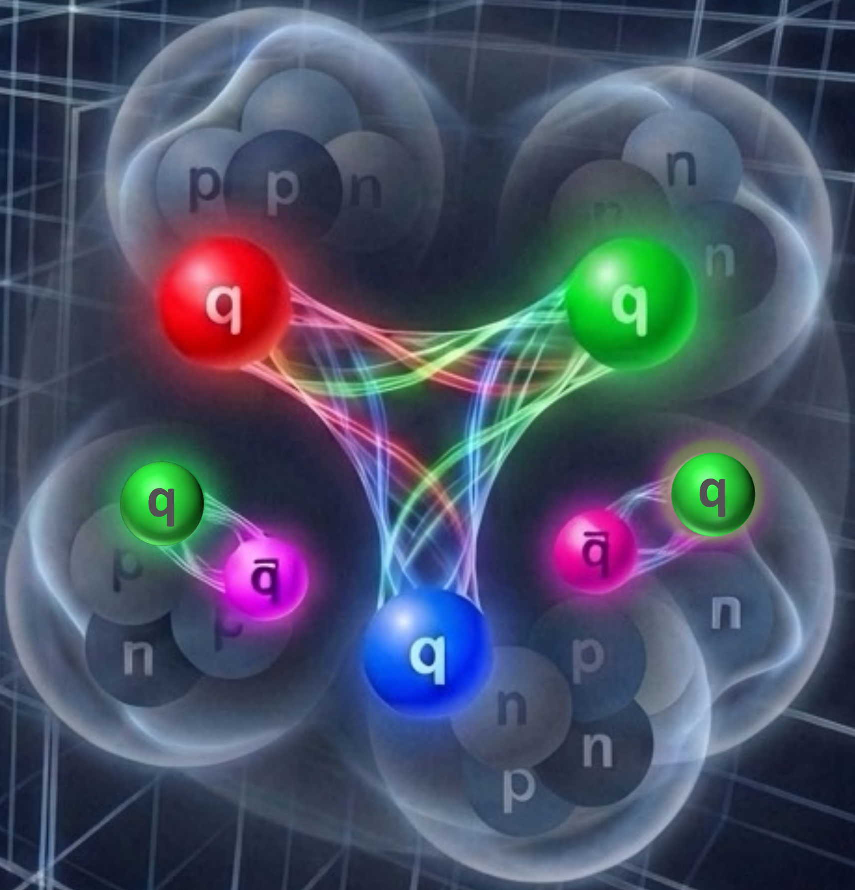
# Workshop on Strongly Interacting Matter

Institute of Theoretical Physics, Chinese Academy of Sciences

2026.01.20



# New CRC

- Supported by the Bureau of Interactional Corporation, Chinese Academy of Sciences
- 1.97M CNY for 5 years, supporting international exchanges and activities

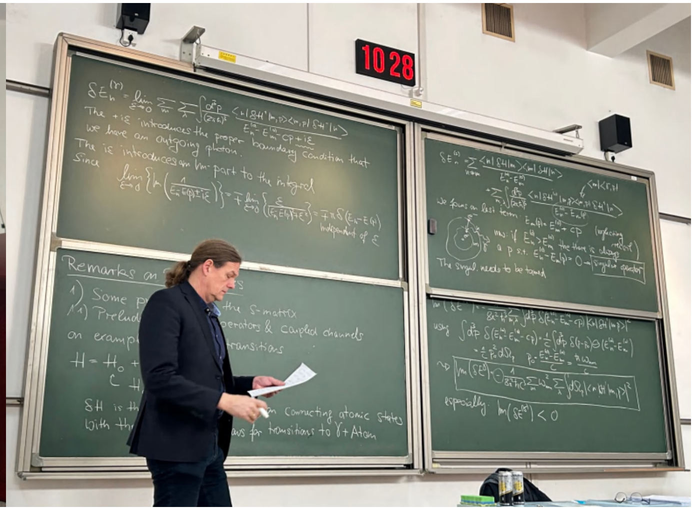
中国科学院国际合作局
Bureau of International Cooperation, Chinese Academy of Sciences

| Hadronic molecules in continuum and finite volume | Theoretical tools for amplitude analysis | Hadronic molecules with strange quark from Jülich model |
|---|---|---|
| • F.-K. Guo<br>• B.-S. Zou<br>• Hanhart<br>• Rusetsky | • F.-K. Guo<br>• Hanhart<br>• Kubis | • B.-S. Zou<br>• Meißner<br>• Rönchen |

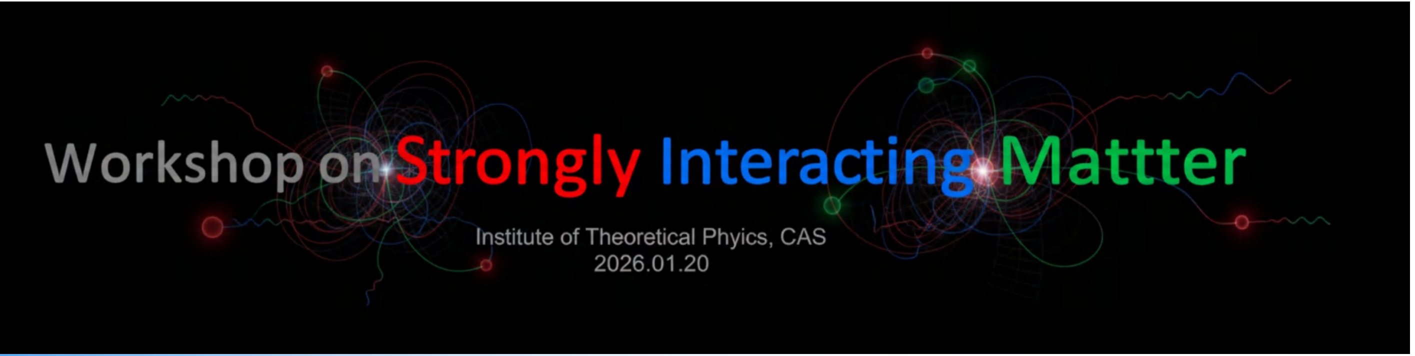
| Deuteron-halo scattering | Hypernuclei |
|---|---|
| • F.-K. Guo<br>• Meißner<br>• S.-H. Shen | • S.-G. Zhou<br>• Meißner<br>• Nogga |

# Activities in 2025

- Lectures on Resonance Theory (by Christoph Hanhart)
- Workshop on NLEFT (Organized by Ulf & collaborators, @ Beihang)
- Visitors
  - Ulf-G. Meißner (U. Bonn/ FZJ)
  - A. Rusetsky, A. Nefediev, X.-K. Dong, T. Ji (U. Bonn)
  - A. Nogga, X.-X. Sun (FZJ)
  - R. Molina, P.-P. Shi (U. Valencia)
  - Y.-H. Lin (TU Darmstadt)
- Student exchanges
  - Anuvind Asokan (FZJ/U. Bonn): two months at ITP
  - Cheng-Cheng Li (ITP): two months at U. Bochum
- Planned activities in 2026
  - Workshop on few-body problems in nuclear and hadron physics
  - Lectures:
    - ➢ Effective Field Theory (Ulf)
    - ➢ Few-body nuclear systems (A. Nogga)




CAS
Congratulations to Ulf for
the 2025 CAS International Corporation Award!




# Predicting isovector charmonium-like states from X(3872) properties

**Feng-Kun Guo**

**Institute of Theoretical Physics, Chinese Academy of Sciences**

2026.01.20

# X(3872) and possible isospin-1 partners

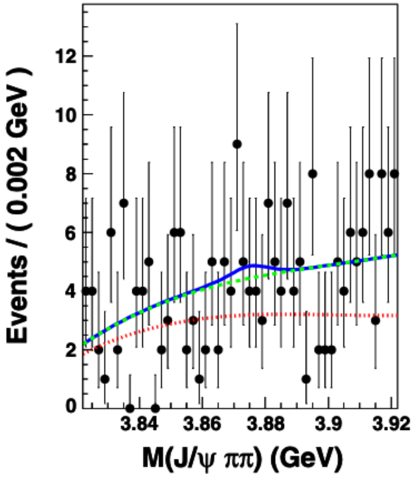
- X(3872) has been discovered by Belle for more than 20 years, debates are still ongoing!
- Excellent observable for distinguishing models: Isospin-1 partners!
  - No, in charmonium model — L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, PRD 71 (2005) 014028
  - Quark bound states, in compact tetraquark model
    - With isospin-independent quark interactions, isoscalar and isovector tetraquarks must coexist

$$I = 1 \text{ multiplet: } [cu][\bar{c}\bar{d}],\ \frac{1}{\sqrt{2}}([cu][\bar{c}\bar{u}] - [cu][\bar{c}\bar{d}]),\ [cd][\bar{c}\bar{u}]$$

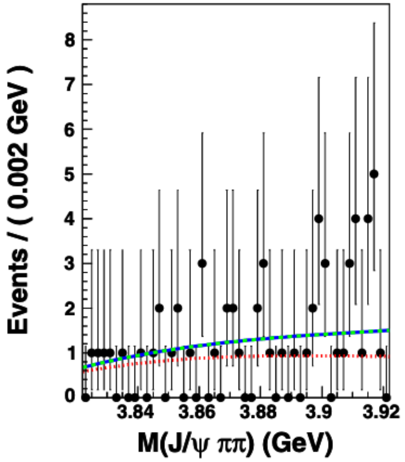
  - How about hadronic molecular picture?
    - Thought to be non-existing, but never carefully investigated
    - Will be shown to exist as virtual states in this talk

# So far negative signal

- No signal in the charged channel so far



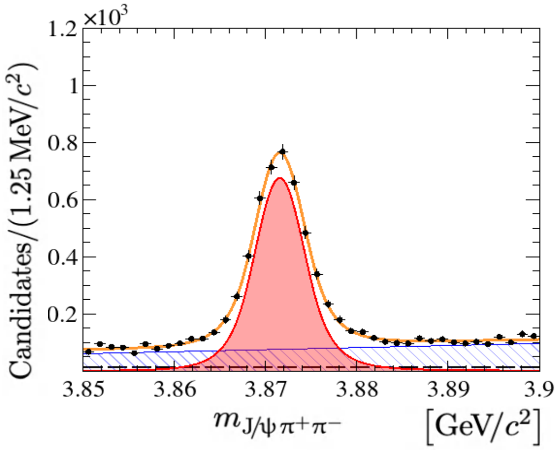

$\bar{B}^0 \to K^- \pi^+ \pi^0 J/\psi$




$B^+ \to K^0 \pi^+ \pi^0 J/\psi$

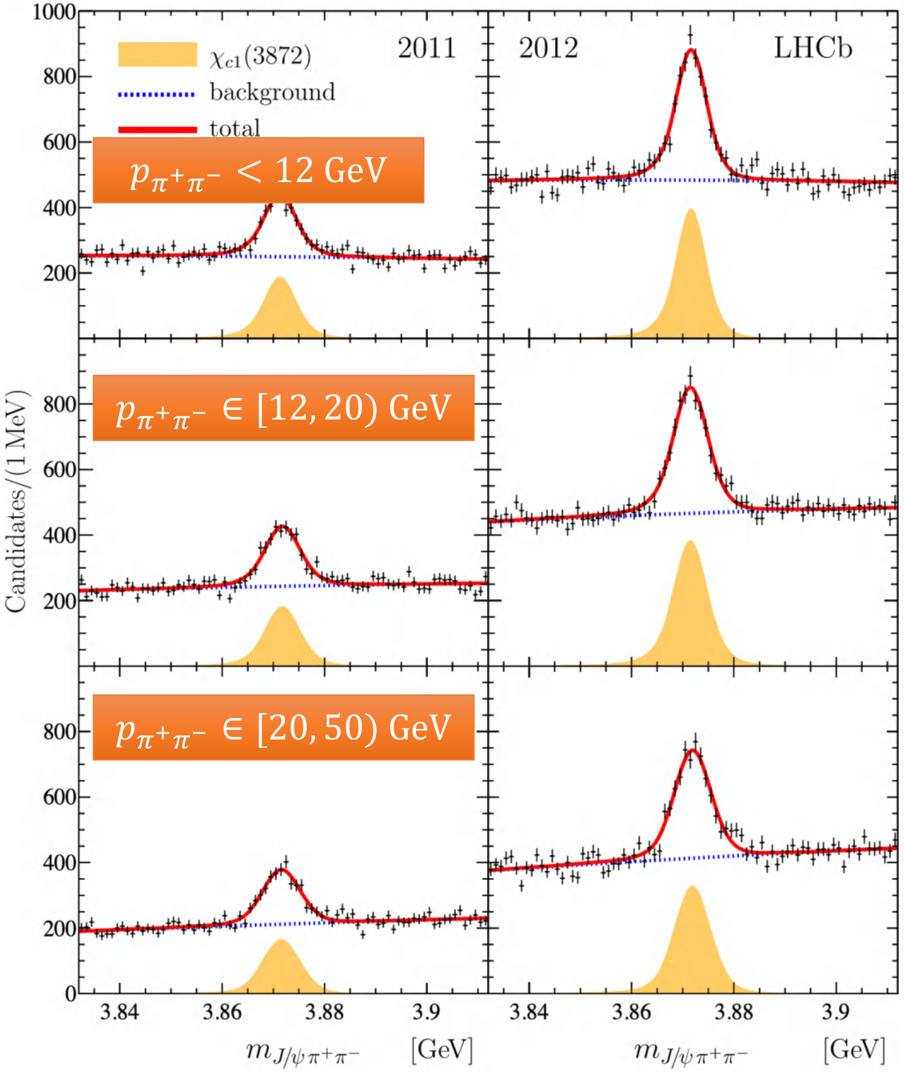
Belle, PRD 84 (2011) 052004

- No signal around the $D^+ D^{*-}$ threshold




$B^+ \to K^+ \pi^+ \pi^- J/\psi$

LHCb, JHEP 08 (2020) 123




$\pi^+ \pi^- J/\psi$ from $b$-hadrons

LHCb, PRD 102 (2020) 092005

# $J^{PC} = 1^{++}$ sector

- Hadronic molecules: consider S-wave interactions between charm and anti-charm mesons
- For each isospin, only two low-energy constants (LECs) at LO in nonrelativistic expansion for S-wave interactions of 6 meson pairs
- For the $J^{PC} = 1^{++}$ sector, also two LECs at LO:
  - $I = 0$: $C_{0X}$; $I = 1$: $C_{1X}$
- Two inputs from $X(3872)$ properties :

➢ Mass

$$M_X = 3871.69^{+0.00+0.05}_{-0.04-0.13}\,\mathrm{MeV}$$ LHCb, PRD 102 (2020) 092005

$$M_{D^0} + M_{D^{*0}} = 3871.69(7)\ \mathrm{MeV}$$ PDG 2024

➢ Isospin breaking in decays LHCb, PRD 108 (2023) L011103

$$R_X = \left|\frac{\mathcal{M}_{X(3872)\to J/\psi\rho^0}}{\mathcal{M}_{X(3872)\to J/\psi\omega}}\right| = 0.29 \pm 0.04$$

➢ Neutral systems $X$ and $W_{c1}^0$: coupled channels

✓ $(D\bar{D}^*)_0 \equiv (D^0\bar{D}^{*0} - \bar{D}^0 D^{*0})/\sqrt{2}$

✓ $(D\bar{D}^*)_\pm \equiv (D^+ D^{*-} - D^- D^{*+})/\sqrt{2}$

➢ Charged systems $W_{c1}^\pm$: single channel

Extracted using BW for resonances;

Updated to 0.26 ± 0.03 using Omnes repr. for $\pi\pi$ P-wave $\mathcal{M}_{X\to J/\psi\pi\pi} = \mathcal{N}\varepsilon_{ijk}\varepsilon_\psi^i\varepsilon_X^j q_\pi^k \underbrace{P(s)\Omega(s)}_{\text{Omnes}}\,[1 + \underbrace{\kappa_X G_\omega(s)}_{\rho\omega\text{ mixing}}]$

J. Dias et al., PRD 111 (2025) 014031

# Lippmann-Schwinger equation (LSE)

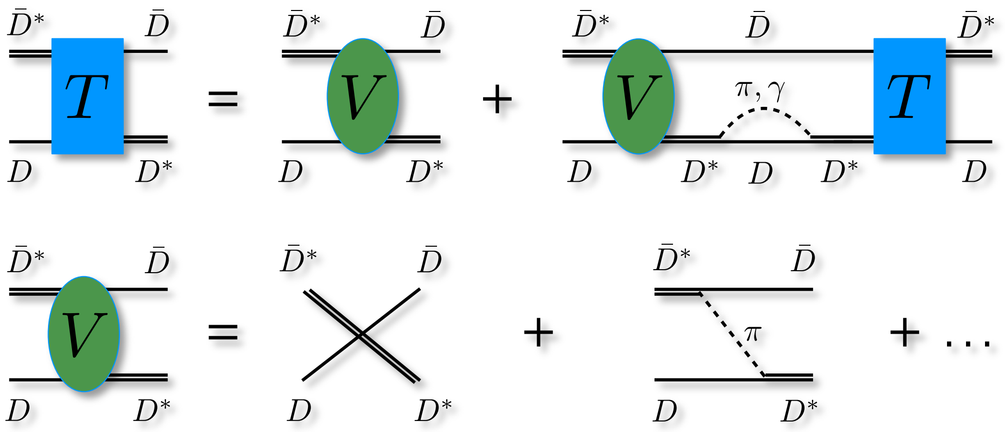
Z.-H. Zhang et al., JHEP 08 (2024) 130

- Coupled channels: $D^0\bar{D}^{*0}, D^+D^{*-}$ with $C=+$
- $T$ matrix is given by the LSE:

$$T(E;p',p) = V(E;p',p) + \int \frac{l^2 dl}{2\pi^2} V(E;p',l)G(E;l)T(E;l;p)$$

Potential: contact term ($C_{0X}, C_{1X}$) + one-pion exchange (OPE)

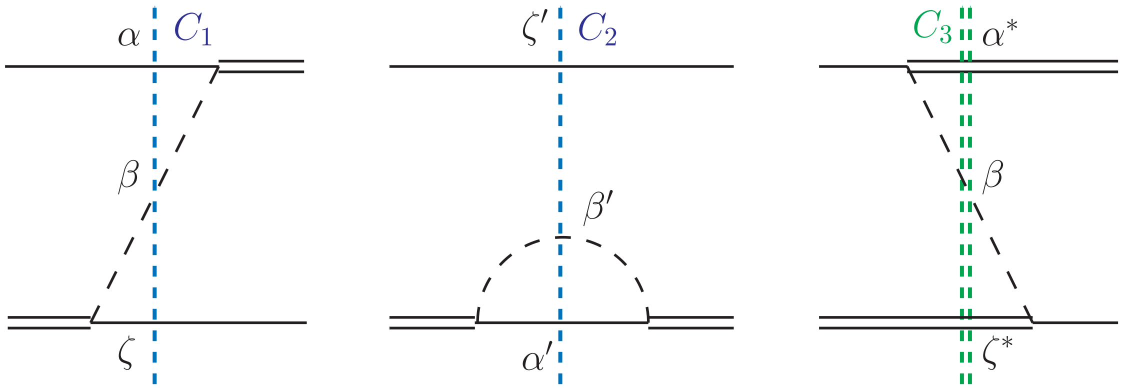
$$V_{\rm ct} = \frac{1}{2}\begin{pmatrix} C_{0X}+C_{1X} & C_{0X}-C_{1X} \\ C_{0X}-C_{1X} & C_{0X}+C_{1X}\end{pmatrix}$$




➢ 3-body effects: OPE, $D^*$ selfenergy

$$V_\pi(E;p',p) = \frac{g^2}{6F_\pi^2}\begin{pmatrix} \frac{1}{2}V^{SS}_{D^0\pi^0\bar{D}^0} & V^{SS}_{D^0\pi^+D^-} \\ V^{SS}_{D^+\pi^-\bar{D}^0} & \frac{1}{2}V^{SS}_{D^+\pi^0D^-}\end{pmatrix}$$

$$V^{SS}_{\alpha\beta\zeta} = \frac{1}{2}\int_{-1}^{1} dz \frac{q^2}{2E_\beta(q)}\left(\frac{1}{D^R_{\alpha\beta\zeta}} + \frac{1}{D^A_{\alpha^*\beta\zeta^*}}\right)$$



(a) (b) (c)

retarded OPE | $D^*$ selfenergy | advanced OPE

# Molecular line shapes at LO

- Scattering length approx.: $k\cot\delta = -\frac{1}{a} + \cdots$
- Poles: bound or virtual state ($\kappa = 1/|a|$)
  - Bound and virtual state can hardly be distinguished above threshold ($E > 0$)

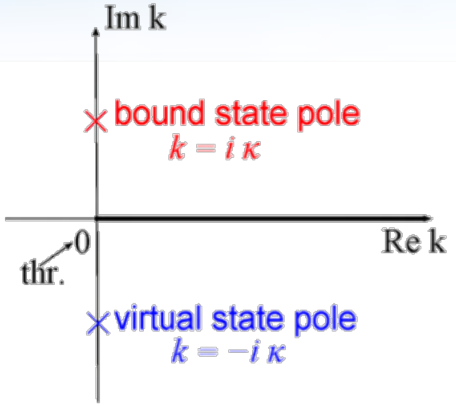
$$|T_{\rm NR}(E)|^2 \propto \left|\frac{1}{\pm\kappa + i\sqrt{2\mu E}}\right|^2 = \frac{1}{\kappa^2 + 2\mu E}$$

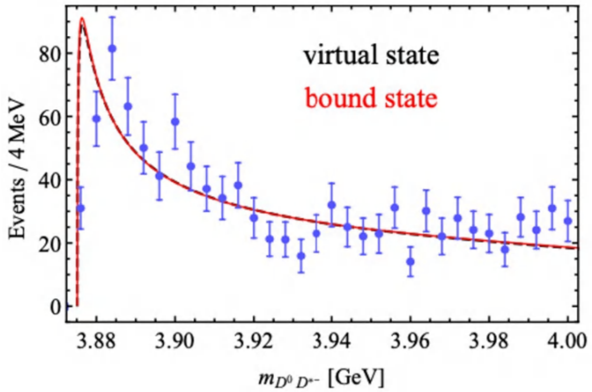
  - Different below threshold ($E < 0$)
    - bound state: peaked below threshold

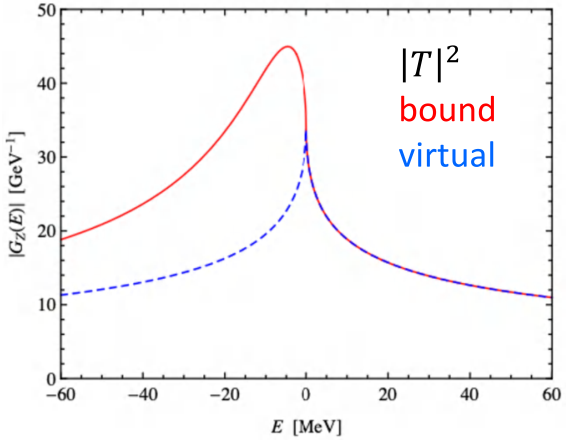
$$|T_{\rm NR}(E)|^2 \propto \frac{1}{(\kappa - \sqrt{-2\mu E})^2}$$

    - virtual state: sharp cusp at threshold

$$|T_{\rm NR}(E)|^2 \propto \frac{1}{(\kappa + \sqrt{-2\mu E})^2}$$

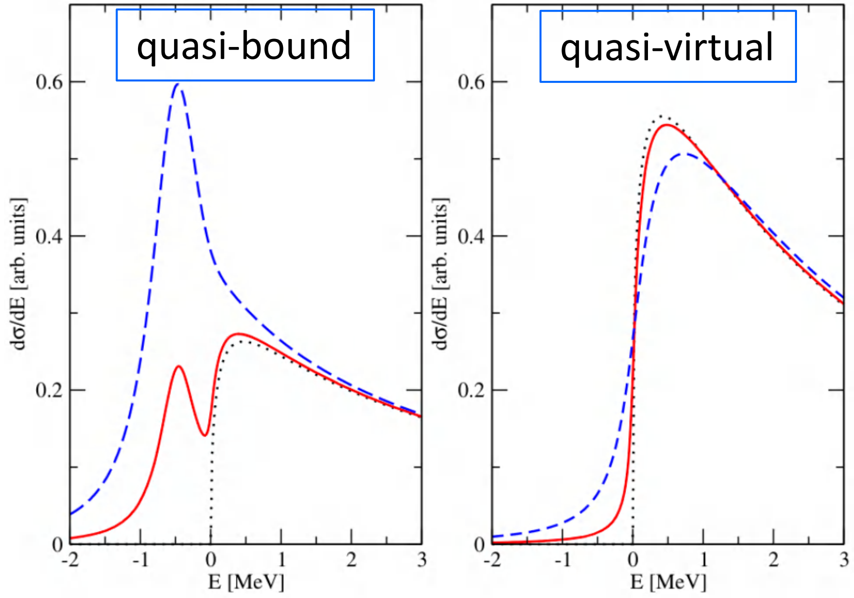



FKG, et al., RMP 90 (2018) 015004;
N. Brambilla et al., Phys.Rept. 873 (2020) 1

line shapes w/ phase space; one unstable constituent:

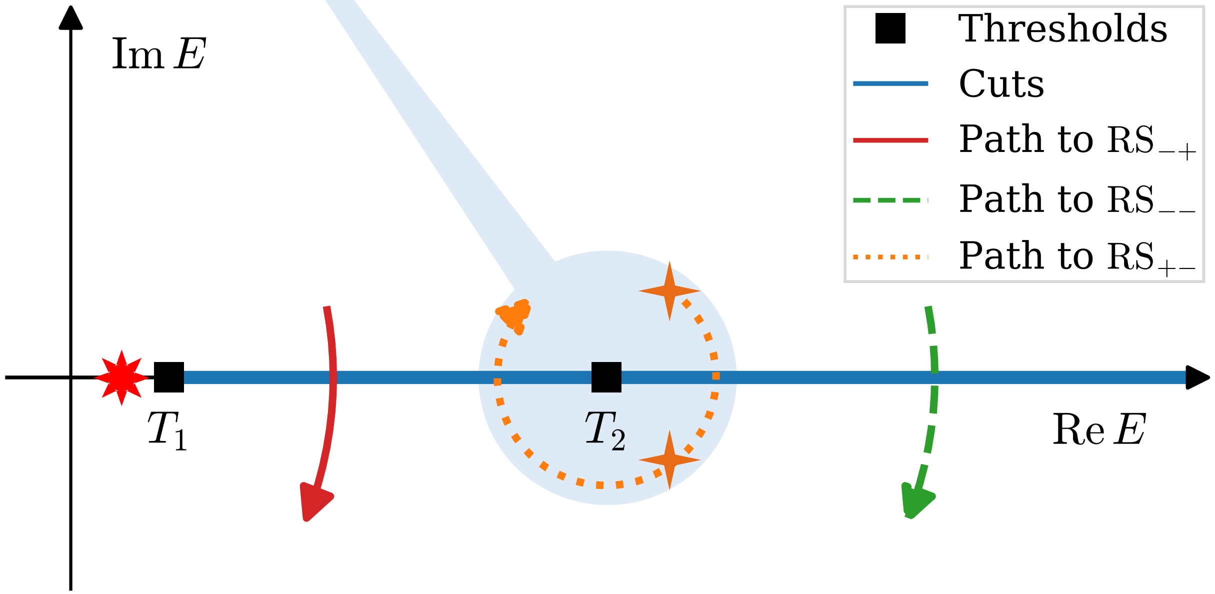



| $\Gamma = 0$ | 0.1 MeV | 1 MeV |
|---|---|---|
| dotted | dashed | solid |

# Lippmann-Schwinger equation (LSE)

Z.-H. Zhang et al., JHEP 08 (2024) 130

- Two poles of the $T$-matrix for the $(D\bar{D}^*)_0 - (D\bar{D}^*)_\pm$ scattering amplitudes (4 Riemann sheets)
  - $X(3872)$ pole on the 1st RS ($\mathrm{RS}_{++}$)
  - $W_{c1}(3880)$ pole on the 4th RS ($\mathrm{RS}_{+-}$)
    - Shaded by $D^+D^{*-}$ threshold
    - Cusp at the $D^+D^{*-}$ threshold!!!



# Prediction of an isospin vector partner of $X(3872)$

- There must be near-threshold isovector $W_{c1}$ states
  - Virtual state pole in the stable $D^*$ limit
    - $W_{c1}^{+}$ in $D^+\bar{D}^{*0}$ single-channel scattering amplitude: pole on the 2nd Riemann sheet (RS), $8^{+8}_{-5}$ MeV below $D^0 D^{*-}$ threshold

$$W_{c1}^{\pm}:\quad 3866.9^{+4.6}_{-7.7} - i(0.07 \pm 0.01)\ \mathrm{MeV}$$

    - $W_{c1}^{0}$ in $(D\bar{D}^*)_0 - (D\bar{D}^*)_{\pm}$ scattering amplitudes: pole on the 4th RS ($\mathrm{RS}_{+-}$), $1.3^{+0.8}_{-0.0}$ MeV above $D^+D^{*-}$ threshold

$$W_{c1}^{0}:\quad 3881.2^{+0.8}_{-0.0} + i1.6^{+0.7}_{-0.9}\ \mathrm{MeV}$$

  - Must appear as threshold cusps!!!
  - Compact tetraquarks (Maiani et al. (2005)) cannot be virtual states as they do not feel the thresholds

Cutoff insensitivity checked: poles relative to thresholds varied within 5% for $\Lambda \in [0.5, 1.0]$ GeV

- Virtual state $W_{c1}$ was confirmed in lattice QCD calculation with $M_\pi = 280$ MeV

M. Sadl et al., PRD 111 (2025) 054513

| $J^{PC}$ | Interpolators | $1/a_0\,[\mathrm{fm}^{-1}]$ | $r_0$ [fm] | $\chi^2/N_{\mathrm{dof}}$ | $\Delta m_V$ [MeV] |
|---|---|---|---|---|---|
| $1^{+-}$ | All | $0.46^{+1.16}_{-0.45}$ | $0.96^{+0.43}_{-0.73}$ | 0.13 | $-3.0^{+3.0}_{-31.1}$ |
| | $\eta_c\rho$ excl | $0.54^{+1.07}_{-0.44}$ | $2.23^{+0.95}_{-1.08}$ | 0.24 | $-2.8^{+2.6}_{-17.1}$ |
| $1^{++}$ | All | $0.62^{+1.30}_{-0.51}$ | $1.78^{+0.25}_{-2.44}$ | 0.18 | $-3.8^{+3.6}$[a] |
| | $J/\psi\rho, \eta_c a_0$ excl | $0.96^{+1.42}_{-0.91}$ | $2.19^{+0.36}_{-1.00}$ | 0.15 | $-6.7^{+6.7}_{-19.5}$ |

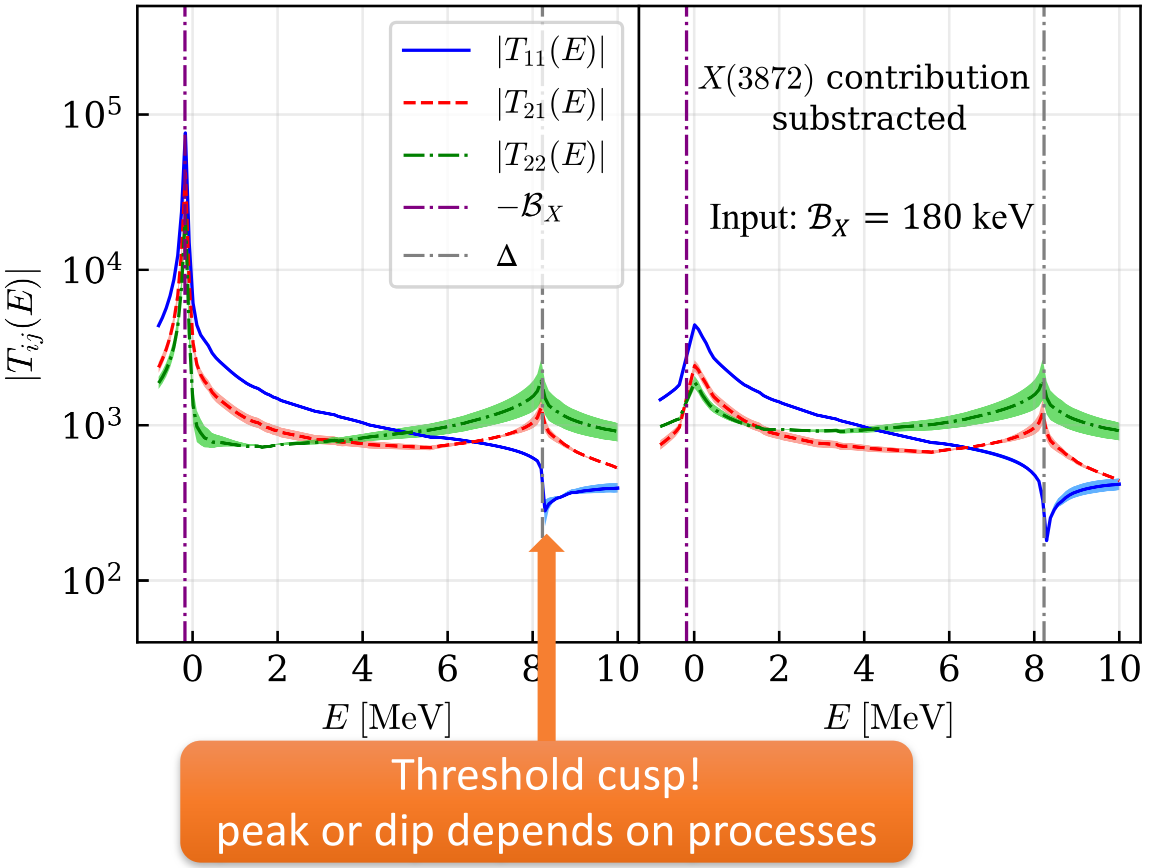
[a]Uncertainty is so large that it is unbounded from below.

sign convention different from ours

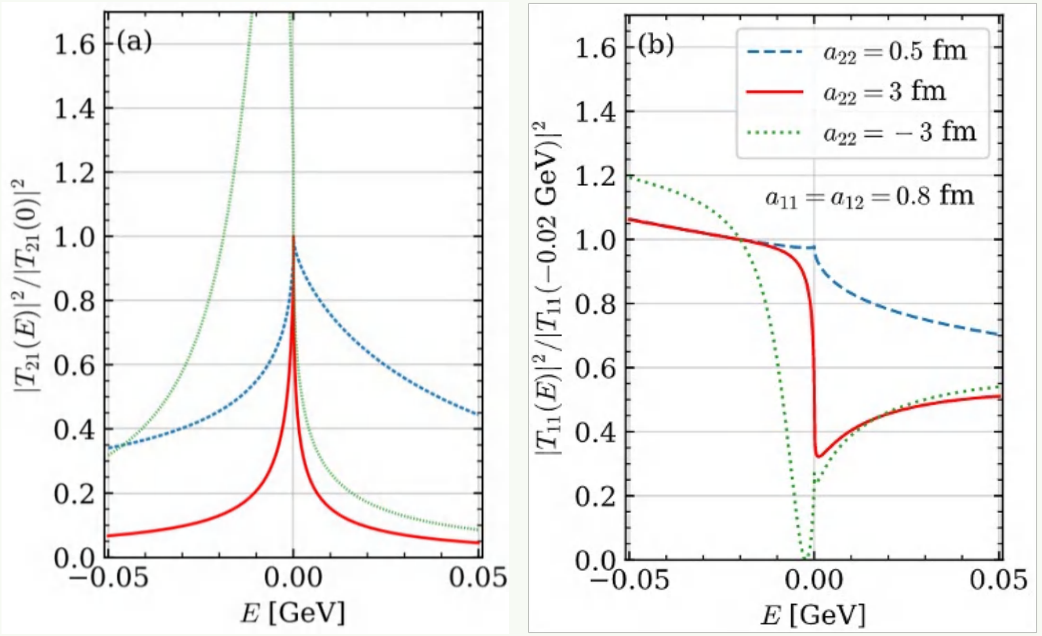
Also obtained in one-boson exchange model in X.-X. Chen, Z.-M. Ding, J. He, PRD 111 (2025) 114008

# Why have they not been observed?

- $W_{c1}^0$ lives in the same amplitudes as the $X(3872)$, effects shielded by $X$
  - $W_{c1}^0$ in $D^0\bar{D}^{*0} - D^+D^{*-}$ scattering amplitudes




- Universality of dip for large scattering length
  - ✓ For strongly interacting channel-2 (large $a_{22}$), there must be a dip around threshold

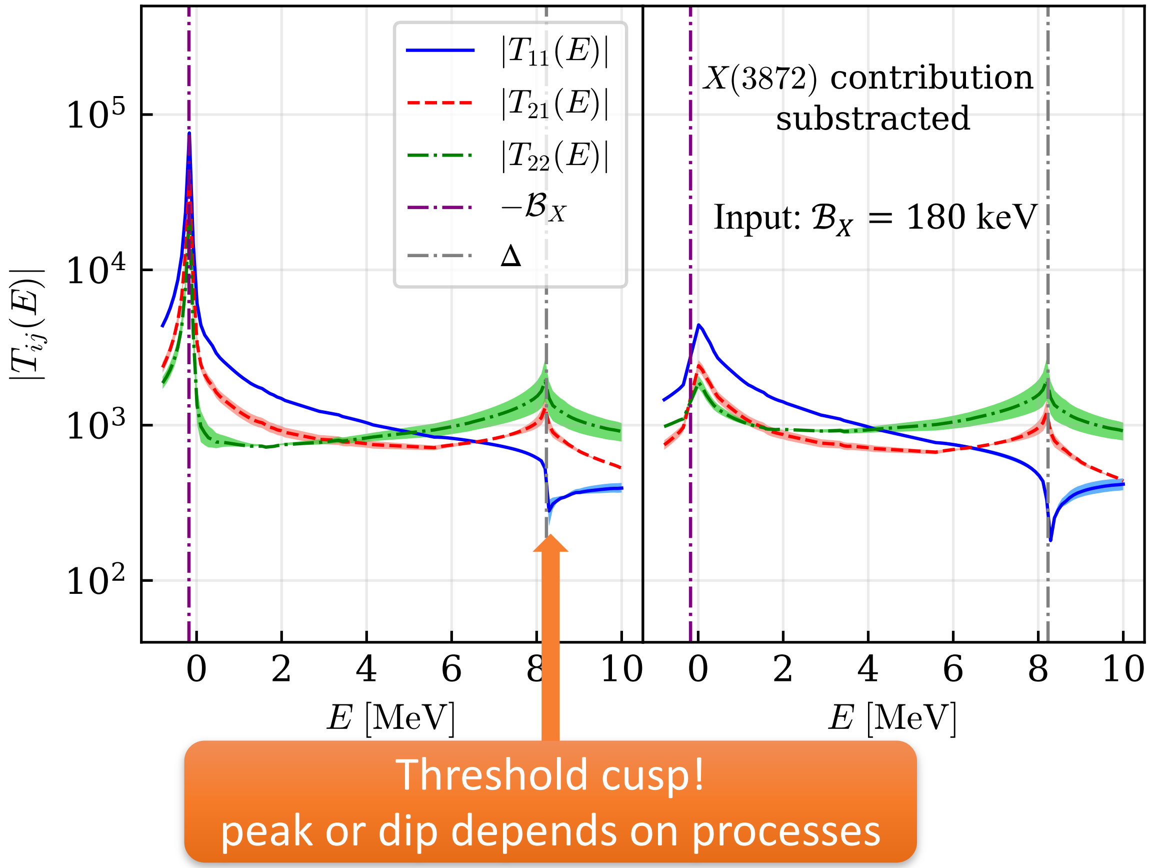



$$T_{11}(E) = \frac{-8\pi\Sigma_2\left(\frac{1}{a_{22}} - i\sqrt{2\mu_2 E}\right)}{\left(\frac{1}{a_{11}} - ik_1\right)\left[\frac{1}{a_{22,\text{ eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]}$$

X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

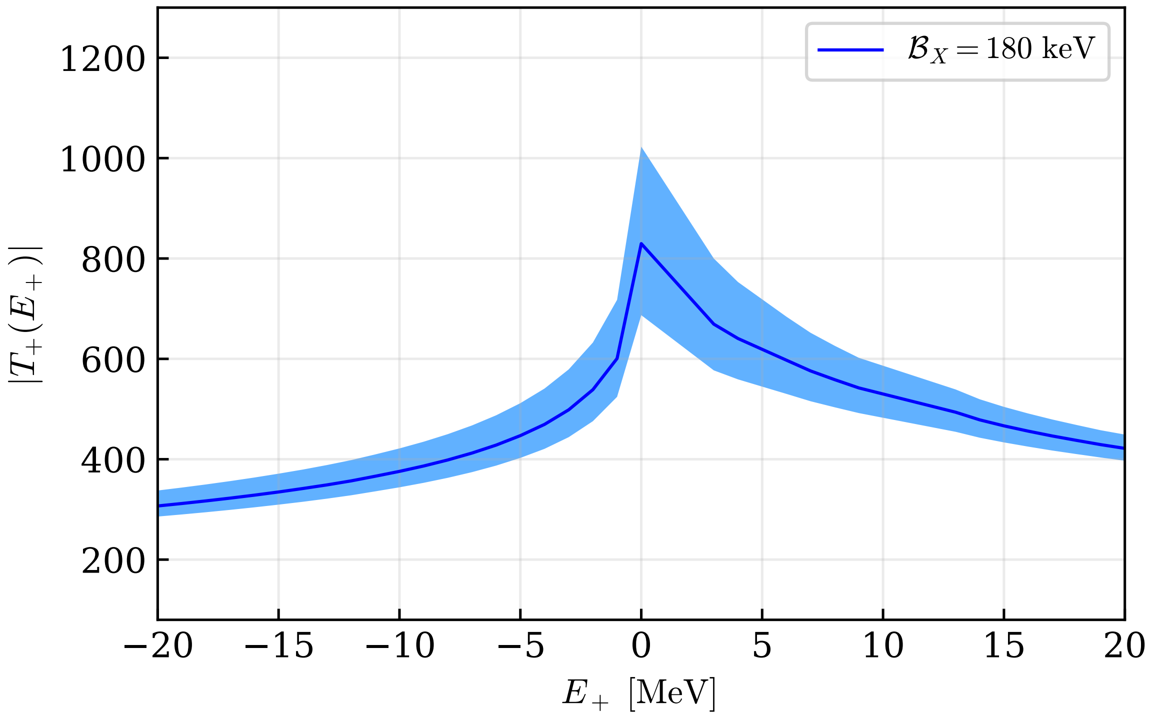
# Why have they not been observed?

- $W_{c1}^0$ lives in the same amplitudes as the $X(3872)$, effects shaded by $X$
  - ➢ $W_{c1}^0$ in $D^0\bar{D}^{*0} - D^+D^{*-}$ scattering amplitudes




X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001

- ➢ Charged $W_{c1}^+$ in $D^+\bar{D}^{*0}$ scattering amplitude: height much lower than the $X$ peak




- ➢ should be searched for in high-statistic $J/\psi\pi^\pm\pi^0$ data

- The observed $X(3872)$ signals should contain the $W_{c1}^0$ contribution as well ⇒ combined analysis !!

# Combined analysis of BESIII and LHCb data for $X(3872)$

Teng Ji et al., arXiv:2502.04458

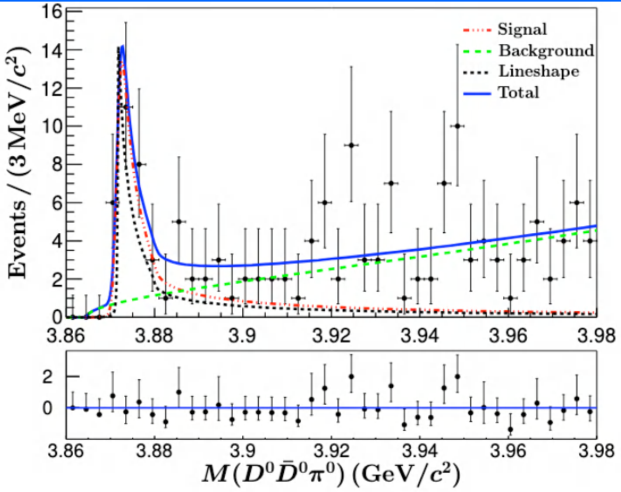
- $X(3872)$ line shapes $\Rightarrow X(3872)$ + possible $W_{c1}(3880)^0$
- $\pi^+\pi^-$ invariant mass distribution $\Rightarrow$ isospin breaking, information on $I=1$

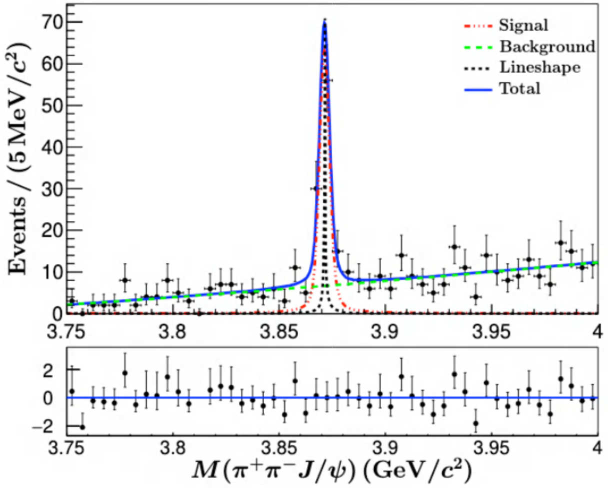
BESIII:

$e^+e^- \to \gamma[D^0\bar{D}^0\pi^0]$

$e^+e^- \to \gamma[J/\psi\pi^+\pi^-]$

BESIII, PRL 132 (2024) 151903

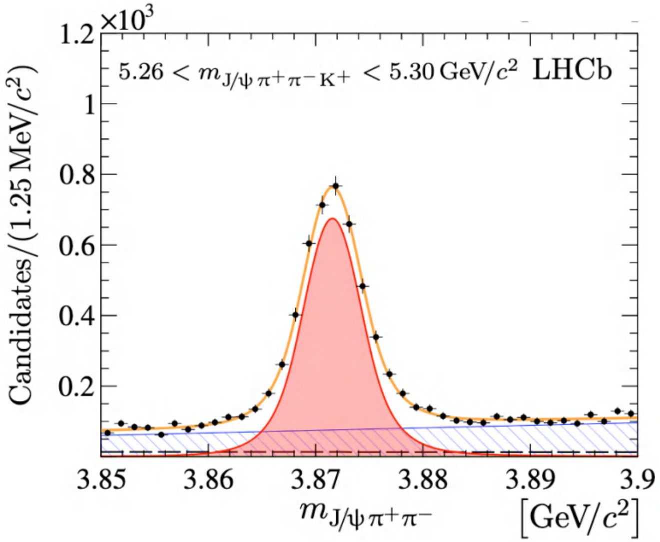


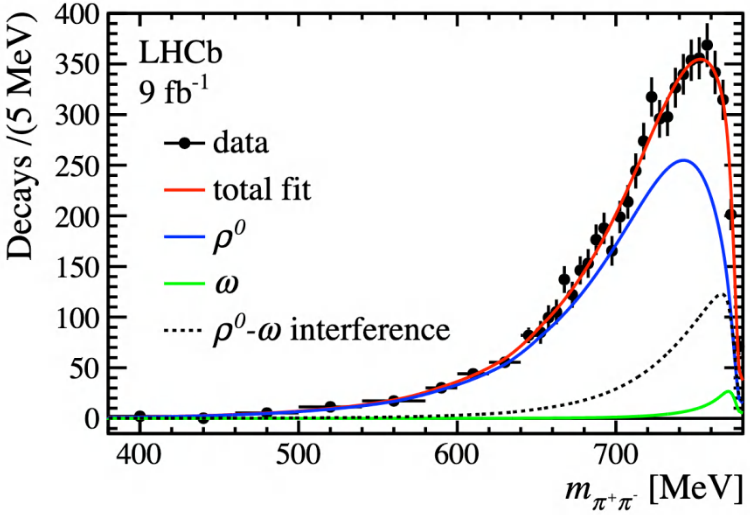




LHCb:

$B^+ \to K^+[J/\psi\pi^+\pi^-]$

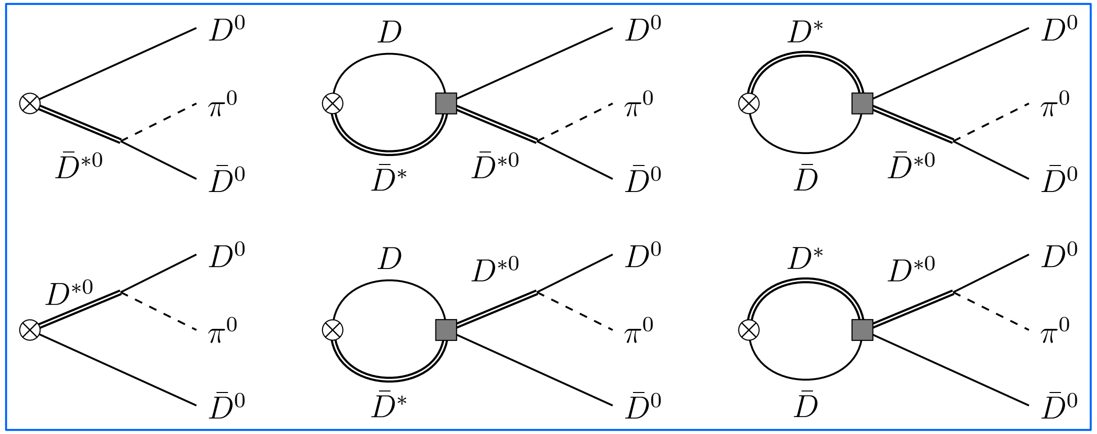
LHCb, JHEP 08 (2020) 123;
PRD 108 (2023) L011103

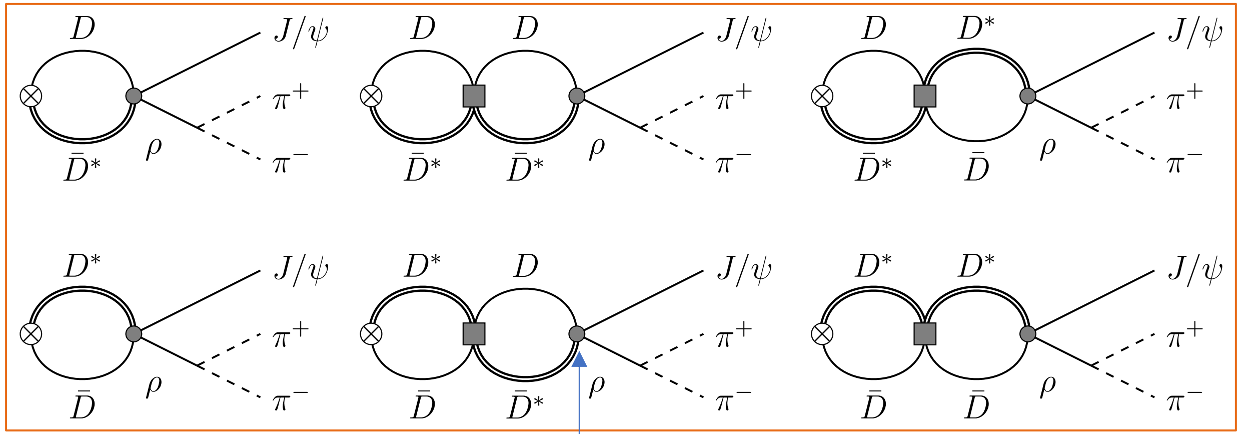


# Combined analysis of BESIII and LHCb data for $X(3872)$

- Coupled channels
  - $(D\bar{D}^*)_0$, $(D\bar{D}^*)_\pm$: contact terms + OPE, $D\bar{D}\pi$ three-body effects considered
  - Inelastic channels:
    - $J/\psi\rho, J/\psi\omega$: $\rho$ included using the Omnes dispersive approach, $\rho$-$\omega$ mixing considered
    - $J/\psi\gamma, \psi'\gamma, \chi_{cJ}(1P)\pi^0$: neglected in the baseline fit, included in uncertainty analysis

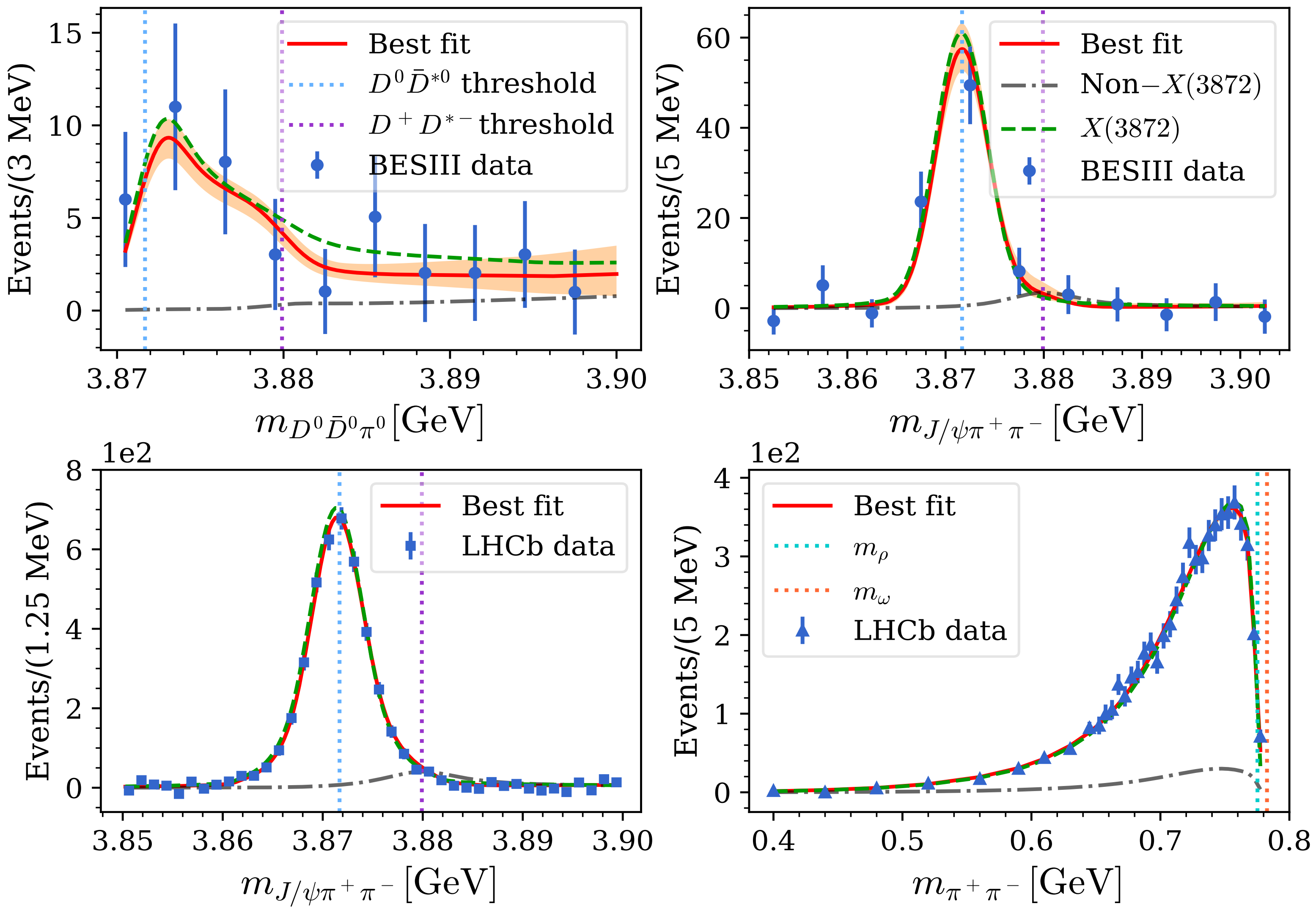



$\rho$-$\omega$ mixing included

# Combined analysis of BESIII and LHCb data for $X(3872)$

## Results updated




Best fit: $\chi^2/\mathrm{dof} = 57/(96-10) = 0.66$

# Pole positions: $X(3872)$ and isovector $W_{c1}(3880)$

- Poles
  - $X(3872)$ as a bound state below $D^0\bar{D}^{*0}$ threshold (2.7$\sigma$)

$$E_X = \left(-160^{+28+32+40}_{-38-28-60} - 125^{+12+14+15}_{-13-22-28}\,i\right)\ \mathrm{keV}$$

$\delta m_{D^0} = \delta m_{D^{*0}} = 50\ \mathrm{keV}, \delta(m_{D^{*0}} - m_{D^0}) = 30\ \mathrm{keV}$

$$M_X = \left(3871.53^{+0.06}_{-0.08} - 0.13^{+0.02}_{-0.04}\,i\right)\ \mathrm{MeV}$$

  - $W_{c1}(3880)^0$ pole on $\mathrm{RS}_{+-}$, relative to the $D^+D^{*-}$ threshold:

$$E_W = \left(3.1 \pm 0.7 + 1.3^{+1.9}_{-0.6}\,i\right)\ \mathrm{MeV}$$

- Residues:

$$g_{X,0} = (0.35 \pm 0.03)e^{0.12^{+0.08}_{-0.04}\,i}\ \mathrm{GeV}^{-\frac{1}{2}},$$

$$g_{X,\pm} = (0.22 \pm 0.02)e^{0.09^{+0.09}_{-0.05}\,i}\ \mathrm{GeV}^{-\frac{1}{2}},$$

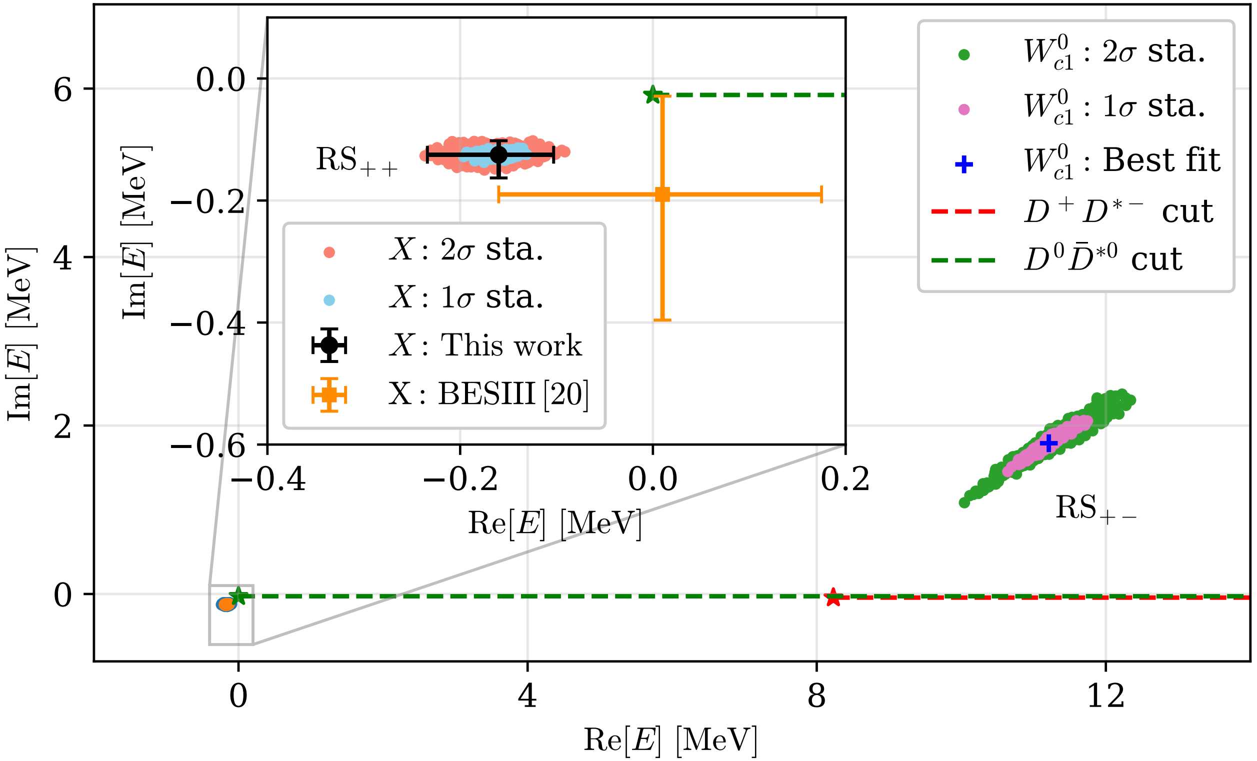
$$g_{W,0} = (0.44 \pm 0.07)e^{-1.1^{+0.05}_{-0.09}\,i}\ \mathrm{GeV}^{-\frac{1}{2}},$$

$$g_{W,\pm} = \left(0.70^{+0.03}_{-0.06}\right) e^{-2.18^{+0.16}_{-0.05}\,i}\ \mathrm{GeV}^{-\frac{1}{2}}.$$

$X(3872)$ couples more strongly to $D^0\bar{D}^{*0}$;

$W_{c1}(3880)$ couples more strongly to $D^+D^{*-}$

➤ Much more precise than previous determination



# Other properties of X(3872)

- Width (twice of the imaginary part of the pole): $250^{+48}_{-62}$ keV
  - Branching fractions computed using the method in L.A. Heuser, G. Chanturia, FKG, C. Hanhart, M. Hoferichter, B. Kubis, EPJC 84 (2024) 599

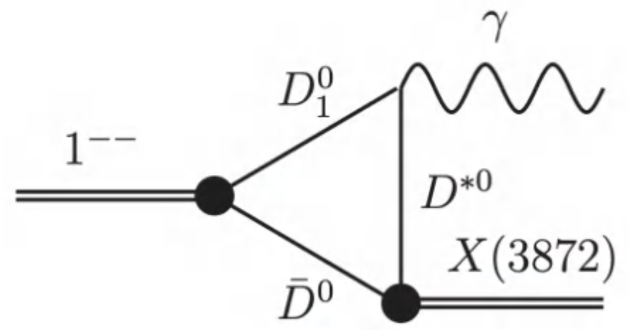
| Mode | $D^0\bar{D}^0\pi^0$ | $D^0\bar{D}^0\gamma$ | $J/\psi\pi^+\pi^-$ | $J/\psi\pi^+\pi^-\pi^0$ | others |
|---|---|---|---|---|---|
| BR(%) | $41^{+3}_{-4}$ | $22\pm 2$ | $5^{+2}_{-1}$ | $16^{+4}_{-3}$ | $16\pm 2$ |

- Isospin breaking ratio $R_X \equiv \left|\frac{g_{XJ/\psi\rho}}{g_{XJ/\psi\omega}}\right| = 0.26(2)$
- Compositeness using a formula including range corrections Y. Li, FKG, J.-Y. Pang, J.-J. Wu, PRD 105 (2022) L071502

$$X = 1 - \exp\left(\frac{1}{\pi}\int_0^\infty dE\,\frac{\mathrm{Re}\,\delta(E)}{E - \mathrm{Re}\,E_X}\right) = 0.97(2)$$

# Signal of $W_{c1}(3880)$?

- Signal of $W_{c1}(3880)^0$ almost invisible in the current data, reasons:
  - Virtual state, threshold cusp
  - $(D\bar{D}^*)_0$ easier produced than $(D\bar{D}^*)_\pm$ for both $e^+e^- \to \gamma D\bar{D}^*$ @ $\sqrt{s} \approx 4.23$ GeV and $B^+ \to K^+ D\bar{D}^*$




FKG et al., PLB 725 (2013) 127

$$\Gamma(D_1^0 \to \gamma D^{*0}) \gg \Gamma(D_1^+ \to \gamma D^{*+})$$

J.G. Korner et al., PRD 47 (1993) 3955;
Fayyazuddin et al., PRD 50 (1994) 2329

- For $B^+$ decays, fit parameters (ratio of production vertices): $P_\pm/P_0 = 0.5 \pm 0.1$

  Data: PDG 2024

$$\frac{\mathrm{Br}\left[B^+ \to K^+\left(D^+D^{*-} + D^-D^{*+}\right)\right]}{\mathrm{Br}\left[B^+ \to K^+\left(D^0\bar{D}^{*0} + \bar{D}^0D^{*0}\right)\right]} = 0.14 \pm 0.02$$

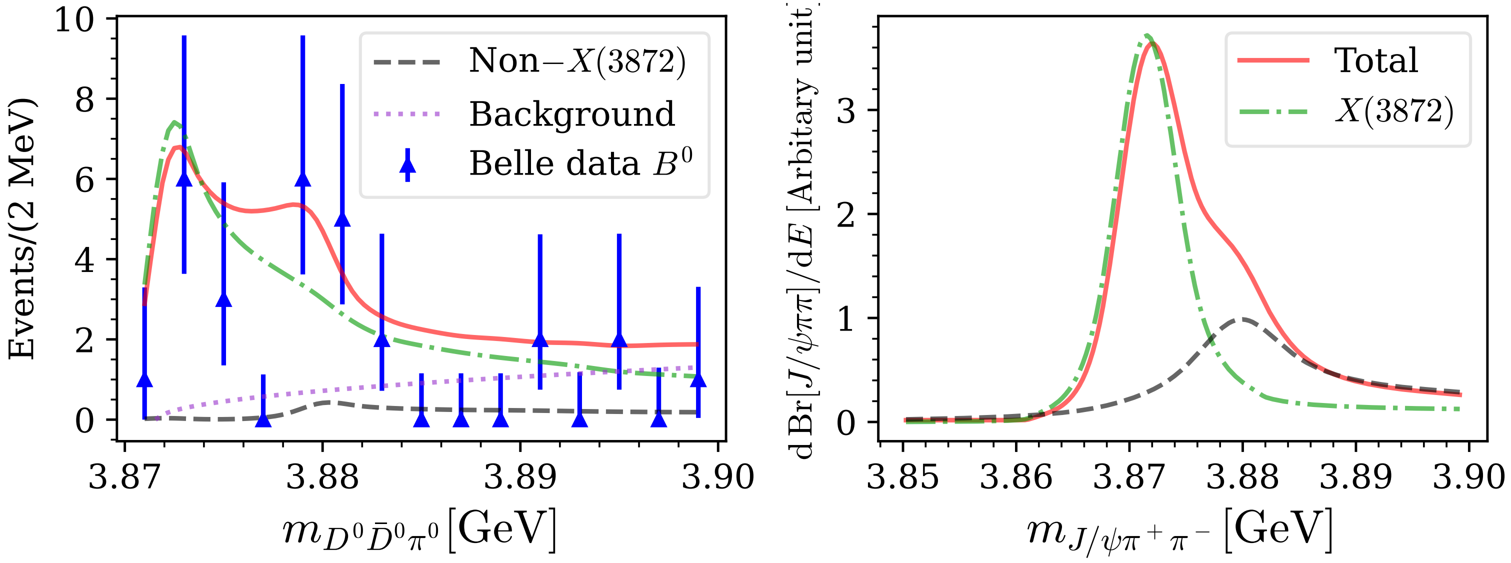
- Switching $u \leftrightarrow d$, situation should be different for $B^0$ decays

  Data: PDG 2024

$$\frac{\mathrm{Br}\left[B^0 \to K^0\left(D^+D^{*-} + D^-D^{*+}\right)\right]}{\mathrm{Br}\left[B^0 \to K^0\left(D^0\bar{D}^{*0} + \bar{D}^0D^{*0}\right)\right]} = 5.8 \pm 2.7$$

# Implications of the existence of $W_{c1}(3880)$

- $W_{c1}(3880)^0$ signal should be stronger in $B^0 \to K^0[D^0\bar{D}^0\pi^0, J/\psi\pi^+\pi^-]$ decays, to be checked @ LHCb, Belle II




- Cusp at $D^+\bar{D}^{*0}$ threshold in $J/\psi\pi^\pm\pi^0$

# Other implications to be explored

- $\mathrm{Br}(X \to \psi'\gamma)/\mathrm{Br}(X \to J/\psi\gamma)$: could be different at different experiments

$\Gamma(\chi_{c1}(3872) \to \gamma\psi(2S))/\Gamma(\chi_{c1}(3872) \to \gamma J/\psi)$

PDG 2024

| VALUE | CL% | EVTS | DOCUMENT ID | | TECN | COMMENT |
|---|---|---|---|---|---|---|
| • • We do not use the following data for averages, fits, limits, etc. • • | | | | | | |
| $< 0.59$ | 90 | | ABLIKIM | 2020W | BES3 | $e^+ e^- \to \gamma\chi_{c1}(3872)$ |
| $2.46 \pm 0.64 \pm 0.29$ | | $36 \pm 9$ | [1] AAIJ | 2014AH | LHCB | $B^+ \to \gamma\psi(2S)K^+$ |
| $< 2.1$ | 90 | | BHARDWAJ | 2011 | BELL | $B^+ \to \gamma\psi(2S)K^+$ |
| $3.4 \pm 1.4$ | | | AUBERT | 2009B | BABR | $B^+ \to \gamma c\bar{c}K'$ |

[1] From $36.4 \pm 9.0$ events of $\chi_{c1}(3872) \to J/\psi\gamma$ decays with a statistical significance of $4.4\sigma$.

- $\Gamma(X \to \chi_{cJ}\pi^0)$ with iso-vector final states: could receive sizable contribution from $W_{c1}(3880)^0$

| | |
|---|---|
| $\pi^0\chi_{c2}$ | $< 5\%$ |
| $\pi^0\chi_{c1}$ | $(3.8^{+1.9}_{-1.7})\%$ |
| $\pi^0\chi_{c0}$ | $< 16\%$ |

PDG 2024

# Summary and outlook

- Pole position of the $X(3872)$ determined to be (relative to the $D^0\overline{D}^{*0}$ threshold)

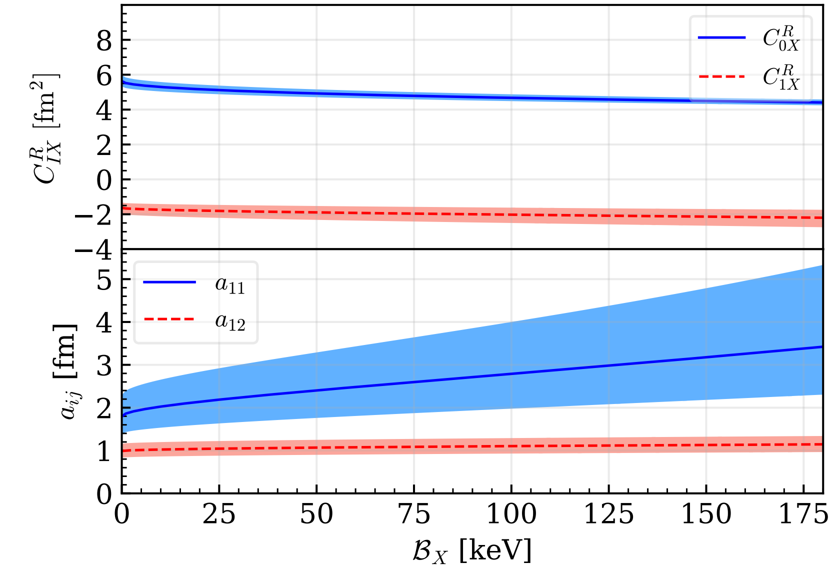
$$E_X = \left(-160^{+43+38}_{-47-57} - 125^{+18+15}_{-25-28} i\right)\ \mathrm{keV}$$

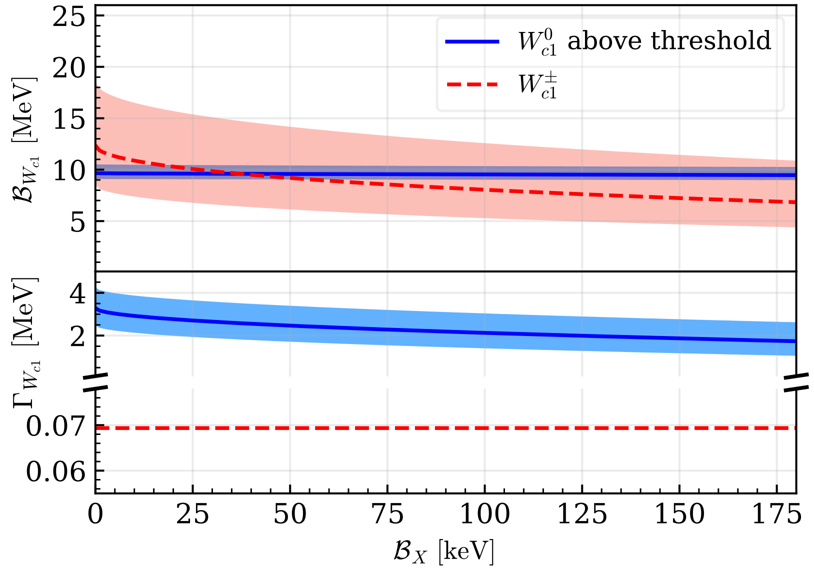
- Compositeness of $X(3872)$: $0.97(2)$
- Existence of an isovector $W_{c1}(3880)$
  - Signal of $W_{c1}(3880)^0$ predicted to be more visible in $B^0 \to K^0[D^0\overline{D}^0\pi^0, J/\psi\pi^+\pi^-]$
  - Signal of $W_{c1}(3880)^\pm$: threshold cusp at $D^+\overline{D}^{*0}$ threshold in $J/\psi\pi^\pm\pi^0$
- Some X(3872) decays need to be reanalyzed

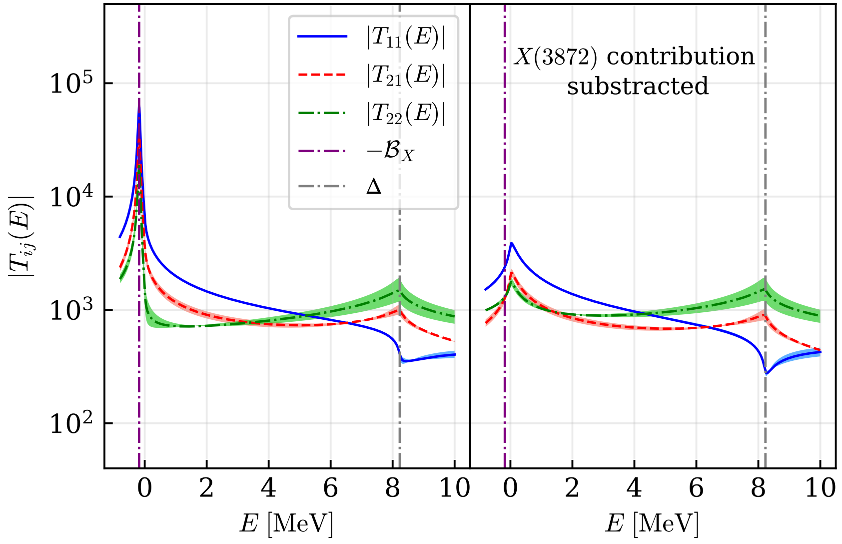
**Thank you for your attention!**

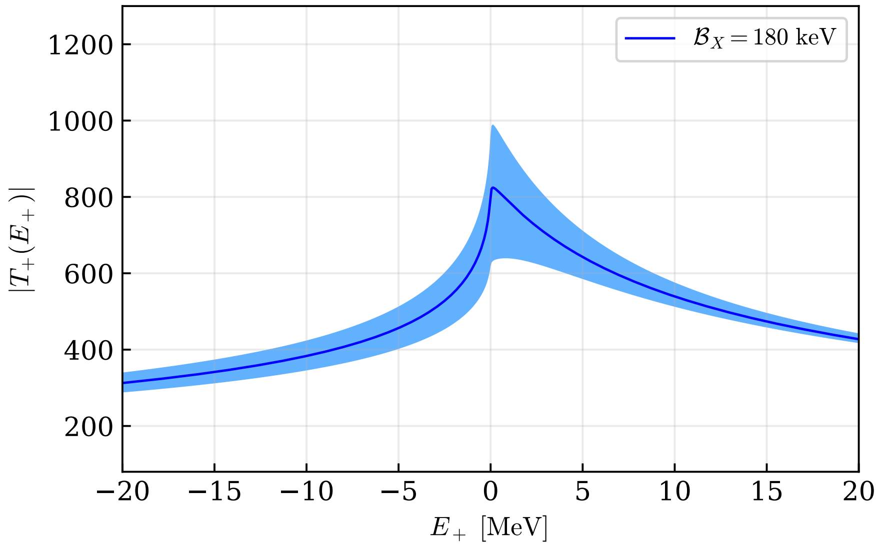
# Results in the pionless theory

- All the qualitative features in the pion-full theory persist in the much simpler pionless theory



# X(3872) line shapes

- Line shapes of a near-threshold resonance depend on reaction mechanism! X.-K. Dong, FKG, B.-S. Zou, PRL 126 (2021) 152001
  - Peak for $|T_{21}|$ (1: lower inelastic channel; 2: elastic channel)

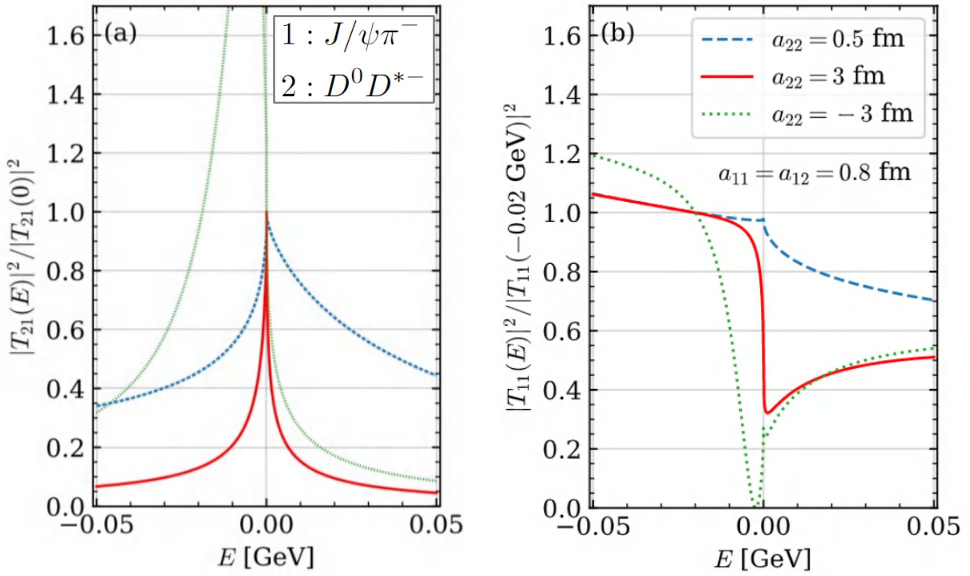
$$T_{21}(E) = \frac{-8\pi\Sigma_2}{a_{12}(1/a_{11} - ik_1)}\left[\frac{1}{a_{22,\text{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]^{-1}.$$

  - Dip for $|T_{11}|$ if scattering length for channel-2 is large

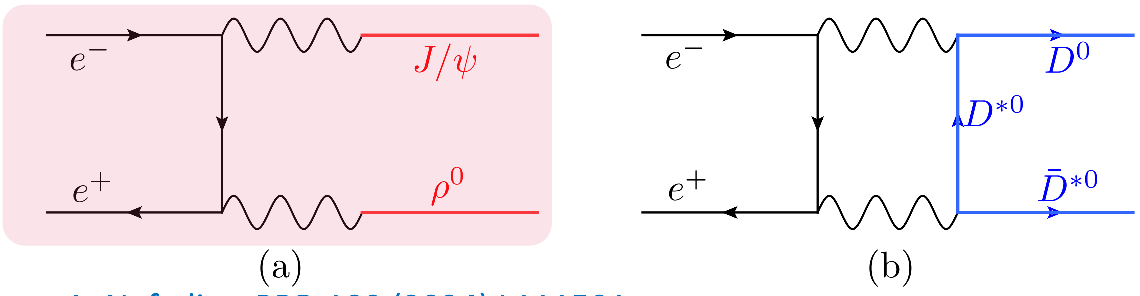
$$T_{11}(E) = \frac{-8\pi\Sigma_2\left(\frac{1}{a_{22}} - i\sqrt{2\mu_2 E}\right)}{\left(\frac{1}{a_{11}} - i\,k_1\right)\left[\frac{1}{a_{22,\text{eff}}} - i\sqrt{2\mu_2 E} + \mathcal{O}(E)\right]}$$

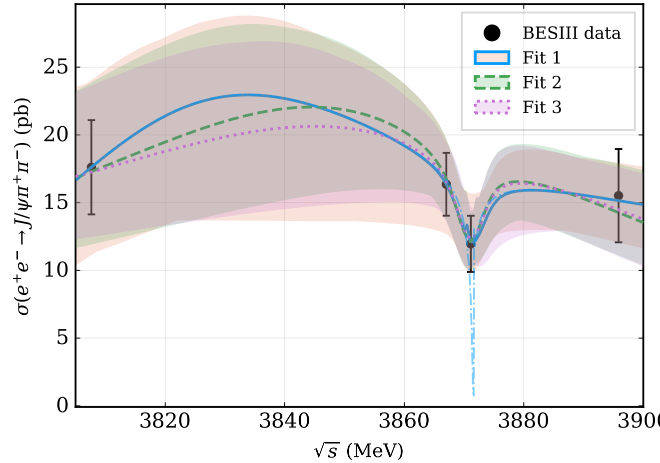
$$= -8\pi E_2^{\text{thr}}\left(\underbrace{\frac{1}{a_{11}^{-1} - ik_1}}_{\text{background}} + \underbrace{\frac{a_{12}^{-2}(a_{11}^{-1} - ik_1)^{-2}}{a_{22,\text{eff}}^{-1} - ik_2}}_{\text{pole term}}\right)$$

The interfering phase is fixed by unitarity!




- $X(3872)$ shows up as a dip in $e^+e^- \to X \to J/\psi\pi\pi$ direct production




V. Baru, FKG, C. Hanhart, A. Nefediev, PRD 109 (2024) L111501