Hyperfine Electronic Bridge in the ^{229}Th Nucleus

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Contents

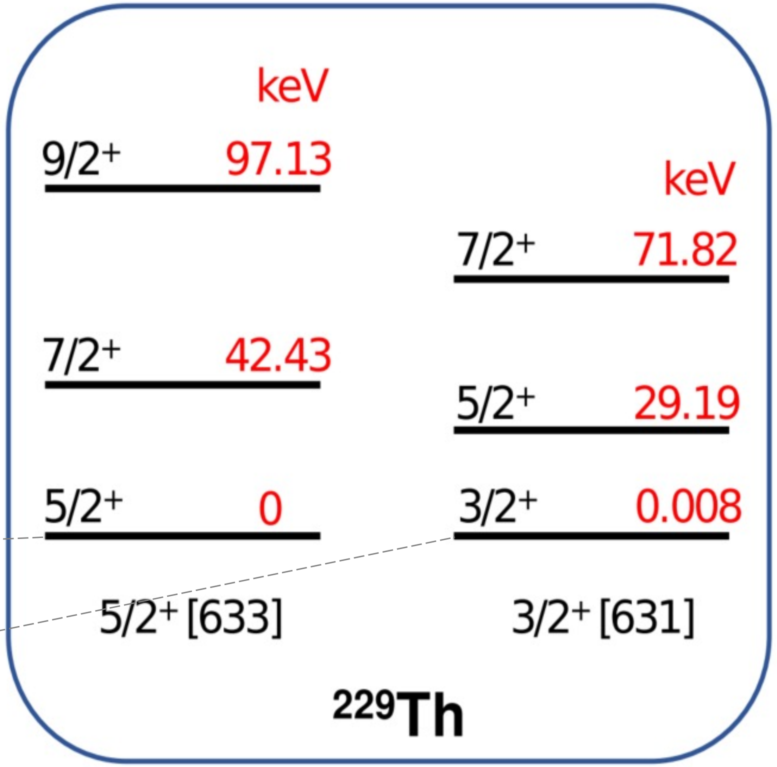
1. Introduction to the ^{229}Th isomer

Thorium (Th): atomic number 90

^{229}Th: a radiative isotope with a half-life of about 7900 yr

^{229m}Th: a unique isomeric state with an energy of about 8 eV

Typical nuclear excitation energies: keV and MeV



Partial nuclear levels of ^{229}Th

W. Wang, H. Zhang, X. Wang, J. Phys. B **54,** 244001 (2021)

1. Introduction to the ^{229}Th isomer

Atomic clock: a frequency reference based on a specific transition between atomic levels

Fractional frequency uncertainty: 10^{-19}

Limitation: external electromagnetic perturbations

A potential solution: ^{229}Th nuclear clock

E. Peik, C. E. Tamm, Europhys. Lett. **61**, 181 (2003)

- High robustness against external electromagnetic perturbations
- Narrow relative linewidth $\Delta E/E \sim 1.6\times10^{-20}$
- Accessible with a vacuum-ultraviolet (VUV) laser: 8 eV ~150 nm

A candidate for next-generation frequency standard

V. V. Flambaum, Phys. Rev. Lett. **97**, 092502 (2006)

Precision measurements, search for variations of fundamental constants

1. Introduction to the ^{229}Th isomer

Weak radiative transition: lifetime ~ 2500 s

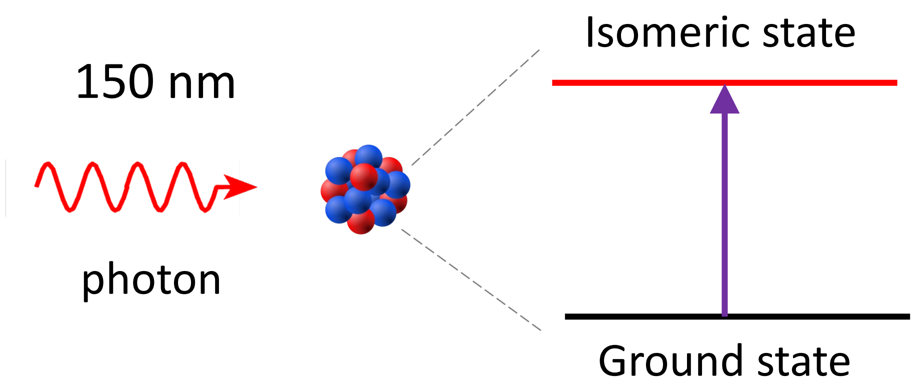
J. Tiedau et al., Phys. Rev. Lett. **132**, 182501 (2024)

It is difficult to excite the ^{229}Th isomer

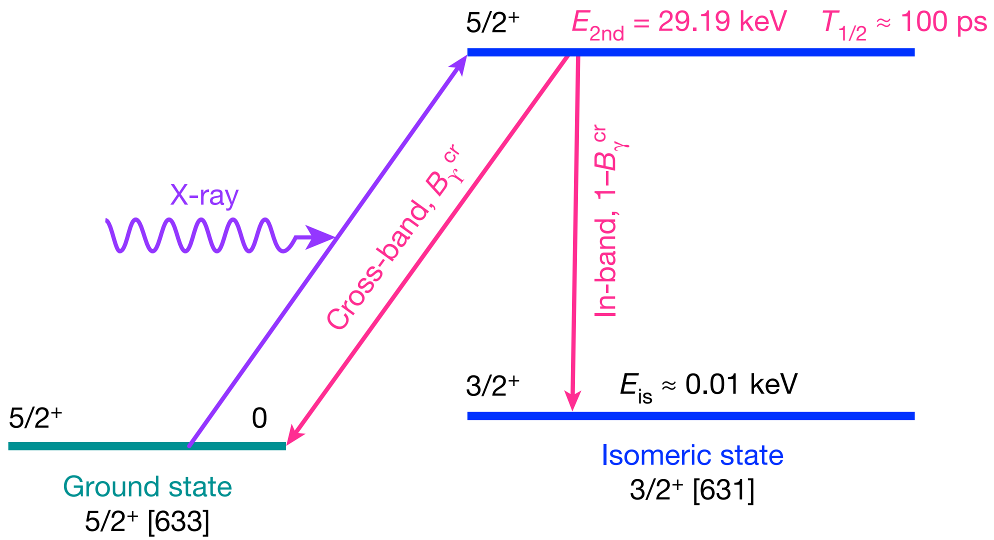
Various methods are proposed

1. Introduction to the ^{229}Th isomer

1. Direct optical excitation：



2. Indirect x-ray pumping:



After many years of effort, it was finally successful.

C. Zhang et al., Nature (London) **633**, 63 (2024)

J. Tiedau et al., Phys. Rev. Lett. **132**, 182501 (2024)

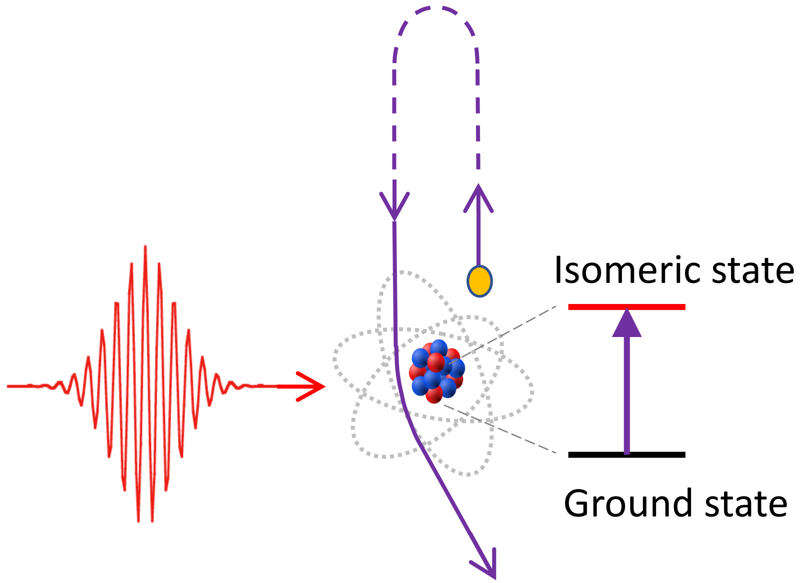
R. Elwell et al., Phys. Rev. Lett. **133**, 013201 (2024)

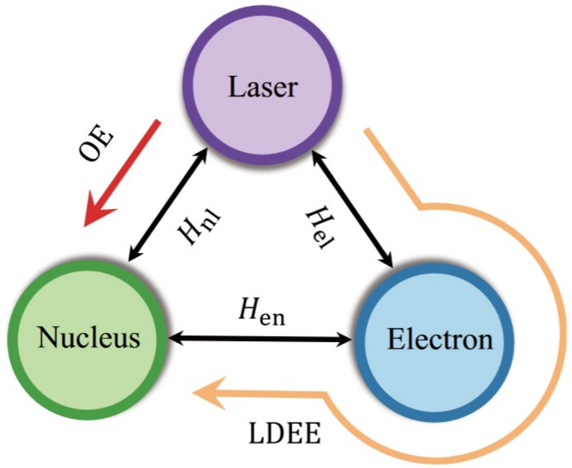
T. Hiraki et al., Nat. Commun. **15**, 5536 (2024)

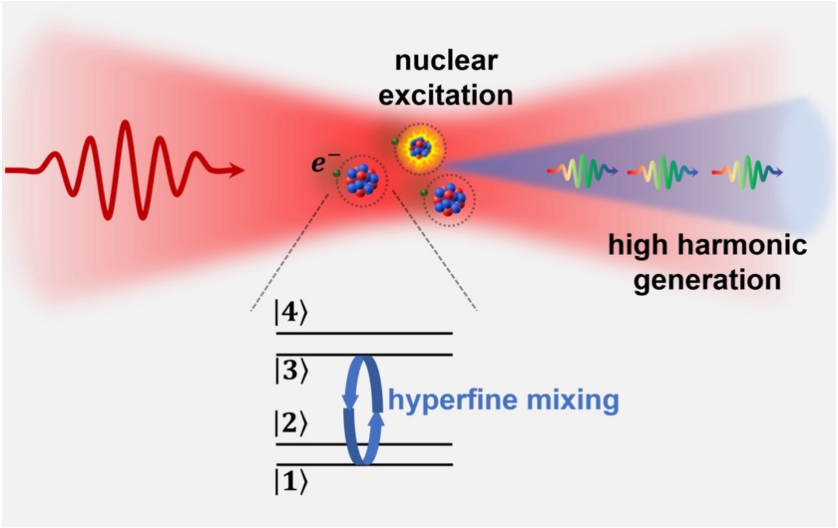
T. Masuda et al., Nature (London) **573**, 238 (2019)

1. Introduction to the ^{229}Th isomer

3. Strong laser field excitation:







Semi-classical method: recollision electron

Quantum theory: laser-nucleus-electron interaction

$Th^{0+,1+,2+,3+,4+} \sim 10^{-11}$ per pulse

Nuclear hyperfine mixing

$Th^{89+} \sim 0.1 - 0.9$ per pulse

W. Wang, J. Zhou, B. Liu, and X. Wang, Phys. Rev. Lett. **127**, 052501 (2021)

W. Wang and X. Wang, Phys. Rev. Res. **5**, 043232 (2023)

H. Zhang, T. Li, and X. Wang, Phys. Rev. Lett. **133**, 152503 (2024); Phys. Rev. C **111**, 044614 (2025)

1. Introduction to the ^{229}Th isomer

4. Electronic bridge

5. Other methods

2. Electronic bridge

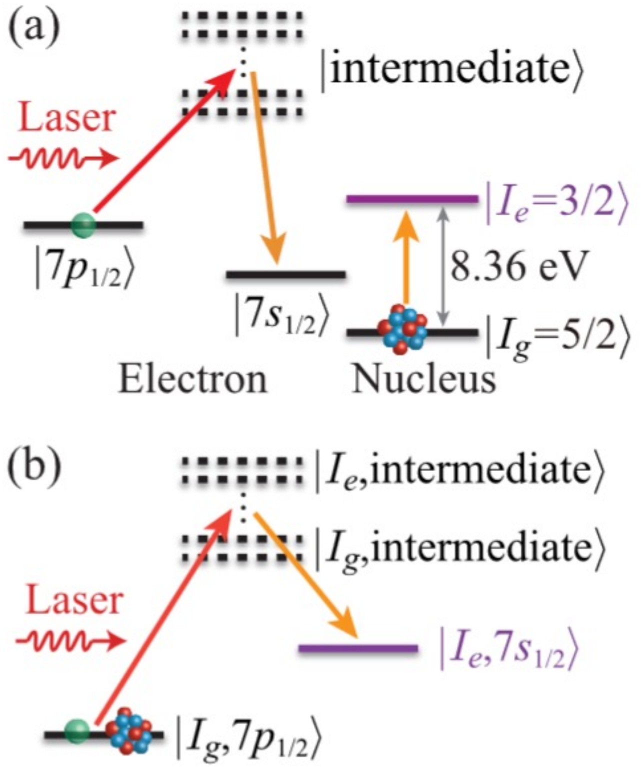
Electronic Bridge (EB):

- Nuclear transition is induced by laser-driven electronic transitions
- Photon energy +Eletronic transition energy=Nuclear energy

I. S. Batkin, Sov. J. Nucl. Phys. **29**, 464 (1979)

E. V. Tkalya, JETP Lett. **55**, 216 (1992)

- EB can enhance the effective coupling between laser and ^{229m}Th
- Two-photon EB: two lasers, wavelength ~ 300 nm



Schematic diagram of EB in $^{229}Th^{3+}$

S. G. Porsev *et al.*, Phys. Rev. Lett. **105**, 182501 (2010)

W. Wang, F. Zou, S. Fritzsche, and Y. Li, Phys. Rev. Lett. **133**, 223001 (2024)

2. Electronic bridge

Limitations of existing research:

1. Neglecting the hyperfine structure

Neglecting the hyperfine splitting of levels, we can represent the total wave function as a product of the nuclear wave function and the electronic wave function. For instance,

S. G. Porsev and V. V. Flambaum, Phys. Rev. A **81**, 032504 (2010)

Hyperfine splitting in $^{229}Th^{3+}$: GHz

Measurement uncertainty of $E_{\rm n}$: kHz

J. Tiedau *et al.*, Phys. Rev. Lett. **132**, 182501 (2024)

C. Zhang et al., Nature (London) **633**, 63 (2024)

2. Excitation rate based on the Fermi's golden rule

Perturbation behavior, incoherent excitation

$$W_{ba}^{\rm in} = W_{ab}\frac{4\pi^3c^2}{\omega^3}I_\omega.$$

3. For two-photon EB, the decay of excited electronic level is neglected

As a result, the actual efficiency of EB method remains unknown

2. Dressed hyperfine state

Atomic level: $(I, \gamma J)$ I is the nuclear spin, J is the electronic angular momentum

γ denotes other electronic quantum numbers

Hyperfine interaction: $H_{\text{n-e}} = \sum_{\tau K} \mathcal{M}^{(\tau K)} \cdot T^{(\tau K)}$,

With $H_{\text{n}-\text{e}}$, hyperfine level: $(I, \gamma J, F)$ F is the total angular momentum

Dressed hyperfine state: $|[I\gamma J]FM\rangle = a|I\gamma J; FM\rangle + \sum_t b_t |I_t \gamma_t J_t; FM\rangle$ ⟷ $(I, \gamma J, F)$

- Eigenstate of the coupled electron–nucleus system

Hyperfine-coupled basis: $|I\gamma J; FM\rangle$

- Mixing of different nuclear states, i.e., nuclear hyperfine mixing

Mixing coefficient: $b_t = \sum_{\tau K} \frac{(-1)^{I+J_t+F}}{E_0 - E_t} \begin{Bmatrix} I_t & J_t & F \\ J & I & K \end{Bmatrix}$
$\times \langle I_t || \mathcal{M}^{(\tau K)} || I \rangle \langle \gamma_t J_t || T^{(\tau K)} || \gamma J \rangle,$

W. Wang, F. Zou, S. Fritzsche, and Y. Li, Phys. Rev. Lett. **133**, 223001 (2024)

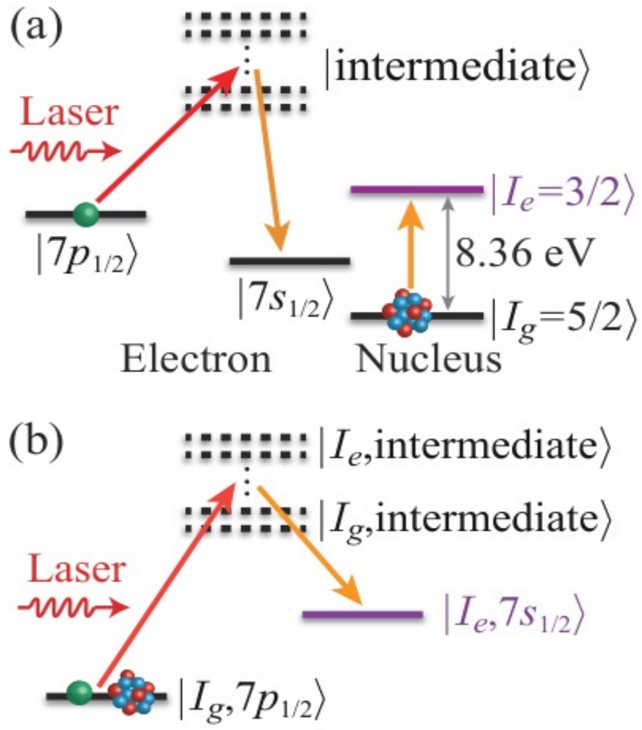
2. Hyperfine electronic bridge

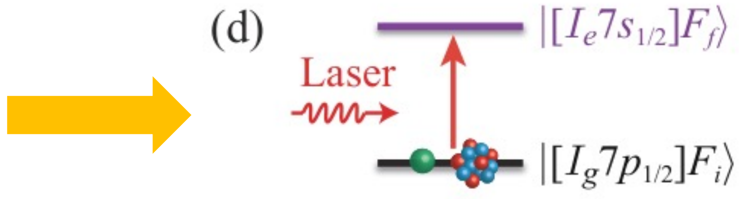
Hyperfine EB (HEB) transition: $|[I_i\gamma_iJ_i]F_iM_i\rangle \rightarrow |[I_f\gamma_fJ_f]F_fM_f\rangle$ or $(I_i,\gamma_iJ_i,F_i) \rightarrow (I_f,\gamma_fJ_f,F_f)$

$I_i \neq I_f,\ \gamma_iJ_i \neq \gamma_fJ_f$

The advantages of HEB:

- Hyperfine structure is considered
- The intermediate state of traditional EB was eliminated
- Easy to combine with the quantum-optical methods





- Two-level transition

W. Wang, F. Zou, S. Fritzsche, and Y. Li, Phys. Rev. Lett. **133**, 223001 (2024) **W. Wang**, S. Fritzsche, and Y. Li, Phys. Rev. A **112**, 022811 (2025)

2. Quantum-optical model based on HEB

With the HEB transition, a quantum-optical model is readily constructed

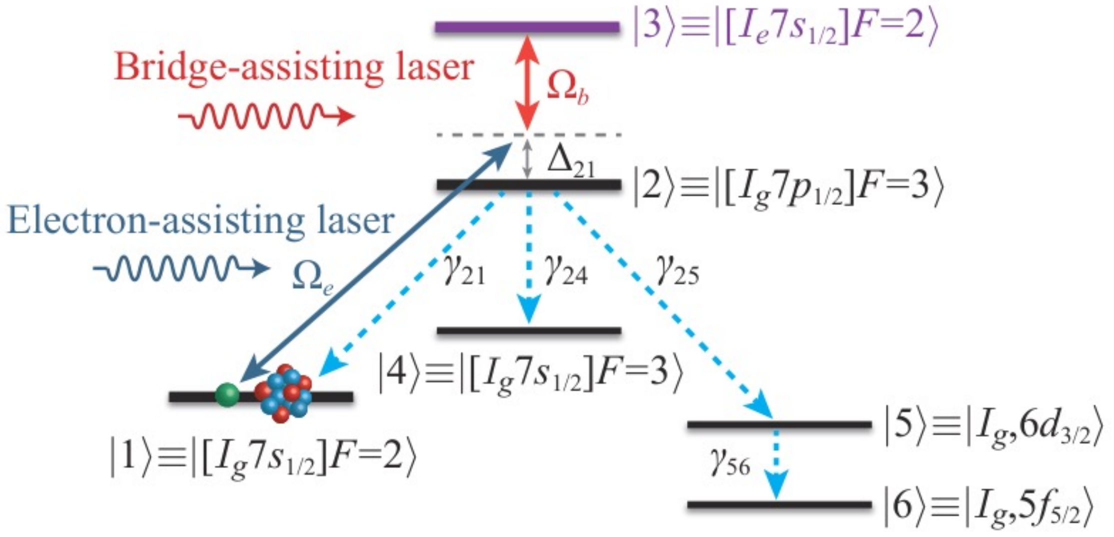
$$^{229}\mathrm{Th}^{3+}: \left(I_g, 7p_{1/2}, F=3\right) \rightarrow \left(I_e, 7s_{1/2}, F=2\right)$$

$\left|\left[I_g 7p_{1/2}\right]F_2\right\rangle \equiv |2\rangle$ is unstable (1.1 ns) with three decay channels

Quantum master equation: $\dot{\rho} = -i[H_{\mathrm{I}}(t), \rho] + \mathcal{L}\rho$

$$H_{\mathrm{I}}(t) = \Delta_{21}\hat{\sigma}_{22} + \left(\Omega_e(t)\hat{\sigma}_{21} + \Omega_b(t)\hat{\sigma}_{32} + \mathrm{H.c.}\right)$$

- Δ_{21} one-photon detuning
- Ω_e, Ω_b Rabi frequencies



A schematic diagram of a six-level HEB scheme in $^{229}\mathrm{Th}^{3+}$

W. Wang, F. Zou, S. Fritzsche, and Y. Li, Phys. Rev. Lett. **133**, 223001 (2024)

2. Isomeric population transfer of ^{229}Th

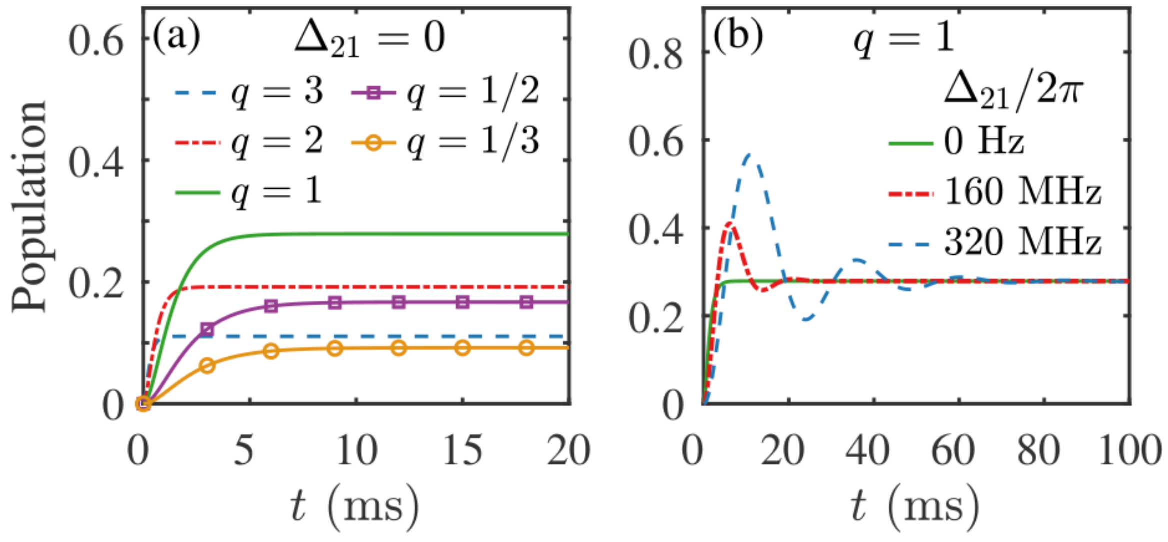
Time-independent scheme: envelop functions of Rabi frequencies $f_b(t) = f_e(t) = 1$

$$q = |\Omega_e/\Omega_b|$$

Maximum isomeric population at $q = 1$: 27.9%

Δ_{21} does not affect the maximum isomeric population

Bridge-assisting laser intensity: 3×10^5 W/cm^2



(a, b) Isomeric population as a function of time

W. Wang, F. Zou, S. Fritzsche, and Y. Li, Phys. Rev. Lett. **133**, 223001 (2024)

2. Isomeric population transfer of ^{229}Th

Time-independent scheme:

stimulated Raman adiabatic passage (STIRAP)

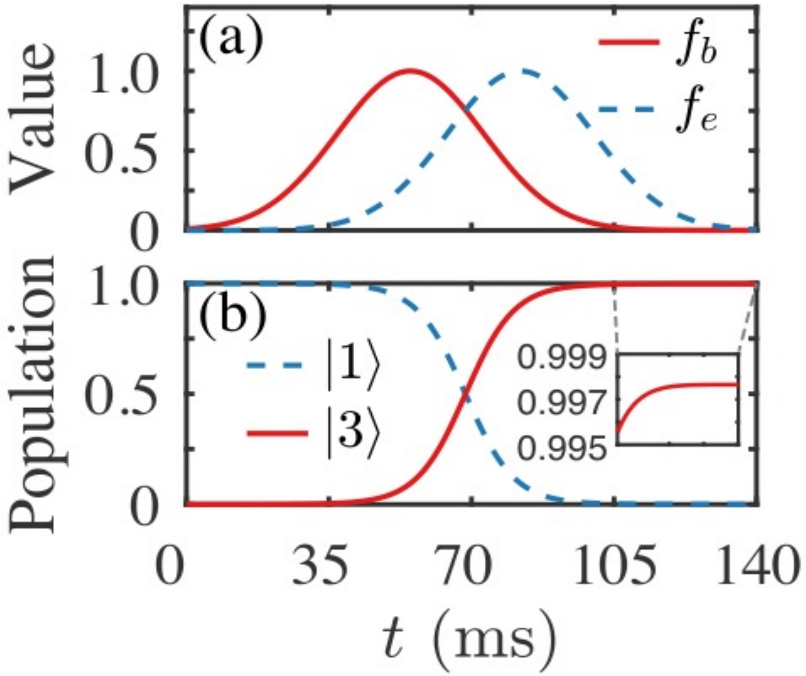
Envelop functions: $f_b(t) = \exp[-(t-t_b)^2/T^2]$

$f_e(t) = \exp[-(t-t_e)^2/T^2]$

$|\Omega_b(t_b)| = |\Omega_e(t_e)|$ adiabatic condition : $\Omega_b^2(t_b)T^2 \gg 1$

Bridge-assisting laser peak intensity: 2×10^7 W/cm^2

At final time 140 ms: isomeric population of 99.7%



(a) Gaussian envelop functions. (b) The population of $|1\rangle$ and $|3\rangle$ (isomeric state) as a function of time

W. Wang, F. Zou, S. Fritzsche, and Y. Li, Phys. Rev. Lett. **133**, 223001 (2024)

3. The variation of fine-structure constant in HEB

Fine-structure constant: $\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$

Some theories beyond the Standard Model predict the possibility that the variation of α

Experiments: cosmological observations, atomic clock measurements

For atomic clock, introduce the sensitivity factor K_α: $\frac{\dot{\omega}}{\omega} = K_\alpha \frac{\dot{\alpha}}{\alpha},$

For ordinary atomic clock, $|K_\alpha| \sim 1$

^{229}Th nuclear clock, $K_\alpha = -8.2\times10^3$

P. Fadeev, J. C. Berengut, and V. V. Flambaum, Phys. Rev. A **102**, 052833 (2020)

3. The variation of fine-structure constant in HEB

In previous studies, electronic and nuclear transitions are treated as independent processes.

If correlation arises, how does this influence the clock performance?

HEB transition provides a natural framework to answer this question

$$K_\alpha = \frac{\Delta E_C}{\omega}\left[1 - \sum_t (|b_{i,t}|^2 + |b_{f,t}|^2)\right]$$

ΔE_C : the Coulomb energy difference between nuclear isomeric and ground states

$b_{i,t}$ and $b_{f,t}$ are the mixing coefficients of the initial and final dressed hyperfine states. Since these mixing coefficients are small. Qualitatively, for a given nucleus, the smaller the transition energy, the larger the sensitivity factor.

W. Wang, S. Fritzsche, and Y. Li, Phys. Rev. A **112**, 022811 (2025)

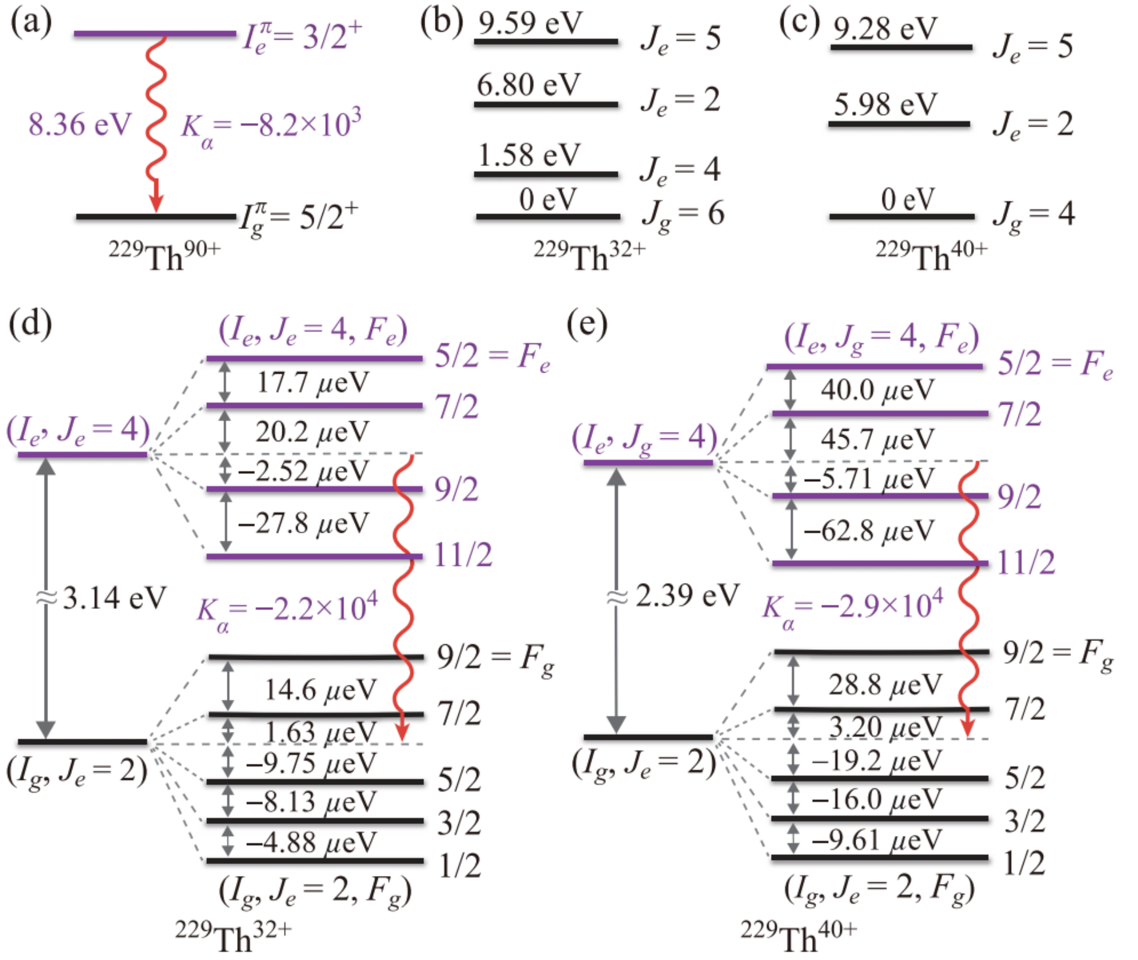
3. The variation of fine-structure constant in HEB

(a) Nuclear levels of $^{229}Th^{90+}$. (b) Partial electronic levels of $^{229}Th^{32+}$. (c) Partial electronic levels of $^{229}Th^{40+}$. (d) HEB transitions in $^{229}Th^{32+}$. (e) HEB transitions in $^{229}Th^{40+}$

3.14-eV HEB transitions in $^{229}Th^{32+}$

2.39-eV HEB transitions in $^{229}Th^{40+}$

Compared with the bare nuclear transition, K_α are enhanced by factors of 2.7 and 3.5, respectively, and the transitions lie in the visible spectrum.



W. Wang, S. Fritzsche, and Y. Li, Phys. Rev. A **112**, 022811 (2025)

3. The variation of fine-structure constant in HEB

- HEB transition rate formula

$$A_{\mathrm{HEB}} = \frac{2[L][F_f]}{[L]!!^2}\left(\frac{\omega}{c}\right)^{[L]}\frac{L+1}{L} \times |\langle [I_f\gamma_f J_f]F_f||\mathcal{O}^{(\tau L)}||[I_i\gamma_i J_i]F_i\rangle|^2,$$

- A general formula for multipole transitions

$$\begin{aligned}
&\langle [I_f\gamma_f J_f]F_f||\mathcal{O}^{(\tau L)}||[I_i\gamma_i J_i]F_i\rangle \\
&\approx \sum_t \Bigg[a_i b^*_{f,t} \begin{Bmatrix} J_t & J_i & L \\ F_i & F_f & I_i \end{Bmatrix} \langle \gamma_t J_t||\mathcal{O}^{(\tau L)}||\gamma_i J_i\rangle \\
&\quad + a^*_f b_{i,t} \begin{Bmatrix} J_f & J_t & L \\ F_i & F_f & I_f \end{Bmatrix} \langle \gamma_f J_f||\mathcal{O}^{(\tau L)}||\gamma_t J_t\rangle \\
&\quad \times (-1)^{J_t - J_i + I_f - I_i} \Bigg].
\end{aligned}$$

W. Wang, S. Fritzsche, and Y. Li, Phys. Rev. A **112**, 022811 (2025)

3. The variation of fine-structure constant in HEB

Results:

TABLE I. The HEB transition rates for different channels in $^{229}Th^{32+}$ ions.

Transition	Type	Rate (s^{-1})
$F_e = 5/2 \to F_g = 3/2$	$M1$	1.3×10^{-9}
$F_e = 5/2 \to F_g = 5/2$	$M1$	9.8×10^{-9}
$F_e = 5/2 \to F_g = 7/2$	$M1$	4.4×10^{-9}
$F_e = 7/2 \to F_g = 5/2$	$M1$	1.5×10^{-9}
$F_e = 7/2 \to F_g = 7/2$	$M1$	3.7×10^{-8}
$F_e = 7/2 \to F_g = 9/2$	$M1$	8.8×10^{-9}
$F_e = 9/2 \to F_g = 7/2$	$M1$	1.2×10^{-10}
$F_e = 9/2 \to F_g = 9/2$	$M1$	6.5×10^{-8}
$F_e = 11/2 \to F_g = 9/2$	$M1$	2.0×10^{-8}

TABLE II. Same as Table I, but for $^{229}Th^{40+}$ ions.

Transition	Type	Rate (s^{-1})
$F_e = 5/2 \to F_g = 3/2$	$M1$	9.1×10^{-10}
$F_e = 5/2 \to F_g = 5/2$	$M1$	2.1×10^{-9}
$F_e = 5/2 \to F_g = 7/2$	$M1$	5.7×10^{-10}
$F_e = 7/2 \to F_g = 5/2$	$M1$	2.1×10^{-9}
$F_e = 7/2 \to F_g = 7/2$	$M1$	5.5×10^{-9}
$F_e = 7/2 \to F_g = 9/2$	$M1$	8.7×10^{-10}
$F_e = 9/2 \to F_g = 7/2$	$M1$	7.6×10^{-10}
$F_e = 9/2 \to F_g = 9/2$	$M1$	6.0×10^{-9}
$F_e = 11/2 \to F_g = 9/2$	$M1$	4.0×10^{-9}

HEB transition rates $\sim 10^{-9}\ s^{-1}$

These HEB transitions can be controlled by the precision spectroscopy

For the $E3$ clock transition of the $^{171}Yb^{+}$ atomic clock, its transition rate is $\sim 10^{-9}\ s^{-1}$

W. Wang, S. Fritzsche, and Y. Li, Phys. Rev. A **112**, 022811 (2025)

4. Summary and outlook

Summary:

1. The HEB transition is proposed and investigated. A quantum-optical model is constructed. By employing quantum control techniques, efficient isomeric population transfer is predicted.
2. In $^{229}Th^{32+}$ and $^{229}Th^{40+}$, HEB transitions lie in the visible spectrum, and are more sensitive to the variation of the fine-structure constant than the bare nuclear transition.

Outlook:

1. By employing different quantum control techniques, the isomeric population transfer can be further optimized.
2. Since ^{235}U has a nuclear excitation energy of about 76.7 eV, investigating HEB transitions in ^{235}U is of interest.

Thank you for your attention!