Hadronic Physics within Light-front approach

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Light-Front Motivations

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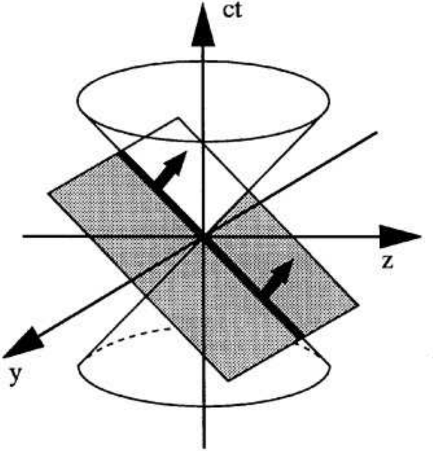
- After Integrate in k^-: Bethe-Salpeter Amplitude (Wave Function)
- LF Lorentz Invariant Hamiltonian: $P^2 = P^+P^- - P_\perp^2$



Light-cone

Global Electron-Ion Colliders: EIC & EIcC

- **The Electron-Ion Collider (EIC - US)**
  - **Mission:** Precisely map the 3D structure of the nucleon (Hadrons/proton/nucleus)
  - **Key Physics:** Understanding the source of nucleon spin and the role of gluons at high energy (α_s)
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- **Collective Impact:** Together, EIC and EIcC will define the next generation of Quantum Chromodynamics (QCD) research, focusing on the fundamental structure of visible matter.

• Light-Front Coordinates

Four-Vector $\implies$ $x^\mu = (x^0, x^1, x^2, x^3) \; = \; (x^+, x^-, x_\perp)$

$x^+ = t + z$ $\quad$ $x^+ = x^0 + x^3$ $\implies$ **Time**

$x^- = t - z$ $\quad$ $x^- = x^0 - x^3$ $\implies$ **Position**

Scalar product

$$x \cdot y = x^\mu y_\mu = x^+ y_+ + x^- y_- + x^1 y_1 + x^2 y_2 = \frac{x^+ y^- + x^- y^+}{2} - \vec{x}_\perp \vec{y}_\perp$$

Metric Tensor:

$$g^{\mu\nu} = \begin{pmatrix} 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \; and \; g_{\mu\nu} = \begin{pmatrix} 0 & 1/2 & 0 & 0 \\ 1/2 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Scalar product

$$x \cdot y = x^{\mu} y_{\mu} = x^{+} y_{+} + x^{-} y_{-} + x^{1} y_{1} + x^{2} y_{2} = \frac{x^{+} y^{-} + x^{-} y^{+}}{2} - \vec{x}_{\perp} \vec{y}_{\perp}$$

$p^{+} = p^{0} + p^{3}\ ,\ p^{-} = p^{0} - p^{3}\ ,\ p^{\perp} = (p^{1}, p^{2})$

$$p_{1}^{\mu} p_{2\nu} = \frac{p_{1}^{+} p_{2}^{-} + p_{1}^{-} p_{2}^{+}}{2} - \vec{p_{1}}_{\perp} \vec{p_{2}}_{\perp}$$

Dirac Matrix and Electromagnetic Current

$\gamma^+ = \gamma^0 + \gamma^3 \implies$ **Electr. Current** $J^+ = J^0 + J^3$
$\gamma^- = \gamma^0 - \gamma^3 \implies$ **Electr. Current** $J^- = J^0 - J^3$
$\gamma^\perp = (\gamma^1, \gamma^2) \implies$ **Electr. Current** $J^\perp = (J^1, J^2)$

$$p^\mu x_\mu = \frac{p^+x^- + p^-x^+}{2} - \vec{p}_\perp \vec{x}_\perp$$

$$x^+, x^-, \vec{x_\perp} \implies p^+, p^-, \vec{p}_\perp$$

$p^- \implies$ **Light-Front Energy**

$$p^2 = p^+ p^- - (\vec{p}_\perp)^2 \implies \quad p^- = \frac{(\vec{p}_\perp)^2 + m^2}{p^+}$$

On-shell

Bosons $\implies \quad S_F(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$

Fermions $\implies \quad S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + \imath\epsilon} + \frac{\gamma^+}{2p^+}$

Review Papers:

- **Phys. Rept. 301, (1998) 299-486, Brodsky, Pauli and Pinsky**
- **A. Harindranath, Pramana, Journal of Indian Academy of Sciences Physics Vol.55, Nos 1 & 2, (2000) 241.**
- **An Introduction to Light-Front Dynamics for Pedestrians Avaroth Harindranath**

Light-Front book organizers: **James Vary and Frank Wolz,(1997)**

Propagadors in Light-Front

$$p^2 = p^+ p^- - (\vec{p}_\perp)^2 \Longrightarrow \quad p^- = \frac{(\vec{p}_\perp)^2 + m^2}{p^+}$$ **(Light-Front Energy)**

On-shell **Bosons** $\Longrightarrow \quad S_B(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$

Fermions $\Longrightarrow \quad S_F(p) = \frac{\not{p} + m}{p^2 - m^2 + \imath\epsilon} + \frac{\gamma^+}{2p^+}$

- **Why?**

$$\frac{\not{p} + m}{p^2 - m^2 + \imath\epsilon} = \frac{1}{2}\frac{\gamma^+ p^- + \gamma^- p^+}{p^+\left(p^- - \frac{\vec{p}_\perp^2 + m^2 - \imath\epsilon}{p^+}\right)} - \frac{\gamma_\perp p_\perp + m}{p^+\left(p^- - \frac{\vec{p}_\perp^2 + m^2 - \imath\epsilon}{p^+}\right)}$$

$$= \frac{1}{2}\frac{\gamma^+ (p^- - \bar{p}^-) + \gamma^- p^+}{p^+\left(p^- - \frac{\vec{p}_\perp^2 + m^2 - \imath\epsilon}{p^+}\right)} - \frac{\gamma_\perp p_\perp + m}{p^+\left(p^- - \frac{\vec{p}_\perp^2 + m^2 - \imath\epsilon}{p^+}\right)} + \frac{\gamma^+ \bar{p}^-}{p^+\left(p^- - \frac{\vec{p}_\perp^2 + m^2 - \imath\epsilon}{p^+}\right)}$$

- **Result:**

$$S(p) = \frac{\not{p} + m}{p^2 - m^2 + \imath\epsilon} + \frac{\gamma^+}{2p^+}$$

Problem: Broken some symmetries

- **Equal time integral:**

$$\int \frac{d^4k}{(k^2 - m^2 + i\epsilon)^3} = -\frac{i\pi^2}{2m^2} \neq 0$$

$\rightrightarrows \twoheadrightarrow \quad d^4k = dk_0 dk_1 dk_2 dk_3$

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- **Light-front integral:**

$$\int \frac{dk^- dk^+ d^2k_\perp}{(k^+k^- - k_\perp^2 - m^2 + i\epsilon)^3} = 0, \ \text{Why?}$$

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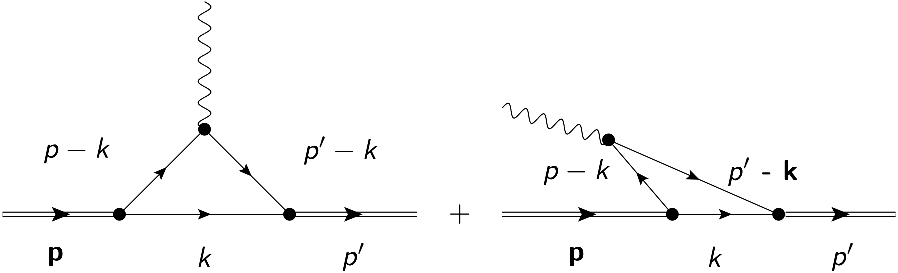
- **Light-front integral:**

$$\int \frac{dk^- dk^+ d^2k_\perp}{(k^+k^- - k_\perp^2 - m^2 + i\epsilon)^3} = 0, \ \text{Why?}$$

- **You'll soon learn to use Cauchy's Theorem and obtain zero in this case!**
- **Parte of the Answer:Take the zero-modes contributions!**



Feynman diagramms for the valence contribution (left panel) and the non-valence contribuition (right panel) for the electromagnetic current

Solution: Pole Dislocation Method

$p^+ \quad \Longrightarrow \quad p'^+ = p^+ + \delta$

Boson Eletromagnetic Current

Breit Frame$\Longrightarrow \quad q^- = 0,\ q^+ \Longrightarrow 0_+,\ \vec{q}_\perp \neq 0$

$J^+ = J^- +$ **restoration covariance term**

$J_\perp \propto q^+ \Rightarrow 0$

Ref:(More Details)

- **Boson Case** $\Longrightarrow$ **de Melo, Sales and Frederico, Nucl. Phys. B631, (1998) 574c-579c**
- Ward-Takahashi Identity $\Longrightarrow$ **Pair Contribuition, Naus, de Melo and Frederico, Few-Body Syst. 24, 1998, 99-107**

Motivations

- Hadronic Form factors: **Important Sources Informations for Hadrons Strucuture**

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- **Pion Electromagnetic Form Factors:** $F_1(Q^2, t)$ and $F_2(Q^2, t)$

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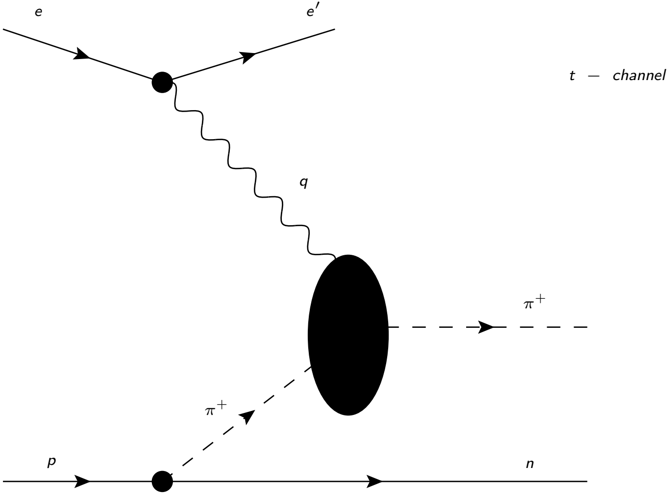
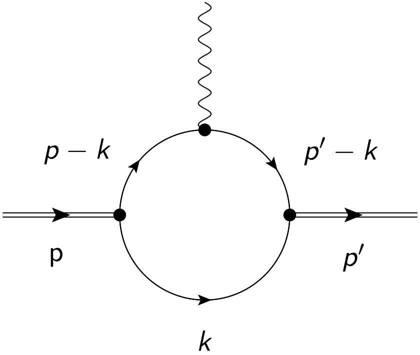
  $\Longrightarrow$ **Extracted from Cross sections**

Data from Experiments: Electroproduction



Electroproduction: Diagrammatic representation of the pion pole contribution to $p(e,e')\pi^+ n$ process
The black blob represents the half-on-mass shell photo absorption amplitude

Electromagnetic Form Factors



Feynman triangular diagram representing the matrix element of the pion EM current within the Mandelstam framework

Electromagnetic Form Factors

$\Longrightarrow$ **Form factors are essential for our understanding of internal hadron structure and the dynamics**

- **On-shell Case:** $\Longrightarrow \; F_1(Q^2, t)$
- **Off-shell Case:**

$\Longrightarrow$ **Two Electromagnetic Form Factors:** $F_1(Q^2, t)$ and $F_2(Q^2, t)$

- **Structure:**

$$< p'|\mathcal{O}|P >= (p' + p)^\mu F_1(Q^2, t) + (p' - p)^\mu F_2(Q^2, t)$$

Effective Lagrangian for the Vertex

- **Coupling of quarks to the pseudoscalar meson field with SU(4) flavor symmetry**

$$\mathcal{L}_I = -\imath g \bar{\Psi} M_{\mathrm{SU(4)}} \gamma^5 \Psi \equiv -\imath \frac{g}{\sqrt{2}} \sum_{i=1}^{15} \left(\bar{\Psi} \lambda_i \gamma^5 \Psi \right) \varphi^i$$

- g **is the coupling constant**
- $\lambda_i (i = 1, \ldots, 15)$ **are the SU(4) matrices (Gell-Mann)**
- φ^i **are the Cartesian components of the pseudoscalar meson fields**
- **Quark fields (**$\Psi^T = (u, d, s, c)$ **(**T**: transpose), decomposed into their quark-flavor components**
- **SU(4), pseudoscalar matrix**

- **Mesons:**

$$\pi^+ = \mathrm{Tr}\left[M_{SU(4)}\lambda_{\pi^+}\right], \quad K^+ = \mathrm{Tr}\left[M_{SU(4)}\lambda_{K^+}\right],$$
$$D^+ = \mathrm{Tr}\left[M_{SU(4)}\lambda_{D^+}\right], \quad D_s^+ = \mathrm{Tr}\left[M_{SU(4)}\lambda_{D_s^+}\right],$$

- **Flavor Matrices**

$$\lambda_{\pi^+} = \frac{1}{\sqrt{2}}(\lambda_1 + \imath\lambda_2), \quad \lambda_{K^+} = \frac{1}{\sqrt{2}}(\lambda_4 + \imath\lambda_5),$$
$$\lambda_{D^+} = \frac{1}{\sqrt{2}}(\lambda_{11} - \imath\lambda_{12}), \quad \lambda_{D_s^+} = \frac{1}{\sqrt{2}}(\lambda_{13} - \imath\lambda_{14}),$$

- **Bethe-Salpeter Amplitude**

$$\Psi_M(k,p) = S_q(k)\gamma^5 g\Lambda_M(k,p)\lambda_M S_q(k-p),$$

- **Quark Propagator**

$$S_q(k) = i\left[\not{k} - \hat{m}_q + \imath\epsilon\right]^{-1}, \quad \mathrm{diag}[\hat{m}_q] = [m_u, m_d, m_s, m_c]$$

- **Vertex for the Pseudoscalar Mesons:** $M = (\pi^+, K^+, D^+, D_s^+)$

$$g\Lambda_M(k,p) = \frac{C_M}{k^2 - \mu_M^2 + \imath\epsilon} + [k \to p - k]$$

- **Infrared (IR) dynamics is at the mass scale** μ_M
- **Ultraviolet (UV) physics is in the analytic vertex**
- **Constant** C_M**: Normalization (BS)**

$$\begin{aligned} 2ip^\mu =& N_c \,\mathrm{Tr} \int \frac{d^4k}{(2\pi)^4} g^2 \Lambda_M^2(k,p) \\ & \times \Big[\gamma^5 \lambda_M S_q(k-p) \gamma^\mu S_q(k-p) \gamma^5 \lambda_M^\dagger S_q(k) \\ & + \gamma^5 \lambda_M^\dagger S_q(k+p) \gamma^\mu S_q(k+p) \gamma^5 \lambda_M S_q(k)\Big] \end{aligned}$$

Ψ_M is the momentum part of the antialigned quark spin component of the pseudoscalar meson valence wave function (after k^- integration)

$$\psi_M(x,\vec{k}_\perp;p^+,\vec{p}_\perp) = \frac{p^+}{m_M}\frac{g\,C_M}{m_M^2-\mathcal{M}^2(m_q,m_{\bar{q}})} \times\left[\frac{1}{(1-x)(m_M^2-\mathcal{M}^2(m_q,\mu_M))} + \frac{1}{x(m_M^2-\mathcal{M}^2(\mu_M,m_{\bar{q}})}\right] + [m_q \leftrightarrow m_{\bar{q}}]$$

where, $x=\frac{k^+}{p^+}$, $0<x<1$, and

$$\mathcal{M}^2(m_1,m_2) = \frac{|\vec{k}_\perp|^2+m_1^2}{x} + \frac{|\vec{p}_\perp-\vec{k}_\perp|^2+m_2^2}{1-x} - |\vec{p}_\perp|^2$$

Observables

- **Decay Constant:** The **Axial Current Operator**

$$\langle 0 | A^j_\mu \left| M^k \right\rangle = \imath p_\mu f_{M^k} \delta^{jk}$$

- $A^j_\mu = \bar{q}(0)\gamma_\mu \gamma^5 \frac{\lambda_j}{2} q(0)$ is the **axial current**.
The indices j and k identify the isospin (flavor) components of the current operator and the pseudoscalar meson

$$\imath\, p^\mu f_M = N_c \int \frac{d^4 k}{(2\pi)^4} \frac{1}{2} \mathrm{Tr}\left[\gamma^\mu \gamma^5 \lambda^\dagger_M \Psi(k,p)\right]$$

$$f_M = \frac{N_c}{4\pi^3} \int d^2 k_\perp \int_0^1 dx\, \psi_M(x, \vec{k}_\perp; m_M, \vec{0}_\perp)$$

- **Electromagnetic Radius:** $\langle r^2 \rangle = 6 \frac{\partial F_M(Q^2)}{\partial q^2}\Big|_{q^2 \sim 0}$

- **Electromagnetic Form Factors**

$$\langle p'; M | j^\mu | p; M \rangle = (p'^\mu + p^\mu) F_M(q^2)$$

- **Flavor Decomposition: Sum of two components (traces)**

$$\mathbf{Tr}[\ ] = 2\left(\frac{2}{3}\Delta^\mu_{a\bar{b}a} + \frac{1}{3}\Delta^\mu_{\bar{b}a\bar{b}}\right),$$

$$\Delta^\mu_{aba} = \mathbf{Tr}\left[\gamma^5 S^a_q(k-p)\,\gamma^\mu S^a_q(k-p') S^b_q(k)\gamma^5\right],$$

- **Current with quark flavor content**

$$\begin{aligned}\langle p'; M | j^\mu | p; M \rangle &= -2iN_c \int \frac{d^4k}{(2\pi)^4} g\Lambda_M(k,p') g\Lambda_M(k,p) \\ &\times \left(\frac{2}{3}\Delta^\mu_{a\bar{b}a} + \frac{1}{3}\Delta^\mu_{\bar{b}a\bar{b}}\right)\end{aligned}$$

Flavor Decomposition: Electromagnetic Form Factor

- **The photon interacts with the quark a in the first term and with the antiquark $\bar{b}$ in the second one.**

$$
\begin{aligned}
F_{\pi^+}(q^2) &= \frac{2}{3}F_{u\bar{d}u}(q^2) + \frac{1}{3}F_{\bar{d}u\bar{d}}(q^2)\,, \\
F_{K^+}(q^2) &= \frac{2}{3}F_{u\bar{s}u}(q^2) + \frac{1}{3}F_{\bar{s}u\bar{s}}(q^2)\,, \\
F_{D^+}(q^2) &= \frac{2}{3}F_{c\bar{d}c}(q^2) + \frac{1}{3}F_{\bar{d}c\bar{d}}(q^2)\,, \\
F_{D_s^+}(q^2) &= \frac{2}{3}F_{c\bar{s}c}(q^2) + \frac{1}{3}F_{\bar{s}c\bar{s}}(q^2)
\end{aligned}
$$

- **Condition (charge conservation):**

$$
\begin{aligned}
F_{u\bar{s}u}(0) = F_{\bar{s}u\bar{s}}(0) = 1, \\
F_{c\bar{c}c}(0) = F_{\bar{d}c\bar{d}}(0) = 1, \\
F_{c\bar{s}c}(0) = F_{\bar{s}c\bar{s}}(0) = 1.
\end{aligned}
$$

- **Elastic photo-absorption transition amplitude**
- **Breit Frame**

  with the choice of initial and final meson four-momentum

$$p^\mu = (p_0, p_x, 0, -\frac{q\sin\alpha}{2}) \quad \text{and} \quad p'^\mu = (p_0, p'_x, 0, \frac{q\sin\alpha}{2})$$

$$p_\perp = (-q\cos\alpha/2, 0), \; p'_\perp = (q\cos\alpha/2, 0), \; q^\mu = (0, q\cos\alpha, 0, q\sin\alpha)$$

$$\Longrightarrow \quad q^+ = 0, \; \underline{\textbf{Drell-Yan Condition: } \alpha = 0^\circ}$$

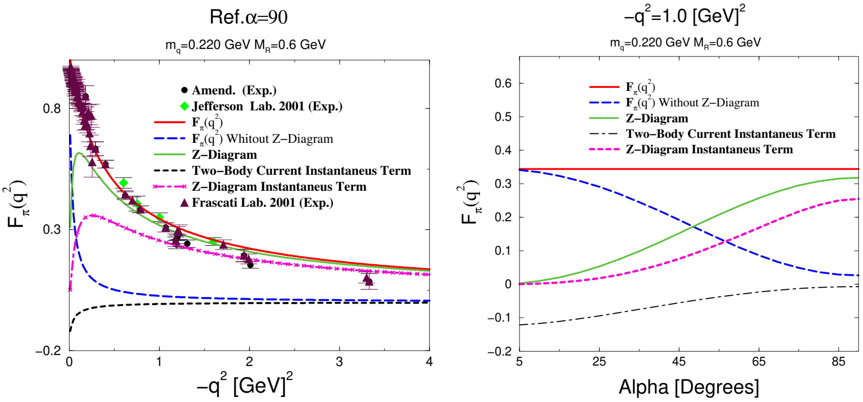
- **J^+ component of the electromagnetic current:** $J^+ = J^0 + J^3$
- **Current associated with $\gamma^+ = \gamma^0 + \gamma^3$**
- **Electromagnetic Form Factor:** $F_M(q^2) = \frac{1}{2p^+}\langle p'|j_M^+|p\rangle$

VIP!! Binding Energy $\Longrightarrow$ $\epsilon_M = m_a + m_{\bar{a}} - m_M > 0$

•**And the Cutkosky rules (see for exemplo, the book by Itzykson and Zuber (2012)** applied to the triangle diagram relevant cuts as function of m_M that have branch points in the regions

$$\mu_M + m_q - m_M > 0 \quad \text{and} \quad \mu_M + m_{\bar{q}} - m_M > 0$$

These branch points are also clear in the analytic form of
for the wave function. The minimum value of the position of the branch point is actually the dominant scale that determines the charge radius.



Pion Electromagnetic Form Factor

- **de Melo, Frederico, Pace and Salmé, Nucl.Phys.A 707 (2002) 399**

Pseudoscalar meson observables (Models A-F)*

Model	Flav.	$I(J^P)$	m_q [MeV]	$m_{\bar{q}}$ [MeV]	m_M [MeV]	ϵ_M [MeV]	μ_M [MeV]	r_M [fm]	f_M [MeV]	r_M^{Exp} [fm]	f_M^{Exp} [MeV]
(A) π^+	$u\bar{d}$	$1(0^-)$	384	384	140	628	225	0.665	92.55	0.672(8)	92.28(7)
(B)			220	220	–	300	600	0.736	92.12	–	–
(C) K^+	$u\bar{s}$	$\frac{1}{2}(0^-)$	384	508	494	398	420	0.551	110.8	0.560(3)	110(1)
(D)			220	440	–	166	600	0.754	110.8	–	–
(E) D^+	$c\bar{d}$	$\frac{1}{2}(0^-)$	1623	384	1869	138	1607	0.505	144.5	–	144(3)
(F) D_s^+	$c\bar{s}$	$0(0^-)$	1623	508	1968	163	1685	0.377	182.7	–	182(3) 179(5)

Models B, D from Ref [1]; A, C, E, F from Ref [2]. Exp data from [3,4].

Ref.

[1] G. H. S. Yabusaki et al.Phys. Rev. D 92, 034017 (2015)
[2] E. Suisso, de Melo, and Frederico, Phys. Rev. D 65, 094009 (2002)
[3] P. A. Zyla et al. (Particle Data Group), PTEP 2020, 083C01 (2020).
[4] M. Ablikim et al. (BESIII), Phys. Rev. Lett. 122, 071802 (2019)
[*] Moita, de Melo, Tsushima and Frederico, Phys.Rev.D 104 (2021) 9, 096020

Comparison of Decay Constants and e.m. Radius*

Decay constants [MeV] and radius [fm] for π^+ and K^+.

Reference	f_{π^+}	f_{K^+}	r_{π^+}	r_{K^+}	f_{K^+}/f_{π^+}
This work*	92.55	110.8	0.665	0.551	1.196
Maris & Tandy	92.62	109.60	0.671	0.615	1.182
Ebert et al.	109.60	165.45	0.66	0.57	1.24
Bashir et al.	101	–	–	–	–
Chen & Chang	93	111	–	–	1.192
Hutauruk et al	93	97	0.629	0.586	1.043
Ivanov et al.	92.14	111.0	–	–	1.20
Silva et al.	101	129	0.672	0.710	1.276
Jia & Vary	142.8	166.7	0.68(5)	0.54(3)	1.166
PDG	**92.28(7)**	**110(1)**	**0.672(8)**	**0.560(3)**	**1.192(14)**

[*] With models [A - Pion, and C - Kaon]
Moita, de Melo, Tsushima and Frederico,
Phys.Rev.D 104 (2021) 9, 096020

Partial ratios for the electromagnetic form factors of pion (A) and kaon (C) at 10 GeV^2, compared with the NJL model [*].

Model	$\frac{F_{u\bar{s}u}}{F_{u\bar{d}u}}$	$\frac{F_{\bar{s}u\bar{s}}}{F_{\bar{d}u\bar{d}}}$	$\frac{e_{\bar{s}} F_{\bar{s}u\bar{s}}}{e_u F_{u\bar{s}u}}$
This work	0.80	1.10	0.69
NJL [*]	0.36	2.74	0.56

[*] P. T. P. Hutauruk, I. C. Cloët, and A. W. Thomas, Phys. Rev. C 94, 035201 (2016).

Ratio of the EFF for pion (A) and kaon (C), compared with calculations and experimental time-like data

Reference	Q^2 [(GeV/c)^2]	F_{π^+}/F_{K^+}
This work	10.0	1.10
	13.48	1.14
	14.2	1.13
	17.4	1.16
Hutauruk et al.[1]	10.0	0.87
Bakulev et al. [2]	13.48	0.53
Shi et al. [3]	17.4	0.81
Pedlar et al.[4]	13.48	1.19(17)
Seth et al. [5]	14.2	1.21(3)
Seth et al. [5]	17.4	1.09(4)

[1] P. T. P. Hutauruk, I. C. Cloët, and A. W. Thomas, Phys. Rev. C 94, 035201 (2016)
[2] A. P. Bakulev, S. V. Mikhailov, and N. G. Stefanis, Phys. Lett. B 508, 279 (2001), [Erratum: Phys.Lett.B 590, 309–310 (2004)]
[3] C. Shi, L. Chang, C. D. Roberts, S. M. Schmidt, P. C. Tandy, and H.-S. Zong, Phys. Lett. B 738, 512 (2014),
[4] T. K. Pedlar et al. (CLEO), Phys. Rev. Lett. 95, 261803 (2005)
[5] K. K. Seth, S. Dobbs, Z. Metreveli, A. Tomaradze, T. Xiao, and G. Bonvicini, Phys. Rev. Lett. 110, 022002 (2013)

Vector meson dominance

$$F_{\pi^+}(q^2) = (e_u + e_{\bar{d}})\frac{m_\rho^2}{m_\rho^2 - q^2} ,$$

$$F_{K^+}(q^2) = e_u \frac{m_\rho^2}{m_\rho^2 - q^2} + e_{\bar{s}} \frac{m_\phi^2}{m_\phi^2 - q^2} ,$$

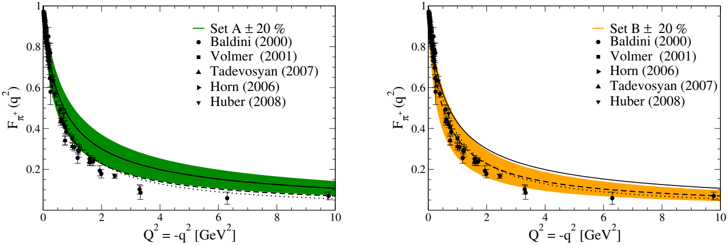
$\Rightarrow$ **Assume SU(3) flavor symmetry:** $g_{\rho,u} = g_{\rho,d} = g_{\phi,s}$

PDG $\begin{cases} m_\rho = 775.26 \pm 0.25 \textbf{ MeV} \\ m_\phi = 1019.461 \pm 0.016 \textbf{ MeV} \end{cases}$

Ref.

• Sakurai, Annals Phys. 11, 1 (1960), and Sakurai, Currents and Mesons (University of Chicago Press, 1969).

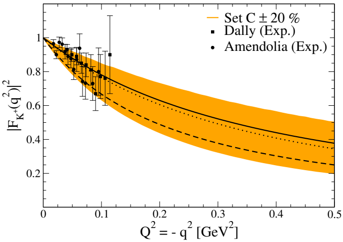
• H. B. O'Connell, B. C. Pearce, A. W. Thomas, and A. G. Williams, Prog. Part. Nucl. Phys. 39, 201 (1997)

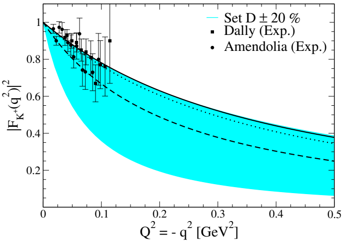


Pion electromagnetic form factor: Left panel: band for set (A) with $\pm 20\%$ variation of the model parameters. Right panel: band for set (B) with $\pm 20\%$ variation of the model parameters. In both panels: set (A) (solid line), set(B) (dashed line) and VMD (dotted line).

Ref. [Exp.]

- R. Baldini et al. Phys. A 666, 38 (2000).
- J. Volmer et al. Phys. Rev. Lett. 86, 1713 (2001).
- T. Horn et al., Phys. Rev. Lett. 97, 192001 (2006).
- V. Tadevosyan, H. Blok, G. Huber et al,Phys. Rev. C 75, 055205 (2007).
- G. Huber, H. Blok, T. Horn, et al,Phys. Rev. C 78, 045203 (2008).
- E. B. Dally et al., Phys. Rev. Lett. 45, 232 (1980).





Kaon electromagnetic form factor square with variation of the model parameters. Left: band for set (C) with $\pm 20\%$ variation of the model parameters. Right: band for set (D) with $\pm 20\%$ variation of the model parameters. In both panels: set (C) (solid line), set(D) (dashed line) and VMD (dotted line).

Experimental data from Refs. Dally and Amendolia.

Ref.

- **E. B. Dally et al., Phys. Rev. Lett. 45, 232 (1980).**
- **S. R. Amendolia et al., Phys. Lett. B 178, 435 (1986).**

D mesons: D^+ and D_s^+

Reference	f_{D^+} [MeV]	$f_{D_s^+}$ [MeV]	$r^2_{D^+}$ [fm^2]	$r^2_{D_s^+}$ [fm^2]	$f_{D_s^+}/f_{D^+}$
This work	**144.5**	**182.7**	**0.505**	**0.377**	**1.265**
Bashir et al.	155.4	205.1	–	–	1.32
Ivanov et al.	145.7	182.2	–	–	1.25
Choi	149.2	179.6	–	–	1.20
Hwang	145.7(6.3)	189(13)	0.406	0.300	1.30(4)
Das et al.	–	–	0.510	0.465	–
Dhiman & Dahiya	147.8	167.6	–	–	1.13
Tang et al.	295(63)	313(67)	–	–	1.06(32)
LQCD					
Aubin et al.	142(14)	176(13)	–	–	1.24(8)
Follana et al.	147(3)	170(2)	–	–	1.16(3)
Chen et al.	143.1(2.4)	182.9(2.2)	–	–	1.28(3)
Carrasco et al.	146.6(2.7)	174.8(3.0)	–	–	1.19(3)
Can et al.	–	–	0.371(17)	–	–
Li & Wu	–	–	0.402(61)	0.286(19)	–
PDG (Exp.)	**144(3)**	**182(3)**	–	–	**1.26(5)**
Ablikin et al. (Exp.)	–	178.8(2.6)	–	–	–

- Ref. **Moita, de Melo, Tsushima and Frederico Phys.Rev.D 104 (2021) 9, 096020**

Pseudoscalar meson static observables with the masses for D and D_s mesons from LQCD ensembles (B1), (C1) used in Refs.[*] to compute the EM form factors; and the present model (E,F) parameters: $m_c = 1623$ MeV, $m_u = 384$ MeV, $m_s = 508$ MeV, $\mu_{D^+} = 1607$ MeV, and $\mu_{D_s^+} = 1685$ MeV.

Inputs	(B1)	(C1)	(E,F)
m_D^+	1737	1824	1869
$m_{D_s^+}$	1801	1880	1968
Static observ.	(B1)	(C1)	(E,F)
r_{D^+} [*]	0.402(61)	0.420(82)	
r_{D^+}	0.347	0.422	0.505
f_{D^+}	205.5	170.3	144.5
$r_{D_s^+}$ [*]	0.286(19)	0.354(18)	
$r_{D_s^+}$	0.281	0.312	0.377
$f_{D_s^+}$	243.3	219.4	182.7

- Masses in [MeV] and radius in [fm]

[*] (LQCD) N. Li and Y.-J. Wu, Eur. Phys. J. A 53, 56 (2017).

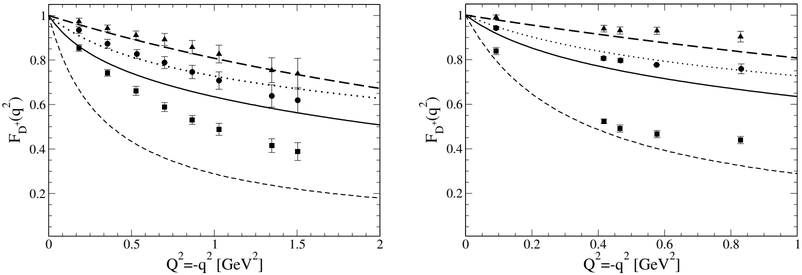
Vector dominance for D^+ and D_s^+

$$
\begin{aligned}
F_{D^+}(q^2) &= \frac{2}{3}\frac{m^2_{J/\psi}}{m^2_{J/\psi}-q^2}+\frac{1}{3}\frac{m^2_{\rho}}{m^2_{\rho}-q^2}\,, \\
F_{D_s^+}(q^2) &= \frac{2}{3}\frac{m^2_{J/\psi}}{m^2_{J/\psi}-q^2}+\frac{1}{3}\frac{m^2_{\phi}}{m^2_{\phi}-q^2}\,.
\end{aligned}
$$

$$
PDG \begin{cases} m_\rho = 775.26 \pm 0.25 \ \textbf{MeV} \\ m_\phi = 1019.461 \pm 0.016 \ \textbf{MeV} \\ m_{j/\psi} = 3096.900 \pm 0.006 \textbf{MeV} \end{cases}
$$

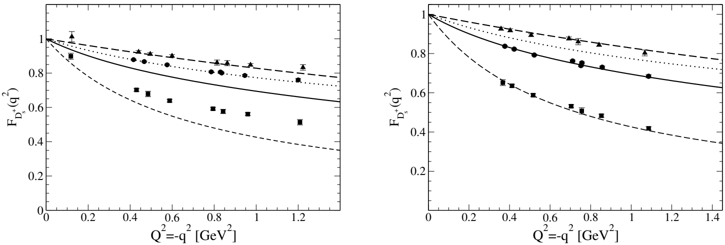
The VMD model form factor gives for D^+ a charge radius of $r_{D^+} = 0.381$ fm and for D_s^+ it gives $r_{D_s^+} = 0.302$ fm.

Close to the present model and also to the lattice results



D^+ electromagnetic form factor with the corresponding quark contributions and comparison with lattice calculations. Our results: D^+ form factor (full line), $\bar{d}$ contribution — $e_{\bar{d}} F_{\bar{d}c\bar{d}}$ (short-dashed) and c contribution — $e_c F_{c\bar{d}c}$ (dashed). VMD (dotted line). LQCD results: D^+ (circles), $\bar{d}$ contribution (squares) and c contribution (triangles). [Left]: Comparison with LQCD Ref. [1], [Right]: LQCD Ref. [2].

Ref.: [1] K. U. Can, G. Erkol, M. Oka, A. Ozpineci, and T. T. Takahashi, Phys. Lett. B 719, 103 (2013).
[2] N. Li and Y.-J. Wu, Eur. Phys. J. A 53, 56 (2017).



Present model: D_s^+ form factor (full line), $\bar{d}$ contribution (short-dashed line) and c contribution (dashed line), VMD (dotted line). LQCD results for D_s^+ form factor (circles), $\bar{s}$ contribution (squares) and c contribution (triangles). Left panel: comparison with LQCD results from ensemble (B1) [**]. Right panel: comparison with LQCD results from ensemble (C1) [**].

[**] N. Li and Y.-J. Wu, Eur. Phys. J. A 53, 56 (2017).

Off-shell form factors

- **Most General Structure**

$$\Gamma_\mu = e[P^\mu \; G_1(q^2, p^2, p') + q^\mu \; G_2(q^2, p^2, p')]$$

- $P^\mu = (p'^\mu + p^\mu)$, and, $q^u = (p'^\mu - p^\mu)$

- $\Longrightarrow \quad G_1 = F_1(p^2, p'^2, q^2)$ **and** $G_2 = F_2(p^2, p'^2, q^2)$

- **Ward-Takahashi Identity-WTI**

$$q^\mu \Gamma_\mu = e\Delta_0^{-1}[\Delta(p) - \Delta(p')]\Delta_0^{-1}(p),$$

where, $\Delta_0(p) = \frac{1}{p^2 - m^2 + \imath\epsilon}$, **and,** $\Delta(p) = \frac{1}{p^2 - m^2 - \Pi(p^2) + \imath\epsilon}$

- **Assuming (Standard renormalization)**

$$\Longrightarrow \quad \{ \quad \Pi(m_\pi^2) = 0$$

- **From the WTI , we have,**

$$(p'^2 - p^2)G_1(q^2, p^2, p'^2) + q^2 G_2(q^2, p^2, p'^2)$$
$$= \Delta^{-1}(p') - \Delta^{-1}(p)$$

- **For the reaction** $^1\mathrm{H}(e, e'\pi^+)n$

- **From the WTI , we have,**

$$(p'^2 - p^2)G_1(q^2, p^2, p'^2) + q^2 G_2(q^2, p^2, p'^2)$$
$$= \Delta^{-1}(p') - \Delta^{-1}(p)$$

- **For the reaction** ${}^1\mathrm{H}(e, e'\pi^+)n$

⋆ **The Final state pion on-mass-shell** $\Longrightarrow$ $p'^2 = m_\pi^2$, **with** $\Delta^{-1}(p') = 0$

- **From the WTI , we have,**

$$(\mathrm{p}'^2 - \mathrm{p}^2)\mathrm{G}_1(\mathrm{q}^2, \mathrm{p}^2, \mathrm{p}'^2) + \mathrm{q}^2\mathrm{G}_2(\mathrm{q}^2, \mathrm{p}^2, \mathrm{p}'^2)$$
$$= \Delta^{-1}(\mathrm{p}') - \Delta^{-1}(\mathrm{p})$$

- **For the reaction** $^1\mathrm{H}(e, e'\pi^+)n$

⋆ **The Final state pion on-mass-shell** $\Longrightarrow$ $p'^2 = m_\pi^2$**, with** $\Delta^{-1}(p') = 0$

- **Real photon with** $q^2 = 0$

$$(p^2 - m_\pi^2)\, G_1(0, p^2, m_\pi^2) = \Delta^{-1}(p)$$

- **And,**

$$G_2(q^2, p^2, m_\pi^2)$$
$$= \frac{(m_\pi^2 - p^2)}{q^2}\left[G_1(0, p^2, m_\pi^2) - G_1(q^2, p^2, m_\pi^2)\right]$$

$F_2(Q^2, t)$ Off-Shell electromagnetic form factor

- **Pion initial state off-mass shell** $(p^2 = t)$
- **Pion Final state on-mass shell** $(p'^2 = m_\pi^2)$

  Then,

$$F_2(Q^2, t) = \frac{t - m_\pi^2}{Q^2} \left[F_1(0, t) - F_1(Q^2, t) \right]$$

- **With** $F_i(Q^2, t) \equiv G_i(q^2, t, m_\pi^2)$ $(i = 1, 2)$
- $Q^2 (= -q^2)$ **is the four-momentum transfer in the spacelike region.**
- **Also** $G_2(q^2 = -Q^2, m_\pi^2, m_\pi^2) = F_2(Q^2, m_\pi^2) = 0$
- **For both the initial and final mesons are on-shell**
- **Implicate** $\Longrightarrow$ **Conservation of the electromagnetic current**

Pion-Photon vertex

- **For half on-shell ($p'^2 = m_\pi^2$) and half off-shell ($p^2 = t$):**

$$\Gamma_\mu(p',p)|_{p'^2=m_\pi^2,p^2=t} = e\Big[(p'+p)^\mu F_1(Q^2,t) + (p'-p)^\mu \frac{(t-m_\pi^2)}{Q^2}\left(F_1(0,t) - F_1(Q^2,t)\right)\Big]$$

- **Pion charge normalization:** $\underline{\underline{F_1(Q^2=0,m_\pi^2) = G_1(0,m_\pi^2,m_\pi^2) = 1}}$

- In the electroproduction process, directly measuring the form factor $F_2(Q^2, t)$

$\Longrightarrow$ Is impractical due to the transversality of the electron current

- But $F_2(Q^2, t)$ tends to zero as $t \to m_\pi^2$

- However:

- **In the electroproduction process, directly measuring the form factor $F_2(Q^2, t)$**

$\Longrightarrow$ **Is impractical due to the transversality of the electron current**

- **But** $F_2(Q^2, t)$ **tends to zero as** $t \to m_\pi^2$

- **However:**

$\Longrightarrow$ **The ratio of** $F_2(Q^2, t)$ **to** $t - m_\pi^2$ **remains nonzero when** t **approaches** m_π^2

New electromagnetic form factor

- **New form factor $g(Q^2, t)$, defined**

$$g(Q^2,t) \equiv \frac{F_2(Q^2,t)}{t - m_\pi^2} = \frac{1}{Q^2}[F_1(0,t) - F_1(Q^2,t)]$$

- **Ref. H.-M. Choi, T. Frederico, C.-R. Ji, and J. P. B. C. de Melo, Pion off-shell electromagnetic form factors: Data extraction and model analysis, Phys. Rev. D 100, 116020 (2019).**
- **H.-M. Choi, C.-R. Ji, T. Frederico, and J. P. B. C. de Melo, 3D imaging of the pion off-shell electromagnetic form factors. Proc. Sci. LC2019 (2020) 035.**
- **J. Leão, J. de Melo, T. Frederico, H.M. Choi and C. -R. Ji Off-shell pion properties: Electromagnetic form factors and light-front wave functions, Phys.Rev.D 110 (2024) 7**

- **Master equation off-shell form factor sum rule****

$$F_1\left(Q^2,t\right) - F_1(0,t) + Q^2 g\left(Q^2,t\right) = 0$$

- **Derivative with respect to $Q^2 \Longrightarrow$ evolution equation:**

$$\frac{\partial}{\partial Q^2} F_1\left(Q^2,t\right) + g\left(Q^2,t\right) + Q^2 \frac{\partial g\left(Q^2,t\right)}{\partial Q^2} = 0.$$

**** Master Equation: Consequence of Ward-Takahashi identity**

In on-mass shell limit, $g(Q^2 = 0, m_\pi^2)$

$\implies \quad t = m_\pi^2$ **and at** $Q^2 = 0$

- $g(Q, t)$ **is connected with charge radius for on-shell pion EFF**

$$g(Q^2 = 0, m_\pi^2) = -\frac{\partial}{\partial Q^2} F_1(Q^2 = 0, m_\pi^2) = \frac{1}{6}\langle r_\pi^2 \rangle$$

- **On-mass shell solution for** $g(Q^2, m_\pi^2)$ **is given by**

$$g(Q^2, m_\pi^2) = \frac{1}{6}\langle r_\pi^2 \rangle + \alpha Q^2 + \cdots$$

- **The master equation allows extract both $F_1(Q^2, t)$ and $F_2(Q^2, t)$**

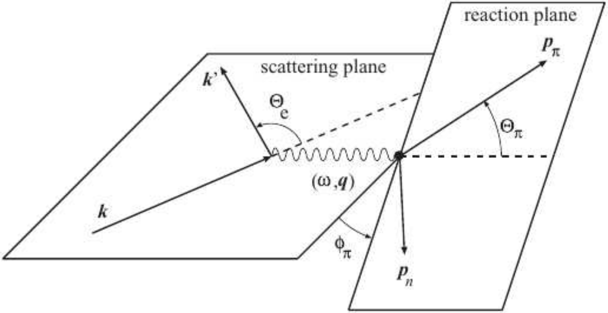
$\Longrightarrow$ **Electroproduction process cannot measure directly $F_2(Q, t)$!!!**

- The master equation allows extract both $F_1(Q^2, t)$ and $F_2(Q^2, t)$

$\Longrightarrow$ **Electroproduction process cannot measure directly $F_2(Q, t)$!!!**

- $g(Q^2, t)$ Is the new observable form factor

Cross-section



- **Figure from ref.** H. P. Blok et al.; Phys.Rev.C 78 (2008) 045202

- **Exclusive reaction:** ${}^{1}\mathrm{H}\,(e, e'\pi^{+})\,n$

$\Longrightarrow$ **Longitudinal (L), Transverse (T), and also interference terms (LT and TT)**

$$(2\pi)\frac{d^2\sigma}{dtd\phi} = \frac{d\sigma_{\mathrm{T}}}{dt} + \epsilon\frac{d\sigma_{\mathrm{L}}}{dt} + \sqrt{2\epsilon(\epsilon+1)}\frac{d\sigma_{\mathrm{LT}}}{dt}\cos\phi + \epsilon\frac{d\sigma_{\mathrm{TT}}}{dt}\cos 2\phi$$

$$\epsilon = \left(1 + \frac{2|\mathrm{q}|^2}{Q^2}\tan^2\frac{\theta_e}{2}\right)^{-1}$$

- ϵ **Polarization of the virtual photon**
- q **is its three momentum**
- θ_e **is the angle between initial and final electron momentum**

- **Sullivan process: Chew-Low formulation**

For small t for the pion pole contribuition

$$N\frac{d\sigma_{\rm L}}{dt} = 4\hbar c\left(eG_{\pi NN}\right)^2 \frac{-tQ^2}{\left(t-m_\pi^2\right)^2} F_\pi^2\left(Q^2\right),$$

- **Flux fator for σ_L:**

$$N = 32\pi\left(W^2-m_p^2\right)\sqrt{\left(W^2-m_p^2\right)^2+Q^4+2Q^2\left(W^2+m_p^2\right)}.$$

- **Invariant mass (virtual photon-nucleon system):**

$$W=\sqrt{M_p^2+2M_p\omega-Q^2}$$

- M_p **proton mass**

- **Pion Nucleon form factor (monopole):**

$$G_{\pi NN}(t) = G_{\pi NN}\left(m_\pi^2\right)\left(\frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 - t}\right),$$

$G_{\pi NN}\left(m_\pi^2\right) = 13.4$ **and** $\Lambda_\pi = 0.80\ \mathrm{GeV}$

- **For extraction of F_π from the Jefferson Lab***

$\star$ **Ref. G. M. Huber et al. (Jefferson Lab F_π Collaboration)**
Phys. Rev. C 78, 045203 (2008)

Obs.: The value of Λ_π is agree with the deuteron proprieties
See T. E. O. Ericson, B. Loiseau, and A. W. Thomas ,
Determination of the pion nucleon coupling constant and scattering lengths,
Phys. Rev. C 66, 014005 (2002)

Models

- **Previus work: Covariant Model***

$$\Gamma_{\pi}^{\mathrm{CON}}(k,p) = g_{\pi\bar{q}\bar{q}},$$

$\Longrightarrow$ $g_{\pi q\bar{q}}$ **pion-quark coupling constant**

- **pointlike vertices**
- **fermion-loop was regulated by dimensional regularization**
- **UV divergence eliminated redefining the renormalized form factor**

$$F_1^{\mathrm{ren}}\left(Q^2,t\right) = 1 + \left(F_1\left(Q^2,t\right) - F_1\left(0,m_\pi^2\right)\right)$$

$\star$ **Ref.; Ho-Meoyng Choi, T. Frederico, Chueng-Ryong Ji and J.P.B.C. de Melo, Phys.Rev.D 100 (2019) 11, 116020**

- **Pion microscospic current:**

Mandelstam amplitude for the photoabsorption

- **Here two constraints**

i) Covariance

- **Pion microscospic current:**

Mandelstam amplitude for the photoabsorption

- **Here two constraints**

i) Covariance

ii) Current conservation (on-mass shell, initial and final pion)

Effective Lagrangian

Effective Lagrangian with pion and quark degrees of freedom

$$\mathcal{L} = -\imath \frac{m}{f_\pi} \vec{\pi} \cdot \bar{q} \gamma_5 \vec{\tau} q,$$

- **Coupling of the constituent quark / pseudoscalar isovector pion field / $\mathrm{SU}(2)$ flavor symmetry**
- **f_π is the pion decay constant and m is the constituent quark mass**

- **Mandelstam formula** $\Longrightarrow$ **Pion-photon absorption amplitude**

$$\Gamma^{\mu}\left(p', p\right) = -\,2ie\frac{m^2}{f_\pi^2}N_c\int\frac{d^4k}{(2\pi)^4}\,\mathrm{Tr}\left[S(k)\gamma^5 S\left(k-p'\right)\right.$$
$$\left.\times\gamma^{\mu}S(k-p)\gamma^5\right]\Gamma_\pi\left(k,p'\right)\Gamma_\pi(k,p),$$

$$\begin{cases} S(p)\ \textit{is}\ \textbf{constituent quark propagator} \\ N_c = 3\ \textbf{is the number of colors} \\ q=(p'-p)\ \textbf{the momentum transfer} \\ k\ \ \textbf{the spectator quark momentum} \end{cases}$$

- **Frame**
- $\begin{cases} \textit{Offshell}: & \textbf{initial pion:} \quad p^2 = t < 0 \\ \textit{On-shell}: & \textbf{final pion} \quad p'^2 = m_\pi^2 \end{cases}$

- **Frame**
- $\begin{cases} \textit{Offshell}: & \textbf{initial pion:} \quad p^2 = t < 0 \\ \textit{On-shell}: & \textbf{final pion} \quad p'^2 = m_\pi^2 \end{cases}$

$\Longrightarrow$ **The equation above:** **Satisfy current conservation:**

$$q_\mu \Gamma^\mu \left(p', p\right)\Big|_{p^2 = p'^2 = m_\pi^2} = 0$$

If both pion in on-shell

- $\Longrightarrow$ $q^+ = q^0 + q^3 = 0$ **(with LF energy $p^- = p^0 - p^3$)**
- $\Longrightarrow$ **Momentum** $(p^+, p_\perp)$ **satisfying**

$$p^- = \frac{p_\perp^2 + t}{p^+}, \quad \textbf{Off-shell}$$

- $\vec{p}'_\perp = \vec{q}_\perp/2$, **pion initial off-shell pion**

$$p'^- = \frac{p'^2_\perp + m_\pi^2}{p^+}, \quad \textbf{On-shell}$$

- $\vec{p}'_\perp = \vec{q}_\perp/2$, **final state on-mass shell pion**
- **Final state on-shell pion momentum:** $p'^+ = p'^-$
- **Defines the kinematics, for** $t < 0$ **and** $t = m_\pi^2$

Bethe-Salpeter amplitude

- **Symmetric (SYM) vertex to smear the $q\bar{q}$ bound-state vertex**

$$\Gamma_\pi^{(\mathrm{SYM})}(k,p) = N\left(\frac{1}{k^2 - m_R^2 + i\epsilon} + \frac{1}{(p-k)^2 - m_R^2 + i\epsilon}\right).$$

$\Longrightarrow$ **Pauli-Villars regularization mass m_R plays the role of a momentum cutoff**

$\Longrightarrow$ **Fixed by fitting the pion decay constant (Exp.) PDG**

- **Model for the BS amplitude**

$$\Psi_i(k,p) = \frac{m}{f_\pi} S(k)\gamma^5 \Gamma(k,p)\tau_i S(k-p).$$

- **de Melo, Naus, and Frederico, Phys. Rev. C 59, 2278 (1999)**
- **de Melo, Frederico, Pace, and Salmé,Nucl. Phys. A707, 399 (2002); ibid., Braz. J. Phys. 33. 301 (2003)**

Wave function

- **Wave function // LF projection SYM vertex**

$$\Psi(x, k_\perp, t = p^+p^- - p_\perp^2)$$
$$= \frac{\mathcal{N}}{t - M_0^2\left(m^2, m^2\right)} \left[\frac{1}{x\left(t - M_0^2\left(m_R^2, m^2\right)\right)} + \frac{1}{(1-x)\left(t - M_0^2\left(m^2, m_R^2\right)\right)}\right]$$

$$M_0^2\left(m_a^2, m_b^2\right) = \frac{k_\perp^2 + m_a^2}{x} + \frac{(p-k)_\perp^2 + m_b^2}{1-x}$$

- $\begin{cases} m_a \textit{ and } m_b \text{ // } \textbf{quark masses} \\ m_R \text{ //}\textbf{regulator mass} \\ x = \frac{k^+}{p^+} \end{cases}$

Wave function Normalização

$\Longrightarrow$ **Normalization constant** $\mathcal{N} = N\sqrt{N_c}\frac{m}{f_\pi}$

- **Number of colors** $N_c = 3$**,** f_π **Weak pion decay constant**
- **Wave Function:** $\begin{cases} \textbf{On-shell for } t = p^+p^- - p_\perp^2 = m_\pi^2 \\ \textbf{Off-shell otherwise} \end{cases}$

$$\int_0^1 dx \int \frac{d^2k_\perp}{16\pi^3} \left|\Psi\left(x, k_\perp\right)\right|^2 = 1.$$

- **TMD - Transverse Momentun Distribution**

$$f\left(x,k_{\perp}\right)=\frac{1}{16\pi^{3}}\left|\Psi\left(x,k_{\perp}\right)\right|^{2}$$

- **PDF - Parton Distribution Function**

$$f(x)=\int d^{2}k_{\perp}f\left(x,k_{\perp}\right)$$

- **Sum Rule**

$$\int dx\int d^{2}k_{\perp}f\left(x,k_{\perp}\right)=\int dxf(x)=1$$

Pion Electromagentic Form Factors

- **Off-mass shell EM form factor** $F_1\left(Q^2, t\right)$

$\Longrightarrow$ **With the final state on-shell, and** $\Gamma^+\left(p', p\right)$ **for the EM-current**

$\Longrightarrow$ **Also** $q^+ = 0$

$$F_1\left(Q^2, t\right) = \frac{\Gamma^+\left(p', p\right)}{2ep^+}.$$

- **Electromagnetic form-factor** $F_2\left(Q^2, t\right)$

$$F_2\left(Q^2, t\right) = \frac{t - m_\pi^2}{Q^2}\left[F_1(0, t) - F_1\left(Q^2, t\right)\right],$$

- **As a reminder, the FFactor** $g(Q^2, t)$ **is obtained by combining** F_1 **and and** F_2
- **VIP: Use of** γ^+ **eliminates the instantaneous terms**

- **Kinematics (on-shell):**

$$\begin{aligned} p^\mu &= (p_0, \frac{-q\cos\alpha}{2}, 0, \frac{-q\sin\alpha}{2}) \\ p'^\mu &= (p_0, \frac{q\cos\alpha}{2}, 0, \frac{q\sin\alpha}{2}), \end{aligned}$$

where $p^0 = \sqrt{m_\pi^2 + \frac{q^2}{4}}$**.**

- **Transfer momentum:** $q^\mu = p'^\mu - p^\mu = (0, q\cos\alpha, 0, 0, q\sin\alpha)$.
- **Light-front coordinates:** $(t+z, t-z, x, y) = (+, -, \perp)$**,**

$$\begin{aligned} p^\mu &= (p_0 - \frac{q\sin\alpha}{2}, p_0 + \frac{q\sin\alpha}{2}, -\frac{q\cos\alpha}{2}) \\ p'^\mu &= (p_0 + \frac{q\sin\alpha}{2}, p_0 - \frac{q\sin\alpha}{2}, \frac{q\cos\alpha}{2}) \end{aligned}$$

Momentum transfer: $q^\mu = (q\sin\alpha, -q\sin\alpha, q\cos\alpha)$.

Model	$\sqrt{\langle r_\pi^2 \rangle}[\mathrm{fm}]$	$f_\pi[\mathrm{MeV}]$	$g\left(0, m_\pi^2\right)\left[\mathrm{GeV}^2\right]$
$\Gamma_\pi^{(\mathrm{SYM})}$	**0.736**	**92.40**	**2.32**
$\Gamma_\pi^{(\mathrm{CON})}$	0.713 ± 0.013	...	2.18 ± 0.08
Exp. [PDG]	0.672(8)	92.28(7)	1.93(5)

- **Quark masses:** $m_u = m_{\bar{d}} = \ 0.220 \ GeV$
- **Regulator mass:** $m_R = \ 0.600 \ GeV$

$\Longrightarrow$ **(fit f_π Exp. value)!!**

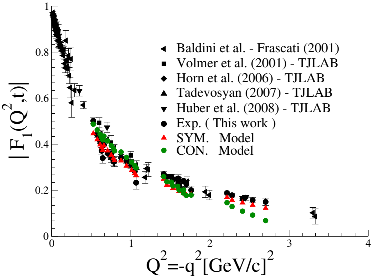
- f_π, and charge radius r_π, for $m_q = 0.22.\mathrm{GeV}$

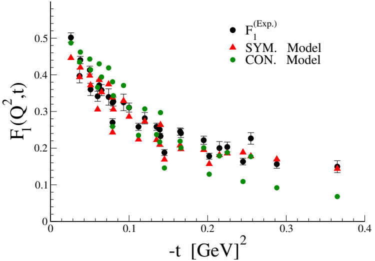
Different values of the regulator mass m_R

The fractional percent deviations are

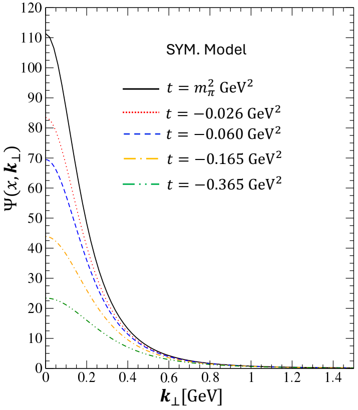
$$\Delta f_\pi = \left| \frac{f_\pi^{f_T^{\mathrm{SM}} - f_\pi^{\mathrm{Exp}}}}{f_\pi^{\mathrm{SYM}}} \right| \times 100 \text{ and } \Delta r_\pi = \left| \frac{r_\pi^{S_T{}^{\mathrm{YM}}} - r_\pi^{\mathrm{Exp}}}{r_\pi^{\mathrm{STM}}} \right| \times 100$$

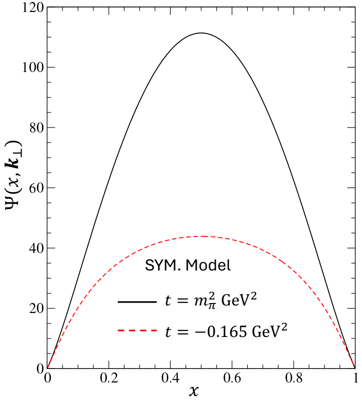
$m_R[\mathrm{GeV}]$	$f_\pi[\mathrm{MeV}]$	$\Delta f_\pi(\%)$	$r_\pi[\mathrm{fm}]$	$\Delta r_\pi(\%)$
0.6	92.4	0.1	0.736	8.7
0.7	97.0	4.9	0.695	3.4
0.8	100.9	8.5	0.675	0.4





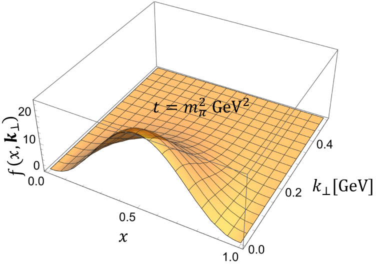
Pion off-shell EM form factor $F_1\left(Q^2, t\right)$.
Left: $F_1\left(Q^2, t\right)$ extracted from the experimental cross sections, and compared with CON, and SYM models.
Right: $F_1\left(Q^2, t\right) \times -t$ for the same models.

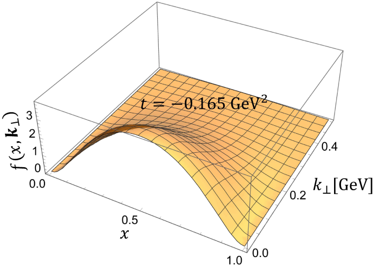




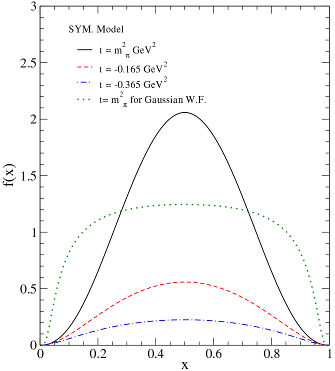
Pion wave function $\Psi(x, k_\perp)$ from the SYM model

- **Left panel: Fixed values $x = 0.5$ for various values of t times $k_\perp$.**
- **Right panel: Fixed values of $k_\perp = 0$ for two value of t times x.**





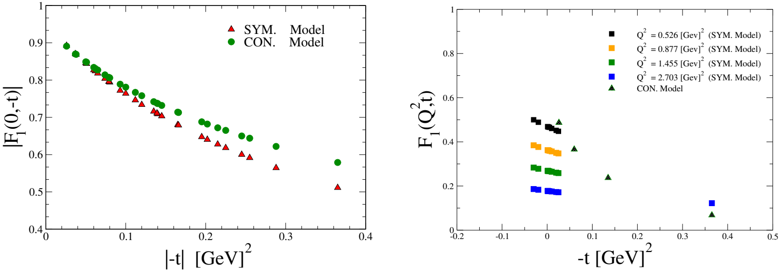
Twist-2 pion TMD $f(x, k_\perp)$ for SYM model; fixed value of $Q^2 = 0$, with two values of $t = m_\pi^2$, and $-0.165\mathrm{GeV}^2$



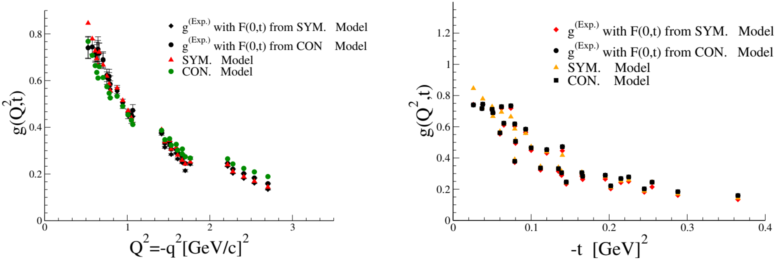
Twist-2 PDF $f(x)$ with SYM model, at fixed $Q^2 = 0$ and some values of t.

Light-front quark model on-shell pion, from the the Gaussian wave function**

$\star\star$ **H.-M. Choi and C.-R. Ji, Phys. Rev. D 110, 014006 (2024)**



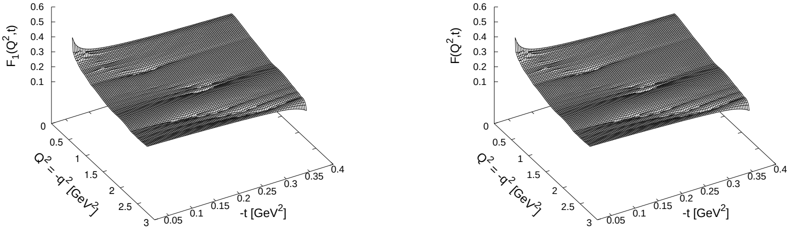
Pion off-shell EM form factor $F_1(Q^2,t)$ from the CON and SYM models versus $-t$
Right panel for $F_1(0,t)$
Left panel: $F_1(Q^2,t)$ for $Q^2 = 0.526\,, 0.877\,, 1.455$ and 2.703 [GeV]^2



Pion off-shell EM form factor $g\left(Q^2,t\right)$.
Right: $g\left(Q^2,t\right)\,\times\,Q^2$, compared with the extracted result from the experimental data* with $F_1(0,t)$ from CON and SYM models

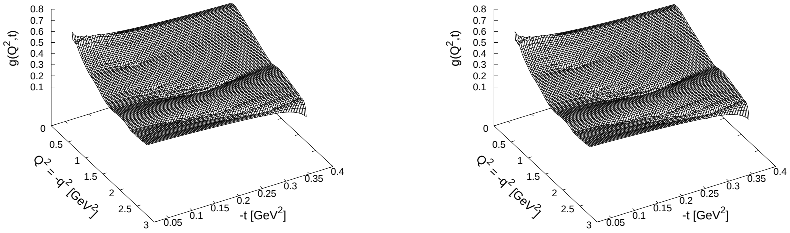
Left: $g\left(Q^2,t\right)\;\times\;-t$ with same models

$\star$ **H. P. Blok et al. (Jefferson Lab F_π Collaboration) Phys. Rev. C 78, 045202 (2008)**



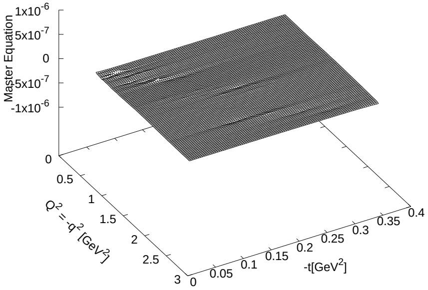
The 3D plots of the form factor $F_1(Q^2, t)$ extracted from the experimental data*. Inputs for "$F_1(0, t)$" from the CON model (left), and SYM model (right panel)

$\star$ H. P. Blok et al. (Jefferson Lab F_π Collaboration) Phys. Rev. C 78, 045202 (2008).



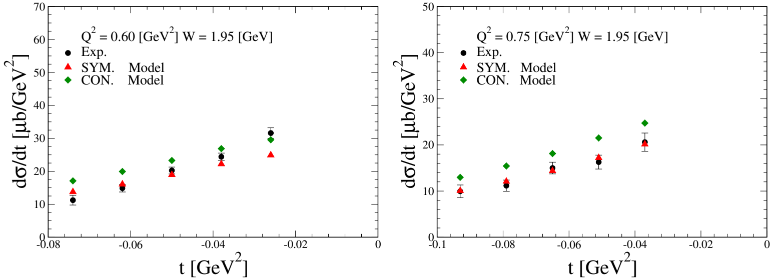
The 3D plots of the form factor $g(Q^2, t)$ extracted from the experimental data*. Inputs for "$F_1(0,t)$" from the CON model (left), and SYM model (right panel)

$\star$ H. P. Blok et al. (Jefferson Lab F_π Collaboration) Phys. Rev. C 78, 045202 (2008).



- **Off-shell form factor sum rule****

$$F_1\left(Q^2,t\right)-F_1(0,t)+Q^2 g\left(Q^2,t\right)=0$$



Cross-sections calculated with CON and SYM models, compared with the experimental data, for fixed Q^2 and W, and varying t

$\star$ **H. P. Blok et al. (Jefferson Lab F_π Collaboration) Phys. Rev. C 78, 045202 (2008).**

Summary

- **Light-front approach correctly describes hadronic bound states**
- **Take New Informations about Bound States**
- **Breaking of the rotational invariance has to be evaluated**
- **Choose** $G_{\pi NN}(t)$ **with two differents parametrizations**
- **Use WTI to related** $F_1(Q^2, t)$ **and** $F_2(Q^2, t)$
- **Use** $F_1(Q^2, t)$ **and** $F_2(Q^2, t)$ **relationship to extract the EMFF** $g(Q^2, t)$
- **The Light-front pion wave function off-shell is very sensitive for the** t **and** x
- **Show** $F_1(Q^2, t) \times |-t|$ **from experiments**
- **Show** $g(Q^2, t) \neq 0$ **no matter what** $G_{\pi NN}(t)$ **is used**

- **The electromagnetic form factors from the models, both on-shell and off-shell F_1 are compared with the experimental data**
- **Also for $g(Q,t)$ electromagnetic form factor (on and off -shell)**
- **The cross-section from the models are compared with the experimental data, giving good results**

Next:

- **Kaon Electromagetic Form Factors / on-shell and off-shell**
- **Parton Distribution Function off-shell regime (on-shell)**
- **Gravitational Form factor (Pion, Kaon)**
- Frame off-shell dependence in the Light-Front

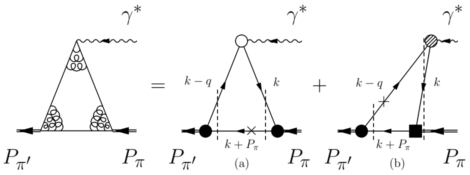
IMP - CAS / Huizhou Research Group - USTC - Hefei

Collaboration with the group on hadronic physics and nuclear matter

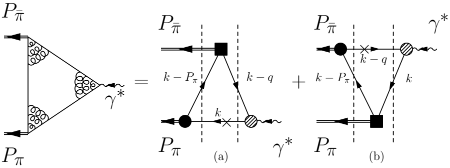
- Nucleon BLFQ in nuclear matter and the equation of state (EoS).
- Generalization of the 't Hooft meson equation.
- Schwinger-Dyson equation for quarks in light-cone gauge and spontaneous chiral symmetry breaking.
- Pion observables in the space-time and time-like regions.
- Non-valence contribution to the electromagnetic current for spin-1 particles
- Off-shell electromagnetic form factors

- We developed a microscopical, Poincaré covariant model for the hadron em form factors in both SL and TL region
- The Quark-photon vertex has been approximated by a microscopical VMD model plus a bare term
- The Z-diagram (higher Fock components) are essential for the pion
- Pion: Results yielded a first successful test for our model.
- de Melo, J., Frederico T., Pace, E., Salme, G. Phys.Lett.B 581 (2004) 75.
- de Melo, J., Frederico T., Pace, E. and Salme, G. Phys.Rev.D 73 (2006) 074013
- Eigenfunctions of a relativistic CQ square mass operator Frederico,T. Pauli, H. C., Zhou, S., PRD 66 (2002) 116011

Through a k^- integration (only the poles of the Dirac propagators taken into account), in a reference frame where $q^+ > 0, \mathrm{q}_\perp = 0$



Space-like



Time-like

- $\Rightarrow$ k **on its mass shell :** $k^-_{on} = \left(m^2 + k^2_\perp\right)/k^+$

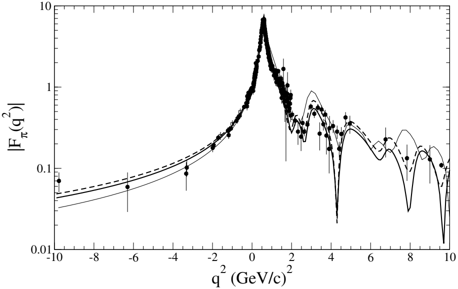
Vector meson masses, M_n, and widths, Γ_n. The corresponding decay widths into e^-e^+ pairs, calculated with the VM valence probabilities $P_{q\bar{q};n}$ and the oscillator strength $\omega = 1.556\ GeV^2$, are compared with the experimental values from PDG[*].

Meson	M_n (MeV)	M_n^{exp} (MeV) [*]	Γ_n (MeV)	Γ_n^{exp} (MeV) [*]	$\Gamma_{e^+e^-}$ (KeV)	$\Gamma^{exp}_{e^+e^-}$ (KeV)[*]
$\rho(770)$	770	775.8 ± 0.5	146.4	146.4 ± 1.5	6.98	7.02 ± 0.11
$\rho(1450)$	1497 [PDG]	1465.0 ± 25.0	226 [1]	400 ± 60	1.04	1.47 ± 0.4
$\rho(1700)$	1720	1720.0 ± 20.0	220	250 ± 100	0.98	$> 0.23 \pm 0.1$
$\rho(2150)$	2149	2149.0 ± 17	230 [2]	363 ± 50	0.65	-

Ref. [1] R. R. Akhmetshin et al., Phys. Lett. B 509, 217 (2001). [2] A. V. Anisovich et al., Phys. Lett. B 542, 8 (2002).

n	0	1	2	3	4	5	6	7	8	9
$P_{q\bar{q};n}$	0.77	0.31	0.29	0.27	0.22	0.18	0.18	0.18	0.17	0.16

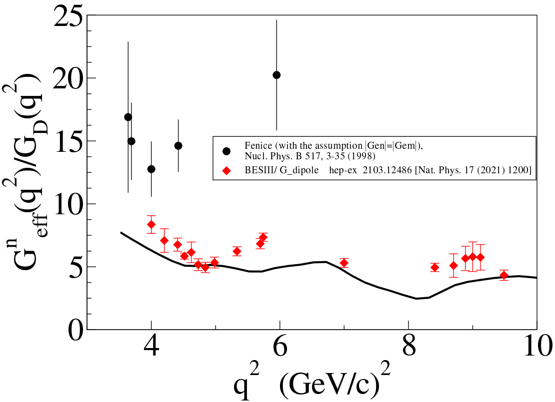
Vector-meson valence probabilities $P_{q\bar{q};n}$ for the first 10 resonances.



Pion electromagnetic form factor as a function of the momentum squared q^2. Results for the asymptotic and the full pion wave functions, obtained with $w_{VM} = -0.7$ and the quantities shown in Table, are indicated by dashed and solid curves, respectively. The thin solid line represents the result with $w_{VM} = -1/3$ and the parameters of Table. Experimental data are from Baldini (full dots) and JLab (open squares).

Ref. [Baldini] R. Baldini et al., Eur. Phys. J. C 11, 709 (1999); Nucl. Phys. A666, 38 (2000); (private communication)
[JLab] J. Volmer et al., Phys. Rev. Lett. 86, 1713 (2001)

Neutron *effective* TL form factor and recent BESIII data



Time- and Spacelike Nucleon Electromagnetic Form Factors beyond Relativistic Constituent Quark Models. de Melo, Frederico, Pace, Pisano and Salme: Phys.Lett.B671 (2009) 153.

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