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Juelich-Bonn Coupled-Channel Approach: a Bridge from World Data to Hadron Spectroscopy and Structures

The 127th Hadron Physics Online Forum

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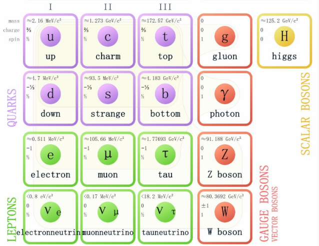
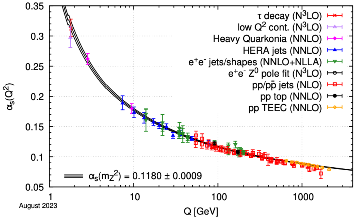
Hadrons

I Introduction

Hadrons

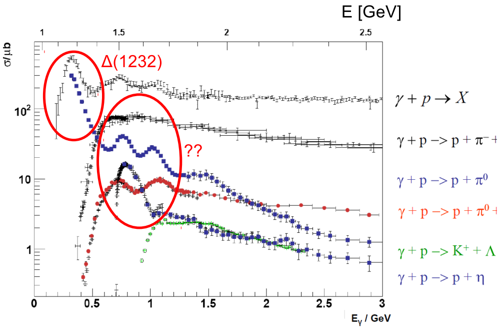
- Hadrons as basic building blocks of the macroscopic world
- Formation $\rightarrow$ quarks & gluons
  - Baryons $\rightarrow$ 3 quarks (qqq)
  - Mesons $\rightarrow$ quark antiquark pair ($q\bar{q}$)
  - ...
- Strong interaction $\rightarrow$ Quantum Chromodynamics (QCD)
- Confinement & asymptotic freedom $\rightarrow$ Perturbative only at high energies





[PDG]

- Intermediate energies
  $\longrightarrow$Abundant experimental observations
- Spectroscopy
  $\longrightarrow$ Failure of the Breit-Wigner description
  - Resonance v.s. background
  - Interference
- Structures of hadrons $\longrightarrow$ extremely difficult
  - Not direct observables
  - Non-perturbative nature of QCD
  - Uncertainties of the hadron spectroscopy
- A bridge between the data and the spectra/structures?
  World data $\longrightarrow$??? $\longrightarrow$ Spectra $\longrightarrow$ Structures



[source: ELSA; data: ELSA, JLab, MAMI]

- Extraction of resonances
  $\longrightarrow$ Partial Wave Analyses (PWA)
- Unitary amplitudes under JLS basis T^{JLS}
  $\longrightarrow$ Resonances with definite quantum numbers
- Recollection of $T^{JLS} \longrightarrow$ observables
- Methods for Baryon Spectra
  - K-matrix Unitarization:
    On-shell intermediate states
    [GWU/SAID, BnGa, Gießen]
  - Unitary isobar models:
    Unitary amplitudes + Breit-Wigner
    [MAID, Yerevan/JLab, KSU]
  - Dynamical Coupled-Channel (DCC) approaches:
    Potentials + scattering equations (off-shell intermediate states)
    [ANL-Osaka (EBAC), Dubna-Mainz-Taipeh]

    & Jülich-Bonn Model

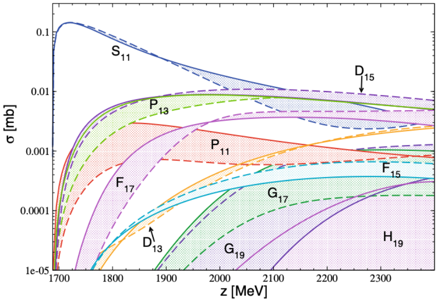


Fig. 24. Partial cross sections of the reaction $\pi^- p \to K^0\Sigma^0$ in the isospin 1/2 sector. Solid lines: fit A; dashed lines: fit B. Note that the partial cross sections for the other $K\Sigma$ final states can be obtained from these curves, fig. 25, and eq. (24).

[Rönchen *et. al.*, EPJA 49, 44 (2013)]

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II Methodology

History of the Jülich-Bonn Model

II Methodology

A comprehensive DCC model describing a worldwide collection of data

Central part ($\pi N \to \cdots$)

- The πN elastic scatterings [Schütz *et al.*, PRC 51, 1374 (1995)] [Schütz *et al.*, PRC 49, 2671 (1994)]
- $\pi\pi N$ and ηN [Schütz *et al.*, PRC 57, 1464 (1998)] [Krehl *et al.*, PRC 62, 025207 (2000)] [Gasparyan *et al.*, PRC 68, 045207 (2003)]
- $K\Lambda$ and $K\Sigma$ [Döring *et al.*, NPA 851, 58 (2011)] [Rönchen *et al.*, EPJA 49, 44 (2013)]
- ωN [Wang *et al.*, PRD 106, 094031 (2022)]
- Analytical continuation [Döring *et al.*, NPA 829, 170 (2009)]

Other sectors

- Photoproduction (γN)
  [Rönchen *et al.*, EPJA 50, 101 (2014)] [Rönchen *et al.*, EPJA 51, 70 (2015)][Rönchen *et al.*, EPJA 54, 110 (2018)] [Rönchen *et al.*, EPJA 558, 229 (2022)]
- Electroproduction ($e^-(\gamma^*)N$) (Jülich-Bonn-Washington Model)
  [Mai *et al.*, PRC 103, 065204 (2021)] [Mai *et al.*, PRC 106, 015201 (2022)] [Mai *et al.*, EPJA 59, 286 (2023)]
- Hidden charm ($J/\psi N \to \cdots$)
  [Wang *et al.*, EPJC 82, 497 (2022)][Shen *et al.*, EPJC 84, 764 (2024)] [Wang *et al.*, PRD 112, 074010 (2025)]

Dynamics I

II Methodology

The Lippmann-Schwinger-like equation (reduced Bethe-Salpeter equation):

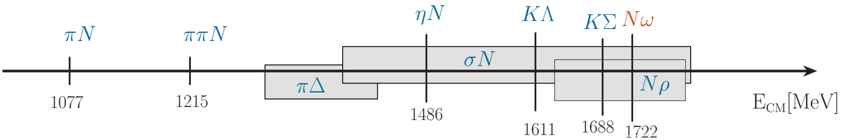
$$T_{\mu\nu}(p'',p',z) = V_{\mu\nu}(p'',p',z) + \sum_\kappa \int_0^\infty p^2 dp V_{\mu\kappa}(p'',p,z) G_\kappa(p,z) T_{\kappa\nu}(p,p',z)$$

- Reaction channels $\nu \to \mu$
  (after PW and isospin projection, JLS basis [Jacob & Wick, Annals Phys. 7, 404 (1959)],$J \leq 9/2$)
- Intermediate channel: κ. Initial (final) momentum: $p'(p'')$. CM energy: z
- Potential (kernel): V. Amplitude: T
- Propagator: G ($\pi\pi N$ channel: effective channels $\rho N, \sigma N, \pi\Delta$. E/ω - energy of the baryon/meson.)

$$G_\kappa(z,p) = \begin{cases} (z - E_\kappa - \omega_\kappa + i0^+)^{-1} & (\text{if } \kappa \text{ is a two-body channel}) , \\ \left[z - E_\kappa - \omega_\kappa - \Sigma_\kappa(z,p) + i0^+\right]^{-1} & (\text{if } \kappa \text{ is an effective channel}) . \end{cases}$$

- Reason for 1D: PW+time-ordered perturbation theory (TOPT)

[Schweber, *An Introduction to Relativistic Quantum Field Theory*, (1964)] [Sterman, *An Introduction to Quantum Field Theory*, 1993]

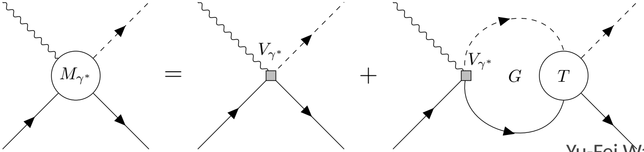


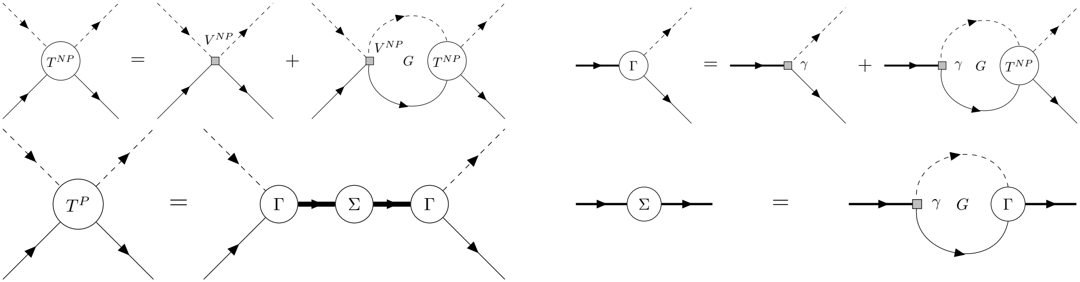
Solving the equation

- Separating the amplitude: $T = T^P + T^{NP}$
- "Non-pole part":
  $T^{NP} = V^{NP} + \sum \int p^2 dp V^{NP} G T^{NP}$,
- "Pole part": Dyson-Schwinger-like equations
  $\rightarrow$ fully-dressed s-channel amplitudes
  $T^P_{\mu\nu}(p'', p', z) = \sum_{i,j} \Gamma^a_{\mu,i}(p'') D_{ij}(z) \Gamma^c_{\nu,j}(p')$
  $(D^{-1})_{ij} = \delta_{ij}(z - m_i^b) - \Sigma_{ij}(z)$
- Dimensionless amplitude $\tau = (S-1)/(2i)$
  $\rightarrow$ Observables

Photo-/electroproductions

- Watson's final state theorem
  $M_{\mu\gamma^*}(Q^2) = V_{\mu\gamma^*} + \sum_\kappa \int p^2 dp T_{\mu\kappa} G_\kappa V_{\kappa\gamma^*}$
- Notations
  - γ^*: the $\gamma^* N$ (electroproduction)
  - Q^2: photon virtuality
  - $V_{\kappa\gamma^*}$: photon potential
  - Photoproduction $\rightarrow Q^2 = 0$





T^{NP}
=
V^{NP}
+
V^{NP}
G
T^{NP}
Γ
=
γ
+
γ
G
T^{NP}
T^{P}
=
Γ
Σ
Γ
Σ
=
γ
G
Γ

Potentials

II Methodology

The *NP* part

- Tree-level potentials (field-theoretical construction)
  - t-channel + u-channel + contact (about 70)
  - Effective Lagrangians → SU(3) flavour symmetry, CP conservation, chiral symmetry
  - Regulators for every vertex $\rightarrow F(q) \sim \left(\frac{\Lambda^2 - m^2}{\Lambda^2 + \mathbf{q}^2}\right)^n$
- Beyond tree-level → correlated two-pion exchanges [Schütz *et al.*, PRC 49, 2671 (1994)] [Schütz *et al.*, PRC 51, 1374 (1995)]

The *P* part

- Effective Lagrangians with CP conservation (tree-level bare vertices)
- Contact terms for distant resonances $\rightarrow D \sim (1 - \Sigma)^{-1}$ [Rönchen *et al.*, EPJA 51, 70 (2015)]
- Renormalization of the nucleon

The photon potentials

- Phenomenologically parametrized
- Kinematic constraints: Siegert's theorem [Siegert, PR 52, 787 (1937)], etc.

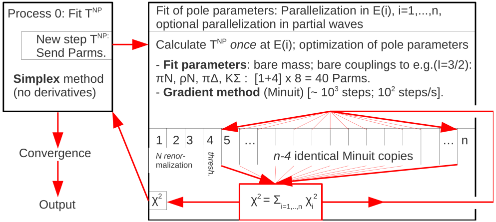
Numerical fits

II Methodology

- Gaussian quadrature $\int f(x)dx \simeq \sum_i w_i f(x_i)$
- Haftl-Tabakin matrix inversion $\rightarrow$ discretization via the Gaussian points [Haftel & Tabakin, NPA 158, 1 (1970)]
- $a, b = 1, \cdots, n$ for Gaussian points; $n+1$ for on-shell point:

$$T^{NP}_{ab} = V^{NP}_{ab} + \sum_{i=1}^{n} p_i^2 w_i V^{NP}_{ai} G_i T^{NP}_{ib} , \quad \hat{T} = (1-\hat{V}\hat{G})^{-1}\hat{V}$$

- Supercomputer JURECA
  [JSC, Journal of large-scale research facilities 7 (2021)]
- Nested fitting strategy
  $\rightarrow$ one step T^{NP} parms v.s. many steps of T^{P}
- Fit parameters:
  cutoffs (V^{NP}), bare masses/couplings (T^{P})

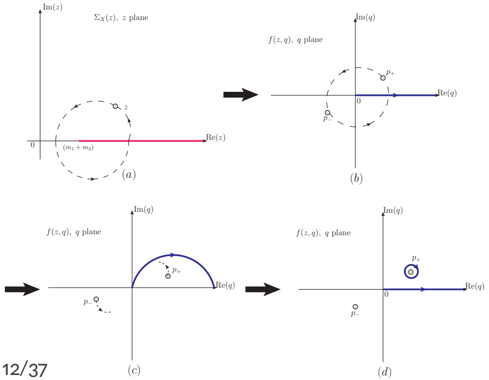


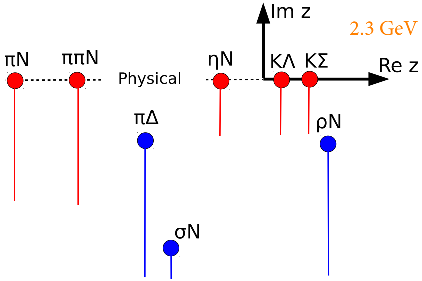
Extraction of resonances

II Methodology

- Resonances $\longrightarrow$ poles on the second Riemann sheet. Analytical continuation $\longrightarrow$ contour deformation.
- Effective three-body channels $\longrightarrow$ Complex branch points [Frazier & Hendry, PR 134, B1307 (1964)] [Cutkosky & Wang, PRD 42, 235 (1990)]
- Pole position $z_r = M_r - i\Gamma_r/2$. Coupling strengths $\longrightarrow$ normalized residues[PDG]

$$\tau^{II}_{\mu\nu} \sim \frac{R_\mu R_\nu}{z_r - z} + \cdots, NR_\mu \equiv \frac{2R_{\pi N}}{\Gamma_r} \times R_\mu$$





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III Application 1: the ωN interaction

Backgrounds and Numerical Details

Physical backgrounds

- Low-density nuclear matter $\rightarrow$ restoration of the chiral symmetry [Brown & Rho, PRL 66, 2720 (1991)]
- Vector meson dominance [Gell-Mann & Zachariasen, PR 124, 953 (1961)] $\rightarrow$ ω in the nuclear matter
- ω plays a very important role in the EOS of the neutron stars [H. Shen *et al.*, NPA 637, 435 (1998)]
- The ωN elastic scattering length $\rightarrow$ in-medium bound states??
  Cannot be measured directly by experiments!!

This study: [Y.F. Wang *et al.*, PRD 106, 094031 (2022)]

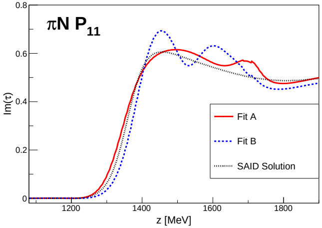
Numerical details

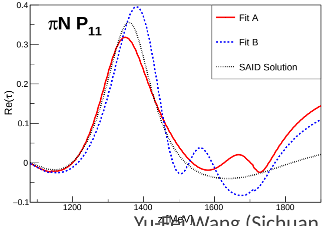
- Database $\rightarrow$ over 9000 points, 174 new of $\pi N \rightarrow \omega N$. Energy $\in [1078, 2300]$ MeV
- Parameters $\rightarrow 79(NP) + 225(P)$

Estimation of the Uncertainties

III Application 1: the ωN interaction

- The statistics
  - Energy-dependent solutions of πN amplitudes [Arndt *et al.*, PRC 74, 045205 (2006)] $\longrightarrow$ no errors
  - Some problematic data points of ηN [Brown *et al.*, NPB 153, 89 (1979)]
  - Extra weights on important data sets (e.g. ωN)
  - Impossible to switch on all parameters in one attempt
  - Estimation of the uncertainties $\longrightarrow$ fits with different initial values
- Uncertainties $\longrightarrow$ two fits with similar qualities
  - Fit A $\longrightarrow$ from intermediate values of [Röchen *et al.*, EPJA 54, 110 (2018)]
  - Fit B $\longrightarrow$ an extra narrow resonance in P_{11} wave ($J^P = \frac{1}{2}^+$, $z_r = 1585 - 35i$ MeV)





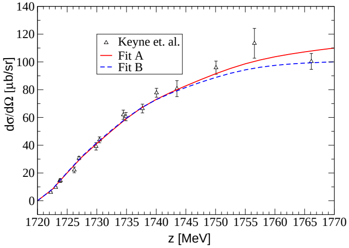
Fit Results

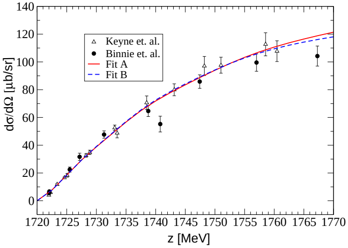
III Application 1: the ωN interaction

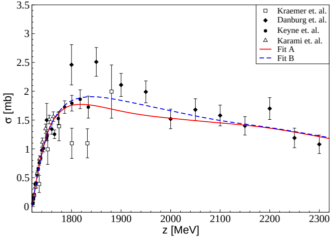
ωN Data: [Danburg *et al.*, PRD 2, 2564 (1970)] [Kraemer *et al.*, PR 136, B496 (1964)] [Binnie *et al.*, PRD 8, 2789 (1973)] [Keyne *et al.*, PRD 14, 28 (1976)] [Karami *et al.*, NPB 154, 503 (1979)]

Other channels: see the website

First: backward differential cross section. Second: forward. Third: total cross section.







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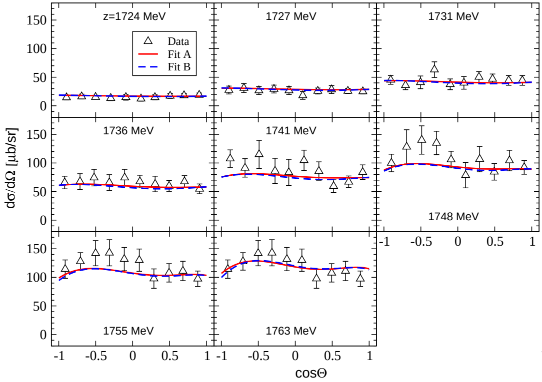
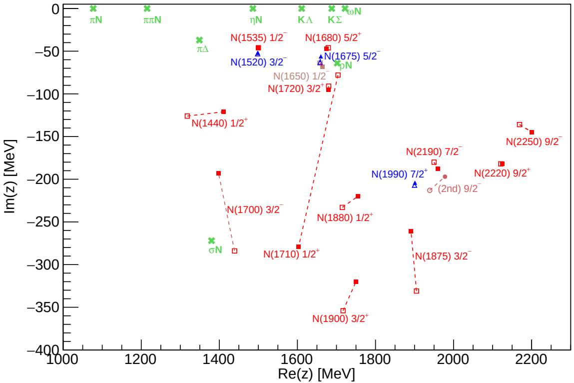


N^* Spectra

III Application 1: the ωN interaction

J^P convention. Empty symbols: fit A. Filled: fit B.



Physical Discussions

III Application 1: the ωN interaction

Our results

- ωN mainly couples to lower states ($|NR| > 0.5$): $N(1535)\,\frac{1}{2}^-$, $N(1710)\,\frac{1}{2}^+$ and $N(1680)\,\frac{5}{2}^+$
- Very large bare couplings $\rightarrow N^*(1535)$ and $N^*(1710)$
- Fit C (constraining the bare couplings) failed $\rightarrow$ left for the future with photonproduction
- Higher states $\rightarrow N(2250)\,\frac{9}{2}^-$ relatively important ($Br > 10\%$)

In the literature: which states are important for ωN

- $N(1720)\,\frac{3}{2}^+$ and $N(1680)\,\frac{5}{2}^+$ [Zhao, PRC 63, 025203 (2001)]
- $N(1535)\,\frac{1}{2}^-$, $N(1650)\,\frac{1}{2}^-$ and $N(1520)\,\frac{3}{2}^-$ [Lutz *et al.*, NPA 706, 431 (2002)]
- $N(1710)\,\frac{1}{2}^+$, $N(1675)\,\frac{5}{2}^-$ and $N(1680)\,\frac{5}{2}^+$

  [Penner & Mosel, PRC 66, 055211 (2002)][Penner & Mosel, PRC 66, 055212 (2002)] [Shklyar *et al.*, PRC 71, 055206 (2005)]
- $N(1675)\,\frac{5}{2}^-$ and $N(1680)\,\frac{5}{2}^+$ $\rightarrow$ very large bare couplings [Muehlich *et al.*, NPA 780, 187 (2006)]

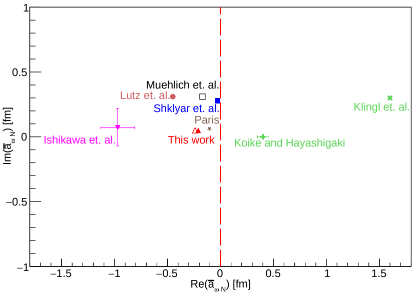
Physical Discussions

III Application 1: the ωN interaction

- Definition $\rightarrow a_\kappa \equiv \lim_{p_\kappa \to 0} p_\kappa^{-1} \tan \tilde{\delta}_\kappa^{(L=0)} = \lim_{p_\kappa \to 0} p_\kappa^{-1} \tau_{\kappa\kappa}^{(L=0)}$ ($\tilde{\delta}$: generalized phase shift)
- Spin average of $\omega N \rightarrow \bar{a}_{\omega N} = \frac{1}{3} a_{\omega N}\left(S = \frac{1}{2}\right) + \frac{2}{3} a_{\omega N}\left(S = \frac{3}{2}\right)$
- Fit A: $\bar{a}_{\omega N} = (-0.24 + 0.05i)$ fm. Fit B: $\bar{a}_{\omega N} = (-0.21 + 0.05i)$ fm.
- Small negative Re$\bar{a}$ $\rightarrow$ not indicating in-medium bound states

Other results: [Koike & Hayashigaki, PTP 98, 631 (1997)][Klingl *et al.*, NPA 650, 299 (1999)] [Lutz *et al.*, NPA 706, 431 (2002)]

[Shklyar *et al.*, PRC 71, 055206 (2005)][Muehlich *et al.*, NPA 780, 187 (2006)][Paris, PRC 79, 025208 (2009)] [Ishikawa *et al.*, PRC 101, 052201 (2020)]



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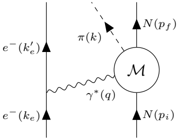
IV Application 2: the Electromagnetic Transition Form Factors of Baryons

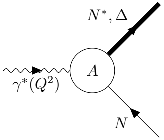
Backgrounds

IV Application 2: the Electromagnetic Transition Form Factors of Baryons

Electromagnetic Probes

- EM interactions $\to$ clean probes of structures
- $\gamma N \to$ finding states coupling weakly to πN
  [Ireland, Pasyuk, Strakovsky, Prog. Part. Nucl. Phys. 111, 103752 (2020)]
- Electroproduction ($\gamma^* N$) $\to$ energy scale $Q^2 \equiv -q^2$
- Transition Form Factors (TFFs) $\to$ "pictures of hadrons"
  [Ramalho, Peña, Prog. Part. Nucl. Phys. 136, 104097 (2024)]
  - Lower Q^2: meson clouds etc.
  - Higher Q^2: the quark core
  - Related to transverse charge densities
    [Tiator & Vanderhaeghen, PLB 672, 344 (2009)]





Towards the TFFs

- Predictions at quark level
  - Quark models & Dyson-Schwinger methods
    [Burkert, Roberts RMP 91, 011003 (2019)]
    [Eichmann *et al.*, Prog. Part. Nucl. Phys. 91, 1 (2016)]
  - Lattice QCD [Agadjanov *et al.*, NPB 886, 1199 (2014)]
- Extraction from data
  - Experimental facilities & data accumulation
    [Mokeev *et al.*, PRC 93, 025206 (2016)]
  - Unitary isobar models $\to$ real-valued, depending on Breit-Wigner parameters
    [Drechsel, Kamalov, Tiator, EPJA 34, 69 (2007)]
    [Tiator *et al.*, EPJST 198, 141 (2011)]
  - DCC models $\to$ complex TFFs at the poles
    [Kamano, Few Body Syst. 59, 24 (2018)]

This study: [Y.F. Wang *et al.*, PRL 133, 101901 (2024)]

Definitions

IV Application 2: the Electromagnetic Transition Form Factors of Baryons

Origianl definition

[Ramalho, Peña, Prog. Part. Nucl. Phys. 136, 104097 (2024)]

$$A_h = \sqrt{\frac{2\pi\alpha}{K}}\left\langle R, h \middle| \epsilon_+ \cdot J \middle| N, h-1 \right\rangle$$

$$S_{\frac{1}{2}} = \frac{|\mathbf{q}|}{Q}\sqrt{\frac{2\pi\alpha}{K}}\left\langle R, \tfrac{1}{2} \middle| \epsilon_0 \cdot J \middle| N, \tfrac{1}{2} \right\rangle$$

- A, S: helicity transition amplitudes
- $h = 1/2, 3/2$: the helicity
- α: fine structure constant
- $\epsilon(J)$: virtual photon polarization vector (current)
- $\mathbf{q}$: 3-momentum of the virtual photon
- $M_R(m_N)$: mass of the excitation state R (nucleon)
- $K = (M_R^2 - m_N^2)/(2M_R)$

At the pole

[Workman, Tiator, Sarantsev, PRC 87, 068201 (2013)]

$$H_h = C_I\sqrt{\frac{p_{\pi N}}{\omega_0}\frac{2\pi(2J+1)z_p}{m_N\widetilde{R}}}\widetilde{\mathcal{H}}_h$$

- H is either A or S
- C_I: isospin factor, $C_{1/2} = -\sqrt{3}$ and $C_{3/2} = \sqrt{2/3}$
- $p_{\pi N}$: πN c.m. momentum
- ω_0: photon energy at $Q^2 = 0$
- $z_p = M_R - i\Gamma_R/2$ the pole position
- $\widetilde{R}, \widetilde{\mathcal{H}}$: the residues of $\pi N, \gamma^* N$ channels
- Understanding: the $|R\rangle \rightarrow |R)$ Gamow state
  [Gamow, Zeitschrift für Physik 51, 204 (1928)]
- Complex-valued

Numerical details

IV Application 2: the Electromagnetic Transition Form Factors of Baryons

The latest JBW results

- $\gamma^* p$ initial state
- Coupled-channel study of πN, ηN, and $K\Lambda$ [Mai *et al.*, EPJA 59, 286 (2023)]
- Based on the JüBo2017 solution [Rönchen *et al.*, EPJA 54, 110 (2018)]
- C.M. energy range $z \in [1.13, 1.8]$ GeV
- Virtuality $Q^2 \in [0, 8]$ GeV^2
- Orbital angular momentum $L \leq 3$

Database & errors

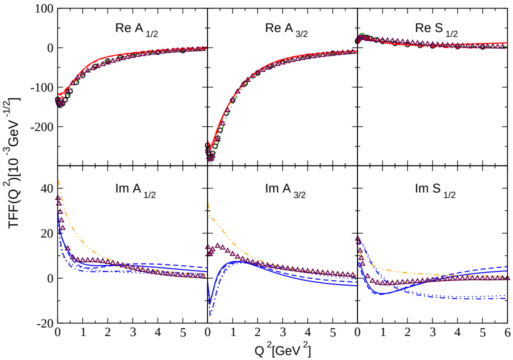
- Database [Mai *et al.*, EPJA 59, 286 (2023)]
  - 10^5 data points vs 533 fit parameters
  - 5×10^4 from photoproduction/hadronic
- Four solutions $\to$ fully explored parameter space
  - weighted vs unweighted χ^2
  - different local minima
- Fit $\to$ supercomputers

	$\chi^2_{\rm dof}$	$\chi^2_{\rm pp}(\pi^0 p)$	$\chi^2_{\rm pp}(\pi^+ n)$	$\chi^2_{\rm pp}(\eta p)$	$\chi^2_{\rm pp}(K^+\Lambda)$
FIT$_1$	1.42	1.40	1.47	1.49	0.70
FIT$_2$	1.35	1.38	1.35	1.40	0.58
	$\chi^2_{\rm wt,dof}$	$\chi^2_{\rm pp}(\pi^0 p)$	$\chi^2_{\rm pp}(\pi^+ n)$	$\chi^2_{\rm pp}(\eta p)$	$\chi^2_{\rm pp}(K^+\Lambda)$
FIT$_3$	1.12	1.44	1.61	1.08	0.33
FIT$_4$	1.06	1.42	1.44	1.09	0.32

Result: $\Delta(1232)$

IV Application 2: the Electromagnetic Transition Form Factors of Baryons

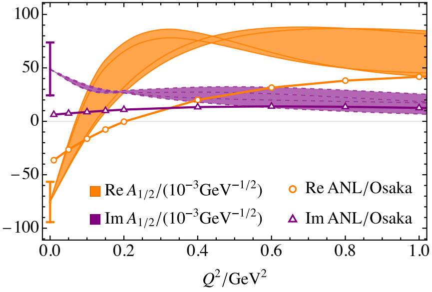
- Solid, dashed, dotted, dash-dotted curves: four fit solutions
- Dash-double-dotted curves: “L+P” extraction from MAID analyses [Workman, Tiator, Sarantsev, PRC 87, 068201 (2013)]
- Triangles: ANL-Osaka[Kamano, Few Body Syst. 59, 24 (2018)]

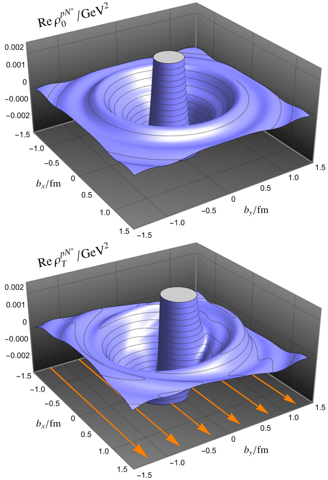


Result: $N^*(1440)$

IV Application 2: the Electromagnetic Transition Form Factors of Baryons

- A zero crossing!!
- ρ_0, ρ_T [Tiator & Vanderhaeghen, PLB 672, 344 (2009)]

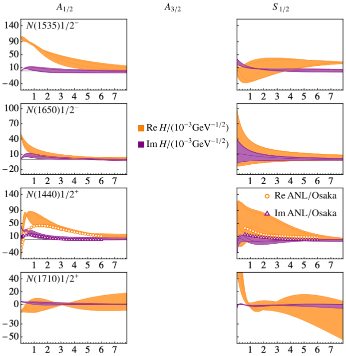


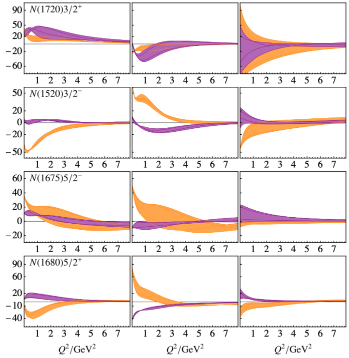


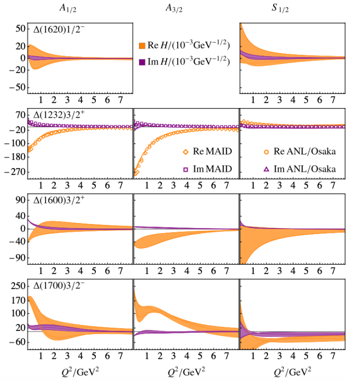
Summary of the results

IV Application 2: the Electromagnetic Transition Form Factors of Baryons

First combined analyses of the TFFs at the poles, based on multi-channel data







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V Application 3: Compositeness of the P_c States

Backgrounds

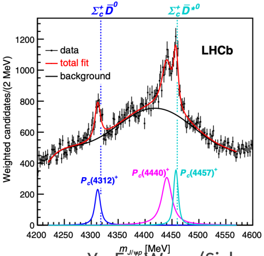
V Application 3: Compositeness of the P_c States

P_c as exotic states

- Exotic hadrons since $X(3872)$ [Belle, PRL 91, 262001 (2003)]
  $\rightarrow$ deviating much from conventional quark models
- Possible pictures
  - Hadronic molecules [Guo *et al.*, RMP 90, 015004 (2018)]: bound states formed by hadron-level interactions (meson exchanges etc.)
  - compact multi-quark states
  - hybrid states, kinematic cusps, etc.
- The P_c states
  - Discovered in $\Lambda_b \to J/\psi p K^-$ by LHCb [LHCb, PRL 115, 072001 (2015)]
    $P_c(4380)$ & $P_c(4450)$
  - Updated results [LHCb, PRL 122, 222001 (2019)]
    $P_c(4312)$, $P_c(4440)$, $P_c(4457)$
    $P_c(4380)$ insignificant

Interpretations

- $\bar{D}^{(*)}\Sigma_c^{(*)}$ molecules [Liu *et al.*, PRL 122, 242001 (2019)][Du *et al.*, PRL 124, 072001 (2020)][Yao *et al.*, Rept. Prog. Phys. 84, 076201 (2021)]
- Compact or cusps [Wang, Int. J. Mod. Phys. 35, 2050003 (2020)][Nakamura, PRD 103, 111503 (2021)][Burns & Swanson, PRD 106, 054029 (2022)]



Composition of P_c

Jülich-Bonn model for P_c

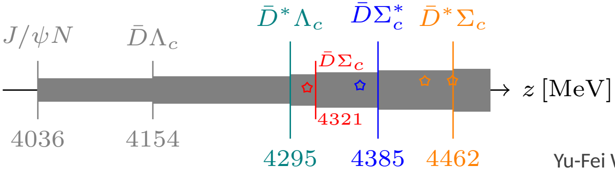
This study:

[Y.F. Wang *et al.*, PRD 112, 074010 (2025)]

- Previous studies
  [Wang *et al.*, EPJC 82, 497 (2022)][Shen *et al.*, EPJC 84, 764 (2024)]
- Three solutions $\longrightarrow$ updated LHCb data (175)
- P_c as S-wave states near $\bar{D}^{(*)}\Sigma_c^{(*)}$ thresholds:
  $P_c(4312)\,1/2^-$, $P_c(4440)\,1/2^-$, $P_c(4380)\,3/2^-$, $P_c(4457)\,3/2^-$
- Criteria $\longrightarrow$ amplitudes, poles, & residues

Compositeness criteria

- Dynamical generation $\longrightarrow$ model dependent
- Origin of compositeness criteria: Weinberg's study of deuteron [Weinberg, PR 137, B672 (1965)]
- Modern extensions
  - Pole-counting rule[Morgan, NPA 543, 632 (1992)]
  - Spectral density functions
    [Baru *et al.*, PLB 586, 53 (2004)]
  - Gamow wave functions
    [Sekihara, PRC 95, 025206 (2017)]



Criterion I

V Application 3: Compositeness of the P_c States

Weinberg's criterion

- Weinberg's criterion on deuteron $|d\rangle$

$$a = -\frac{2(1-Z)}{2-Z} R + \mathcal{O}(L)$$
$$r = -\frac{Z}{1-Z} R + \mathcal{O}(L)$$

Z: "elementariness". $R \sim 4.3$ fm: deuteron radius.
$L \sim m_\pi^{-1}$ interaction range.
a: scattering length. r: effective range.

- $Z = 1 - \int d\alpha |\langle \alpha | d \rangle|^2$. α: two-nucleon continuum
- Experimental results $\to Z \simeq 0$
- Conditions: S-wave, near-threshold, stable
- Model-independent [van Kolck, Symmetry 14, 1884 (2022)]

Pole-counting rule

- Near-threshold single-channel amplitudes: $T \sim (rp^2/2 - ip + 1/a)^{-1}$
- One bound state p_- and one virtual state p_+: $|p_+| + |p_-| \geq \frac{2}{|r|}$
- A molecule $\to$ quantum mechanical potentials
  $\to |r|$ being the interaction range
  $\to$ only one pole inside $1/|r|$
- A genuine state $\to |r|$ extremely large,
  $|p_+| \simeq |p_-|$ [Castillejo *et al.*, PR 101, 453 (1956)]
  $\to$ two nearby poles
- Can be extended to coupled-channel scatterings
- Only for S-wave near threshold states
- Applications [Meng *et al.*, PRD 92, 034020 (2015)]
  [Gong *et al.*, PRD 94, 114019 (2016)][Cao *et al.*, CPC 45, 103102 (2021)]

Criterion II

Spectral Density Functions

- Weinberg's Z for bound states: $Z = |\langle\psi_0|\Psi_B\rangle|^2$
- Lower decay channels: $Z \to w(E) = -\mathrm{Im}D(E)/\pi$
  $\to$ distribution on energy E
- Narrow resonance $E = E_R - i\Gamma_R/2$:
  $w(E) \simeq \frac{Z_B}{\pi}\frac{\Gamma_R/2}{(E-E_R)^2+(\Gamma_R/2)^2}$
- S-wave nearby channels:
  $w(E) \simeq -\frac{1}{\pi}\mathrm{Im}(E - E_0 - \sum_j i g_j^2 p_j/2)^{-1}$
  j: channels. E_0 and g_j's: from residues (couplings) and pole positions
- Elementariness $\to$ deviation of $w(E)$ from "Breit-Wigner":
  $BW(E) = \frac{1}{\pi}\frac{\Gamma_R/2}{(E-E_R)^2+(\Gamma_R/2)^2}$
- Applications [Kalashnikova and Nefediev, PRD 80, 074004 (2009)]
  [Baru *et al.*, EPJA 44, 93 (2010)][Hanhart *et al.*, EPJA 47, 101 (2011)]

Complex Compositeness

- Resonances
  - Unphysical Gamow states
    [Gamow, Zeitschrift für Physik 51, 204 (1928)]
    [Civitarese and Gadella, Physics Reports 396, 41(2004)]
  - Complex eigenstates of the Hamiltonian
    $\hat{H}|\Psi_R) = E_R|\Psi_R)$
  - Non-normalizable [Xiao and Zhou, PRD 94, 076006 (2016), J. Math. Phys. 58, 062110 (2017)]
- Complex elementariness: $Z_R = (\Psi_R^*|\psi_0\rangle\langle\psi_0|\Psi_R)$, complex-valued
- Complex compositeness $\to$ off-shell residues $r(k)$
  $T(E, p_i, p_f) \sim \frac{r(p_i)r(p_f)}{E-E_R}$
  $X = 1 - Z_R = \int_C \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{r^2(k)}{[\mathcal{E}_R-k^2/(2\mu)]^2}$
  [Sekihara, PRC 95, 025206 (2017)]
- Naive measure, e.g. $\tilde{X}_j = \frac{|X_j|}{\sum_n |X_n|+|Z_R|}$

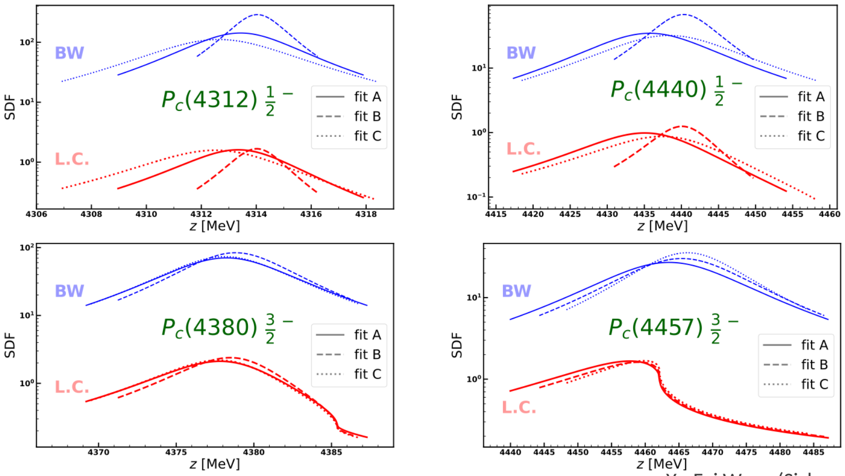
Results I

V Application 3: Compositeness of the P_c States

- Pole-counting rule: always only one pole
- Effective ranges: at fm level

States	Channels	a	r
$P_c(4312)\ 1/2^-$	$\bar{D}\Sigma_c$	$-2.0+0.39i$	$-3.0+2.9i$
		$-2.2+0.29i$	$-3.0+2.9i$
		$-1.9+0.40i$	$-3.0+2.9i$
$P_c(4440)\ 1/2^-$	$\bar{D}^*\Sigma_c$	$-1.2+0.14i$	$0.36-1.9i$
		$-1.3+0.099i$	$0.44-1.8i$
		$-1.3+0.17i$	$0.43-1.9i$
$P_c(4380)\ 3/2^-$	$\bar{D}\Sigma_c^*$	$-1.9+0.29i$	$-0.84-1.4i$
		$-2.0+0.26i$	$-0.82-1.3i$
		$-1.9+0.27i$	$-0.83-1.4i$
$P_c(4457)\ 3/2^-$	$\bar{D}^*\Sigma_c$	$-1.2+0.96i$	$0.78-1.6i$
		$-1.2+1.1i$	$0.77-1.6i$
		$-1.1+1.2i$	$0.76-1.7i$

Spectral density functions: one/two orders of magnitude smaller than BW!



Results III

V Application 3: Compositeness of the P_c States

- Gamow wave functions $\rightarrow$ fairly small elementariness
- Analyses of the compositions $\rightarrow$ the closest S-wave channels
  - $P_c(4312)\,1/2^-$: $\bar{D}\Sigma_c$ ($80\% \sim 81\%$)
  - $P_c(4440)\,1/2^-$: $\bar{D}^*\Sigma_c$ ($72\% \sim 77\%$)
  - $P_c(4380)\,3/2^-$: $\bar{D}\Sigma_c^*$ ($80\% \sim 81\%$)
  - $P_c(4457)\,3/2^-$: $\bar{D}^*\Sigma_c$ ($66\% \sim 71\%$), $\bar{D}\Sigma_c^*$ ($9\% \sim 12\%$)
- Relatively large $\bar{D}\Sigma_c^*$ configuration of $P_c(4457)$ $\rightarrow$ large coupled-channel effects
- Independent of model parameterization
- Further confirmation of the molecular interpretation!!
- A similar study on the N^* and Δ states $\rightarrow$ much more complicated
  [Y.F. Wang *et al.*, PRC 109, 015202 (2024)]

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VI Conclusion and Outlook

Conclusion & Outlook

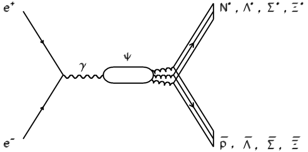
VI Conclusion and Outlook

Conclusion

- Hadrons at intermediate energies $\rightarrow$ intrinsic difficulty
- The "data-driven" direction $\rightarrow$ PWA via comprehensive DCC approaches
- The Jülich-Bonn model: decades of development, describing world data, with basic aspects of QCD, unitarity/analyticity
- Three applications
  - Hadronic ωN interaction: more than 9000 data
    [Y.F. Wang *et al.*, PRD 106, 094031 (2022)]
  - Baryon electromagnetic transition form factors: more than 10^5 data
    first multi-channel combined study
    [Y.F. Wang *et al.*, PRL 133, 101901 (2024)]
  - Compositeness of the P_c states: extension to the exotic hadrons
    [Y.F. Wang *et al.*, PRD 112, 074010 (2025)]

Outlook

- The model itself
  - $\gamma N \rightarrow \omega N$: more modern data & ω in medium
  - Other channels, more exotic hadrons...
  - Improvement of statistics
- Broader applications of PWA
  - BES: $J/\psi(\psi') \rightarrow \bar{N}N^*$ [BES, PRL 97, 062001 (2006)] [BES, PRL 110, 022001 (2013)] [BESIII, CPC 44, 040001 (2020)]
  - HHaS @ HIAF: π beams for πN interaction
    [Chen *et al.*, arXiv: 2511.22864 (2025)]




Thank you