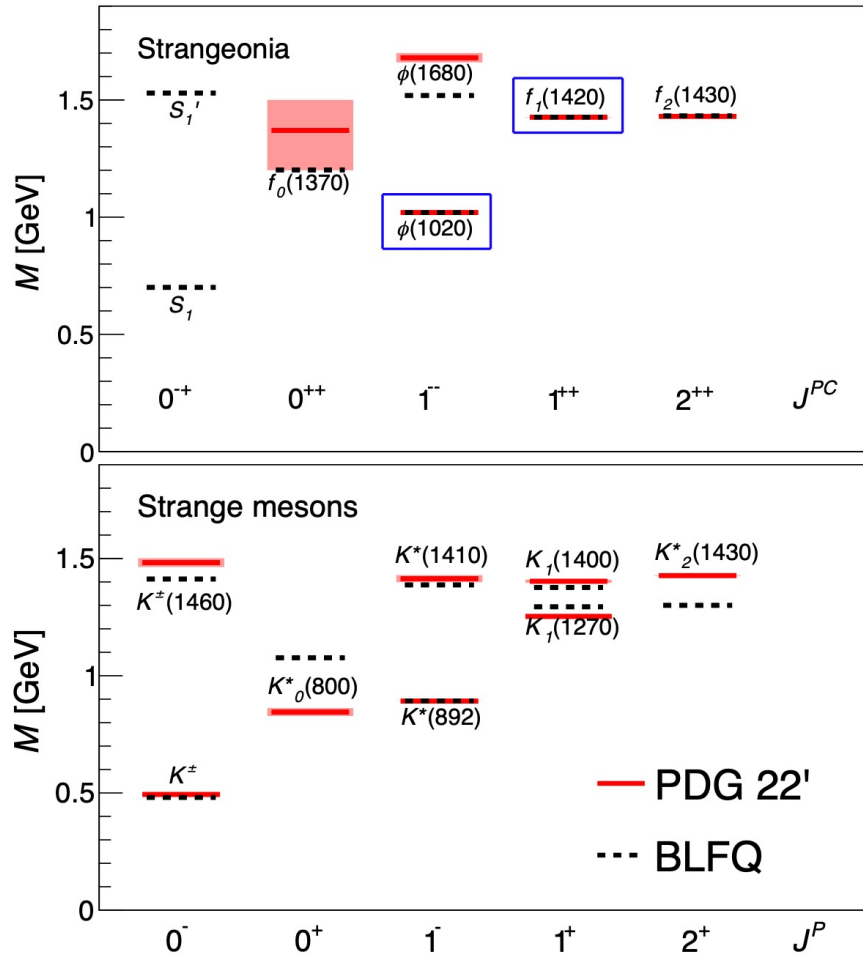
$$H_\pi^g(x, 0, -t) = \sum_{\mathcal{N}, \lambda_i} \int_{\mathcal{N}} \Psi_{\{x_i, \vec{k}'_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_J=0*} \Psi_{\{x_i, \vec{k}_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_J=0} \delta(x - x_g),$$

$$\vec{k}'_{\perp i} = \begin{cases} \vec{k}_{\perp i} + (1 - x_i)\vec{\Delta}_{\perp}, & \text{for the struck parton,} \\ \vec{k}_{\perp i} - x_i\vec{\Delta}_{\perp}, & \text{for spectator partons.} \end{cases}$$



- Quark content enhanced at small x with $|q\bar{q}g\rangle$
- Falls slowly at larger x
- Emerge at larger x range for larger $-t$

Strange meson mass spectrum



m_q	m_{f_q}	m_g	b	κ	g_s	m_s	m_{f_s}
0.39	5.69	0.60	0.29	0.65	1.92	0.55	7.13

- Agree with experimental data (K, K^*)
- The DC of kaon: 156.9 MeV
(Exp. $155.6 \pm 0.4 \text{ MeV}$)

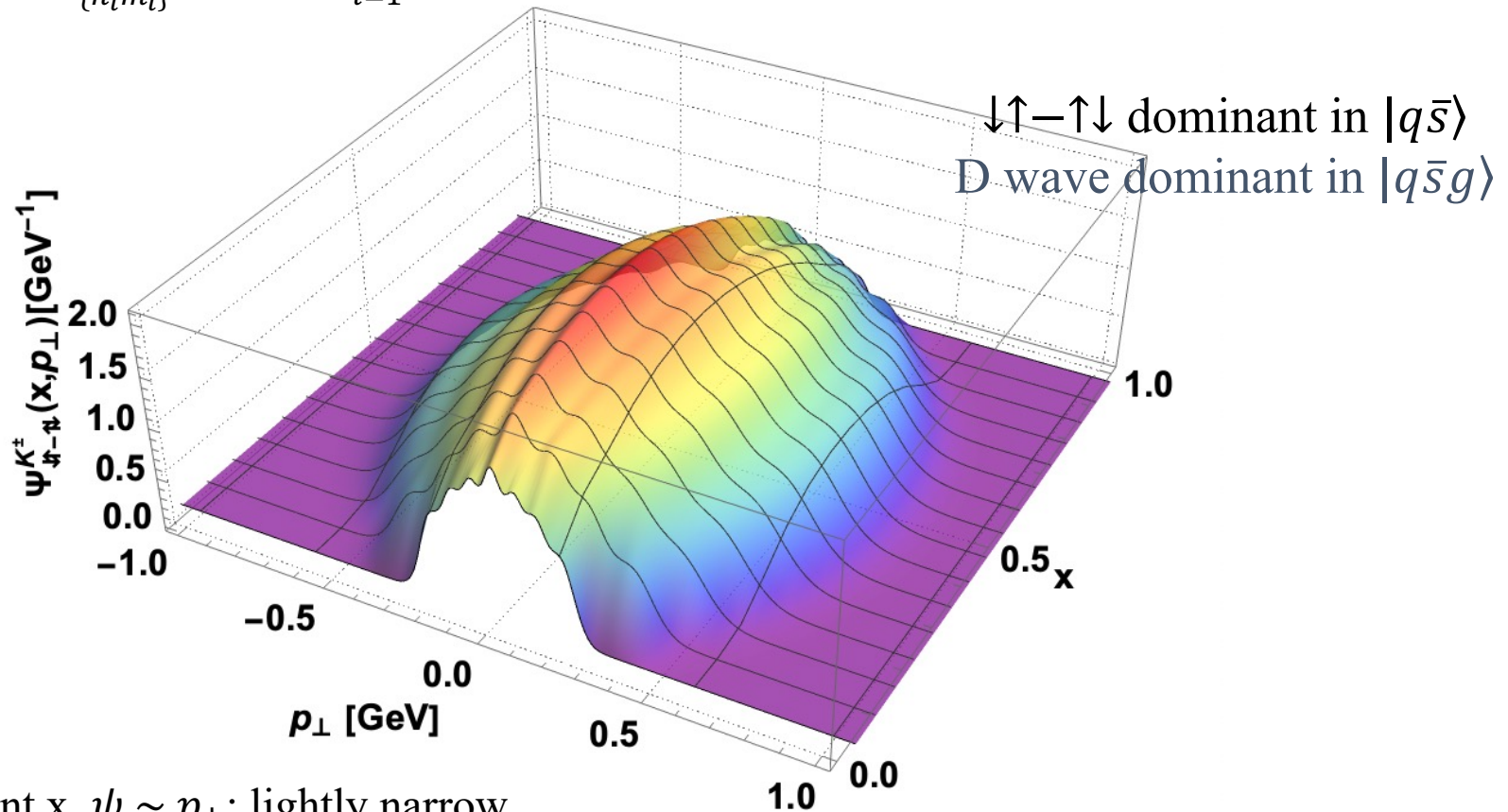
$$|\text{meson}\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

[J. Lan et al, Phys.Lett.B 868 (2025) 139654]

The Wave Function in Leading Fock Sector

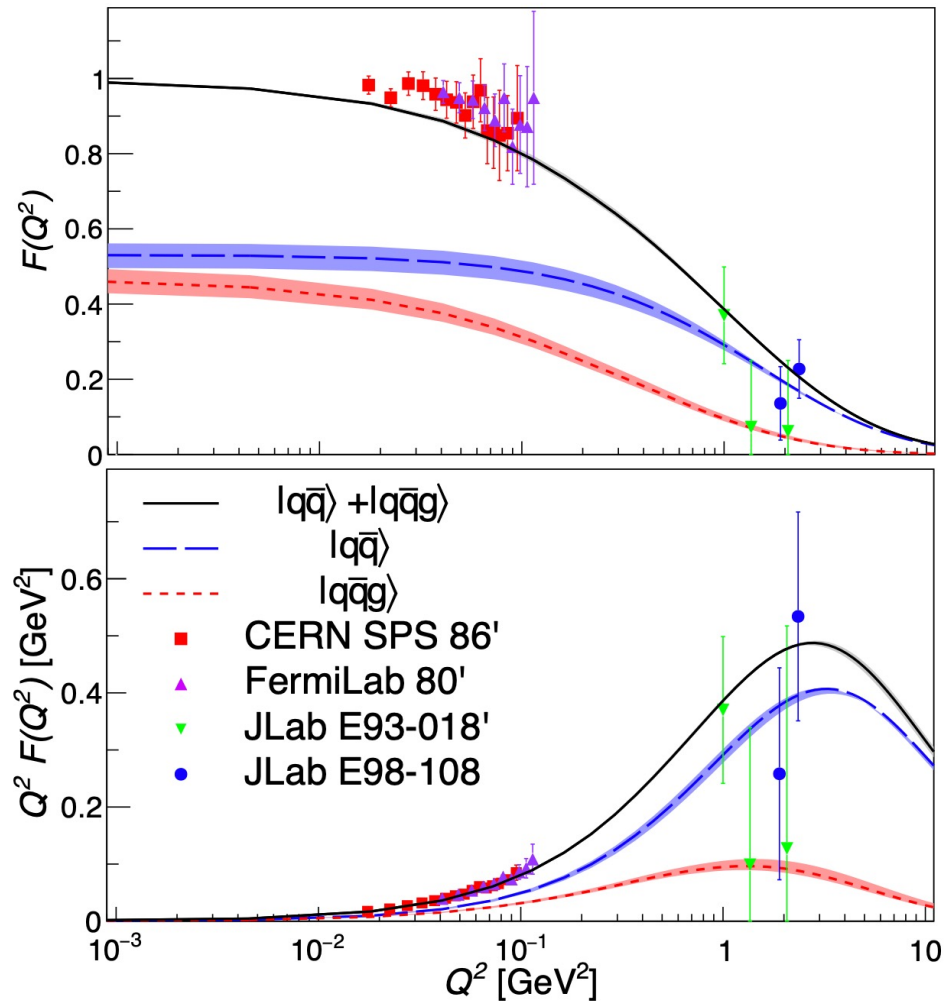
$$\Psi_{\{x_i, \vec{p}_{\perp i}, \lambda_i\}}^{\mathcal{N}, M_J} = \sum_{\{n_i m_i\}} \psi^{\mathcal{N}}(\{\vec{\alpha}_i\}) \prod_{i=1}^{\mathcal{N}} \phi_{n_i m_i}(\vec{p}_{\perp i}, b)$$

$$|K\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \dots$$



- At endpoint x , $\psi \sim p_{\perp}$: lightly narrow
- At middle x , $\psi \sim p_{\perp}$: a little bit wide
- The peak slightly less than $x=1/2$

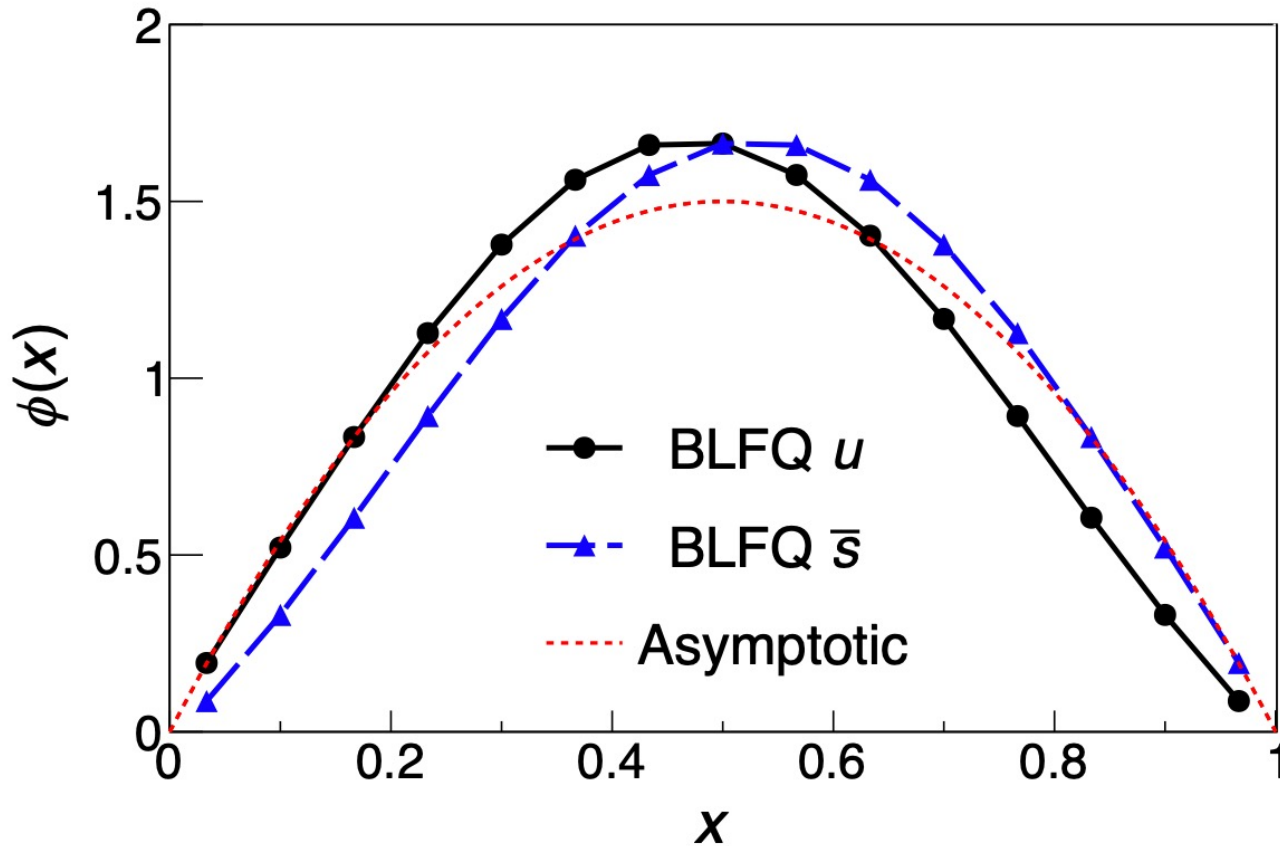
Kaon form factor



- FF is in reasonable agreement with experimental data
- $F(Q^2) \propto 1/Q^2$ for large Q^2

Kaon PDA

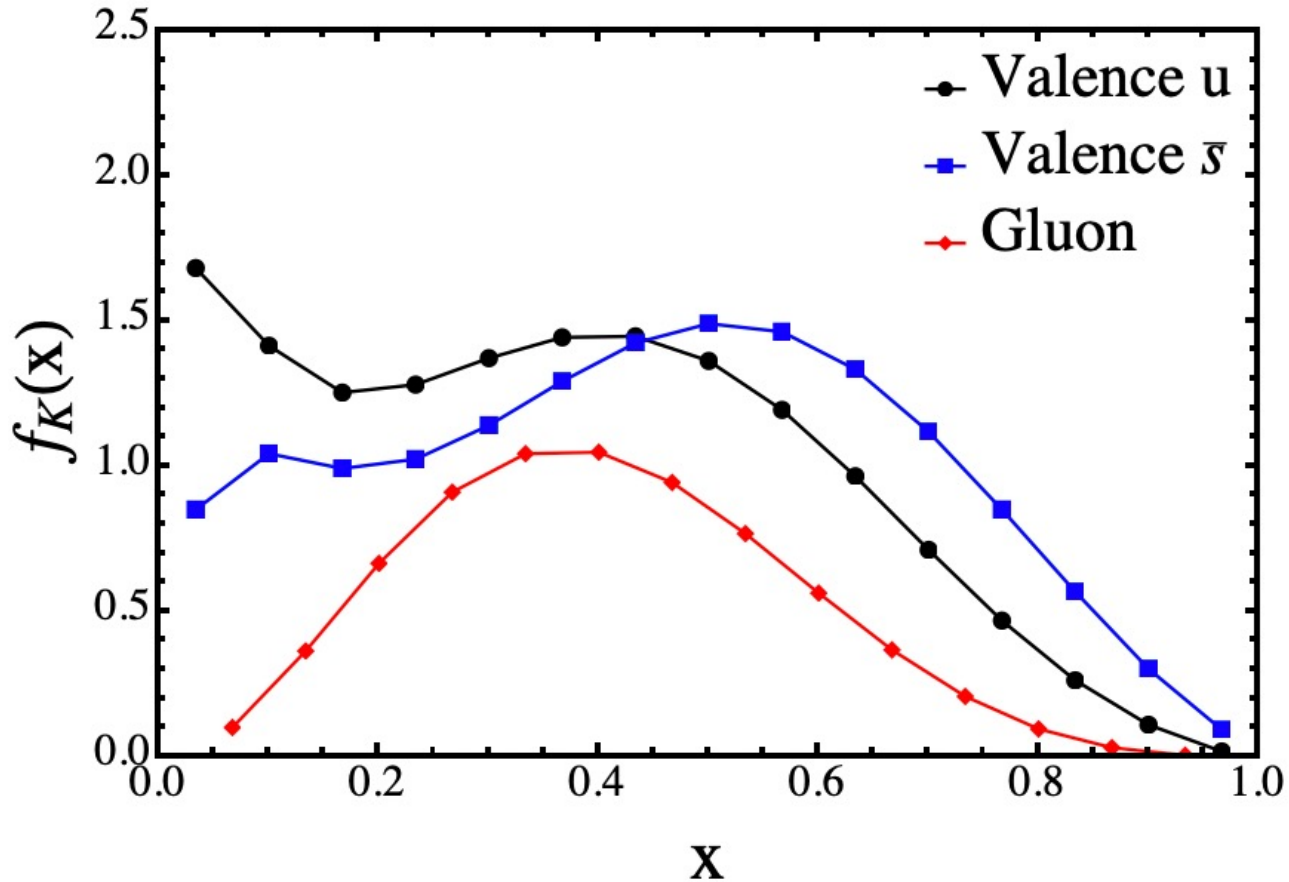
$$|K\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \dots$$



- Endpoint behavior almost agrees with pQCD

[J. Lan et al, Phys.Lett.B 868 (2025) 139654]

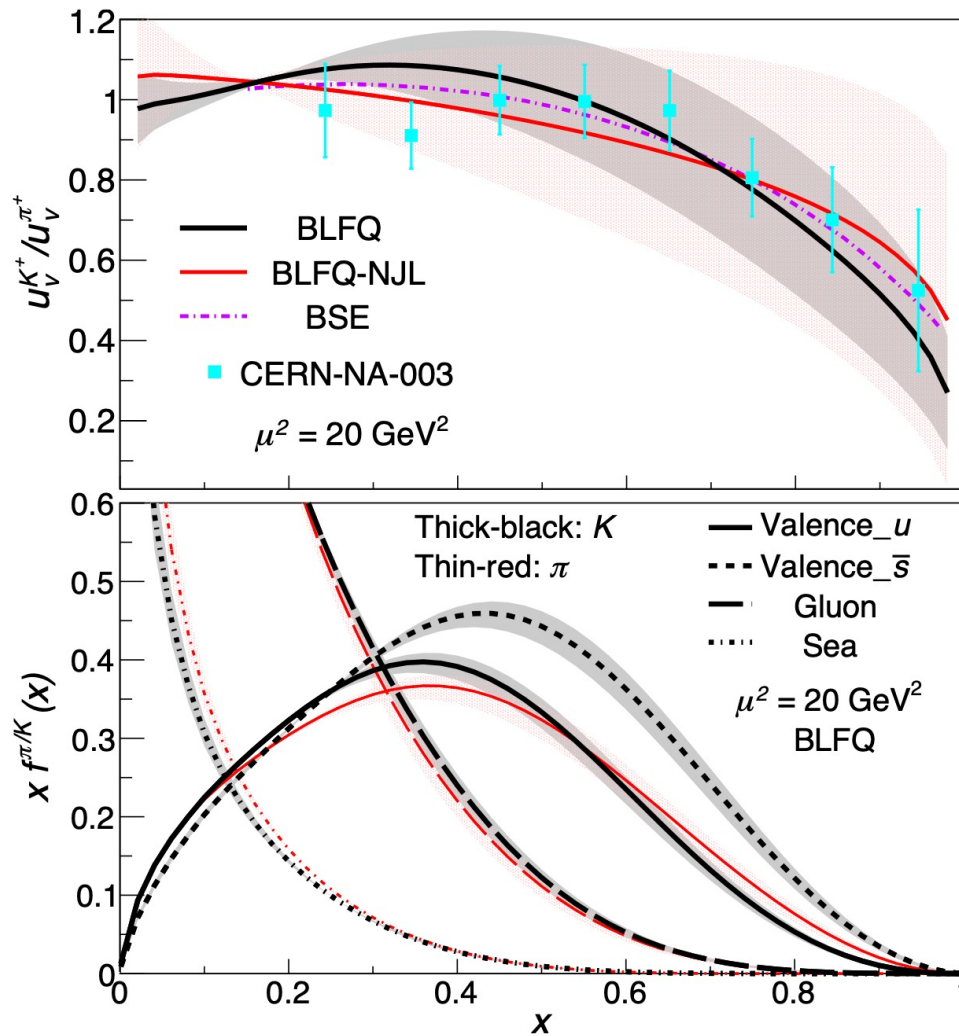
Kaon PDF at model scale



$\pi: \mu_{0\text{BLFQ}}^2 = 0.43 \text{ GeV}^2$ $\langle x \rangle_{\text{gluon}} = 0.216;$ $\langle x \rangle_{\text{val. } u} = 0.392$
 $K: \mu_{0\text{BLFQ}}^2 = 0.52 \text{ GeV}^2$ $\langle x \rangle_{\text{gluon}} = 0.193;$ $\langle x \rangle_{\text{val. } u} = 0.363;$ $\langle x \rangle_{\text{val. } \bar{s}} = 0.444$

[J. Lan et al, Phys.Lett.B 868 (2025) 139654]

Kaon PDF with QCD evolution



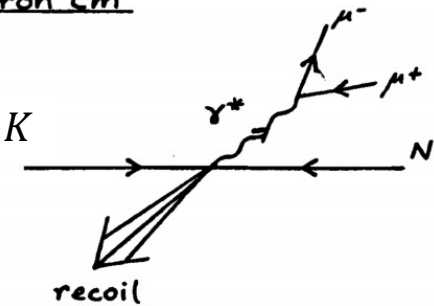
$$|K\rangle = a|q\bar{s}\rangle + b|q\bar{s}g\rangle + \dots$$

- Large-x behavior
 u: $(1-x)^{2.24}$
 s: $(1-x)^{1.72}$

$\langle x \rangle @ 20 \text{ GeV}^2$	BLFQ
Valence u	0.21
Valence s	0.26
Gluon	0.42
Sea	0.11

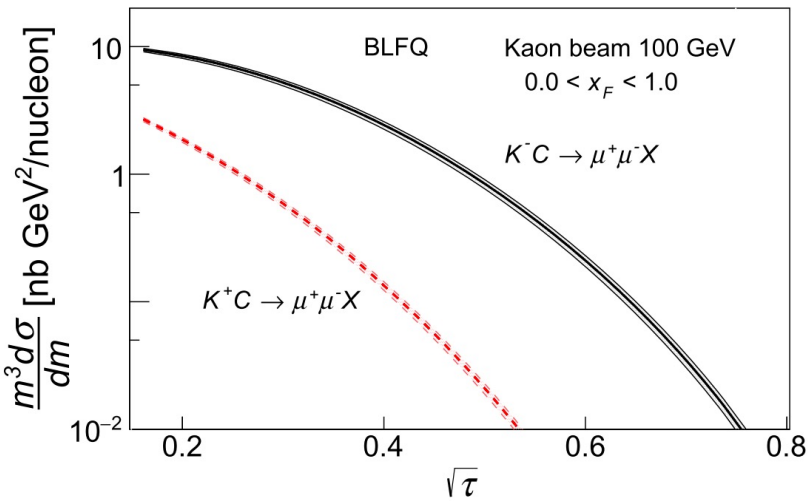
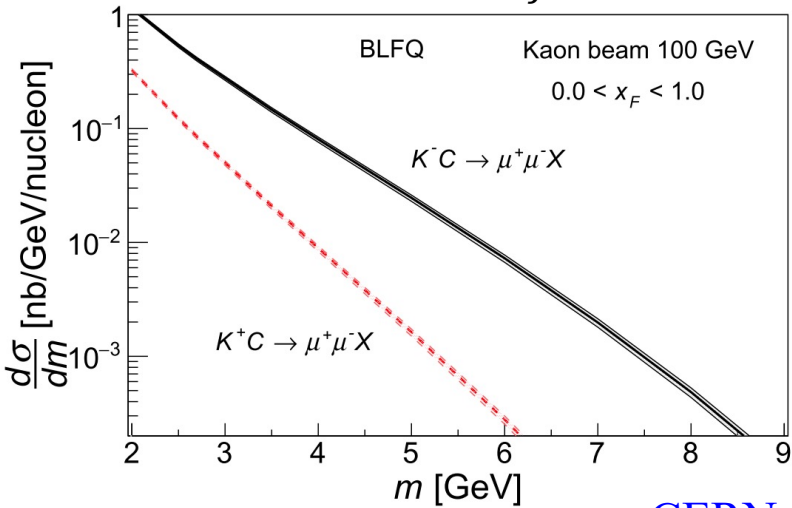
Drell-Yan Cross Section

hadron cm



[S. D. Drell and T.-M. Yan, PRL (1970)]
[T. Becher et al, JKEP07(2008)030]; [P. C. Barry et al, PRL121(2018)152001]
[C. Anastasiou et al, PRL91(2003)182002]

$$\frac{m^3 d^2 \sigma}{dm dY} = \frac{8\pi \alpha^2}{9} \frac{m^2}{s} \sum_{ij} dx_1 dx_2 \tilde{C}_{ij}(x_1, x_2, s, m, \mu_f) f_{i/K}(x_1, \mu_f) f_{j/N}(x_2, \mu_f)$$



CERN-AMBER [CERN-SPSC-2019-022 /SPSC-P-360]

Program	Physics Goals	Beam Energy [GeV]	Beam Intensity [s ⁻¹]	Trigger Rate [kHz]	Beam Type	Target	Earliest start time, duration	Hardware additions
Drell-Yan (RF)	Kaon PDFs & Nucleon TMDs	~100	10 ⁸	25-50	K [±] , p̄	NH ₃ , C/W	2026 2-3 years	"active absorber", vertex detector
Primakoff (RF)	Kaon polarisability & pion life time	~100	5 · 10 ⁶	> 10	K ⁻	Ni	non-exclusive 2026 1 year	
Prompt Photons (RF)	Meson gluon PDFs	≥ 100	5 · 10 ⁶	10-100	K [±] , π [±]	LH2, Ni	non-exclusive 2026 1-2 years	hodoscope
K-induced Spectroscopy (RF)	High-precision strange-meson spectrum	50-100	5 · 10 ⁶	25	K ⁻	LH2	2026 1 year	recoil TOF, forward PID
Vector mesons (RF)	Spin Density Matrix Elements	50-100	5 · 10 ⁶	10-100	K [±] , π [±]	from H to Pb	2026 1 year	

Experiment	Target type	Beam type	Beam intensity (part/sec)	Beam energy (GeV)	DY mass (GeV/c ²)	DY events	
						μ ⁺ μ ⁻	e ⁺ e ⁻
NA3	6 cm Pt	K ⁻		200	4.2 – 8.5	700	0
This exp.	100 cm C	K ⁻	2.1 × 10 ⁷	80	4.0 – 8.5	25,000	13,700
				100	4.0 – 8.5	40,000	17,700
				120	4.0 – 8.5	54,000	20,700
		K ⁺	2.1 × 10 ⁷	80	4.0 – 8.5	2,800	1,300
This exp.	100 cm C	π ⁻	4.8 × 10 ⁷	100	4.0 – 8.5	5,200	2,000
				120	4.0 – 8.5	8,000	2,400
				120	4.0 – 8.5	123,600	39,800

[J. Lan et al, Phys.Lett.B 868 (2025) 139654]

The pion $q\bar{q}g$ BLFQ .vs. BSE

BLFQ

Phys. Lett. B 825 (2022) 136890

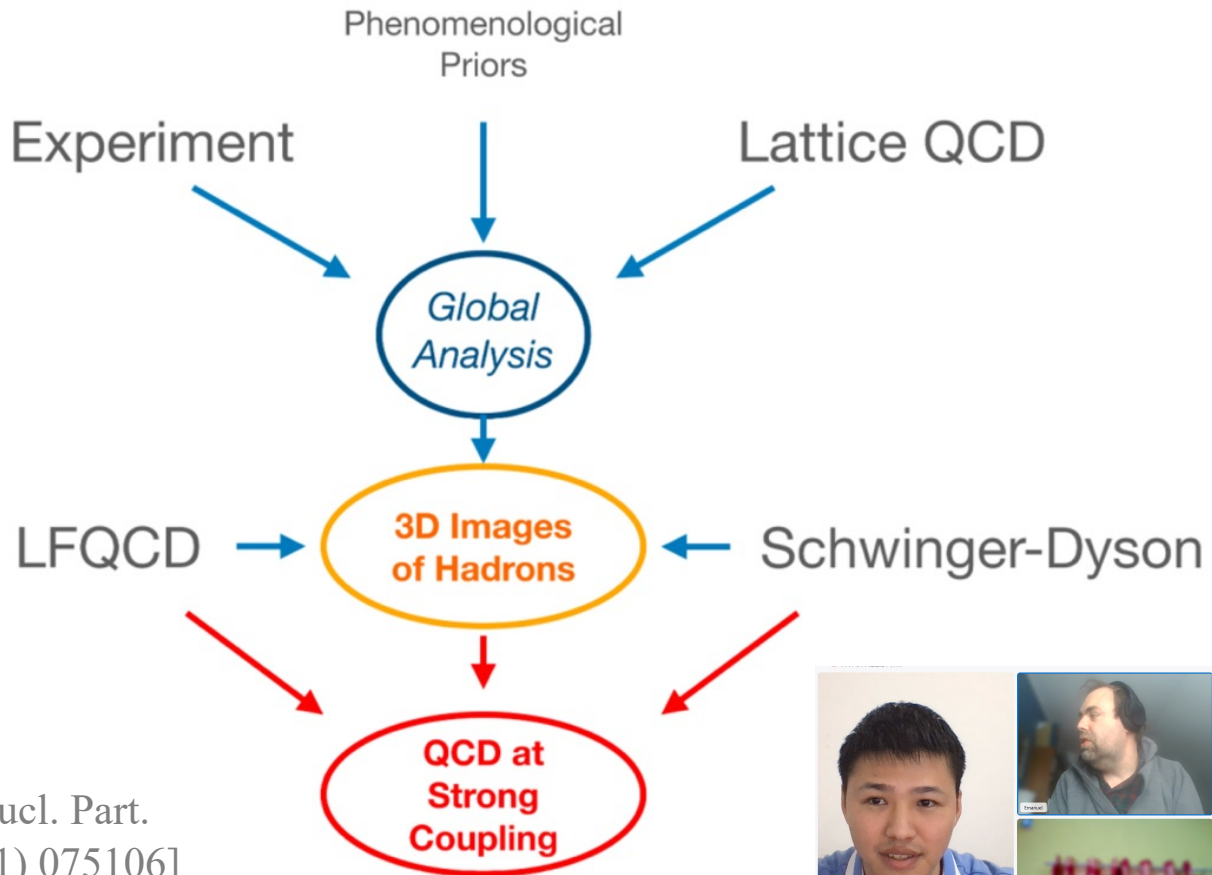
$$|\pi\rangle = a|q\bar{q}\rangle + b|q\bar{q}g\rangle + \dots$$

BSE in Minkowski space

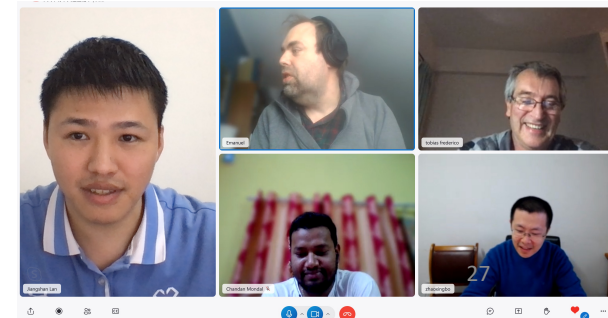
Phys. Rev. D 103 (2021) 014002

Phys. Rev. D 105 (2022) L071505

Phys. Lett. B 820 (2021) 136494



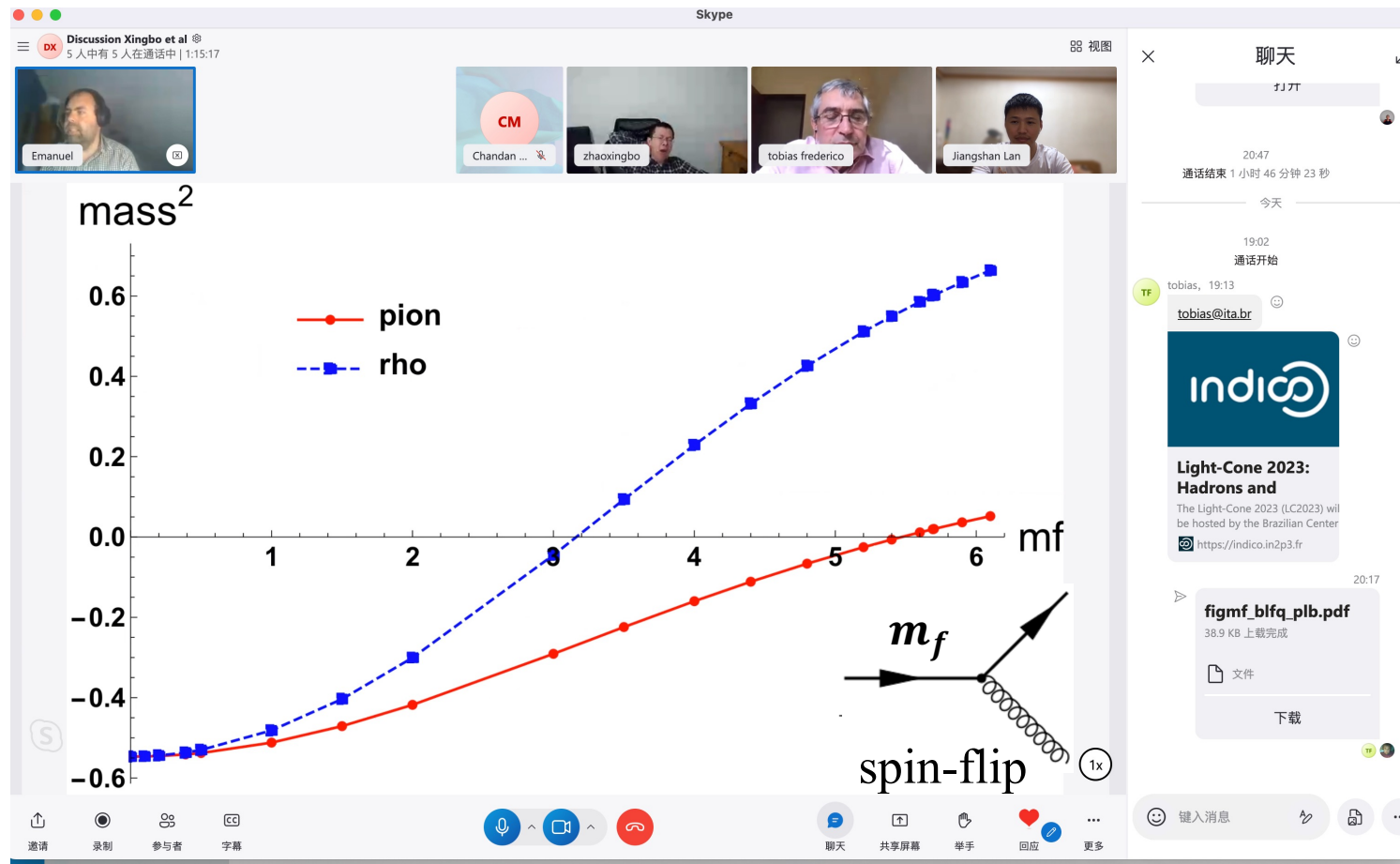
[J. Phys. G: Nucl. Part.
Phys. 48 (2021) 075106]



The pion $q\bar{q}g$ BLFQ .vs. BSE

We explicitly show an enhancement of the low- x contribution in the quark PDF is directly associated with the large spin-flip matrix element, necessary to provide the $\pi - \rho$ mass splitting. [M. Burkardt, *Dynamical vertex mass generation and chiral symmetry breaking on the light front*, Phys. Rev. D 58 (1998) 096015].

It was proposed to enhance the spin flip matrix element of the effective quark-gluon coupling QCD LF-Hamiltonian by introducing a large effective quark vertex mass (m_f).

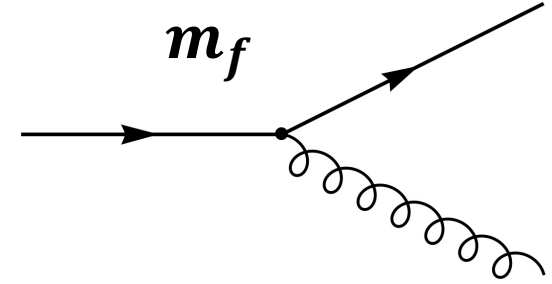


Pion $q\bar{q}g$ BLFQ .vs. BSE

We introduce a LF model:

$$|\text{Meson}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle$$

$$\psi_{q\bar{q}g;(i_q,i_{\bar{q}},a)}^{(s_q,s_{\bar{q}},\lambda)} = \frac{1}{M_\pi^2 - M_{0,q\bar{q}g}^2} \left[V \psi_{q\bar{q};(i_q,i_{\bar{q}})}^{(s_q,s_{\bar{q}})} \right],$$



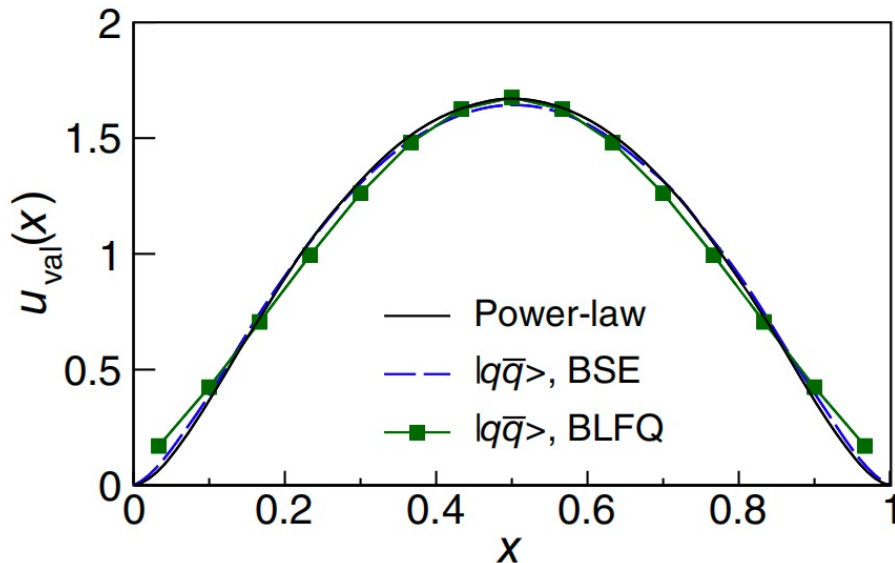
V : interaction connecting $q\bar{q}$ and $q\bar{q}g$ sector

$$\begin{aligned} [V \psi_{q\bar{q};(i_q,i_{\bar{q}})}^{(s_q,s_{\bar{q}})}] &= \frac{g_s \sqrt{2}}{x_q + x_{\bar{q}}} \sum_{i_1 s_1} T_{i_q i_1}^a W_\lambda^{(s_q,s_1)}(p_q, p_g) \\ &\times \psi_{q\bar{q};(i_1,i_{\bar{q}})}^{(s_1,s_{\bar{q}})}(x_{\bar{q}}, \vec{p}_{\bar{q}\perp}) - \frac{g_s \sqrt{2}}{x_{\bar{q}} + x_g} \\ &\times \sum_{i_1 s_1} T_{i_1 i_{\bar{q}}}^a \bar{W}_\lambda^{(s_1,s_{\bar{q}})}(p_{\bar{q}}, p_g) \psi_{q\bar{q};(i_q,i_1)}^{(s_q,s_1)}(x_q, \vec{p}_{q\perp}). \end{aligned}$$

- To get the LFWF for $|q\bar{q}g\rangle$ based on the LFWF for $|q\bar{q}\rangle$ with the interaction V .

Pion $q\bar{q}g$ BLFQ .vs. BSE

To get LFWF for $|q\bar{q}g\rangle$



[Phys. Rev. D 50 (1994) 6895]

Take a power-law form for simplicity:

$$\psi_{pl}(x_q, \mathbf{p}_{q\perp}) = N \left[1 + \frac{A_{0,eff}(x_q, \mathbf{p}_{q\perp})/4 - m_q^2}{\beta^2} \right]^{-s}$$

$$A_{0,eff}(x_q, \mathbf{p}_{q\perp}) = \frac{\mathbf{p}_{q\perp}^2 + m_q^2}{x_q} + \frac{\mathbf{p}_{\bar{q}\perp}^2 + m_{\bar{q}}^2}{x_{\bar{q}}}$$

Fitting the valence PDF of both the BLFQ and the BSE:

$$s = 1.4$$

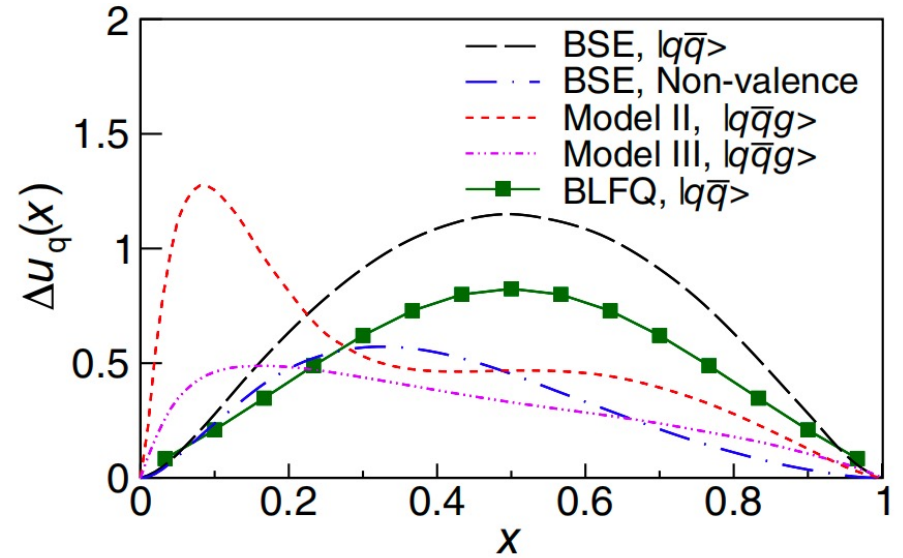
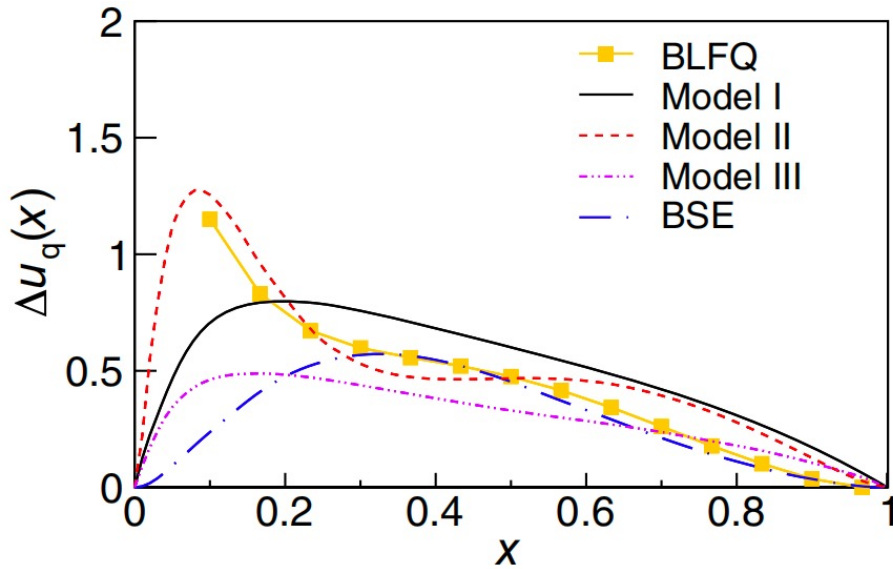
$$\beta/m_q = 1.16$$

BLFQ: [PLB 825 (2022) 136890]

BSE: [PRD 103 (2021) 014002]

[J. Lan et al, Phys.Rev.D 111 (2025) 11, L111903]

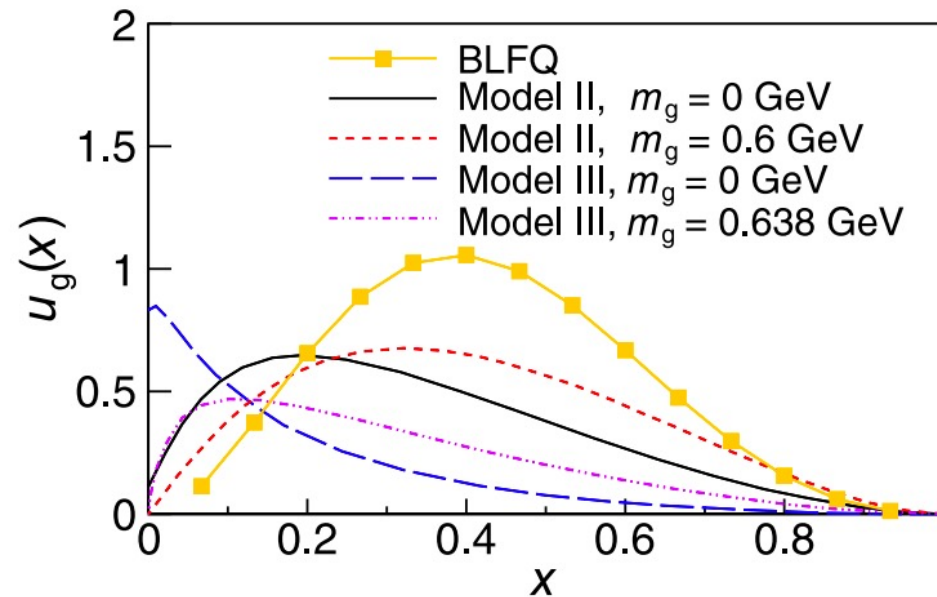
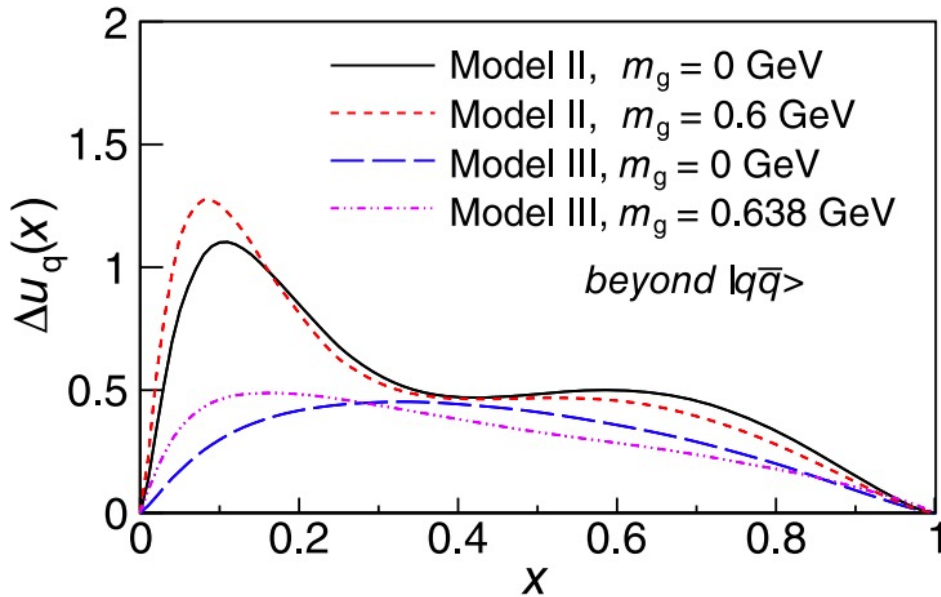
Pion $q\bar{q}g$ BLFQ .vs. BSE



- Model II: input from BLFQ [PLB 22'](#)
- Model III: input from BSE [PRD 21'](#)
- Model I .vs. II: bump is related to large value of m_f
- Model II agrees with BLFQ: a large bump at low- x is reproduced
- BSE result differs from Model III: BSE result contains contributions from $q\bar{q}ng$, where $n = 1, 2, \dots, \infty$
- $q\bar{q}g$ Fock sector is mostly important at small- x

Model	m_q [GeV]	m_f [GeV]	m_g [GeV]	$1-P_{val}$
I	0.390	0.390	0.600	0.508
II	0.390	5.69	0.600	0.508
III	0.255	0.255	0.638	0.300

Pion $q\bar{q}g$ BLFQ .vs. BSE



Impact of m_g on pion PDFs:

- Model II: input from BLFQ [PLB 22'](#)
- Model III: input from BSE [PRD 21'](#)
- An increase of m_g leads to a shift of quark PDF to lower values of x
- A larger m_g gives a gluon PDF shifted towards larger- x
- Perturbative results agree qualitatively with the BLFQ

Model	m_q [GeV]	m_f [GeV]	m_g [GeV]	$1-P_{val}$
I	0.390	0.390	0.600	0.508
II	0.390	5.69	0.600	0.508
III	0.255	0.255	0.638	0.300

Conclusion & Outlook

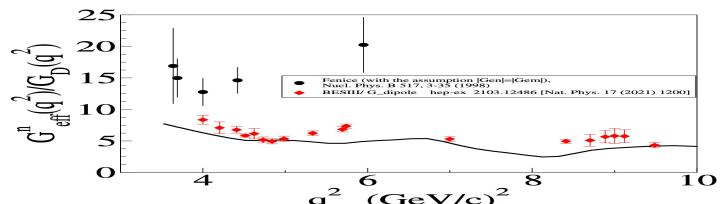
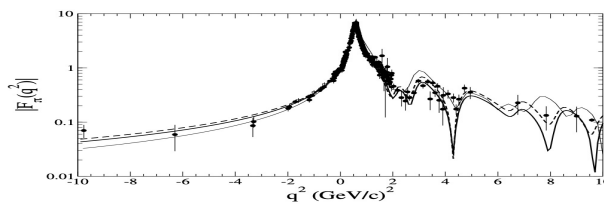
- BLFQ : A non-perturbative approach based on light-front QCD Hamiltonian
- LF QCD Hamiltonian $\Rightarrow$ Wave functions $\Rightarrow$ Observables
- Solved for light mesons structure based on $q\bar{q} + q\bar{q}g$ Fock components, EMFF, PDF, TMD, GPD...
- $\pi - \rho$ mass splitting $\sim$ a large spin-flip matrix element
 $\sim$ a peak at low- x for quark PDF

Plan 1

$$|\text{Meson}\rangle = |q\bar{q}\rangle + |q\bar{q}g\rangle + |q\bar{q}q\bar{q}\rangle + |q\bar{q}gg\rangle + |q\bar{q}q\bar{q}g\rangle + |q\bar{q}ggg\rangle + |gg\rangle + \dots$$

Plan 2

Combine different perturbative methods $\Rightarrow$ Timelike Physics, ...



with Tobias, Pacheco, Xiaorong, ..., in future



Good health!

Smooth work!

Happy New Year!

Thank you!