磁约束等离子体中高能量粒子激发不稳定性与输运

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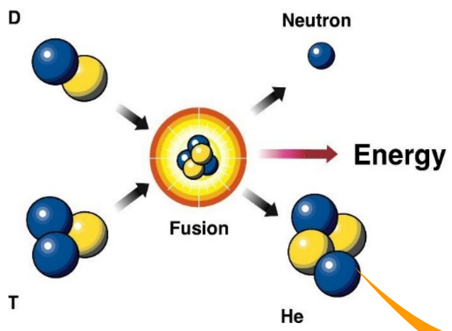
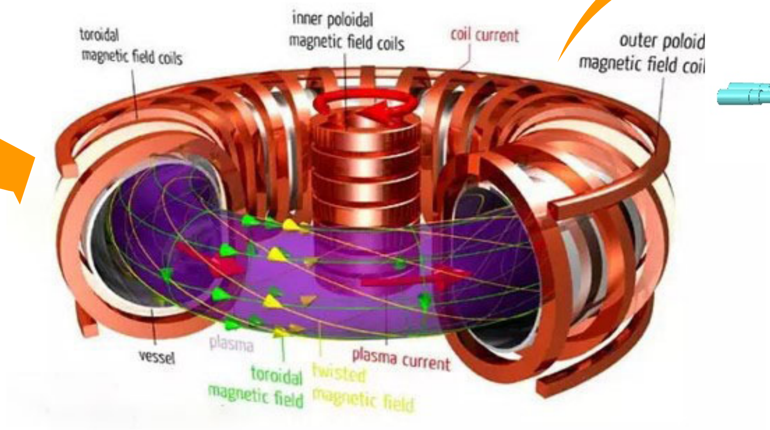
Outline: energetic particles in fusion plasmas

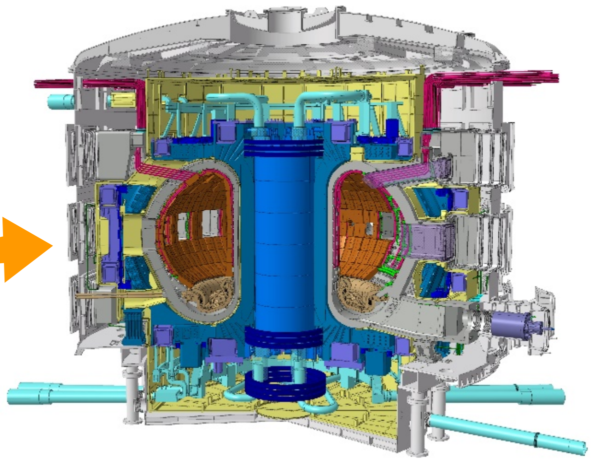
- Energetic particles (EP) in fusion plasmas
  - Single particle orbit
  - Shear Alfven waves in torus
- Kinetic excitation of SAW instabilities by EPs
  - Beam-plasma instability: paradigm for wave-particle interaction
  - Excitation of “fishbone” instability by trapped EPs
  - Excitation of toroidal Alfven eigenmode by circulating EPs
- Transport of EPs
  - Convective transport due to phase-locking
  - diffusive transport due to resonance overlapping
- Nonlinear spectrum evolution

Energetic particles in fusion plasmas



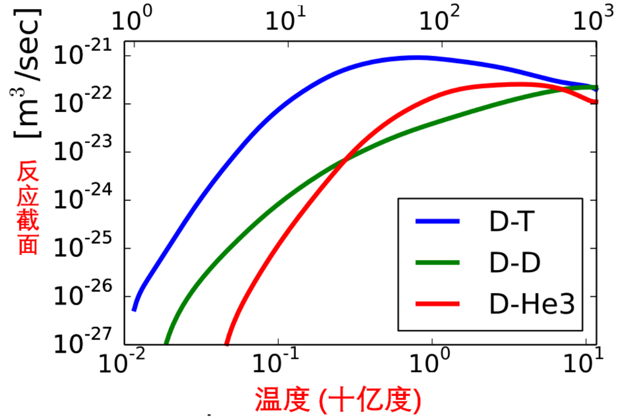
Tokamak





ITER

Q=10, 400s

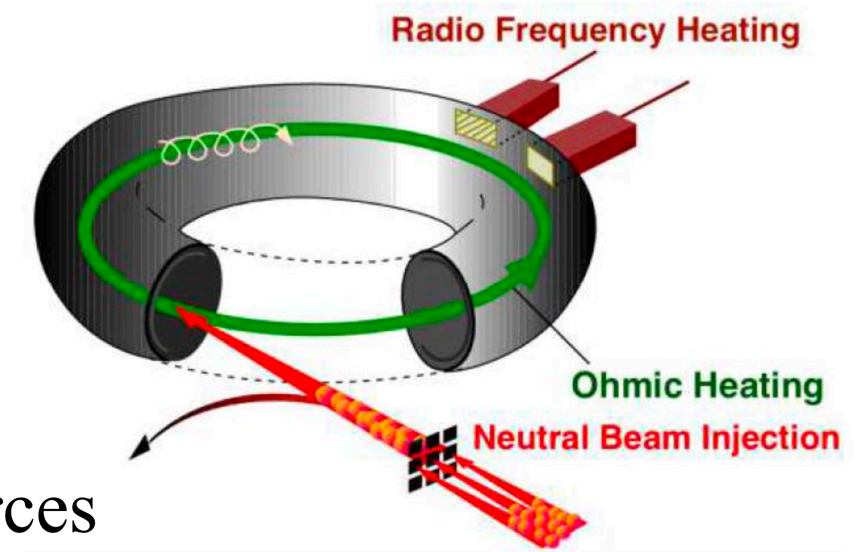


$$D + T \rightarrow {}^{4}H_e(3.52\text{Mev}) + n(14.06\text{Mev})$$

α heating of fuel ions ⇒ sustained burning

Energetic particles in fusion plasmas

- In addition to thermal ions and electrons, plasma has a super-thermal species: "energetic particles" (or "hot particles", "fast particles")
  - highly energetic ($T_h \gg T_{th} \sim 10\ KeV$)
  - much smaller density ($n_h \ll n_{th}$), comparable pressure
  - orbit deviate significantly from magnetic field lines
  - negligible deflection by collisions ($\nu \propto T^{-3/2}$)
  - velocity comparable to Alfven velocity



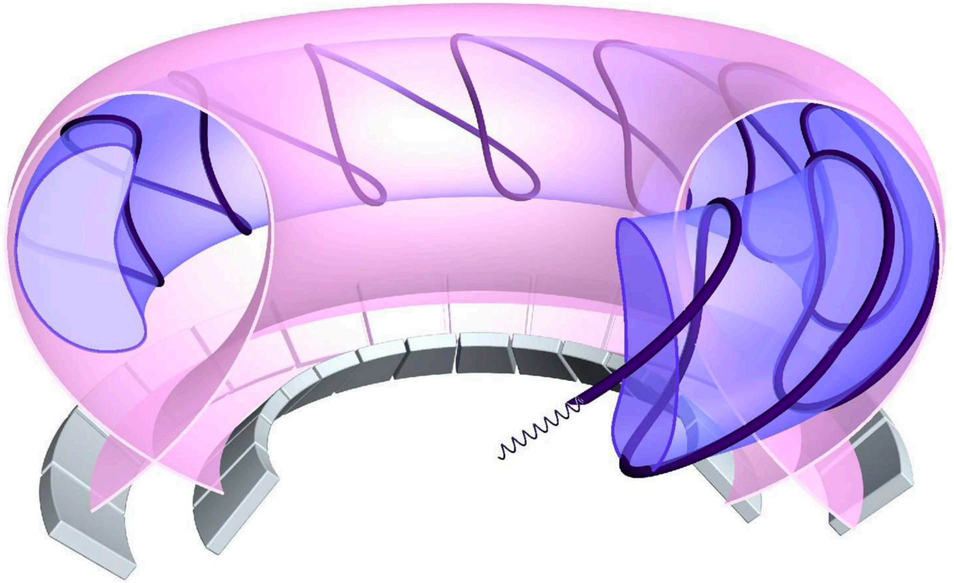
- Energetic particles can be created from different sources
  - Externally: highly energetic tail of distribution function ($\sim 100\ KeV$)
    - neutral beam injection (NBI)
    - electron/ion radio frequency wave heating (ICRH, LHW, ECRH)
  - Internally: alpha particles from fusion reactions (3.5 MeV), runaway electrons (> MeV)

Roles of energetic particles in fusion plasmas

- Roles of EPs in fusion plasmas - "major minority"
  - heat thermal plasma via Coulomb collision ⇒ sustained burning
  - destabilize symmetry breaking collective modes (shear Alfven modes) via wave-particle interactions ⇒ EP anomalous transport loss
  - affect thermal plasma confinement via regulation of micro-scale turbulence
  - EP loss/redistribution could degrade plasma heating and damage tokamak wall
- To understand SAW instabilities excitation by EPs
  - wave-particle resonance condition
  - Shear Alfven wave spectrum in torus
  - Excitation of instabilities
  - EP anomalous transport

Single particle orbit

- Single particle motion



$$m\dot{\mathbf{v}} = q\mathbf{v}\times\mathbf{B}/c+\mathbf{F}$$

$$\mathbf{B} = \frac{B_0}{1+(r/R_0)\cos\theta}\mathbf{e}_\phi + \frac{rB_0}{qR_0}\mathbf{e}_\theta$$

$$\Rightarrow \mathbf{v} = v_\perp(\hat{\mathbf{x}}\cos\Omega t+\hat{\mathbf{y}}\sin\Omega t)+v_\parallel\hat{\mathbf{b}}-\frac{c}{Bq}\hat{\mathbf{b}}\times\mathbf{F}$$

- Symmetry in geometry ⇒ constants of motion

periodic motions	constants of motion	characteristic frequency
cyclotron motion	magnetic moment	$\Omega_c = eB_0/m$
poloidal bounce	energy	$\omega_b \propto v_\parallel/R_0$
toroidal precession	toroidal angular momentum	$\bar{\omega}_d \propto E/R_0$

- For axi-symmetric torus, particles are confined as long as orbit width is not too large ($P_\phi \equiv e\psi + mv_\parallel R_0$ conservation)
- Toroidal symmetry breaking by ripples, external 3D perturbations, and MHD/SAW modes can cause EP transport loss

Transport: breaking of CoM by wave-particle resonance

- Equilibrium toroidally symmetric: banana center fixed ⇒ no radial transport (unless orbit is too big)

$$\dot{r}_0 \propto \dot{P}_\phi = \frac{\partial}{\partial\phi} H_0 = 0$$

- Transport: breaking of CoM due to wave-particle resonance with toroidally symmetry breaking ($n \neq 0$) electromagnetic perturbations

$$\delta\dot{r} \propto \delta\dot{P}_\phi = \frac{\partial}{\partial\phi}(H_0 + \delta H) \neq 0$$

- Resonant particle most efficient ($\delta H \equiv \delta\hat{H} \exp(-i\omega t + in\phi - im\theta)$)

$$\delta r \propto \frac{n}{\omega - n\dot{\phi} + m\dot{\theta}} \delta\hat{H} \exp(-i\omega t + in\phi - im\theta)$$

- symmetry breaking
- wave-particle resonance
- perturbation amplitude

➡ nonlinear turbulence spectrum

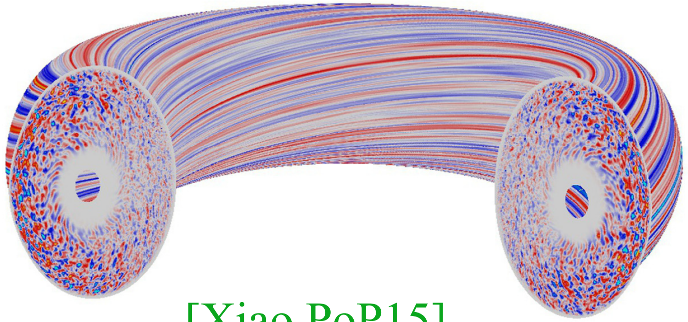
Anomalous transport by turbulence

- Wave-particle resonance condition in torus (with periodicity in θ/ϕ)

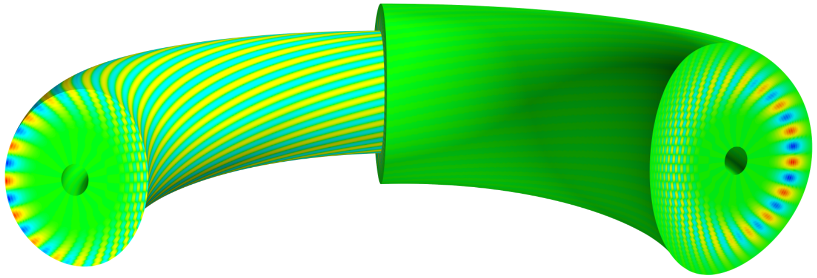
circulating: $\omega = k_{||}v_{||} + l\omega_t + n\bar{\omega}_d$

trapped: $\omega = l\omega_b + n\bar{\omega}_d$

- Electromagnetic perturbations
  - Thermal electrons/ions ⇐ drift wave turbulence (high-k, low-ω)
  - Energetic particles ⇐ shear Alfven waves (low-k, high-ω)



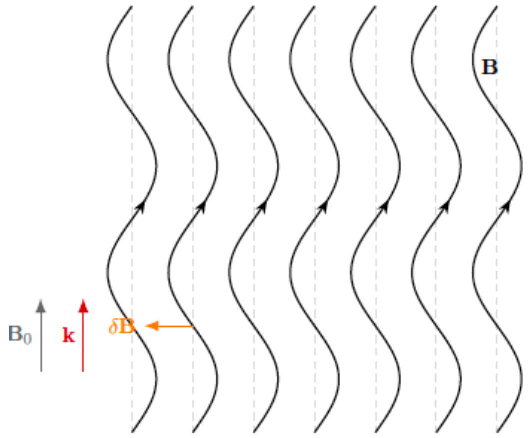
[Xiao PoP15]

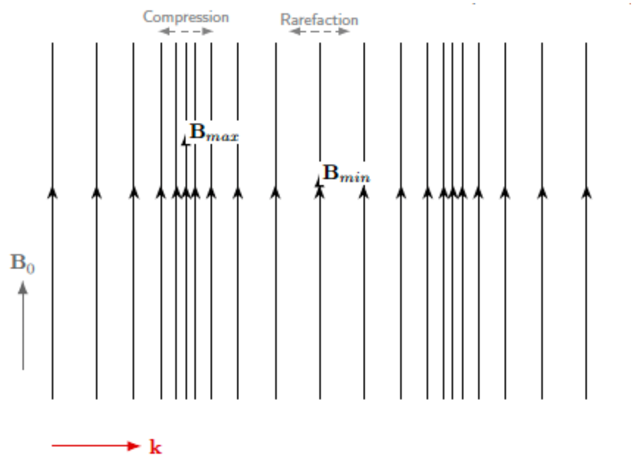


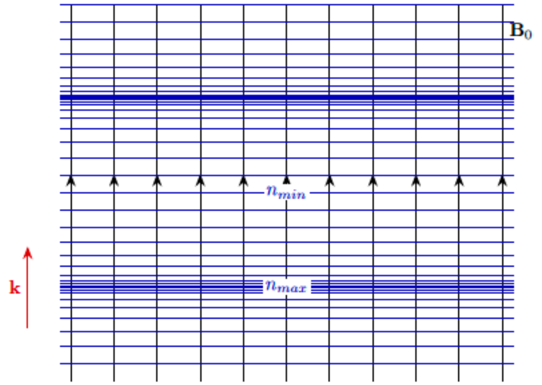
[Wei PoP24]

Three fundamental MHD waves

- Three fundamental Magnetohydrodynamic (MHD) waves







Shear Alfven wave

- $\omega \simeq k_{\parallel} V_A$
- Anisotropic
- Propagate along $\vec{b}$
- Incompressible
- Easier to excite

Compressional Alfven wave

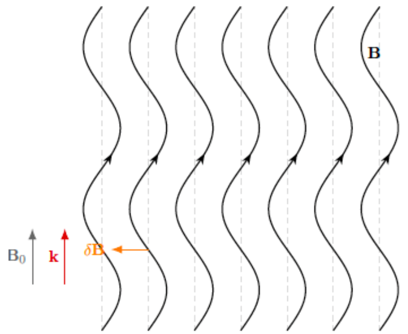
- $\omega \simeq k V_A$
- Isotropic
- Compressible in B and n
- Difficult to excite ($k \gg k_{\parallel}$)

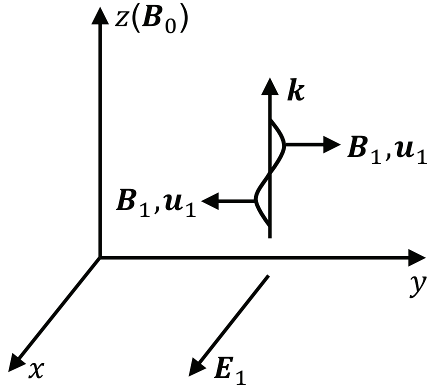
Ion acoustic wave

- $\omega \simeq k_{\parallel} C_S$
- Isotropic
- Compressible in n
- Heavily ion Landau damped

SAW: from cylinder to torus

- SAW: e&m oscillation in the presence of equilibrium B, transverse wave propagate along B, ~ incompressible



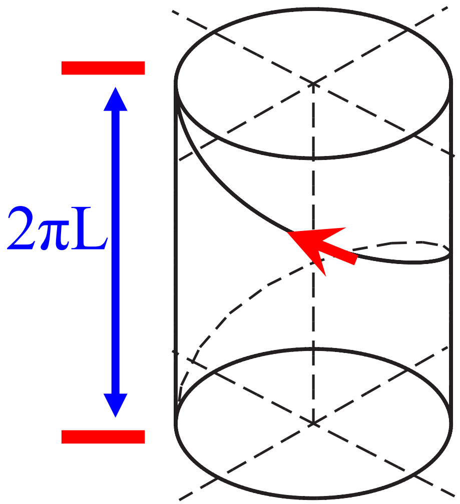


$$\omega = k_{\|} V_A$$
$$V_A \equiv B/\sqrt{4\pi\rho}$$

Hannes Alfvén

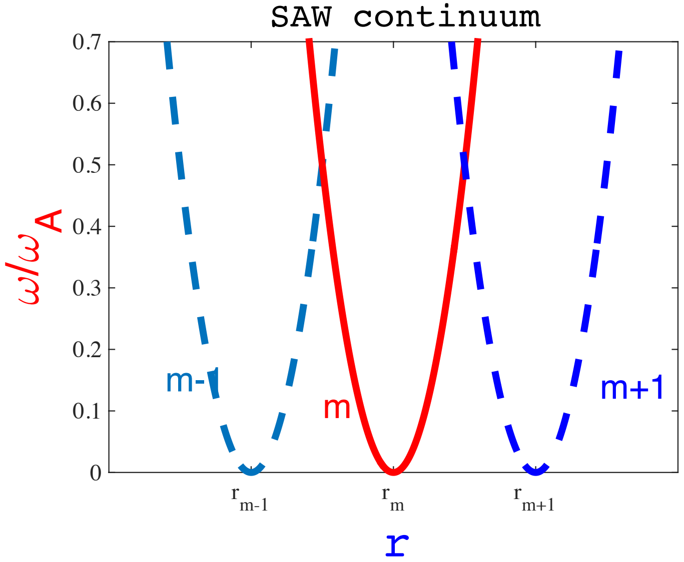
Nobel Laureate 1970

- In a cylinder with finite magnetic shear $\Rightarrow$ $\delta\phi \propto \exp(in(z/L) - im\theta)$



$\omega = \omega(r)$

"continuum"

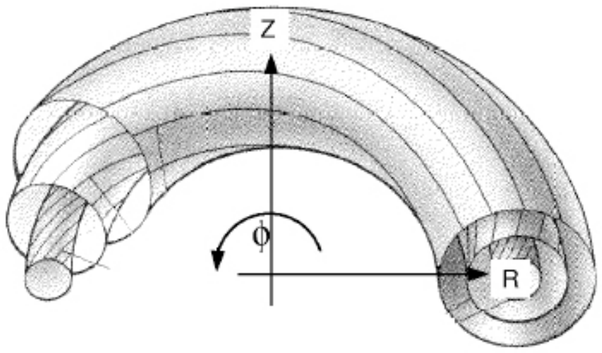


SAW: from cylinder to torus

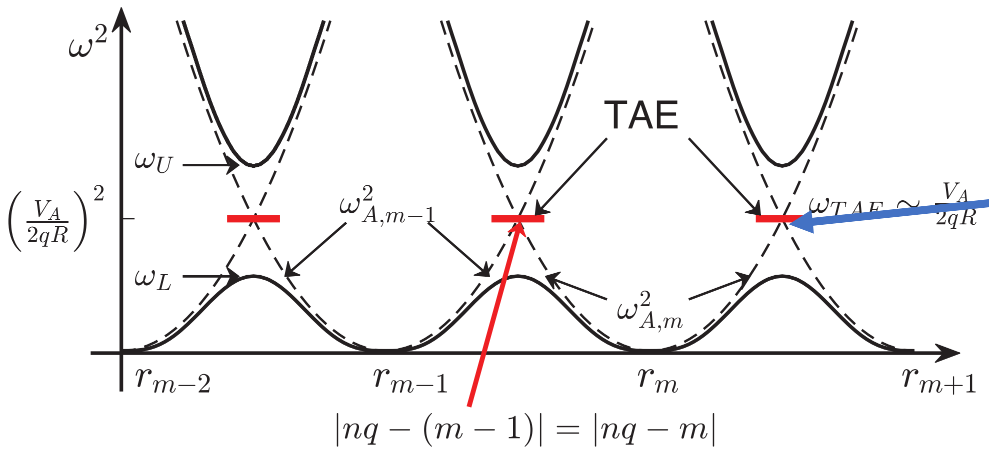
- In tokamaks: $V_A \propto 1 - \epsilon cos\theta$ modulated periodically as SAW propagates along B_0

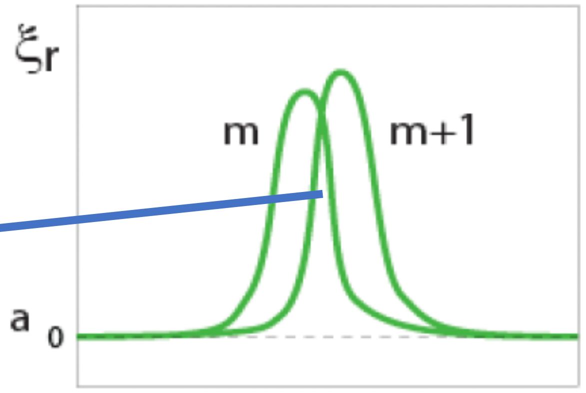
⇒ forbidden gap formation in the SAW continuum as Bragg condition satisfied
  - discretized Alfvén eigenmode: toroidal Alfvén eigenmode (TAE) [Cheng AP85]
  - TAE excited (by EPs) inside this gap [Fu PoFB89]: minimized continuum damping

  ⇒ one of the most dangerous causes of EP transport in reactors

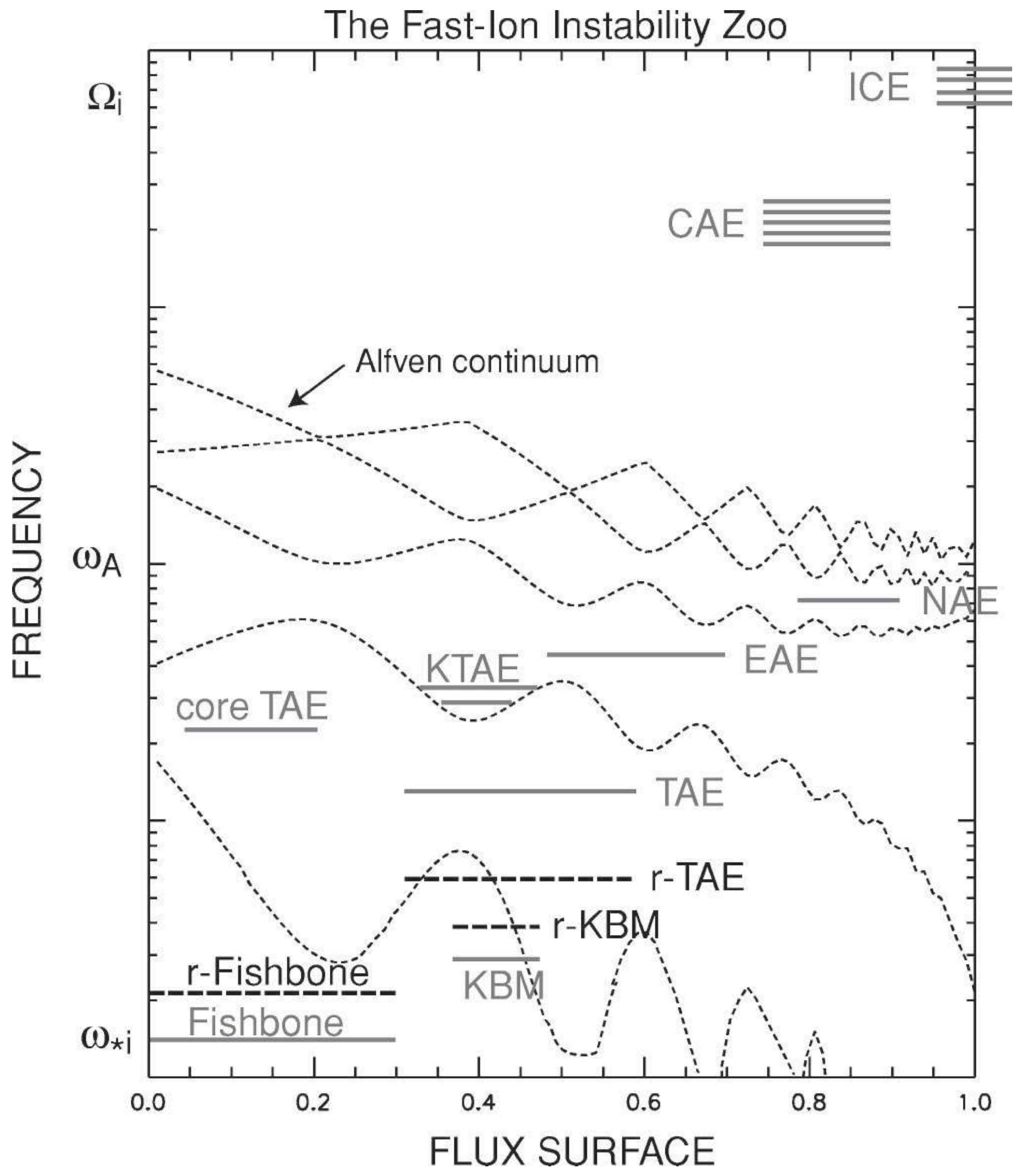


$B \propto 1 - (r/R_0)\cos\theta$

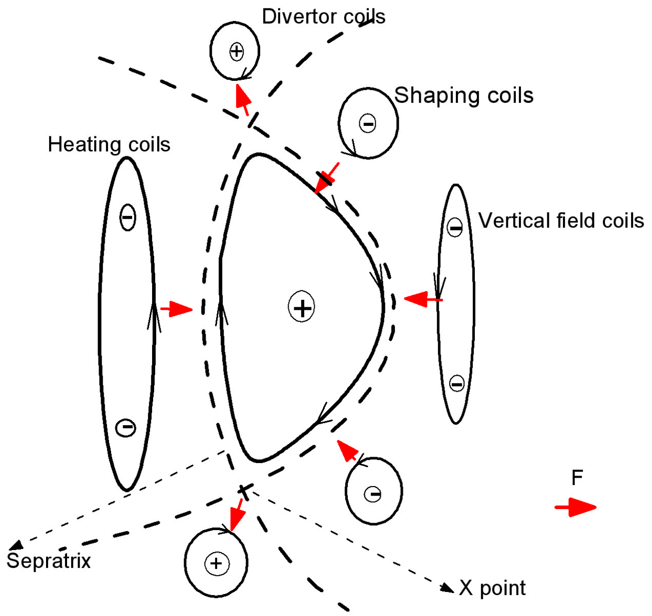




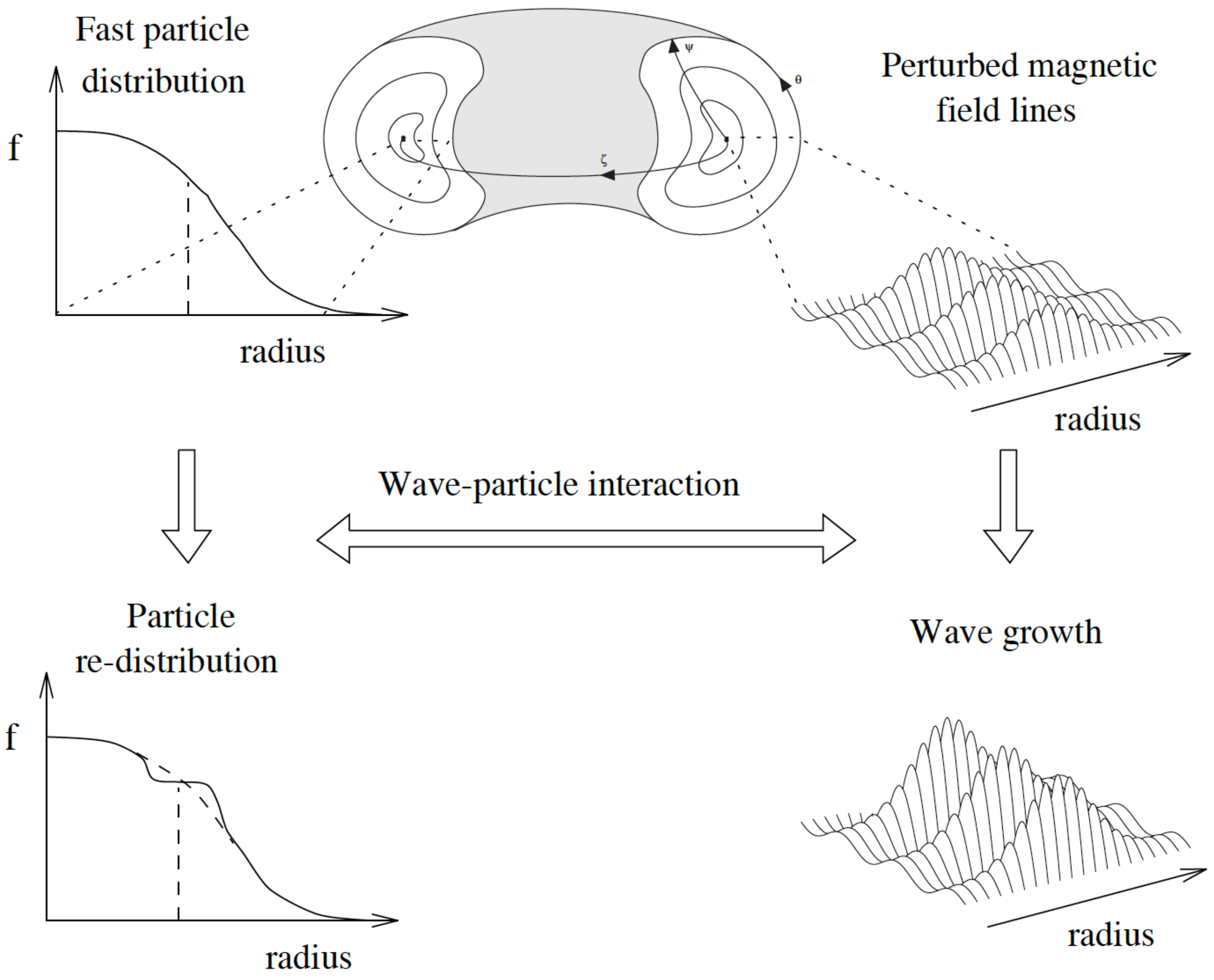
Alfven "zoology"



TAE	toroidicity	m & m+1
EAE	elongation	m & m+2
NAE	non-circular	m & m+3
HAE	helicity	stellarator
BAE	beta	compressibility
RSAE	reversed shear	reversed-q



Crucial issues in EP stability and transport



- ☐ Wave-particle resonance
- ☐ SAW instability excitation
- ☐ EP transport
- ☐ Nonlinear spectrum evolution

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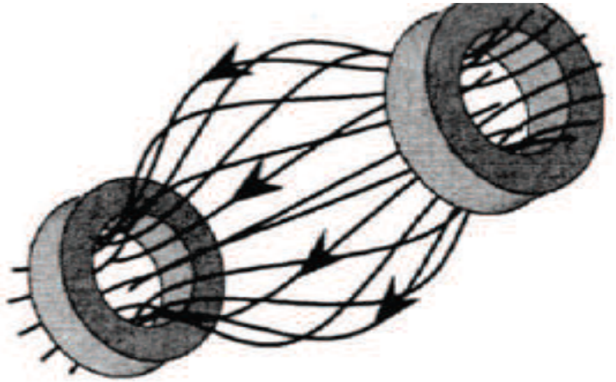
Beam-plasma instability: paradigm for w-p interaction

- Beam-plasma instability: electron Langmuir wave (plasma wave) driven unstable by energetic electrons beam
- System of a super-thermal electron beam interacting with plasma in a strongly axial magnetic field
  - 1-D, electrostatic
  - linear wave-particle interaction



- Kinetic theory needed for wave-particle interaction
- Governing equations (1-D, e.s.): Vlasov equation and Poisson's equation

$$\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x} - \frac{q}{m}\frac{\partial \phi}{\partial x}\frac{\partial}{\partial v}\right) f = 0$$

$$\nabla^2 \phi = -\sum_s q \int f dv$$

- $f = f(x, v, t)$: distribution function, ϕ: scalar potential

Beam-plasma instability: dispersion relation

- Linearization: $f = f_0 + \delta f$, with $\delta f \ll f_0$, and take $\delta f \sim \delta\hat{f}\, e^{i(kx-\omega t)}$

$$\delta f = -\frac{q}{m} k\delta\phi \frac{\partial f_0/\partial v}{\omega - kv}$$

- Substituting into Poisson's equation $\Rightarrow$

$$1 + \frac{1}{k^2}\sum_s \frac{q^2}{m}\int \frac{\partial f_0/\partial v}{\omega/k - v} dv = 0$$

- For beam-plasma system with three species, Maxwellian thermal electrons/ions and beam electron with $f = n_b\delta(v - v_b)$, we assume $kv_i \ll \omega \ll k\, v_e \Rightarrow$ dispersion relation:

$$\mathscr{E}_{b-p} \equiv 1 - \frac{\omega_{pe}^2}{\omega^2} - \frac{\omega_{pb}^2}{(\omega - kv_b)^2} = 0$$

def: $\omega_{pe}^2 \equiv n_0 e^2/m_e$, $\omega_{pb}^2 \equiv n_b e^2/m_e \ll \omega_{pe}^2$

- Plasma mode $\omega^2 = \omega_{pe}^2$ and beam mode ($\omega \simeq kv_b$) couple due to finite n_b/n_0

$$\left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)\left(1 - \frac{kv_b}{\omega}\right)^2 = \frac{\omega_{pb}^2}{\omega^2} \ll 1$$

Beam-plasma instability: stability analysis

- Mode unstable for $\omega < \omega_{pe}$:

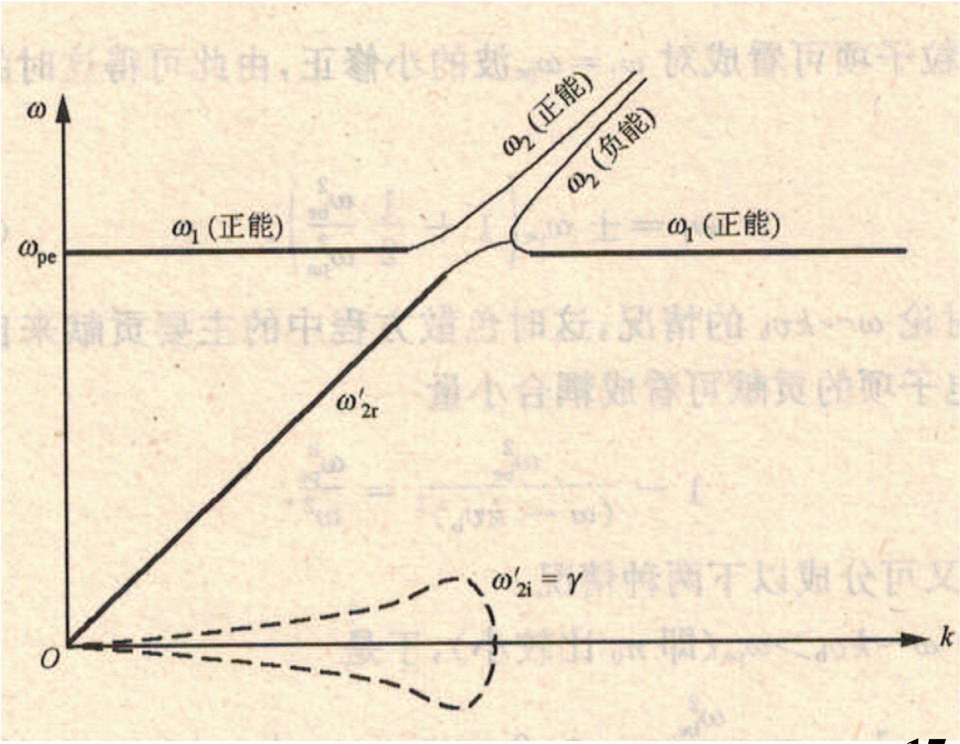
$$(\omega - kv_b)^2 = \frac{\omega_{pb}^2}{1 - \omega_{pe}^2/\omega^2} < 0 \quad \text{for } \omega < \omega_{pe}$$

- Reactive type instability: coupling of negative and positive energy modes. Similar to Reyleigh-Taylor instability.

- Most unstable for $\omega \simeq \omega_{pe} \simeq kv_b$: beam-mode and plasma-mode strongly coupled.
- Letting $\omega = \omega_{pe} + \Delta = kv_b + \Delta \Rightarrow$

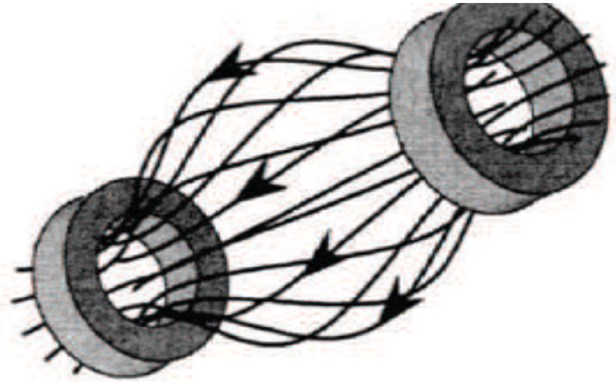
$$1 - \frac{\omega_{pe}^2}{(\omega_{pe} + \Delta)^2} - \frac{\omega_{pb}^2}{(kv_b + \Delta - kv_b)^2} = 0 \Rightarrow \Delta^3 = \omega_{pe}\omega_{pb}^2$$

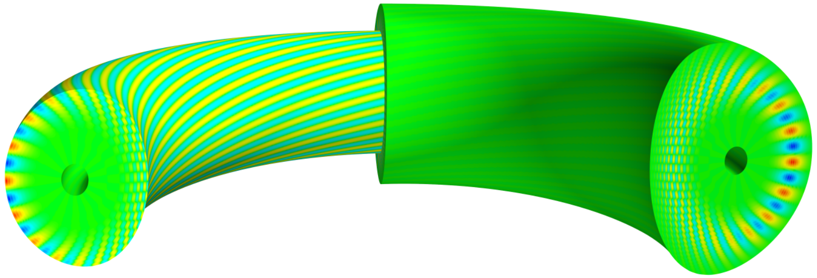
$$\Rightarrow \quad \omega = \omega_{pe}\left(1 + \left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)\left(\frac{n_b}{n_0}\right)^{1/3}\right) \qquad \gamma \propto \left(\frac{n_b}{n_0}\right)^{1/3}$$



One-to-one analogy of SAW instability and beam-plasma instability

beam-plasma instability	SAW instability
Langmiur wave	SAW wave
energetic electron beam	energetic particles (usually ions)
plasma mode	Alfvén eigenmode
beam mode	energetic particle mode
$\mathbf{v}_{\parallel} \cdot \mathbf{E}_{\parallel}$	$\mathbf{v}_B \cdot \mathbf{E}_{\perp}$
$\omega = kv_b$	$\omega = n\omega_\phi + m\omega_\theta$

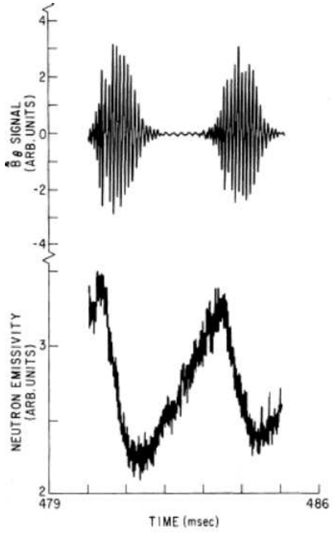


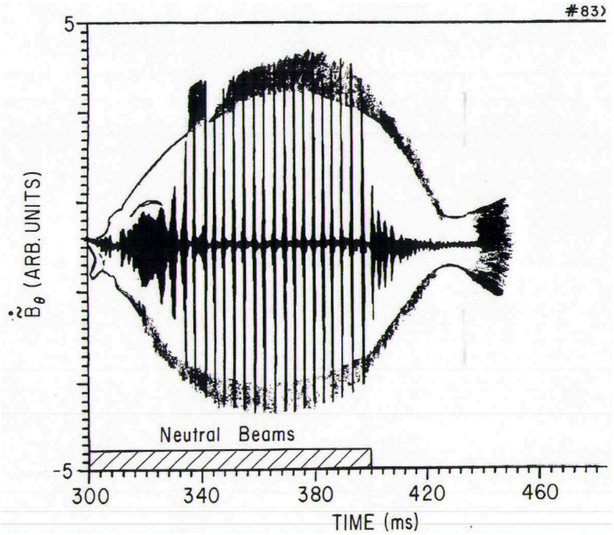


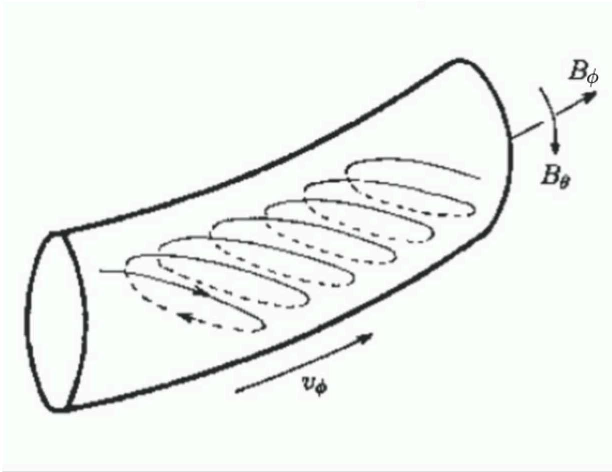
[Wei PoP24]

Kinetic excitation of "fishbone" instability

- Fishbone instability in PDX [McGuire 1983, Chen 1984]: first example of EPM
  - SAW fluctuations excited during perpendicular (to B) NBI heating experiments
  - symmetry-breaking perturbations ⇒ significantly (>30%) loss of energetic beam ions







- Fishbone excitation: wave-particle interactions tapping finite ∇P_{EP} of EPs
  - magnetically trapped particles precess in ϕ
  - precession frequency $\bar{\omega}_d \propto E \equiv v^2/2$
  - $\omega = k_\phi v_\phi = \bar{\omega}_d$ resonant EPs secularly move in r
  - $\mu \equiv v_\perp^2/B$ conserved ⇒ move radially outward ⇔ particle loss energy

Fishbone excitation by trapped particles

- Fishbone dispersion relation (kinetic energy principle) [Chen PRL84, RMP16]

$$i\Lambda(\omega) = \delta W_f + \delta W_k(\omega)$$

- $\Lambda(\omega)$: "inertia" (kinetic energy) due to background plasma
  $\Rightarrow$ structure of continuum, forbidden gaps, and resonant absorption
- δW_f: potential energy due to background plasmas
  $\Rightarrow$ existence of discrete AEs
- $\delta W_k(\omega)$: active potential energy due to EPs
  $\Rightarrow$ instability mechanisms & new unstable branches

- Simple limit:
  - $\Lambda(\omega) = \omega/\omega_A$ with $\omega_A \equiv V_A/qR_0$, $\delta W_f \simeq 0$
  - $\delta W_k \propto \langle \frac{E\,\overline{\omega}_d}{\overline{\omega}_d - \omega} \frac{\partial F_{EP}}{\partial r} \rangle$ $\quad$ $\overline{\omega}_d - \omega = 0$: w-p resonance $\Rightarrow$ energy transfer
  - $Im(\delta W_k) > \omega$: threshold in $\partial_r P_{EP}$ $\Rightarrow$ threshold in beam-injection power
  - $Re(\delta W_k) \simeq 0 \Rightarrow \omega \simeq \overline{\omega}_d(E_{inj})$:
    - unstable discrete mode intrinsically due to EP
    - Energetic particle mode (EPM)

TAE excitation by circulating particles

- Governing equation from gyrokinetic vorticity equation (ideal MHD assumed):

$$\frac{c^2}{4\pi\omega^2}B\frac{\partial}{\partial l}\frac{k_\perp^2}{B}\frac{\partial}{\partial l}\delta\phi_k+\frac{e^2}{T_i}\langle(1-J_k^2)F_0\rangle\delta\phi_k-\sum\left\langle\frac{q}{\omega}J_k\omega_d\delta H\right\rangle=0$$

- EP response to TAE derived from gyrokinetic equation:

$$\left(\partial_t+v_\parallel\partial_l+i\omega_d\right)\delta H_k=-i\frac{q}{T}(\omega-\omega_*)F_0J_k(\delta\phi-v_\parallel\delta A_\parallel/c)_k$$

- EP drive from curvature coupling:

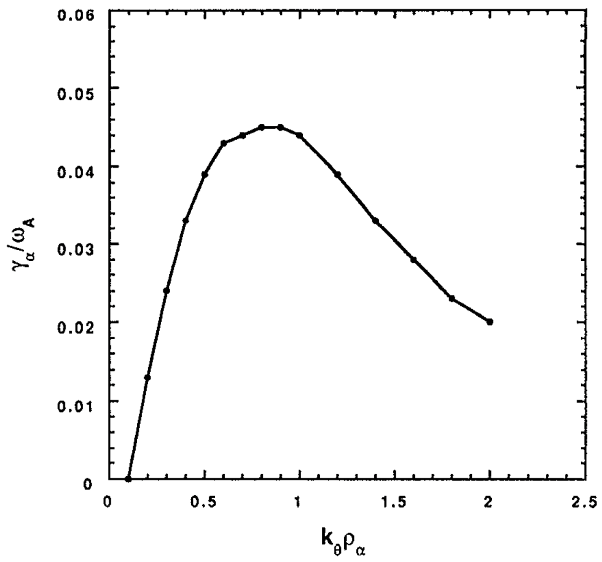
$$\langle\overline{\omega_d\delta H}\rangle_v\propto\left\langle\sum_l\frac{l^2\omega_{tr}^2J_l^2(k_\theta\hat{\rho}_d)}{\omega-k_\parallel v_\parallel-l\omega_{tr}}(\omega-\omega_{*,E})F_{0,E}\right\rangle_v$$

- TAE D.R. in the WKB limit ($l=\pm1$):

$$\frac{1}{2}k_\perp^2\rho_i^2\left(1-\frac{k_\parallel^2v_A^2}{\omega^2}\right)+i\frac{\pi}{4}\frac{n_E}{n_0}\frac{k_\theta^2}{\omega^2}\left\langle(\omega-\omega_{*E})\frac{F_{0E}}{n_E}\hat{v}_d^2\delta(\omega-k_\parallel v_\parallel\pm\omega_{tr})\right\rangle=0$$

$$\Rightarrow\quad\frac{\gamma}{\omega_r}\propto\left(\frac{\omega_{*,E}}{\omega}-1\right)\left[\delta(v_\parallel-V_A)+\delta(v_\parallel-V_A/3)\right]$$

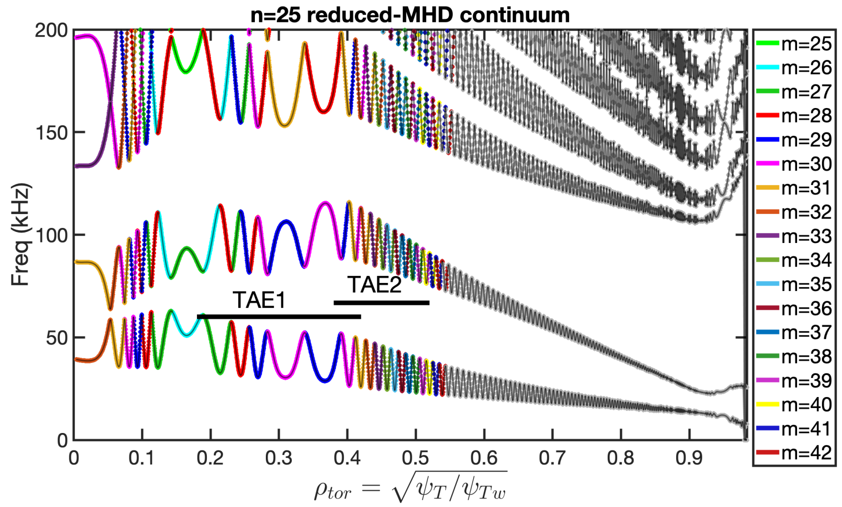
Most unstable mode for $k_{\perp}\rho_h \sim O(1)$

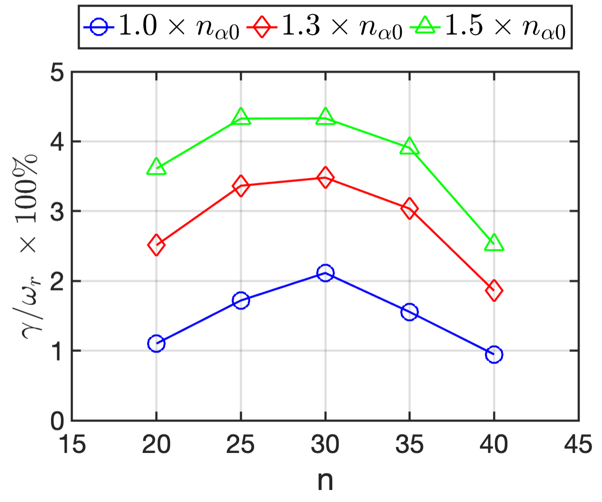


- $k_\theta \equiv nq/r$: n-toroidal mode number
- $\omega_* \propto n$: drive from ∇P_{EP} increases with n
- $J_l^2(k_\perp\rho_h) \propto 1/\sqrt{k_\perp\rho_h}$ suppress short wave length (high-n) modes

$\Rightarrow$ $k_\perp\rho_h \sim O(1)$ for most unstable modes

- For ITER/CFEDR: most unstable $n \sim 20 - 30$





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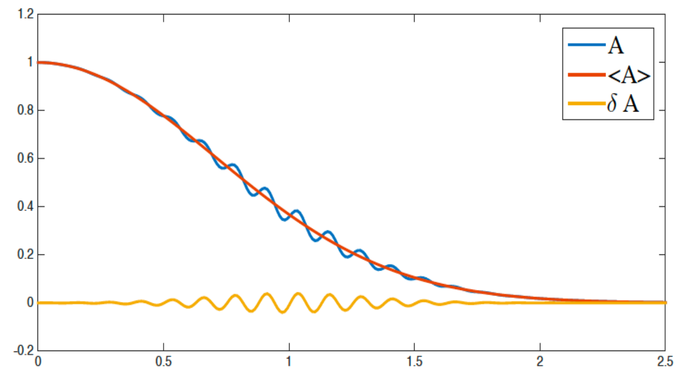
Anomalous transport of EPs

- Anomalous transport of EPs by e&m perturbations: mean-field theory
- Sketched derivation: $f = \langle f \rangle + \delta f$, Vlasov equation separated into

$$(\partial_t + v\partial_x)\delta f + \frac{q}{m}\delta E \partial_v \langle f \rangle + \underbrace{\frac{q}{m}\frac{\partial}{\partial v}\left[\delta E \delta f - \langle \delta E \delta f \rangle\right]}_{mode-mode\ coupling} = 0$$

$$\partial_t \langle f \rangle = -\frac{q}{m}\partial_v \langle \delta E \delta f \rangle$$



- A
- $\langle A \rangle$
- δA

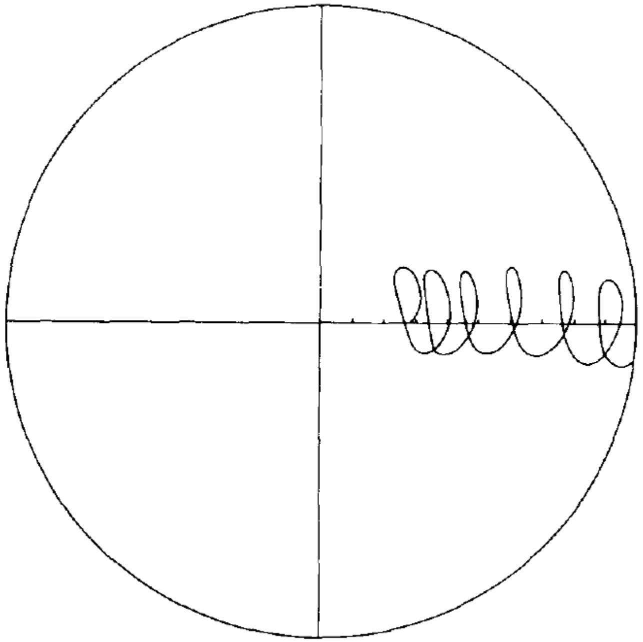
- EP transport due to stochasticity (Brownian motion) induced by resonant island overlapping $\Rightarrow$ diffusive transport

$$\partial_t \langle f \rangle = \frac{q^2}{m^2}\frac{\partial}{\partial v}\left[I_0 + \frac{d}{dt}I_1\right]\frac{\partial}{\partial v}\langle f \rangle \qquad I_0 \equiv \sum_{k>0} 2\pi\delta(kv - \omega_r)|\delta \hat{E}_k|^2$$

- What if $I_0 \propto \partial_t^{-1}$? Convective transport typical of EPM like mode
- Transport scaling $\nu \propto \delta\phi^{\alpha}$, with $\alpha = 1,2$ for convective/diffusive transport

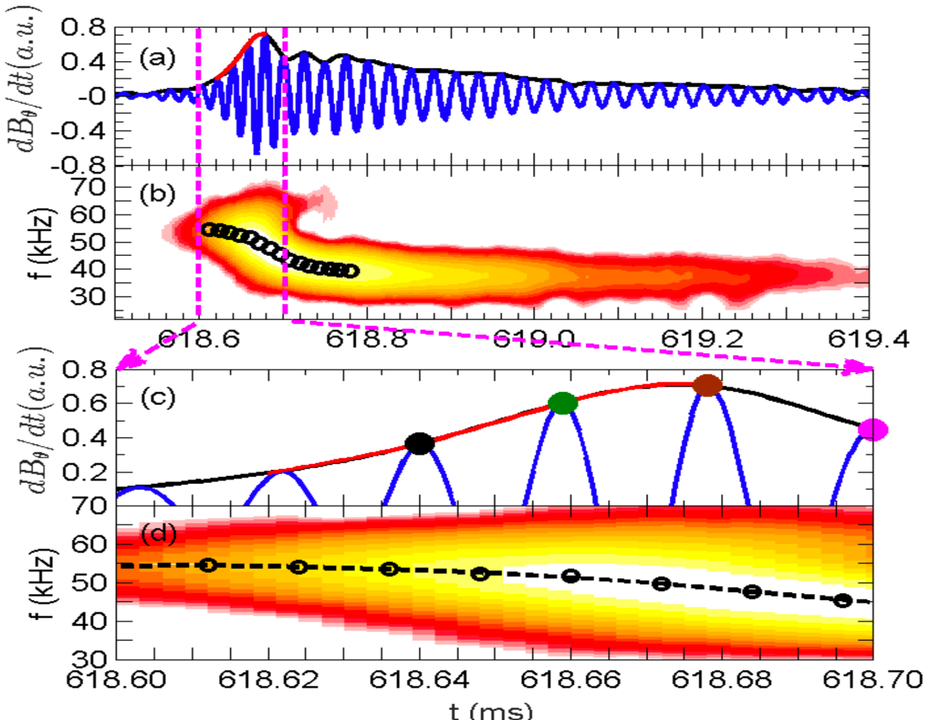
Convective transport due to resonant phase locking

fishbone induced loss orbit in the PDX tokamak



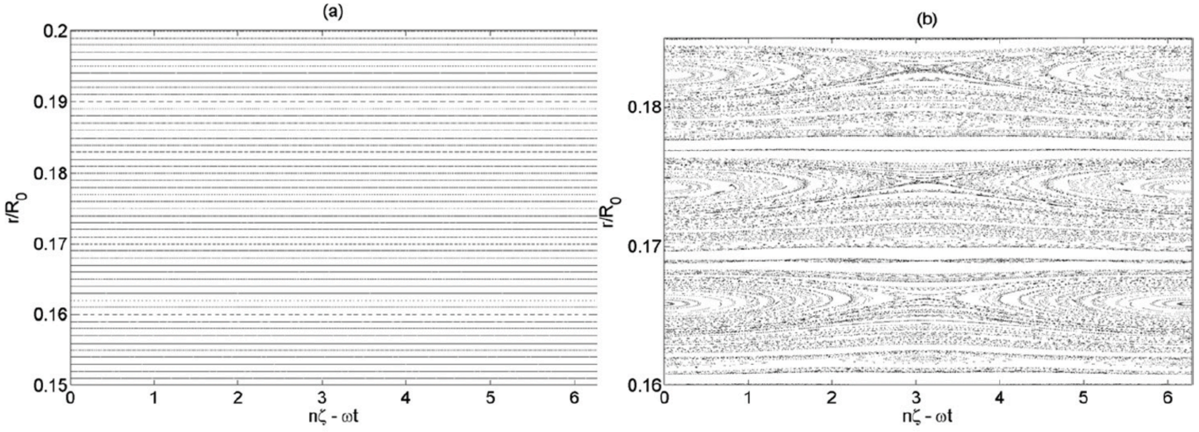
[White PoFB 1983]

- Fishbone: globally extended, low-frequency mode
- The mode extracts energy from the fast ions ⇒ particle move radially outward
- $\omega = \bar{\omega}_d \propto E$: frequency chirping down to maintain resonance with EP (phase-locking)
- Main loss mechanism: convective E x B radial transport

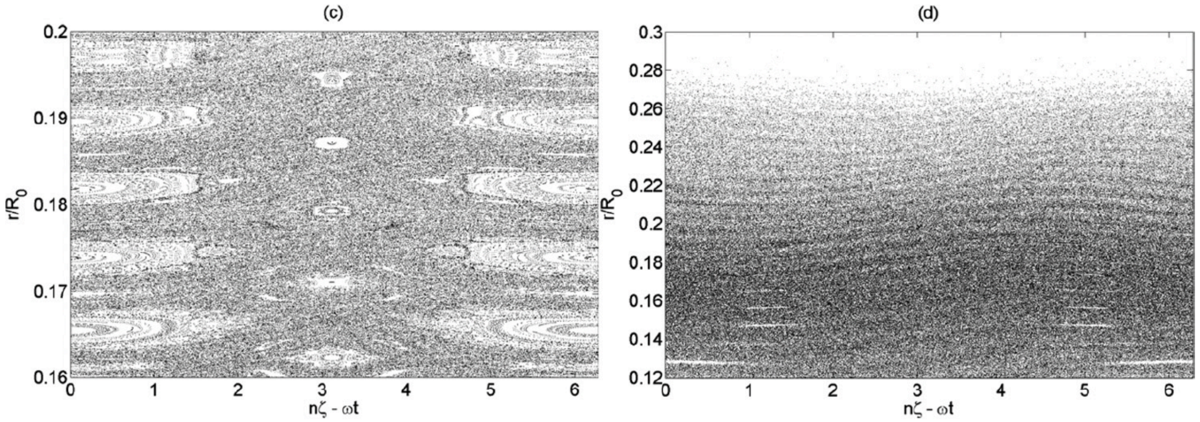


Diffusive transport due to resonant islands overlapping

Test particle transport by small scale ITG

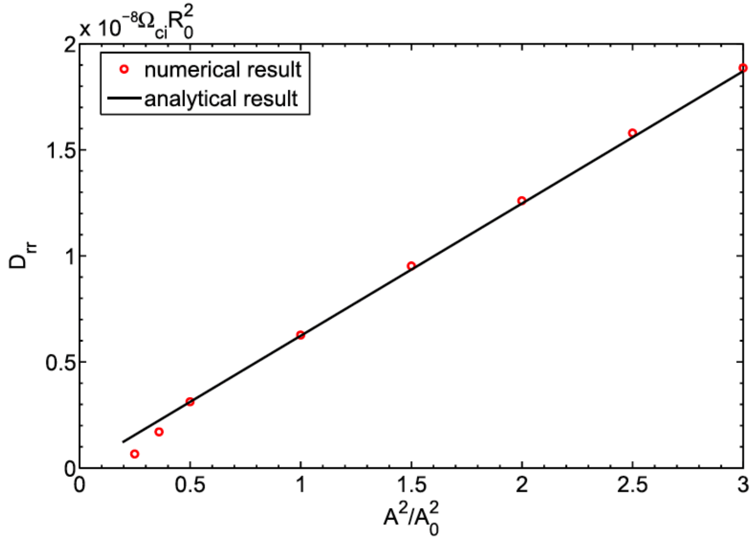


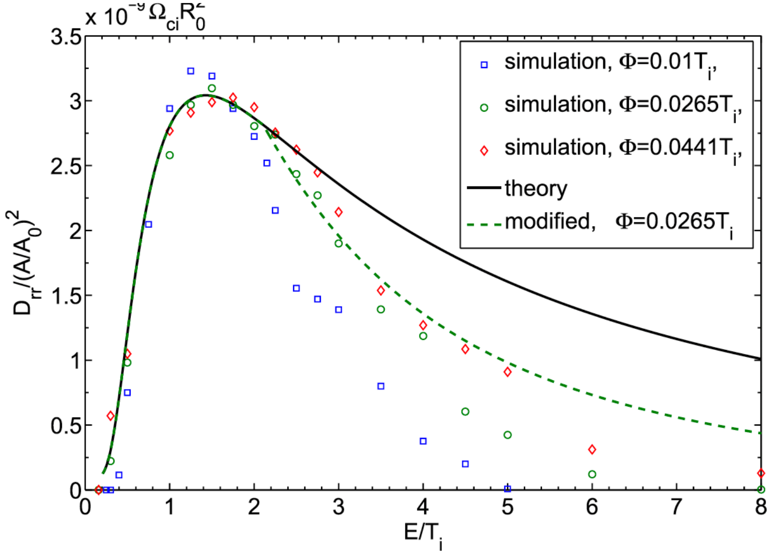
(a) $e\Phi/T_i = 0,\ E_t = T_i$ (b) $e\Phi/T_i = 0.001,\ E_t = T_i$



(c) $e\Phi/T_i = 0.003,\ E_t = T_i$ (d) $e\Phi/T_i = 0.01,\ E_t = T_i$

Orbits become stochastic as ITG amplitude increase

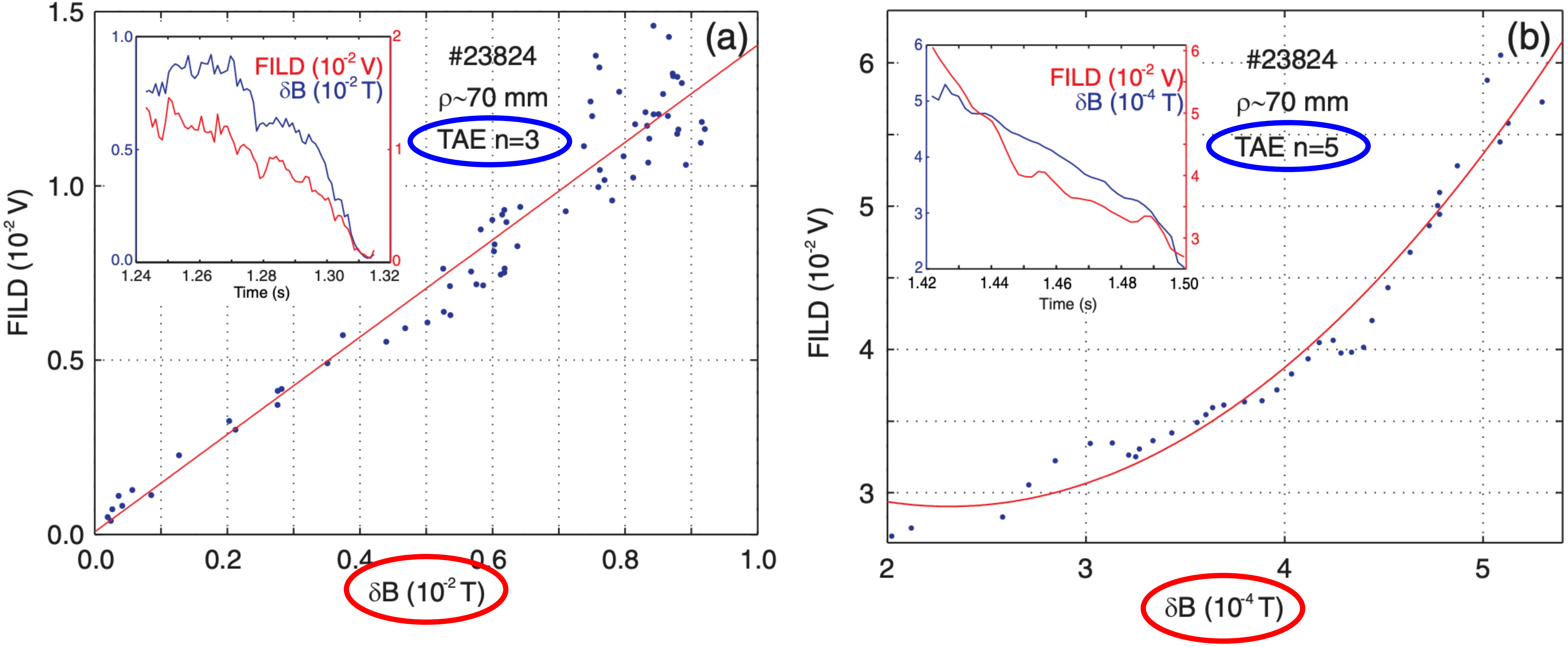




[Feng PoP 2013]

Convective v.s. diffusive transport by TAE observed

- Convective v.s. diffusive transport of EPs

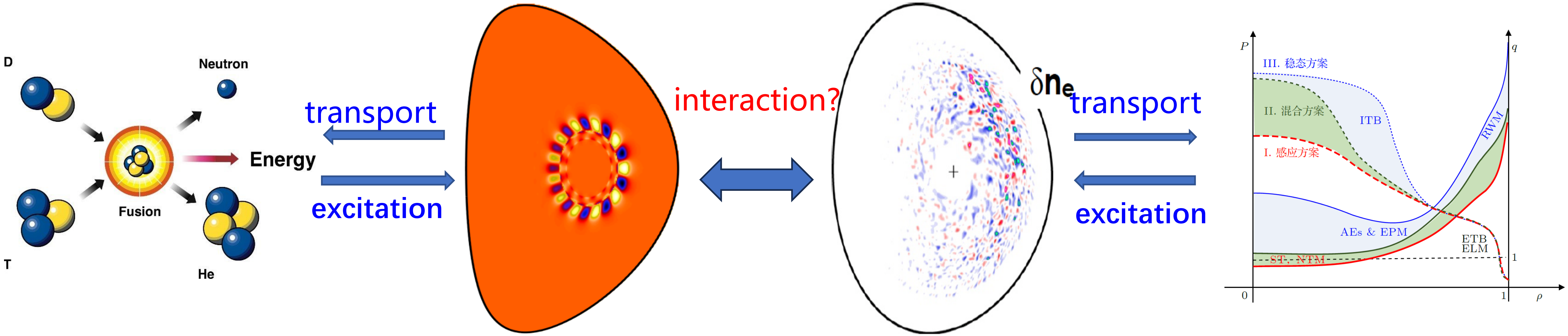


[Garcia-Munoz, PRL 2010]

Outline: energetic particles in fusion plasmas

- Energetic particles (EP) in fusion plasmas
  - Single particle orbit
  - Shear Alfven waves in torus
- Kinetic excitation of SAW instabilities by EPs
  - Beam-plasma instability: paradigm for wave-particle interaction
  - Excitation of “fishbone” instability by trapped EPs
  - Excitation of toroidal Alfven eigenmode by circulating EPs
- Transport of EPs
  - Convective transport due to phase-locking
  - diffusive transport due to resonance overlapping
- Nonlinear spectrum evolution

Nonlinear spectrum due to cross-scale interactions



Nonlinear SAW instability spectrum due to cross-scale interactions?

- zonal flow/current generation? [PRL 12]
- scattering by micro-scale turbulence? [NF 22&23]
- scattering by ion acoustic wave? [NF 19&24]

⇒ Saturation spectrum of SAW instability?

⇒ Compatibility of EP & bulk plasma confinement.

Energetic particle stability and transport in fusion plasmas

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Thank you!