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强子物理在线论坛

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# Nucleon-nucleon interaction in manifestly Lorentz-invariant ChEFT

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# OUTLINE

- Introduction
- Theoretical framework
- Results and discussion
- Summary

# Nuclear force

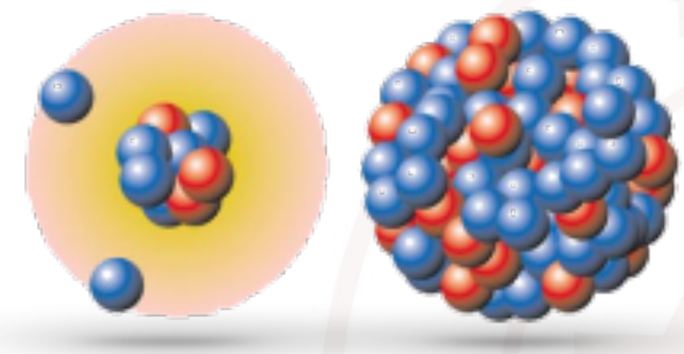
- Acts between two or more nucleons
- Binds protons and neutrons into atomic nuclei
- Plays an important role in whole nuclear physics
  - Ab-initio calculation *R. Machleidt, arXiv:2307.06416*

**ONLY input:** **Realistic Nuclear force**

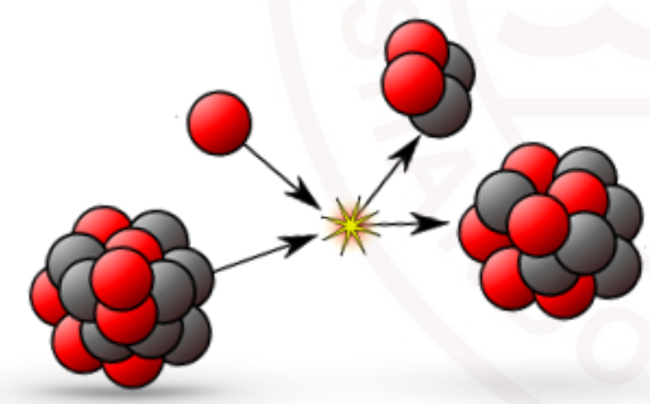
e.g. NN: AV18, CD-Bonn, Reid93, $N^3LO$…
3N: Tucson-Melbourne, NNLO…

**Exactly Solve:** **Many-Body Hamiltonian**

e.g. no-core shell model,
Green's function MC method,
(R) Breuckner-Hartree-Fock
Nuclear Lattice EFT…



Nuclear structure

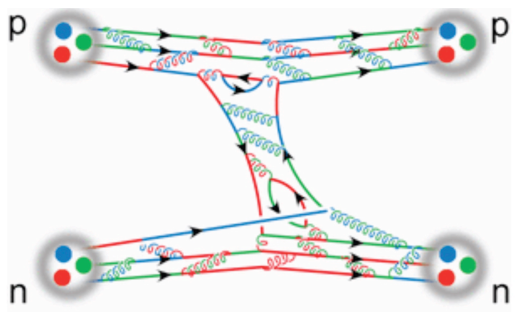


Nuclear reaction

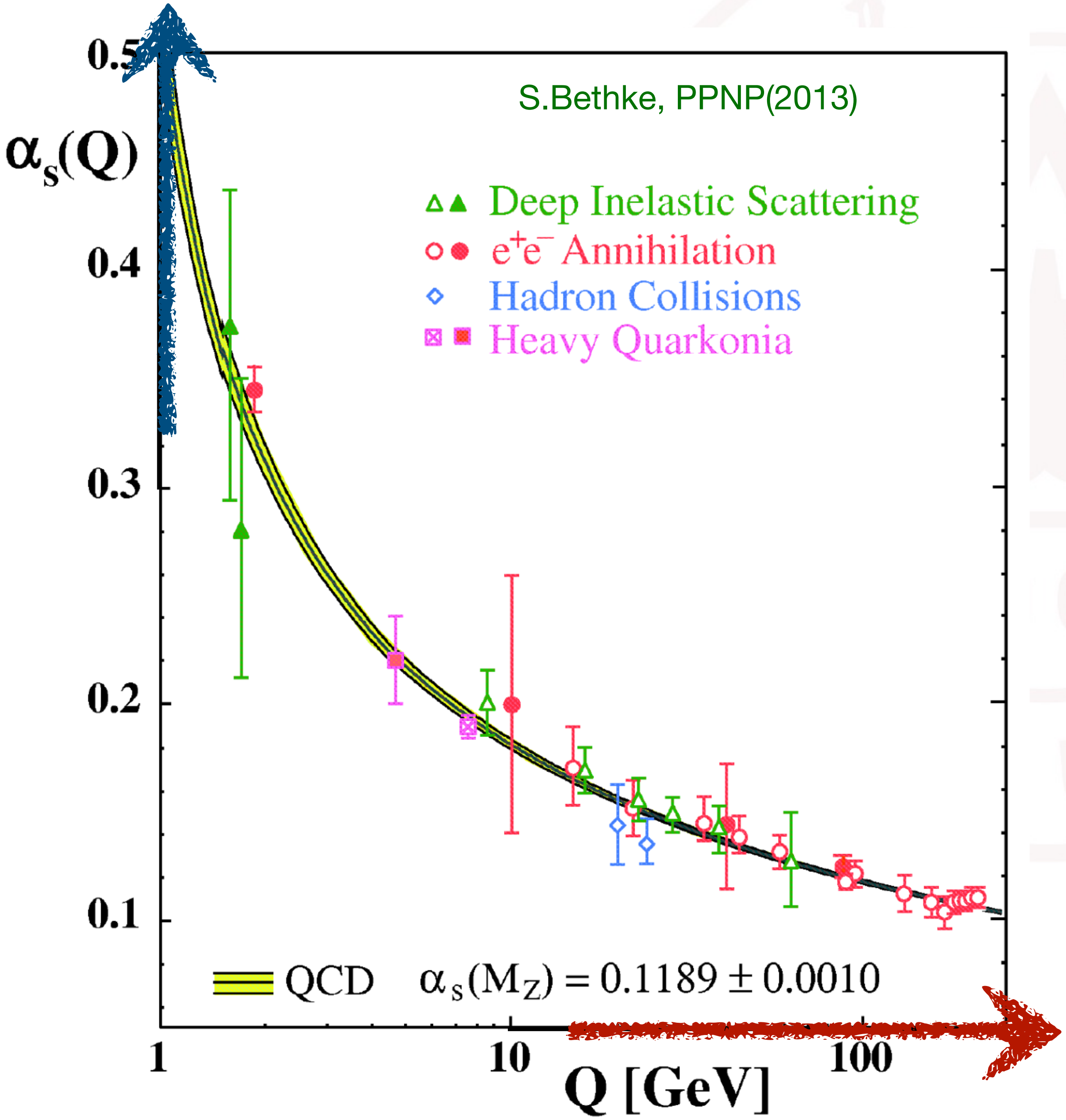
**Detailed understanding of the strong nuclear force is essential!**

# Nuclear force from QCD

- **Residual** quark-gluon strong interaction




- Understood from **Q**uantum **C**hromo-**D**ynamics
  - Fundamental theory for strong interactions
  - In the low-energy region
    - ✓ Running coupling constant
    - ✓ Non-perturbative QCD $\alpha_s > 1$

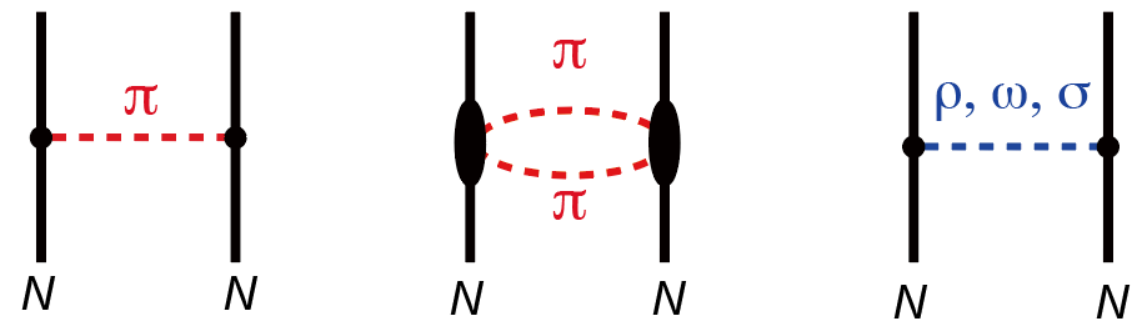


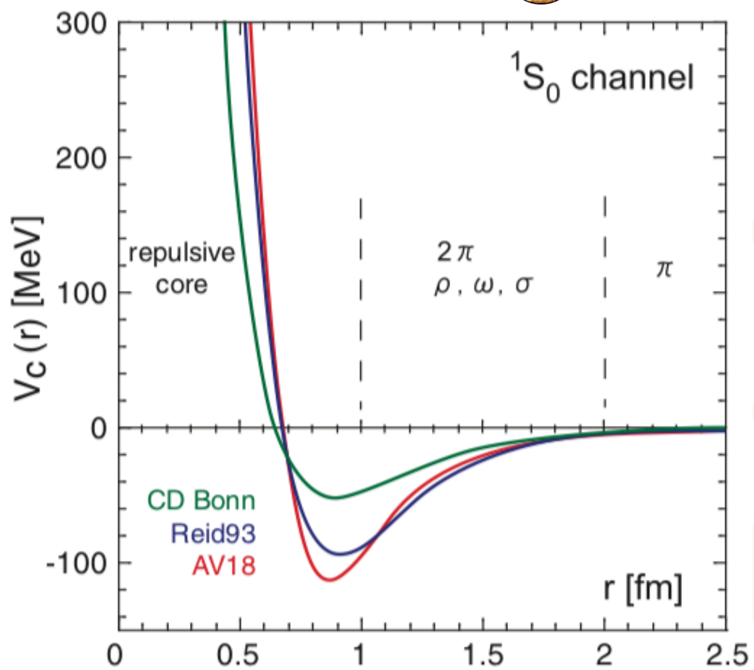

## → Low-energy phenomena

- Phenomenological models
- Lattice QCD simulations
- **Chiral effective field theory**

# Nuclear force studies

- NF from phenomenological models (since 1935, Yukawa OPE )
  - Meson “theory”: CD-Bonn *R. Machleidt,PRC2001*



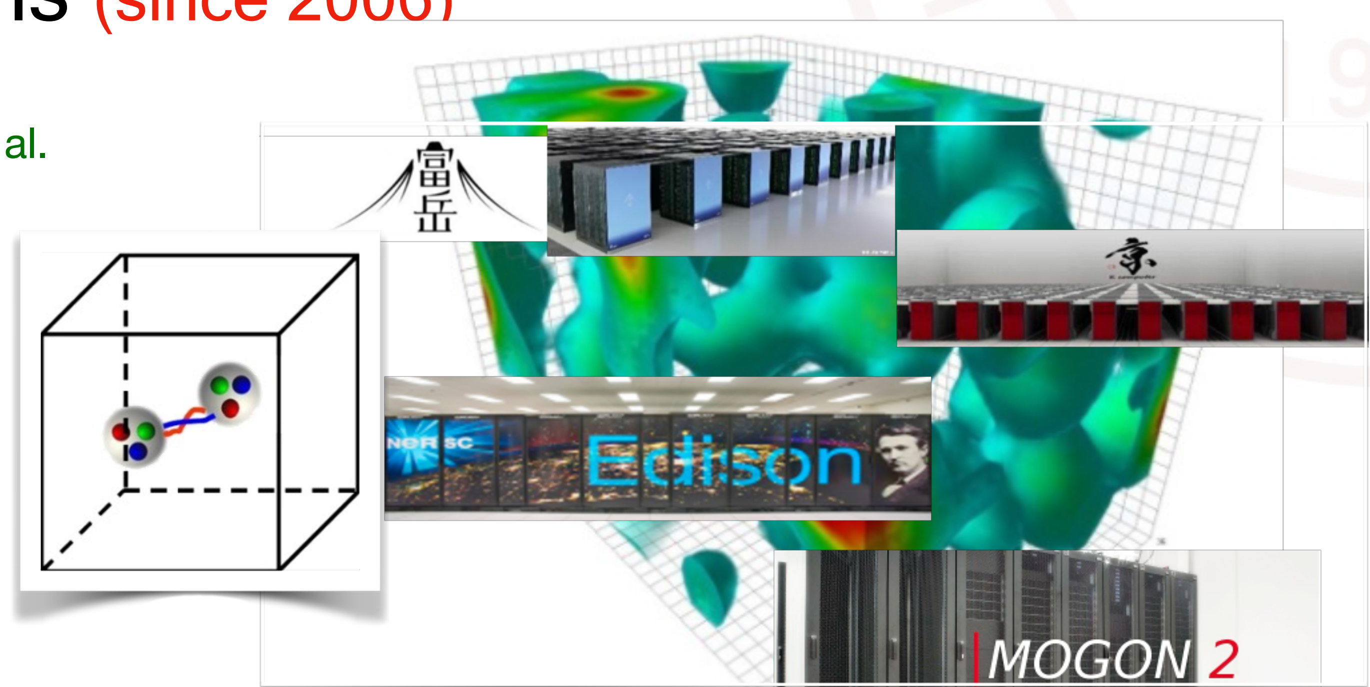




N. Ishii et al. PRL(2007)

  - Operators parameterization: Reid93, *V. Stoks, PRC(1994)*
    AV18, *R. Wiringa,PRC(1994)*

$$V_{NN} = V_c(r)\,\hat{1} + V_\sigma(r)\,\sigma_1\cdot\sigma_2 + V_{LS}(r)\,L\cdot S + V_T(r)\,\sigma_1\cdot q\sigma_2\cdot q + \cdots$$

- NF from lattice QCD simulations (since 2006)
  - HAL QCD coll. T. Hatsuda, S. Aoki, T. Doi et al.
  - NPLQCD coll. S. Beane, M. Savage et al.
    - ✓ CalLat coll. / sLapHnn coll.
    - ✓ T. Yamazaki et al.
  - Mainz coll. H. Wittig, H. Meyer et al.



# Nuclear force — Weinberg's seminal work

Nuclear forces from chiral lagrangians

Steven Weinberg [1]
*Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA*

Received 14 August 1990 PLB251(1990)288-292

EFFECTIVE CHIRAL LAGRANGIANS FOR NUCLEON-PION INTERACTIONS AND NUCLEAR FORCES

Steven WEINBERG*
*Theory Group, Department of Physics, University of Texas, Austin, TX 78712, USA*

Received 2 April 1991 NPB363(1991)3-18

- **Self-consistently** include many-body forces

$$V = V_{2N} + V_{3N} + V_{4N} + \cdots$$

- **Systematically improve** order by order (heavy baryon ChEFT)

$$V_{iN} = V_{iN}^{\rm LO} + V_{iN}^{\rm NLO} + V_{iN}^{\rm NNLO} + \cdots$$

- Scattering amplitude: **Schrödinger / Lippmann-Schwinger Eq.**

$$\left[\left(\sum_{i=1}^{A} -\frac{\nabla_i^2}{2m_N}\right) + V_{2N} + V_{3N} + V_{4N} + \ldots\right] |\Psi\rangle = E|\Psi\rangle$$

□ Provide **a systematic and solid theoretical approach** to study the few-nucleon scattering

# Renormalization issue of chiral force

- Renormalizability: important feature of an EFT

  - Iteration of the chiral NN potential within LSE

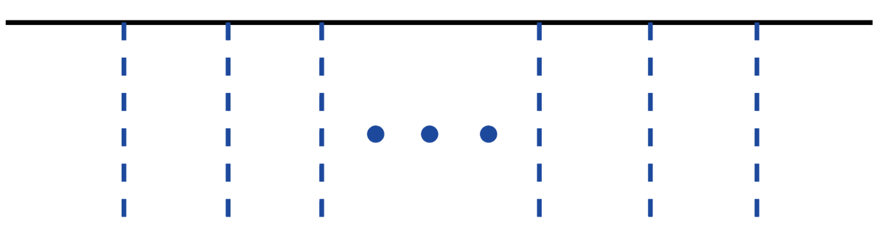
$$T(\boldsymbol{p}',\boldsymbol{p}) = V(\boldsymbol{p}',\boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}',\boldsymbol{k}) \frac{m_N}{\boldsymbol{p}^2-\boldsymbol{k}^2+i\epsilon} T(\boldsymbol{k},\boldsymbol{p})$$

➡ **UV divergencies cannot be absorbed by contact terms!**

- Leading order NN potential

$$V_{\rm LO} = C_S + C_T \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 - \frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2 \frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\,\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{\boldsymbol{q}^2+m_\pi^2}$$

- Iterated one-pion exchange potential (ladder diagrams)



M. Savage, arXiv:nucl-th/9804034

$k \to \infty$ → Spin-triplet

**Logarithmic Divergence**

$$\sim (Q m_N)^n$$

cannot be absorbed by $C_S$, $C_T$

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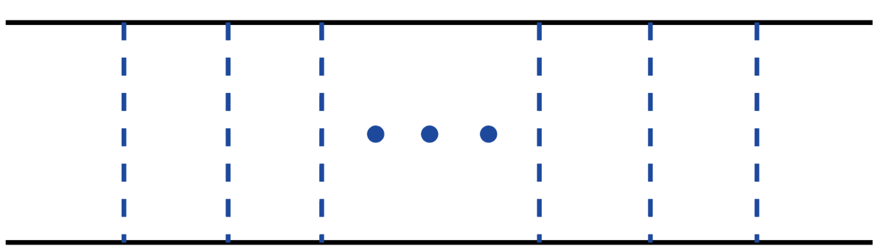

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$k \to \infty$

Spin-triplet

**Logarithmic Divergence**

$\sim (Qm_N)^n$

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**Weinberg's proposal is inconsistent with renormalization, even at LO!**

# Deal with the renormalization issue

## Possible solutions

- **Weinberg power counting**

  ✓ Chiral potential $V = V_{\text{LO}} + V_{\text{NLO}} + \dots$ iterated in LSE

  ✓ Keep finite cutoff lower than hard scale: $\Lambda \leq \Lambda_{\chi PT} \sim 1\,\text{GeV}$

$$T\left(\boldsymbol{p}', \boldsymbol{p}\right) = V\left(\boldsymbol{p}', \boldsymbol{p}\right) + \int^{\Lambda} \frac{d^3k}{(2\pi)^3} V\left(\boldsymbol{p}', \boldsymbol{k}\right) \frac{m_N}{\boldsymbol{p}^2 - \boldsymbol{k}^2 + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p})$$

  ✓ WPC is consistent *G.P. Lepage, nucl-th/9706029; E.Epelbaum, J.Gegelia, Ulf-G. Meißner, NPB925(2017)161*

  - Renormalization achieved only at infinite chiral order
  - Towards a formal proof *A.M. Gasparyan, E. Epelbaum PRC105(2022)024001; 107 (2023) 044002*

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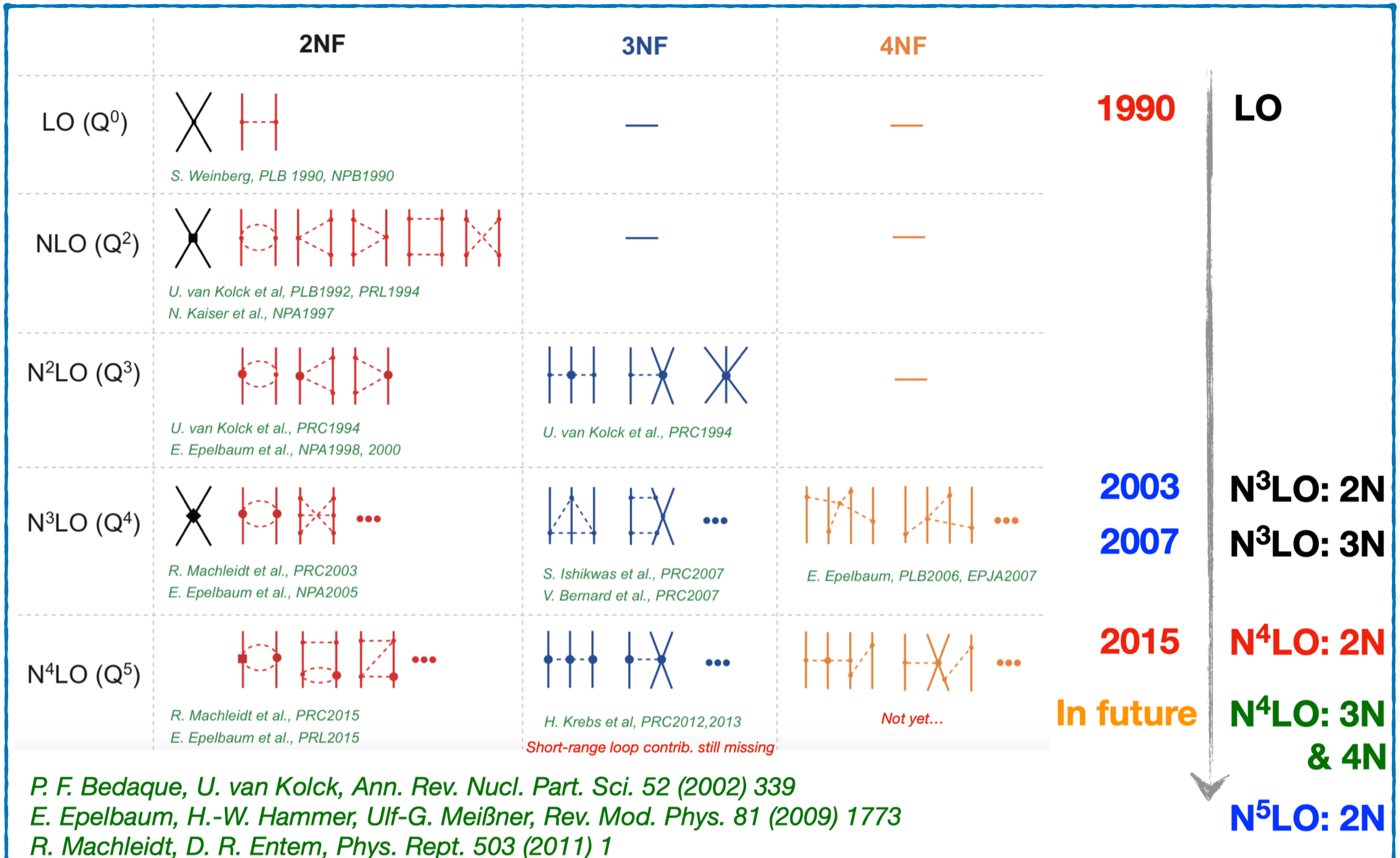

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      - Renormalization achieved only at infinite chiral order
      - Towards a formal proof *A.M. Gasparyan, E. Epelbaum PRC105(2022)024001; 107 (2023) 044002*
    - ✓ High precision chiral nuclear force

| | Phenomenological forces | | | Non-Rel. Chiral nuclear force | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **Reid93** | **AV18** | **CD-Bonn** | LO | NLO | NNLO | **N$^3$LO** | **N$^4$LO$^+$** include 4 ct. in F-waves | |
| No. of para. | **50** | **40** | **38** | 2+2 | 9+2 | 9+2 | 24+2 (3 redundant) | **24+3+4** (3 redundant) | |
| np — $\chi^2$/datum *np 0-300 MeV* | **1.03** | **1.04** | **1.02** | 94 | 36.7 | 5.28 | 1.27 | **1.10** | **Idaho** |
| | | | | | | *D. Entem, et al., PRC96(2017)024004* | | | |
| | | | | 75 | 14 | 4.2 | 2.01 | **1.06** | **Bochum/Juelich** |
| | | | | | | *P. Reinert, et al., EPJA54(2018)86* | | | |

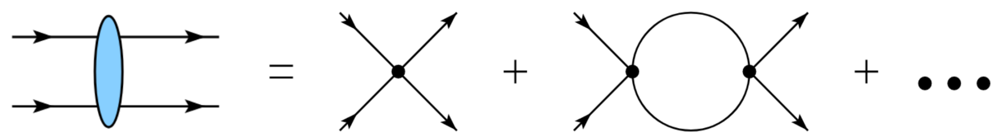
# Deal with the renormalization issue

## Possible solutions

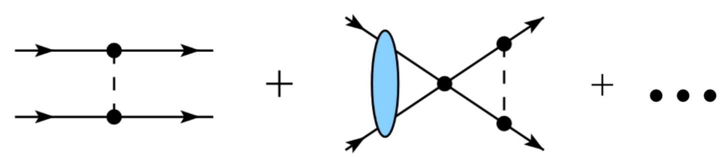
- **Weinberg power counting**
- **Kaplan, Savage, and Wise (KSW) power counting**

✓ Treat the exchange of pions perturbatively *D.B. Kaplan, M.J. Savage, M.B. Wise, PLB424(1998)390*

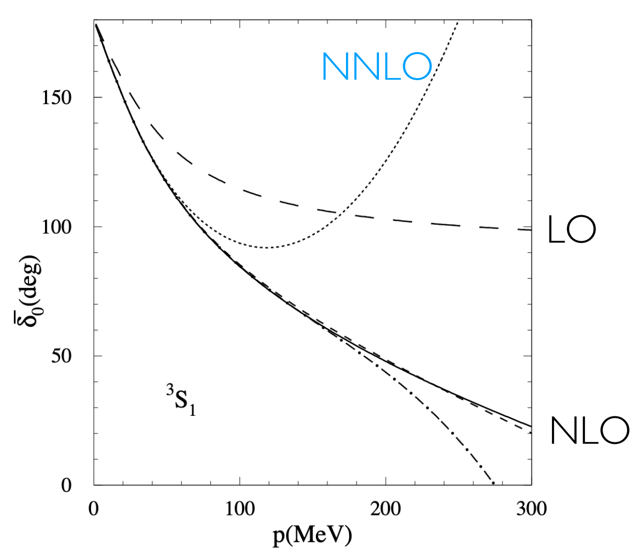
LO: only contacts

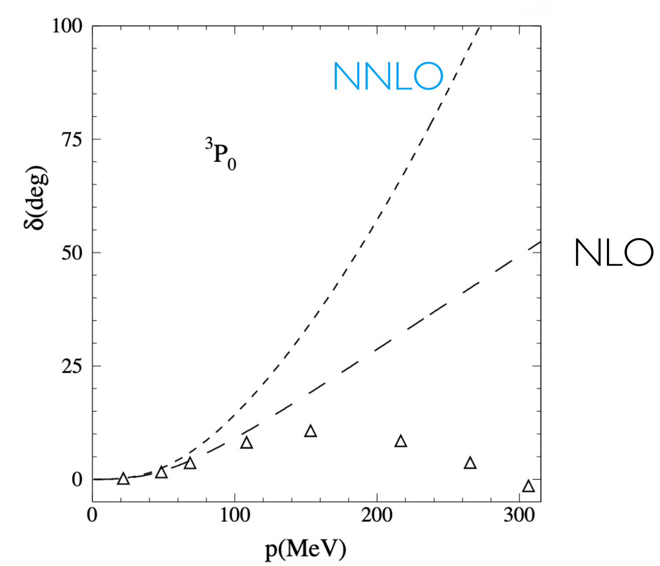


NLO: perturbative OPE



✓ Fail to converge in certain spin-triplet channels *S. Fleming, et al., Nucl.Phys. A677 (2000) 313* *D.B. Kaplan, PRC102(2020)034004*




✓ Perturbative pion scheme with re-organized contacts *Bingwei Long et al. CD2024, in progress*

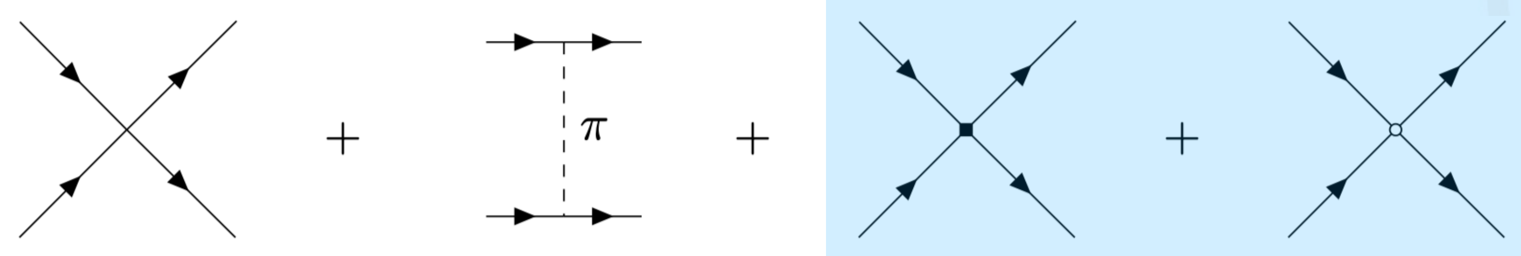
# Deal with the renormalization issue

## Possible solutions

- **Weinberg power counting**
- **Kaplan, Savage, and Wise (KSW) power counting**
- **Modified Weinberg power counting**
  - ✓ Promote the higher order contact terms to the lower chiral order

    *A. Nogga, et al., PRC72(2005)054006  M. C. Birse, PRC74(2006)014003  M. Pavon Valderrama,PRC72(2005) 054002.*
    *B. Long and C.-J. Yang, PRC84(2011)057001 …*
    *H. W. Hammer, S. König, U. van Kolck, Rev. Mod. Phys. 92(2), 025004 (2020)*

  - ✓ Renormalization achieved at every chiral order with $\Lambda \gg \Lambda_{\chi PT}$

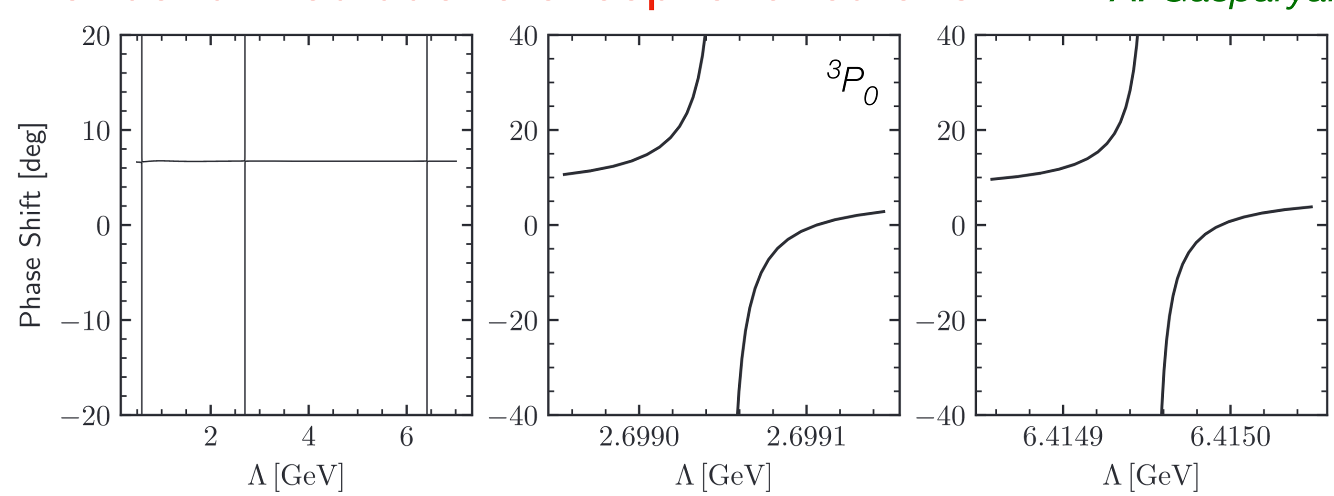
$V_{\rm MWPC}^{[\rm LO]} =$ 



Higher order contacts

  - ▸ Seemingly cannot be fulfilled due to exceptional cutoffs

*A. Gasparyan, E.Epelbaum, PRC107, 034001 (2023)*
*C.-J. Yang, PRC 112, 014004 (2025)*





Elab=130 MeV

# Deal with the renormalization issue

- Possible solutions
  - **Weinberg power counting**
  - **Kaplan, Savage, and Wise (KSW) power counting**
  - **Modified Weinberg power counting**

**Still under debate !!!**

**Nuclear Forces for Precision Nuclear Physics: A Collection of Perspectives**

Few-Body Syst (2022) 63:67

The collection represents the reflections of a vibrant and engaged community of researchers on the status of theoretical research in low-energy nuclear physics, …

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**10 Nuclear Forces in a Manifestly Lorentz-Invariant Formulation of Chiral Effective Field Theory by Xiu-Lei Ren, Evgeny Epelbaum, Jambul Gegelia**

We outline the advantages and disadvantages of manifestly Lorentz-invariant formulation of chiral effective field theory ($\chi$EFT) for the nuclear forces compared to the non-relativistic formalism.

# Chiral forces in Lorentz invariant framework

- Initial idea: modified Weinberg approach E. Epelbaum and J. Gegelia, PLB716(2012)338-344
  - Employ the covariant chiral Lagrangian
  - Apply Weinberg power counting to organize the NN potential

    ✓ Relativistic corrections are perturbatively included

$$V(p', p) = \bar{u}_1 \bar{u}_2 \mathcal{A} u_1 u_2, \quad \text{with} \quad u = u_0 + u_1 + u_2 + \cdots$$

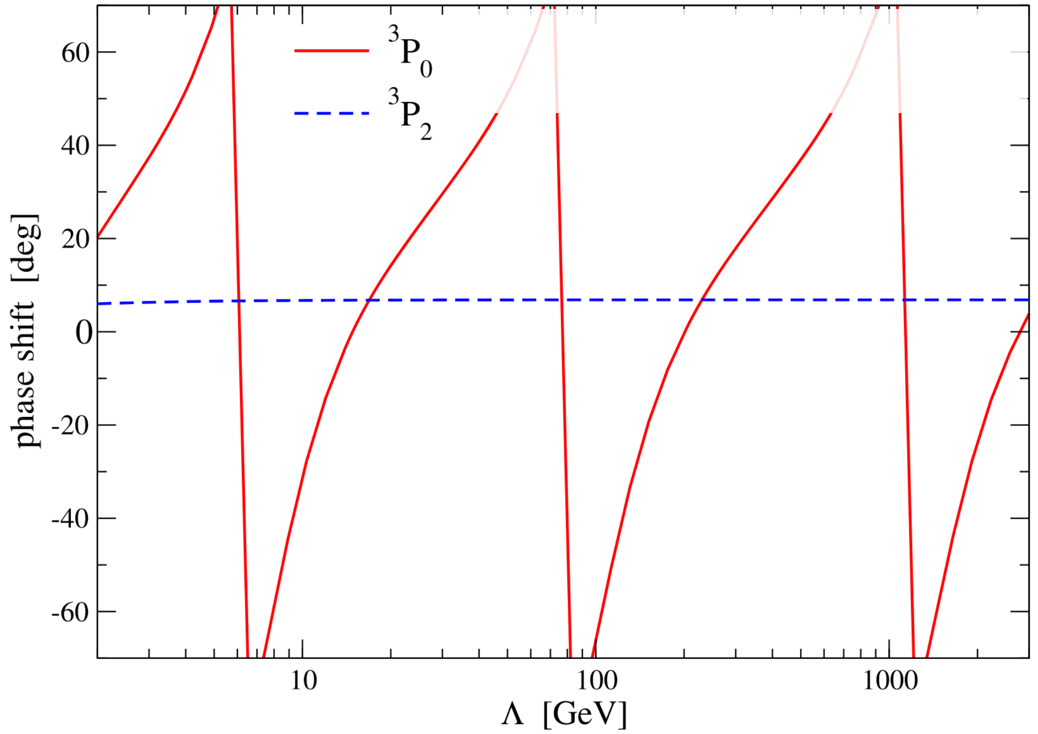
  - Use Kadyshevsky equation to calculate the scattering T-matrix

V. Kadyshevsky, NPB (1968)

$$T(\boldsymbol{p}', \boldsymbol{p}) = V(\boldsymbol{p}', \boldsymbol{p}) + \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p}', \boldsymbol{k}) \frac{m_N^2}{2(\boldsymbol{k}^2 + m_N^2)} \frac{1}{\sqrt{\boldsymbol{p}^2 + m_N^2} - \sqrt{\boldsymbol{k}^2 + m_N^2} + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p})$$

  - **LO study: a renormalizable framework**

    - **No need a finite cutoff** to numerically solve the scattering equation
    - **A good starting point** to investigate the renormalization issue in rel. scheme



# Chiral forces in Lorentz invariant framework

- But, why Kadyshevsky equation for the relativistic NN scattering
  - Bethe-Salpeter equation
    - ✓ 4D form and hard to solve it exactly (open question)
  - Reduction BSE to its 3D forms *R. M. Woloshyn and A. D. Jackson, NPB 64, 269 (1973)*
    - ✓ In principal, there are infinity numbers of 3D forms
    - ✓ Blankenbecler-Sugar eq.; Thompson I, II eqs.; Gross eq.; …
    - ✓ Kadyshevsky eq.; Erkelenz-Holinde eq., …
- **A systematic framework of chiral force should have a unique choice of scattering equation**
  - Non-rel. scheme with the Lippmann-Schwinger equation
  - **Important for the discussion of the renormalization issue**

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  - Non-rel. scheme with the Lippmann-Schwinger equation
  - **Important for the discussion of the renormalization issue**

- We proposed **a systematic framework** within the time-ordered perturbation theory (TOPT) using covariant chiral Lagrangians
  - Formulate the NN interaction up to next-to-next-to-leading order

# Theoretical framework

# Major procedures of chiral NF

① Effective Lagrangians from chiral perturbation theory

② Drive the NF from Lagrangian (power counting, unknown coupling constants)

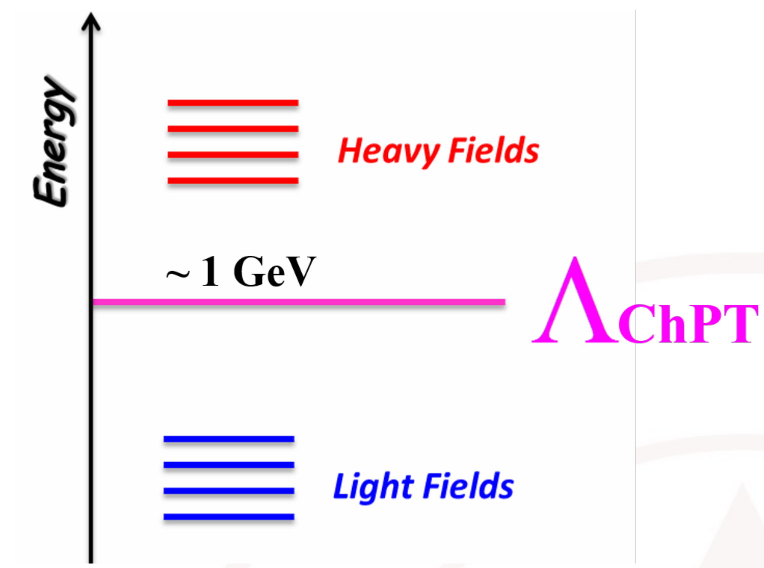
③ Obtain the scattering amplitude by solving the Schrödinger / Lippmann-Schwinger equation (other scattering equations)

④ Describe partial wave phase shifts, scattering data

# Chiral Perturbation Theory

- Effective field theory of low-energy QCD

S. Weinberg, Phys.A 96(1979)327




- Chiral symmetry $SU(2)_L \times SU(2)_R$
- Spontaneous and explicit symmetry breaking

  ⇨ Pseudo-Goldstone bosons (GBs): pion…
- Map u, d quark d.o.f.s to GBs

$$\mathcal{L}_{\mathrm{QCD}}[q,\bar{q};G] \Longrightarrow \mathcal{L}_{\mathrm{ChPT}}[U,\partial U,B,\mathcal{M},\cdots].$$

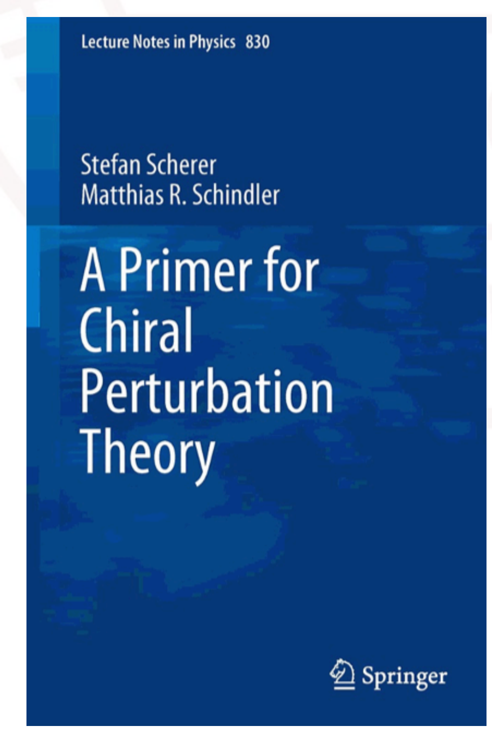
Callan, Coleman, Wess, Zumino, Phys.Rev. 177(1969)177

- Expand scattering amplitude in powers of Q

$$\mathbf{Q} = \frac{\text{momentum of pions and nucleons or } M_\pi \sim 140 \text{ MeV}}{\text{hard scales } [\Lambda_{\mathrm{ChPT}} = 4\pi F_\pi \sim 1\mathrm{GeV}]}$$

- (Chiral) Effective Lagrangian

$$\begin{aligned}\mathcal{L}_{\mathrm{eff}} &= \mathcal{L}_\pi + \mathcal{L}_{\pi N} + \mathcal{L}_{NN} \\ &= \frac{F_0^2}{4}\langle u_\mu u^\mu + \chi_+\rangle + \cdots \\ &+ \bar{N}\left(i\gamma_\mu D^\mu - m\right)N + \frac{1}{2}\bar{N}(g_A\gamma_\mu\gamma_5 u^\mu)N + \cdots \\ &+ \frac{1}{2}C_S(\bar{N}N)(\bar{N}N) + C_A(\bar{N}\gamma_5 N)(\bar{N}\gamma_5 N) + \cdots\end{aligned}$$



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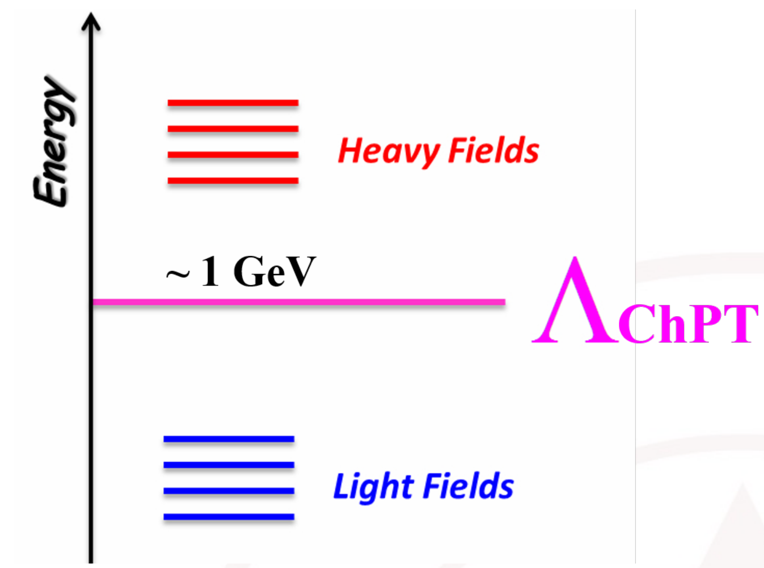


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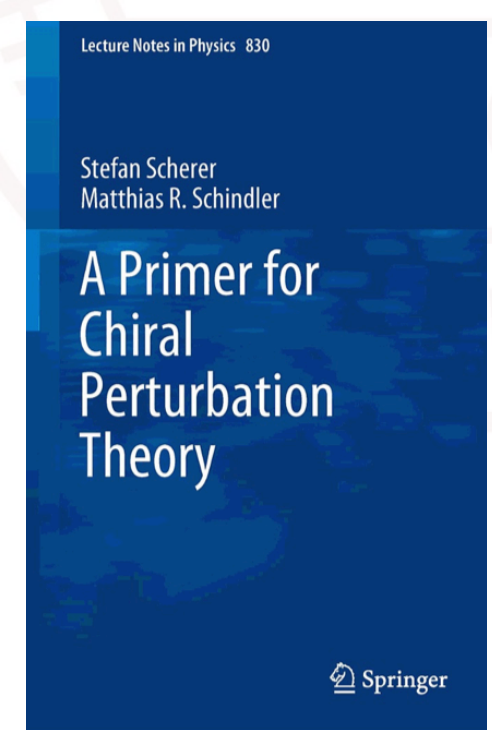
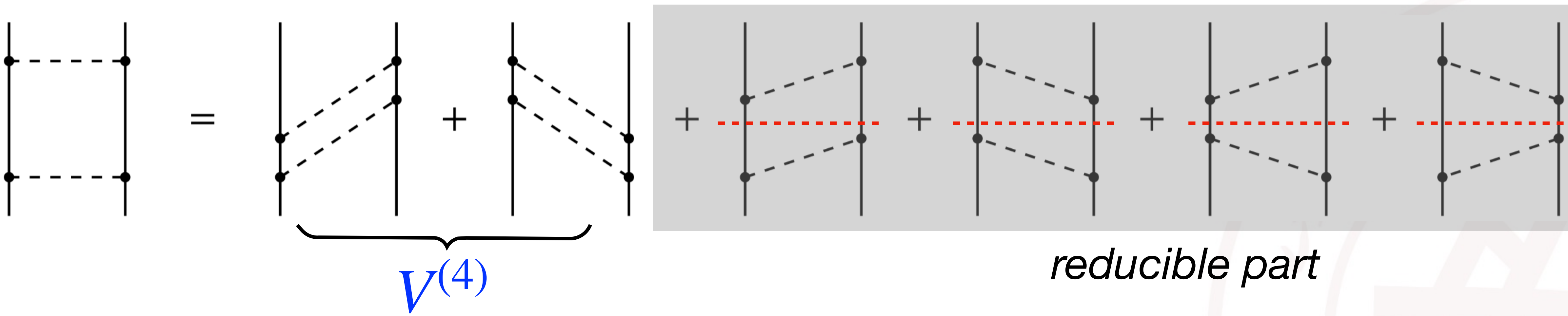


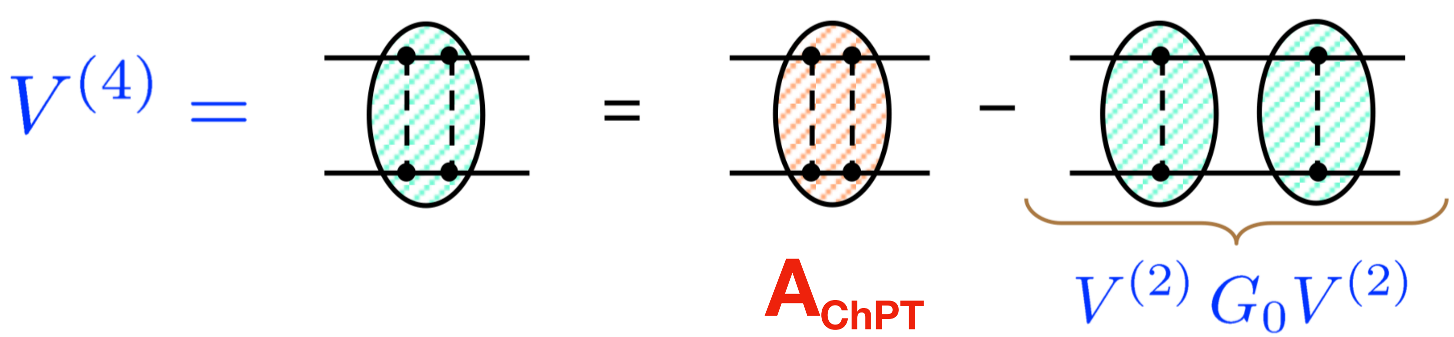
# How to obtain chiral forces?

## Nuclear force from Chiral Lagrangians

- Irreducible time-ordered diagrams *Weinberg '90; van Kolck et al. '93, ...*
  - ✓ Box diagram, two pion exchange contribution:




- Matching to the amplitude (perturbatively) *Kaiser '97, Machleidt, '03 ...*




- Decouple pion states via a suitable unitary transition in the Fock space *Epelbaum, Glockle, Meissner, '98*

**Lead to the same results**

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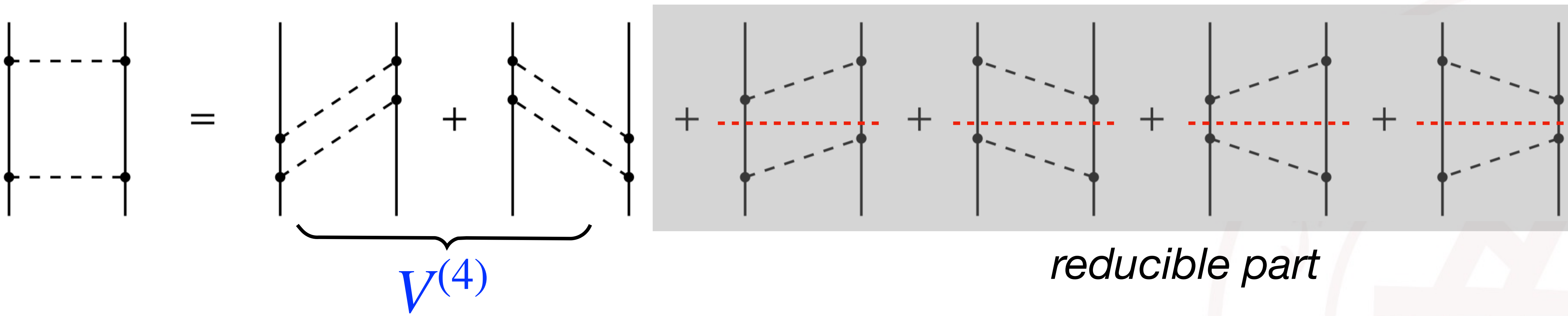


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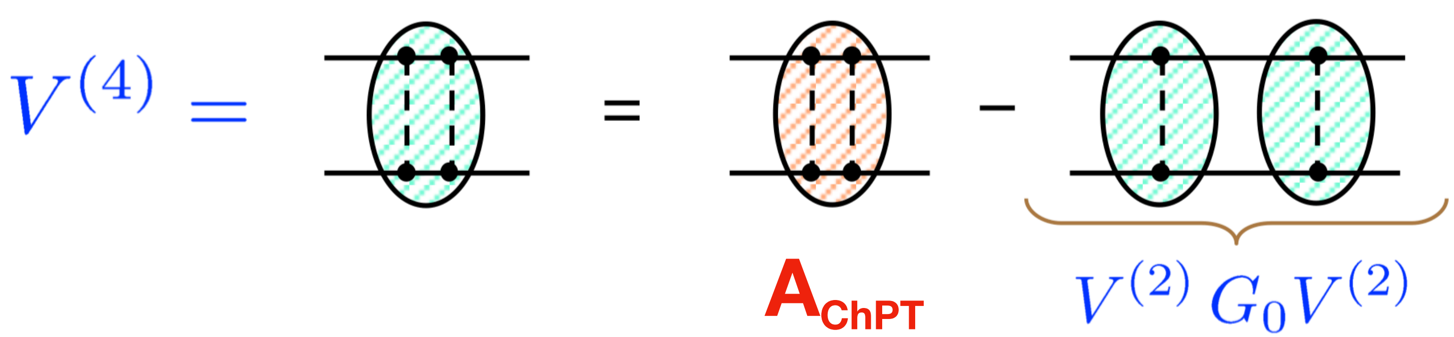
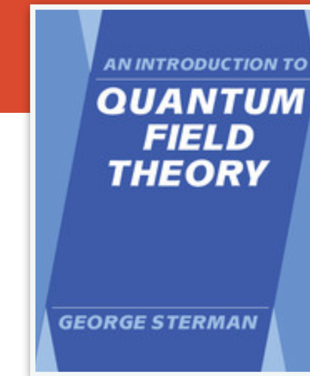



- Decouple pion states via a suitable unitary transition in the Fock space *Epelbaum, Glockle, Meißner, '98*

➡ **Lead to the same results**

# TOPT with covariant Lagrangian



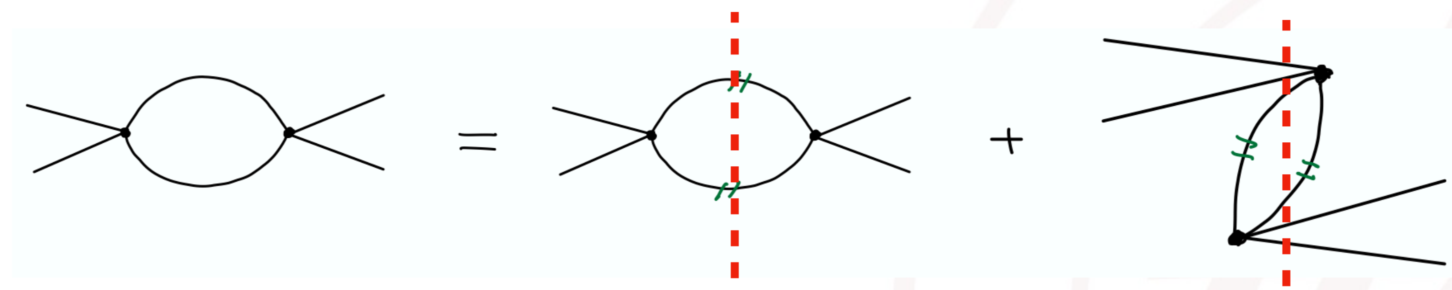

## Time-ordered perturbation theory (TOPT)

- Definition

*S. Weinberg, Phys.Rev.150(1966)1313*
*G.F. Sterman, "An introduction to quantum field theory", Cambridge (1993)*

✓ Re-express the Feynman integral in a form that makes the connection with on-mass-shell (off-energy shell) state explicit. This form is called **TOPT or old-fashioned PT**

✓ (In short) Instead the propagators for internal lines as the energy denominators for intermediate states



- Advantages

✓ Explicitly show the unitarity

✓ One-to-one relation between internal lines and intermediate states

✓ Easily to tell the contributions of a particular diagram

- Derive the rules for time-ordered diagrams

✓ Perform Feynman integrations over the zeroth components of the loop momenta

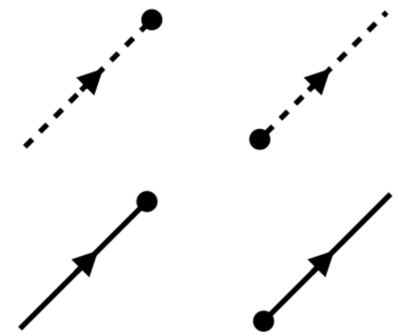
✓ Decompose Feynman diagram into sums of time-ordered diagrams

✓ Match to the rules of time-ordered diagrams

# Diagrammatic rules in TOPT

XLR, PoS(CD2021)007

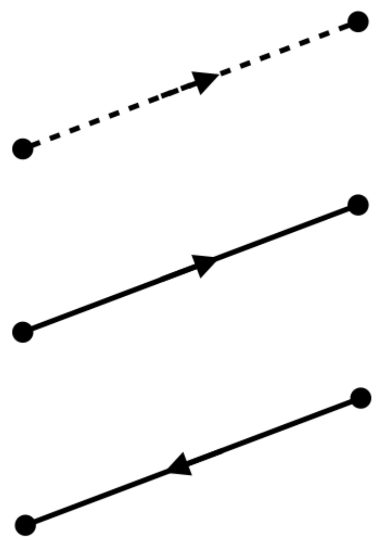
- **External lines**



Spin 0 boson (in, out) — **1**

Spin 1/2 fermion (in, out) — $u(\mathbf{p}), \quad \bar{u}(\mathbf{p}')$

- **Internal lines**



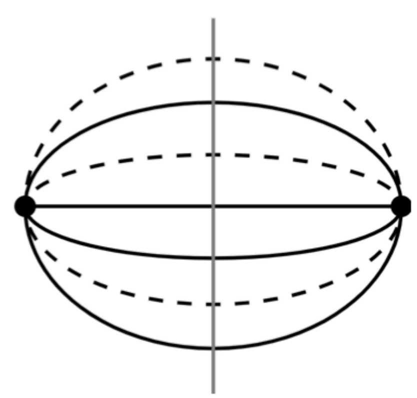
Spin 0 (anti-)boson — $\dfrac{1}{2\,\epsilon_q}$ $\qquad \epsilon_q \equiv \sqrt{\mathbf{q}^2 + M^2}$

Spin 1/2 fermion — $\dfrac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p})$ $\qquad \omega_p \equiv \sqrt{\mathbf{p}^2 + m^2}$

anti-fermion — $\dfrac{m}{\omega_p} \sum u(\mathbf{p})\bar{u}(\mathbf{p}) - \gamma_0$

- **Intermediate state**

A set of lines between two vertices



$$\frac{1}{E - \sum_i \omega_{p_i} - \sum_j \epsilon_{q_j} + i\epsilon}$$

- **Interaction vertices:** the standard Feynman rules
  - Zeroth components of integration momenta

✓ particle $\quad p^0 \to \omega(p, m)$

✓ antiparticle $\quad p^0 \to -\omega(p, m)$

# Chiral potential in TOPT

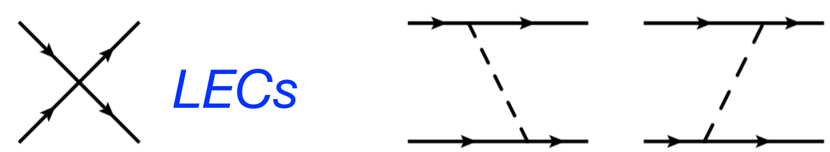
## Chiral potential V

- **Define:** sum up the two-nucleon **irreducible time-ordered diagrams**
- **Power counting: systematic ordering of all graphs**
  - ✓ Employ the Weinberg power counting to perturbatively calculate potential

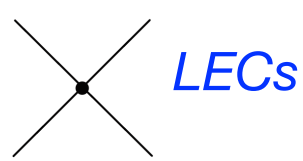
$$V_{\rm eff} = \sum_{\nu} (Q/\Lambda_\chi)^\nu \mathcal{V}_\nu$$

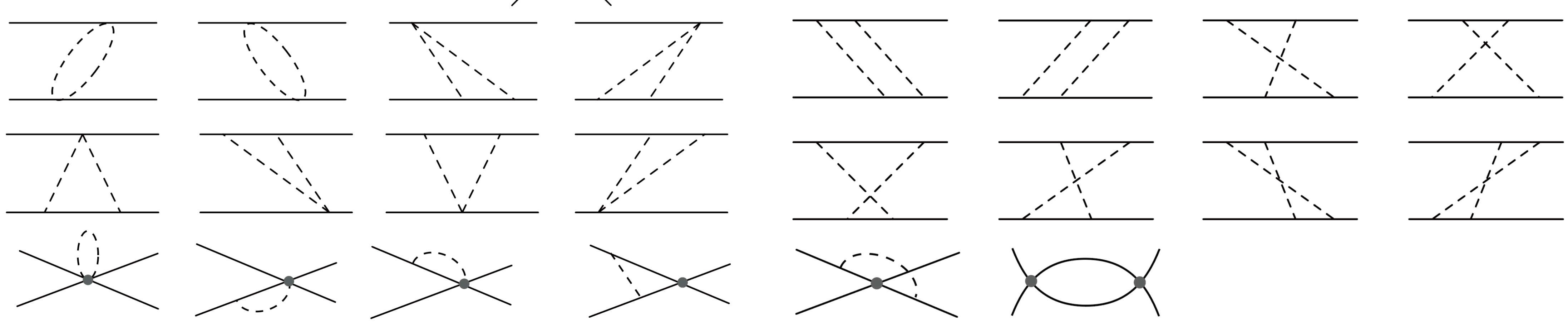
$$\nu = 2 - \frac{1}{2}N + 2L + \sum_i v_i \Delta_i, \quad \Delta_i = d_i + \frac{1}{2}n_i - 2$$

- Leading Order:



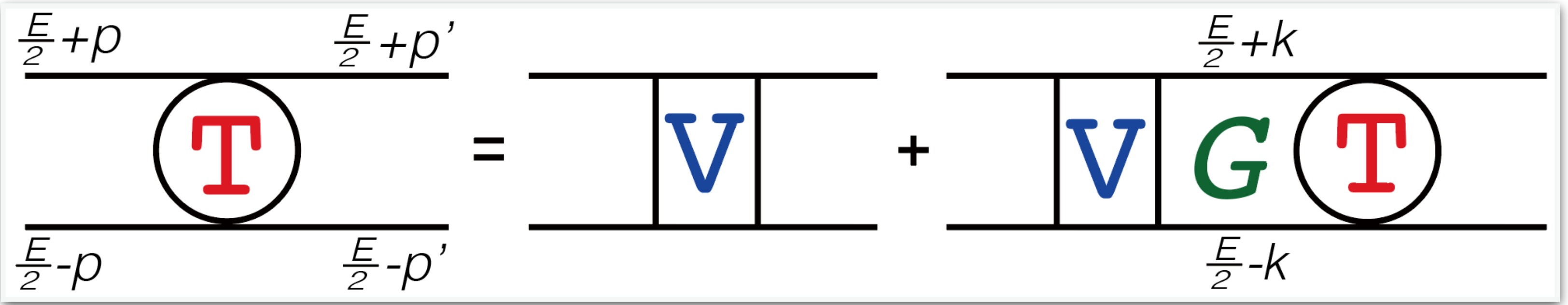

- Next-to-Leading Order:



# Scattering equation in TOPT

- Scattering amplitude T (non-perturbative)




- **Non.-Rel.: Lippmann-Schwinger equation**

$$T\left(\boldsymbol{p}', \boldsymbol{p}\right) = V\left(\boldsymbol{p}', \boldsymbol{p}\right) + \int \frac{d^3k}{(2\pi)^3} V\left(\boldsymbol{p}', \boldsymbol{k}\right) \frac{m_N}{\boldsymbol{p}^2 - \boldsymbol{k}^2 + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p})$$

- **Rel.: Kadyshevsky equation** (SELF-CONSISTENTLY obtained in TOPT)
  - ✓ Two-body Green functions *G*:

$$G_{ij}(E) = \frac{1}{E(k, m_i)\, E(k, m_j)} \frac{m_i m_j}{E - E(k, m_i) - E(k, m_j) + i\epsilon}$$

  - ✓ Kady. equation for NN scattering *V. Kadyshevsky, NPB (1968)*

$$T\left(\boldsymbol{p}', \boldsymbol{p}\right) = V\left(\boldsymbol{p}', \boldsymbol{p}\right) + \int \frac{d^3k}{(2\pi)^3} V\left(\boldsymbol{p}', \boldsymbol{k}\right) \frac{m_N^2}{2E_k^2} \frac{1}{E_p - E_k + i\epsilon} T(\boldsymbol{k}, \boldsymbol{p})$$

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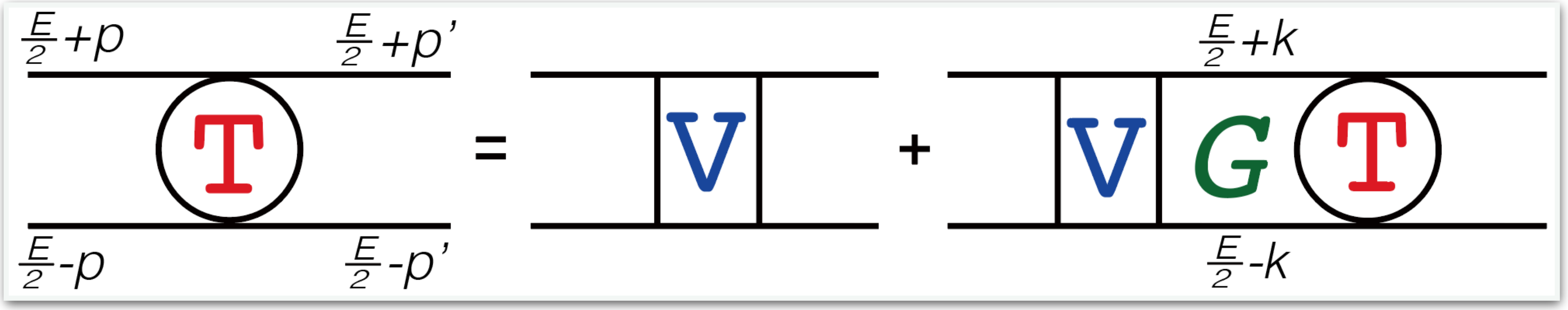


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# Partial wave phase shifts

- Perform the p.w. decomposition of potential

$$\langle p'|V_{\rm LO}|p\rangle \xrightarrow[\text{conservation of total spin}]{\text{rotation invariant}} \langle L'SJ|V_{\rm LO}|LSJ\rangle$$

- e.g. p.w. channels for NN scattering

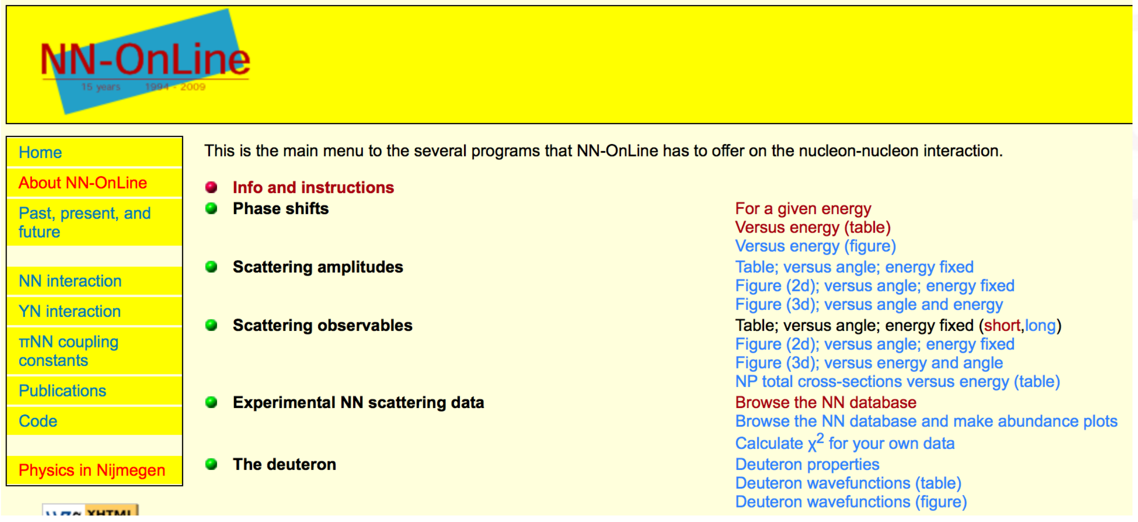
| $^{2S+1}L_J$ | $S=0$ | $S=1$ |
|---|---|---|
| $J=0$ | $^1S_0$ | $^3P_0$ |
| $J=1$ | $^1P_1$ | $^3P_1$, $^3D_1$, $^3S_1$, $^3D_1$-$^3S_1$ |
| $J=2$ | $^1D_2$ | $^3D_2$, $^3F_2$, $^3P_2$, $^3F_2$-$^3P_2$ |
| …… | …… | …… |

- p.w. phase shifts
  - ✓ Solve Kadyshevsky equation in LSJ-basis
  - ✓ Obtain the p.w. scattering T-, S-matrix
  - ✓ e.g. Single channel phase shifts

$$S^{0J}_{JJ}=\exp(2i\delta^{0J}_J)$$

*H. P. Stapp, et al., Phys. Rev., 105: 302 (1957)*

http://nn-online.org



# Partial wave phase shifts

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| $J=2$ | $^1D_2$ | $^3D_2$, $^3F_2$, $^3P_2$, $^3F_2$-$^3P_2$ |
| …… | …… | …… |

## ④ p.w. phase shifts

✓ Solve Kadyshevsky equation in LSJ-basis

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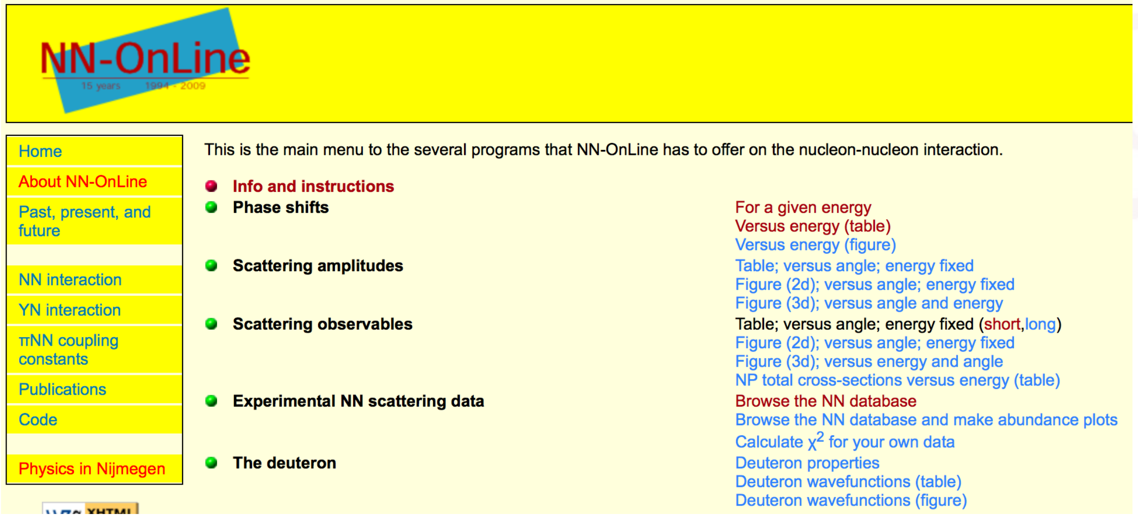

# Major procedures of chiral NF

① Effective Lagrangians from chiral perturbation theory

- Lorentz invariant chiral Lagrangian

② Drive the NF from Lagrangian

- Irreducible time-ordered diagrams
- Weinberg power counting

③ Obtain the scattering amplitude by solving the Kadyshevsky equation

- **Self-consistently** obtained in TOPT

④ Describe partial wave phase shifts, scattering data

# Related tutorials & lecture notes

— general: **EE, Nuclear forces from chiral EFT: A primer, arXiv:1001.3229**

— renormalization:

**Lepage, How to renormalize the Schrödinger equation, nucl/th:9706029**

— RG analysis:

**Birse, The renormalization group and nuclear forces, Phil. Trans. Roy. Soc. Lond. A369 (2011) 2662**

— Uncertainty quantification:

**Weselowski et al., Bayesian parameter estimation for effective field theories, J.Phys. G43 (2016) 074001**

**Grießhammer, Assessing Theory Uncertainties in EFT Power Countings from Residual Cutoff Dependence, arXiv:1511.00490**

Slides from E. Epelbaum


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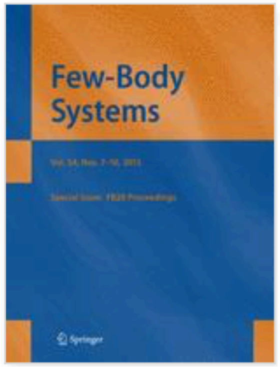
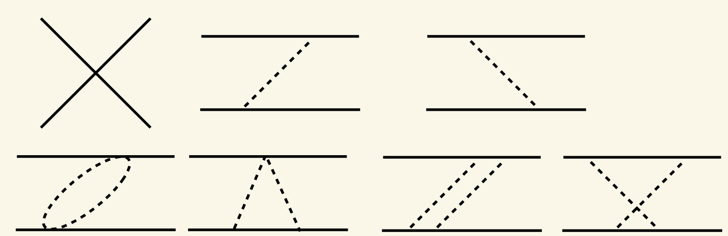
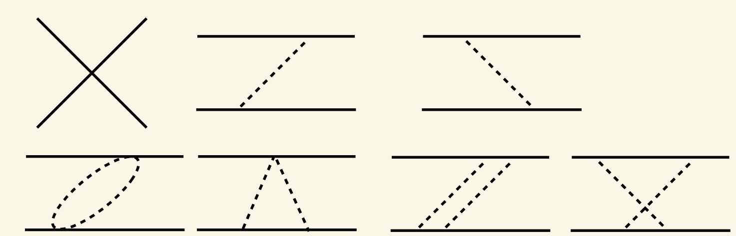


# Short summary for theoretical framework

- Follow the standard procedure of formulating chiral forces in time-ordered perturbation theory

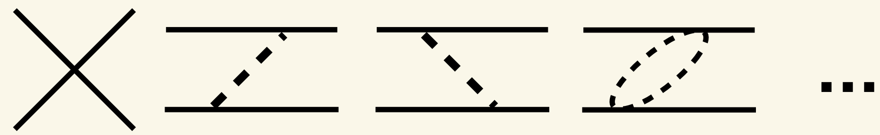
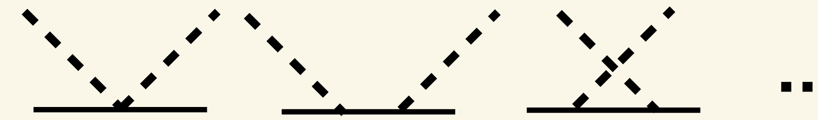
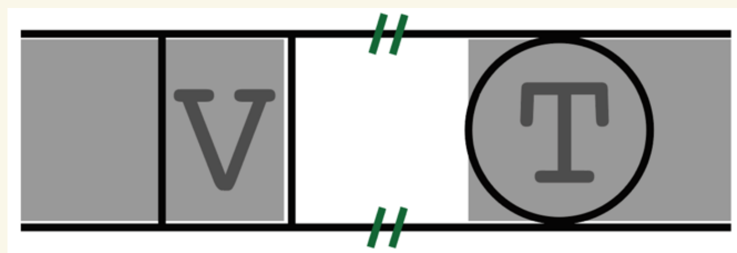
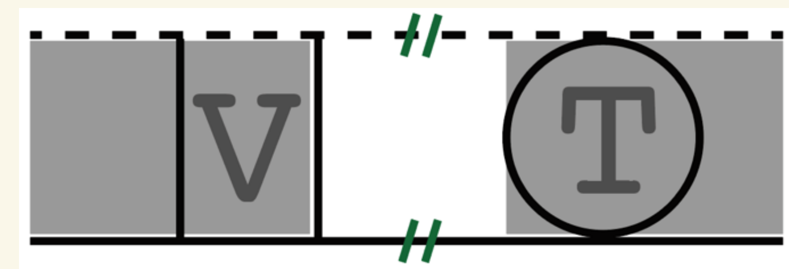
S. Weinberg, PLB1990, NPB1991;
C. Ordóñez, U. van Kolck PLB(1992)
…

| | **Non-relativistic** (Heavy-baryon) | **Lorentz invariant** |
|---|---|---|
| Chiral Lagrangians | $N^\dagger\left[i(v\cdot D)+g_A(S\cdot u)\right]N$ <br> $-\frac{1}{2}C_S\left(N^\dagger N\right)\left(N^\dagger N\right)-\frac{1}{2}C_T\left(N^\dagger\vec{\sigma}N\right)\left(N^\dagger\vec{\sigma}N\right)+\cdots$ | $\bar{\Psi}_N\left\{i\gamma_\mu D^\mu-m_N+\frac{1}{2}g_A\psi\gamma^5\right\}\Psi_N$ <br> $+\frac{1}{2}\Big[C_S(\bar{\Psi}_N\Psi_N)(\bar{\Psi}_N\Psi_N)+C_A\left(\bar{\Psi}_N\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma_5\Psi_N\right)$ <br> $+C_V\left(\bar{\Psi}_N\gamma_\mu\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\Psi_N\right)+C_{AV}\left(\bar{\Psi}_N\gamma_\mu\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\gamma_5\Psi_N\right)$ <br> $+C_T\left(\bar{\Psi}_N\sigma_{\mu\nu}\Psi_N\right)\left(\bar{\Psi}_N\sigma^{\mu\nu}\Psi_N\right)\Big]+\ldots$ |
| Potential TOPT diagrams |  |  |
| Scattering equations (T = V + VGT) | Lippmann-Schwinger eq. | Kadyshevsky eq. |
| Power counting | Weinberg p.c. | Weinberg p.c. |

$1/m_N$

XLR, PoS(CD2021)007

# Extend to BB and MB scatterings

| | Baryon-baryon scattering | Meson-baryon scattering |
|---|---|---|
| Potential<br>TOPT diagrams |  |  |
| Green function | <br>$G^{BB}_{ij}(E) = \frac{m_i m_j}{\omega_{m_i}\omega_{m_j}} \frac{1}{E - \omega_{m_i} - \omega_{m_j} + i\epsilon}$ | <br>$G^{MB}(E) = \frac{m}{2\omega_M \omega_m} \frac{1}{E - \omega_M - \omega_m + i\epsilon}$ |



- **Unify the description of SU(3) baryon-baryon and meson-baryon scatterings within our TOPT framework**
  - $S = -1$ baryon-baryon interaction at LO

    XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 101, 034001 (2020)
  - $S = -1$ meson-baryon interaction at LO and NLO / $\Lambda(1405)$

    XLR, E. Epelbaum, J. Gegelia and U.-G. Meißner, EPJC 80 (2020) 406; 81 (2021) 582;
    XLR, Phys. Lett. B 855,138802 (2024)
    XLR et al., work in progress

# Results and discussion

# Chiral Lagrangian up to NNLO

## Lorentz-invariant effective Lagrangians

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{\pi\pi}^{(2)} + \mathcal{L}_{\pi N}^{(1)} + \mathcal{L}_{\pi N}^{(2)} + \mathcal{L}_{NN}^{(0)} + \mathcal{L}_{NN}^{(2)}.$$

- Purely pionic sector *J.Gasser, H. Leutwyler, Ann.Phys.(1984)*

$$\mathcal{L}_{\pi\pi}^{(2)} = \frac{f_\pi^2}{4}\langle u_\mu u^\mu + \chi_+\rangle.$$

- One-nucleon sector *J. Gasser, M. E. Sainio, and A. Svarc, NPB(1988)*

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left\{ i\not{D} - m_N + \frac{1}{2} g_A \not{u} \gamma^5 \right\} \Psi_N$$

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N \left\{ c_1 \langle \chi + \rangle - \frac{c_2}{4m_N^2} \langle u^\mu u^\nu \rangle \left( D_\mu D_\nu + \text{ h.c. } \right) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu \left[ u_\mu, u_\nu \right] \right\} \Psi_N$$

✓ $f_\pi = 92.4$ MeV, $g_A = 1.267$, $c_{1,2,3,4}$ determined by $\pi N$ scattering data

- Two-nucleon sector (with unknown LECs) *N.Fettes, U.-G. Meißner, S. Steininger, NPA(1998)*

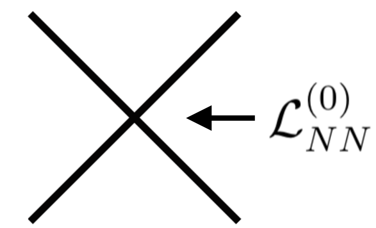
$$\mathcal{L}_{NN}^{(0)} = \frac{1}{2}\left[ C_S (\bar{\Psi}_N \Psi_N)(\bar{\Psi}_N \Psi_N) + C_A \left(\bar{\Psi}_N \gamma_5 \Psi_N\right)\left(\bar{\Psi}_N \gamma_5 \Psi_N\right) + C_V \left(\bar{\Psi}_N \gamma_\mu \Psi_N\right)\left(\bar{\Psi}_N \gamma^\mu \Psi_N\right) \right.$$
$$\left. + C_{AV} \left(\bar{\Psi}_N \gamma_\mu \gamma_5 \Psi_N\right)\left(\bar{\Psi}_N \gamma^\mu \gamma_5 \Psi_N\right) + C_T \left(\bar{\Psi}_N \sigma_{\mu\nu} \Psi_N\right)\left(\bar{\Psi}_N \sigma^{\mu\nu} \Psi_N\right) \right]$$

$$\mathcal{L}_{NN}^{(2)} = \sum_{i=1} \bar{\Psi}_N \bar{\Psi}_N \mathcal{O}_i \Psi_N \Psi_N$$

*L.Girlanda, S. Pastore, R. Schiavilla, M. Viviani, PRC(2010)*
*Yang Xiao, Li-Sheng Geng, XLR, PRC(2019)*
*E. Filandri, L. Girlanda, PLB (2023)*

# Leading order potential

- Contact nucleon-nucleon interaction




  - According to our TOPT rules

$$V_{0,C} = C_S(\bar{u}_3\, u_1)(\bar{u}_4\, u_2) + C_A(\bar{u}_3\, \gamma_5\, u_1)(\bar{u}_4 \gamma_5 u_2) + C_V(\bar{u}_3\, \gamma_\mu\, u_1)(\bar{u}_4\, \gamma^\mu\, u_2)$$
$$+ C_{AV}(\bar{u}_3\, \gamma_\mu \gamma_5\, u_1)(\bar{u}_4\, \gamma^\mu \gamma_5\, u_2) + C_T(\bar{u}_3\, \sigma_{\mu\nu}\, u_1)(\bar{u}_4\, \sigma^{\mu\nu}\, u_3)$$

  - Contain higher order contributions according to Weinberg P.C.
  - Perform the expansion for the nucleon energies

$$\sqrt{\omega(p, m_N) + m_N} = \sqrt{2m_N} + \mathcal{O}(p^2)$$

$$V_{LO,C} = (C_S + C_V) - (C_{AV} - 2C_T)\, \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2$$

✓ Two independent parameters to be fixed

✓ Consistent with the non-relativistic contact terms

*S. Weinberg, PLB251(1990)288-292*

# Leading order potential

- One-pion-exchange (OPE) potential

  - According to our TOPT rules

$$V_{0,\mathrm{OPE}} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{1}{2\omega(q,M_\pi)}\left[\frac{(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1)(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2)}{\omega(p,m_N)+\omega(p',m_N)+\omega(q,M_\pi)-E-i\epsilon}\right.$$
$$\left.+\frac{(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1)(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2)}{\omega(p,m_N)+\omega(p',m_N)+\omega(q,M_\pi)-E-i\epsilon}\right]$$

  - Contains higher order contributions according to Weinberg P.C.
  - Perform the expansion for the nucleon energies in numerator

$$\sqrt{\omega(p,m_N)+m_N} = \sqrt{2m_N} + \mathcal{O}(p^2)$$

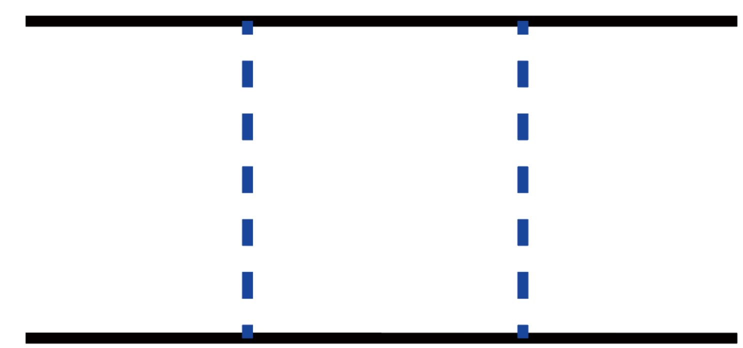
$$V_{\mathrm{OPE}} = -\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{4m_N^2}{\omega(q,M_\pi)\,(m_N+\omega(p,m_N))\,(m_N+\omega(p',m_N))}$$
$$\times\frac{\boldsymbol{\sigma}_1\cdot\boldsymbol{q}\,\boldsymbol{\sigma}_2\cdot\boldsymbol{q}}{\omega(p,m_N)+\omega(p',m_N)+\omega(q,M_\pi)-E-i\epsilon}$$

  - Milder UV behaviour than that of the non-relativisitc OPEP

$$V_{\mathrm{OPE}}(p',k)\ \xrightarrow{k\longrightarrow\infty}\ \text{Our}\ \frac{1}{k}\quad \text{vs.}\quad \text{Non-Rel.}\ \frac{1}{1}$$

# UV Behavior of the long-range potential

- One-loop integral $V\,G\,V$:



$$I_{VGV} = \int \frac{d^3k}{(2\pi)^3} V_{\rm OPE}\, G(E)\, V_{\rm OPE}$$

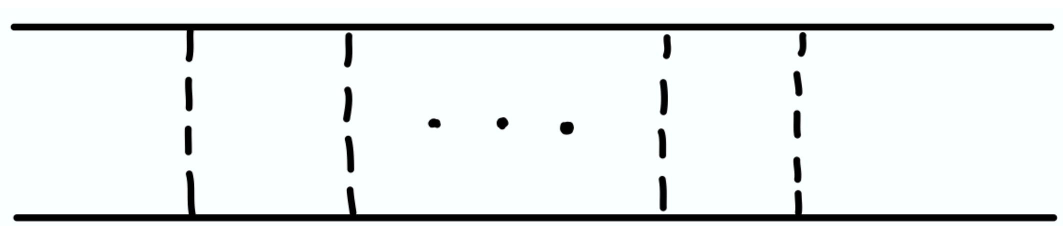
$$\xrightarrow{k\to\infty}$$

$$\begin{cases} \text{Our:} \quad I_{VGV}^{\rm Our} \to \int dk^3 \frac{1}{k}\frac{1}{k^3}\frac{1}{k} = \int dk^3 \frac{1}{k^5} \\ \text{NR:} \quad I_{VGV}^{\rm NR} \to \int dk^3\, 1\, \frac{1}{k^2}\, 1 = \int dk^3 \frac{1}{k^2} \end{cases}$$

**Ultraviolet convergent!**

**Ultraviolet divergent!**

- Iteration of our OPEP



$$\xrightarrow{k\to\infty}$$ Finite diagram!

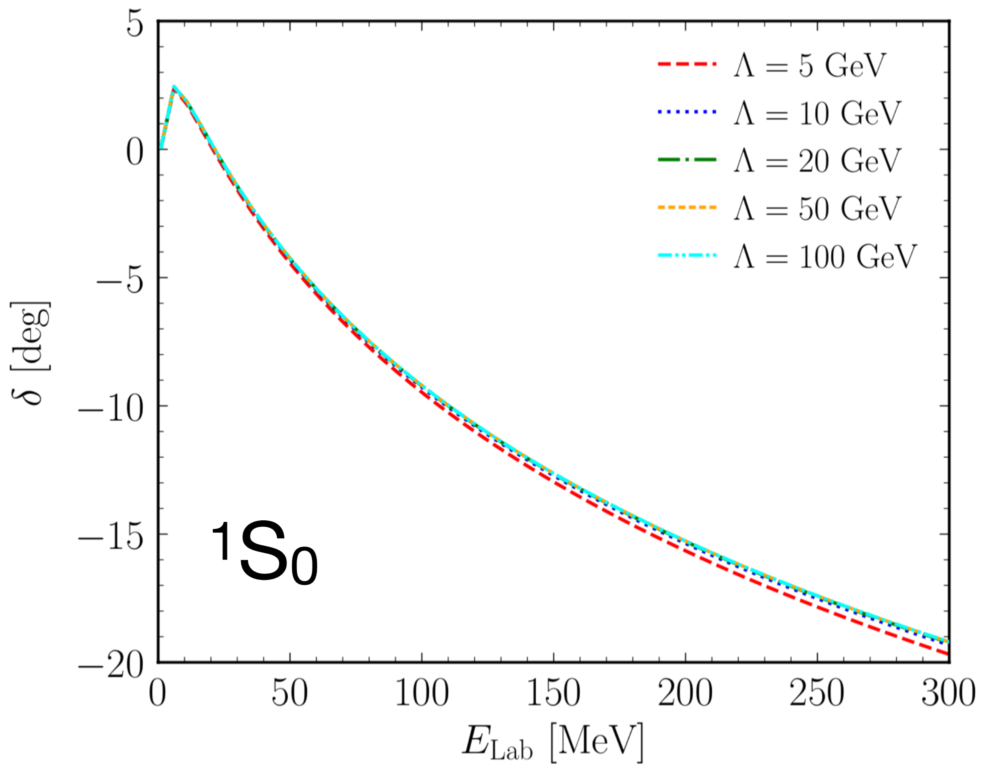
- Scattering amplitude from OPEP is cutoff independent

$$T_{\rm OPE} = V_{\rm OPE} + V_{\rm OPE}\, G\, T_{\rm OPE}$$

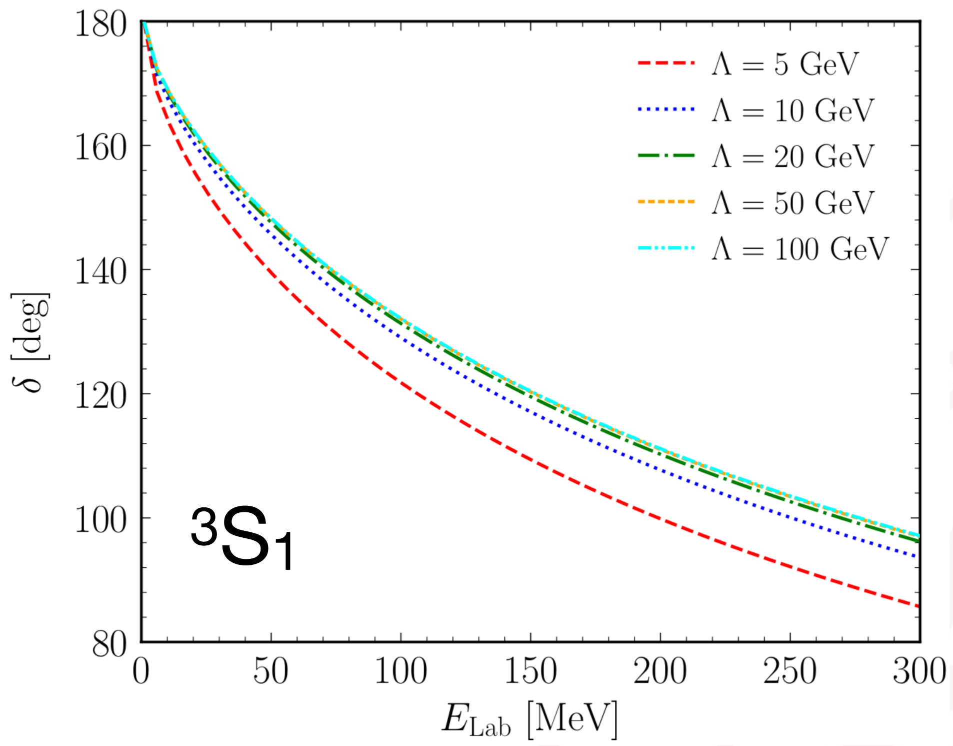
**Renormalizable!**

# Phase shifts: cutoff-independent

- NN single channel: e.g. 1S0




- NN couple channels: e.g. 3S1

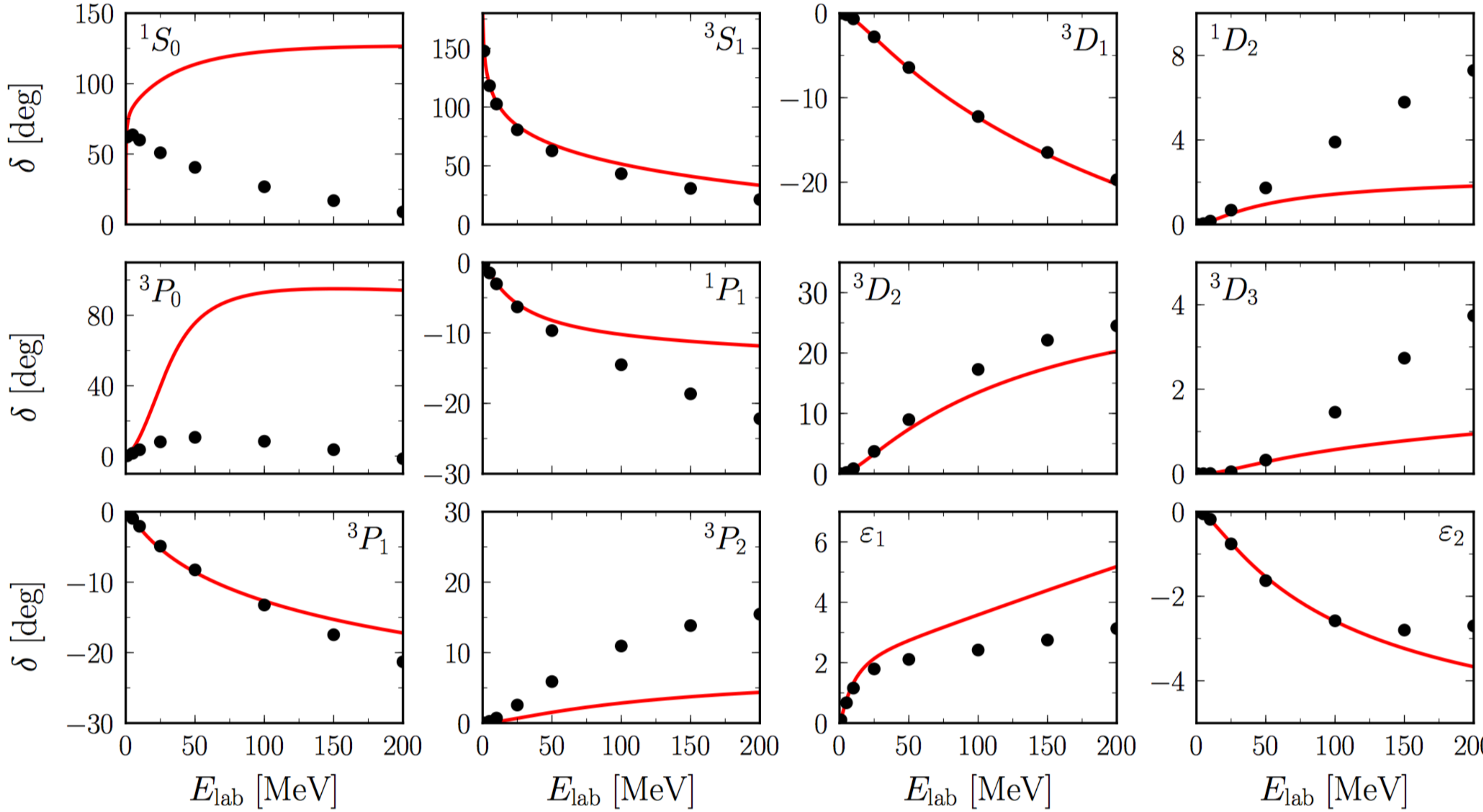



✤ Our LO potential is perturbatively renormalizable!

- All divergences appearing from its iterations can be absorbed in the coupling constant of the contact interaction
- Scattering equation has unique solutions for all partial waves
- Avoid finite-cutoff artefacts inherent to the conventional non-relativistic framework

# Phase shifts at LO

- Two LECs: fixed by scattering lengths of $^1S_0$ and $^3S_1$ $(\Lambda = 20\ \mathrm{GeV})$




- Provides a reasonable description of the empirical phase shifts
  - ✓ $^1S_0$ and $^3P_0$ : Large deviation
  - ✓ Part of the subleading corrections must be treated non-perturbatively

Beyond LO

V. Baru, E. Epelbaum, J. Gegelia, XLR, Phys. Lett. B 798, 134987 (2019)

# Beyond Leading order studies

- Two strategies to include higher orders
  - Restricting the non-perturbative treatment to the (non-singular) LO potential and higher-order interactions are treated perturbatively
    - ✓ Systematically remove all divergences from the amplitude
  - Full effective potential (LO + higher orders) are treated non-perturbatively
    - ✓ Milder UV behavior offers a larger flexibility regarding admissible cutoff
    - ✓ Direct input for few-/many-body problems
- Here, we focus on the second strategy (as a first step)
  - **Formulate the chiral nuclear potential up to NLO and NNLO**
    - ✓ Higher order contributions is computationally more demanding
  - Calculate the two-pion exchange contribution at one-loop level

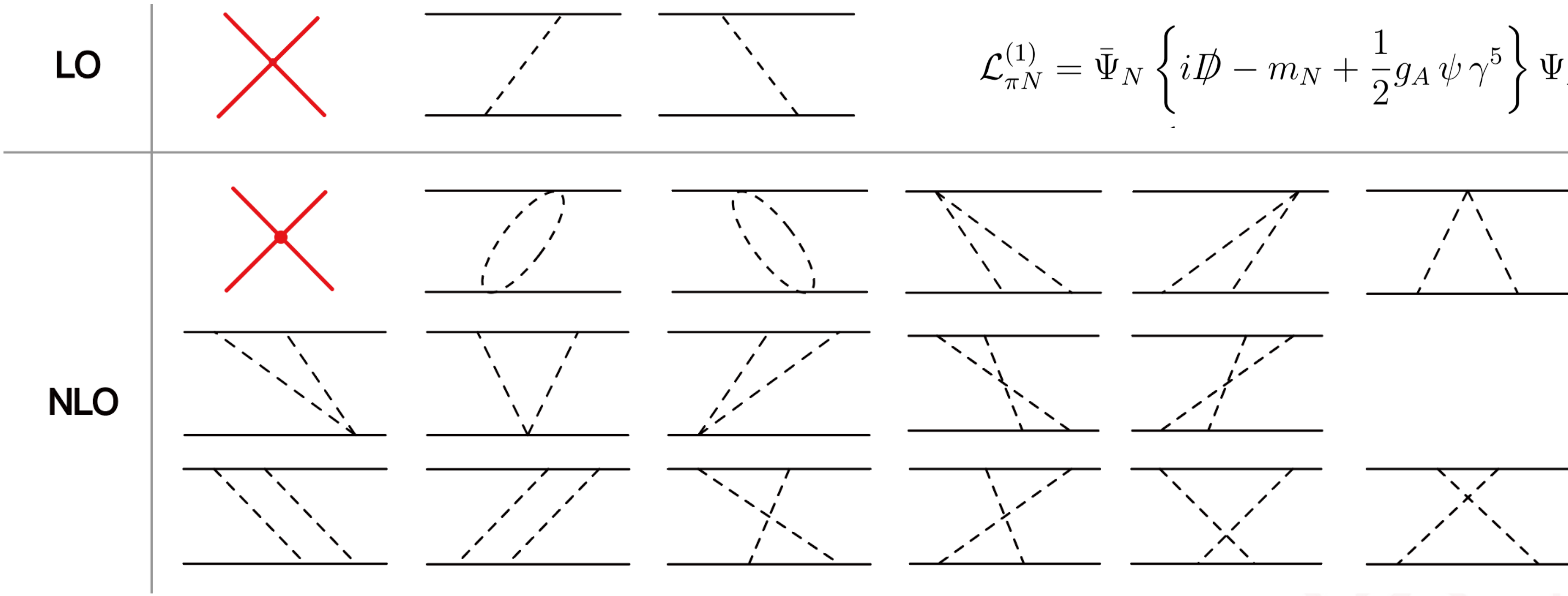
XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022);

XLR, E. Epelbaum, J. Gegelia, 2510.22648 [nucl-th]

XLR, et al., in preparation

# Study of NLO potential in TOPT

- Time ordered diagrams up to NLO



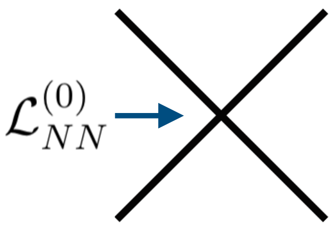

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left\{ i\not{D} - m_N + \frac{1}{2} g_A \not{u} \gamma^5 \right\} \Psi_N$$

$$\mathcal{L}_{NN}^{(0)} = \frac{1}{2} \left[ C_S (\bar{\Psi}_N \Psi_N)(\bar{\Psi}_N \Psi_N) + C_A \left(\bar{\Psi}_N \gamma_5 \Psi_N\right) \left(\bar{\Psi}_N \gamma_5 \Psi_N\right) + C_V \left(\bar{\Psi}_N \gamma_\mu \Psi_N\right) \left(\bar{\Psi}_N \gamma^\mu \Psi_N\right) \right.$$
$$\left. + C_{AV} \left(\bar{\Psi}_N \gamma_\mu \gamma_5 \Psi_N\right) \left(\bar{\Psi}_N \gamma^\mu \gamma_5 \Psi_N\right) + C_T \left(\bar{\Psi}_N \sigma_{\mu\nu} \Psi_N\right) \left(\bar{\Psi}_N \sigma^{\mu\nu} \Psi_N\right) \right]$$

$$\mathcal{L}_{NN}^{(2)} = \sum_{i=1} \bar{\Psi}_N \bar{\Psi}_N \mathcal{O}_i \Psi_N \Psi_N$$

# Contact terms up to NLO

## LO contact term (5 LECs)




$$V_{\rm LO} = C_S(\bar{u}_3 u_1)(\bar{u}_4 u_2) + C_A(\bar{u}_3 \gamma_5 u_1)(\bar{u}_4\gamma_5 u_2) + C_V(\bar{u}_3 \gamma_\mu u_1)(\bar{u}_4 \gamma^\mu u_2)$$
$$+ C_{AV}(\bar{u}_3 \gamma_\mu\gamma_5 u_1)(\bar{u}_4 \gamma^\mu\gamma_5 u_2) + C_T(\bar{u}_3 \sigma_{\mu\nu} u_1)(\bar{u}_4 \sigma^{\mu\nu} u_3)$$

- Expand the nucleon energy up to $\mathcal{O}(p^2)$ / NLO

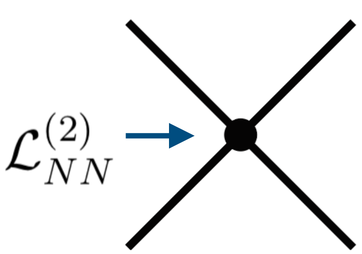
$$\sqrt{\omega(p, m_N) + m_N} = \sqrt{2m_N} + \frac{p^2}{4\sqrt{2}\, m_N^{3/2}} + \mathcal{O}(p^4)$$

✓ For simplicity, we include higher orders $\mathcal{O}(p^4)$ for LO contact terms

➡ Keep the full form of Dirac spinors

## NLO contact term

- Expand the nucleon energy $\sqrt{\omega(p, m_N) + m_N} = \sqrt{2m_N} + \mathcal{O}(p^2)$
- Same form as the non-relativistic case with 7 LECs




$$V_{\rm NLO} = C_1 \boldsymbol{q}^2 + C_2 \boldsymbol{P}^2 + \left(C_3 \boldsymbol{q}^2 + C_4 \boldsymbol{P}^2\right)(\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) + \frac{i}{2} C_5 (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \boldsymbol{n}$$
$$+ C_6 (\boldsymbol{q} \cdot \boldsymbol{\sigma}_1)(\boldsymbol{q} \cdot \boldsymbol{\sigma}_2) + C_7 (\boldsymbol{P} \cdot \boldsymbol{\sigma}_1)(\boldsymbol{P} \cdot \boldsymbol{\sigma}_2)$$

# Partial wave decomposition for contact terms

- J=0: 1S0 and 3P0 partial waves

$$V(^1S_0) = \xi_N\left[\tilde{C}_{^1S_0} + \tilde{C}_{^1S_0} R_p^2 R_{p'}^2 + C_{^1S_0}\left(R_p^2 + R_{p'}^2\right)\right]$$

$$V(^3P_0) = C_{^3P_0}\, p\, p'$$

- J=1: 3S1-3D1, 1P1, 3P1, partial waves

$$V(^3S_1) = \xi_N\left[\tilde{C}_{^3S_1} + \frac{\tilde{C}_{^3S_1}}{9} R_p^2 R_{p'}^2 + \frac{C_{^3S_1}}{9}\left(R_p^2 + R_{p'}^2\right)\right]$$

$$V(^1P_1) = C_{^1P_1}\, p\, p'$$

$$V(^3P_1) = C_{^3P_1}\, p\, p'$$

$$V(^3D_1) = \frac{8\xi_N}{9}\tilde{C}_{^3S_1} R_p^2 R_{p'}^2$$

$$V(^3S_1 - {}^3D_1) = \xi_N\left[C_{\varepsilon_1} R_p^2 + \frac{2\sqrt{2}}{9}\tilde{C}_{^3S_1} R_p^2 R_{p'}^2\right]$$

$$V(^3D_1 - {}^3S_1) = \xi_N\left[C_{\varepsilon_1} R_{p'}^2 + \frac{2\sqrt{2}}{9}\tilde{C}_{^3S_1} R_p^2 R_{p'}^2\right]$$

$$\xi_N = \frac{(\omega_p + m_N)(\omega_{p'} + m_N)}{4m_N^2}$$

$$R_p = \frac{p}{\omega_p + m_N}$$

$$R_{p'} = \frac{p'}{\omega_{p'} + m_N}$$

- J=2: 3P2 partial wave

$$V(^3P_2) = C_{^3P_2}\, p\, p'$$

**9 LECs to be fixed:** $C_{^1S_0}, C_{^3S_1}, \tilde{C}_{^1S_0}, C_{^3P_0}, C_{^1P_1}, C_{^3P_1}, \tilde{C}_{^3S_1}, C_{^3D_1-^3S_1}, C_{^3P_2}$

**Same number of contact terms as the non-relativistic NLO case**

**XLR, E. Epelbaum, J. Gegelia, 2510.22648 [nucl-th]**

# In comparison with covariant power counting

- Covariant power counting
  - Keep the small component of Dirac spinor

$$u_i(\vec{p}, s) = \sqrt{\frac{E_N + M_N}{2M_N}} \begin{pmatrix} 1 \\ \frac{\vec{\sigma}_1 \cdot \vec{p}}{\epsilon_p} \end{pmatrix} \chi_{s,i}$$

  - Up to NLO with 17 LECs

TABLE II. LECs (in units of $10^4$ GeV$^{-2}$) for the relativistic LO, NLO, and NNLO results shown in Fig. 2.

| | $O_1$ | $O_2$ | $O_3$ | $O_4$ | $O_5$ | $O_6$ | $O_7$ | $O_8$ | $O_9$ | $O_{10}$ | $O_{11}$ | $O_{12}$ | $O_{13}$ | $O_{14}$ | $O_{15}$ | $O_{16}$ | $O_{17}$ | $D_1$ | $D_2$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| LO | −1.32 | −0.21 | −0.93 | 0.31 | | | | | | | | | | | | | | | |
| NLO | −2.62 | 9.45 | −5.42 | −6.05 | 30.09 | 9.02 | −9.19 | 8.74 | 4.74 | 7.02 | 3.52 | 11.42 | −6.03 | −20.55 | −4.99 | −12.80 | 6.30 | 0.42 | 0.28 |
| NNLO | −14.83 | −2.25 | −4.85 | 6.24 | −0.82 | 1.96 | −6.89 | 7.19 | 1.44 | 3.50 | −8.10 | −9.38 | −4.33 | −12.89 | −12.26 | −11.69 | 3.86 | −1.88 | −0.63 |

**J.-X. Lu, C.-X. Wang, Y. Xiao, L.-S. Geng, J. Meng, P. Ring, PRL 128, 142002 (2022)**

# One-Pion exchange potential up to NLO

## OPE potential

$$V_{\mathrm{OPE}}=-\frac{g_A^2}{4f_\pi^2}\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2\frac{1}{\omega_q}\frac{\left(\bar{u}_3\gamma^\mu\gamma_5 q_\mu u_1\right)\left(\bar{u}_4\gamma^\nu\gamma_5 q_\nu u_2\right)}{\omega_p+\omega_{p'}+\omega_q-E-i\epsilon}$$

- Expand the nucleon energy expansion for OPEP at NLO

$$\sqrt{\omega(p,m_N)+m_N}=\sqrt{2m_N}+\frac{p^2}{4\sqrt{2}\,m_N^{3/2}}+\mathcal{O}(p^4)$$

  ✓ For simplicity, we include higher orders $\mathcal{O}(p^4)$ for OPE potential

  ➡ Keep the full form of Dirac spinors

- Eliminate the energy dependence of OPEP (avoid the pole contribution)

  ✓ Expand E at $\omega_p+\omega_p'$, then, we obtain contribution of OPEP at NLO

$$V_{\mathrm{OPE}}^{\not{E}}=-\frac{g_A^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4f_\pi^2}\frac{1}{\omega_q^2}\left(\bar{u}_3\gamma_\mu\gamma_5 q^\mu u_1\right)\left(\bar{u}_4\gamma_\nu\gamma_5 q^\nu u_2\right)$$

⟶ LO correction $V_{\mathrm{OPE},\not{E}}^{(0)}$

NLO correction $V_{2\pi,\not{E}}^{(2)}$

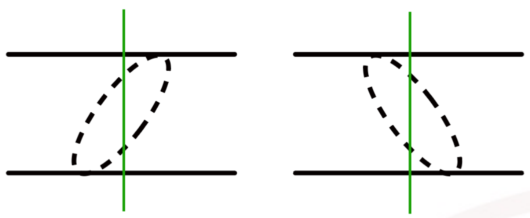
$$+\frac{1}{2}\left(\frac{g_A^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{4f_\pi^2}\right)^2\int\frac{d^3k}{(2\pi)^3}\frac{m_N^2}{k^2+m_N^2}\frac{\omega_{p'-k}+\omega_{p-k}}{\omega_{p'-k}^3\omega_{p-k}^3}\times\left[\boldsymbol{\sigma}_1\cdot(\boldsymbol{p}'-\boldsymbol{k})\boldsymbol{\sigma}_1\cdot(\boldsymbol{k}-\boldsymbol{p})\right]\left[\boldsymbol{\sigma}_2\cdot(\boldsymbol{p}'-\boldsymbol{k})\boldsymbol{\sigma}_2\cdot(\boldsymbol{k}-\boldsymbol{p})\right].$$

# Two-pion exchange potential at NLO

- Follow our TOPT rules:
  - Football diagram

$$V_F = \frac{1}{16 f_\pi^4} \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2 \int \frac{d^3k}{(2\pi)^3} \frac{(\omega_k + \omega_{k+q})(\omega_p + \omega_{p'}) + 4\omega_k \omega_{k+q} - E(\omega_k + \omega_{k+q})}{2\omega_k \omega_{k+q} \left(\omega_k + \omega_{k+q} + \omega_p + \omega_{p'} - E\right)} .$$

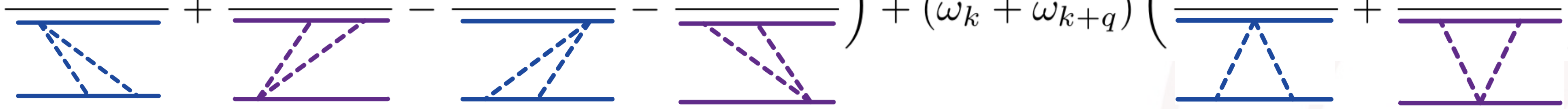
Energy denominator of football diagram



  - Triangle diagrams

$$V_{T+\tilde{T}}^{NN} = \frac{4 m_N g_A^2 \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2}{128 f_\pi^4} \int \frac{d^3k}{(2\pi)^3} \left[ \left(\boldsymbol{k}^2 + (\boldsymbol{p}' - \boldsymbol{p}) \cdot \boldsymbol{k}\right) + \frac{i}{2} (\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \boldsymbol{n} (a+b) \right] \frac{1}{\omega_k \omega_{k+q} \omega_{p-k}}$$

$\times \Big[ (\omega_{k+q} - \omega_k) \Big( \frac{1}{\cdot} + \frac{1}{\cdot} - \frac{1}{\cdot} - \frac{1}{\cdot} \Big) + (\omega_k + \omega_{k+q}) \Big( \frac{1}{\cdot} + \frac{1}{\cdot} \Big) \Big]$

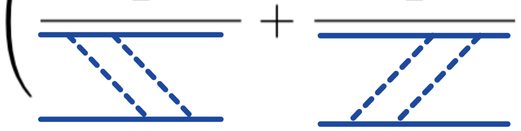


Energy denominator

  - Planar and crossed box diagrams

$$V_B = \frac{m_N^2 g_A^4 (3 - 2\, \boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)}{64 f_\pi^4} \int \frac{d^3k}{(2\pi)^3} \left[ X_1 + X_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + X_3 \frac{i\,(\boldsymbol{\sigma}_1 + \boldsymbol{\sigma}_2) \cdot \boldsymbol{n}}{2} + X_4 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{n})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{n}) + X_5 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) \right]$$

$\times \frac{1}{\omega_k \omega_{k+q} \omega_{p-k}^2} \Big( \frac{1}{\cdot} + \frac{1}{\cdot} \Big)$



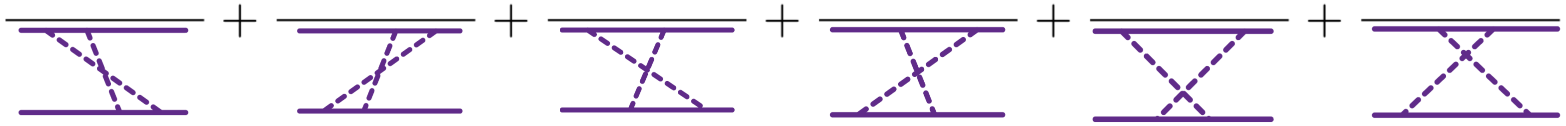
$$\boldsymbol{k} = a\,\boldsymbol{p} + b\,\boldsymbol{p}' + c\,(\boldsymbol{p}' \times \boldsymbol{p})$$

$X_1 = \left[\boldsymbol{k}^2 + \boldsymbol{q} \cdot \boldsymbol{k}\right]^2, \quad X_2 = -c^2 \boldsymbol{q}^2 \left[\boldsymbol{P}^2 \boldsymbol{q}^2 - (\boldsymbol{q} \cdot \boldsymbol{P})^2\right], \quad X_3 = -2(a+b)\left(\boldsymbol{k}^2 + (\boldsymbol{p}' - \boldsymbol{p}) \cdot \boldsymbol{k}\right),$

$X_4 = -(a+b)^2 + c^2 \boldsymbol{q}^2, \quad X_5 c^2 \left[\boldsymbol{P}^2 \boldsymbol{q}^2 - (\boldsymbol{q} \cdot \boldsymbol{P})^2\right]$

$$V_{\tilde{B}} = \frac{m_N^2 g_A^4 (3 + 2\boldsymbol{\tau}_1 \cdot \boldsymbol{\tau}_2)}{64 f_\pi^4} \int \frac{d^3k}{(2\pi)^3} \left[ X_1 + X_2 \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + X_4 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{n})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{n}) + X_5 (\boldsymbol{\sigma}_1 \cdot \boldsymbol{q})(\boldsymbol{\sigma}_2 \cdot \boldsymbol{q}) \right]$$

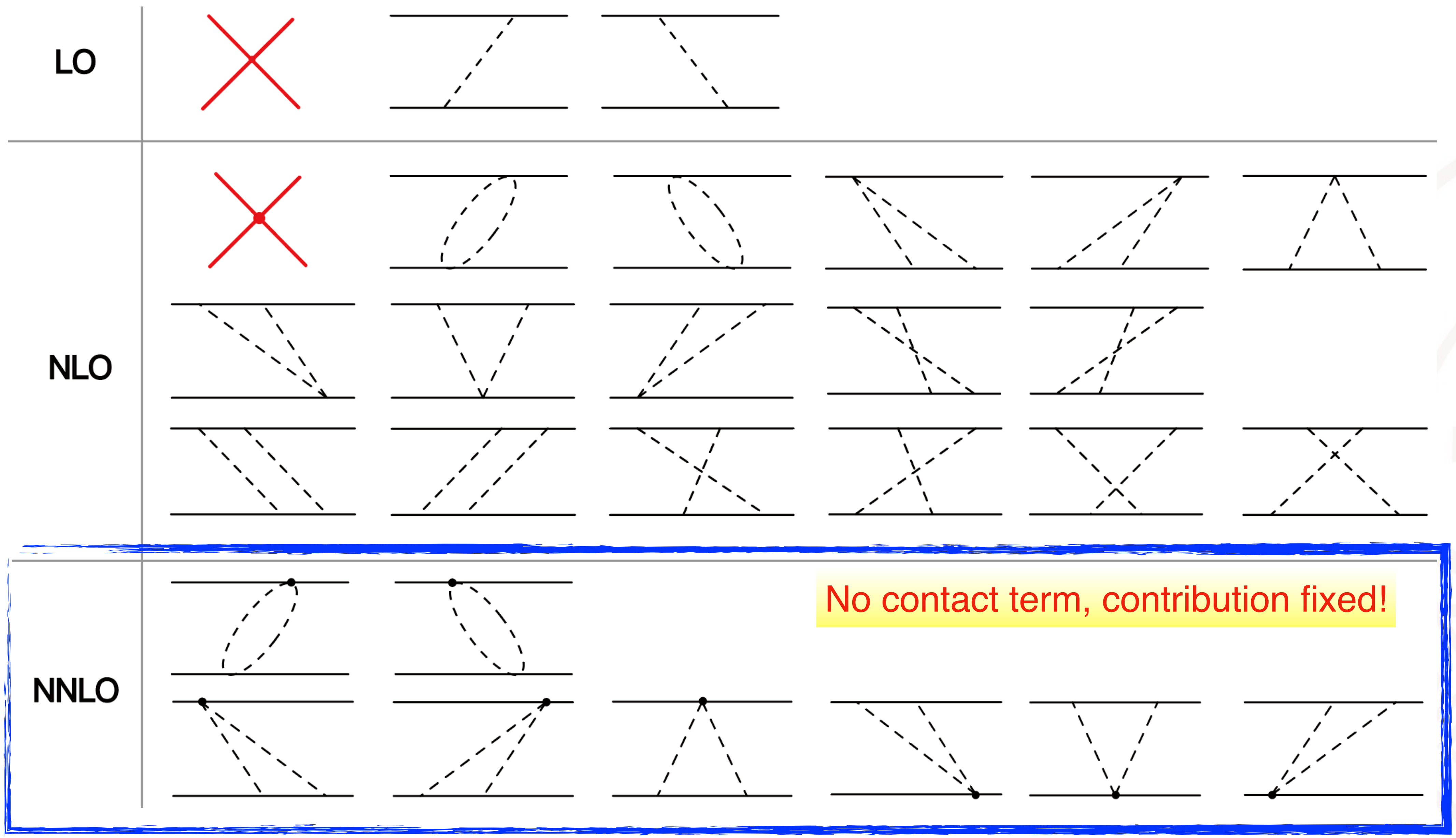
$\times \frac{1}{\omega_k \omega_{k+q} \omega_{p-k} \omega_{p'+k}} \Big( \frac{1}{\cdot} + \frac{1}{\cdot} + \frac{1}{\cdot} + \frac{1}{\cdot} + \frac{1}{\cdot} + \frac{1}{\cdot} \Big)$



UV Divergent terms and power counting breaking terms are removed by using **the subtractive renormalization**

# Study of NNLO potential in TOPT

- Time ordered diagrams up to NNLO



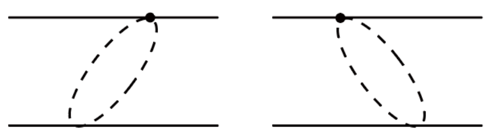

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}_N \left\{ i\not{D} - m_N + \frac{1}{2} g_A \not{u} \gamma^5 \right\} \Psi_N$$

$$\mathcal{L}_{\pi N}^{(2)} = \bar{\Psi}_N \left\{ c_1 \langle \chi + \rangle - \frac{c_2}{4m_N^2} \langle u^\mu u^\nu \rangle \left( D_\mu D_\nu + \text{ h.c. } \right) + \frac{c_3}{2} \langle u^\mu u_\mu \rangle - \frac{c_4}{4} \gamma^\mu \gamma^\nu \left[ u_\mu, u_\nu \right] \right\} \Psi_N$$

$c_1 = -0.74,\ c_2 = 1.81,\ c_3 = -3.61,\ c_4 = 2.17\ \text{GeV}^{-1}$

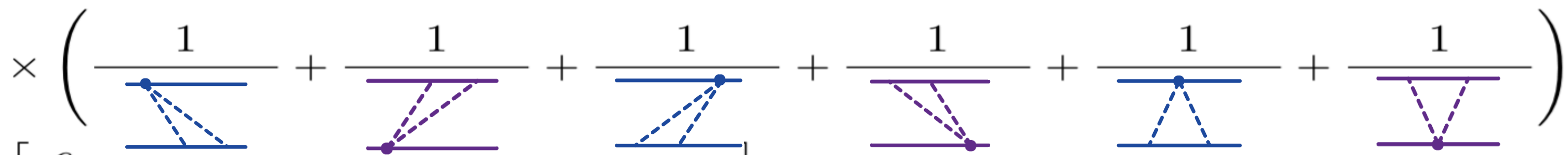
*D. Siemens, et al., PLB770 (2017) 27-34*

# Two-pion exchange potential at NNLO

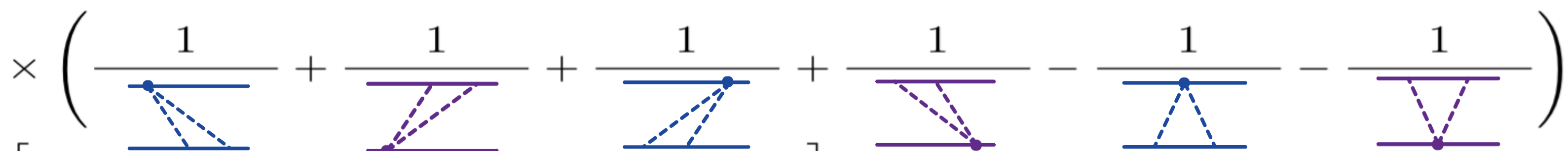
- Follow our TOPT rules:
  - Football diagrams  No contribution!
  - Triangle diagrams

$$V_{T+\tilde{T}} = \frac{3 m_N g_A^2}{16 f_\pi^4}\int \frac{d^3k}{(2\pi)^3}\left[(\boldsymbol{k}^2+(\boldsymbol{p}'-\boldsymbol{p})\cdot\boldsymbol{k})-(a+b)\frac{i(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{n}}{2}\right]\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}}$$

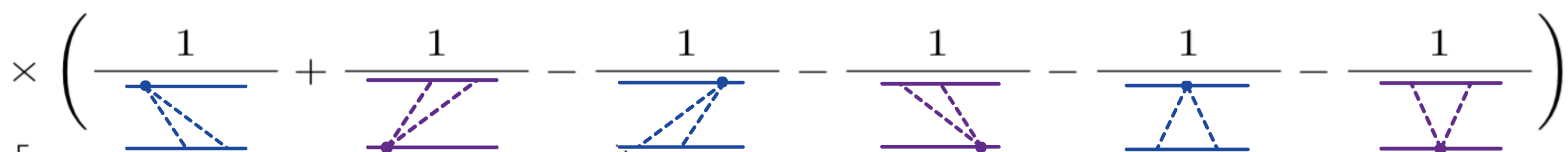
$$\times\left\{\left[4c_1M_\pi^2-\frac{c_2}{m_N^2}\Big(\boldsymbol{p}\cdot\boldsymbol{k}\,\boldsymbol{p}\cdot(\boldsymbol{k}+\boldsymbol{q})+\boldsymbol{p}'\cdot\boldsymbol{k}\,\boldsymbol{p}'\cdot(\boldsymbol{k}+\boldsymbol{q})\Big)+2c_3\boldsymbol{k}\cdot(\boldsymbol{k}+\boldsymbol{q})\right]\right.$$



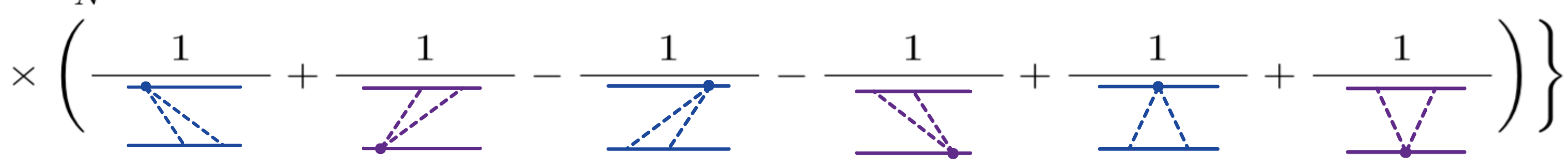

$$+\left[\frac{c_2}{m_N^2}\omega_k\omega_{k+q}(\omega_p+\omega_{p'})+2c_3\omega_k\omega_{k+q}\right]$$




$$+\left[\frac{c_2}{m_N^2}\omega_k\left(\omega_p\boldsymbol{p}\cdot(\boldsymbol{k}+\boldsymbol{q})+\omega_{p'}\boldsymbol{p}'\cdot(\boldsymbol{k}+\boldsymbol{q})\right)\right]$$

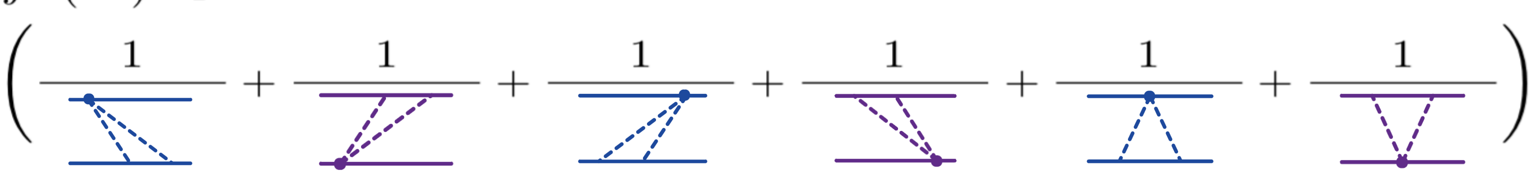



$$-\left[\frac{c_2}{m_N^2}\omega_{k+q}\left(\omega_p\boldsymbol{p}\cdot\boldsymbol{k}+\omega_{p'}\boldsymbol{p}'\cdot\boldsymbol{k}\right)\right]$$




$$+\frac{c_4 m_N g_A^2\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_2}{8f_\pi^4}\int\frac{d^3k}{(2\pi)^3}\left[X_2\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2+\frac{X_3}{2}\frac{i\,(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\boldsymbol{n}}{2}+X_4\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{n})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{n})+X_5\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{q})\,(\boldsymbol{\sigma}_2\cdot\boldsymbol{q})\right]$$

$$\times\frac{1}{\omega_k\omega_{k+q}\omega_{p-k}}$$




- UV Divergent terms
- Power-counting breaking terms
- are removed by using **the subtractive renormalization**

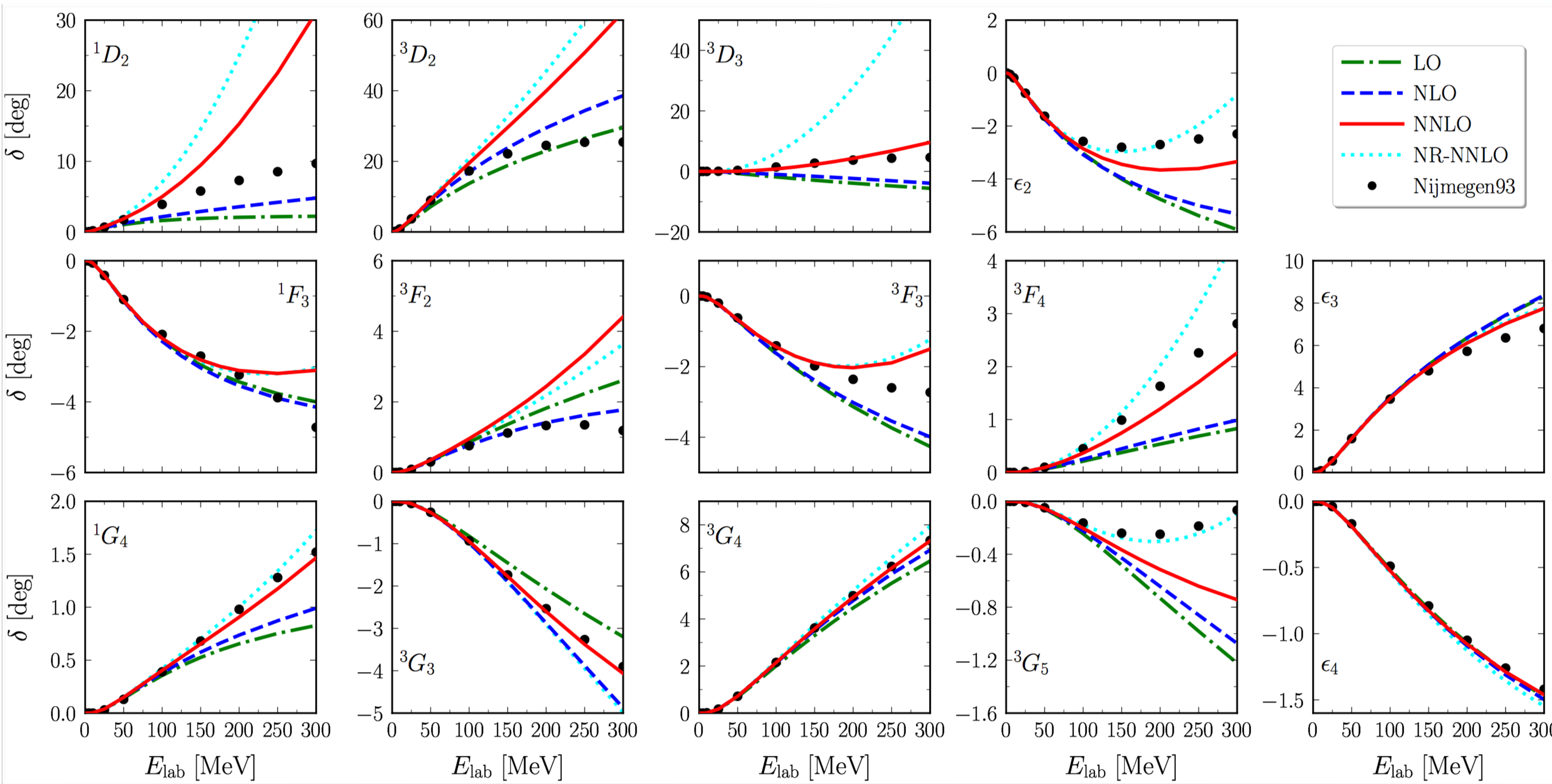
2022.04018

# Pion-exchange contribution

- On-shell T-matrix under the Born approximation

$$T(p',p) = V_{\rm OPE}(p',p) + V^{(2)}_{2\pi,irr}(p',p) + V^{(3)}_{2\pi,irr}(p',p) + V_{\rm OPE}\, G\, V_{\rm OPE}$$

- **Prediction: phase shifts of D, F, G waves**




✓ Improve the description of D waves; globally similar results for F, G waves

- $^3G_5$: non-rel. result is accidental, $c_i/m_N$ effect ($N^4LO$) is large *D. Entem, et al., PRC 91, 014002 (2015)*

**XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022)**

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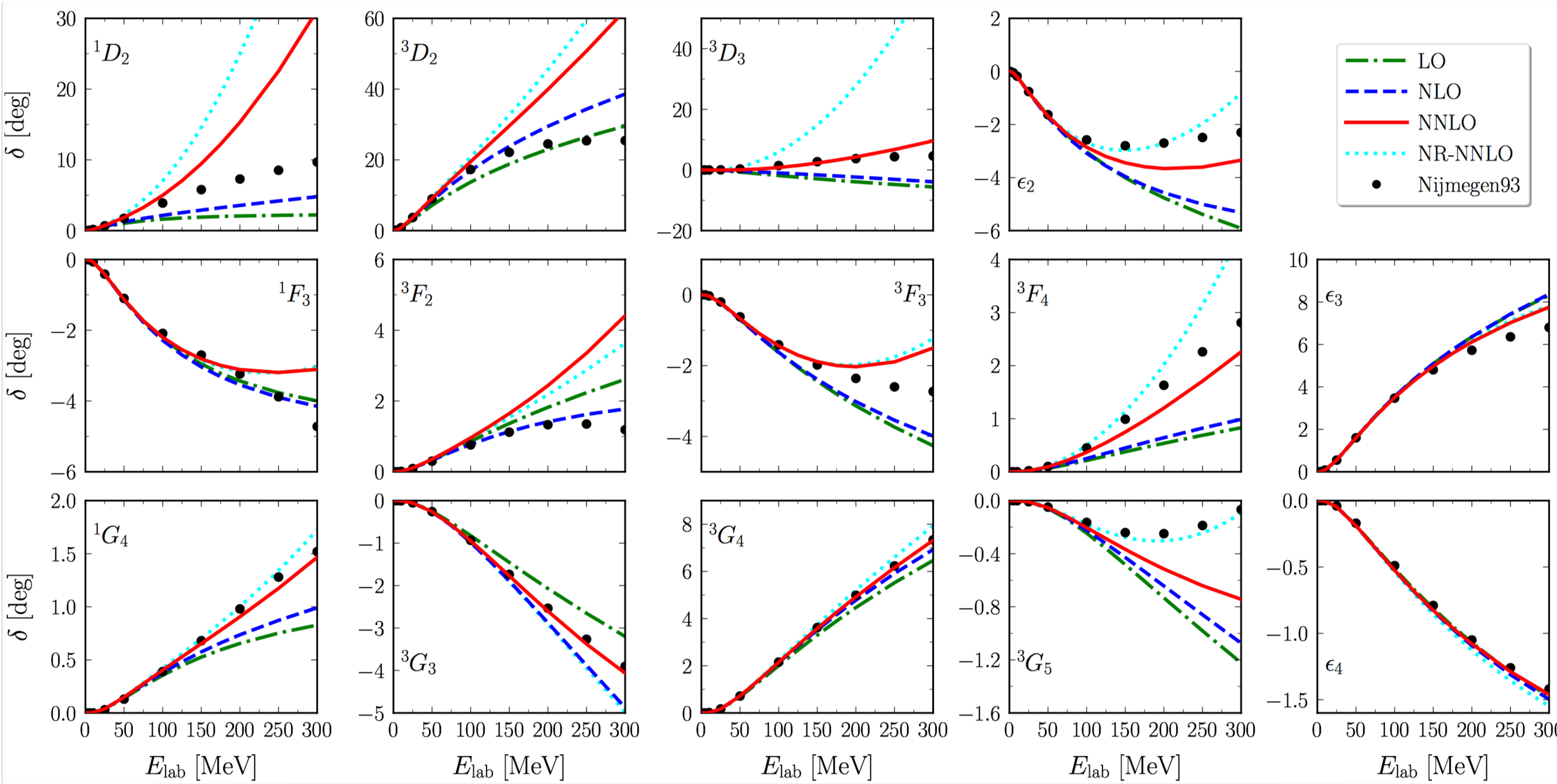


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**XLR, E. Epelbaum, J. Gegelia, Phys. Rev. C 106, 034001 (2022)**

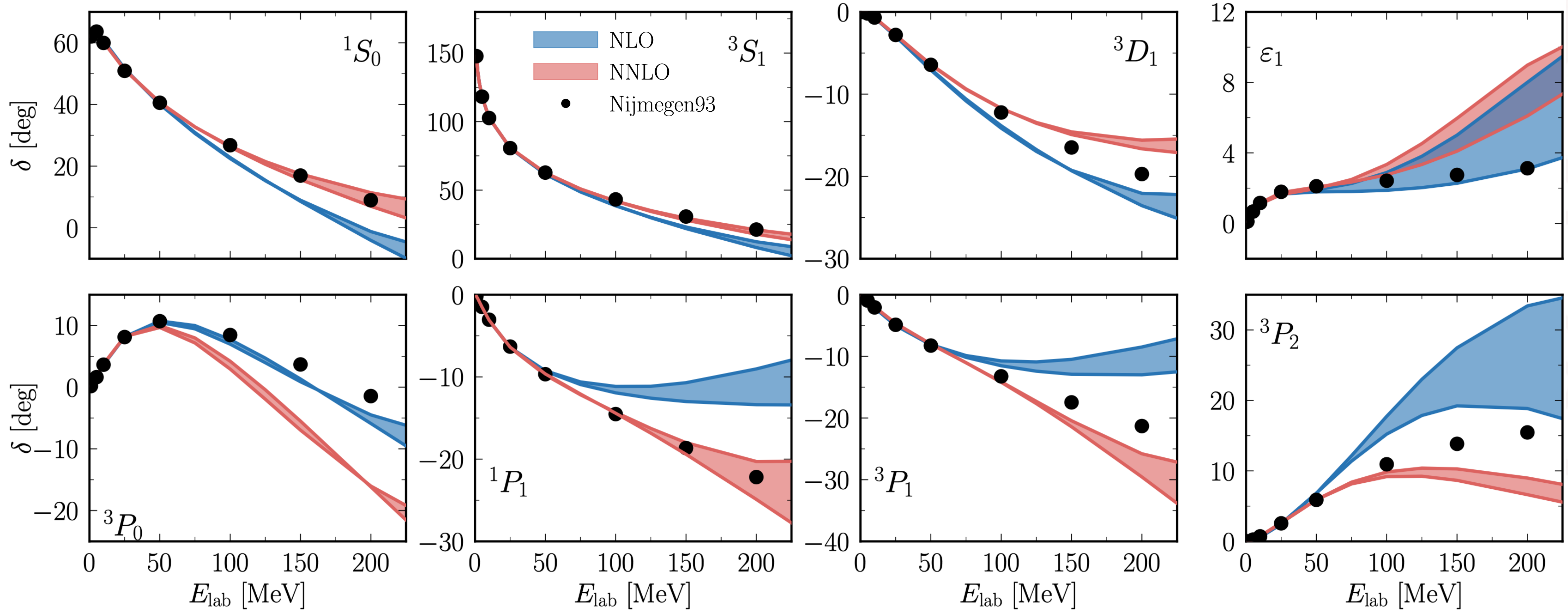
# NNLO: contact + pion exchanges

- Partial wave T-matrix
  - $V_{\rm NNLO}$ non-perturbatively iterated in the Kadyshevsky equation

$$T_{ll'}^{sj}(p',p) = V_{ll'}^{sj}(p',p) + \sum_{l''}\int \frac{d^3k}{(2\pi)^3} V_{ll''}^{sj}(p',k)\frac{m_N^2}{2(k^2+m_N^2)}\frac{1}{\sqrt{p^2+m_N^2}-\sqrt{k^2+m_N^2}+i\epsilon} T_{l''l'}^{sj}(k,p)$$

  - Pion-loop potential: cutoff regularization with $k_{\rm max.} = 500$ MeV
  - Exponential regulator: $F(p) = \exp(-p^{2n}/\Lambda^{2n})$, with $n=2$, $\Lambda = 400 \sim 550$ MeV

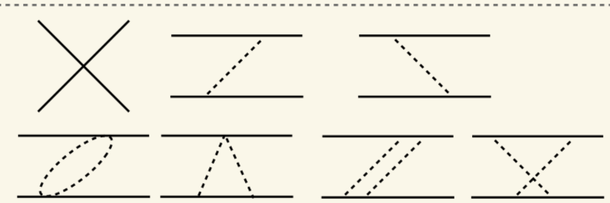
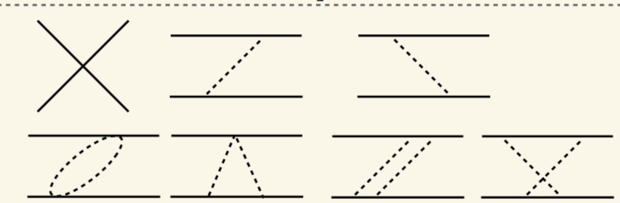
- Phase shifts: Fit NPWA ($E_{\rm lab} \leq 100$ MeV)




- Deuteron binding energy NLO $-2.16$ MeV; NNLO $-2.18$ GeV; no deeply bound states

# Summary

- Proposed a systematic framework to formulate chiral forces

| **Time-ordered perturbation theory** | **Non-relativistic** (Heavy-baryon) | **Manifestly Lorentz invariant** |
|---|---|---|
| Chiral Lagrangians | $N^\dagger\left[i(v\cdot D)+g_A(S\cdot u)\right]N$ <br> $-\frac{1}{2}C_S\left(N^\dagger N\right)\left(N^\dagger N\right)-\frac{1}{2}C_T\left(N^\dagger\vec{\sigma}N\right)\left(N^\dagger\vec{\sigma}N\right)+\cdots$ | $\bar{\Psi}_N\left\{i\gamma_\mu D^\mu-m_N+\frac{1}{2}g_A\psi\gamma^5\right\}\Psi_N$ <br> $+\frac{1}{2}\Big[C_S(\bar{\Psi}_N\Psi_N)(\bar{\Psi}_N\Psi_N)+C_A\left(\bar{\Psi}_N\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma_5\Psi_N\right)$ <br> $+C_V\left(\bar{\Psi}_N\gamma_\mu\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\Psi_N\right)+C_{AV}\left(\bar{\Psi}_N\gamma_\mu\gamma_5\Psi_N\right)\left(\bar{\Psi}_N\gamma^\mu\gamma_5\Psi_N\right)$ <br> $+C_T\left(\bar{\Psi}_N\sigma_{\mu\nu}\Psi_N\right)\left(\bar{\Psi}_N\sigma^{\mu\nu}\Psi_N\right)\Big]+\ldots$ |
| Potential TOPT diagrams |  |  |
| Scattering equations (T = V + VGT) | Lippmann-Schwinger eq. | Kadyshevsky eq. |
| Power counting | Weinberg p.c. | Weinberg p.c. |

- **Uniquely determined** the scattering equation
  - ✓ Chiral potential and scattering equation are obtained within the same framework
- Obtained non-singular LO potential
  - ✓ Avoid finite-cutoff artefacts and take cutoff $\Lambda\to\infty$
- Formulated the chiral potential up to NNLO
  - ✓ Calculated the two-pion-exchange potential at one-loop level
  - ✓ Achieved a rather reasonable description of NN phase shifts

# Future perspectives

- Perturbatively include NLO/NNLO contributions
  - Based on our non-singular LO potential, all divergences of the amplitude can be systematically removed ($\Lambda \sim \infty$)
- In the long run, apply **<u>symmetry preserving regularization</u>** to investigate the chiral potential
  - Maintain the chiral symmetry and gauge symmetry
  - e.g. higher-derivative approach *A. A. Slavnov, PNB31, 301-315 (1971)* *D. Djukanovic, et al., PRD72,045002(2005)*
  - e.g. gradient flow method *D. Kaplan, HHIQCD 2015* *H. Krebs, E. Epelbaum, PRC110, 044004(2024)*

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Thank you for your attention!