Ab initio charge form factors and radii of light nuclei

Jan 20, 2026, ITP, CAS, Beijing

Ab initio nuclear mass–radius challenge

No–Core Shell–Model (NCSM)

Two–body charge density

Charge form factor and radius

Summary and perspective

Xiang–Xiang Sun (IAS–4, FZJ)

JÜLICH Forschungszentrum

XXS, Vadim Baru, Arseniy A. Filin, Evgeny Epelbaum, Hermann Krebs, Ulf–G. Meißner, Andreas Nogga, arXiv2601.09614

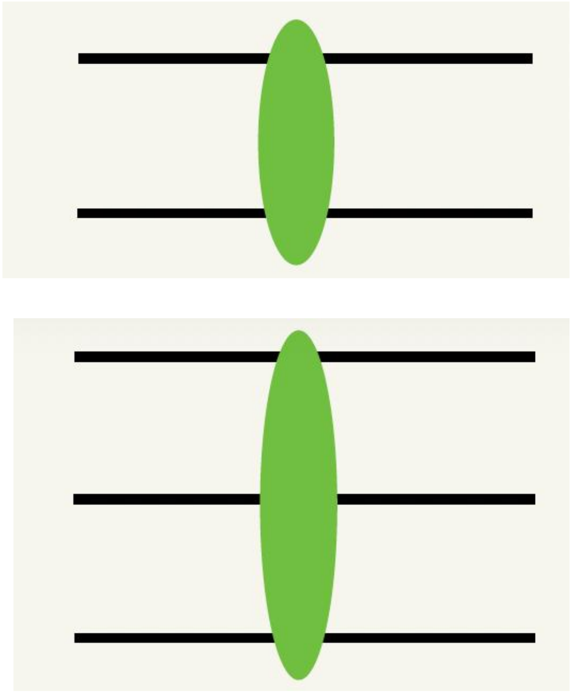
Calculations using Jureca

Ab initio No-Core Shell-Model

$$H|\Psi\rangle = E|\Psi\rangle$$

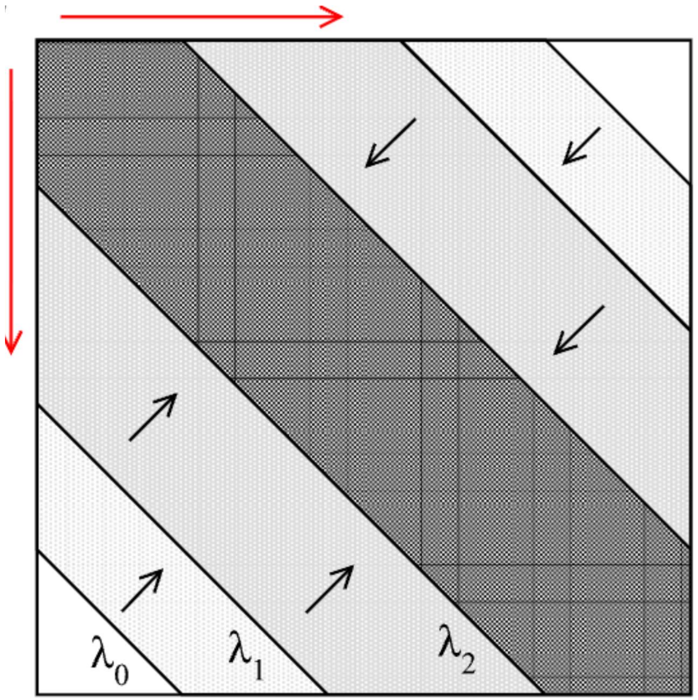
quantum many–body Schrödinger equation with realistic nuclear forces

Hamiltonian



Chiral 2N + 3N forces

Pre–conditions



Soften of interactions (SRG)

Many–body method

$$\sum_{\beta} H_{\alpha\beta} c_{\beta} = E c_{\alpha}$$

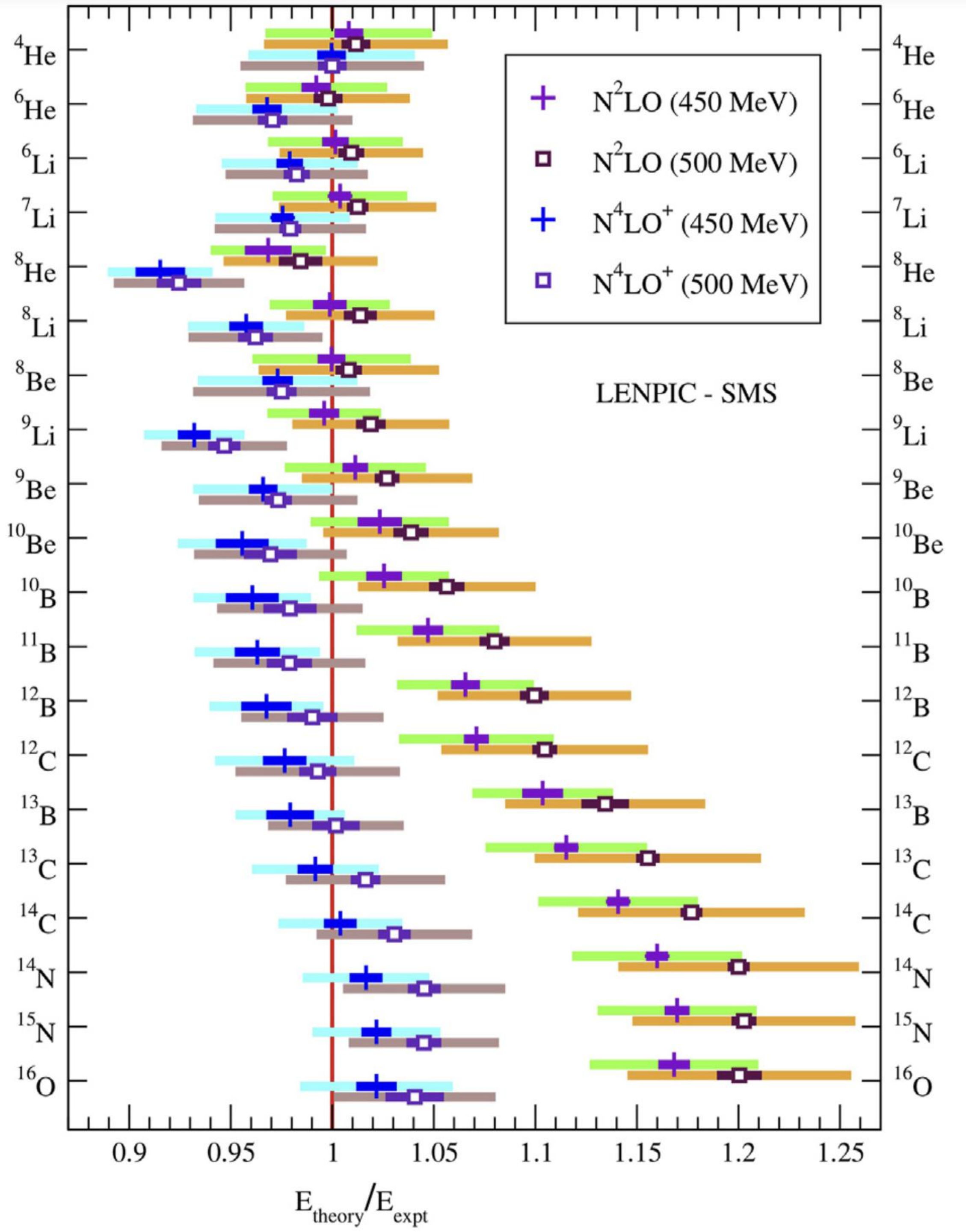
Dimension of the basis
Non–zero metrix elements

Light nuclei with SMS interactions

- SMS: **S**emi–**M**omentum–**S**pace regularised chiral interactions
  LENPIC collaborations

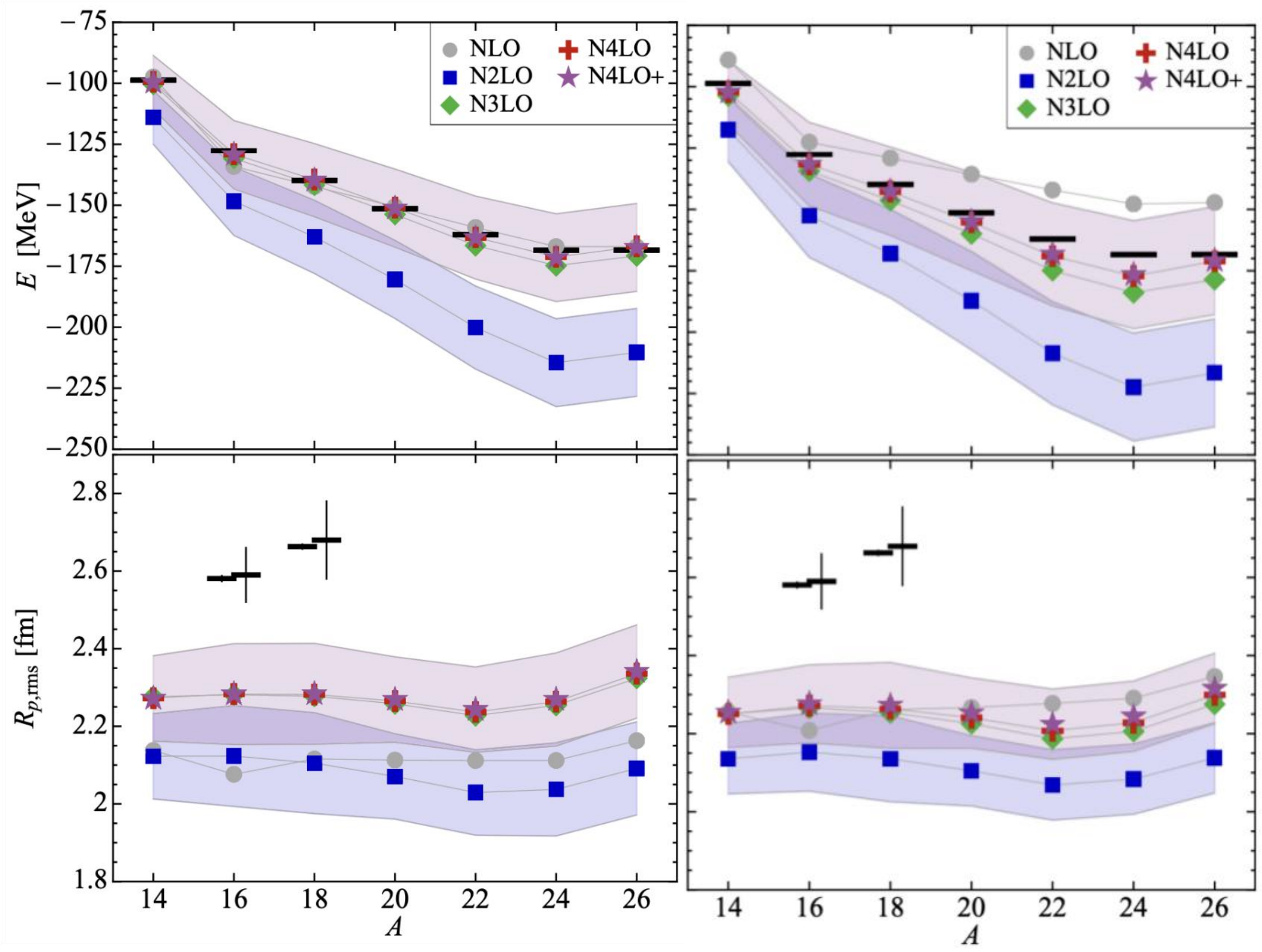
- NN (N^4LO^+): perfect description for nucleon–nucleon scattering data
  Reinert et al. PRL 126, 092501 (2021)
- 3N (N^2LO): Nd scattering, ^{3}He binding energy
  Epelbaum, Krebs, Reinert, FrontPhys 8, 98 (2022)
- 2N Chiral electromagnetic charge and currents (general N^2LO; isoscalar N^4LO)
  – N^2LO (isoscalar N^4LO) is derived and regularised consistently with the chiral NN forces
  – Consistent regularisation of N^3LO (isovector) is in progress
  Krebs, Epelbaum, Meißner FBS 60, 31 (2019)
  Krebs EPJA 56, 9 (2020) (Review)
- Nearly perfect description for light systems

LENPIC PRC 106, 064002 (2022)
Maris, Le, Nogga, et al. Front. Phys. 11, 1098262 (2023)



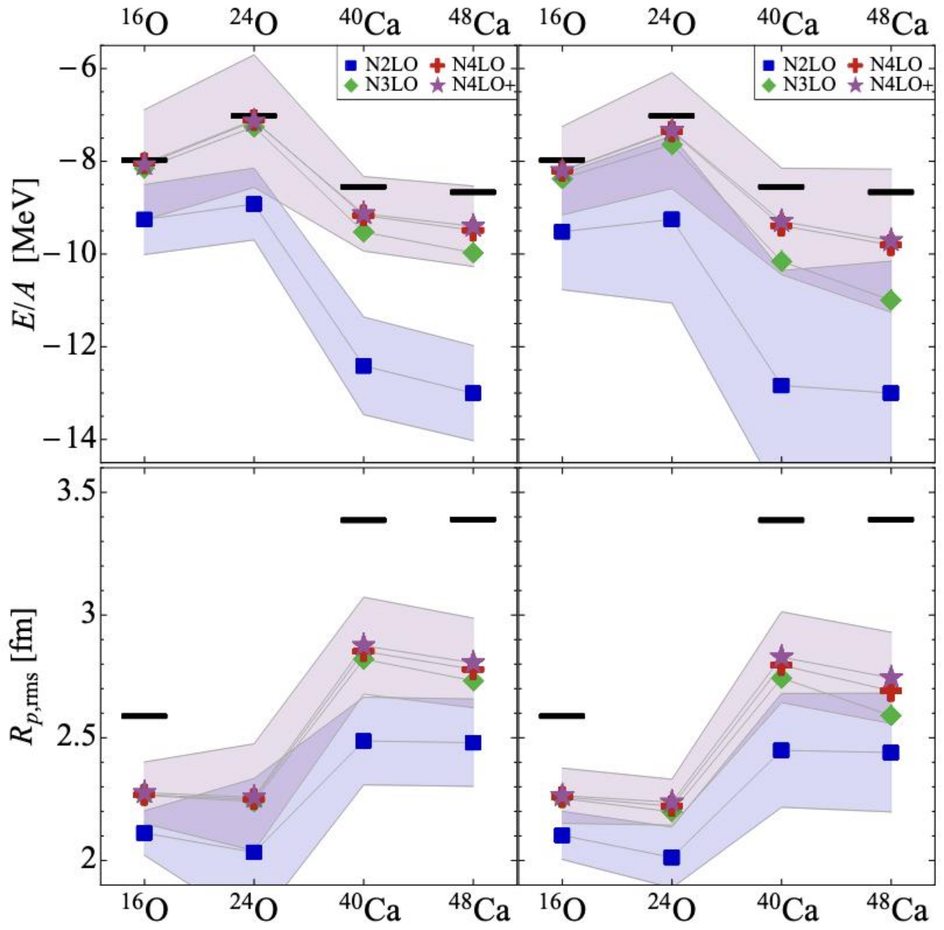
Nuclear *ab initio* calculations: Mass-radius challenge

Can we get accurate energy, radius, shape, … of atomic nuclei with realistic forces?



SMS interactions N^{4}LO$^+$+N^{2}LO description for O isotopes
left: 450 MeV and right 500 MeV

LENPIC PRC 106, 064002 (2022)



- Nearly perfact descrption for BE
- Underestimate nuclear size, over ~10%

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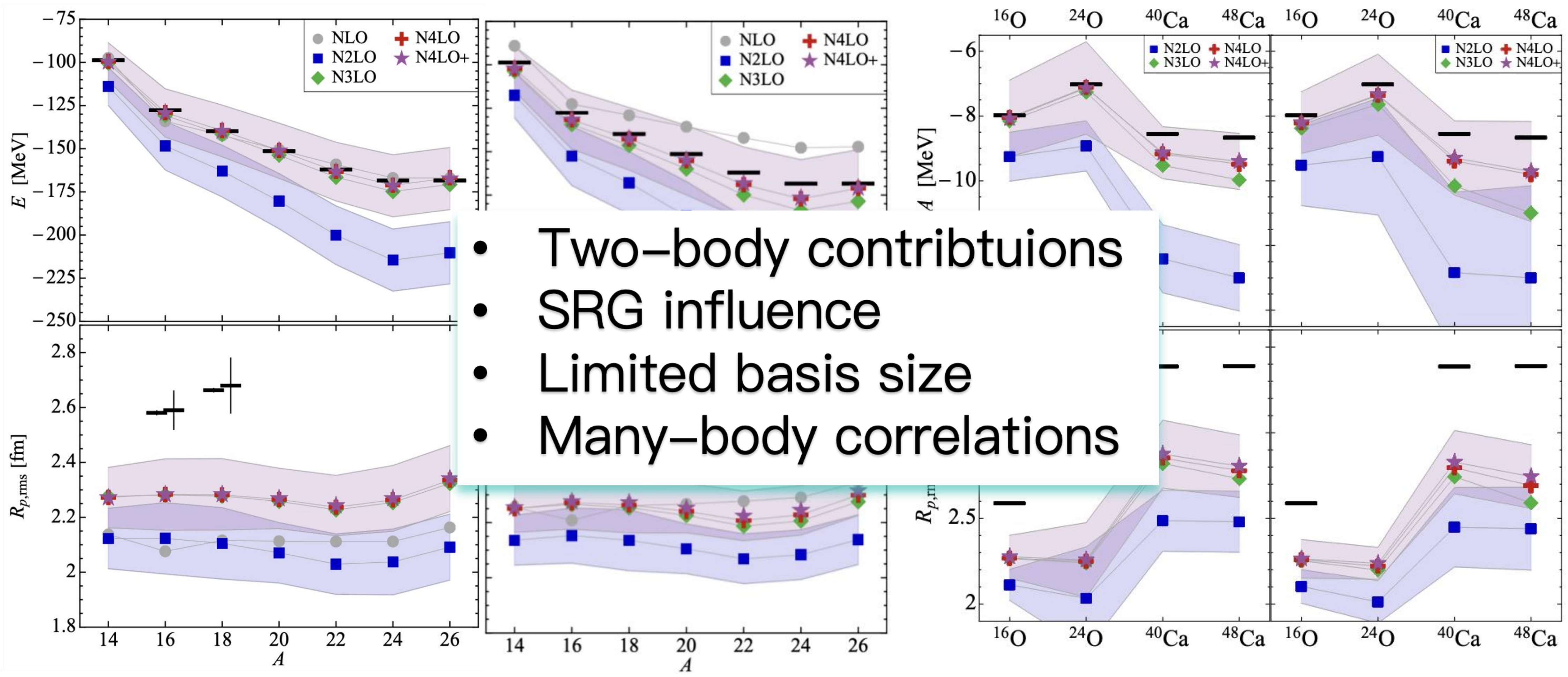

- Two–body contribtuions
- SRG influence
- Limited basis size
- Many–body correlations

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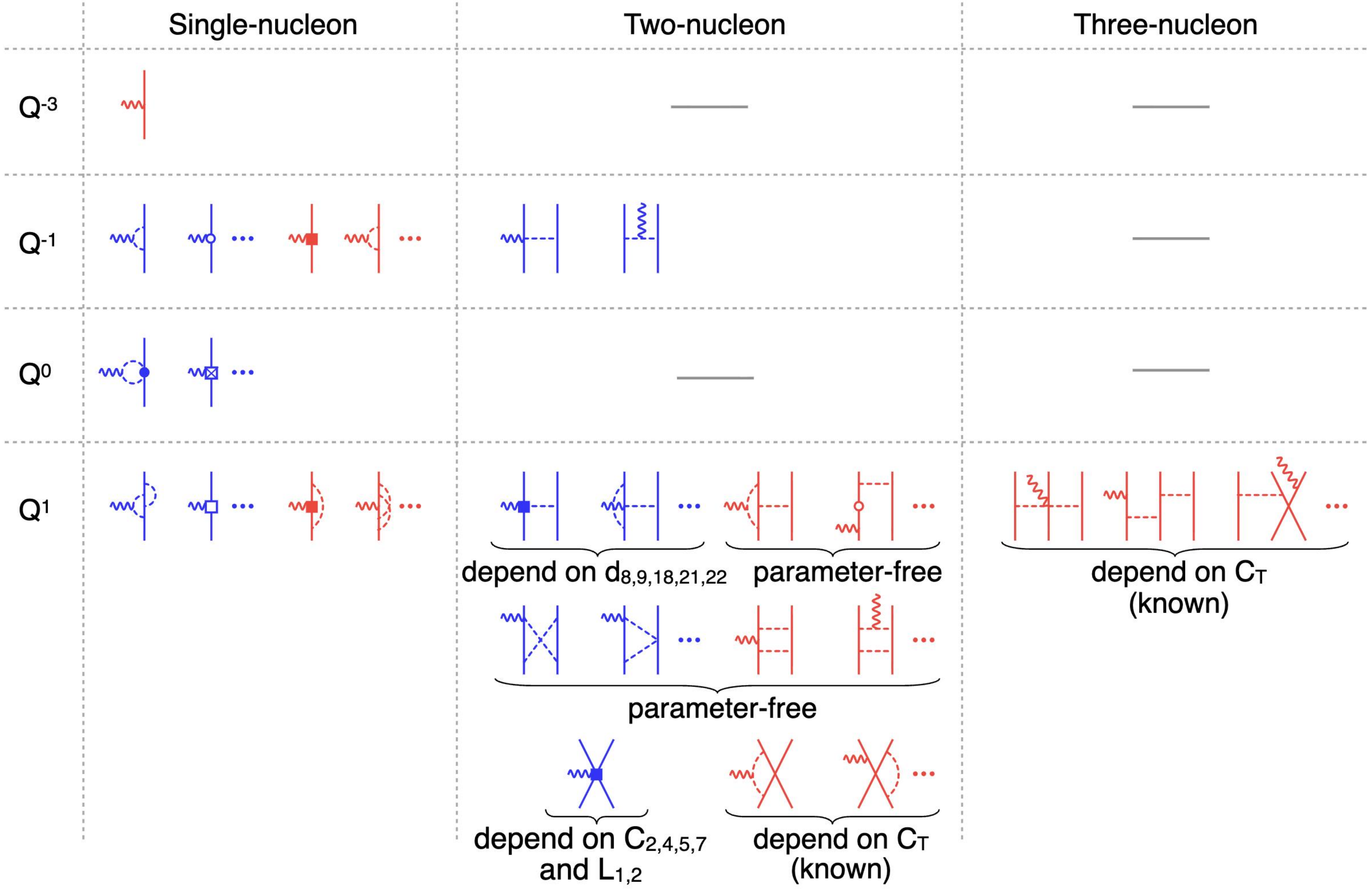
Electromagnetic observables: FFs, radii, moments, ...



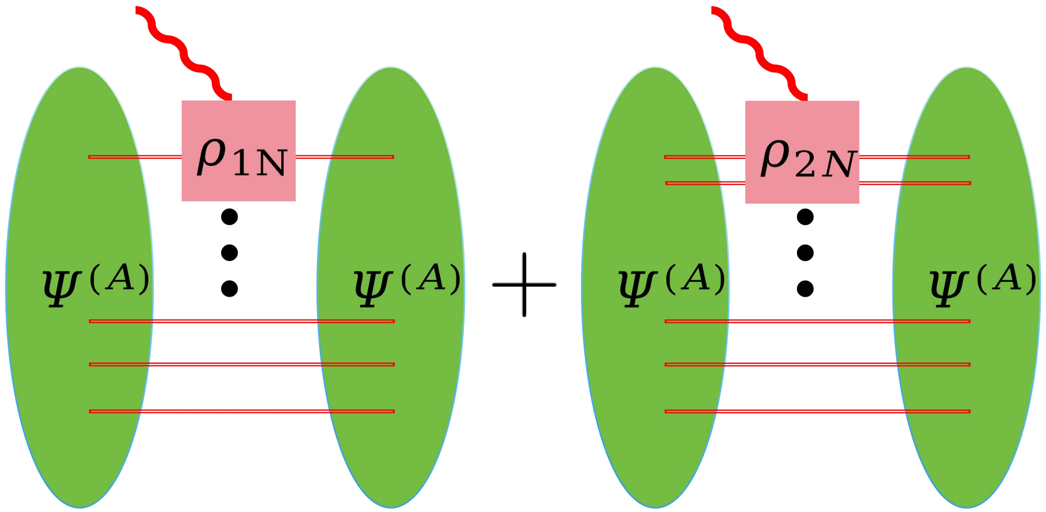
Chiral EFT allows us a systematic expansion for charge and current operators

Consistent regularization for the charge and current operators

arXiv:1908.09349, arXiv:1902.06839

Chiral EFT calculations for charge FFs and radius

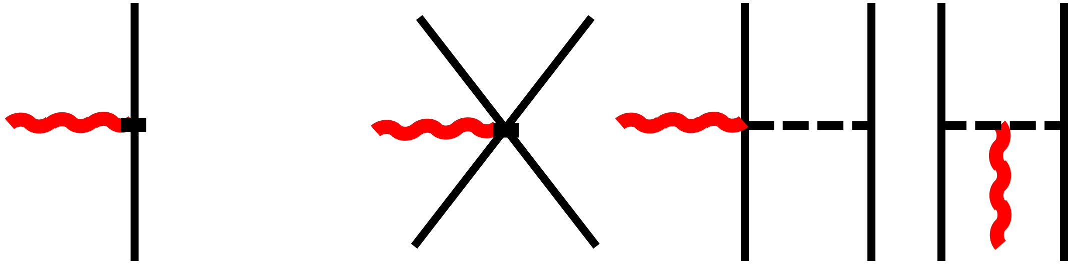
Charge radius r_C is related to the charge form factor $F_C(Q^2)$

$F_C(Q^2) =$  $+ \cdots$

G_E: Lin, Hammer, Meißner PRL128, 052002(2022)

r_p=0.84075(64) fm CODATA2022

r^2_n=−0.0105 fm^2 PRL 124 (2020) 082501



Contact term, 3 LECs   OPE, parameter free

$$r_C^2 = -6\,\frac{\partial F_C(Q^2)}{\partial Q^2}\Big|_{Q=0} = \underbrace{R_p^2 + r_p^2 + r_n^2 + r_{\rm DF}^2 + r_{\rm SO}^2}_{\rm 1N} + \underbrace{r_{1\pi}^2 + r_{\rm Cont}^2}_{\rm 2N}$$

Missing 2N contributions for almost all calculations for charge radius!!!

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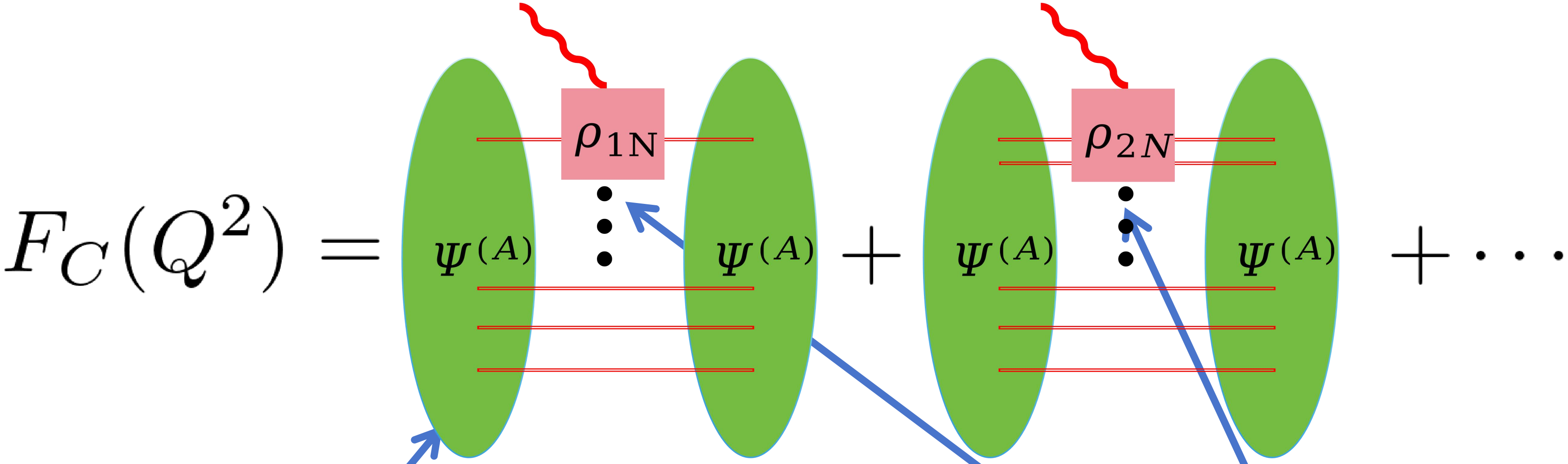

$$\langle JM' | \binom{A}{1} \rho_{1\mathrm{N}}(\vec{q}, \vec{k}) + \binom{A}{2} \rho_{2\mathrm{N}}(\vec{q}, \vec{k}) | JM \rangle$$

WFs from FY/NCSM
with SMS interactions

depend on systems

1N&2N charge operator
matrix elements

share for all the systems

LECs in the contact charge operator

GES: Lin, Hammer, Meißner PRL128, 052002(2022)
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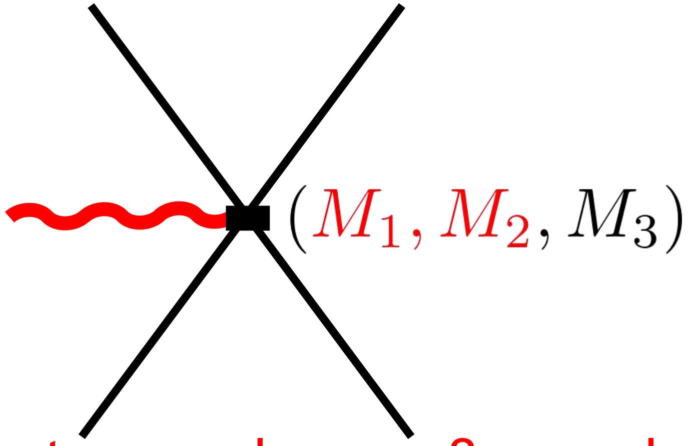
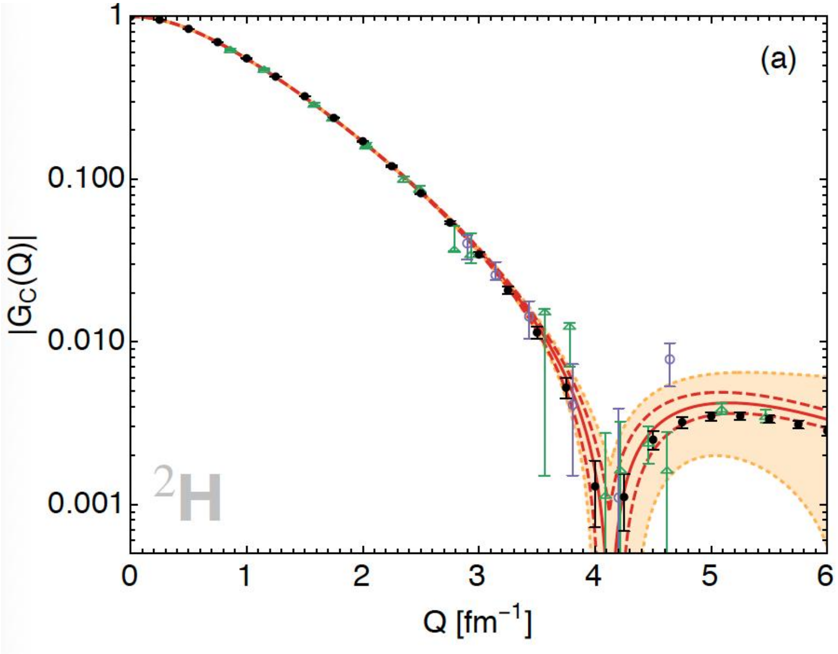
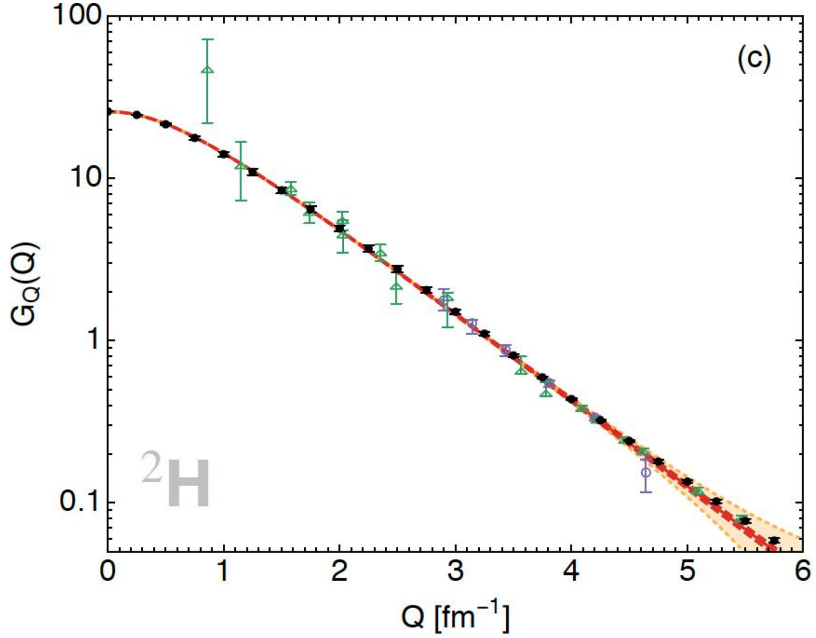
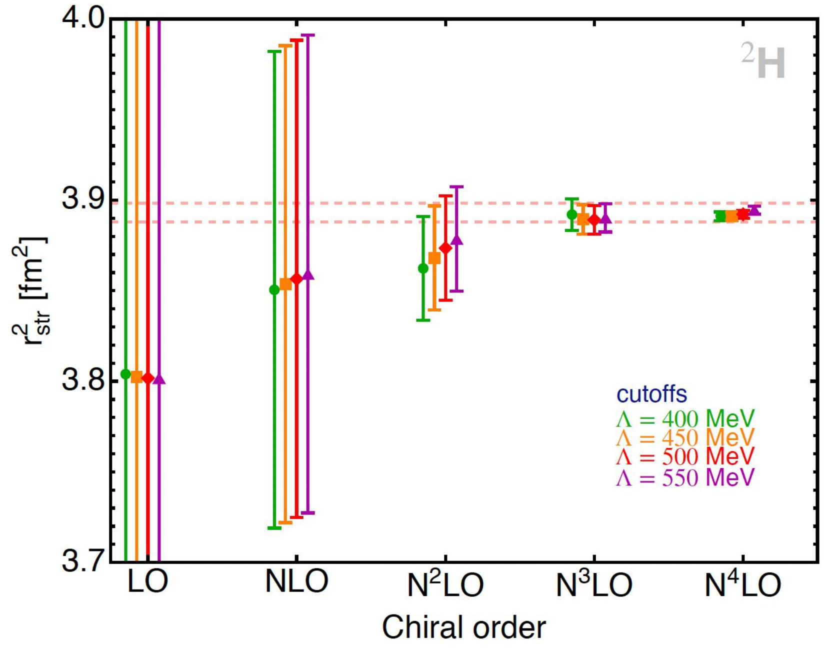


$$\rho_{\text{Cont}}(\boldsymbol{k}) = 2eG_E^S(\boldsymbol{k}^2)\Bigg[M_1 \frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+3}{4}\frac{1-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4}\boldsymbol{k}^2 + M_2\frac{1-\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4}\left((\boldsymbol{k}\cdot\sigma_1)(\boldsymbol{k}\cdot\sigma_2)-\frac{1}{3}(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\boldsymbol{k}^2\right) + M_3\frac{1-\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2}{4}\left(\frac{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2+3}{4}\right)\boldsymbol{k}^2\Bigg],$$

Deuteron charge & quadrupole form factor M_1 and M_2







Fillin, Möller, Baru, et al., PRL 124 (2020) 082501 & PRC 103 (2021) 024313

$$r^2_{\text{str}} = R_p^2 + r^2_{\text{SO}} + r^2_{1\pi} + r^2_{\text{Cont}}$$

LECs in the contact charge operator

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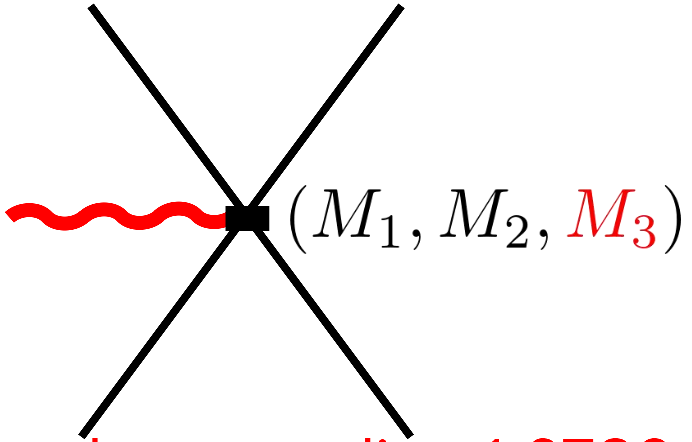

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^{4}He charge radius 1.67824(83) Nature 589 527–531 (2021) for M_3

LECs in the contact charge operator

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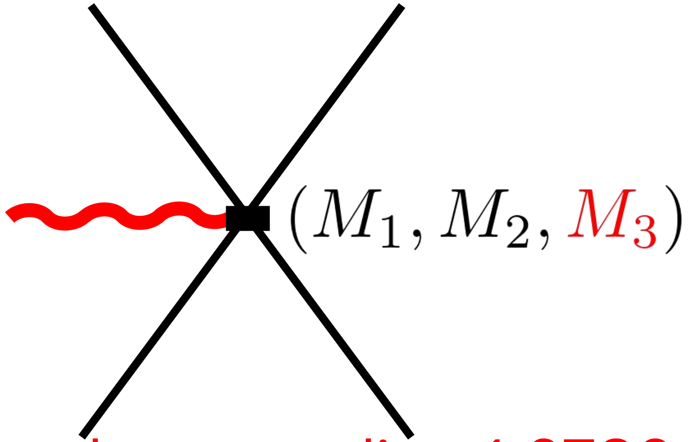
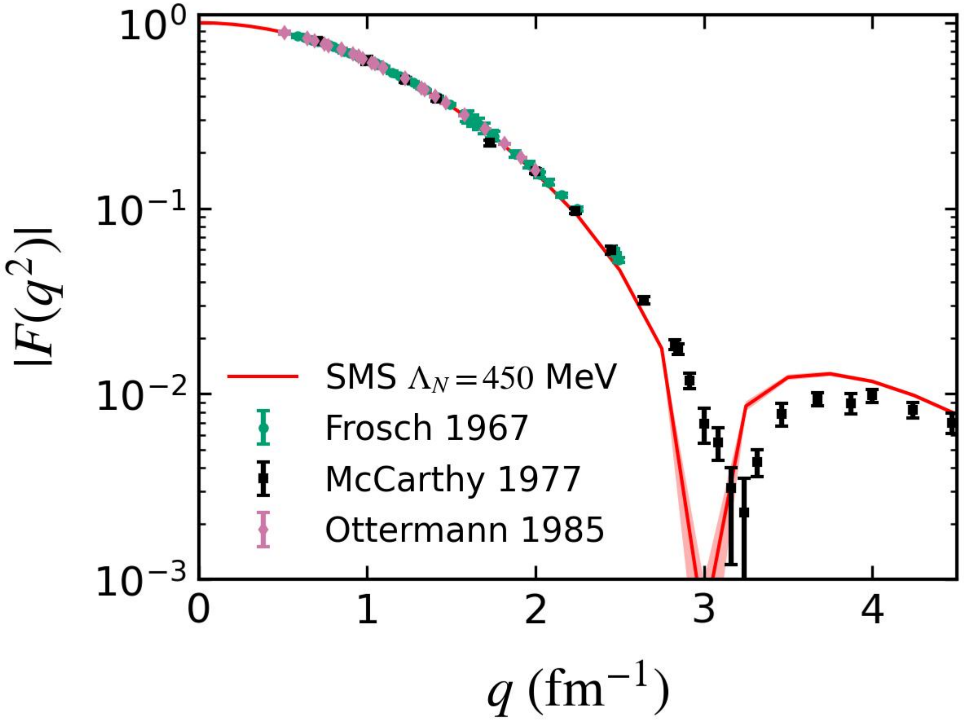
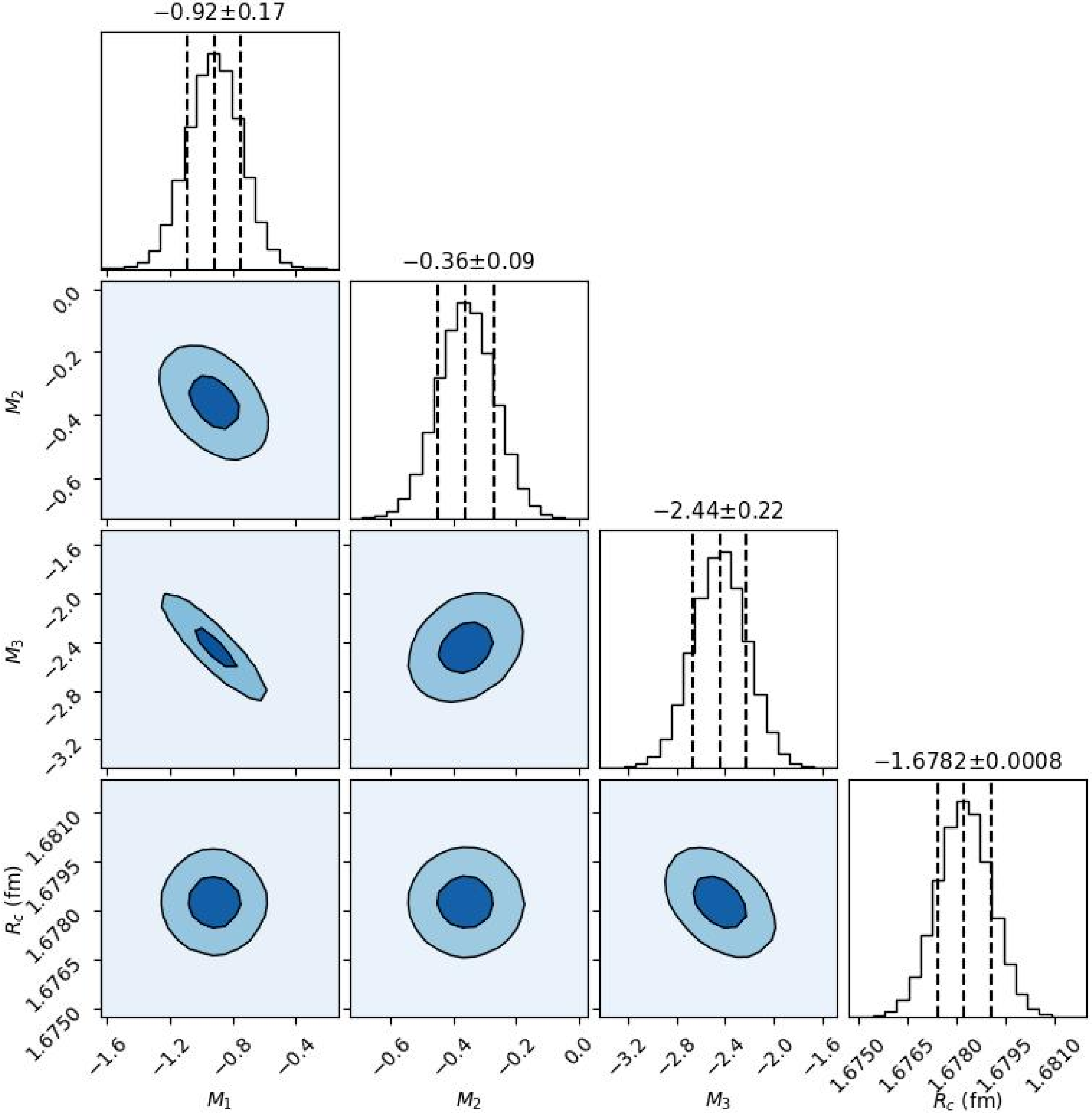


$\rho_{\text{Cont}}(\boldsymbol{k}) = 2eG_E^S$

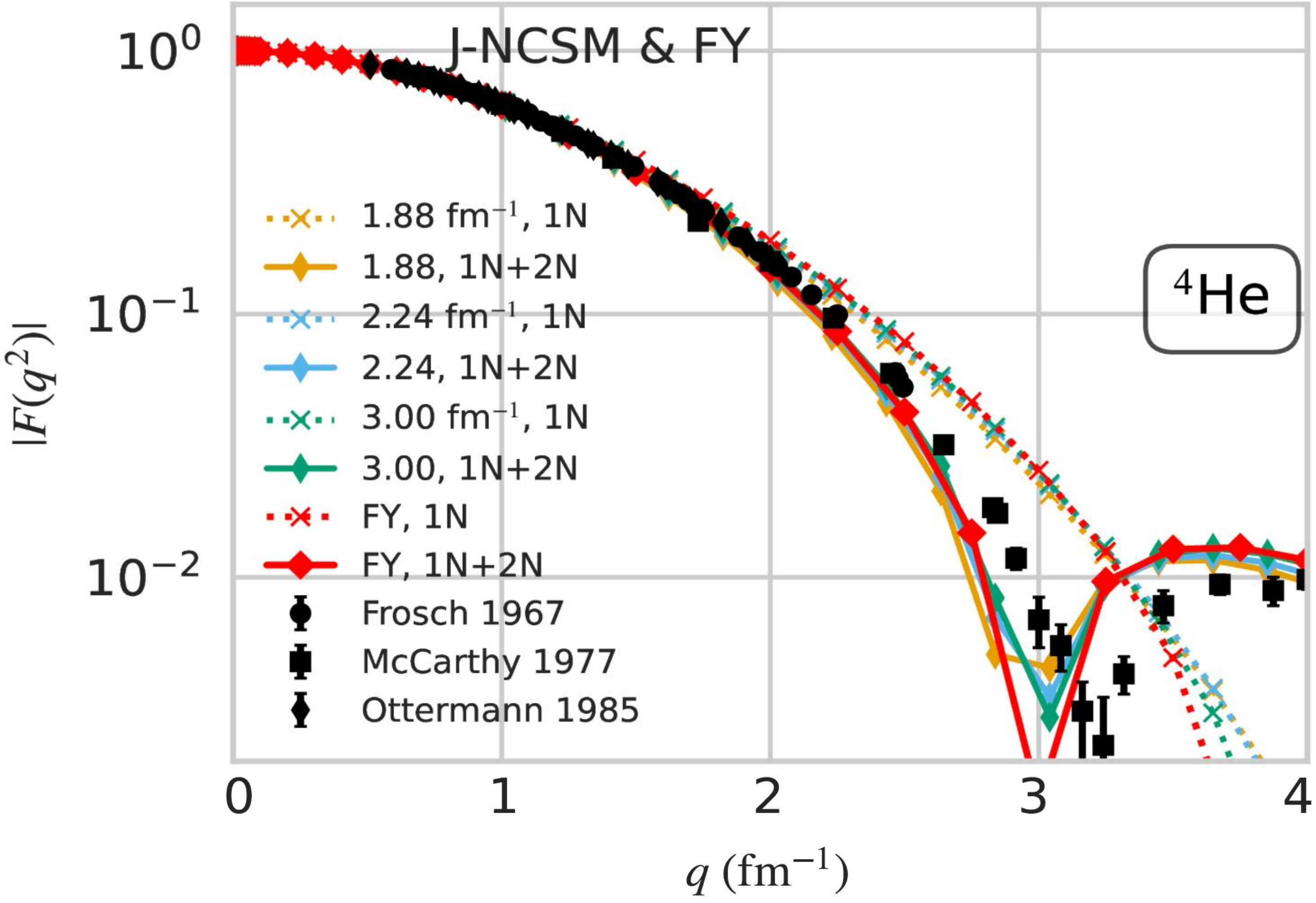
^{4}He charge radius 1.67824(83) Nature 589 52





Charge FFs and radius for ^{4}He

FY: Faddeev–Yakubovsk, accurate solution with bare SM Sinteractions
J–NCSM: Jacobi–NCSM with SRG evolved SMS interactions
SRG transformed two–body observables



- E_B (–28.3 MeV)
- Important 2N contribution in high–momentum transfers
- SRG flow parameter dependences
- Difference comes from one–body parts without SRG transformed

Meson exchange calculations have already shown its importance at large momentum transfer region.
Meissner&Gari, PLB 125(1983)364, Wiringa&Schiavilla, PRL81(1998)4317

SMS N^{4}LO$^+$ NN + N^{2}LO 3N, 450 MeV

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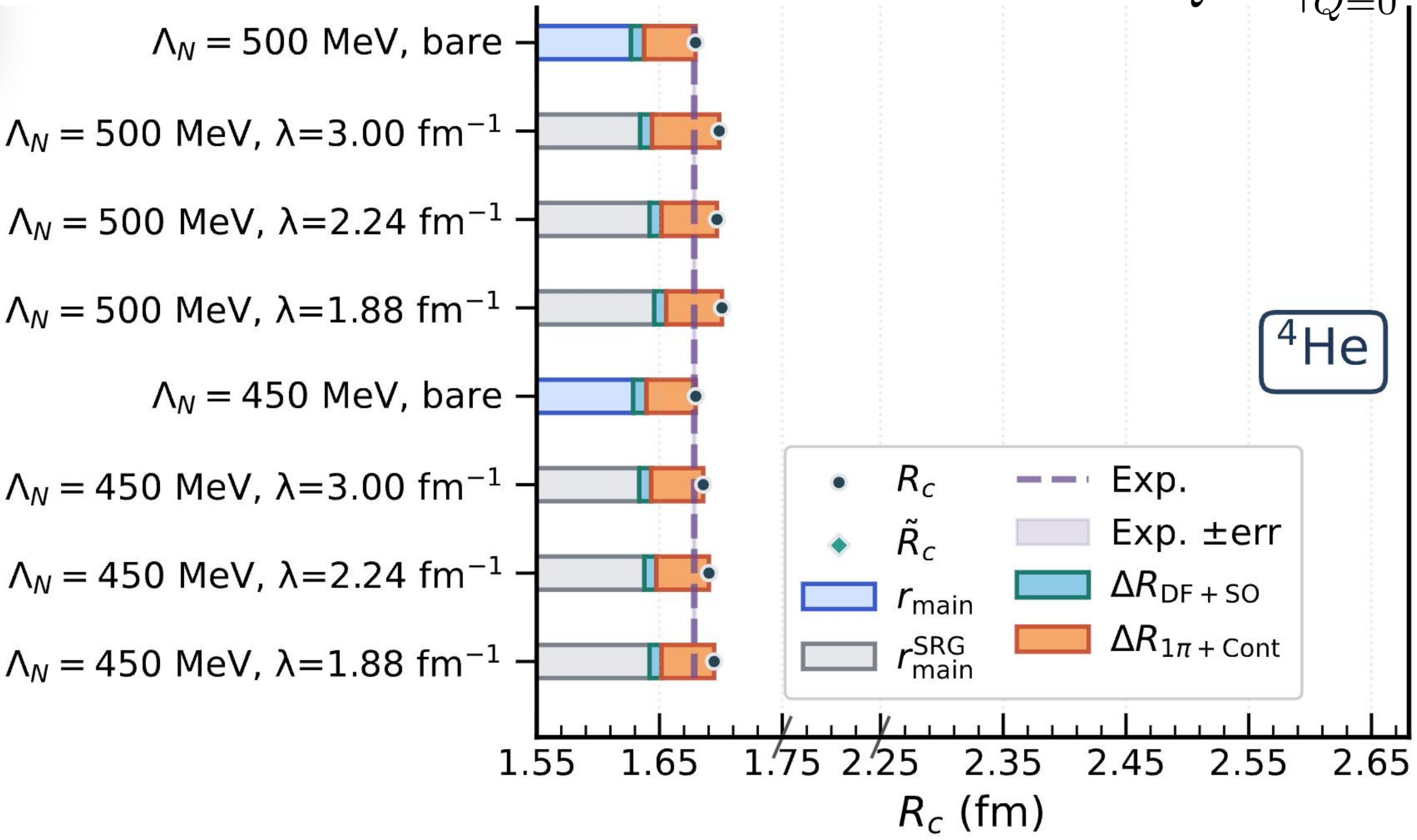


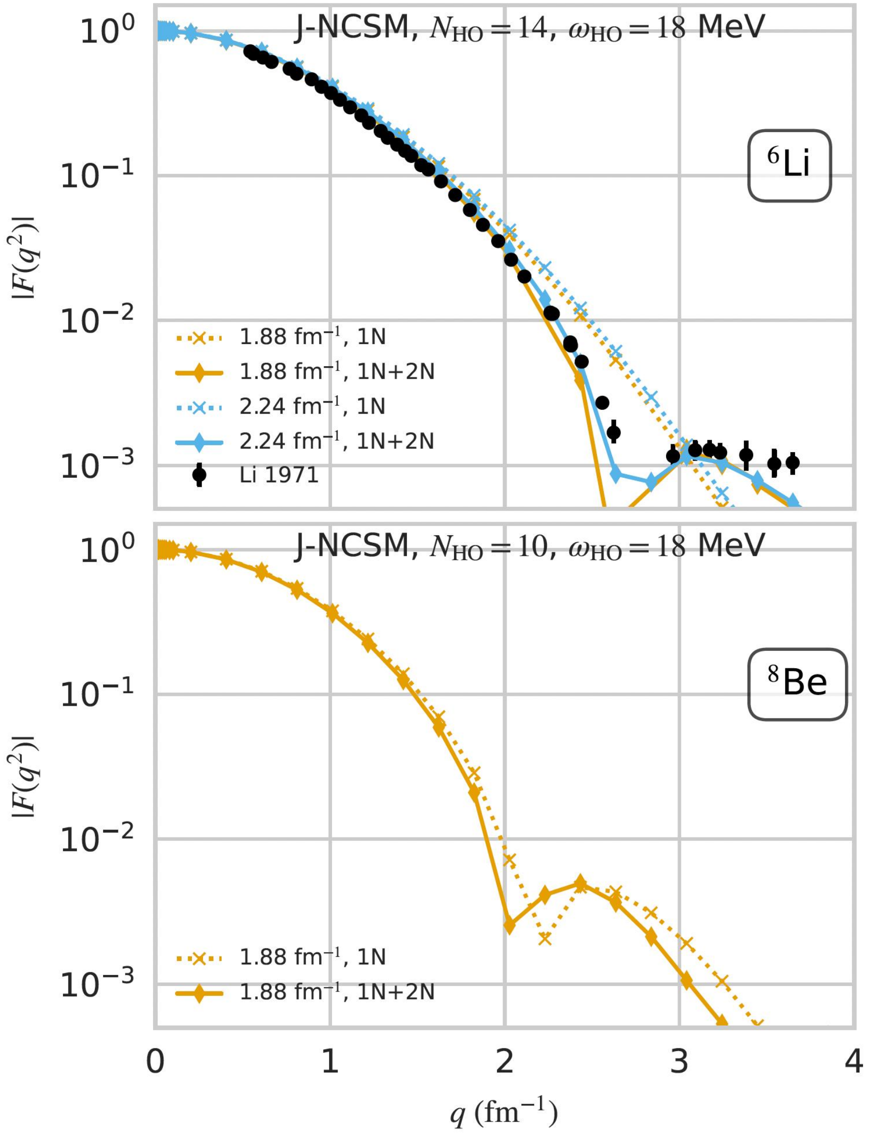
- 1N ~ 97.5%, 2N ~ 2.5%
- 2N (0.042 fm) > DF+SO (0.01 fm)
- SRG transfromation makes radius larger

- Uncertainties from LECs ~ 0.0008 fm
- From chiral truncation

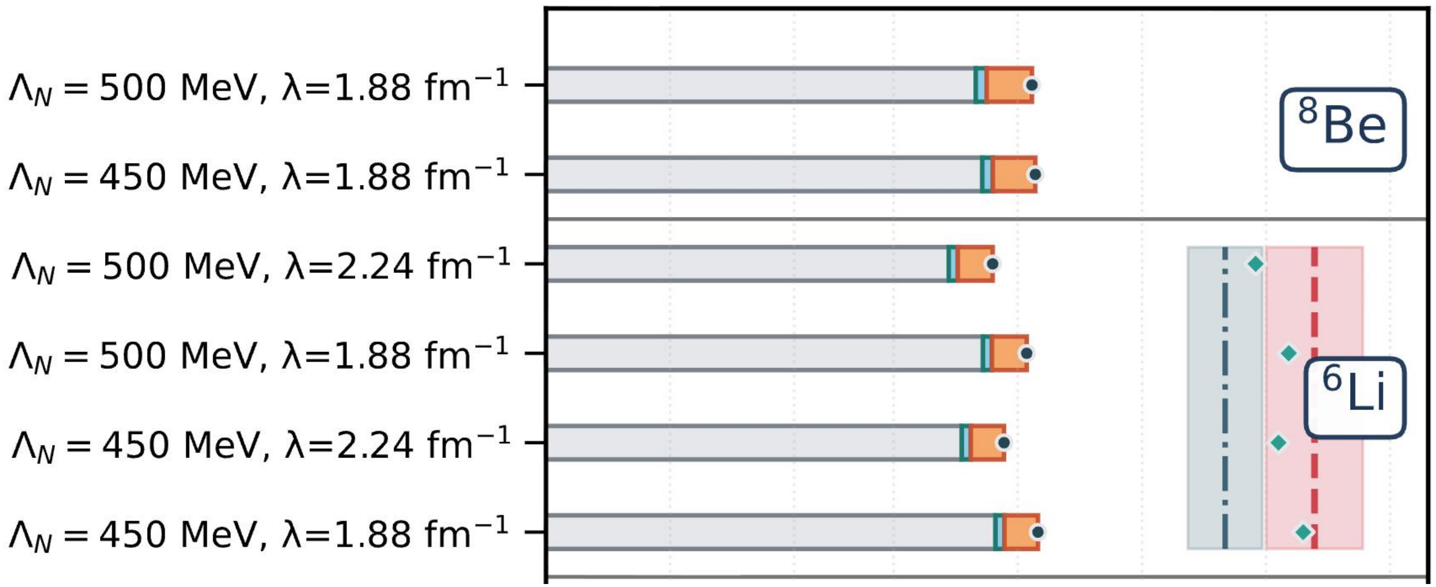
$$\left(\frac{m_\pi}{\Lambda_b}\right)^3 \times R_c \approx 0.0015 \text{ fm}$$

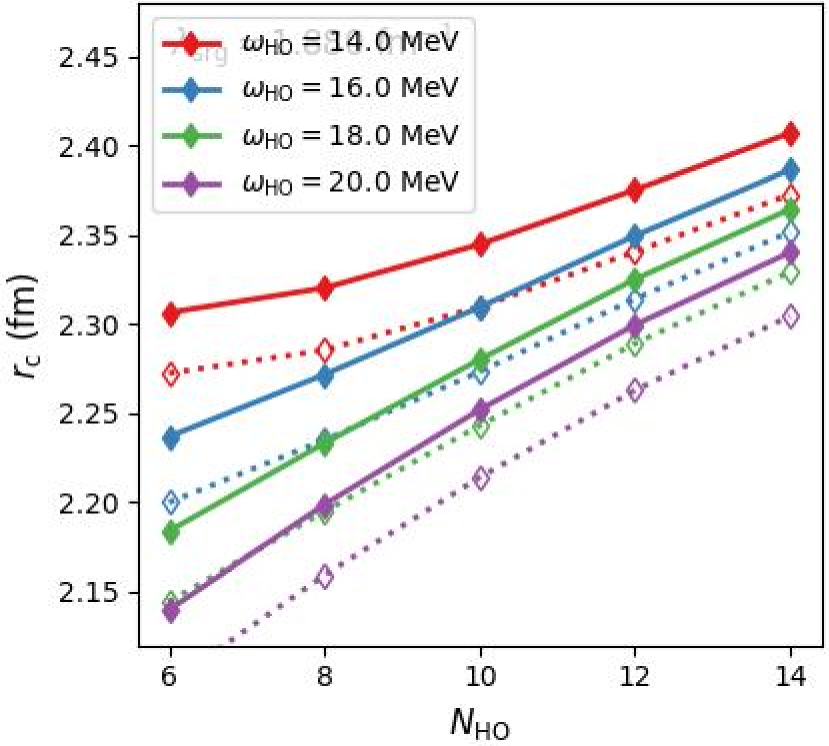
J–NCSM with SRG evolved interactions converged with N=28
SMS N^{4}LO$^+$ NN+ N^{2}LO 3N, 450 MeV
SRG back transformation SUN, Le, Meißner, Nogga, arXiv2512.15454

Charge FFs and radius for ^{6}Li and ^{8}Be



Charge FFs and radius for ^{6}Li and ^{8}Be





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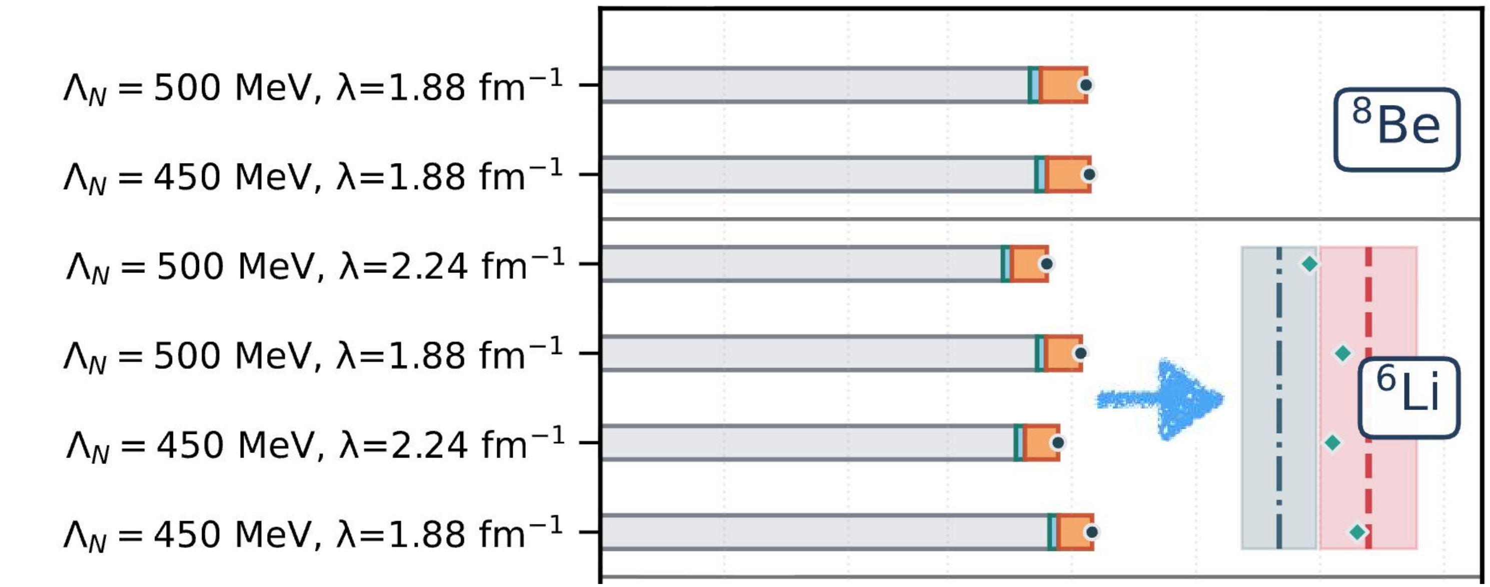
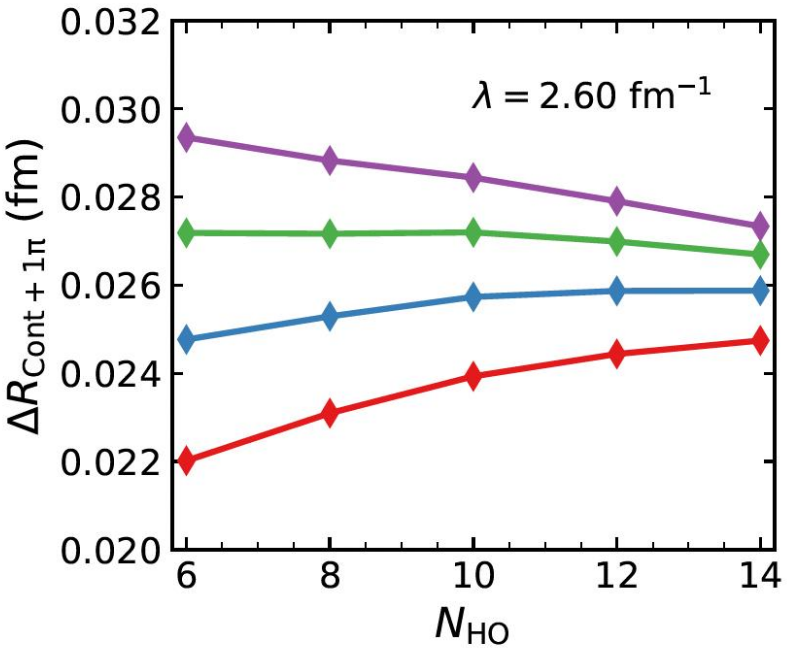
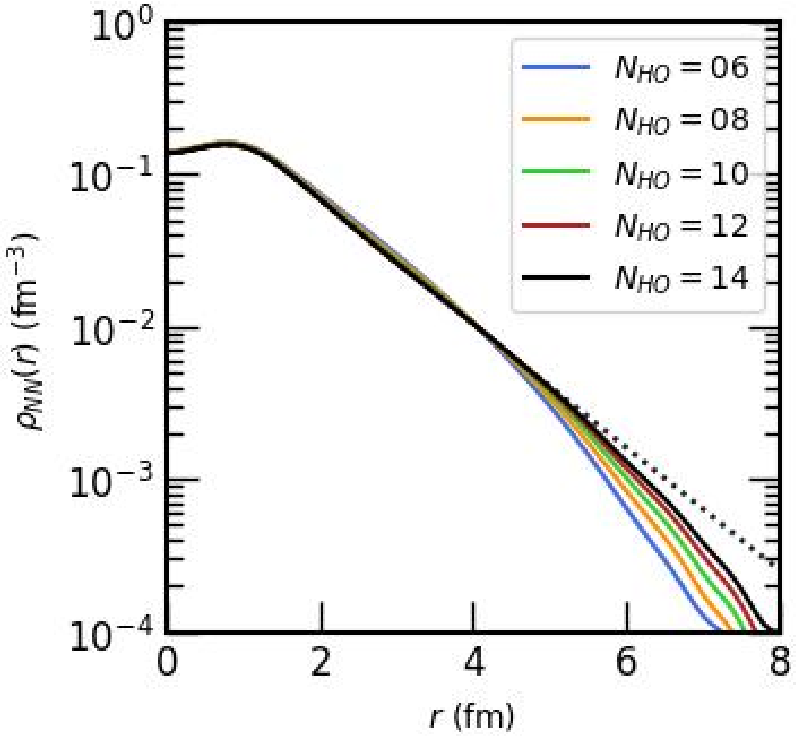




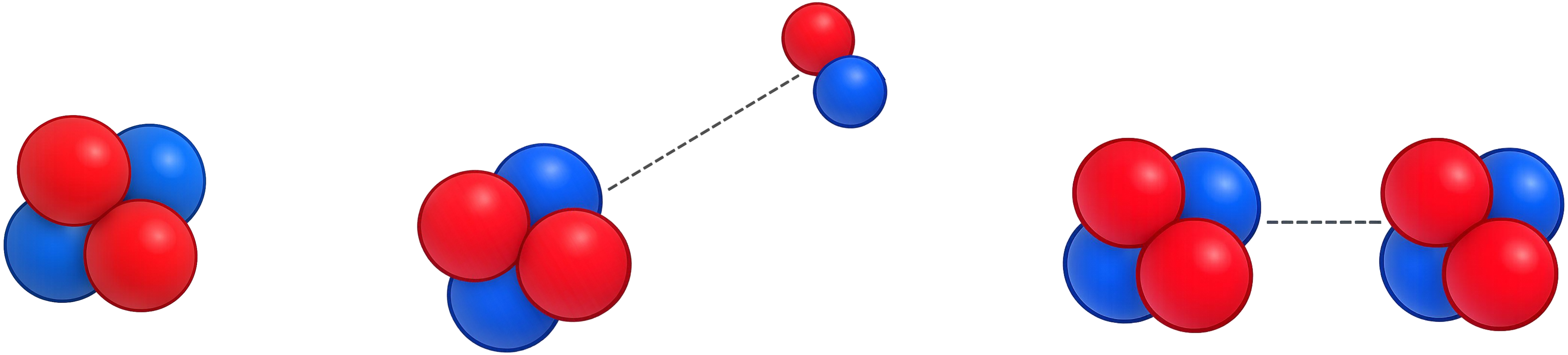


2N: 0.028 fm for ^{6}Li & 0.035 fm for ^{8}Be

- J-NCSM with SRG evolved interactions connot converge with N=14 for ^{6}Li
- Long-range operators
- Tail correction
- 2N contribution converged
- 2N > DF+SO

SUN, Le, Meißner, Nogga, PRC 112 (2025) 024317

2N contribution for ^{4}He, ^{6}Li, ^{8}Be, cluster structure



^{4}He $^{6}Li = {}^{4}He + d$ $^{8}Be = {}^{4}He + {}^{4}He$

- 2N contribution mainly from 4 nucleons in the *s*–shell
- Cluster structure (long–range) is not affected by 2N
- Increasing of R_C is mainly from swelling of ^{4}He

Summary and perspective

- *Ab initio* charge form factors and charge radius using SMS interactions and self–consistent charge operators
- Two–body charge operators are important to large momentum region and accurately extract charge radius
- Two–body contrbutions for charge radius is about 0.03–0.04 fm for ^{4}He, ^{6}Li, and ^{8}Be, but much larger than SO+DF terms
- In NCSM, basis limitation, 2N controbution, SRG influence, DF+SO terms

- Isovector charge operators and also current operators
- Heavy systems such as ^{12}C, ^{16}O (cluster structure)
- Mechine learning, considering 2N controbutions, mass–radius issue for *ab initio* NCSM

THANK YOU!