Two-Photon-Exchange Effect in Elastic *ep* Scattering

Dispersion Relation vs. Hadronic model

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Outline

1. Introduction: Ex of ep, estimation of TPE

2. TPE in toy models

3. Discussion and conclusion

4. Further studies

Introduction: the EM form factors

The electromagnetic (EM) form factors of proton are defined as

$$< P(p_4) | J_\mu^\gamma | P(p_2) > \triangleq \bar{u}(p_4)[F_1(Q^2)\gamma_\mu + F_2(Q^2)\frac{i\sigma_{\mu\nu}}{2M_N}q^\nu]u(p_2)$$

EM current

EM form factors

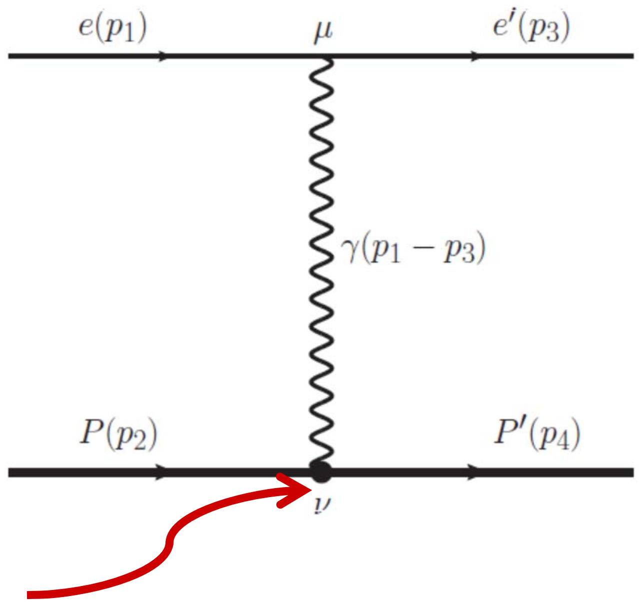
Question:

How to measure the EM FFs? How to relate the physical cross section with the matrix element.

$q \equiv p_4 - p_2, Q^2 \equiv -q^2$

FFs by unpolarized *ep* scattering

Before 1995, the unpolarized *ep* scattering by assuming one-photon-exchange is used to measure $F_1(Q^2), F_2(Q^2)$ (Rosenbluth method)



$$\Gamma^{\mu}_{\gamma N \to N} = ie[F_1(Q^2)\gamma^{\mu} + F_2(Q^2)\frac{i\sigma^{\mu\nu}}{2M_N}q_{\nu}]$$

EM radiative corrections are also considered and ***soft photon approximation*** is used in TPE before 2003.

FFs by unpolarized *ep* scattering

Rosenbluth method: extract $F_1(Q^2), F_2(Q^2)$ from the unpolarized OPE cross section

$$\sigma_R^{\mathrm{Ex}} = \sigma_R^{1\gamma} \equiv G_M^2(Q^2) + \frac{\varepsilon}{\tau} G_E^2(Q^2)$$

fixed

In pQCD, one has

$$R \equiv \mu_P \frac{G_E(Q^2)}{G_M(Q^2)} \xrightarrow[pQCD]{Q^2 \to \infty} 1$$

$$\tau \triangleq Q^2 / 4M_N^2, \varepsilon = [1 + 2(1 + \tau \tan^2 \theta_e)]^{-1},$$

$$G_E \triangleq F_1 - \tau F_2, G_M \triangleq F_1 + F_2,$$

PRD49(1994)5671,PRD50(1994)5491.

FFs by polarized *ep* scattering

About in 2000, JLab measured $\mu_p R$ from polarized *ep* scattering *e*(λ)*p*->*ep*($s_{t,l}$) at fixed ε (***polarization transfer method***),

$$\lambda P_{t,l} \equiv \frac{\sigma_{t,l}^{+}(\lambda)-\sigma_{t,l}^{-}(\lambda)}{\sigma_{t,l}^{+}(\lambda)+\sigma_{t,l}^{-}(\lambda)}$$

+ and – correspond to parallel or antiparallel

λ

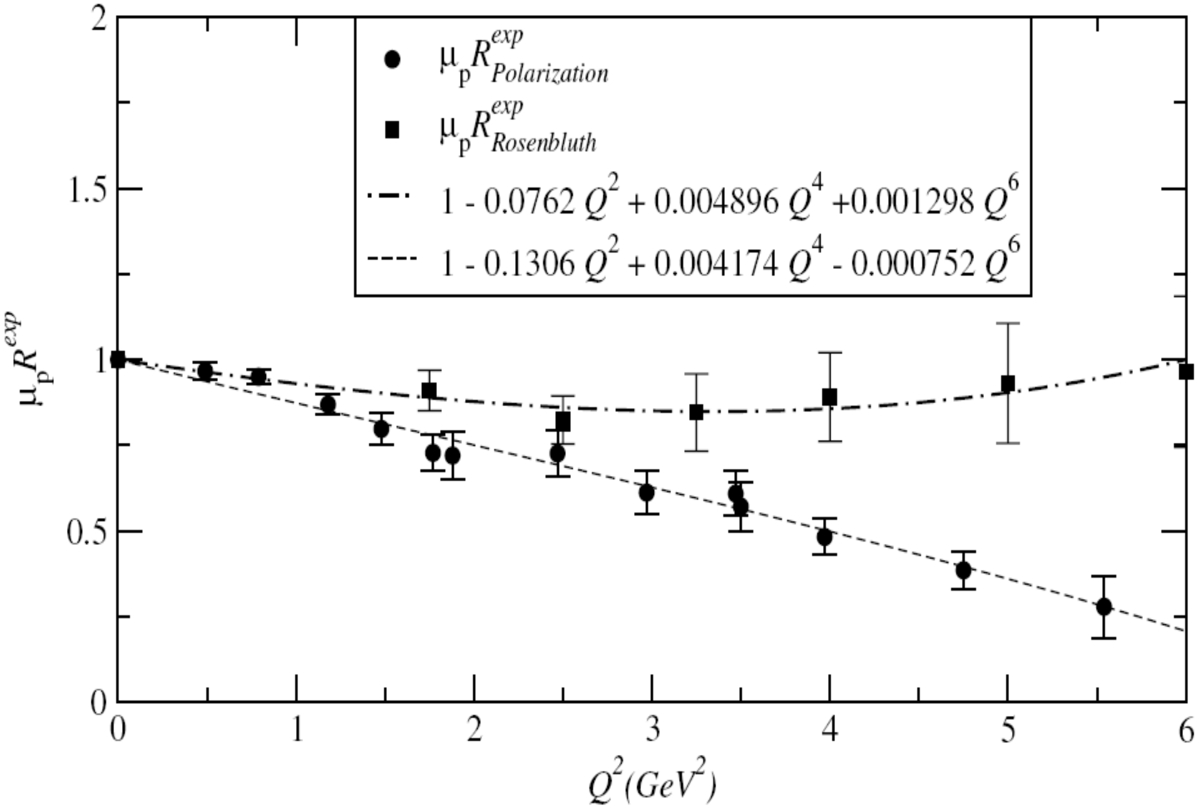
under OPE approximation

$$P_t^{(1\gamma)} = -\frac{1}{\sigma_R}\sqrt{\frac{2\varepsilon(1-\varepsilon)}{\tau}}G_E G_M, \quad P_l^{(1\gamma)} = \frac{1}{\sigma_R}\sqrt{1-\varepsilon^2}G_M^2$$

The results show large ***discrepancy*** with the ***Rosenbluth method***.

PRL84(2000)1398,PRL104(2010)232401.

Rosenbluth vs. polarized



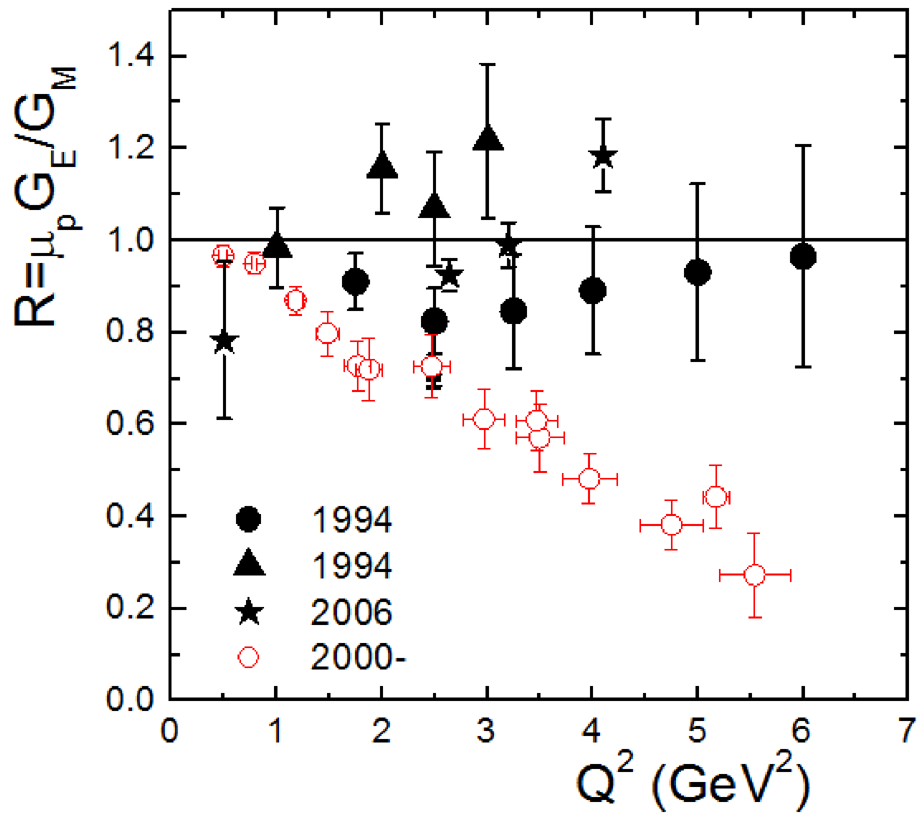
Rosenbluth method

Polarization method

experimental values of μ_pR by Rosenbluth method and polarization method. references in PRL91,142304(2003)

Super-Rosenbluth

In 2005, a more precise measurement by Rosenbluth method was presented and shows:



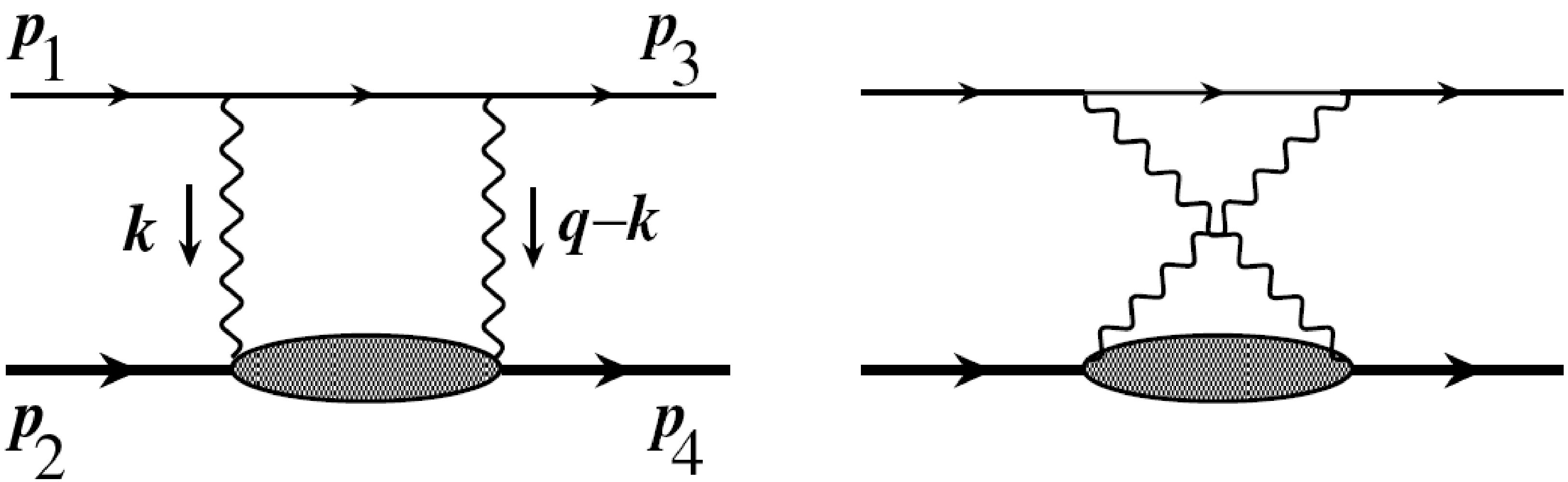
Rosenbluth method

Polarization method

summary of the discrepancy between the experimental results

Possible reason : TPE in *ep* scattering

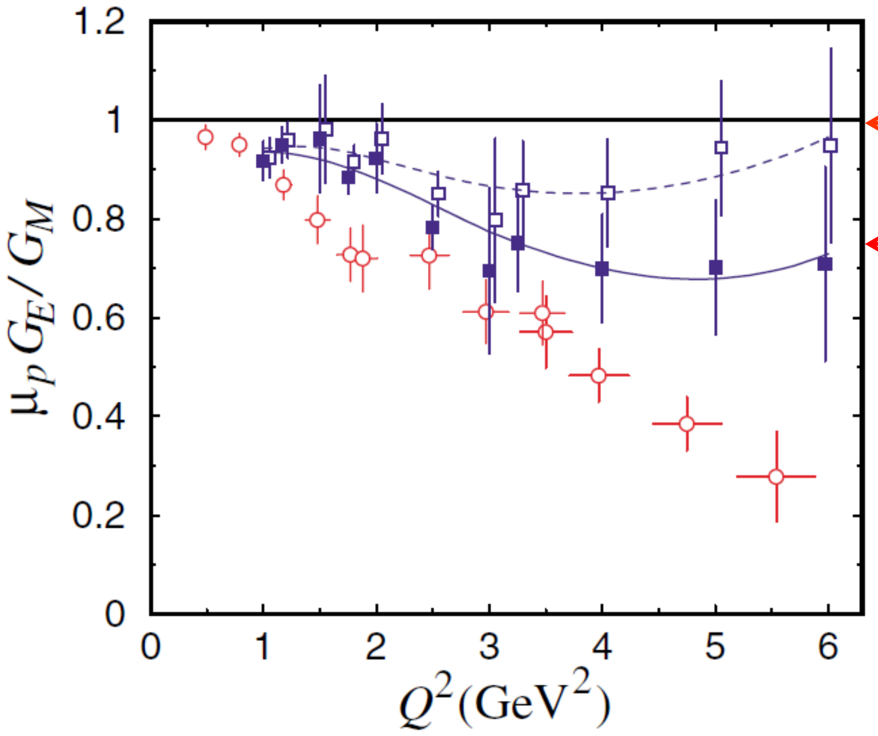
2003,two-photon-exchange (TPE) effects in *ep->ep* are suggested to explain this *discrepancy*.



TEP exchange contribution with finite k

one example: model dependent estimation

numerical results for the TPE corrections to $\mu_p R$



hollow:OPE

solid :OPE+TPE

P.G. Blunden, W. Melnitchouk and J. A. Tjon, PRL91 (2003) 142304

Why does TPE give large corrections?

OPE: Ex data + fitting formula

$$\sigma_R^{\mathrm{Ex}} = \sigma_R^{1\gamma} \equiv G_M^2\left(Q^2\right)\left[1+\frac{\varepsilon}{\mu_p\tau}R^2\left(Q^2\right)\right]$$

TPE: Corrected Ex data + fitting formula

$$\sigma_R^{\mathrm{Ex}} = \sigma_R^{1\gamma}(1+\delta_\varepsilon^{(2\gamma)}) \Rightarrow \overline{\sigma}_R^{\mathrm{Ex}} \equiv \sigma_R^{\mathrm{Ex}}(1-\delta_{2\gamma})$$

$$\overline{\sigma}_R^{\mathrm{Ex}} = \sigma_R^{1\gamma} \equiv G_M^2\left(Q^2\right)\left[1+\frac{\varepsilon}{\mu_p\tau}\overline{R}^2\left(Q^2\right)\right]$$

Although $\delta^{(2\gamma)}$ is about 1%, but the new fitted R may be very different from the old R.

$\delta_\varepsilon^{(2\gamma)}$) the TPE corrections to the unpolairzed cross sections.

Methods used to estimate TPE

many model dependent methods are used to estimate the TPE effects in the literature.

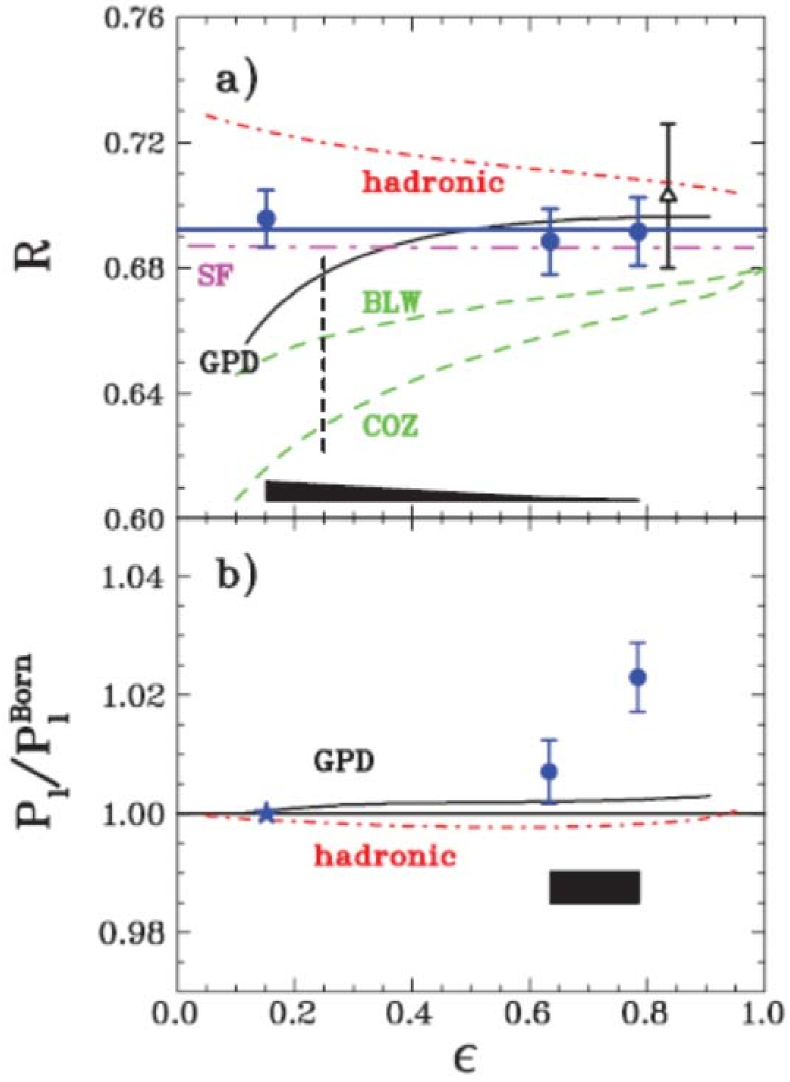
(1) hadronic model:	Blunden....	(2003)
(2) GPDs:	Vanderhaeghen	(2004)
(3) dispersion relation:	Borisyuk ...	(2006,2015,2017)
(4) pQCD:	Borisyuk ...	(2009)
(5) SCEF:	Vanderhaeghen	(2013)
(6) ChpT:	Talukdar	(2020)

PRL91 (2003) 142304; PRL93(2004) 12230;
PRC 78 (2008) 025208; PRL103 (2009) 092004.

Measurements of $\mu_p R$ at different ε

In 2011, $\mu_p R$ at different ε with Q^2=2.49GeV^2 by Polarized transfer methods were firstly measured.

$$R \equiv \mu_p \frac{G_E}{G_M} = -\mu_p \sqrt{\frac{\tau(1+\epsilon)}{2\epsilon}} \frac{P_t^{(1\gamma)}}{P_l^{(1\gamma)}}$$



PRL106(2011) 132501, GEp2γ Collaboration

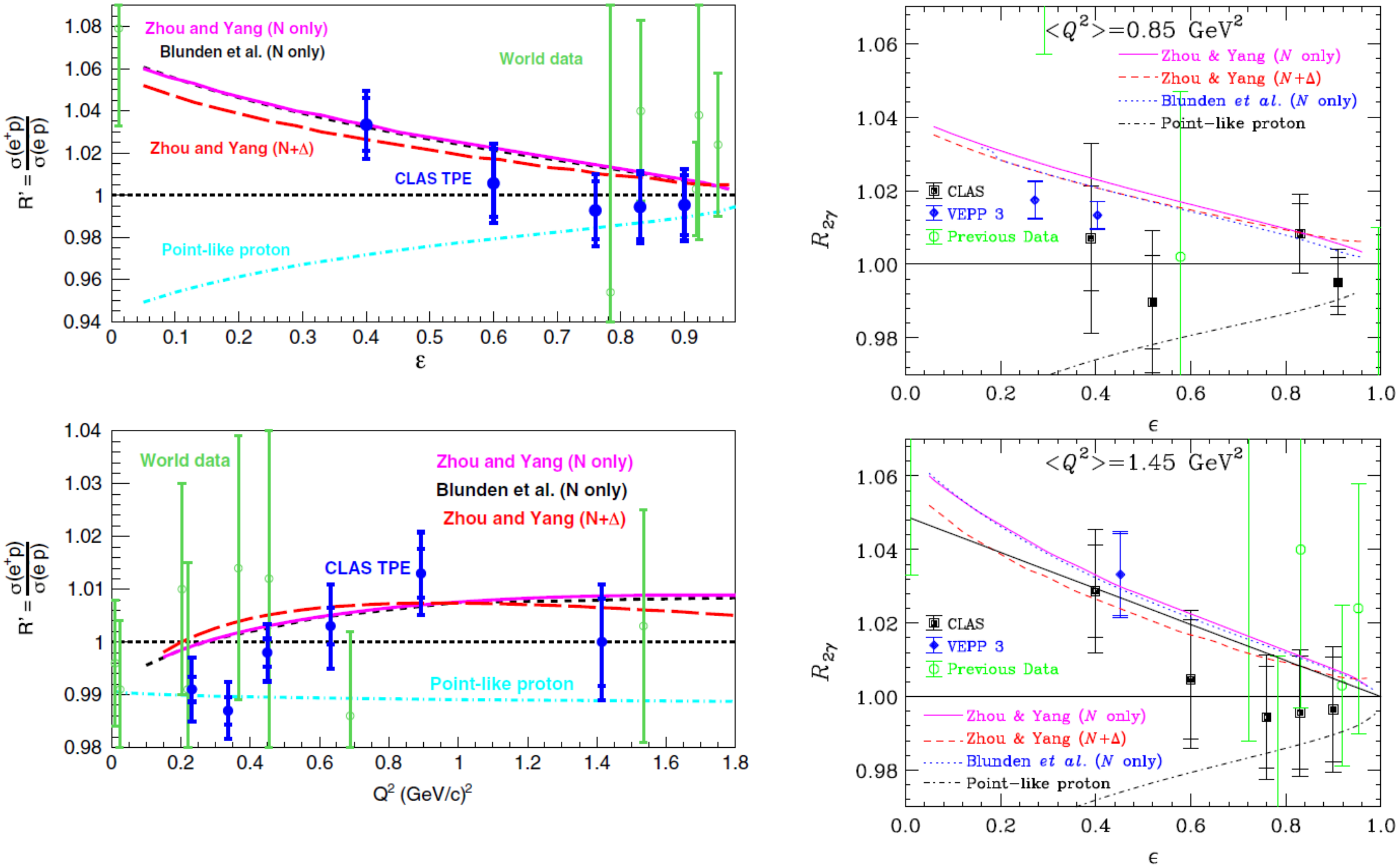
Measurements of $R^{(2\gamma)}$

To study the TPE effects directly, the experiment e^+p scattering is suggested.

$$R^{(2\gamma)} \equiv \frac{\sigma\left(e^+p \to e^+p\right)}{\sigma\left(e^-p \to e^-p\right)}$$

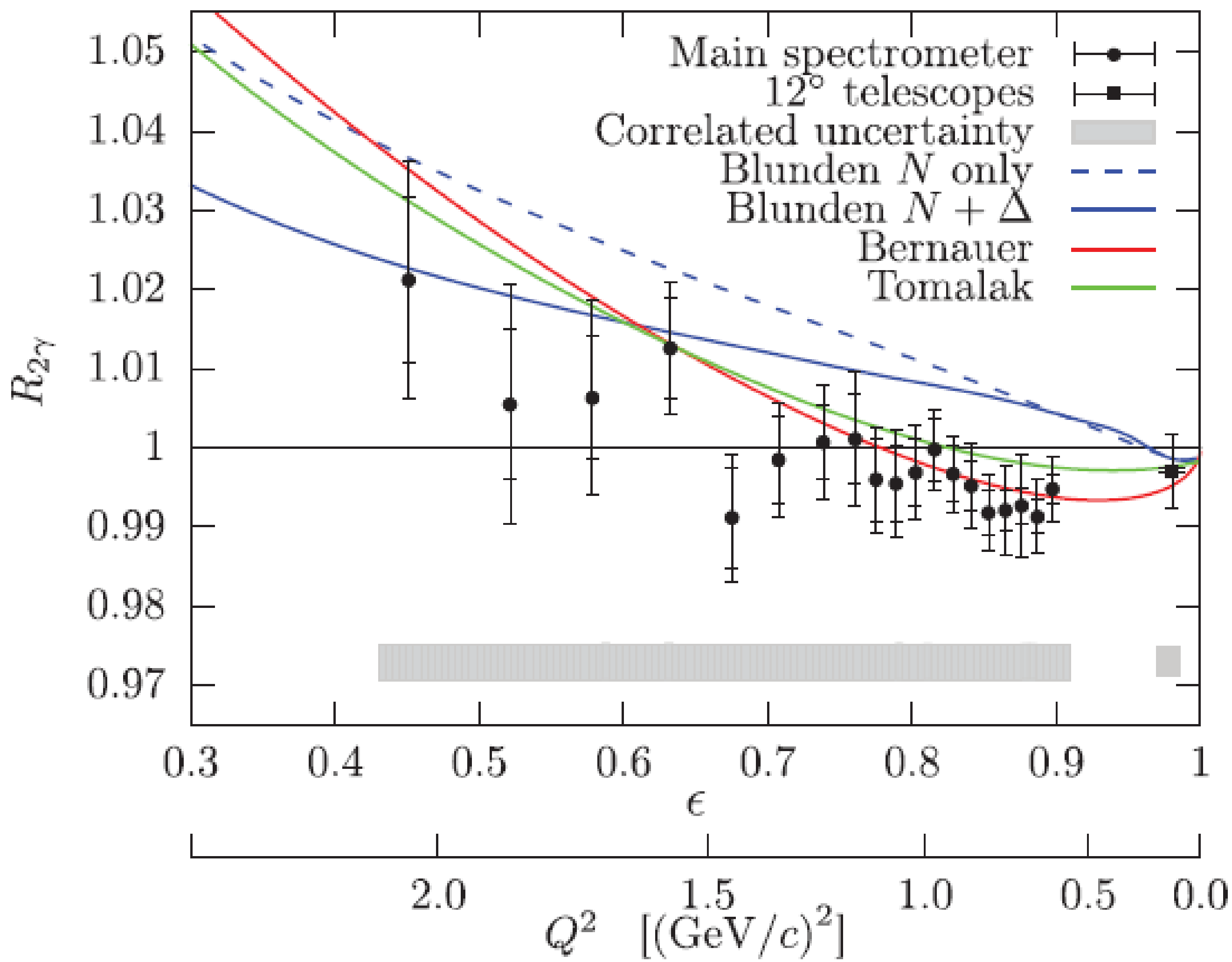
CLAS : 2015
VEPP-3 : 2015
OLYMPUS : 2017

Measurements of $R^{2\gamma}$



PRL114(2015)062003 by CLAS, PRC95 (2017) 065201 by CLAS.

Measurements of $R^{2\gamma}$



PRL118(2017) 092501 by OLYMPUS.

Measurements of $R^{2\gamma}$

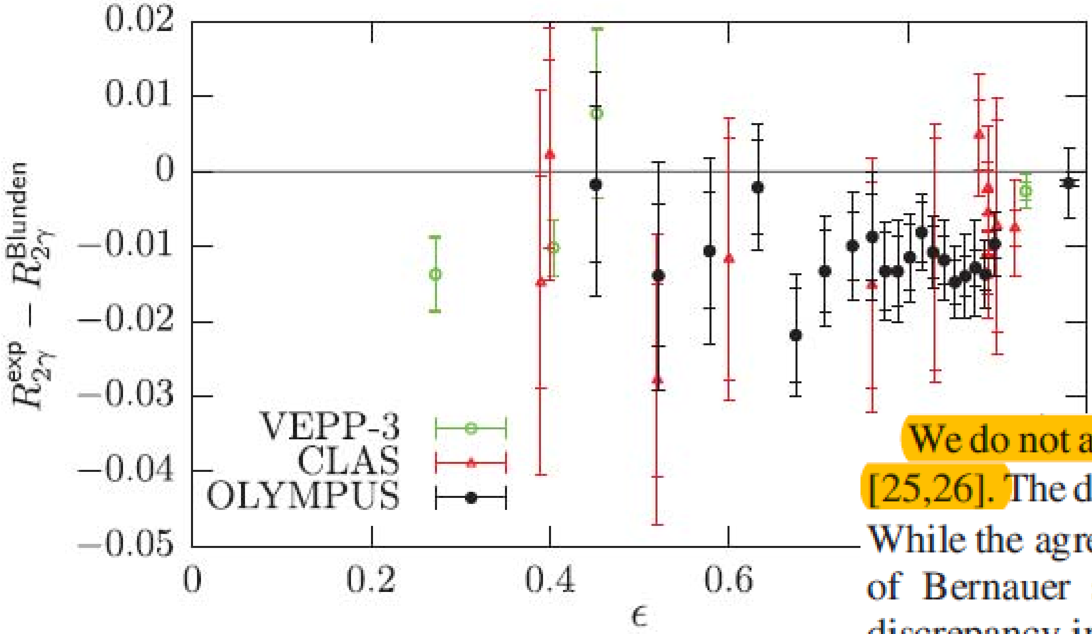


FIG. 3. Comparison of the recent results to the Blunden. The data are in good agreement, but below the prediction. Please note that data at simila been measured at different Q^2. Also note that the have been normalized to the calculation at high ϵ.

We do not agree with the conclusions of the earlier Letters [25,26]. The data shown in Fig. 3 clearly favor a smaller $R_{2\gamma}$. While the agreement with the phenomenological prediction of Bernauer suggests that TPE is causing most of the discrepancy in the form factor ratio in the measured range, the theoretical calculation of Blunden, which shows roughly enough strength to explain the discrepancy at larger Q^2, does not match the data in this regime. To clarify the situation, the size of TPE at large Q^2 has to be determined in future measurements.

PRL118(2017) 092501 by OLYMPUS, Blunden's calculation is taken at 2017 using DR.

TPE in other Processes: application

$$ep \rightarrow e\Delta$$

$$e\pi \rightarrow e\pi$$

$$\mu p \rightarrow \mu p$$

$$ep \rightarrow e\Delta \rightarrow eN\pi$$

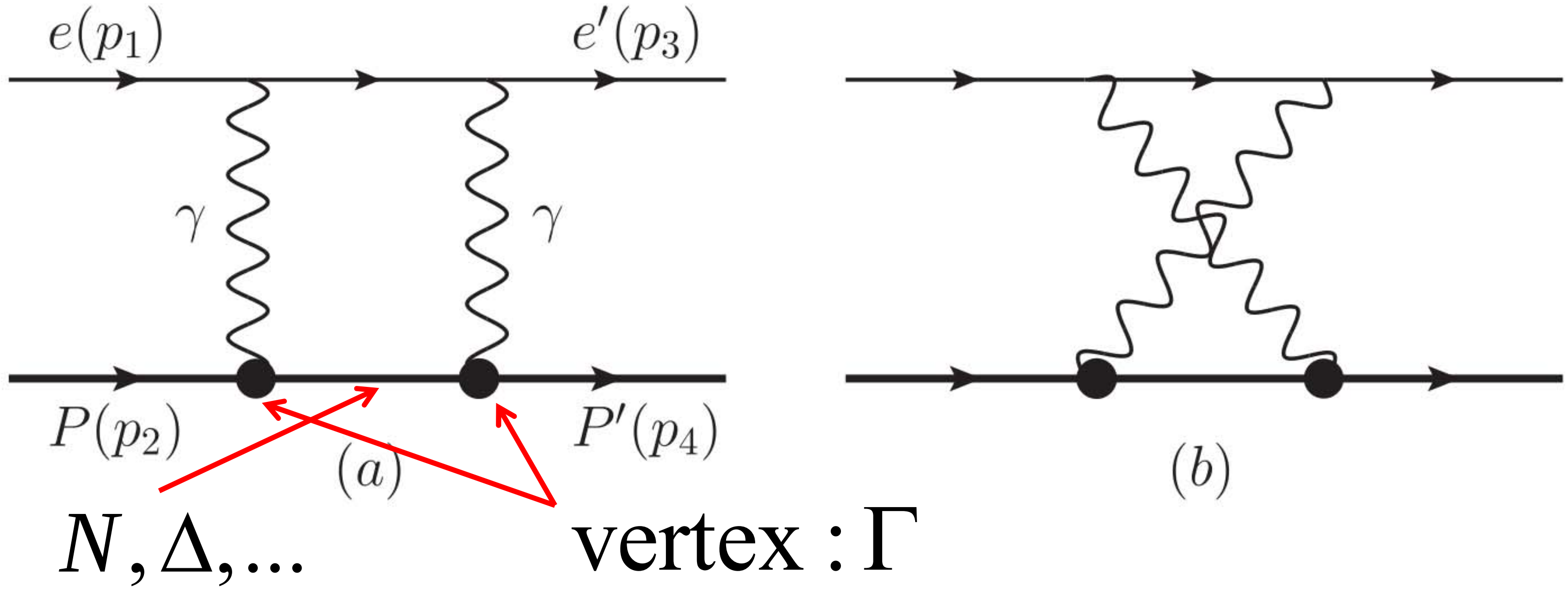
$$ep \rightarrow en\pi^{+}$$

$$e^{+}e^{-} \rightarrow p\bar{p}, \pi^{+}\pi^{-}$$

Estimation of TPE in ep: HM

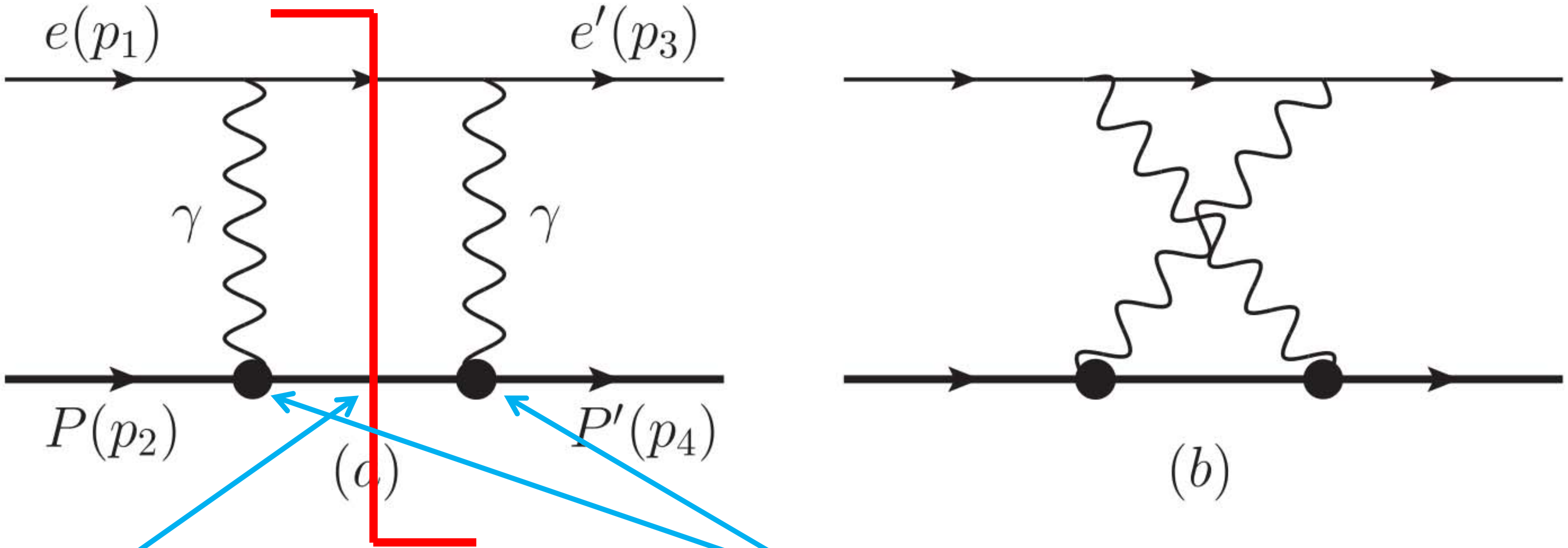
When Q^2 < a few GeV^2, dispersion relation and hadronic model are applied.

HM: intermediate states + vertex (with ph FFs)
=> amplitude



Estimation of TPE in ep: DR

DR: cut => imaginary part of the TPE amplitude
DR => real part of the TPE amplitude



physical: $N, \Delta, \pi N, \ldots$

physical FFs

TPE in ep: DR vs. HD

DR: why DR? "model independent"

Before 2015, un-subtracted DRs are used.

In 2015, once-subtracted DR is used.

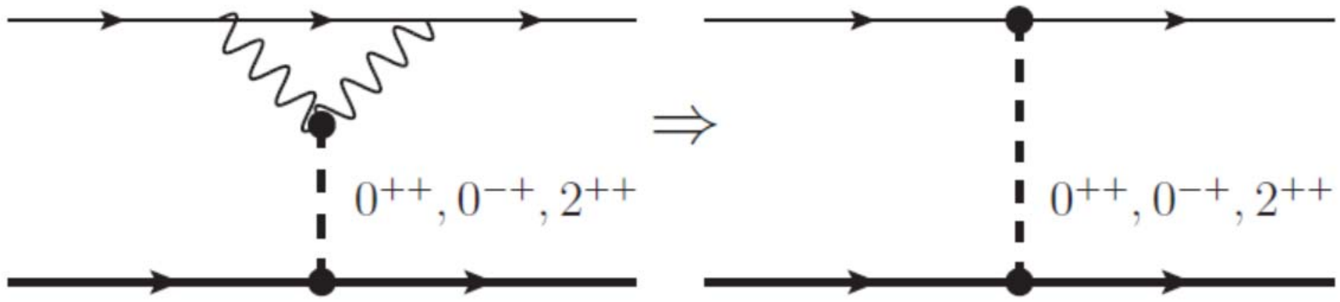
In 2017: unsubtracted DRs are still used by Blunden to analyze the $R^{2\gamma}$ data.

HM: when Δ is included, TPE/OPE -> ∞ when s-> ∞.

In 2014, we suggested the meson-exchange effect.

TPE in ep: DR vs. HD

meson-exchange effect



(1)Their imaginary parts are exact zero, and the un-subtracted DRs give zero results.

(2) TPE/OPE -> ∞ when s-> ∞ if normal propagator for 2^{++} meson is used. Regge form was used in 2014.

PRC90(2014)045205

Short summary on the Ex and Th

$d\sigma_{un}^{ep}$:	1994,2005,	$Q^2 = 2.5, 3.2, 4\ GeV^2$	
P_t / P_l:	2011,	$Q^2 = 2.49\ GeV^2$	*
$R^{2\gamma}$:	2015,2017,	$Q^2 < 2\ GeV^2$	*
$d\sigma_{un}^{e^+e^- \to p\bar{p}}$:	2020	$\sqrt{s} = 2.0 \sim 3.08\ GeV$	
$B_n \propto \mathrm{Im}[\mathcal{M}^{2\gamma}]$:	2020	$Q^2 < 0.613\ GeV^2$	$N\pi\pi$

DR: unsubtracted DR or once-subtracted DR?
HM: unphysical behavior and meson-exchange effect?

DR vs. HM which is reasonable?

toy models are used to try to answer this question.

TPE in ep: general properties at fix t

In the mass less limit, the general amplitude with C,P,T invariance can be written as

$$\mathcal{M}_{ep\to ep} \equiv \sum_{i=1}^{3} \mathcal{F}_i(t,\nu)\mathcal{M}_i$$

After some algebra calculation, $\mathcal{F}_i$ can be written as

$$\mathcal{F}_i(t,\nu) \equiv \sum_{j=1}^{3} \left(\mathcal{D}^{-1}\right)_{ij} \sum_{helity} \mathcal{M}_{ep\to ep}\mathcal{M}_j^*$$

$t = -Q^2, \nu = 2s - 2M_N^2 + t$

TPE in ep: general properties at fixed t

singularity, asymptotic behavior, branch cut.

(1) singularities

DR : $\mathcal{F}_i^{(2\gamma)}(t,\nu)$ have no any singularities.

HD : $\mathcal{D}^{-1}$ has two signularities at $\nu \to \pm\nu_s \equiv \pm\sqrt{-t(4M_N^2 - t)}$.

(2) asymptotic behavior, assumed by DRs

ubsubtructed DRs : $\mathcal{F}_i^{(a+b)}(t,\nu) \overset{\nu\to\infty}{\to} 0$

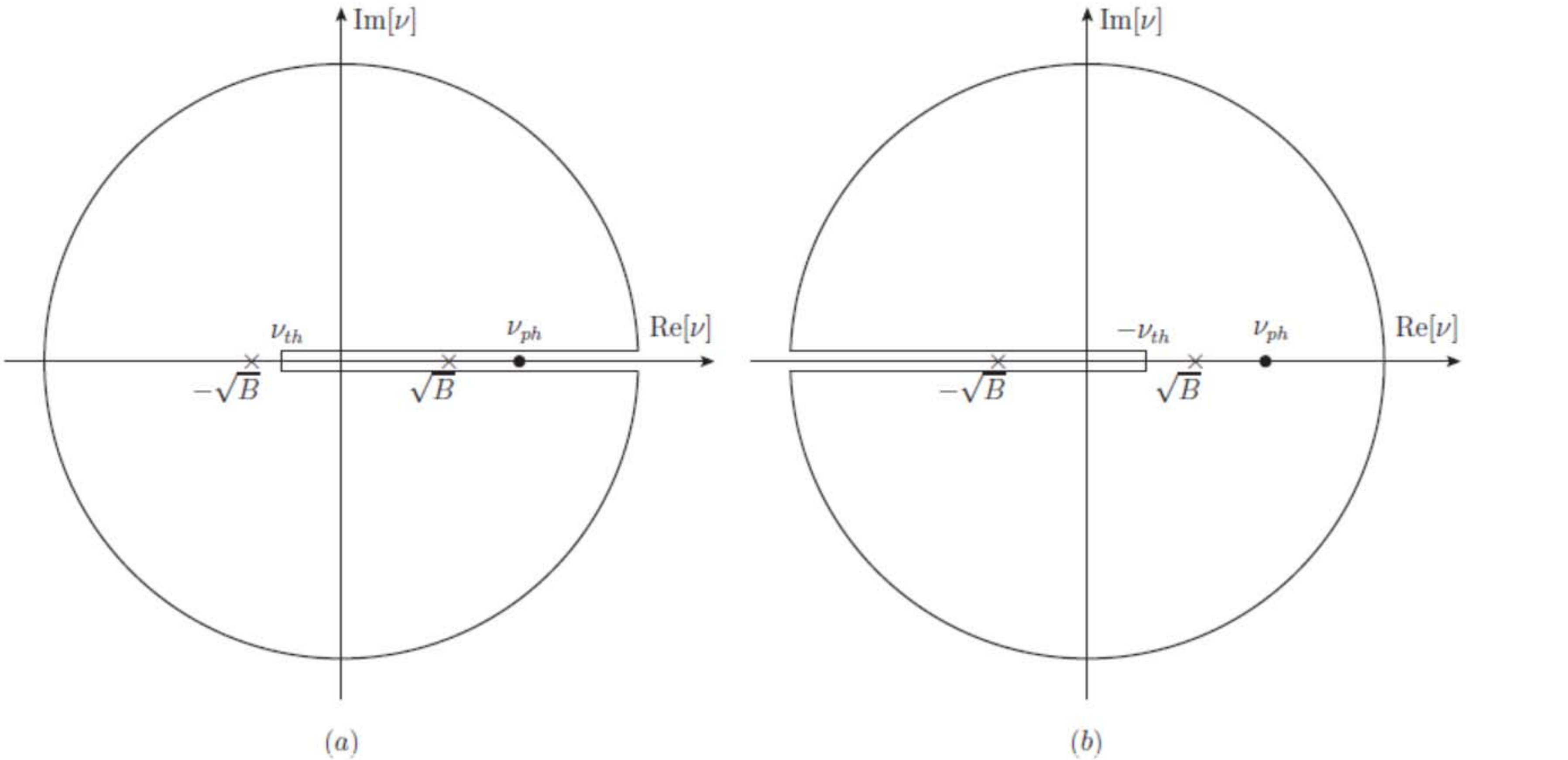
once - subtructed DRs : $\mathcal{F}_{1,2}^{(a+b)}(t,\nu) \overset{\nu\to\infty}{\to} 0,\ \mathcal{F}_3^{(a+b)}(t,\nu) \overset{\nu\to\infty}{\to} c$

HD sometimes (Δ) : detemined by the models

sometimes $\mathcal{F}_i^{(a+b)}(t,\nu) \overset{\nu\to\infty}{\to} \infty$

TPE in ep: general properties at fix t

(3) branch cuts when t<0 (only N is considered)



$v_{th} = t, B = v_s^2$

when t>0, there is a new branch cut, which is corresponding to the TPE in e^+e^-->ppbar.

TPE in ep: general properties at fix t

(4) crossing symmetry when t<0.

$$\mathcal{F}_{1,2}^{(a,c,d)}(t,\nu^+) = -\mathcal{F}_{1,2}^{(b,c,d)}(t,-\nu^+),$$
$$\mathcal{F}_{3}^{(a,c,d)}(t,\nu^+) = \mathcal{F}_{3}^{(b,c,d)}(t,-\nu^+),$$

(5) singularity + asymptotic + branch cut
=> unsubtracted or n^{th}-subtracted DRs.

It is natural that the results by the direct loop calculation should satisfy some DRs.

new diagrams (c,d) are showed below.

TPE in ep: DRs with different assumptions

un-subtracted DRs used in the literature — inputs

$$\mathrm{Re}[\mathcal{F}_{1,2}^{\mathrm{DR1}}(t,\nu)] \overset{def}{=} \frac{2\nu}{\pi}\mathrm{P}\Big[\int_{\nu_{th}}^{\infty}\frac{\mathrm{Im}[\mathcal{F}_{1,2}^{(a)}(t,\overline{\nu}^{+})]}{\overline{\nu}^2-\nu^2}d\overline{\nu}\Big]$$

$$\mathrm{Re}[\mathcal{F}_{3}^{\mathrm{DR1}}(t,\nu)] \overset{def}{=} \frac{2}{\pi}\mathrm{P}\Big[\int_{\nu_{th}}^{\infty}\frac{\overline{\nu}\mathrm{Im}[\mathcal{F}_{3}^{(a)}(t,\overline{\nu}^{+})]}{\overline{\nu}^2-\nu^2}d\overline{\nu}\Big]$$

once-subtracted DR used for $\mathcal{F}_3$ in the literature

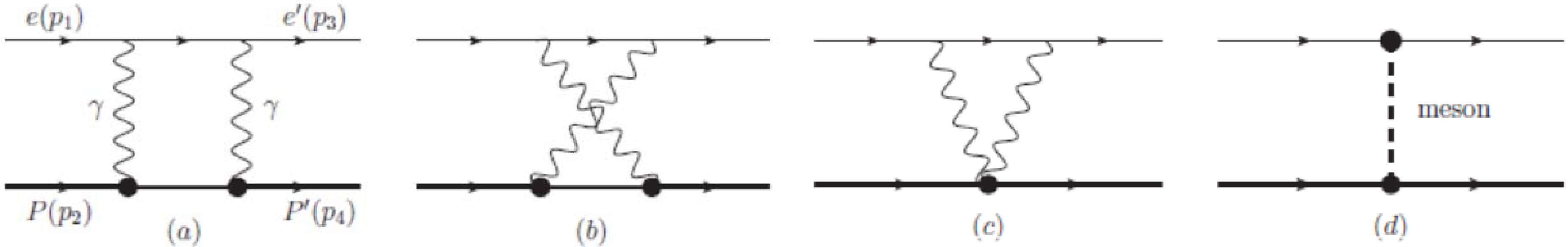
$$\mathrm{Re}[\mathcal{F}_{3}^{\mathrm{DR2}}(t,\nu)] \overset{def}{=} \mathrm{Re}[\mathcal{F}_{3}^{\mathrm{DR2}}(t,\nu_0)] + \frac{2(\nu^2-\nu_0^2)}{\pi}\times \mathrm{P}\Big[\int_{\nu_{th}}^{\infty}\frac{\overline{\nu}\mathrm{Im}[\mathcal{F}_{3}^{(a)}(t,\overline{\nu}^{+})]}{(\overline{\nu}^2-\nu^2)(\overline{\nu}^2-\nu_0^2)}d\overline{\nu}\Big],$$

what will happen in the toy models?

PRC74,065203;PRC89,025204;PRC95,065209. EPJA51,24

TPE in toy models

Our opinion: there are other contributions.
One can check these DRs in some toy models at first.



$$\mathcal{L}_E \stackrel{def}{=} -e\overline{\psi}_p\gamma^\mu\psi_p A_\mu,$$

$$\mathcal{L}_M \stackrel{def}{=} -\frac{e\kappa}{4M_N}\overline{\psi}_p\sigma^{\mu\nu}\psi_p F_{\mu\nu},$$

$$\mathcal{L}_S \stackrel{def}{=} -\frac{2\pi}{M_N^2}(\partial_\mu\overline{\psi}_p)(\partial_\nu\psi_p)(\alpha_{E1}F^{\mu\rho}F_\rho^\nu + \beta_{M1}\widetilde{F}^{\mu\rho}\widetilde{F}_\rho^\nu),$$

$$\mathcal{L}_T \stackrel{def}{=} ig_{\mathrm{Tpp}}[(\partial_\mu\overline{\psi}_p)\gamma_\nu\psi_p - \overline{\psi}_p\gamma_\nu(\partial_\mu\psi_p)]\phi^{\mu\nu} + ig_{\mathrm{Tee}}[(\partial_\mu\overline{\psi}_e)\gamma_\nu\psi_e - \overline{\psi}_e\gamma_\nu(\partial_\mu\psi_e)]\phi^{\mu\nu},$$

TPE in toy models

$\mathcal{L}_{E,S,T}$: no singulalrity /singularities in D^{-1} are cancelled.

$\mathcal{L}_{M}$: have singularities.

$$\mathrm{Re}\left[\mathcal{F}_{E1}^{(a)}(t,\nu)\right] \overset{\nu\to\infty}{\longrightarrow} -\frac{4\alpha_e^2}{M_N t}\left[\left(\frac{1}{\tilde{\epsilon}_{\mathrm{IR}}}+\ln\frac{\bar{\mu}_{\mathrm{IR}}^2}{-t}\right)\ln\nu + c_1\right]$$

$$\mathrm{Im}\left[\mathcal{F}_{E1}^{(a)}(t,\nu^+)\right] \overset{\nu\to\infty}{\longrightarrow} \frac{4\pi\alpha_e^2}{M_N t}\left(\frac{1}{\tilde{\epsilon}_{\mathrm{IR}}}+\ln\frac{\bar{\mu}_{\mathrm{IR}}^2}{-t}\right)$$

$$\mathcal{F}_{E2,E3}^{(a)} \overset{\nu\to\infty}{\longrightarrow} 0$$

After applying the crossing symmetry, one can check $\mathcal{F}_{Ei}^{(a+b)}$ satisfy DR1.

TPE in toy models

$\mathcal{L}_M$ case

$$\mathrm{Im}\left[\mathcal{F}_{M1}^{(a)}(t,\nu^+)\right] \stackrel{\nu\to\infty}{\longrightarrow} \frac{\pi\alpha_e^2\kappa^2}{2M_N^3}\left[\log\frac{\nu}{-t} - (1+\log 2)\right]$$

$$\mathrm{Im}\left[\mathcal{F}_{M2}^{(a)}(t,\nu^+)\right] \stackrel{\nu\to\infty}{\longrightarrow} 0$$

$$\mathrm{Im}\left[\mathcal{F}_{M3}^{(a)}(t,\nu^+)\right] \stackrel{\nu\to\infty}{\longrightarrow} 0$$

$$\mathrm{Re}\left[\mathcal{F}_{M1}^{(a)}(t,\nu)\right] \stackrel{\nu\to\infty}{\longrightarrow} -\frac{\alpha_e^2\kappa^2}{4M_N^3}\left[\log^2\nu - 2(1+\log 2 - t)\log\nu + c_{M10} + \frac{3}{4}\frac{1}{\tilde{\epsilon}_{\mathrm{UV}}}\right]$$

$$\mathrm{Re}\left[\mathcal{F}_{M2}^{(a)}(t,\nu)\right] \stackrel{\nu\to\infty}{\longrightarrow} \frac{\alpha_e^2\kappa^2}{4M_N^3}c_{M20}$$

$$\mathrm{Re}\left[\mathcal{F}_{M3}^{(a)}(t,\nu)\right] \stackrel{\nu\to\infty}{\longrightarrow} -\frac{\alpha_e^2\kappa^2}{8M_N^3}\left(5 + 3\log\frac{\bar{\mu}_{\mathrm{UV}}^2}{-t} + 3\frac{1}{\tilde{\epsilon}_{\mathrm{UV}}}\right)$$

TPE in toy models

Subtracting the terms with singularities and define

$$\overline{\mathcal{F}}_{Mi}^{(a,b)}(t,\nu) \stackrel{def}{=} \mathcal{F}_{Mi}^{(a,b)}(t,\nu) - \frac{\mathrm{Res}_{Mi}^{(a,b)}}{(\nu^2 - B)^2},$$

After applying the crossing symmetry, one can check $\overline{\mathcal{F}}_{M1,M2}^{(a+b)}$ satisfy DR1, $\overline{\mathcal{F}}_{M3}^{(a+b)}$ satisfy DR2.

Physically, it can be understood by the UV behavior.

$$\mathcal{F}_{M1,M2}^{\mathrm{UV}}(t,\nu) = 0,$$

$$\mathcal{F}_{M3}^{\mathrm{UV}}(t,\nu) = -\frac{3\alpha_e^2\kappa^2}{4M_N^3}\frac{1}{\widetilde{\epsilon}_{\mathrm{UV}}}$$

$B = -t(4M_N^2 - t)$

TPE in toy models

The UV divergence means some contact interactions should be included to absorb the UV divergence. It also introduces corresponding finite contributions with unknown finite coupling. This means

$$\mathrm{Re}[\mathcal{F}_{M3}^{\mathrm{DR2}}(t,\nu)] = \mathrm{Re}[\overline{\mathcal{F}}_{M3}^{(\mathrm{a+b})}(t,\nu)] - \mathcal{F}_{M3}^{\mathrm{UV}}(t,\nu) + f_3(t),$$

TPE in toy models

On the terms with singularities:

we find they do not dependent on the mass of photon, which mean that if one add monopole FFs to the vertex, then the singularities are cancelled.

$$\Gamma^{\mu}_{M}(k) \rightarrow \Gamma^{\mu}_{M}(k)F(k)$$

$$F(k) = \sum_{j} \frac{d_j}{\left(k^2 - \Lambda_j^2\right)^{n_j}}$$

$$\frac{N}{(k^2 - z_1^2)(k^2 - z_2^2)} = \frac{1}{z_1^2 - z_2^2}\left[\frac{N}{k^2 - z_1^2} - \frac{N}{k^2 - z_2^2}\right].$$

TPE in toy models

$\mathcal{L}_S$ case

$$\mathrm{Re}\left[\mathcal{F}_{S2}^{(c)}(t,\nu)\right] = -\frac{\alpha_e(\alpha_{E1}+\beta_{E1})}{72M_N^2}\left(17 + 12\log\frac{\bar{\mu}_{\mathrm{UV}}^2}{-t} + 12\frac{1}{\tilde{\epsilon}_{\mathrm{UV}}}\right)\nu$$

$$\mathrm{Im}\left[\mathcal{F}_{Si}^{(c)}(t,\nu^+)\right] = 0$$

$$\mathrm{Re}\left[\mathcal{F}_{S1,S3}^{(c)}(t,\nu)\right] = 0$$

Similarly, $\mathcal{F}_{S2}^{(c)}$ satisfies twice-subtracted DR.

TPE in toy models

$\mathcal{L}_T$ case

$$\begin{aligned}
\mathrm{Re}[\mathcal{F}^{(d)}_{\mathrm{T1}}(t,\nu)] &= \frac{g_{\mathrm{Tee}} g_{\mathrm{Tpp}}}{M_N(M_T^2 - t)}\nu, \\
\mathrm{Re}[\mathcal{F}^{(d)}_{\mathrm{T3}}(t,\nu)] &= \frac{g_{\mathrm{Tee}} g_{\mathrm{Tpp}}}{2M_N(M_T^2 - t)}t \\
\text{other} &= 0
\end{aligned}$$

Similarly, $\mathcal{F}^{(d)}_{T1}$ satisfy twice-subtracted DR, $\mathcal{F}^{(d)}_{T3}$ satisfy once-subtracted DR like DR2.

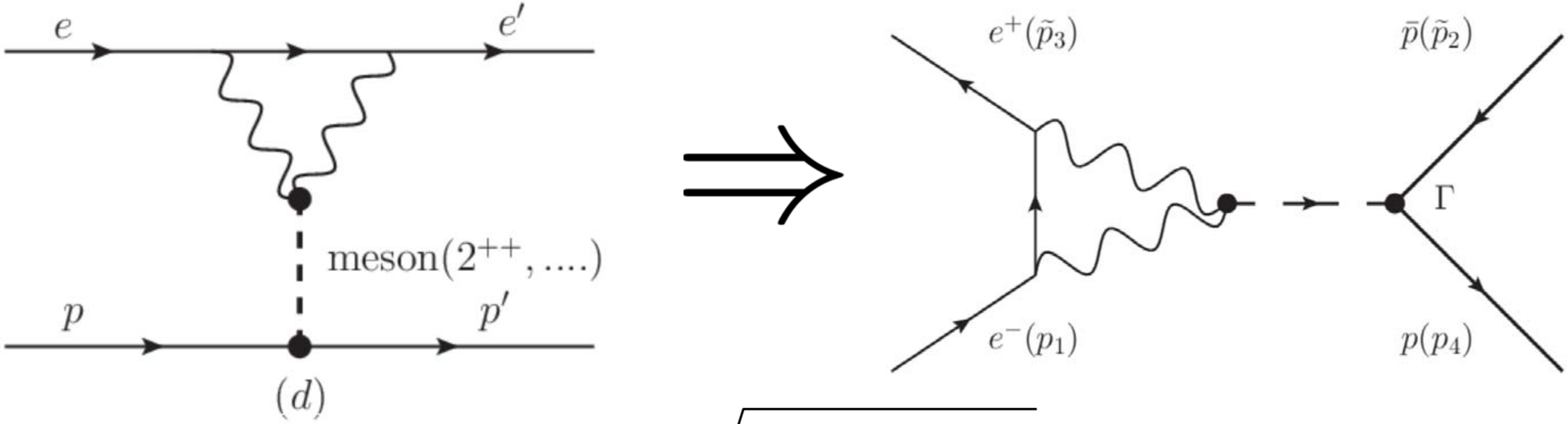
TPE in toy models

(1) interactions with more derivatives does not change the v dependence of the results (meson-exchange).
(2) all the off-shell related contributions can be expressed as some polynomial functions on v. (also other mesons)

(3) the behaviors of these new contributions are valid at low energy and give un-physical behaviors at high energy since the exchange-mesons are composite particles.
How to continue these contributions to high energy?

Discussion: continue the TPE to high energy

The physical meaning/properties of the meson-exchange effects are much simpler in the s-channel.



$$t \to \widetilde{s}, \nu \to -\sqrt{\widetilde{s}(\widetilde{s}-4M_N^2)}cos\theta_p$$

2^{++} meson-exchange means $^3P_2$ state of ppbar, whose amplitude is just cosθ. The results (also other mesons) are valid when $|\nu| < \sqrt{t(t-4M_N^2)}$

Discussion: continue the TPE to high energy

All the contributions from the seagull interaction, the meson-exchange interactions, the off-shell effects can be expressed as polynomial functions on ν. Their sum is convergent when $|\nu| < \sqrt{t(t-4M_N^2)} = \nu_s$

the singularitied found in $\mathcal{L}_M$ case

$$\sum_{j=0} c_{1j,2j}(t)\nu^{2j+1} = \sum_{j=1} \frac{g_{1j,2j}(t)\nu}{(\nu^2-\nu_s^2)^j} \approx \frac{f_{1,2}(t)\nu}{\nu^2-\nu_s^2},$$

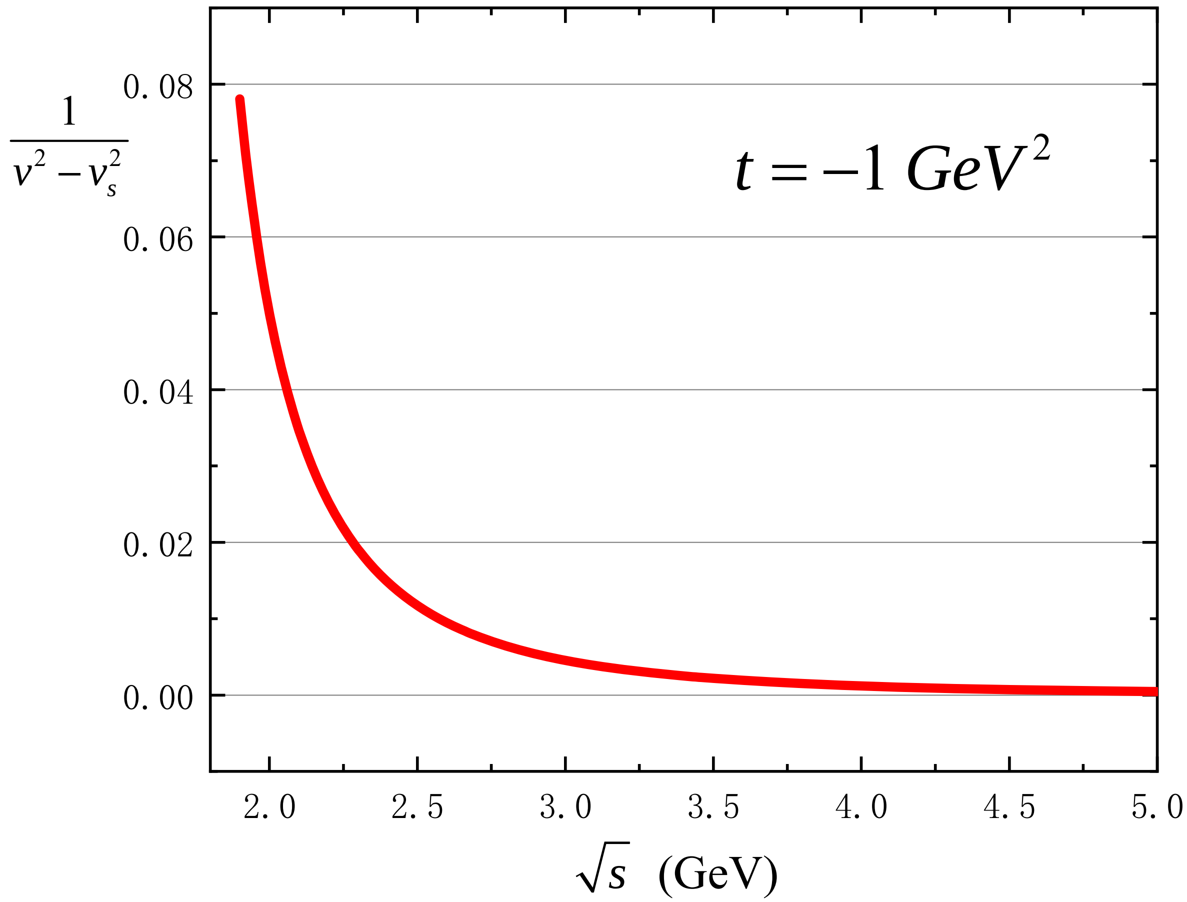
$$\sum_{j=0} c_{3j}(t)\nu^{2j} = \sum_{j=0} \frac{g_{3j}(t)}{(\nu^2-\nu_s^2)^j} \approx f_3(t),$$

$\mathcal{F}_{1,2}^{(2\gamma)}$ are odd funcitons of ν

$\mathcal{F}_{3}^{(2\gamma)}$ is even function of ν

$$\nu_{ep\to ep} \geq \nu_{ph} \stackrel{def}{=} \frac{(2M_N^2-t)(-t+\sqrt{t(t-4M_N^2)})}{2M_N^2}$$

Discussion: continue the TPE to high energy



This means the higher orders can be neglected.

DRs for TPE including meson-exchange etc.

After taking the leading order, one has new DRs

$$\mathrm{Re}[\mathcal{F}_{1,2}^{\mathrm{DR3}}(t,\nu)] \overset{def}{=} \frac{f_{1,2}(t)\nu}{(\nu^2-B)} + \frac{2\nu}{\pi}\mathrm{P}\Big[\int_{\nu_{th}}^{\infty}\frac{\mathrm{Im}[\mathcal{F}_{1,2}^{(a)}(t,\overline{\nu}^+)]}{\overline{\nu}^2-\nu^2}d\overline{\nu}\Big]$$

$$\mathrm{Re}[\mathcal{F}_{3}^{\mathrm{DR3}}(t,\nu)] \overset{def}{=} f_3(t,\nu_0) + \frac{f_{31}(t,\nu_0)}{(\nu^2-B)} + \frac{2(\nu^2-\nu_0^2)}{\pi}\mathrm{P}\Big[\int_{\nu_{th}}^{\infty}\frac{\overline{\nu}\mathrm{Im}[\mathcal{F}_{3}^{(a)}(Q^2,\overline{\nu}^+)]}{(\overline{\nu}^2-\nu^2)(\overline{\nu}^2-\nu_0^2)}d\overline{\nu}\Big]$$

also has the relations

$$\mathcal{F}_{1,2}^{\mathrm{DR3}}(t,\nu) \overset{def}{=} \frac{H_{1,2}(t)\nu}{(\nu^2-B)} + \mathcal{F}_{1,2}^{(a+b)}(t,\nu) + \sum_{j=0} h_{1j2j}(t)\nu^{2j+1},$$

$$\mathcal{F}_{3}^{\mathrm{DR3}}(t,\nu) \overset{def}{=} \mathcal{F}_{3}^{(a+b)}(t,\nu) + H_3(t) + \sum_{j=1} h_{3j}(t)\nu^{2j}$$

h_{ij} are chosen to cancel the similar contributions in (a+b), terms with $1/(\nu^2\text{-}B)$^2…

Conclusion

(1). The new TPE forms include the contributions from the seagull interactions, meson-exchange effects, contact interactions and off-shell effects.

(2). The new TPE forms suggest that there are additional three unknown factors and they should be included to analyze the elastic *ep* scattering data sets.

Further studies

(1) Analyze the ep data sets.
(2) DRs in $e\pi \to e\pi$, $ep \to en\pi^+$ and FF of pion.
extraction of the FF of pion is more difficult.
(3) DRs in P-violated $ep \to ep$.
the weak charge and strange FF of proton.
(4)DRs in the complex t plane/TPE in $e^+e^- \to p\bar{p}$
directly test the TPE (time-like)
(1) ChpT + DRs + HD: ChpT maybe can give some constrains on the behaviors of $f_i(t)$.
(2)At low energy, contributions from ep bound states?
TPE in e^-p vs. e^+p
(3) TPE in $e^+\mu^- \to e^+\mu^-$: the role of $e^+\mu^-$ bound states is similar with the meson-exchange. (double counting)

………

Thanks!

Any comments, suggestions, and discussion are

Welcome, Welcome!

请大家批评指正！

Appendix: definition of M_i

$$\mathcal{M}_1 \stackrel{def}{=} M_N[\overline{u}_3\gamma_\mu u_1][\overline{u}_4\gamma^\mu u_2],$$

$$\mathcal{M}_2 \stackrel{def}{=} [\overline{u}_3(\not{p}_2+\not{p}_4)u_1][\overline{u}_4 u_2],$$

$$\mathcal{M}_3 \stackrel{def}{=} M_N[\overline{u}_3\gamma_5\gamma_\mu u_1][\overline{u}_4\gamma_5\gamma^\mu u_2],$$

$$\mathcal{D}^{-1} = \frac{1}{4M_N^2Q^2(\nu^2-B)^2}\begin{pmatrix} \overline{d}_{11} & \overline{d}_{12} & \overline{d}_{13} \\ \overline{d}_{12} & \overline{d}_{22} & \overline{d}_{23} \\ \overline{d}_{31} & \overline{d}_{23} & \overline{d}_{33} \end{pmatrix},$$

$$\begin{aligned}
\overline{d}_{11} &= 4(M_N^2+Q^2)(\nu^2+B), \\
\overline{d}_{22} &= M_N^2(\nu^2+A), \\
\overline{d}_{33} &= Q^2(\nu^2+B), \\
\overline{d}_{12} &= \overline{d}_{21} = -2M_N^2(\nu^2+B), \\
\overline{d}_{13} &= \overline{d}_{31} = -2Q^2(4M_N^2+Q^2)\nu, \\
\overline{d}_{23} &= \overline{d}_{32} = 4M_N^2Q^2\nu.
\end{aligned}$$

Appendix：original DRs

$$\mathrm{Re}[\mathcal{F}_{E1}^{(a)}(t,\nu)] - \mathrm{Re}[\mathcal{F}_{E1}^{(a)}(t,\nu_1)] = \frac{\nu-\nu_1}{\pi}\mathrm{P}\Big[\int_{\nu_{th}}^{\infty}\frac{\mathrm{Im}[\mathcal{F}_{E1}^{(a)}(t,\overline{\nu}^+)]}{(\overline{\nu}-\nu)(\overline{\nu}-\nu_1)}d\overline{\nu}\Big],$$

$$\mathrm{Re}[\mathcal{F}_{E1}^{(b)}(t,\nu)] - \mathrm{Re}[\mathcal{F}_{E1}^{(b)}(t,\nu_2)] = -\frac{\nu-\nu_2}{\pi}\mathrm{P}\Big[\int_{-\infty}^{-\nu_{th}}\frac{\mathrm{Im}[\mathcal{F}_{E1}^{(b)}(t,\overline{\nu}^-)]}{(\overline{\nu}-\nu)(\overline{\nu}-\nu_2)}d\overline{\nu}\Big].$$

$$\begin{aligned}
\mathrm{Re}[\mathcal{F}_{E1}^{(a+b)}(t,\nu)] &= \frac{\nu-\nu_1}{\pi}\mathrm{P}\Big[\int_{\nu_{th}}^{\infty}\frac{\mathrm{Im}[\mathcal{F}_{E1}^{(a)}(t,\overline{\nu}^+)]}{(\overline{\nu}-\nu)(\overline{\nu}-\nu_1)}d\overline{\nu}\Big] - \frac{\nu+\nu_1}{\pi}\mathrm{P}\Big[\int_{-\infty}^{-\nu_{th}}\frac{\mathrm{Im}[\mathcal{F}_{E1}^{(b)}(t,\overline{\nu}^-)]}{(\overline{\nu}-\nu)(\overline{\nu}+\nu_1)}d\overline{\nu}\Big] \\
&= \frac{\nu-\nu_1}{\pi}\mathrm{P}\Big[\int_{\nu_{th}}^{\infty}\frac{\mathrm{Im}[\mathcal{F}_{E1}^{(a)}(t,\overline{\nu}^+)]}{(\overline{\nu}-\nu)(\overline{\nu}-\nu_1)}d\overline{\nu}\Big] + \frac{\nu+\nu_1}{\pi}\mathrm{P}\Big[\int_{-\infty}^{-\nu_{th}}\frac{\mathrm{Im}[\mathcal{F}_{E1}^{(a)}(t,-\overline{\nu}^-)]}{(\overline{\nu}-\nu)(\overline{\nu}+\nu_1)}d\overline{\nu}\Big] \\
&= \frac{\nu-\nu_1}{\pi}\mathrm{P}\Big[\int_{\nu_{th}}^{\infty}\frac{\mathrm{Im}[\mathcal{F}_{E1}^{(a)}(t,\overline{\nu}^+)]}{(\overline{\nu}-\nu)(\overline{\nu}-\nu_1)}d\overline{\nu}\Big] + \frac{\nu+\nu_1}{\pi}\mathrm{P}\Big[\int_{\nu_{th}}^{\infty}\frac{\mathrm{Im}[\mathcal{F}_{E1}^{(a)}(t,\overline{\nu}^+)]}{(\overline{\nu}+\nu)(\overline{\nu}-\nu_1)}d\overline{\nu}\Big] \\
&= \frac{2\nu}{\pi}\mathrm{P}\Big[\int_{\nu_{th}}^{\infty}\frac{\mathrm{Im}[\mathcal{F}_{E1}^{(a)}(t,\overline{\nu}^+)]}{\overline{\nu}^2-\nu^2}d\overline{\nu}\Big].
\end{aligned}$$

Appendix： Ex results by BESIII

TABLE I. The integrated luminosity, the number of $p\bar{p}$ events, the Born cross section $\sigma_{p\bar{p}}$, $|G_E/G_M|$, $|G_{\text{eff}}|$, $|G_E|$, and $|G_M|$.

$\sqrt{s}$[GeV]	$\mathcal{L}$[pb^{-1}]	N_{obs}	$\sigma_{p\bar{p}}$[pb]	$\lvert G_{\text{eff}}\rvert[10^{-2}]$	$\lvert G_E/G_M\rvert$	$\lvert G_E\rvert[10^{-2}]$	$\lvert G_M\rvert[10^{-2}]$
2.0000	10.1 ± 0.1	5321	$841.3 \pm 11.5 \pm 24.8$	$27.46 \pm 0.19 \pm 0.40$	$1.38 \pm 0.10 \pm 0.03$	$33.66 \pm 1.23 \pm 0.31$	$24.38 \pm 0.99 \pm 0.26$
2.0500	3.34 ± 0.03	1703	$753.4 \pm 18.3 \pm 23.5$	$24.94 \pm 0.30 \pm 0.39$	$1.24 \pm 0.16 \pm 0.04$	$29.10 \pm 2.08 \pm 0.40$	$23.48 \pm 1.43 \pm 0.42$
2.1000	12.2 ± 0.1	5993	$712.6 \pm 9.2 \pm 21.4$	$23.73 \pm 0.15 \pm 0.36$	$1.27 \pm 0.09 \pm 0.02$	$28.07 \pm 1.10 \pm 0.31$	$22.08 \pm 0.74 \pm 0.17$
2.1250	108 ± 1	50312	$660.0 \pm 3.0 \pm 19.7$	$22.69 \pm 0.05 \pm 0.34$	$1.18 \pm 0.04 \pm 0.01$	$25.62 \pm 0.49 \pm 0.18$	$21.65 \pm 0.31 \pm 0.13$
2.1500	2.84 ± 0.02	1189	$588.8 \pm 17.1 \pm 17.8$	$21.34 \pm 0.31 \pm 0.32$	$1.62 \pm 0.24 \pm 0.06$	$28.32 \pm 1.89 \pm 0.46$	$17.48 \pm 1.51 \pm 0.37$
2.1750	10.6 ± 0.1	3762	$491.0 \pm 8.0 \pm 14.8$	$19.44 \pm 0.16 \pm 0.29$	$1.19 \pm 0.12 \pm 0.02$	$22.08 \pm 1.28 \pm 0.28$	$18.55 \pm 0.75 \pm 0.16$
2.2000	13.7 ± 0.1	4092	$411.6 \pm 6.4 \pm 12.3$	$17.78 \pm 0.14 \pm 0.27$	$1.08 \pm 0.10 \pm 0.02$	$18.93 \pm 1.20 \pm 0.28$	$17.60 \pm 0.63 \pm 0.12$
2.2324	14.5 ± 0.1	3644	$341.9 \pm 5.7 \pm 10.1$	$16.21 \pm 0.13 \pm 0.24$	$0.85 \pm 0.11 \pm 0.03$	$14.48 \pm 1.39 \pm 0.42$	$16.98 \pm 0.57 \pm 0.17$
2.3094	21.1 ± 0.1	2336	$148.0 \pm 3.1 \pm 5.7$	$10.74 \pm 0.11 \pm 0.21$	$0.55 \pm 0.16 \pm 0.02$	$6.61 \pm 1.72 \pm 0.25$	$11.99 \pm 0.44 \pm 0.14$
2.3864	22.5 ± 0.2	1851	$122.0 \pm 2.8 \pm 3.6$	$9.87 \pm 0.11 \pm 0.15$	$0.54 \pm 0.19 \pm 0.02$	$5.98 \pm 1.87 \pm 0.19$	$10.99 \pm 0.44 \pm 0.07$
2.3960	66.9 ± 0.5	5514	$121.9 \pm 1.6 \pm 3.6$	$9.89 \pm 0.07 \pm 0.15$	$0.76 \pm 0.10 \pm 0.02$	$7.93 \pm 0.86 \pm 0.21$	$10.48 \pm 0.27 \pm 0.07$
2.5000	1.10 ± 0.01	55	$77.9 \pm 10.5 \pm 4.1$	$8.08 \pm 0.55 \pm 0.21$	...	...	...
2.6444	33.7 ± 0.2	867	$39.7 \pm 1.3 \pm 1.2$	$5.98 \pm 0.10 \pm 0.09$	$0.97 \pm 0.24 \pm 0.05$	$5.84 \pm 1.13 \pm 0.24$	$5.99 \pm 0.37 \pm 0.11$
2.6464	34.0 ± 0.3	838	$38.2 \pm 1.3 \pm 1.2$	$5.87 \pm 0.10 \pm 0.10$	$0.87 \pm 0.27 \pm 0.04$	$5.18 \pm 1.30 \pm 0.21$	$5.99 \pm 0.37 \pm 0.11$
2.7000	1.03 ± 0.01	20	$29.8 \pm 6.7 \pm 1.6$	$5.26 \pm 0.59 \pm 0.14$	...	...	...
2.8000	4.76 ± 0.03	68	$22.0 \pm 2.7 \pm 1.0$	$4.65 \pm 0.28 \pm 0.11$	...	...	...
2.9000	105 ± 1	1010	$15.0 \pm 0.5 \pm 0.5$	$3.95 \pm 0.06 \pm 0.06$	$0.54 \pm 0.34 \pm 0.03$	$2.31 \pm 1.39 \pm 0.11$	$4.29 \pm 0.21 \pm 0.06$
2.9500	15.9 ± 0.1	118	$11.7 \pm 1.1 \pm 0.4$	$3.53 \pm 0.16 \pm 0.07$	$0.96 \pm 0.39 \pm 0.06$	$3.25 \pm 1.09 \pm 0.17$	$3.37 \pm 0.28 \pm 0.06$
2.9810	16.1 ± 0.1	131	$12.9 \pm 1.1 \pm 0.5$	$3.75 \pm 0.16 \pm 0.07$			
3.0000	15.9 ± 0.1	92	$9.2 \pm 1.0 \pm 0.3$	$3.19 \pm 0.17 \pm 0.06$			
3.0200	17.3 ± 0.1	97	$9.0 \pm 0.9 \pm 0.3$	$3.16 \pm 0.16 \pm 0.05$			
3.0800	157 ± 1	858	$9.0 \pm 0.3 \pm 0.3$	$3.22 \pm 0.05 \pm 0.05$	$0.47 \pm 0.45 \pm 0.04$	$1.64 \pm 1.53 \pm 0.12$	$3.47 \pm 0.18 \pm 0.03$

Appendix

$$\mathrm{Res}_{\mathrm{B1}}^{\mathrm{I}(a)} = -\alpha_e^2\kappa^2\frac{(4M_N^2 + -t)(2\nu + 3 - t)}{8M_N^3}(\nu^2 - B),$$

\rho

$$\mathrm{Res}_{\mathrm{B2}}^{\mathrm{I}(a)} = \alpha_e^2\kappa^2\frac{2\nu + 3 - t}{4M_N}(\nu^2 - B),$$

$$\mathrm{Res}_{\mathrm{B3}}^{\mathrm{I}(a)} = \alpha_e^2\kappa^2\frac{(8M_N^2 + 2 - t + 3\nu) - t}{8M_N^3}(\nu^2 - B),$$

$$\mathrm{Res}_{\mathrm{B1}}^{\mathrm{II}(a)} = -\alpha_e^2\kappa^2\frac{4M_N^2 + -t}{8M_N^3}\Big[2 - t(4M_N^2 + -t)(7 - t - 10\nu) + (11 - t - 4\nu)(\nu^2 - B)\Big],$$

$$\mathrm{Res}_{\mathrm{B2}}^{\mathrm{II}(a)} = \alpha_e^2\kappa^2\frac{1}{4M_N}\Big[2 - t(4M_N^2 + -t)(7 - t - 10\nu) + (11 - t - 4\nu)(\nu^2 - B)\Big],$$

$$\mathrm{Res}_{\mathrm{B3}}^{\mathrm{II}(a)} = -\alpha_e^2\kappa^2\frac{-t}{4M_N^3}\Big[-t(4M_N^2 + -t)(40M_N^2 + 10 - t - 7\nu) + (28M_N^2 + 7 - t - 2\nu)(\nu^2 - B)\Big].$$

Appendix: some conclusion in references

In 2007, Arrington etc. give Global analysis of proton elastic form factor data with two-photon exchange corrections and conclude ……

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