Few-body universality in neutron halo nuclei

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ITP/CAS, Beijing, Feb 05, 2026

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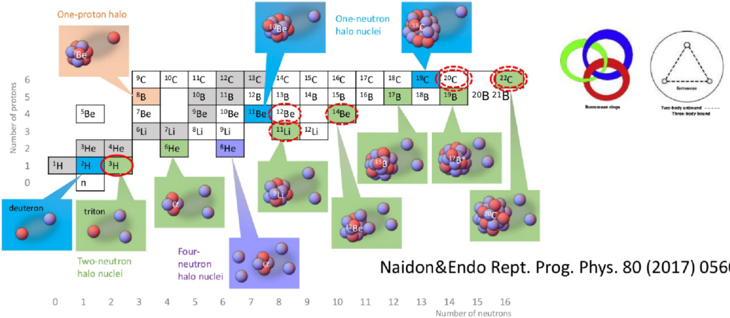


Naidon&Endo Rept. Prog. Phys. 80 (2017) 056001

s-wave 2n Borromean halo ^{11}Li, ^{14}Be, ^{19}B, ^{20}C, ^{22}C → S_{2n}, a_{nn}, a_{nc}

^{20}B(^{17}B+3n & ^{19}B+n) ^{21}B(^{17}B+4n)

Universality = model independence & Efimov/Thomas effect

s-wave 2n halo ^{6}He: S_{2n}, a_{nn}, $^{2}P_{3/2}$ (narrow resonance)

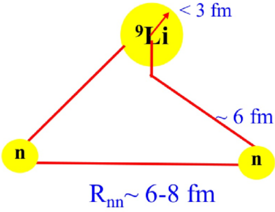
TF, Delfino, Tomio, Yamashita,
"Universal aspects of light halo nuclei" Prog. Part. Nucl. Phys. 67 (2012) 939

Two-neutron weakly bound s-wave three-body halo nuclei

Borromean

^{11}Li

Tanihata et al., PRL**55**, 2676 (1985)



$\mathbf{S_{2n} = 369}$ **keV**

Smith et al. PRL101, 202501 (2008)

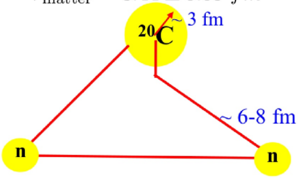
^{22}C

Tanaka et al., PRL104, 062701 (2010)

$$r_{\rm matter} = 5.4 \pm 0.9 \; fm$$

Togano et al., Phys. Lett. B 761, 412 (2016)

$$r_{\rm matter} = 3.44 \pm 0.08 \; fm$$



$\mathbf{S_{2n} < 70}$ **keV**

Mosby et al. NPA 909, 69 (2013)

^{19}B=^{17}B+n+n: $S_{2n} = 0.14 \pm 0.39\,\mathrm{MeV}$

Gaudefroy etal PRL109(2012) 202503

Hiyama, Lazauskas, Marqués, Carbonell, PRC100 (2019) 011603

Hiyama, Lazauskas, Carbonell, TF PRC106 (2022) 064001

Weakly bound quantum systems - large and dilute

$$(E - H_0)\psi = 0$$

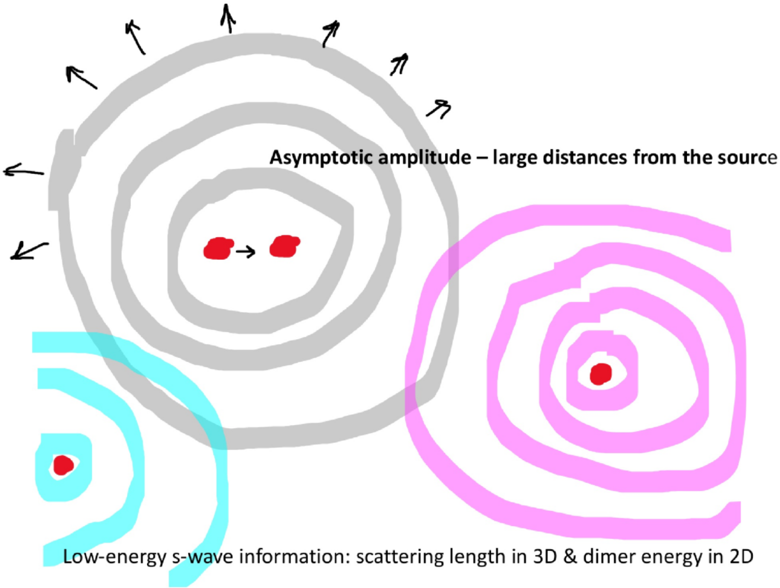
- Almost everywhere the wf is an eigenstate of H_0 - **short-range force;**
- **Scale invariance in the UV (E=0);**
- **Physics:** symmetry (fermions, bosons), scales and dimension (& mass ratios);

➢ **Universality = model independence;**

➢ **Limiting case: contact interactions;**

➢ **How many scales** are needed to describe the physics of these systems ???????

Naive picture - Asymptotic waves: outside the interaction range



Two-body s-wave phase-shift 3D (large scatt. lenghts)

$$k\cot(\delta) = -\frac{1}{a} + \frac{r_0}{2}k^2 + \cdots \qquad |a| >> r_0$$

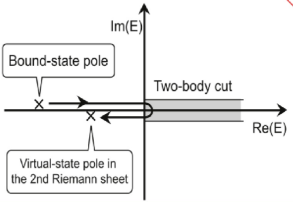
S-wave scatt. amplitude

$$f_{\ell=0}(k) = \frac{1}{k\cot(\delta) - \imath\, k}$$

$$k = \sqrt{\frac{m}{\hbar^2}E}$$

$a > 0$

$a < 0$



- ^{4}He-^{4}He **bound** state: 151.9±13.3 neV (a = 90.4 **Å**)
- Feshbach resonance in cold atomic traps large |a| >> R_{vdW}
- $^1S_0$ nn **virtual** state: $E_{nn}^{virtual}$ = -143 keV (a = -17fm)
- S-wave n-core state: **virtual** (^{10}Li ~ -50 keV) or **bound** (^{19}C ~ -580 keV)

6

Three-boson system: Subtle three-body phenomenum in L_{total}=0:

The Efimov effect

Nuclear Physics

Vitaly Efimov Phys. Lett. B 33, 563 (1970).

Vol 440|16 March 2006|doi:10.1038/nature04626 nature

LETTERS

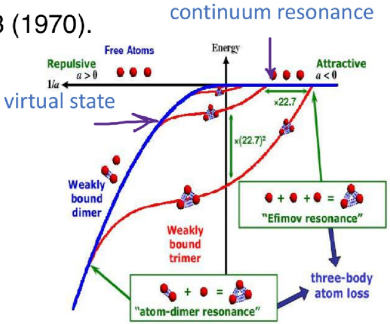
Evidence for Efimov quantum states in an ultracold gas of caesium atoms

T. Kraemer[1], M. Mark[1], P. Waldburger[1], J. G. Danzl[1], C. Chin[1,2], B. Engeser[1], A. D. Lange[1], K. Pilch[1], A. Jaakkola[1], H.-C. Nägerl[1] & R. Grimm[1,3]

nature physics LETTERS
PUBLISHED ONLINE: 22 FEBRUARY 2009 | DOI: 10.1038/NPHYS1203

Observation of an Efimov-like trimer resonance in ultracold atom-dimer scattering

S. Knoop[1]*, F. Ferlaino[1], M. Mark[1], M. Berninger[1], H. Schöbel[1], H.-C. Nägerl[1] and R. Grimm[1,2]



"Fall to the center" in the $1/r^2$ potential

$$-\frac{d^2}{dr^2}\psi(r) - \frac{K}{r^2}\psi(r) = E\psi(r)$$

Continuous symmetry breaking to a discrete one & r^* dimensional scale

$$\psi(r)|_{r\sqrt{E}\ll 1} \to r^{\frac{1}{2}} \sin(\underbrace{\sqrt{4K-1}}_{\mathrm{S}} \log(r/r^*))$$

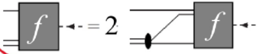
$$K > \frac{1}{4} \text{ discrete scale symmetry } \text{\& log-peridocity}$$

("fall to the center" Landau Quantum Mechanics)

- Boundary condition: Geometric ratio $E_n/E_{n+1} = e^{2\pi/s}$
- *Efimov effect & Thomas collapse →*
  + one three-body scale
- **Adding angular momentum avoids the "fall to the center"**

Continuous Scale invariance and breaking

Skorniakov and Ter-Martirosian equations (1956)



$$f(\vec{q}) = \frac{\pi^{-2}}{\pm\sqrt{E_2} - \sqrt{E - \frac{3}{4}q^2}} \int d^3k \frac{1}{E - q^2 - k^2 - \vec{q}\cdot\vec{k}} f(\vec{k})$$

$\hbar = m = 1$ (+) dimer bound & (−) dimer virtual

❖ SKM integral equation for large momentum

invariant under: $k \to \xi k$ and $p \to \xi p$

❖ $f(k)$ and $f(\xi k)$ solutions for s-wave → $f(k) = k^{\eta}$

Danilov, Sov. Phys. JETP 13 (1961) 349:

$$\boxed{f(y) = a_+ y^{\imath s_0 - 2} + a_- y^{-\imath s_0 - 2}}$$

$$\chi(y) = y^{-2} \sin(s_0 \ln y + c)$$

Efimov equation:

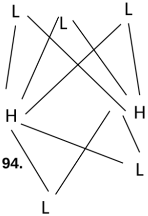
$$\boxed{1 = \frac{8}{\sqrt{3}s} \frac{\sin(\pi s/6)}{\cos(\pi s/2)} \qquad s = \pm i s_0 \qquad s_0 \approx 1.00624}$$

Minlos & Faddeev, Sov. Phys. JETP14, 1315 (1962) infinity number of states geometrically separated

LLHH, LLLHH ... systems: B.O. approximation

LLHH Naidon, Few-Body Syst. 59, 64 (2018)

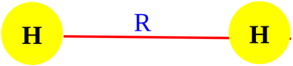
$m_H >> m_L$ ***L-H interaction only***



“Interwoven limit cycles in the spectra of mass imbalanced many-boson system“
De Paula, Delfino, TF, Tomio, JPB53, 205301 (2020)

Born-Oppenheimer approx.

Fonseca, Redish, Shanley, Nucl. Phys. A320 (1979) 273
Bhaduri, Chatterjee, van Zyl, Am. J. Phys. 79 (2011) 274-281; Am. J. Phys. 80 (2012) 94.



$$\left[\frac{d^2}{dR^2} + \frac{s_N^2 + \frac{1}{4}}{R^2} - \mathcal{B}_N\right] u = 0 \quad (N \geq 3)$$

$$\boxed{s_N \equiv s_N(A) \equiv \sqrt{\left(\tfrac{2+A}{4A}\right)(N-2)\gamma^2 - \tfrac{1}{4}}}$$

$A = m_L / m_H$

$\gamma = e^{-\gamma} = 0.5671433$

$(N-2)$ Light bosons

New limit cycles beyond 3-body

- Interwoven limit cycles with different geometric ratios!

Universal Scaling functions, limit cycles & discrete scaling

TF, Delfino, Tomio, Yamashita, PPNP 67 (2012) 939

$$\mathcal{O}_3(E, E_3, \{\pm\sqrt{|E_2|}\}) = |E_3|^\eta \mathcal{F}_3\left(\frac{E}{E_3}, \left\{\pm\sqrt{\frac{E_2}{E_3}}\right\}\right)$$

3-BOSONS L=0

- Discrete scale invariance: scaling E, E_2, E_3 by $e^{\frac{2\pi n}{s_0}}$ ($s_0 = 1.00624\ldots$)
- ABC systems
- First example late 60's: Universal Phillips plot $E[^3H]$ v.s. a_2^{nd}
  Efimov explained as a limit cycle 80's
- Generalization to ABCD systems

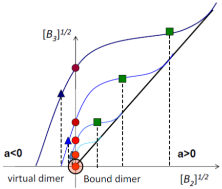
Yamashita, Tomio, Delfino, TF, EPL 75, 555 (2006)

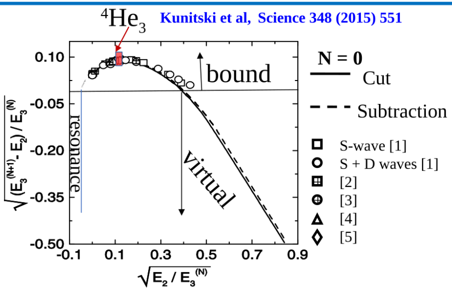
Hadizadeh, Yamashita, Tomio, TF, PRL107, 135304 (2011)

What do we know in 3D for three-bosons:
Scaling function as a limit cycle for the trimer bound-state

TF, Tomio, Delfino, Amorim. *Phys. Rev.* **A60**, R9 (1999). Yamashita et al PRA66(2003)052702

Thomas-Efimov states
Discrete scale symmetry → *Scaling plot - universal correlation*





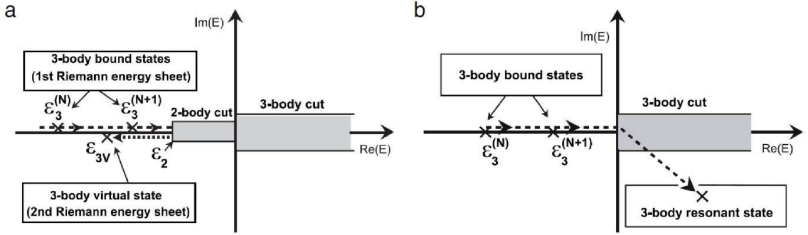
Efimov/zero-range limits

$$\boxed{|a|/r_0 \to \infty}$$

[1] Cornelius & Glöckle. JCP 85, 1 (1996).
[2] Huber. PRA31, 3981 (1985).
[3] Barletta & Kievsky. PRAA64, 042514 (2001).
[4] Fedorov & Jensen. JPA34, 6003 (2001).
[5] Kolganova, Motovilov, Sofianos. PRA56, R1686 (1997).

Range correction: Thogersen, Fedorov, Jensen PRA78(2008)020501(R) 23

Analytic structure & Efimov state trajectory



S.K. Adhikari and L. Tomio, Phys. Rev. C **26**, 83 (1982); S.K. Adhikari, A.C. Fonseca, and L. Tomio, *ibid.* **26**, 77 (1982).
Yamashita et al PRA66 (2002) 052702 (atoms)
Rupak, Vaghani, Higa,van Kolck PLB(2018)

Bringas, Yamashita,TF PRA69 (2004) 040702R
Jonsell EPJL 76 (2006) 8
Deltuva PRC102 (2020) 034003 (separable s-wave pot.)
Dietz,Hammer,Konig,Schwen, PRC105(2022)064002
Dawid, Islam, Briceno,Jackura, PRA109(2024) 043325

Continuum resonances of Borromean systems: observation in atomic traps!

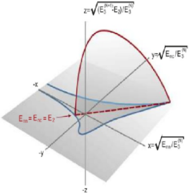
Resonant 3-body recombination (Innsbruck, Rice, Heidelberg, Bar-Ilan, Florence...)

$\alpha\alpha\beta$ system

Delfino, TF, Tomio

7876 J. Chem. Phys., Vol. 113, No. 18, 8 November 2000

$A = m_\beta / m_\alpha$



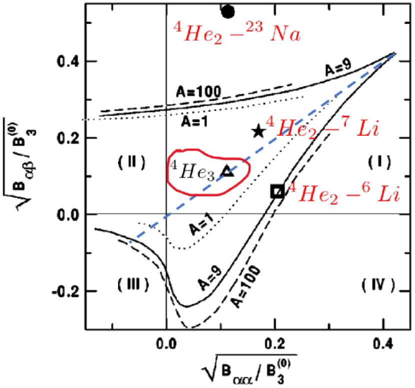


FIG. 1. The scaling approach for the three-body system $\alpha-\alpha-\beta$. The coordinates of the systems with $\alpha \equiv {}^4$He and $\beta = {}^4$He, ^{7}Li, ^{6}Li, and ^{23}Na are respectively, represented by a triangle, a star, a square, and a full circle.

Boundary for one excited Efimov state

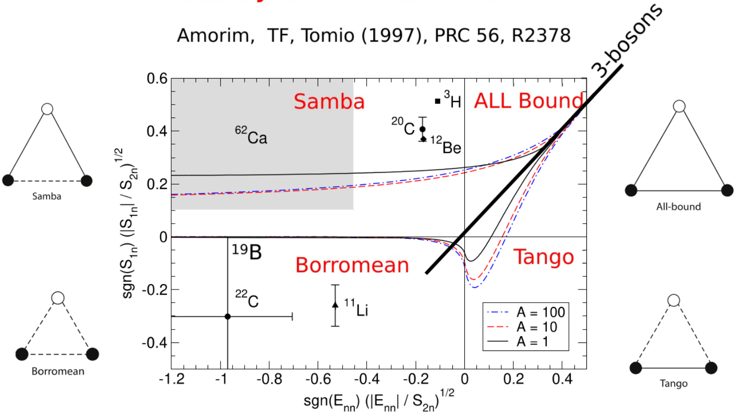


Fig. 5 The contour plot in $\mathrm{sgn}(E_{nn})\sqrt{|E_{nn}|/S_{2n}}$ versus $\mathrm{sgn}(S_{1n})\sqrt{|S_{1n}|/S_{2n}}$ for the ground-state $2n$ halos with core-neutron mass ratios $A = 1, 10, 100$. The hypothetical bound dineutron regime with $E_{nn} > 0$ is also included in the theoretical calculation to complete the contour.

Based on the plot from Hammer, "Handbook of Nuclear Physics", Springer, 2022, ed. by I. Tanihata, H. Toki, and T. Kajino arXiv:2203.13074 [nucl-th]

- Efimov virtual state: ^{20}C @ 100 keV, (P-wave: 40 keV...) Yamashita et al PLB660, 339 (2008)

Virtual states of ^{20}C

Yamashita et al. / Physics Letters B 660 (2008) 339

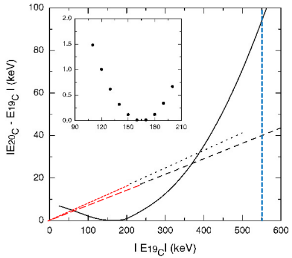


Fig. 2. Three-body n–n–^{18}C results for the first excited state, with respect to the threshold ($|E_{^{20}C} - E_{^{19}C}|$) for varying ^{19}C binding energies. Three-body bound (virtual) states occur when $|E_{^{19}C}|$ is approximately smaller (larger) than 170 keV. s-wave results (solid line) are also presented in the inset (with dots). Results for the p- and d-waves, divided by a factor 10, are shown with dashed and dotted lines, respectively.

Virtual states

S-wave ~ 100 KeV (Efimov nature)

P-wave ~ 40 KeV (continuous scale invariance)

D-wave ~ 45 KeV

F-wave ...

$E_{^{19}C} = -500$ keV

Configuration space two-neutron halo wave function (2n spin singlet) L=0 [^{11}Li, ^{14}Be, ^{19}B, ^{22}C]

$$\hat{\mathcal{H}}_{zr} = -\frac{\hbar^2}{2m_{r_x}}\nabla^2_{r_x} - \frac{\hbar^2}{2m_{r_y}}\nabla^2_{r_y} + \sum_{i<j}\lambda_{ij}\delta(\boldsymbol{r}_i - \boldsymbol{r}_j)$$

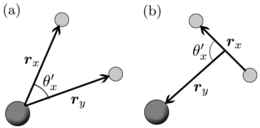


FIG. 4. Definition of the polar angles θ'_x used for the two possible sets of Jacobi coordinates [panels (a) and (b)] in ^{22}C.

nn: spin singlet state

$$\Psi(\boldsymbol{r}_n,\boldsymbol{r}_{n'}) = \int d\boldsymbol{q}\, \frac{e^{-\kappa_{nn}|\boldsymbol{r}_n-\boldsymbol{r}_{n'}|}}{|\boldsymbol{r}_n-\boldsymbol{r}_{n'}|} e^{i\boldsymbol{q}.\boldsymbol{r}_{c,nn}} f_{nn}(\boldsymbol{q}) + \left\{\int d\boldsymbol{q}\, \frac{e^{-\kappa_{nc}|\boldsymbol{r}_n-\boldsymbol{r}_c|}}{|\boldsymbol{r}_n-\boldsymbol{r}_c|} e^{i\boldsymbol{q}.\boldsymbol{r}_{n',nc}} f_{nc}(\boldsymbol{q}) + (n\leftrightarrow n')\right\}$$

$$\kappa_{nn} = \sqrt{2\frac{\mu_{nn}}{\hbar^2}\left(S_{2n} + \frac{\hbar^2 q^2}{2\mu_{nn,c}}\right)}$$

$$\kappa_{nc} = \sqrt{2\frac{\mu_{nc}}{\hbar^2}\left(S_{2n} + \frac{\hbar^2 q^2}{2\mu_{nc,n}}\right)}$$

S, P, D… waves

@ unitarity:

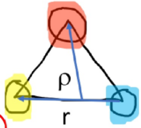
analytic bound-state wave function: Rosa, TF, Krein, Yamashita PRA 106, 023311(2022)

Analytic bound-state wave function for abc systems@unitarity limit

[scatt lengths→ ∞]

Rosa, TF, Krein, Yamashita PRA 106, 023311 (2022)
Rosa, Francisco, TF, Krein, and Yamashita PRA 110, 042803 (2024)

$$\textit{Faddeev equation: } \Psi = \sum_{i=1}^{3} G_0 V_i \Psi = \sum_{i=1}^{3} \psi_i$$

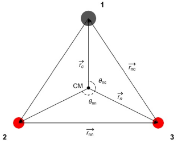


Bethe-Peierls Boundary condition

$$\left[\frac{\partial}{\partial r_i} r_i^{\frac{D-1}{2}} \Psi(\boldsymbol{r}_i, \boldsymbol{\rho}_i)\right]_{r_i \to 0} = \frac{3-D}{2}\left[\frac{\Psi(\boldsymbol{r}_i, \boldsymbol{\rho}_i)}{r_i^{\frac{3-D}{2}}}\right]_{r_i \to 0}$$

$$\Psi(r,\rho) = \frac{K_{is_0}\left(\sqrt{2}\,\kappa_0 \sqrt{r^2+\rho^2}\right)}{(r^2+\rho^2)} \sum_{j=1}^{3} C^{(j)} \frac{\sin\left(is_0\left(\frac{\pi}{2} - \arctan\left(\frac{r_j}{\rho_j}\right)\right)\right)}{\sin\left(2\arctan\left(\frac{r_j}{\rho_j}\right)\right)}$$

where K_{is_0} is the modified Bessel function of the third kind of pure imaginary order is_0, being s_0 the Efimov scale parameter

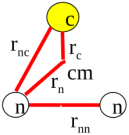


$$-\kappa_0^2 = 2E$$

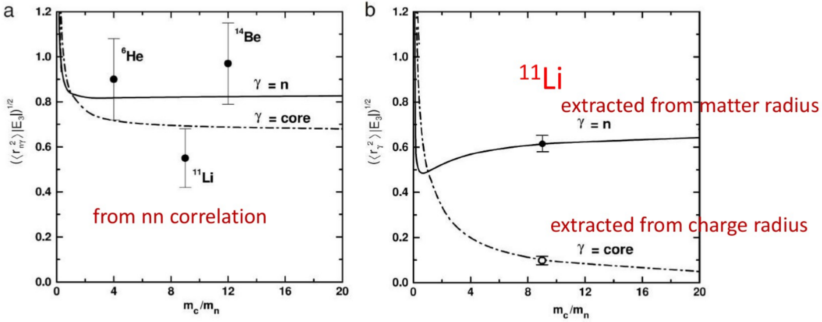
- Unatomic trimer: Francisco, Rosa, TF, Krein, Yamashita, PRA 112, 033315 (2025)
- Application for two-neutron halo nuclei (D=3)

Root mean square radii: Core+neutron+neutron

$a = \infty$ *unitary limit*



exp data nn correlation F.M. Marqués, et al., Phys. Lett. B 476 (2000) 219.
F.M. Marqués, et al., Phys. Rev. C 64 (2001) 061301.



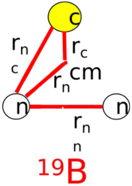
S_{2n}[^{11}Li]=369.15(65) KeV -- Smith etal PRL101(08)202501
neutron halo radius ^{11}Li [6.54(38) fm] -- Egelhof et al EJPA15 (02) 27
Charge radius ^{11}Li [2.217(35) fm] and ^{9}Li [2.467(37) fm]-- Sanchez et al PRL96(96)03302

TF, Delfino, Tomio, Yamashita, Prog. Part. Nucl. Phys. 67 (2012) 939

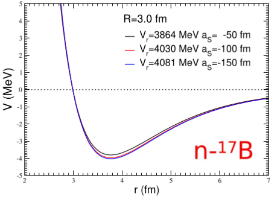
Root mean square radii: ^{19}B= ^{17}B +n+n close to the unitary limit

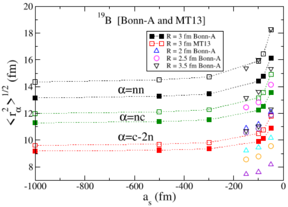
Hiyama, Lazauskas, Carbonell, TF, PRC106, 064001 (2022)

Model from Hiyama, Lazauskas, Marqués, Carbonell, PRC100 (2019) 011603



$$V_{n^{17}\mathrm{B}}(r) \;=\; V_r \,\frac{e^{-2\mu r}}{r}\,\left(1 - e^{-\mu(R-r)}\right)$$





$a_s < -50$ fm

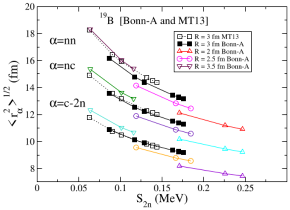
A. Spyrou et al, Phys. Lett. B **683**, 129 (2010)

$S_{2n} = 0.14 \pm 0.39\,\mathrm{MeV}$

L. Gaudefroy et al., Phys. Rev. Lett. **109** 202503 (2012).

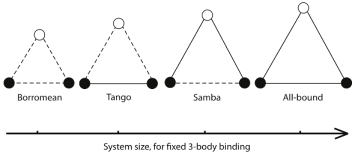
$S_{2n} = 0.09 \pm 0.56$ MeV

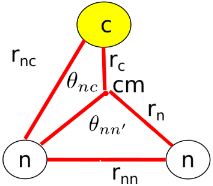
M. Wang, G. Audi, F. G. Kondev, W. Huang, S. Naimi, and X. Xu, Chin. Phys. C **41**, 030003 (2017).

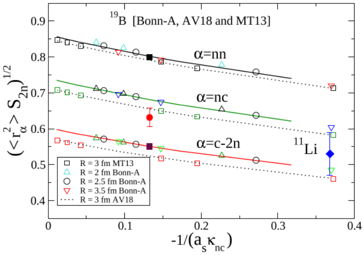


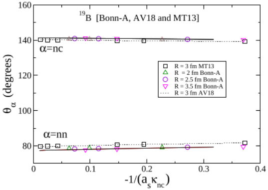
rms radii scaling functions: ^{19}B= ^{17}B +n+n

T. Frederico et al. / Progress in Particle and Nuclear Physics 67 (2012) 939–994

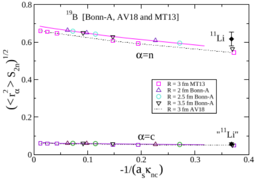








nn attractive < 90º !



full square: Casal & Garrido PRC102 (2020)051304

full circle: $\sqrt{\langle r^2_{c-2n}\rangle} = 5.75 \pm 0.11\,(\text{stat}) \pm 0.21\,(\text{sys})\ \text{fm}$

$$S_{2n} = 0.5\,\text{MeV} \quad a_s = -50\,\text{fm}$$

Cook et al., Phys. Rev. Lett. 124, 212503 (2020)

Hiyama, Lazauskas, Carbonell, TF
exploratory calculations

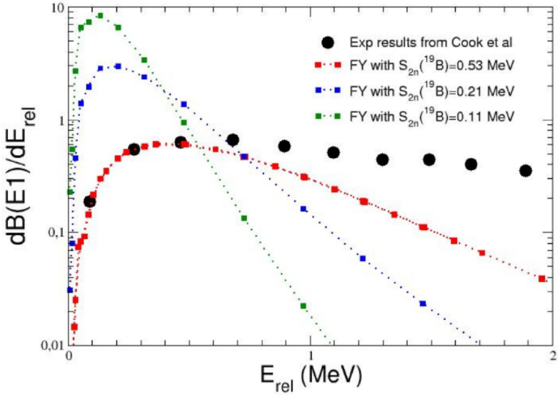
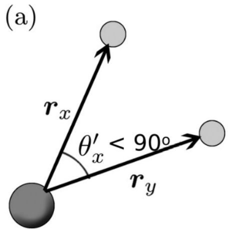


Figure 5: The Coulomb dissociation cross section of $^{19}\mathrm{B}\to{}^{17}\mathrm{B}+n+n$ as a function of the relative energy E_{rel} is compared to experimental results from [22], for several choices of the ^{19}B binding energy. For all of them V_r is adjusted to reproduce $a_S = -150$ fm.

$^{22}C = n + n + ^{20}C$

Souza, Garrido, TF PHYSICAL REVIEW C **94**, 064002 (2016)



nn attractive < 90º

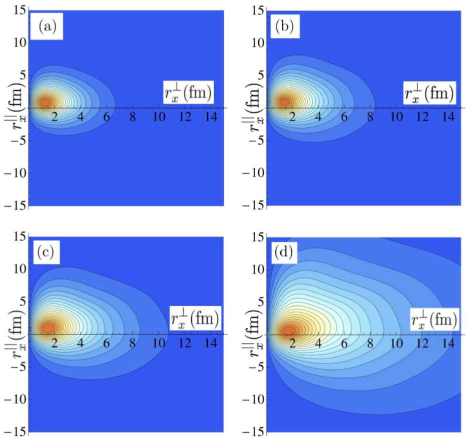
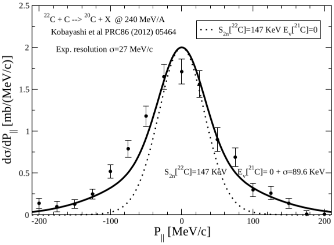
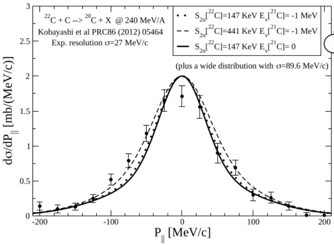


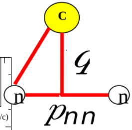
FIG. 7. Contour plot of the density $F(r_x^{\perp}, r_x^{||})$ defined in Eq. (22) for the polar angle θ'_x defined as in Fig. 4(a). Panels (a), (b), (c), and (d) correspond to a ^{22}C two-neutron separation energy of 1000, 500, 250, and 50 keV, respectively.

Core momentum distribution $^{22}C = n + n + {}^{20}C$

L. A. Souza et al PLB757 (2016) 368 & FBS57 (2016)361







$E_v[^{21}C]$=1 MeV Mosby et al. NPA 909, 69 (2013) – MSU - $|a_s| < 2.8$ fm (^{21}C virtual state)

$$\boxed{100\,KeV \lesssim S_{2n}[^{22}C] \lesssim 400\,KeV}$$

$$\boxed{\tilde{r}_m^{22}C \lesssim 4\,\mathrm{fm}}$$

$\boxed{5.4 \pm 0.9 \text{ fm}}$

Tanaka et al PRL 104, 062701 (2010)

$\boxed{3.44 \pm 0.08 \text{ fm}}$

Togano et al PLB 761, 412 (2016)

Core Momentum distribution nnc = AAB: ^{11}Li ,^{14}Be

L. A. Souza et al PLB757 (2016) 368 & FBS57 (2016)361

Limit cycle: Scaling function for the FWHM @ unitarity

$$\frac{\sigma}{\sqrt{S_{2n}}} = \mathcal{S}_c\left(\pm\sqrt{\frac{E_{nn}}{S_{2n}}}, \pm\sqrt{\frac{E_{nc}}{S_{2n}}}; A\right) \qquad \text{FWHM} = 2\sqrt{2\ln 2}\,\sigma$$

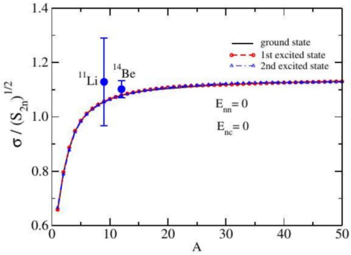
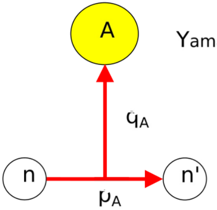


Fig. 1. Scaling plot for the core recoil momentum distribution σ in the Efimov limit as a function of the core mass number A. Experimental widths are from Refs. [1] and [9], for ^{11}Li and ^{14}Be, respectively.

[^{11}Li] I. Tanihata, J. Phys. G 22 (1996) 157;

[^{14}Be] M. Zahar, et al., Phys. Rev. C 48 (1993) R1484.

Neutron-neutron correlation function: ^{11}Li & ^{14}Be



Yamashita, TF, Tomio PRC 72, 011601(R) (2005)

Sudden Breakup amplitude including the nn FSI

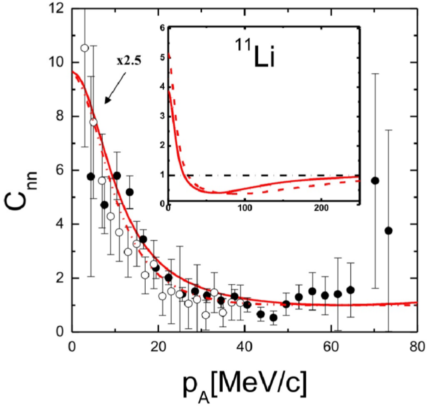
$$\boxed{\Phi = \Psi(\vec{q}_A, \vec{p}_A) + \frac{1/(2\pi^2)}{\sqrt{E_{nn}} - i p_A} \int d^3 p \, \frac{\Psi(\vec{q}_A, \vec{p})}{p_A^2 - p^2 + i\varepsilon}}$$

$$\boxed{C_{nn}(\vec{p}_A) = \frac{\int d^3 q_A |\Phi(\vec{q}_A, \vec{p}_A)|^2}{\int d^3 q_A \rho(\vec{q}_n') \rho(\vec{q}_n)}}$$

$$\begin{cases} \vec{q}_n = -\vec{p}_A - \dfrac{\vec{q}_A}{2} \\ \vec{q}_{n'} = \vec{p}_A - \dfrac{\vec{q}_A}{2} \end{cases}$$

One-body density $\rho(\vec{q}_{nA}) = \int d^3 q_{n'A} \left| \Phi\left(-\vec{q}_{nA} - \vec{q}_{n'A}, \frac{\vec{q}_{nA} - \vec{q}_{n'A}}{2} \right) \right|^2$

Neutron-neutron correlation function: ^{11}Li



●

F. M. Marqués et al.
Phys. Rev. C **64**, 061301 (2001)

○ — · · ·

M. Petrascu et al.
Nucl. Phys. A **738**, 503 (2004)

—— E_3 = 0.29 MeV, E_{nA} = 0.05 MeV

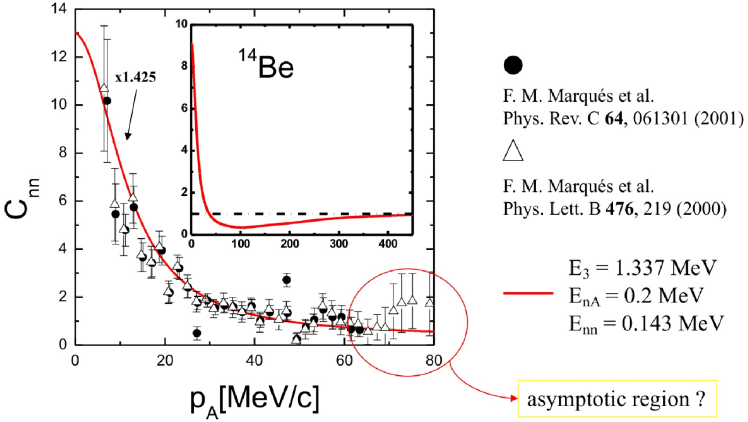
– – – E_3 = 0.37 MeV, E_{nA} = 0.8 MeV

········ E_3 = 0.37 MeV, E_{nA} = 0.05 MeV

Enn = 0.143 MeV

Yamashita, TF, Tomio PRC 72, 011601(R) (2005)

Neutron-neutron correlation function: ^{14}Be



Yamashita, TF, Tomio PRC 72, 011601(R) (2005)

^{6}He sudden breakup

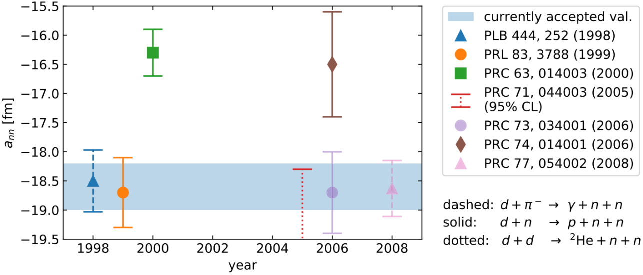
Göbel, Aumann, Bertulani, TF, Hammer, Phillips, PRC104, 024001 (2021)

^{6}He [0^+] = n+n+^{4}He S_{2n}= 975.45(5) KeV

^{5}He: n+^{4}He $^2P_{3/2}$ resonance [735(20) KeV]

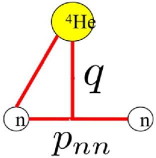
^{6}He(p,pα)2n knockout reactions at around 200 MeV/nucleon

FSI model (coherent model & Watson-Migdal enhancement factor)



T. Aumann *et al.*, Proposal for a Nuclear-Physics Experiment at the RI Beam Factory: Determination of the nn scattering length from a high-resolution measurement of the nn relative-energy spectrum produced in the $^6\text{He}(p,p\alpha)^2n$, $t(p,2p)^2n$, and $d(^7\text{Li},^7\text{Be})^2n$ reactions, proposal No. NP2012-SAMURAI55R1 (2020).

Framework to study the model dependence breakup ^{6}He:



- nn S-wave and ^{5}He $^2P_{3/2}$: EFT@LO & Yamaguchi 1-term potential

$$g_l(p) = p^l \Theta(\Lambda - p) \qquad g_{\mathrm{YG};l}(p) = p^l \left(1 + \frac{p^2}{\beta_l^2}\right)^{-l-1}$$

$$\langle p, l | V_{ij} | p', l' \rangle = 4\pi \delta_{l,l'} \delta_{l_{ij},l} g_{l_{ij}}(p) \lambda_{ij} g_{l_{ij}}(p')$$

- LGM (Local Gaussian Models) + 3BF.

$$V_c^{(l)}(r) := \bar{V}_c^{(l)} \exp\left(-r^2/\left(a_{c;l}^2\right)\right), \qquad V_{3\mathrm{B}}(\rho) := \frac{s_{3\mathrm{B}}}{1.0 + (\rho/\rho_{3\mathrm{B}})^{a_{3\mathrm{B}}}}$$

$$V_{SO}^{(l)}(r) := \bar{V}_{SO}^{(l)} \boldsymbol{LS} \exp\left(-r^2/\left(a_{SO;l}^2\right)\right). \qquad \rho_{3\mathrm{B}} = 5.0\,\mathrm{fm} \text{ and } a_{3\mathrm{B}} = 3$$

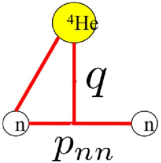
- LGM1: standard version of LGM input parameters for ^{6}He;
- LGM2: LGM1, but nc interaction only in $^2P_{3/2}$ (nc interaction in s-wave, d-wave and $^2P_{1/2}$ is turned off);
- LGM3: LGM2, but three-body potential deactivated.

Final State Interaction

$$\rho(p_{nn}) := \int \mathrm{d}q\, q^2 p_{nn}^2 \,|\Psi_c(p_{nn},q)|^2$$

$$G_1(p) = \frac{((p^2+\alpha^2)r_{nn}/2)^2}{\left(-\frac{1}{a_{nn}}+\frac{r_{nn}}{2}p^2\right)^2+p^2} \quad \alpha = 1/r_{nn}(1+\sqrt{1-2r_{nn}/a_{nn}})$$

[Enhancement factor]



$$\Psi_c^{(\mathrm{wFSI})}(p,q) = \Psi_c(p,q) + \frac{2}{\pi} g_0(p) \frac{1}{a_{nn}^{-1} - \frac{r_{nn}}{2}p^2 + \mathrm{i}p} \int \mathrm{d}p'\, p'^2 g_0(p') \left(p^2 - p'^2 + \mathrm{i}\epsilon\right)^{-1} \Psi_c(p',q)$$

Sudden breakup model [Yamashita, TF, Tomio PRC 72, 011601(R) (2005)]

$$a_{nn}^{(+)} = -16.7\,\mathrm{fm}\,, \qquad a_{nn}^{(0)} = -18.7\,\mathrm{fm}\,, \qquad a_{nn}^{(-)} = -20.7\,\mathrm{fm}$$

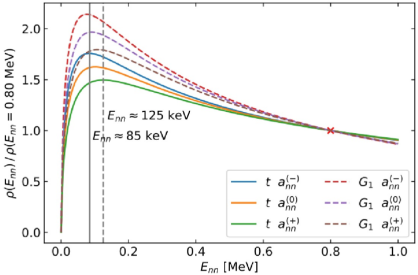
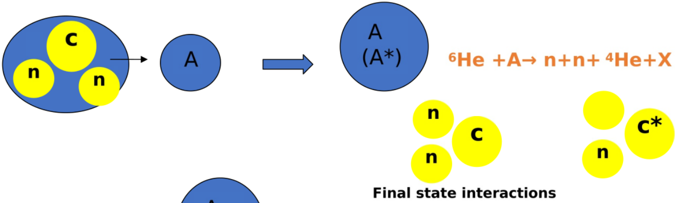


FIG. 5. Comparison of nn relative-energy distributions for different nn scattering lengths obtained with different FSI schemes. The calculation using the nn t-matrix is labeled as "t." $r_{nn} = 2.73$ fm is used. All results are computed using the projection $\Psi_c(p,q)$ and $\Lambda = 1500$ MeV. Uncertainty bands based on comparison with calculation with half as many mesh points and $\Lambda = 1000$ MeV are negligible. In order to be independent of the normalization the distribution is divided by its value at the energy indicated by the red cross. The solid and dashed vertical lines indicate the approximate positions of the maxima in the t-matrix based FSI scheme for $a_{nn}^{(-)}$ and $a_{nn}^{(+)}$, respectively.

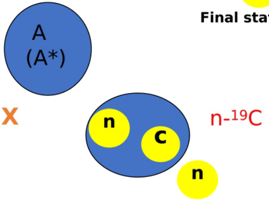
Probing the halo dynamics: low relative momentum (weakly bound)

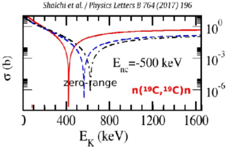
Perspectives on few-body cluster structures in exotic nuclei
Bazin, Becker, Bonaiti, Elster, Fossez, TF...FBS64 (2023) 25

Borromean, Samba, (Tango), All-bound



$^{19}C + A \to {}^{19}C + n + X$

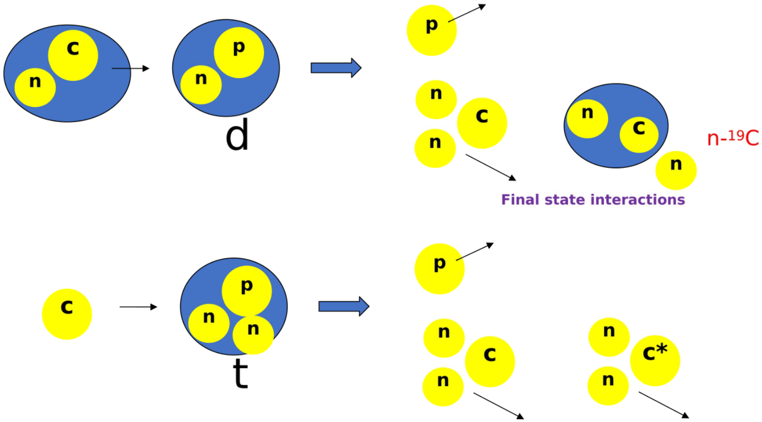




Probing the halo dynamics: low relative momentum; (d,p) and (t,p) reactions

Perspectives on few-body cluster structures in exotic nuclei
Bazin, Becker, Bonaiti, Elster, Fossez, TF...FBS64 (2023) 25

^{19}C + d -> ^{18}C + n + n + p



Beyond 2n halos

Hiyama, Lazauskas, Carbonell, TF

	How many scales determines the halo state?
^{21}B = ^{17}B + 4n ^{22}C = ^{18}C + 4n	- S_{2n} for ^{19}B = ^{17}B + 2n ^{20}C = ^{18}C + 2n - a_{nn} and a_{nc} - Enough for ^{17}B + 3n and ^{17}B + 4n ^{18}C + 3n and ^{18}C + 4n
^{8}He = ^{4}H + 4n	- S_{2n} for ^{6}He = ^{4}He + 2n - a_{nn} and ^{5}He = ^{4}He + n ($P_{3/2}$) @ 735(20) keV - Enough for ^{6}He + 4n !
^{28}O = ^{24}O + 4n	- S_{2n} for ^{26}O = ^{24}O + 2n - a_{nn} and ^{25}O = ^{24}B + n ($D_{3/2}$) @ 749(10) keV & width 88(6) keV - Possibly enough for ^{24}O + 4n !

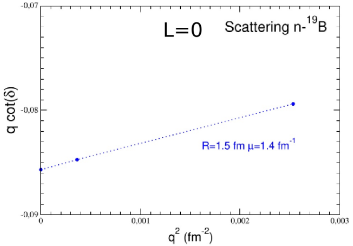
Beyond 2n halos: initial explorations

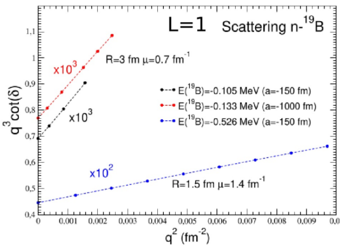
Hiyama, Lazauskas, Carbonell, TF

$$V_{n^{17}\mathrm{B}}(r) \;=\; V_r\,\frac{e^{-2\mu r}}{r}\,\left(1-e^{-\mu(R-r)}\right)$$

^{17}B + 3n: no-bound state

^{19}B + n -> ^{19}B + n: elastic scattering below B.U. thres.
Faddeev-Yakuboski calculations





^{17}B + 3n: no-bound state with FY

Leblond et al. First observation of ^{20}B and ^{21}B. PRL 121, 262502 (2018)

- No new scales beyond core+2n system (Pauli principle)!
- Scaling functions?

f+(ᴬI+2f) scattering @unitarity

Hiyama, Lazauskas, Carbonell, TF

Minnesota potential for f-f and I-f: range ~ 1fm

A	$E[I+2f]$ [MeV]	$a_{L=0}$ [fm]	$a_{L=1}$ [fm^3]	$E^{L=1}_{\mathrm{res.}}[I+3f]$ [KeV]	$R^{L=1}_{4,3}$
1	-2.598	4.977	-272.2	321 - 124i	0.124 - 0.0477i
2	-2.220	4.872	-138.4	558 - 318i	0.251 - 0.143i
3	-2.130	4.757	-113.2	622 - 379i	0.292 - 0.178i
5	-2.079	4.613	-94.45	665 - 413i	0.320 - 0.199i
16	-2.058	4.400	-75.61	695 - 425i	0.338 - 0.207i

TABLE VI: Faddeev-Yakubovski calculations for the $I+2f$ and $I+3f$ systems (the neutron is the adopted fermion). Minnesota potential at unitarity with V_a=98.40275 MeV and m_n=939.5654 MeV. First column: ground state energy for the $I+2f$ in MeV. Second column: scattering length for the $f+(I+2f)$ collision. Third column: scattering volume for the $f+(I+2f)$ collision. Fourth column: p-wave resonance position, $E^{\mathrm{res.}}_{L=1}$, in KeV. Fifth column: $R^{L=1}_{4,3} = E^{L=1}_{\mathrm{res.}}[I+3f]/|E[I+2f]|$ is the ratio of the resonance energy with the three-body ground state energy.

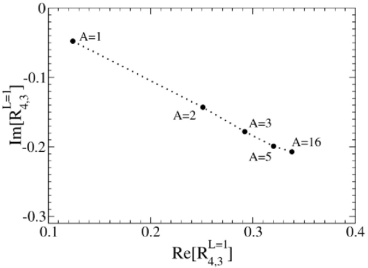


FIG. 4: Position of the p-wave resonance of the elastic fermion and fermion+impurity scattering. The plot presents the ratic between the real and imaginary parts of resonance energy with the binding energy of the $I+2f$ ground state computed close to unitarity (see Table VI). The dotted line guide the eyes.

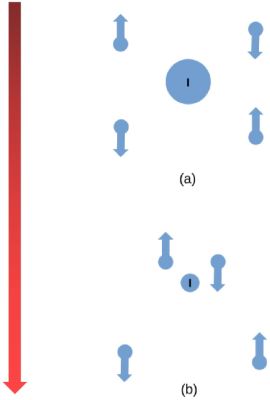
No bound I+3f!

Preliminary calculations for the l+4f systems

Hiyama, Lazauskas, Carbonell, TF

Binding of 2f in (l+2f)

Halo structure rises!



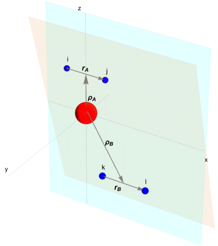
$^{22}C = {}^{18}C + 4n$?

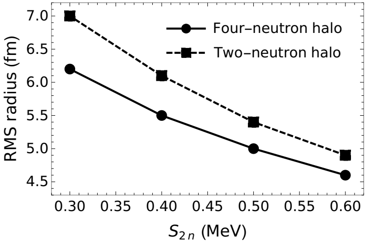
simplified model: in collaboration with Francisco, Issifu, Rosa, Yamashita and Hupin

$^{22}C = {}^{18}C+4n$ @ unitarity

Francisco, Issifu, Rosa, Yamashita, Hupin, TF

$$\Psi_{4n} = \mathcal{A}[\Psi^A_{ij} \otimes \Psi^B_{kl}]$$





Neutron RMS[^{22}C] vs. $S_{2n}[^{22}C]$. ($S_{2n}[^{20}$C]=3.5 MeV). $(S_{2n}[^{22}C] = 0.56^{+0.27}_{-0.20}\,\text{MeV}^\dagger)$

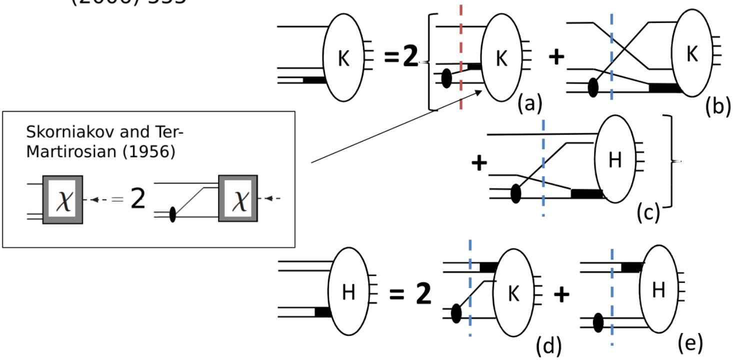
$\langle r_n^2\rangle^{\frac{1}{2}} \simeq 6.4\,\text{fm} \quad (^{22}C = {}^{20}C + 2n) \quad \& \quad \langle r_n^2\rangle^{\frac{1}{2}} \simeq 5.3\,\text{fm} \quad (^{22}C = {}^{18}C + 4n)$

† Togano et al. PLB761(2016)412: $r_m[^{20}C] = 2.97^{+0.03}_{-0.05}\,\text{fm}$ & $r_m[^{22}C] = 3.44 \pm 0.08\,\text{fm}$
Kanungo et al. PRL117(2016)102501: $r_m[^{18}C] = 2.86 \pm 0.04\,\text{fm}$

Faddeev-Yakubovski eqs (4-bosons), zero-range int., reg. and renor.: Scaling-plot

Collapse of the 4B system & 3B energy fixed

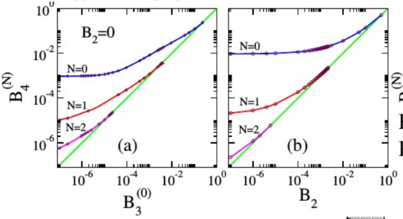
Yamashita, Tomio, Delfino, TF, EPL 75 (2006) 555



Subtracted Green's Functions: $G_0^{(N)} = \frac{1}{E-H_0} - \frac{1}{-\mu_N^2-H_0}$

with μ_3 (RED): 3B scale & μ_4 (BLUE): 4B scale

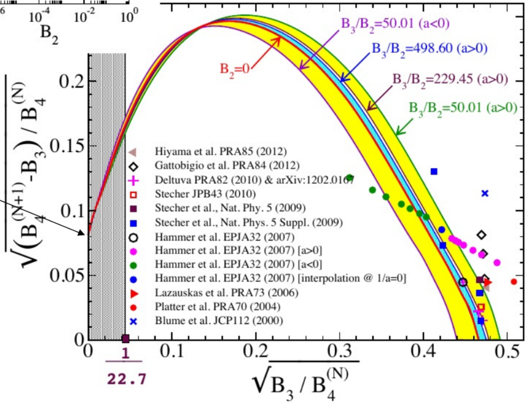
$a = \infty$



Scaling plot

Hadizadeh, Yamashita, Tomio, Delfino, TF, PRL107, 135304 (2011)

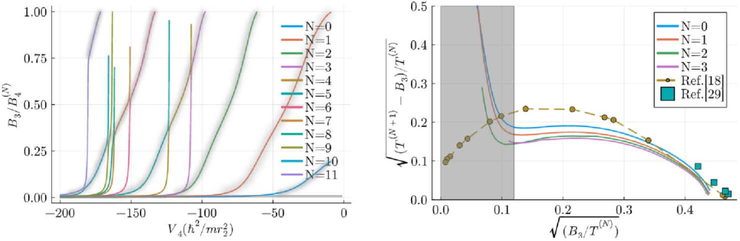
$$\boxed{B_4^{(N)}/B_4^{(N+1)} \sim 151}$$



Universal tetramer limit-cycle at the unitary limit

TF & M. Gattobigio, PRA 108, 033302 (2023)

Gaussian two, three and four-body potentials @ unitarity



- - - Hadizadeh et al PRL 107, 135304 (2011)

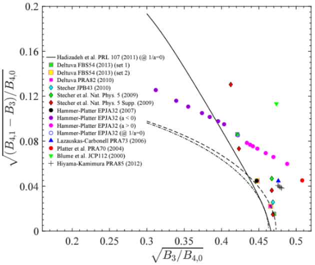
■ Deltuva PRA82 (2010) & arXiv 1202.0167

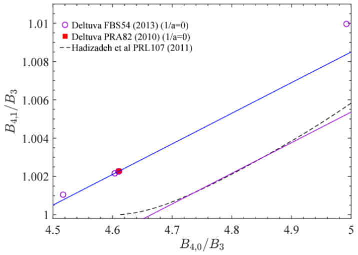
Limit Cycle in potential models

4B Scaling function: EFT fitting

Wu, TF, Higa, van Kolck, PRA 109, 043301 (2024)

Four-boson first excited state near two-body unitarity





$$\frac{B_{4,1}}{B_3} \simeq 0.9255 + 0.0159741\,\frac{B_{4,0}}{B_3}$$

In EFT 4-body scale @ NLO
Perturbative effect of the Four-body force

Summary

- Few-body framework for dilute neutron halos: dominance of few scales;
- Universal scaling functions (limit cycles) depending on few parameters: weakly bound systems / zero-range interactions / close to unitarity (Efimov limit);
- Significant model independence in the halo properties, scalings as seen in the examples:

$$^{6}\mathrm{He},\ ^{11}\mathrm{Li},\ ^{14}\mathrm{Be},\ ^{19}\mathrm{B},\ ^{20}\mathrm{C},\ ^{22}\mathrm{C}$$

- Neutron-halo systems close to the unitarity limit carry the scales of the 2n-core system (Pauli principle);
- Extension to dilute neutron halos – $4n$: possible dominance of few scales.
- Interwoven limit cycles for increasing boson number (seen in Hamiltonian model with 4B interaction);

Outlook..

- Structure & spectrum:

$$(\mathrm{core}+n+n),\ (\mathrm{core}+n+n+n),\ (\mathrm{core}+n+n+n+n),\ldots$$

- Universal properties of multi-neutron halos, e.g. ^{22}C?
- FSI between fragments of the halo in the continuum with low relative energies:

$$(\mathrm{core}+n+n),\ (\mathrm{core}^*+n+n),\ (\mathrm{core}+n+n+n),\ (\mathrm{core}+n+n+n+n),\ldots$$

- Probe the halo in the continuum (description within a few-body framework);
- **RIKEN experiments...**

CAS President's International Fellowship Initiative (PIFI)

Thanks to Collaborators:

S Adhikari
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F Bringas
C Bertulani
B Carlson
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A Delfino+
E Garrido
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M Gobel
M Hadizadeh
HW Hammer
E Hiyama
M Hussein+
G Krein
R Lazauskas
L Madeira
D Phillips
D Rosa
M Schalchi
L A Souza
V Timóteo
L Tomio
M Yamashita

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IJCLab & ITA Project CAPES/COFECUB 2020-2024
+ other French & Brazilian Institutions
U. van Kolck, J. Carbonell, R. Lazauskas,
M Gattobigio

THANK YOU!

Special thanks to the hospitality of Institute of Theoretical Physics/CAS, Institute of Modern Physics/CAS, Department of Modern Physics, University of Science and Technology of China.

Critical Stability of Few-Body Quantum Systems

19 – 23 de out. de 2026
Mainz Institute for Theoretical Physics, Johannes Gutenberg University
Fuso horário Europe/Berlin

Digite o seu termo de pesquisa

- Visão Geral
- General Information
- Application Form
- Travel Information
- Code of Conduct

Contact @ MITP: Inke Klabunde
CSQ2026@uni-mainz.de

Registration closes 19 June 11:59 P.M. CET.

The topical workshop "Critical Stability of Few-Body Quantum Systems" has a broad scope embracing few-body quantum systems at the edge between stability and instability, in atomic, nuclear and hadronic physics. The main purpose is to bring people together from the different communities, experimental as well as theoretical researchers, to focus on the interdisciplinary aspects of techniques, methods, concepts and ideas surpassing the specialization of the different sub-fields of physics. Inside the critical stability region many systems show universal behavior and, in spite of the difficulty to identify similarities among the approaches in several fields, the effort in this direction may be very useful. The progress in science would clearly be faster if we could "speak the same language" and immediately distribute the knowledge from one field to another. Developing collaborations crossing the sub-field barriers is an efficient method to spread new insights. The aim of this workshop is the improvement of the flux of information between the subareas of Few-Body Physics and, at the same time, between the experimental and the theoretical research.

Começar (Iniciar) 19 de out. de 2026 10:00
Fim 23 de out. de 2026 17:00
Europe/Berlin

Mainz Institute for Theoretical Physics, Johannes Gutenberg University
02.430
Staudingerweg 9 / 2nd floor, 55128 Mainz
Ir para o mapa

Organized by Alejandro Kievsky (INFN), Tobias Frederico (ITA, diversity coordinator), Hans Otto Uldall Fynbo (Aarhus Univ.), Jean-Marc Richard (IP2I Lyon), and Sonia Bacca (JGU Mainz).

MITP supports equal opportunities in science.

BACKUP MATERIAL

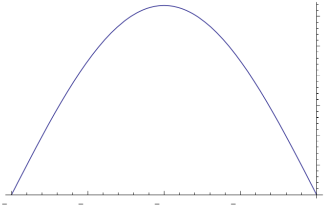
Scale invariance and Breaking: example integral equation

$$f(k) = \lambda \int_0^\infty dp\, p \frac{f(p)}{k^2 + p^2}$$

int. eq. invariant under: $k \to \xi k$ and $p \to \xi p$

$f(k)$ and $f(\xi k)$ solutions $f(k) = k^\eta$

transcendental eq.: $\lambda = -\frac{2}{\pi} \sin\left(\frac{\pi\eta}{2}\right)$



$$\lambda < \lambda_c = \frac{2}{\pi} \text{ for } -2 < \eta < 0$$

Maximum value

28

$$\text{IF } \lambda > \lambda_c = \frac{2}{\pi} \quad \text{then} \quad \eta = -1 + \imath s_0$$

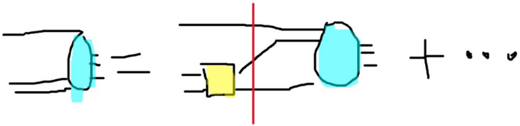
$$\text{Solution: } f(k) = k^{-1} \sin(s_0 \log k/k^*)$$

Continuous symmetry breaking to a discrete one & k*dimensional scale

$$k \to \exp(n\pi/s_0)\, k$$

Applies to 3bosons, 4bosons @unitarity, relativistic bound states (Bethe-Salpeter eq.)

Regularization: Skorniakov and Ter-Martirosyan equations (1956)



- *Thomas-Efimov effect (s-wave SR→ ZR interactions)*
- *continuous scale symmetry broken to a discrete one*
- $a_{nn}\,(e_{nn})$, $a_{nc}\,(e_{nc})$, S_{2n} *(define all low-energy observables)*
- *Correlations between observables: scaling functions (limit cycles discrete scaling)*

Adhikari, TF, Goldman, PRL74, 487 (1995); Adhikari, TF, *ibid.* 74, 4572 (1995)(Subtracted Eqs.)

TF, Timóteo, Tomio NPA 653, 209 (1999) – Weinberg delta+OPEP (Subtracted Eqs.)

Bedaque, Hammer, Van Kolck PRL 82, 463(1999) (EFT)

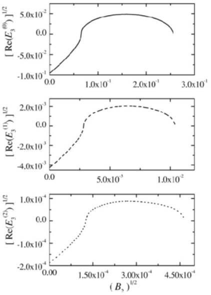
Subtracted equations equivalent to the introduction of a three-body potential in H (EFT)

Scaling function & Limit Cycle - Resonances

TRIATOMIC CONTINUUM RESONANCES FOR LARGE...

Bringas, Yamashita,TF

PHYSICAL REVIEW A **69**, 040702(R) (2004)



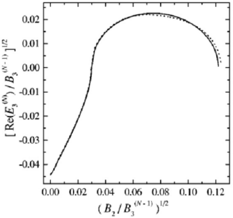


FIG. 3. Ratio of the real part of $E_3^{(N)}$ and $B_3^{(N-1)}$ as a function of $B_2/B_3^{(N-1)}$. The solid line is the results for $N=1$ and the dashed line for $N=2$.

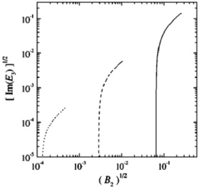


FIG. 2. Imaginary part of the three-body resonance energy as a function of the two-body virtual state energy in units of $\mu=1$. The curves are labeled as in Fig. 1.

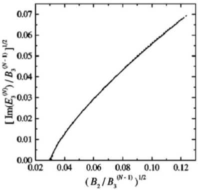
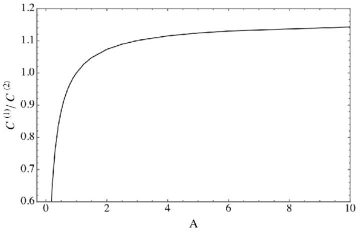
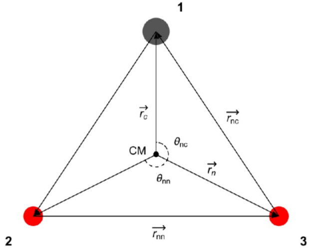


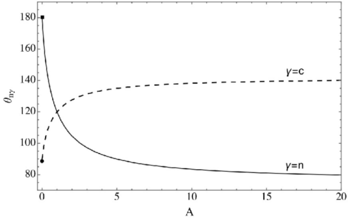
FIG. 4. Ratio of the imaginary part of $E_3^{(N)}$ and $B_3^{(N-1)}$ as a function of $B_2/B_3^{(N-1)}$. The curves are labeled as in Fig. 3.

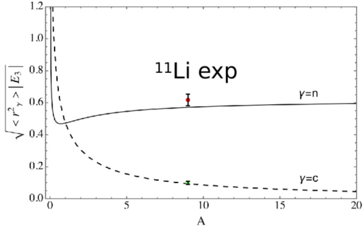
$C^{(1)}$ c-(nn) ->0 for A->0
$C^{(2)} = C^{(3)}$ n-(nc)

credits to R. M. Francisco (PD/ITA)









Four-Boson continuous scale symmetry breaking

TF, de Paula, Delfino, Tomio, FBS60, 46 (2019)

Solution: UV FY eqs.

$$\mathcal{K}(q,p) = \frac{q^\eta}{\sqrt{\frac{3}{4}q^2 + \frac{2}{3}p^2}} f_K\left(q/p\right) \quad \text{and} \quad \mathcal{H}(q,p) = \frac{q^\eta}{\sqrt{\frac{1}{2}q^2 + p^2}} g_H\left(p/q\right)$$

Approximate UV form of FY eqs.:

3-boson eq. SKTM

$$f_K\left(x\right) = \overbrace{\frac{4\,x^{-\eta}}{\sqrt{3}}\int d\Omega dy\, \frac{y^{1+\eta}}{x^2+y^2+xy\,z_0}\left\{f_K\left(y\right)\right.}^{\text{3-boson eq. SKTM}} \left. + \left(\tfrac{1}{3}\right)^\eta f_K\left(\tfrac{1}{3}\right) + g_H\left(\tfrac{1}{2}\right)\right\}$$

$$g_H\left(x\right) = \frac{2}{\pi}\int_0^\infty \frac{dy\,y}{y^2+x^2}\left\{g_H\left(y\right) + 2\left(\tfrac{2y}{3}\right)^\eta f_K\left(\tfrac{2}{3}\right)\right\}.$$

$$\boxed{f_K(x) = f_0 \quad \& \quad g_H(x) = x^\eta}$$

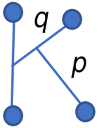
Efimov parameter for geometric series of bound states

$$\boxed{1 = \frac{8}{\sqrt{3}\,(\eta+1)}\frac{\sin\left[(\eta+1)\frac{\pi}{6}\right]}{\cos\left[(\eta+1)\frac{\pi}{2}\right]}\left\{1 - \frac{1}{3^\eta}\left[\frac{1-\sin\left(\eta\frac{\pi}{2}\right)}{1+\sin\left(\eta\frac{\pi}{2}\right)}\right]\right\}}$$

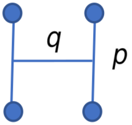
Scale symmetry breaking to a Discrete scale symmetry

Solution of the transcendental equation $\eta = -0.346 \pm 1.552\mathrm{i}$

$$\boxed{\mathcal{K}(q,p) \approx f_0 \frac{\sin(s_4 \log q/\Lambda^*)}{\sqrt{\frac{3}{4}q^2 + \frac{2}{3}p^2}} \quad \& \quad \mathcal{H}(q,p) \approx \frac{\sin(s_4 \log p/\Lambda^*)}{\sqrt{\frac{1}{2}q^2 + p^2}}}$$



4-body scale



Compare to imaginary part: ***1.25 i*** *from Hadizadeh et al. PRL107 (2011) 135304*

$$B_4^{(N)}/B_4^{(N+1)} = 151 = \exp(2\pi/s_4) \quad \& \quad s_4 = 1.25$$

In EFT 4-body scale @ NLO - Bazak et al. PRL 122 (2019) 143001