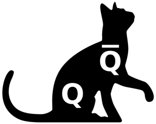
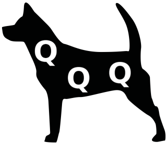
Physics of narrow near-threshold exotic states

A.V. Nefediev

Ordinary hadrons



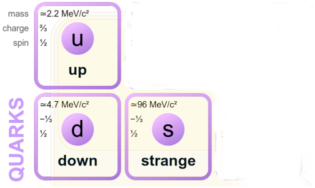


Quark model: The structure of hadrons

1964 — Quark model by Gell-Mann & Zweig $\Longrightarrow$ $SU(3)$ multiplets

"Ordinary" hadrons*:

- Meson consists of quark and antiquark
- Baryon consists of 3 quarks



* Compact "exotic" hadrons anticipated

All hadrons understood $\Longrightarrow$ No "misterious" states

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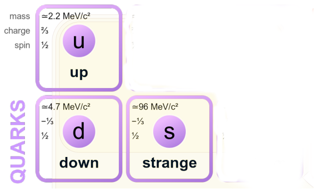

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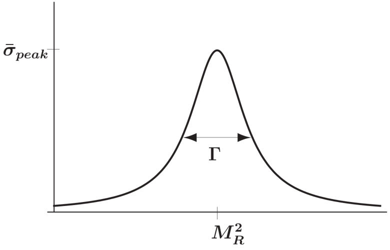
Contemporary status — input for the quark model

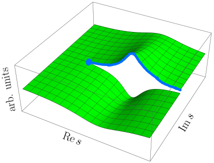
- 6 quarks belonging to 3 generations
- 3 light quarks (u, d, s with $m_u \approx 2$ MeV, $m_d \approx 5$ MeV, $m_s \approx 94$ MeV)
- 2 heavy quarks (c, b with $m_c \approx 1.3$ GeV, $m_b \approx 4.2$ GeV) that form hadrons
- t quark with the mass $m_t \approx 170$ GeV that decays too fast to form hadrons

Breit-Wigner parametrisation: Mass, Width, Poles

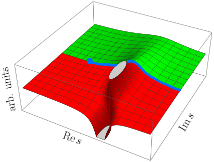


$$\mathcal{A}_{\rm BW} \propto \frac{1}{s - M_R^2 + i\sqrt{s}\Gamma}$$

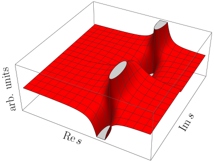
Poles: $\begin{cases} s_{\rm pole} \approx M_R^2 - iM_R\Gamma \\ s^*_{\rm pole} \approx M_R^2 + iM_R\Gamma \end{cases}$



first Riemann sheet

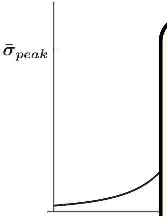


transition from first to second Riemann sheet



second Riemann sheet

Breit-Wigner parametrisation: Mass, Width, Poles



$$\frac{1}{\ldots M_R^2 + i\sqrt{s}\Gamma}$$

$M_R^2 - iM_R\Gamma$

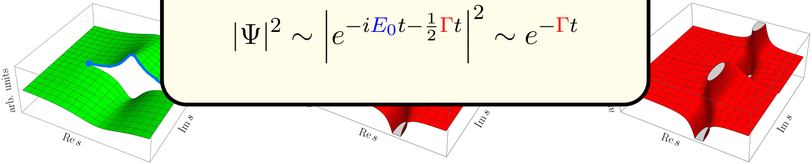
$M_R^2 + iM_R\Gamma$

Nonrelativistic expansion:

$$\sqrt{s} = M_{\rm th} + E$$
$$M_R = M_{\rm th} + E_0$$

$$\mathcal{A}_{\rm BW}^{\rm nr} \propto \frac{1}{E - E_0 + \frac{i}{2}\Gamma}$$

$$|\Psi|^2 \sim \left|e^{-iE_0 t - \frac{1}{2}\Gamma t}\right|^2 \sim e^{-\Gamma t}$$



first Riemann sheet

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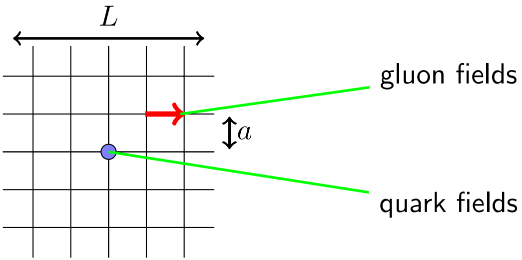
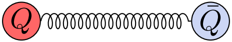
Lattice simulations



$$C_{ij}(t) = \langle 0|O_i(t)O_j(0)|0\rangle = \sum_n \frac{e^{-E_n t}}{2E_n}\langle 0|O_i(0)|n\rangle\langle n|O_j^\dagger(0)|0\rangle$$

- Continuum limit $\Longrightarrow a \to 0$
- Infinite box $\Longrightarrow L \to \infty$
- Unphysical light quark mass $\Longrightarrow$ Chiral extrapolation

Quark model calculations



$$\hat{H}\psi = M_R\psi$$

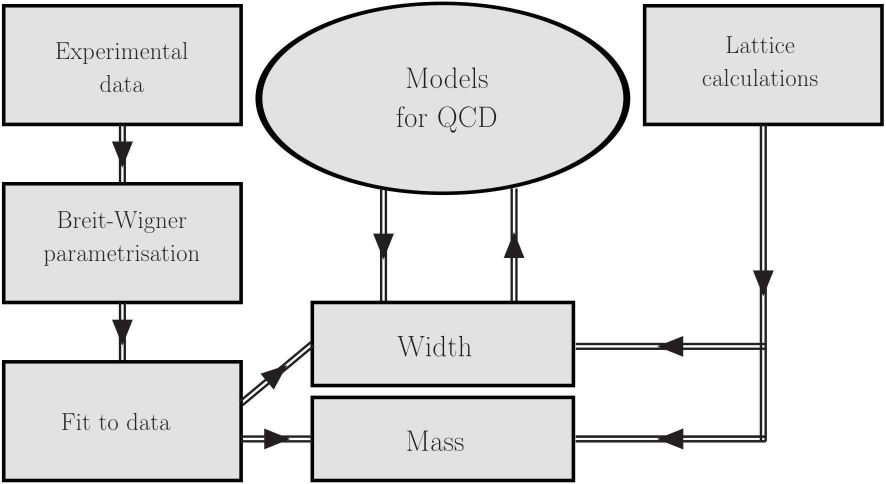
$$\hat{H} = 2m_Q + \frac{p^2}{m_Q} + V_0(r) + V_{\rm SD}(r)$$

$$V_0(r) = \sigma r - \frac{\frac{4}{3}\alpha_s}{r} + C_0 \qquad \text{(Cornell potential)}$$

$$V_{SD}(r) = \underbrace{V_{LS}(r)(\boldsymbol{L}\cdot(\boldsymbol{S}_Q + \boldsymbol{S}_{\bar{Q}}))}_{\text{fine structure}} + \underbrace{V_{SS}(r)(\boldsymbol{S}_Q\cdot\boldsymbol{S}_{\bar{Q}})}_{\text{hyperfine structure}}$$

$$+\underbrace{V_{ST}(r)\Big((\boldsymbol{S}_Q\cdot\boldsymbol{S}_{\bar{Q}}) - 3(\boldsymbol{S}_Q\cdot\boldsymbol{n})(\boldsymbol{S}_{\bar{Q}}\cdot\boldsymbol{n})\Big)}_{\text{spin}-\text{tensor force}} \quad \propto \quad \frac{1}{m_Q^2}$$

Approach to ordinary hadrons



Hadronic physics: Consensus before 2003

- Quark model provides a decent description of low-lying hadrons
- Quark model works surprisingly well even for light flavours
- Heavy flavours (c and b) comply with nonrelativistic theory
- Relativistic corrections improve the description
- Experiment gradually fills "missing states"
- Lattice provides additional/alternative source of information

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General conclusion: Hadronic physics is well understood

Exotic states with heavy quarks

"Exotic animal is more unusual and rare than normal domesticated pets like cats or dogs"

Revolution of 2003: Enfant terrible $X(3872)$

- $I = 0$, $J^{PC} = 1^{++}$, contains $c\bar{c}$
- Too light compared with Quark Model prediction

$$M^{\text{QM}}_{\chi_{c1}(2P)} - M^{\text{exp}}_{X} \sim 100 \text{ MeV}$$

- Strongly attracted to $D\bar{D}^*$ threshold

$$M^{\text{exp}}_{X} - (M_{D^0} + M_{\bar{D}^{*0}}) \sim 0$$

- Large ($\sim 40\%$) probability of the decay into $D\bar{D}^*$
- Strong isospin violation

$$Br(X \to \pi^+\pi^-\pi^0 J/\psi) \approx Br(X \to \pi^+\pi^- J/\psi)$$

Revolution of 2003: Enfant terrible $X(3872)$

- $I = 0$, $J^{PC} = 1^{++}$, contains $c\bar{c}$

> - $\sim$2500 citations (the most cited paper by Belle)
> - $J^{PC} = 1^{++}$ unambiguously established by LHCb in 2013
> - Nature of $X(3872)$ still under debate
> - New name by PDG — $\chi_{c1}(3872)$

$$M_X - (M_{D^0} + M_{\bar{D}^{*0}}) \sim 0$$

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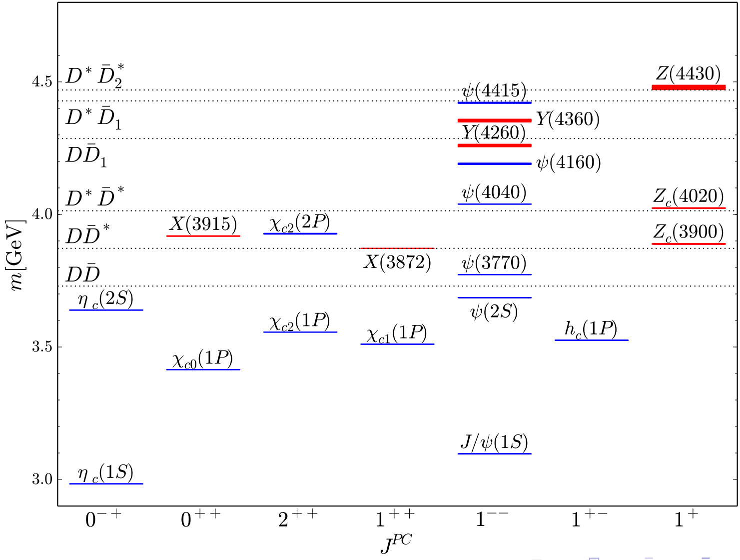
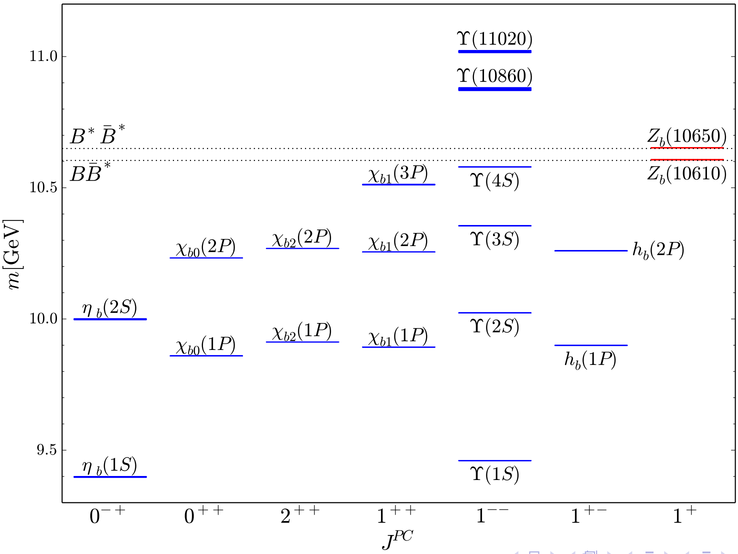
Spectrum of charmonium



Spectrum of bottomonium



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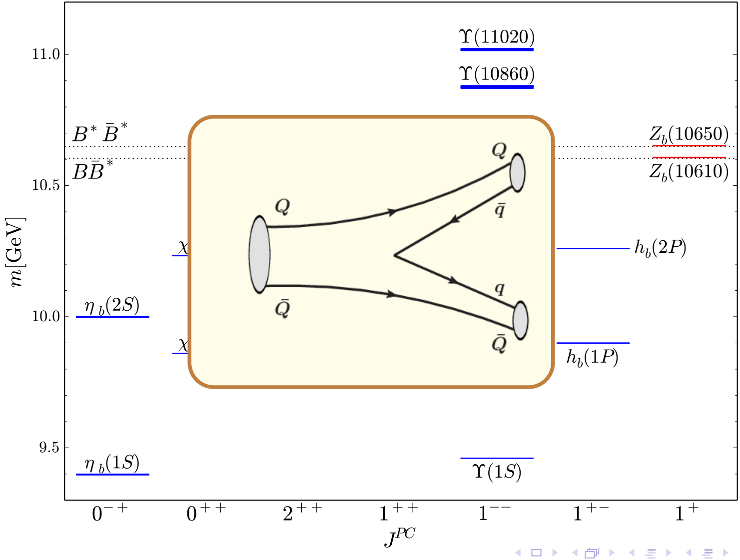


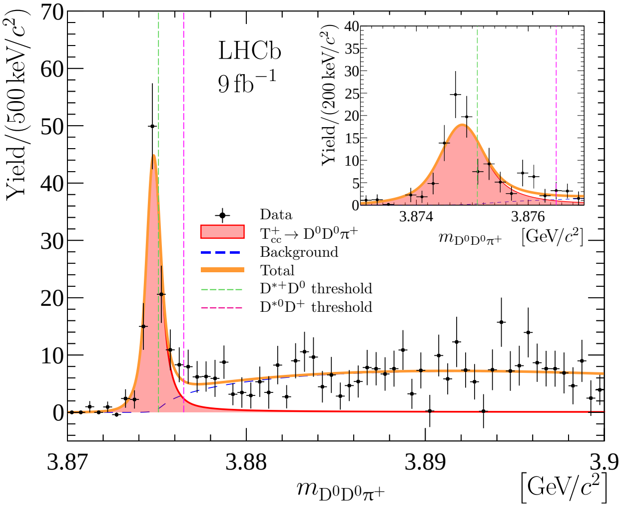
Double-charm state T_{cc}^+

$$I = 0 \quad J^P = 1^+$$

Minimal quark content: $cc\bar{u}\bar{d}$

$$T_{cc}^+ \to D^0 D^0 \pi^+,\ D^+ D^0 \pi^0,\ D^+ D^0 \gamma$$
$$T_{cc}^{++} \to D^+ D^0 \pi^+,\ D^+ D^+ \pi^0 \quad \Longrightarrow \quad \text{No signal}$$

T_{cc}^+ @ LHCb (Nature Phys. 18 (2022) 7, 751)



$$\delta m_{\rm BW} = -273 \pm 61 \pm 5^{+11}_{-14} \text{ keV} \quad \Gamma_{\rm BW} = 410 \pm 165 \pm 43^{+18}_{-38} \text{ keV}$$

T_{cc}^+ versus $X(3872)$

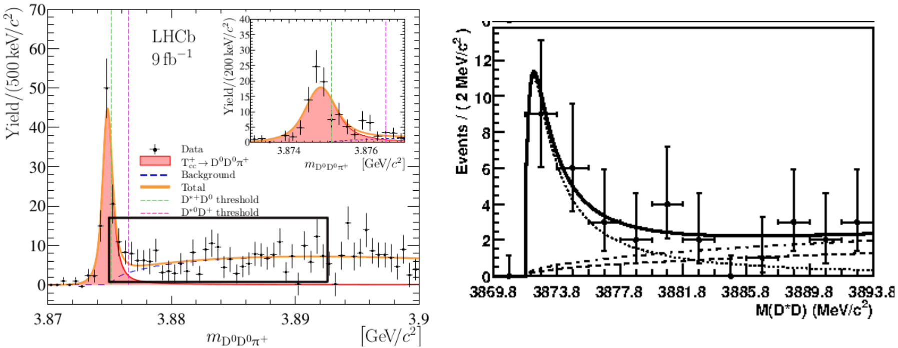
Common features:

- Isoscalars ($X(3872) = c\bar{c}/c\bar{c}u\bar{u}/c\bar{c}d\bar{d}$, $T_{cc}^+ = cc\bar{u}\bar{d}$)
- Definitely contain a pair of heavy quarks
- Reside incredibly close to 2-hadron thresholds $DD^*/D\bar{D}^*$
- Decay to open-charm final states $DD\pi/D\bar{D}\pi$ and $DD\gamma/D\bar{D}\gamma$
- Important consequences from $D^* \to D\pi$ decay

Difference:

- X contains $c\bar{c}$ pair while T_{cc}^+ contains cc
- X decays to hidden-charm states while T_{cc}^+ does not
- Short-range core (if any): $c\bar{c}$ charmonium vs tetraquark $cc\bar{u}\bar{d}$
- The main observation modes: $\pi\pi J/\psi$ for X and $DD\pi$ for T_{cc}^+

Experimental progress: 2021 versus 2003

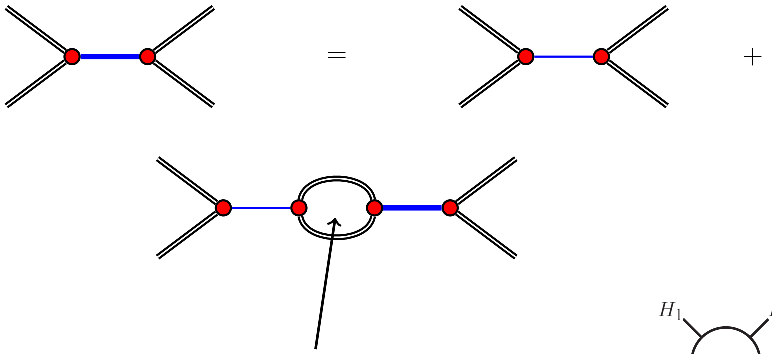


- Higher precision: smaller bins & better known resolution function
- Larger statistics: small uncertainties
- Data below two-body threshold: clear below-threshold peak

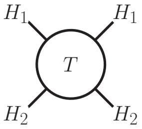
Generalities

Effect of hadronic loops

$$|\Psi\rangle = \begin{pmatrix} \sqrt{Z}|\psi_0\rangle \\ \chi(\boldsymbol{p})|H_1H_2\rangle_{L=0} \end{pmatrix}$$



Flatté: $\dfrac{1}{E - E_f + \frac{i}{2}(gp + \Gamma)}$ $\Longrightarrow$ parameterised



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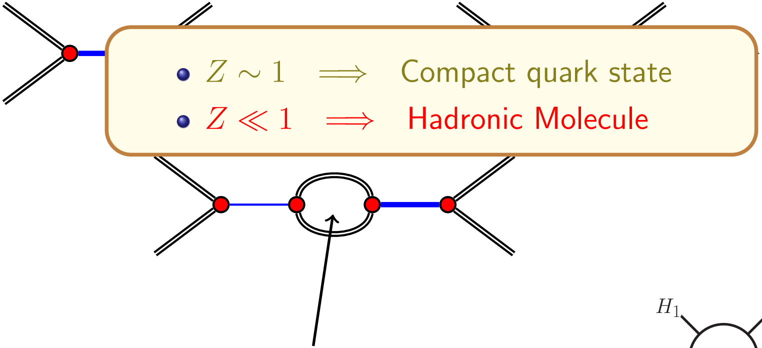
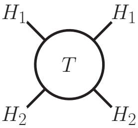


- $Z \sim 1 \implies$ Compact quark state
- $Z \ll 1 \implies$ Hadronic Molecule

Flatté: $\dfrac{1}{E - E_f + \frac{i}{2}(gp + \Gamma)} \implies$ parameterised



Effect of hadronic loops

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$$E - E_f + \frac{i}{2}(gp + \Gamma) = -\frac{g}{2}\left\{\underbrace{\left[\frac{2}{g}E_f - \frac{i}{g}\Gamma\right]}_{a^{-1}} + \frac{1}{2}\underbrace{\left[-\frac{2}{\mu g}\right]}_{r} p^2 - ip\right\}$$

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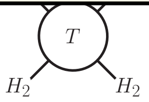

Composite or elementary?

Effective range expansion: $a^{-1} + \frac{1}{2} r p^2 - ip$

$$a = -\frac{2(1-Z)}{(2-Z)} \frac{1}{\sqrt{2\mu E_B}} + O\left(\frac{1}{\beta}\right) \qquad r = -\frac{Z}{(1-Z)} \frac{1}{\sqrt{2\mu E_B}} + O\left(\frac{1}{\beta}\right)$$

β ($\gg p$) — (inverse) range of force

(Weinberg'1960s)

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$$\boxed{Z = 1 - \frac{1}{\sqrt{1+2r/a}}}$$

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β ($\gg p$) — (inverse) range of force

(Weinberg'1960s)

$$Z=1-\frac{1}{\sqrt{1+2r/a}}$$

Elementary (confined) state

- Two near-threshold poles

Composite (molecular) state

- One near-threshold pole

$\Longrightarrow$ pole counting rules (Morgan'1992)

Effective range of a molecule

- Smorodinsky: $r > 0$ for finite-range negative potential (Smorodinsky'1948,Esposito et al.'2021)
- Wigner: causality bounds r from above ($r < 0$ for zero-range potentials) (Wigner'1955)
- Molecule: r is defined by range corrections (Weinberg'1960s)

$$r = \underbrace{-\frac{Z}{(1-Z)}\frac{1}{\sqrt{2\mu E_B}}}_{\text{small for } Z \to 0} + \Delta r(\beta)$$

For zero-range potential $\beta \sim m_\pi$

$$\Delta r(\beta \sim m_\pi) \sim \frac{1}{m_\pi} \sim 1 \text{ fm}$$

Weinberg(like) analysis in physics of heavy flavours

- Resonances reside near S-wave two-body threshold (Yes)
- Bound states (Not always)

  Solution: Employ suitable generalisation of Weinberg formulae

  Spectral density for continuum spectrum (Bogdanova et al'1991)

  $\bar{X} = 1 - Z \to 1/\sqrt{1+2|r/a|}$ (Matuschek et al.'2021)

  $$\bar{X} = 1 - \exp\left(\frac{1}{\pi}\int_0^\infty dE\,\frac{\delta_B(E)}{E-E_B}\right)$$ (Li et al.'2021)

- Stable constituents (Almost never)

  Solution: $p_{\rm eff} = \sqrt{2\mu(E+i\frac{\Gamma}{2})} \Longrightarrow$ ERE at complex point (Braaten et al.'2010)
- No additional thresholds near by (Rarely)

  Solution: Expand contributions from additional channels at $p_1 \to 0$ (caution !!!)
- No additional singularities (Matter of luck)
  - Poles $p\cot\delta$
  - Left-hand cuts
  - ...

  Solution: No general solution...

Interplay of different dynamics

$$|\Psi\rangle = \begin{pmatrix} \sqrt{Z}|\psi_0\rangle \\ \chi(\boldsymbol{p})|H_1 H_2\rangle_{L=0} \end{pmatrix} \qquad H = \begin{pmatrix} E_0 & v(\boldsymbol{p}) \\ v(\boldsymbol{p}) & \frac{p^2}{2\mu} + V_{H_1 H_2} \end{pmatrix}$$

Scattering amplitude $H_1 H_2 \to H_1 H_2$ via $V_{H_1 H_2}$

$$f_V(E) \approx \frac{1}{-\gamma_V - ip} \qquad p = \sqrt{2\mu E}$$

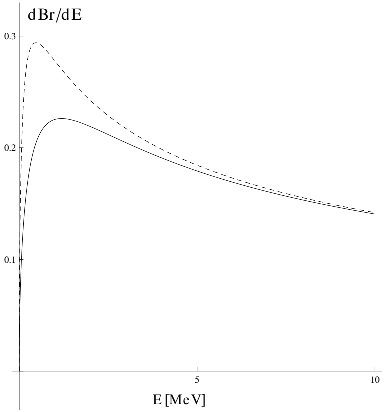
Full scattering amplitude $H_1 H_2 \to H_1 H_2$ (effective coupling g is related to $v(0)$)

$$f(E) = \frac{1}{p\cot\delta - ip}$$

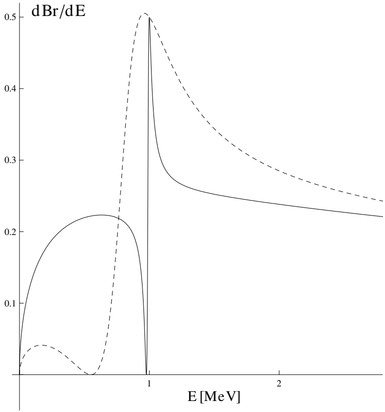
$$p\cot\delta = \frac{2}{g}\left(E - E_f - \frac{(E-E_f)^2}{E-E_C}\right) \quad \text{with} \quad E_C = E_f - \frac{1}{2}g\gamma_V$$

Interplay of different dynamics

$|E_C| \gg |E_f|$



$|E_C| \sim |E_f|$



Generalisation to two-channel case

- Naive expansion

$$E - E_f + \frac{i}{2}g(p_1 + p_2) = \frac{p_1^2}{2\mu} - E_f + \frac{i}{2}g\Big(p_1 + \underbrace{\sqrt{2\mu\Delta - p_1^2}}_{\text{expand for } p_1 \to 0}\Big)$$

$$r = r_0 + \delta r \qquad r_0 = -\frac{2}{\mu g} \qquad \delta r = -\frac{1}{\sqrt{2\mu\Delta}} \underset{\Delta\to 0}{\longrightarrow} \infty \ (!!!)$$

- Educated expansion: Use exact two-channel expression

$$Z = \left(1 - \frac{1}{r_0}\left(\frac{1}{\sqrt{2\mu E_B}} + \frac{1}{\sqrt{2\mu(E_B+\Delta)}}\right)\right)^{-1}$$

in Weinberg formula for r

$$r = r_0 \frac{\sqrt{E_B+\Delta}}{\sqrt{E_B} + \sqrt{E_B+\Delta}} \underset{\Delta\gg E_B}{\longrightarrow} r_0$$

Can we do without ERE?

Probalility to observe resonance in the α-th channel $(\alpha = 1, 2)$

(Hyodo et al,'2012,Aceti & Oset'2012)

$$X_\alpha = g_\alpha^2 \left[\frac{d}{dM^2} \int \frac{d^3p}{(2\pi)^3} G_\alpha(M,p) \right]_{|M=M_{\rm pole}}$$

with the couplings defined as residues

$$g_\alpha g_\beta = \lim_{M \to M_{\rm pole}} (M^2 - M_{\rm pole}^2) T_{\alpha\beta}(M)$$

In neglect of constituents widths

$$X_1 = \frac{\sqrt{E_B + \Delta}}{\sqrt{E_B} + \sqrt{E_B + \Delta}} \qquad X_2 = \frac{\sqrt{E_B}}{\sqrt{E_B} + \sqrt{E_B + \Delta}}$$

Generalisation to compact component

Single hadronic channel

$$Z \propto \sqrt{E_B} \qquad X = 1 - Z$$

Two hadronic channels ($\mu_1 = \mu_2 = \mu$)

$$Z = \frac{R_0}{R_0 + R_1 + R_2} \qquad X_1 = \frac{R_1}{R_0 + R_1 + R_2} \qquad X_2 = \frac{R_2}{R_0 + R_1 + R_2}$$

where

$$R_0 = \frac{2}{\mu g} = |r_0| \qquad R_1 = \frac{1}{\sqrt{2\mu E_B}} \qquad R_2 = \frac{1}{\sqrt{2\mu(E_B + \Delta)}}$$

$$\Delta = M_2^{\rm th} - M_1^{\rm th}$$

Heavy-quark spin symmetry

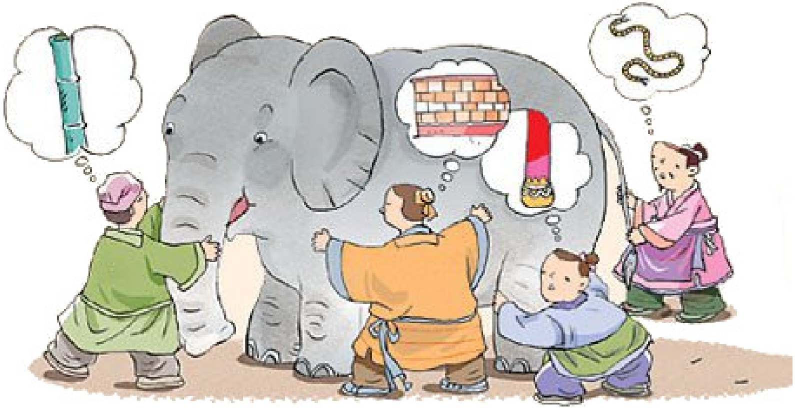
- Exotic states contain heavy quarks (HQ)
- In the limit $m_Q \to \infty$ $(m_Q \gg \Lambda_{\rm QCD})$ spin of HQ decouples

  $\Longrightarrow$ Heavy Quark Spin Symmetry (HQSS)
- For realistic m_Q's HQSS is approximate but accurate symmetry of QCD

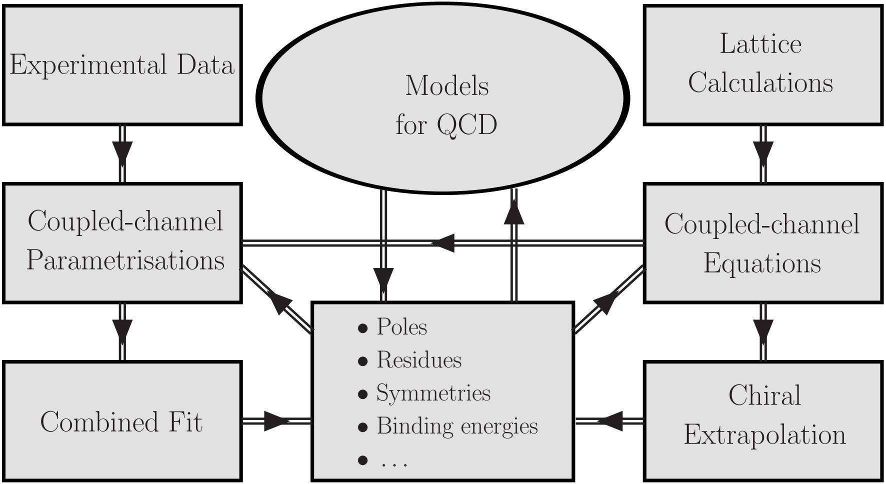
  $\Longrightarrow$ Relations between parameters of the theory
- HQSS $=$ tool to relate properties of states with different HQ spin orientation

  $\Longrightarrow$ Spin partners

Combined analysis



Approach to near-threshold exotic states



Analysis of experimental data on T_{cc}^{+}

Simple Flatté fit ($\chi^2/N_{\rm dof} \approx 1$)

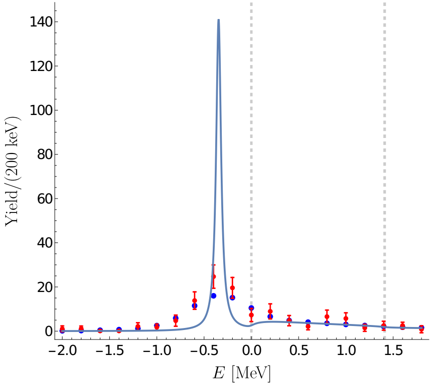
$$\mathcal{A} = \frac{\sqrt{\mathcal{N}}}{E - E_f + \frac{i}{2}\left[g(\tilde{p}_1 + \tilde{p}_2) + \Gamma\right]}$$

$$\tilde{p} = \sqrt{2\mu\left[E - m_D - \left(m_{D^*} - \frac{i}{2}\Gamma_{D^*}\right)\right]}$$

$\Gamma^{\rm fit} = 0 \implies$ No compact component

Pole position:

$$E_{\rm pole} = (-347 - i31)\ \text{keV}$$



In neglect of D^* width:

$$X_1 = \frac{\sqrt{E_B + \Delta}}{\sqrt{E_B} + \sqrt{E_B + \Delta}} \qquad X_2 = \frac{\sqrt{E_B}}{\sqrt{E_B} + \sqrt{E_B + \Delta}}$$

For $E_B = 347$ keV and $\Delta = 1.41$ MeV: $X_1 = 0.7$ $X_2 = 0.3$

Simple Flatté fit ($\chi^2/N_{\rm dof} \approx 1$)

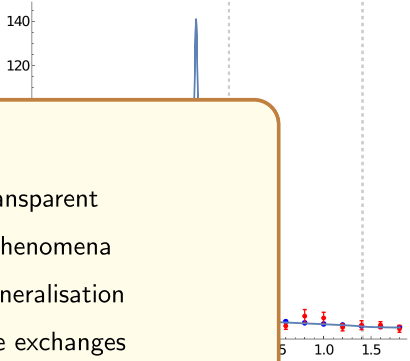
$$\mathcal{A} = \frac{\sqrt{\mathcal{N}}}{E - E_f + \frac{i}{2}\left[g(\tilde{p}_1 + \tilde{p}_2) + \Gamma\right]}$$

$\tilde{p} = \sqrt{2\mu\left[\right.}$

$\Gamma^{\rm fit} = 0$ =

Pole posi

$E_{\rm pol}$



Flatté parametrisation:

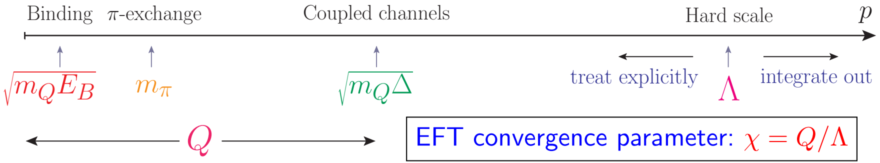
- \+ Simple and physically transparent
- \+ Accounts for threshold phenomena
- – Difficult multichannel generalisation
- – Obscure effect of particle exchanges
- – Not systematically improvable

In neglect

$$X_1 = \frac{\sqrt{E_B + \Delta}}{\sqrt{E_B} + \sqrt{E_B + \Delta}} \qquad X_2 = \frac{\sqrt{E_B}}{\sqrt{E_B} + \sqrt{E_B + \Delta}}$$

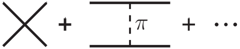
For $E_B = 347$ keV and $\Delta = 1.41$ MeV: $X_1 = 0.7$ $X_2 = 0.3$

Effective field theory for hadronic molecules



Interaction potential between heavy hadrons:

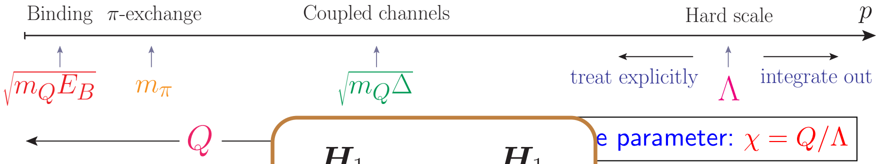
- Includes all relevant interactions

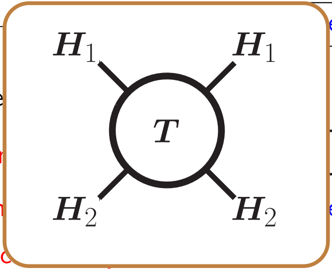


- Complies with relevant symmetries (chiral, HQSS, etc)
- Incorporates coupled-channel dynamics
- Expanded in powers of p^2/Λ^2 and truncated at necessary order (LO, NLO...)
- Iterated to all orders via (multichannel) Lippmann-Schwinger equation

$$T = V - VGT$$

Effective field theory for hadronic molecules





Interaction potential betwe

- Includes all relevant i ... $+ \cdots$
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- Iterated to all orders via (multichannel) Lippmann-Schwinger equation

$$T = V - VGT$$

Effective field theory for hadronic molecules

Free parameters:

- Low-energy constants
- (Bare) couplings to hadronic channels

Input (combined analysis):

- Line shapes (Dalitz plots)
- Partial branchings

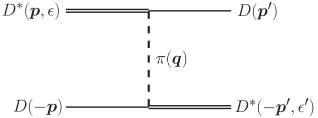
Output:

- Pole position M_0 ("mass" $=\mathrm{Re}(M_0)$, "width" $= 2\times \mathrm{Im}(M_0)$)
- Residues at the poles (dressed couplings)

Predictions:

- New properties of state: line shapes, partial widths,...
- Spin partners: poles, line shapes, partial widths,...
- Chiral extrapolations

Pion exchange in $I=0$ DD^* system



$$V_\pi(\boldsymbol{p},\boldsymbol{p}') = \left(\frac{g_c}{2f_\pi}\right)^2 \langle \boldsymbol{\tau}\cdot\boldsymbol{\tau}\rangle \frac{(\boldsymbol{\epsilon}\cdot\boldsymbol{q})(\boldsymbol{q}\cdot\boldsymbol{\epsilon}'^*)}{u-m_\pi^2}$$

$\Longrightarrow$ $I=0$, central, ~~recoil~~

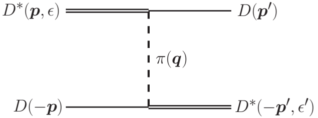
$$\left(\frac{g_c}{2f_\pi}\right)^2 \left(-1 + \overbrace{\frac{\mu_\pi^2}{\boldsymbol{q}^2 + \underbrace{[m_\pi^2 - (m_{D^*}-m_D)^2]}_{\text{Effective mass } \mu_\pi^2}}}^{\text{Long-range OPE}}\right)$$

- Short-range OPE absorbed by (re-fitted) contact interaction
- Perturbative (?) long-range OPE as per

$$\alpha_\pi^{\text{eff}} = \frac{g_c^2|\mu_\pi^2|}{f_\pi^2} \ll 1$$

(XEFT: Voloshin'2004,Fleming et al.'2007,...)

Pion exchange in $I = 0$ DD^* system



$$V_\pi(\boldsymbol{p}, \boldsymbol{p}') = \left(\frac{g_c}{2f_\pi}\right)^2 \langle \boldsymbol{\tau} \cdot \boldsymbol{\tau} \rangle \frac{(\boldsymbol{\epsilon} \cdot \boldsymbol{q})(\boldsymbol{q} \cdot \boldsymbol{\epsilon}'^*)}{u - m_\pi^2}$$

$$\Longrightarrow \left(\frac{g_c}{2f_\pi}\right)^2 \left(-1 + \overbrace{\frac{\mu_\pi^2}{q^2 + [m_\pi^2 - (\ldots)^2]}}^{\text{Long-range OPE}}\right)$$

Is pion exchange important in T_{cc}^+?

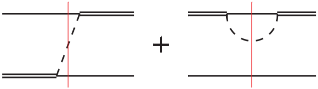
- Short-range OPE absorbed by (re-fitted) contact interaction
- Perturbative (?) long-range OPE as per

$$\alpha_\pi^{\text{eff}} = \frac{g_c^2 |\mu_\pi^2|}{f_\pi^2} \ll 1$$

(XEFT: Voloshin'2004, Fleming et al.'2007,...)

Comment on pion exchange in T_{cc}^+

- Physical T_{cc}^+ ($m_\pi < m_{D^*} - m_D \implies \mu_\pi^2 < 0$ & $|\mu_\pi| \ll m_\pi$):

$\implies$ 3-body unitarity: 

$\implies$ T_{cc}^+ spin partner at D^*D^* threshold

$$\alpha_\pi^{D\text{-wave}} \simeq g_c^2 q_{\rm typ}^2/f_\pi^2 \simeq g_c^2 m_D(m_{D^*} - m_D)/f_\pi^2 > 1$$

Comment on pion exchange in T_{cc}^+

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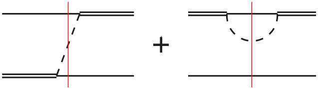
$\implies$ 3-body unitarity: 

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$$\alpha_\pi^{D\text{-wave}} \simeq g_c^2 q_{\rm typ}^2/f_\pi^2 \simeq g_c^2 m_D (m_{D^*} - m_D)/f_\pi^2 > 1$$

- Lattice T_{cc}^+ ($m_\pi^{\rm lat} > m_{D^*}^{\rm lat} - m_D^{\rm lat} \implies (\mu_\pi^{\rm lat})^2 > 0$ & $\mu_\pi^{\rm lat} > m_\pi^{\rm ph}$):

$\implies$ $\alpha_\pi = g_c^2 \mu_\pi^2 / f_\pi^2 \sim 1$

$\implies$ Left-hand cut in partial-wave amplitudes

$$\int d\Omega_{\hat{k}\hat{k}'} V_\pi(\boldsymbol{k} - \boldsymbol{k}') \sim \log \frac{\mu_\pi^2 + (k+k')^2}{\mu_\pi^2 + (k-k')^2} \underset{k'=k=p}{\Longrightarrow} \log\left(1 + \frac{4p^2}{\mu_\pi^2}\right)$$

EFT approach to physical T_{cc}^+

$$\left.\begin{aligned}
\gamma_B &= \sqrt{m_D E_B} \simeq 25\ \text{MeV} \\
p_\pi &\simeq |\mu_\pi| \simeq 40\ \text{MeV} \\
p_{\text{data}}^{\text{max}} &= \sqrt{m_D\, \Delta E_{\text{data}}} \simeq 100\ \text{MeV} \\
p_{\text{coupl.ch.}} &= \sqrt{m_D(m_{D^*} - m_D)} \simeq 500\ \text{MeV}
\end{aligned}\right\} \Longrightarrow$$

$\Lambda = 500$ MeV
Potential at LO
OPE included
No couple channels

EFT approach to physical T_{cc}^+

$$\left.\begin{aligned} \gamma_B &= \sqrt{m_D E_B} \simeq 25 \text{ MeV} \\ p_\pi &\simeq |\mu_\pi| \simeq 40 \text{ MeV} \\ p_{\text{data}}^{\text{max}} &= \sqrt{m_D \, \Delta E_{\text{data}}} \simeq 100 \text{ MeV} \\ p_{\text{coupl.ch.}} &= \sqrt{m_D(m_{D^*} - m_D)} \simeq 500 \text{ MeV} \end{aligned}\right\} \Longrightarrow$$

$\Lambda = 500$ MeV
Potential at LO
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No couple channels

- Lippmann-Schwinger equation for scattering amplitude (1 free parameter)

$$T(M,p,p') = V(M,p,p') - \int \frac{d^3q}{(2\pi)^3} V(M,p,q) G(M,q) T(M,q,p')$$

$$V(M,p,p') = v_0 + V_{\text{OPE}}$$

- Production amplitude (1 additional free parameter: P = point-like source)

$$U(M,p) = P - \int \frac{d^3q}{(2\pi)^3} T(M,p,q) G(M,q) P$$

EFT approach to physical T_{cc}^+

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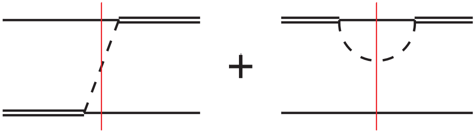
No couple channels

- Lippmann-S ... parameter)

$T(M,$ ... $T(M,q,p')$

3-body effects:

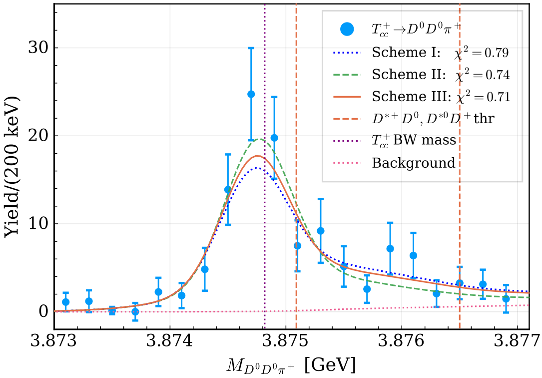


- Production amplitude (1 additional free parameter: P = point-like source)

$$U(M,p) = P - \int \frac{d^3q}{(2\pi)^3} T(M,p,q) G(M,q) P$$

Fitting schemes, results, and conclusions

	Γ_{D^*}=const, ~~OPE~~	$\Gamma_{D^*}(p,M)$, ~~OPE~~	$\Gamma_{D^*}(p,M)$, OPE
χ^2/d.o.f.	0.79	0.74	0.71
v_0 [GeV^{-2}]	-23.34 ± 0.08	$-22.88^{+0.08}_{-0.06}$	$-5.04^{+0.10}_{-0.08}$
Pole [keV]	$-368^{+43}_{-42} - i(37 \pm 0)$	$-333^{+41}_{-36} - i(18 \pm 1)$	$-356^{+39}_{-38} - i(28 \pm 1)$



- (Quasi)bound state just below $D^{*+}D^0$ threshold
- Compositeness: 70% & 30%

Fitting schemes, results, and conclusions

	Γ_{D^*}=const, ~~OPE~~	$\Gamma_{D^*}(p,M)$, ~~OPE~~	$\Gamma_{D^*}(p,M)$, OPE
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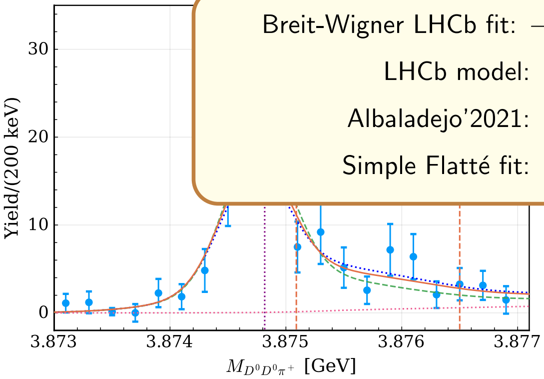

Breit-Wigner LHCb fit: $-273 - i410$ keV

LHCb model: $-360 - i24$ keV

Albaladejo'2021: $-356 - i39$ keV

Simple Flatté fit: $-347 - i31$ keV

state just below

old

- Compositeness: 70% & 30%

Spin partner T_{cc}^{*+}

HQSS: $V^{I=0}(D^*D^* \to D^*D^*, 1^+) = V^{I=0}(D^*D \to D^*D, 1^+) = v_0$

T_{cc}^{+} at D^*D threshold hints existence of T_{cc}^{*+} at D^*D^* threshold

Scheme I:	$\delta_{cc}^{*+} = -1.4$ MeV
Scheme II:	$\delta_{cc}^{*+} = -1.1$ MeV
Scheme III:	$\delta_{cc}^{*+} = -0.5$ MeV

where $\delta_{cc}^{*+} = m_{T_{cc}^{*+}} - m_c^* - m_0^*$

Spin partner T_{cc}^{*+}

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where $\delta_{cc}^{*+} = m_{T_{cc}^{*+}} - m_c^* - m_0^*$

Disclaimer:

- Coupled-channel effects D^*D-D^*D^* neglected
- Multi-body effects & OPE included not selfconsistently
- Experimental signal in $D^*D^* \to DD\pi\pi$ channel may be erroneously attributed to $DD^* \to DD\pi$ channel if one soft pion is lost

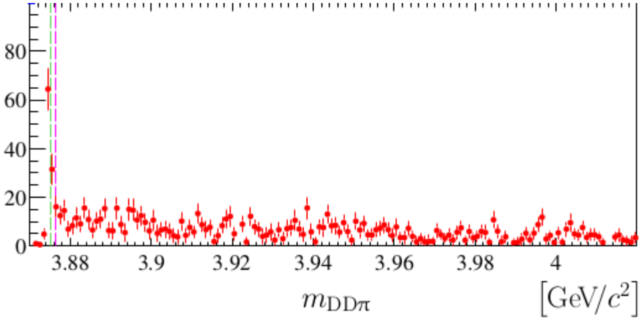
Conclusion: T_{cc}^{*+} is likely to exist but no reliable prediction is possible yet

Spin partner T_{cc}^{*+}

HQSS: $V^{I=0}(D^*D^* \to D^*D^*, 1^+) = V^{I=0}(D^*D \to D^*D, 1^+) = v_0$

T_{cc}^+ at

LHCb Collab., Nature Communications, **13**, 3351 (2022)



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Conclusion: T_{cc}^{*+} is likely to exist but no reliable prediction is possible yet

Analysis of lattice data on T_{cc}^{+}

Lattice studies of T_{cc}^+

- “Signature of a Doubly Charm Tetraquark Pole in DD^* Scattering on Lattice,” M. Padmanath and S. Prelovsek, Phys. Rev. Lett. **129**, 032002 (2022)
  “Towards the quark mass dependence of T_{cc}^+ from lattice QCD, S. Collins, A. Nefediev, M. Padmanath and S. Prelovsek, Phys. Rev. D **109**, 9 (2024)

  $m_\pi = 280$ MeV   5 points in m_c

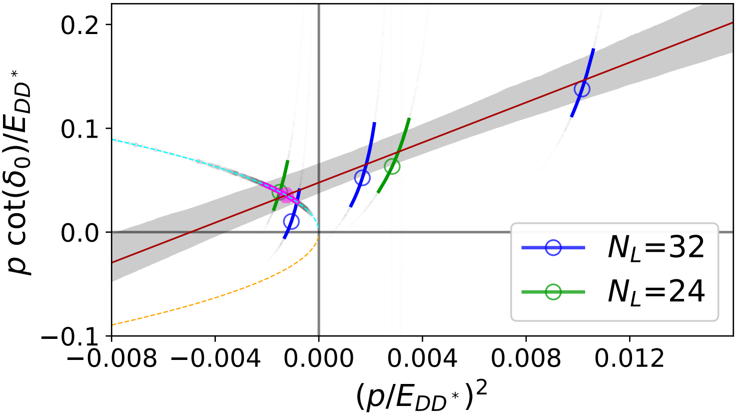
- “$T_{cc}^+(3875)$ relevant DD^* scattering from $N_f = 2$ lattice QCD,” S. Chen, C. Shi, Y. Chen, M. Gong, Z. Liu, W. Sun and R. Zhang, Phys. Lett. B **833**, 137391 (2022)

  $m_\pi = 348$ MeV

- “Doubly Charmed Tetraquark T_{cc}^+ from Lattice QCD near Physical Point,” Y. Lyu, S. Aoki, T. Doi, T. Hatsuda, Y. Ikeda and J. Meng, Phys. Rev. Lett. **131**, 161901 (2023)

  $m_\pi = 146$ MeV   HALQCD technique

ERE analysis of lattice data for T_{cc}^+



(Padmanath & Prelovsek'2022)

$$-\frac{2\pi}{\mu}T^{-1}(E) = p\cot\delta - ip = \frac{1}{a} + \frac{1}{2}rp^2 - ip$$

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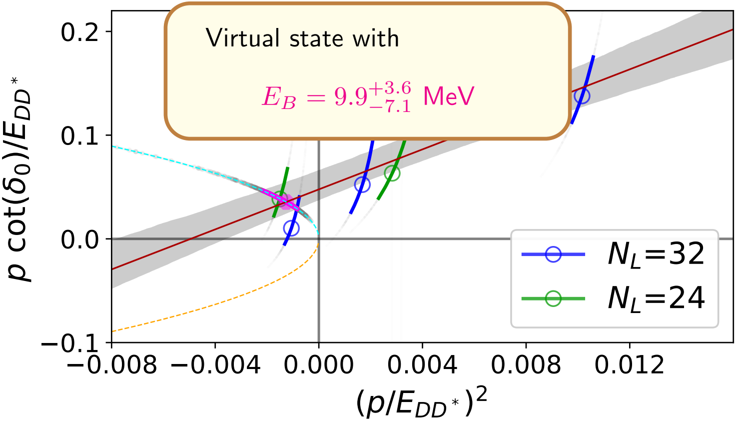

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$$-\frac{2\pi}{\mu}T^{-1}(E) = p\cot\delta - ip = \frac{1}{a} + \frac{1}{2}rp^2 - ip$$

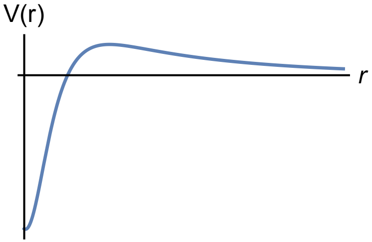
EFT analysis of lattice data for T_{cc}^+

Lippmann–Schwinger equation

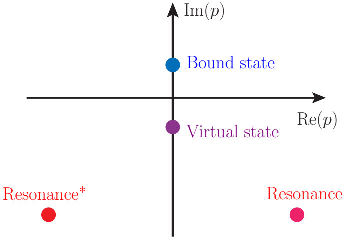
$$T(\boldsymbol{p},\boldsymbol{p}';E) = V(\boldsymbol{p},\boldsymbol{p}') - \int \frac{d^3k}{(2\pi)^3} V(\boldsymbol{p},\boldsymbol{k}) G(\boldsymbol{k};E) T(\boldsymbol{k},\boldsymbol{p}';E)$$

$$V(\boldsymbol{p},\boldsymbol{p}') = \underbrace{\left[2c_0 + 2c_2(p^2+p'^2)\right]}_{\text{Contact interactions}} + \underbrace{V_\pi^S(p,p')}_{S\text{-wave OPE}}$$

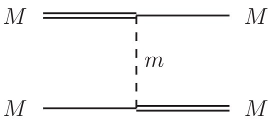
Sketch of full potential ($c_2 = 0$)

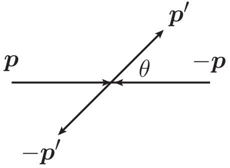


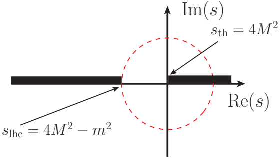
Types of supported poles



Left-hand cut





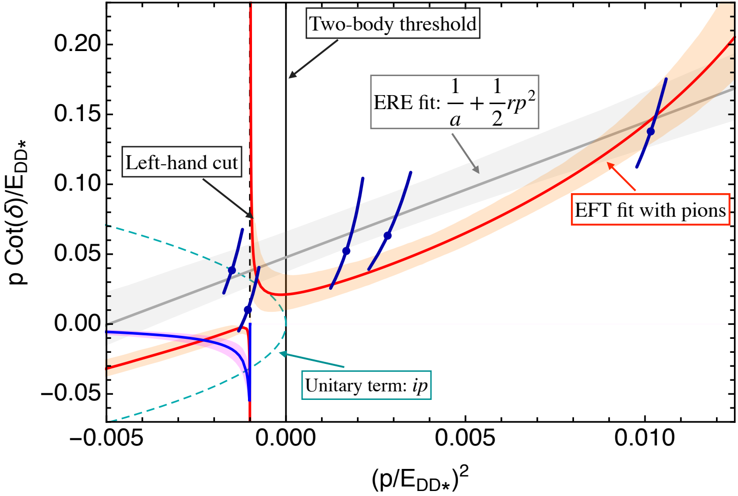


$$\mathcal{A} = \frac{1}{u - m^2} = -\frac{1}{m^2 + 2p^2(1 - \cos\theta)}$$

$$s = (p_1 + p_2)^2 = 4(p^2 + M^2) \quad \Longrightarrow \quad s_{\rm th} = 4M^2$$

$$\mathcal{A}_S = \int \frac{d\Omega}{4\pi}\mathcal{A} = \frac{1}{4p^2}\log\frac{m^2 + 4p^2}{m^2} \quad \Longrightarrow \quad s_{\rm lhc} = 4M^2 - m^2$$

EFT analysis of lattice data for T_{cc}^+



Lattice data: Padmanath & Prelovsek, Phys.Rev.Lett. 129 (2022), 032002
Theoretical curve: Du et al., Phys.Rev.Lett. 131 (2023), 131903

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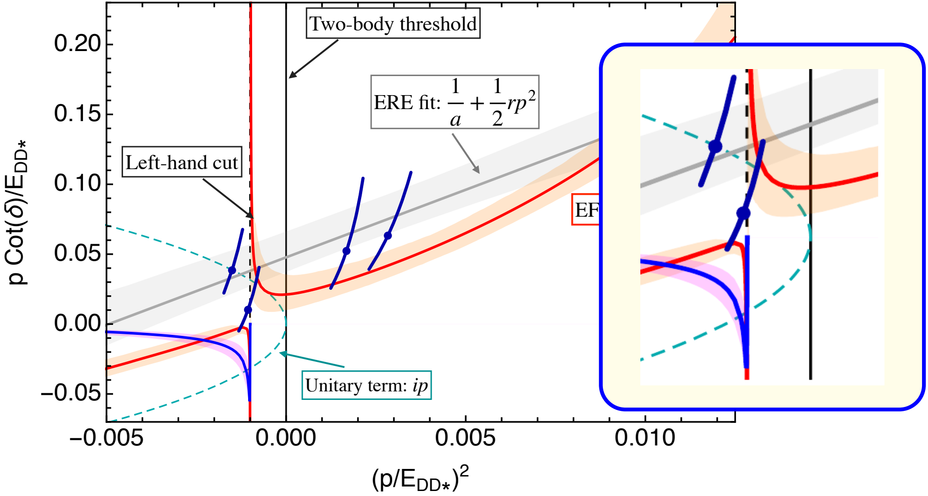

Lattice data: Padmanath & Prelovsek, Phys.Rev.Lett. 129 (2022), 032002
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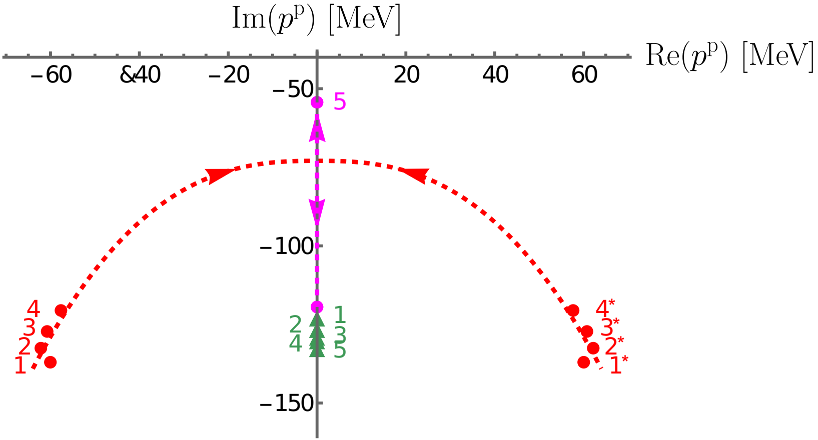
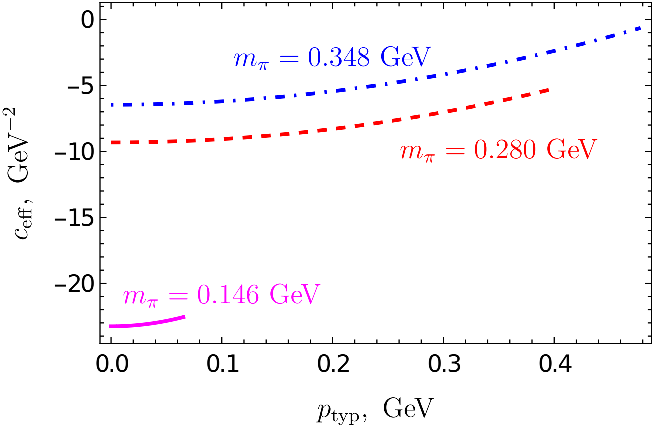
Lattice T_{cc}^+ pole dependence on m_c



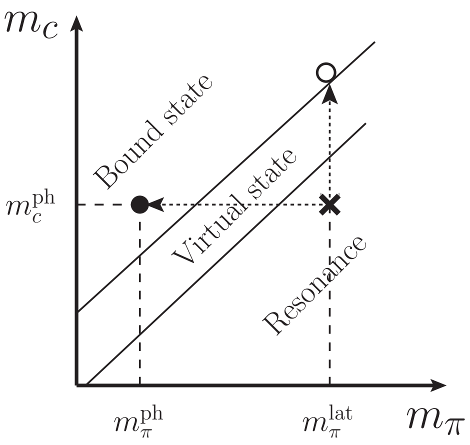
Comment on lattice T_{cc}^+ pole dependence on m_π

$$c_{\rm eff}(p_{\rm typ}) = 2c_0 + 4c_2 p_{\rm typ}^2$$



T_{cc}^+ pole motion across (m_c, m_π) plane

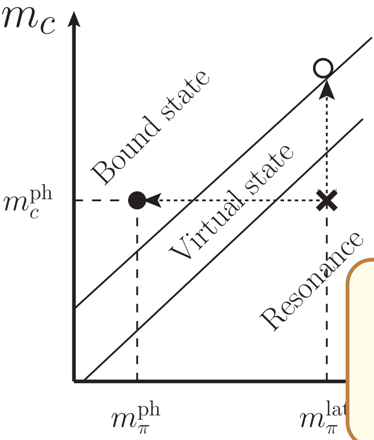
(Collins et al.'2024)



- Filled circle — physical T_{cc}^+
- Cross — starting lattice point
- Open circle — lattice T_{cc}^+ as shallow bound state

T_{cc}^+ pole motion across (m_c, m_π) plane

(Collins et al.'2024)



- Filled circle — physical T_{cc}^+
- Cross — starting lattice point
- Open circle — lattice T_{cc}^+ as shallow bound state

The pole position of the physical T_{cc}^+ comes as a result of a delicate interplay of the light quark mass m_q and the charmed quark mass m_c

Conclusions

- Physics of narrow near-threshold exotic states is dynamical and exciting branch of contemporary hadronic physics
- Two complementary sources of information: experiment & lattice
- Well established theoretical tools $\Longrightarrow$ reliable conclusions
- T_{cc}^+ — new surprise from experiment (though predicted theoretically in 2004)
- Another prominent example of hadronic molecule (bound state)
- Physical T_{cc}^+ comes as a result of fine tuned of m_q/m_c in Nature
- What can we learn/find out on T_{bc} (lattice & experiment) and T_{bb} (lattice)?

References & acknowledgments

Collaboration with colleagues from China, Germany, India, Slovenia, Spain is gratefully acknowledged!

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- M. L. Du, V. Baru, X. K. Dong, A. Filin, F. K. Guo, C. Hanhart, A. Nefediev, J. Nieves, Q. Wang, "Coupled-channel approach to T_{cc}^{+} including three-body effects," Phys. Rev. D **105**, 014024 (2022)
- V. Baru, X. K. Dong, M. L. Du, A. Filin, F. K. Guo, C. Hanhart, A. Nefediev, J. Nieves, Q. Wang, "Effective range expansion for narrow near-threshold resonances," Phys. Lett. B **833**, 137290 (2022)
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Backup

Spectral density

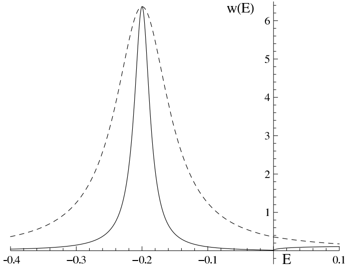
Hint: Extract information from continuum w.f. $(E = k^2/(2\mu))$

$$|\Psi\rangle = C_k|\psi_0\rangle + \chi_k(p)|H_1H_2\rangle$$

$$w(E) = 4\pi\mu k|C_k|^2\Theta(E - E_{\rm th}^{\rm min}) = \frac{1}{2\pi i}\left[\frac{1}{E - E_0 + \mathcal{G}^*(E)} - \text{c.c.}\right]$$

(Bogdanova et al.'1991, Baru *et al*'2004)

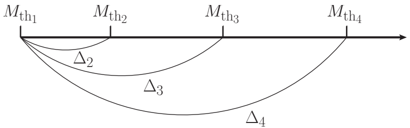
$$\text{“}Z\text{”} = W = \int_{E_{\rm th}-\delta}^{E_{\rm th}+\delta} w(E)dE \qquad (\delta \text{ is not well defined})$$



$$W_{\rm solid} = \int_{-0.6\,{\rm MeV}}^{0.2\,{\rm MeV}} w_{\rm solid}(E)dE \approx 0.3$$

$$W_{\rm dashed} = \int_{-0.6\,{\rm MeV}}^{0.2\,{\rm MeV}} w_{\rm dashed}(E)dE \approx 0.9$$

Generalisation to multiple hadronic channels



$$|\Psi\rangle = \begin{pmatrix} \sqrt{Z}|\psi_0\rangle \\ \chi_1(\boldsymbol{p})|H_{11}H_{12}\rangle \\ \chi_2(\boldsymbol{p})|H_{21}H_{22}\rangle \\ \cdots \end{pmatrix} \qquad H = \begin{pmatrix} E_0 & f_1 & f_2 & \cdots \\ f_1 & H_{h_1} & V_{12} & \cdots \\ f_2 & V_{21} & H_{h_2} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix}$$

$$H_{h_i}(\boldsymbol{p}, \boldsymbol{p}') = \left(\Delta_i + \frac{p^2}{2\mu_i}\right) \delta^{(3)}(\boldsymbol{p} - \boldsymbol{p}') + V_{ii}(\boldsymbol{p}, \boldsymbol{p}')$$

For two channels V_{ij} $(i, j = 1, 2)$ is parametrised through the singlet and triplet inversed scattering lengths γ_s and γ_t:

- γ_s governs the position of the zero E_C
- γ_t governs the relevance of the term $k_1 k_2$

Solution of the Lippmann-Schwinger equation

$$t_s = \frac{1}{2}(t_{11} + t_{22}) + t_{12} = \frac{(E - E_C)\,(2\gamma_t + i(k_1 + k_2))}{4\pi^2\mu\ D(E)}$$

$$t_t = \frac{1}{2}(t_{11} + t_{22}) - t_{12} = \frac{2\gamma_s(E - E_f) + i(k_1 + k_2)(E - E_C)}{4\pi^2\mu\ D(E)}$$

$$t_{st} = \frac{1}{2}(t_{11} - t_{22}) = \frac{i(k_2 - k_1)(E - E_C)}{4\pi^2\mu\ D(E)}$$

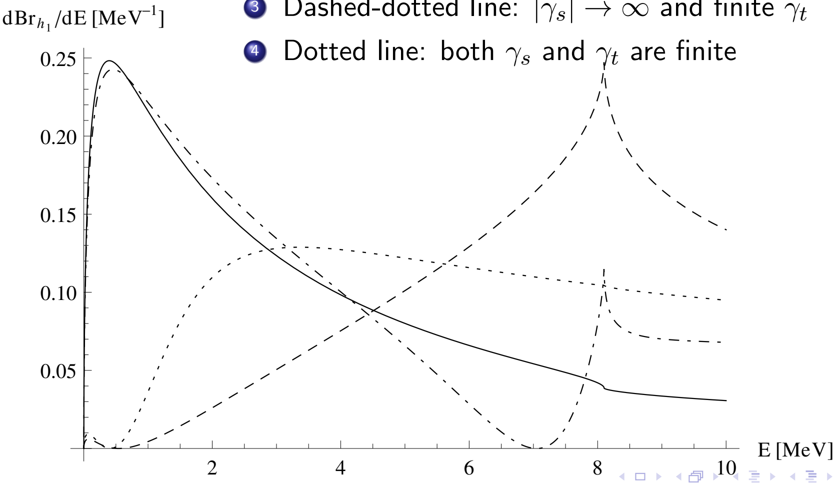
$$D(E) = \gamma_s\Big(2\gamma_t + i(k_1 + k_2)\Big)(E - E_f) - \Big(2k_1 k_2 - i\gamma_t(k_1 + k_2)\Big)(E - E_C)$$

$$E_C = E_f - \frac{1}{2}g\gamma_s$$

(Artoisenet et al.'2010, Hanhart et al.'2011)

Examples of the line shapes

1. Solid line: $|\gamma_s| \to \infty$ and $|\gamma_t| \to \infty$
2. Dashed line: finite γ_s and $|\gamma_t| \to \infty$
3. Dashed-dotted line: $|\gamma_s| \to \infty$ and finite γ_t
4. Dotted line: both γ_s and γ_t are finite



OPE sign

	$I=0$	$I=1$
PV	3	1
$(P\bar{V})_{C=\pm}$	$3C$	$-C$

$X(3872)$ $(I=0,\, C=+)$	T_{cc} $(I=0)$	Z_b $(I=1,\, C=-)$	W_{bJ} $(I=1,\, C=+)$
$+3$	$+3$	$+1$	-1

Comment on lattice studies of T_{cc}^{+}