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Progress on chiral EFT studies of eta and eta'

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Reminder of chiral symmetry of QCD

- QCD with $m_{u,d,s} = 0$ at classic level: $U_L(3) \otimes U_R(3)$.
- $U_V(1) \equiv U_{L+R}$: conserved baryon number.
- $U_A(1) \equiv U_{L-R}$: violated at the quantum level

  QCD $U(1)_A$ anomaly $\Rightarrow$ massive η_0.
- $SU_L(3) \otimes SU_R(3) \to SU_V(3)$: Goldstone π, K, η_8 [$SU(3)$ χPT].

  [Gasser and Leutwyler, NPB'85]
- $N_C \to \infty$: $M^2_{\eta_0} \sim \mathcal{O}(1/N_C)$, $\therefore$ η_0 becomes Goldstone.

  [Witten, NPB'79]
- $U(3)$ χPT: π, K, η_8 and η_0, $\quad \delta \sim p^2 \sim m_q \sim 1/N_C$.
- Main decay channels: $\eta \to \gamma\gamma$ ($\sim 40\%$) and $\eta' \to \rho\gamma$ ($\sim 30\%$)

  Driven by the EM anomaly

η/η': ideal laboratory to study many striking features (chiral symmetry, large Nc, axial anomalies) of SM in the low energy

A quick glance at η and η'

- **Narrow-width hadrons:**

  $\Gamma_\eta = 1.31$ **keV** ($M_\eta = 548$ **MeV**), $\Gamma_{\eta'} = 188$ **keV** ($M_{\eta'} = 958$ **MeV**) ,

  to be compared with $\Gamma_\rho = 140$ **MeV** ($M_\rho = 775$ **MeV**).

- **Quantum numbers** $I^G J^{PC} = 0^+ \, 0^{-+}$

  C **&** ***P*** **eigenstates;**

  all additive quantum numbers are zero: $I=J=S=C=B=Q=L=0$

- **Precicion study in η/η' : discrete symmetry (*C, P, CP, G*, ...) test in flavor-conserving processes (weak interaction from SM highly suppressed).**

  Complentary to flavor-changing cases, such as those in *K, D, B*, ... hadrons.

- **Opportunities to search the light BSM particles in the MeV-GeV range.**

U(3) χPT: a reliable framework for π, K, η & η'

- **Power counting:** $\delta \sim p^2 \sim m_q \sim 1/N_C$ **, also dubbed as large N_C χPT**
- **Axion can be also systematcially included in U(3) χPT**

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

Two ways to proceed:

(1) Remove the $aG\tilde{G}$ term via the quark axial transformation

$$q \to e^{i\frac{a}{2f_a}\gamma_5 Q_a} q$$

$\mathrm{Tr}(Q_a) = 1$

$$-\frac{a\alpha_s}{8\pi f_a}G\tilde{G} - \frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma_5 Q_a q \qquad M_q \to M_q(a) = e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a}$$

Mapping to χPT $\quad \mathcal{L}_2 = \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + \chi_a U^\dagger + U\chi_a^\dagger \rangle + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\rm LO}$

$$\chi_a = 2B_0 e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \qquad J_A^\mu|_{\rm LO} = -i\frac{F^2}{2}\langle Q_a(\partial^\mu U U^\dagger + U^\dagger \partial^\mu U)\rangle$$

- $Q_a = M_q^{-1}/\mathrm{Tr}(M_q^{-1})$ [Georgi,Kaplan,Randall, PLB'86]
- $J_A^\mu \partial_\mu a$ [Bauer, et al., PRL'21]

(2) Explicitly keep the $aG\widetilde{G}$ term and match it to χPT

Reminiscent:

QCD $U(1)_A$ anomaly that is caused by topological charge density $\omega(x) = \alpha_s G_{\mu\nu}\tilde{G}^{\mu\nu}/(8\pi)$ **is responsible for the massive singlet η_0.**

Axion could be similarly included as the η_0 mass via the U(3) χPT:

$$\mathcal{L}^{\rm LO} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{12} M_0^2 X^2$$

$$\mathcal{L}^{\rm NLO} = L_5\langle u^\mu u_\mu \chi_+\rangle + \frac{L_8}{2}\langle \chi_+\chi_+ + \chi_-\chi_-\rangle - \frac{F^2\,\Lambda_1}{12} D^\mu X D_\mu X - \frac{F^2\,\Lambda_2}{12} X\langle\chi_-\rangle\,,$$

$$U = u^2 = e^{i\frac{\sqrt{2}\Phi}{F}}\,, \qquad \chi = 2B(s+ip)\,, \qquad \chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u$$

$$u_\mu = i u^\dagger D_\mu U u^\dagger\,, \quad D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$$

$$X = \log(\det U) - i\,\frac{a}{f_a} \qquad\qquad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \overline{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

- **Q_a is not needed in U(3) χPT.**
- **$M_0^2 = 6\tau/F^2$, with τ the topological susceptibility. Note that $M_0^2 \sim O(1/N_c)$.**
- **Axion interactions enter via the axion-meson mixing terms at LO.**

π-η-η'-a mixing in U(3) AχPT

[Gao,ZHG,Oller,Zhou,JHEP'23] [Gao,Hao,ZHG,et al.,EPJC'25]

LO

(mass mixing only)

$$\begin{pmatrix} \pi^0 \\ \eta \\ \eta' \\ a \end{pmatrix} = \begin{pmatrix} 1+v_{11} & -v_{12} & -v_{13} & -v_{14} \\ v_{12} & 1+v_{22} & -v_{23} & -v_{24} \\ v_{13} & v_{23} & 1+v_{33} & -v_{34} \\ v_{41} & v_{42} & v_{43} & 1+v_{44} \end{pmatrix} \begin{pmatrix} \pi^0 \\ \mathring{\eta} \\ \mathring{\eta}' \\ a \end{pmatrix} \qquad \begin{pmatrix} \mathring{\eta} \\ \mathring{\eta}' \end{pmatrix} = \begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

$$v_{12} = -\frac{\epsilon}{\sqrt{3}} \frac{c_\theta - \sqrt{2}s_\theta}{m_\pi^2 - m_{\mathring{\eta}}^2}, \qquad v_{13} = -\frac{\epsilon}{\sqrt{3}} \frac{\sqrt{2}c_\theta + s_\theta}{m_\pi^2 - m_{\mathring{\eta}'}^2}, \qquad v_{23} = \frac{\sqrt{2}s_\theta^2 + c_\theta s_\theta - \sqrt{2}c_\theta^2}{3(m_{\mathring{\eta}'}^2 - m_{\mathring{\eta}}^2)}\epsilon, \qquad v_{41} = -\frac{M_0^2\epsilon}{6(m_a^2 - m_\pi^2)} \frac{F}{f_a}\left[-\frac{(\sqrt{2}c_\theta - 2s_\theta)s_\theta}{m_a^2 - m_{\mathring{\eta}}^2} + \frac{c_\theta(2c_\theta + \sqrt{2}s_\theta)}{m_a^2 - m_{\mathring{\eta}'}^2}\right]$$

$$v_{42} = \frac{M_0^2 s_\theta}{\sqrt{6}(m_a^2 - m_{\mathring{\eta}}^2)}\frac{F}{f_a} - \frac{M_0^2\epsilon}{3\sqrt{6}(m_a^2 - m_{\mathring{\eta}}^2)}\frac{F}{f_a}\left[\frac{c_\theta(-\sqrt{2}c_\theta^2 + c_\theta s_\theta + \sqrt{2}s_\theta^2)}{m_a^2 - m_{\mathring{\eta}'}^2} - \frac{s_\theta(2c_\theta^2 + 2\sqrt{2}c_\theta s_\theta + s_\theta^2)}{m_a^2 - m_{\mathring{\eta}}^2}\right]$$

$$v_{43} = -\frac{M_0^2 c_\theta}{\sqrt{6}(m_a^2 - m_{\mathring{\eta}'}^2)}\frac{F}{f_a} - \frac{M_0^2\epsilon}{3\sqrt{6}(m_a^2 - m_{\mathring{\eta}'}^2)}\frac{F}{f_a}\left[\frac{c_\theta(c_\theta^2 - 2\sqrt{2}c_\theta s_\theta + 2s_\theta^2)}{m_a^2 - m_{\mathring{\eta}'}^2} - \frac{s_\theta(-\sqrt{2}c_\theta^2 + c_\theta s_\theta + \sqrt{2}s_\theta^2)}{m_a^2 - m_{\mathring{\eta}}^2}\right]$$

... ...

with $$m_{\mathring{\eta}}^2 = \frac{M_0^2}{2} + m_K^2 - \frac{\sqrt{M_0^4 - \frac{4M_0^2\Delta^2}{3} + 4\Delta^4}}{2}, \qquad m_{\mathring{\eta}'}^2 = \frac{M_0^2}{2} + m_K^2 + \frac{\sqrt{M_0^4 - \frac{4M_0^2\Delta^2}{3} + 4\Delta^4}}{2}, \qquad \sin\theta = -\left(\sqrt{1 + \frac{\left(3M_0^2 - 2\Delta^2 + \sqrt{9M_0^4 - 12M_0^2\Delta^2 + 36\Delta^4}\right)^2}{32\Delta^4}}\right)^{-1}$$

Physical masses after diagnolization

$$m_\eta^2 = m_{\mathring{\eta}}^2 + \frac{\epsilon}{3}(\sqrt{2}c_\theta + s_\theta)^2 + O(\epsilon^2) \qquad m_{\eta'}^2 = m_{\mathring{\eta}'}^2 + \frac{\epsilon}{3}(c_\theta - \sqrt{2}s_\theta)^2 + O(\epsilon^2)$$

$$\begin{aligned} m_a^2 = m_{a,0}^2 &+ \frac{M_0^2F^2}{6f_a^2}\left[1 + \frac{c_\theta^2 M_0^2}{m_{a,0}^2 - m_{\mathring{\eta}'}^2} + \frac{s_\theta^2 M_0^2}{m_{a,0}^2 - m_{\mathring{\eta}}^2}\right] \\ &+ \frac{M_0^4F^2\epsilon}{9f_a^2}\left[\frac{s_\theta^2(\sqrt{2}c_\theta + s_\theta)^2}{2(m_{a,0}^2 - m_{\mathring{\eta}}^2)^2} + \frac{c_\theta^2(c_\theta - \sqrt{2}s_\theta)^2}{2(m_{a,0}^2 - m_{\mathring{\eta}'}^2)^2} \right. \\ &\left. + \frac{c_\theta s_\theta(\sqrt{2}c_\theta^2 - c_\theta s_\theta - \sqrt{2}s_\theta^2)}{(m_{a,0}^2 - m_{\mathring{\eta}}^2)(m_{a,0}^2 - m_{\mathring{\eta}'}^2)}\right] + O(\epsilon^2), \end{aligned}$$

$$\boldsymbol{m_a^2 = \frac{m_\pi^2 F^2}{4f_a^2}}$$

[Weinberg,PRL'78]

(keep LO terms in m_π/m_K & m_π/M_0 & ε expansions)

NLO: (kinetic & mass mixing)

$$\mathcal{L} = \frac{1+\delta_k^{\eta}}{2}\partial_\mu\overline{\eta}\partial^\mu\overline{\eta} + \frac{1+\delta_k^{\eta'}}{2}\partial_\mu\overline{\eta}'\partial^\mu\overline{\eta}' + \delta_k^{\eta\eta'}\partial_\mu\overline{\eta}\partial^\mu\overline{\eta}' - \frac{m_{\overline{\eta}}^2+\delta_{m_{\overline{\eta}}^2}}{2}\overline{\eta}\,\overline{\eta} - \frac{m_{\overline{\eta}'}^2+\delta_{m_{\overline{\eta}'}^2}}{2}\overline{\eta}'\,\overline{\eta}' - \delta_{m^2}^{\eta\eta'}\overline{\eta}\,\overline{\eta}'$$
$$+ \frac{1+\delta_k^{\pi}}{2}\partial_\mu\overline{\pi}^0\partial^\mu\overline{\pi}^0 + \delta_k^{\pi\eta}\partial_\mu\overline{\pi}^0\partial^\mu\overline{\eta} + \delta_k^{\pi\eta'}\partial_\mu\overline{\pi}^0\partial^\mu\overline{\eta}' - \frac{m_{\overline{\pi}}^2+\delta_{m_{\overline{\pi}}^2}}{2}\overline{\pi}^0\,\overline{\pi}^0 - \delta_{m^2}^{\pi\eta}\overline{\pi}^0\overline{\eta} - \delta_{m^2}^{\pi\eta'}\overline{\pi}^0\overline{\eta}'$$
$$+ \frac{1+\delta_k^{a}}{2}\partial_\mu\overline{a}\partial^\mu\overline{a} + \delta_k^{a\pi}\partial_\mu\overline{a}\partial^\mu\overline{\pi}^0 + \delta_k^{a\eta}\partial_\mu\overline{a}\partial^\mu\overline{\eta} + \delta_k^{a\eta'}\partial_\mu\overline{a}\partial^\mu\overline{\eta}' - \frac{m_{\overline{a}}^2+\delta_{m_{\overline{a}}^2}}{2}\overline{a}\,\overline{a} - \delta_{m^2}^{a\pi}\overline{a}\,\overline{\pi}^0$$
$$- \delta_{m^2}^{a\eta}\overline{a}\,\overline{\eta} - \delta_{m^2}^{a\eta'}\overline{a}\,\overline{\eta}'$$

Separately handle the kinetic (*x*$_{ij}$) and mass (*y*$_{ij}$) mixing terms: x$_{ij}$, y$_{ij}$~ L$_5$, L$_8$, Λ$_1$, Λ$_2$

$$\begin{pmatrix}\hat{\pi}^0\\ \hat{\eta}\\ \hat{\eta}'\\ \hat{a}\end{pmatrix} = \begin{pmatrix}1 & -y_{12} & -y_{13} & -y_{14}\\ y_{12} & 1 & -y_{23} & -y_{24}\\ y_{13} & y_{23} & 1 & -y_{34}\\ y_{14} & y_{24} & y_{34} & 1\end{pmatrix} \times \begin{pmatrix}1-x_{11} & -x_{12} & -x_{13} & -x_{14}\\ -x_{12} & 1-x_{22} & -x_{23} & -x_{24}\\ -x_{13} & -x_{23} & 1-x_{33} & -x_{34}\\ -x_{14} & -x_{24} & -x_{34} & 1-x_{44}\end{pmatrix}\begin{pmatrix}\overline{\pi}^0\\ \overline{\eta}\\ \overline{\eta}'\\ \overline{a}\end{pmatrix}$$

naturally leading to the so-called two-mixing-angle formula

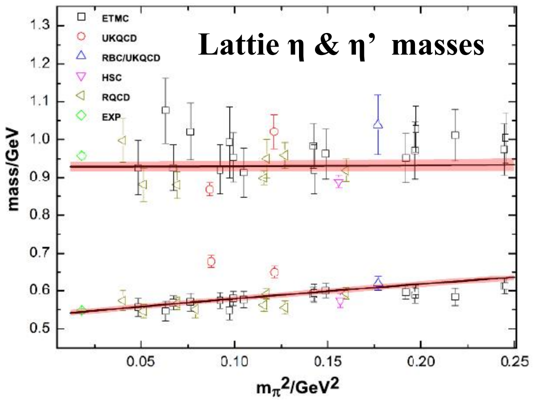
$$\begin{pmatrix}\hat{\eta}\\ \hat{\eta}'\end{pmatrix} = \frac{1}{F}\begin{pmatrix}F_8\cos\theta_8 & -F_0\sin\theta_0\\ F_8\sin\theta_8 & F_0\cos\theta_0\end{pmatrix}\begin{pmatrix}\eta_8\\ \eta_0\end{pmatrix}$$

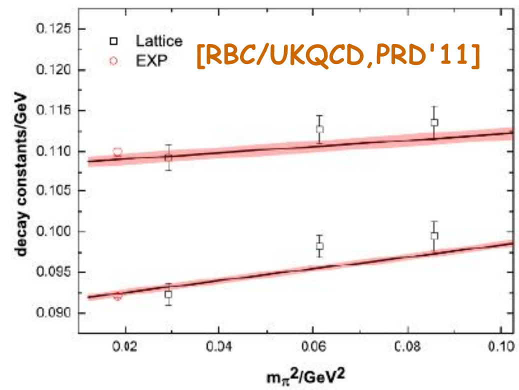
- **F_0 F_8 θ_0 θ_8 are now predicted in terms of the chiral low energy constants.**
- **Two-mixing-angle formalism is needed in phenomenologies of *η* & *η'* .** [Chen, ZHG, et al., PRD'12] [Chen, ZHG, et al., PRD'15] [Yan, Chen, et al., PRD'23]
- **For NNLO *η*-*η'* mixing studies:** [X.K.Guo, ZHG, et al., JHEP'15] [Gu,Duan,ZHG,PRD'18]

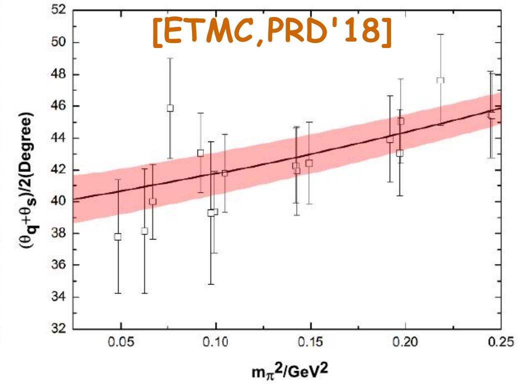
Fit to lattice data

[Gao,ZHG,Oller,Zhou,JHEP'23] [Gao,Hao,ZHG,et al.,EPJC'25]

Parameters	NLO Fit
F(MeV)	$91.05^{+0.42}_{-0.44}$
$10^3 \times L_5$	$1.68^{+0.05}_{-0.06}$
$10^3 \times L_8$	$0.88^{+0.04}_{-0.04}$
Λ_1	$-0.17^{+0.05}_{-0.05}$
Λ_2	$0.06^{+0.08}_{-0.09}$
χ^2/(d.o.f.)	219.9/(111–5)







Mixing pattern@NLO

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = M^{\rm LO+NLO} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix}$$

$$M^{\rm LO+NLO} = \begin{pmatrix} 1+(0.015\pm0.001) & 0.017+(-0.007\pm0.001) & 0.009+(-0.011\pm0.001) & \frac{-12.8+(-0.13\pm0.02)}{f_a} \\ -0.019+(0.005\pm0.001) & 0.94+(0.21\pm0.01) & 0.33+(-0.21\pm0.03) & \frac{-34.3+(1.7^{+0.8}_{-0.7})}{f_a} \\ -0.003+(-0.001\pm0.000) & -0.33+(-0.18\pm0.02) & 0.94+(0.13^{+0.01}_{-0.02}) & \frac{-25.9+(0.2^{+0.4}_{-0.3})}{f_a} \\ \frac{12.1+(0.5\pm0.1)}{f_a} & \frac{23.8+(1.0^{+0.2}_{-0.1})}{f_a} & \frac{35.7+(1.7^{+0.2}_{-0.1})}{f_a} & 1+\frac{-921.5+(-56.6^{+7.9}_{-9.6})}{f_a^2} \end{pmatrix}$$

Mass decomposition@NLO

$$m_{\hat{\pi}} = \left[134.9+(0.1\pm0.07)\right]\,\text{MeV},$$
$$m_{\hat{K}} = \left[492.1+(5.1^{+3.4}_{-3.3})\right]\,\text{MeV},$$
$$m_{\hat{\eta}} = \left[490.4+(61.1^{+10.0}_{-8.7})\right]\,\text{MeV},$$
$$m_{\hat{\eta}'} = \left[954.5+(-28.5^{+11.9}_{-10.9})\right]\,\text{MeV},$$
$$m_{\hat{a}} = \left[5.96+(0.12\pm0.02)\right]\mu\,\text{eV}\,\frac{10^{12}\,\text{GeV}}{f_a},$$

Two-photon couplings (driven by the EM anomaly)

$$\mathcal{L}_{WZW}^{\rm LO} = -\frac{3\sqrt{2}}{8\pi^2 F}\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2\Phi\rangle, \qquad Q = {\rm Diag}(\tfrac{2e}{3}, -\tfrac{e}{3}, -\tfrac{e}{3})$$

$$\mathcal{L}_{WZW}^{\rm NLO} = t_1 \frac{32\sqrt{2}B}{F}\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle (M_q\Phi + \Phi M_q) Q^2\rangle + 16 k_3 \varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2\rangle \left(\frac{\sqrt{2}}{F}\langle\Phi\rangle - \frac{a}{f_a}\right)$$

* **Note: one needs the π-η-η'-*a* mixing as input to calculate $g_{a\gamma\gamma}$**

$$F_{\pi^0\gamma\gamma}^{\rm Exp} = 0.274 \pm 0.002 {\rm GeV}^{-1},$$
$$F_{\eta\gamma\gamma}^{\rm Exp} = 0.274 \pm 0.006 {\rm GeV}^{-1},$$
$$F_{\eta'\gamma\gamma}^{\rm Exp} = 0.344 \pm 0.008 {\rm GeV}^{-1},$$

→

$$t_1 = -(3.8 \pm 2.4)\times 10^{-4} {\rm GeV}^{-2},$$
$$k_3 = (1.21 \pm 0.23)\times 10^{-4}$$

isospin limit(LO) isospin breaking(LO) NLO

→

$$F_{a\gamma\gamma} = \frac{20.1 + 3.4 + (0.5 \pm 0.2)}{f_a} \times 10^{-3},$$

(IB corrections amount to be around 15%!)

$$g_{a\gamma\gamma} = 4\pi\alpha_{em} F_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}(1.89 \pm 0.02).$$

which can be compared to: 1.92±0.04 [Grilli de Cortona, et al., JHEP'16] **and 2.05±0.03** [Lu, et al., JHEP'20]

η/η' decays as probes of SM precision test and new physics

(1) $\eta \to \pi^0\pi^+\pi^-$

$\eta\ (I^G=0^+) \to \pi^0\pi^+\pi^-\ (I^G=1^-)$: G-parity violation

- **QED correction negligible**
- **Dominated by strong IB effect (m_u-m_d)**

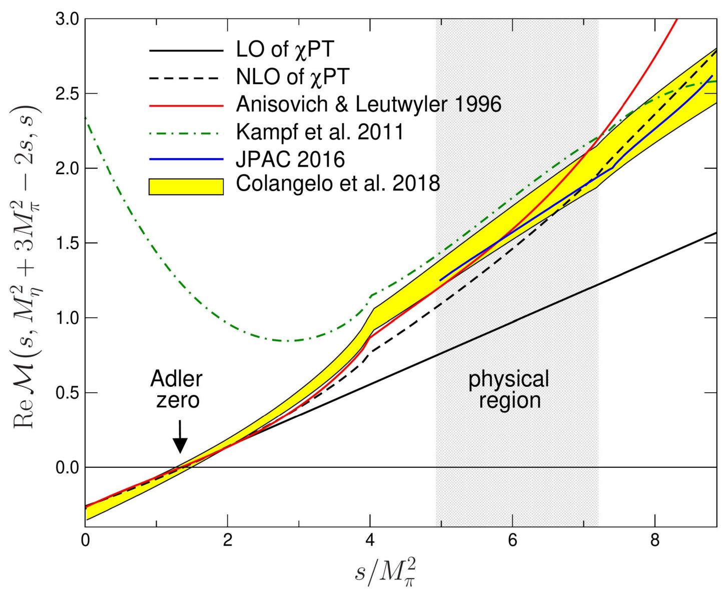
$$\Gamma(\eta \to \pi^+\pi^-\pi^0) = \frac{1}{Q^4}\frac{M_K^4(M_K^2 - M_\pi^2)^2}{6912\pi^3 M_\eta^3 M_\pi^4 F_\pi^4}\int_{s_{\min}}^{s_{\max}} \mathrm{d}s \int_{u_-(s)}^{u_+(s)} \mathrm{d}u\, |\mathcal{M}(s,t,u)|^2$$

$$Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

- **χPT converges rather slowly**
- **$\pi\pi$ rescattering is significant**

Other ways to extract Q

- **Kaon mass splitting**
- **Lattice QCD**

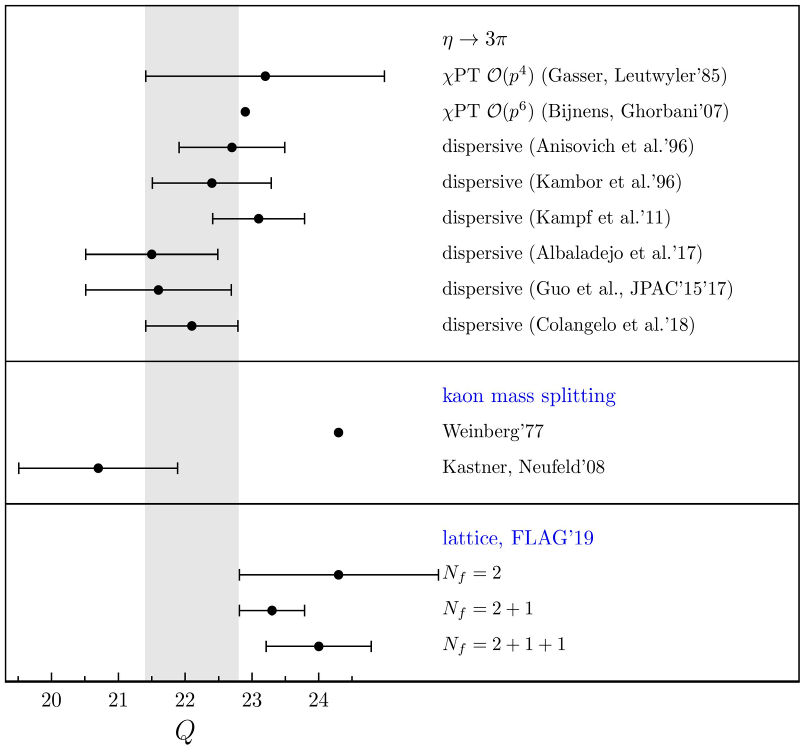


[Gan,et al., PhysRept'22]

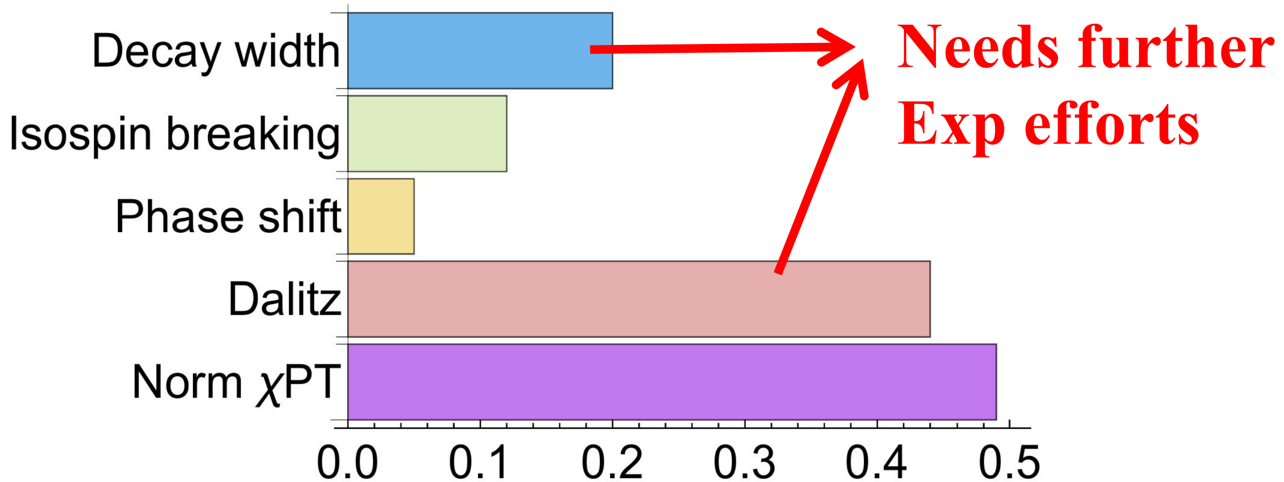
Current determinations of Q

$$Q^2 = \frac{m_s^2 - \hat{m}^2}{m_d^2 - m_u^2}$$

[Gan,et al., PhysRept'22]



Error budgets:
Q=22.04(72)



- $\eta' \to 3\pi$: BR suppressed; less prescise than that of η; more complicated in theory as well; future improvements in Exp and Theo are definitely required.

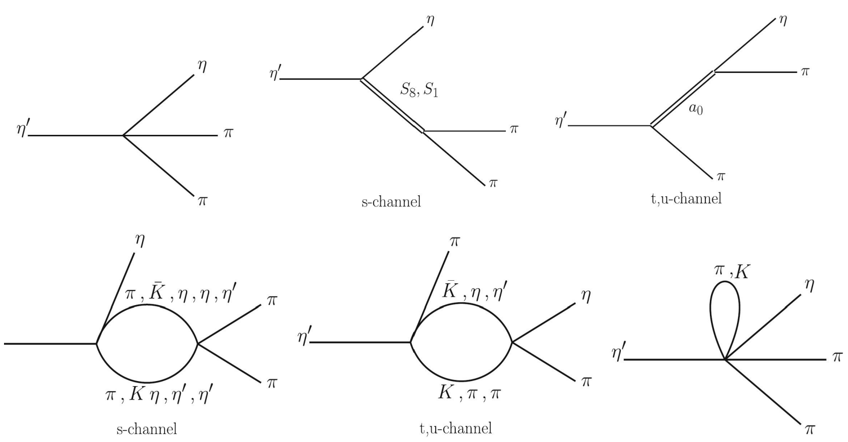
(2) $\eta' \to \eta\pi^+\pi^-$ ($\pi^0\pi^0$)

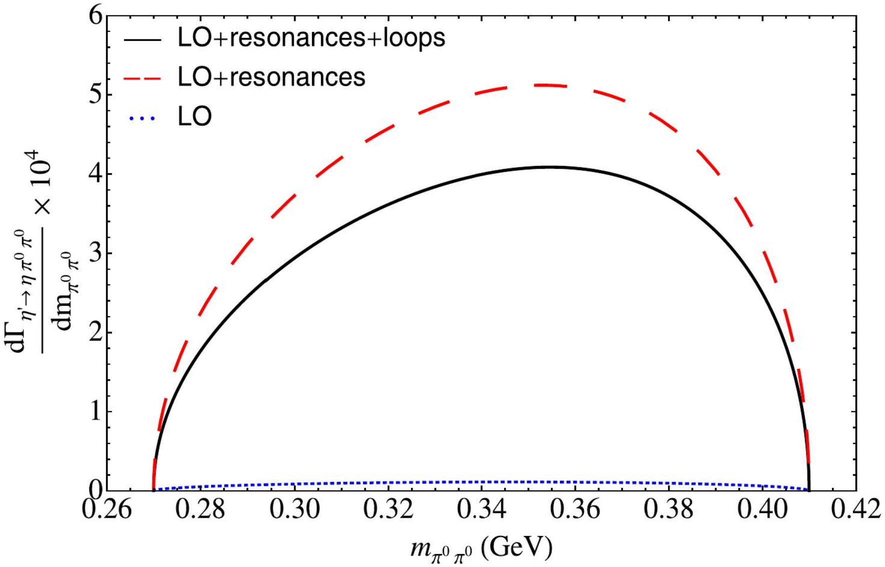
- **G-parity allowed: no IB suppresion**
- **Largest BRs for η' : $\eta' \to \eta\pi^+\pi^-$ (~ 43%) $\eta' \to \eta\pi^0\pi^0$ (~22%)**
- **Interesting subjects: resonances (ππ & πη rescattering), large Nc χPT, η-η' mixing**

Amplitude from leading-order χPT is small:

$$\mathcal{A}^{\chi PT}_{\eta'\to\eta\pi\pi}\Big|_{LO} = \frac{M_\pi^2}{6F^2}\left[2\sqrt{2}\cos(2\theta_P) - \sin(2\theta_P)\right]$$ **(no energy dependence)**

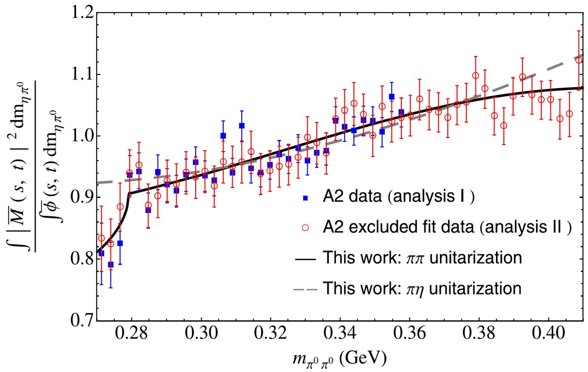
Higher-order contributions are essential: [Gonzalez-Solis, Passemar, EPJC'18]

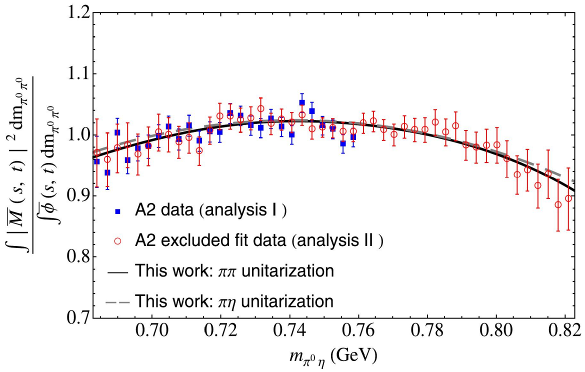


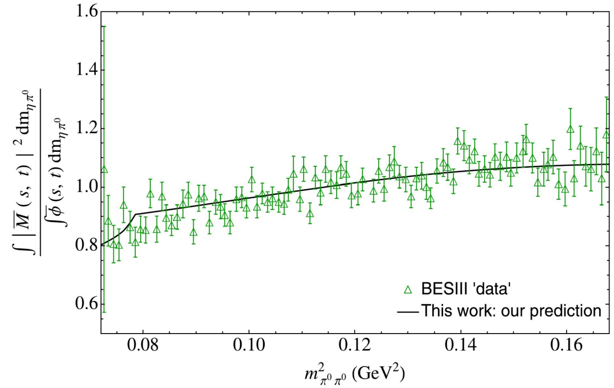


➢ **Dispersive method** [Isken,et al., EPJC'17] **: ππ & ηπ phase shifts as inputs**

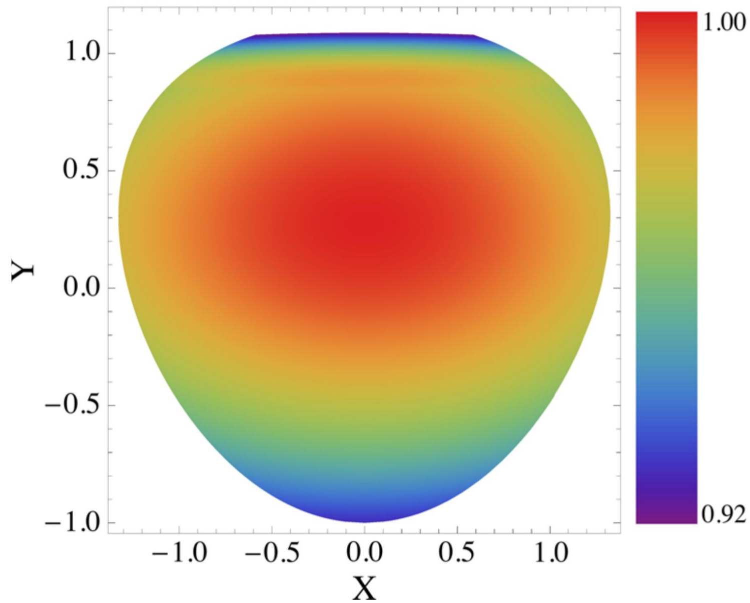
➢ Invairant-mass distributions [Gonzalez-Solis, Passemar, EPJC'18]



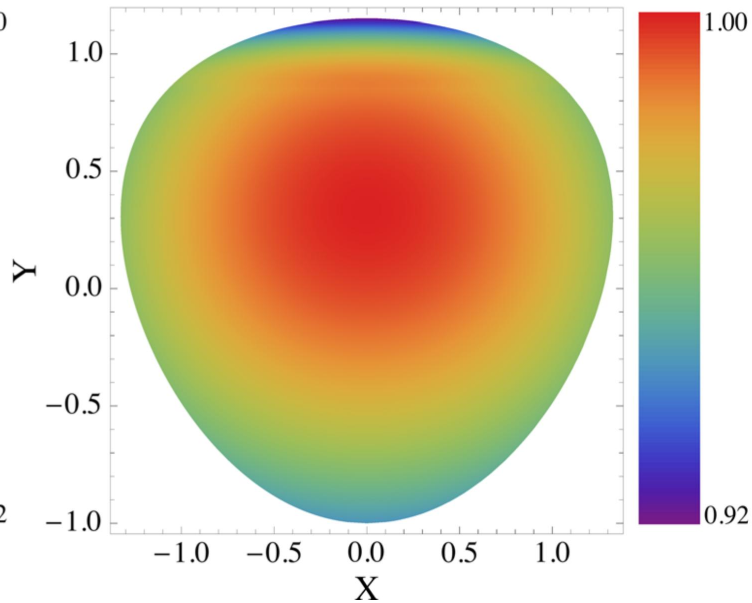




➢ ππ D-wave are found to be important for Dalitz parameters.



with D-wave

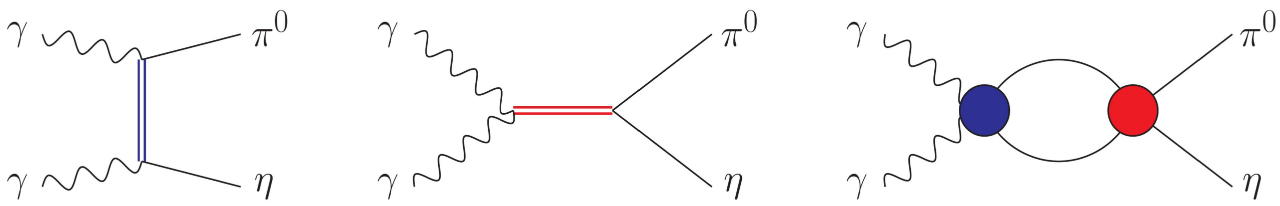


without D-wave

More precise Exp measurement are required for confirmation.

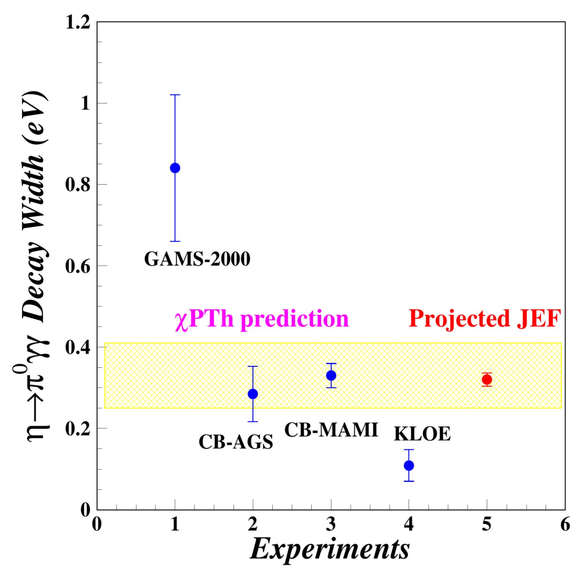
(3) $\eta/\eta' \to \pi^0\gamma\gamma$ & $\eta' \to \eta\gamma\gamma$

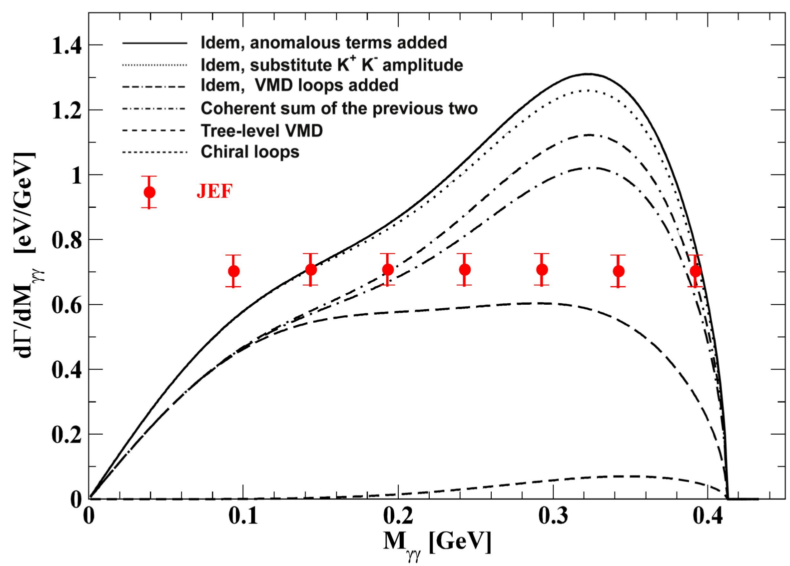
- **Leading-order χPT contributions either vanish or strongly suppressed.**
- **Higher-order effects: interplay between vector and scalar resonances**
- **Renewed interests in searching for B-boson [U(1) gauges symm. of baryon number]**



(a) **vector exchange** (b) **scalar & tensor** (c) **loops & rescattering**

[Gan,et al., PhysRept'22] [Oset,et al., PRD'03'08]

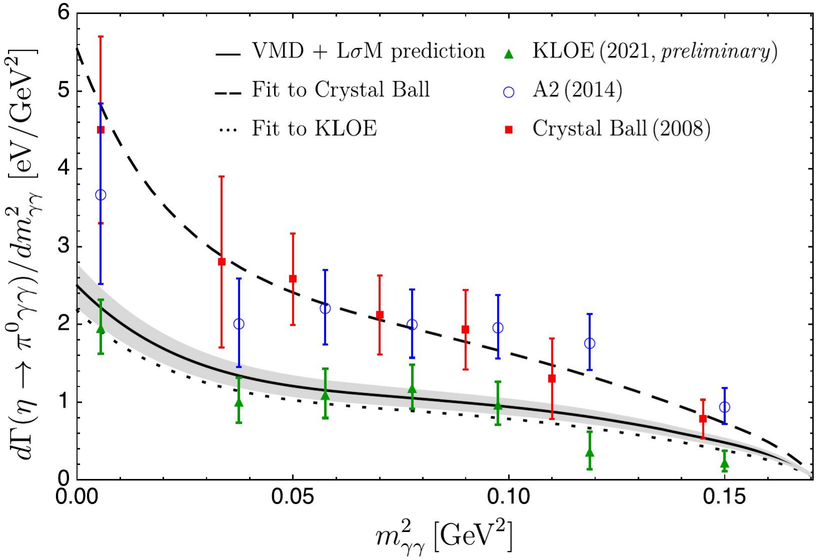


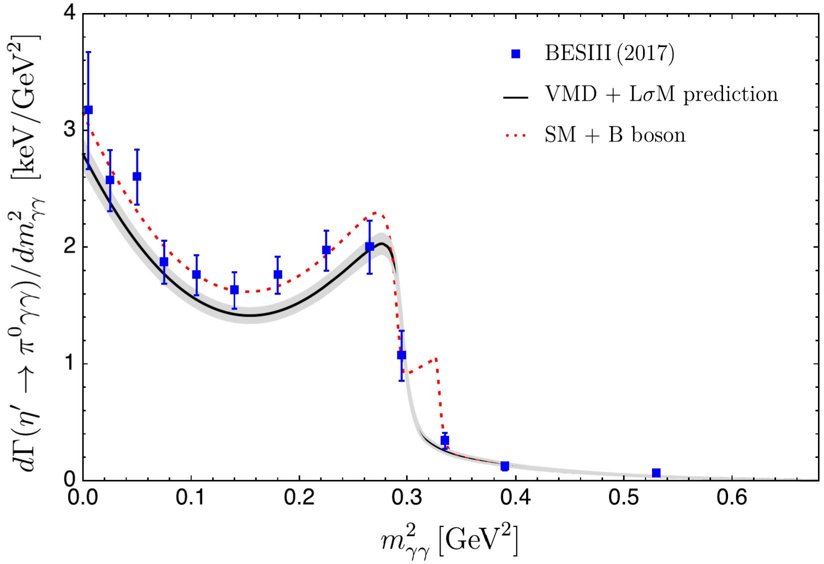


➢ **Discrepancies among various BRs need to be clarified in future Exp.**

- **Sensitivities to search for BSM B-boson** [Escribano,et al., PRD'20 '22]

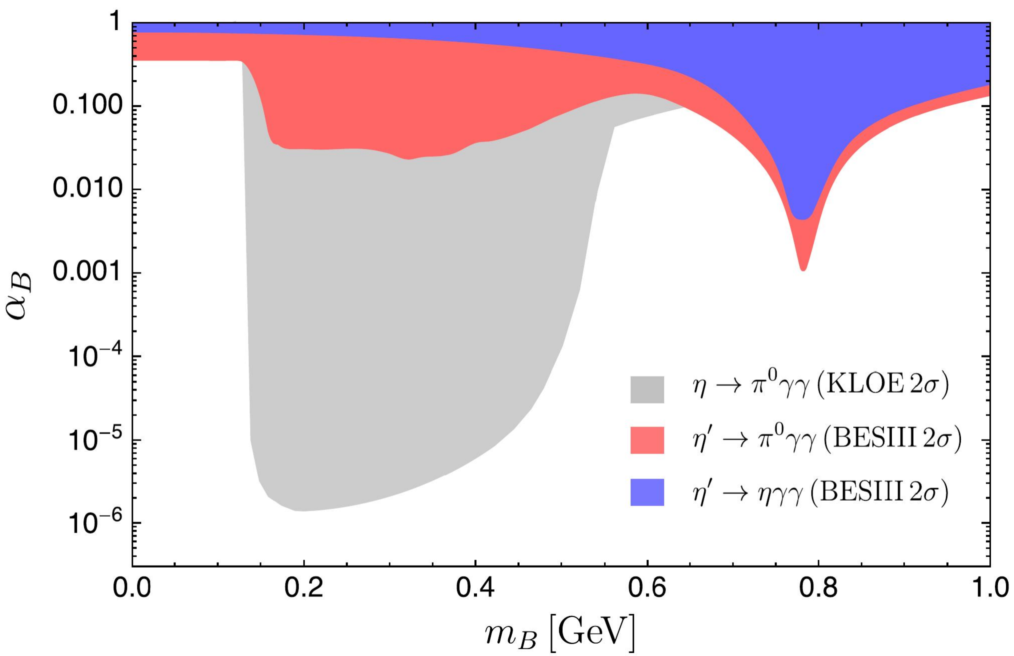
VMD + Linear Sigma Model: unable to simultaneously describe BRs of $\eta/\eta' \to \pi^0\gamma\gamma$





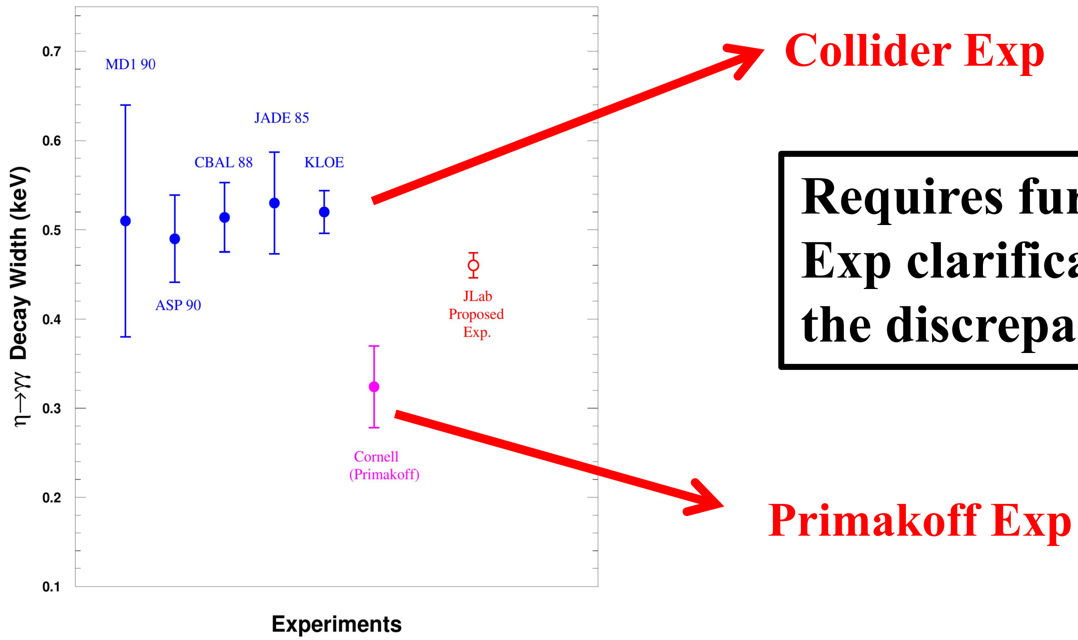
Exclusion limits on B-boson in γγ spectra

$$\frac{1}{3} g_B \ \bar{q}\gamma^\mu q B_\mu \quad \alpha_B = g_B^2/4\pi$$



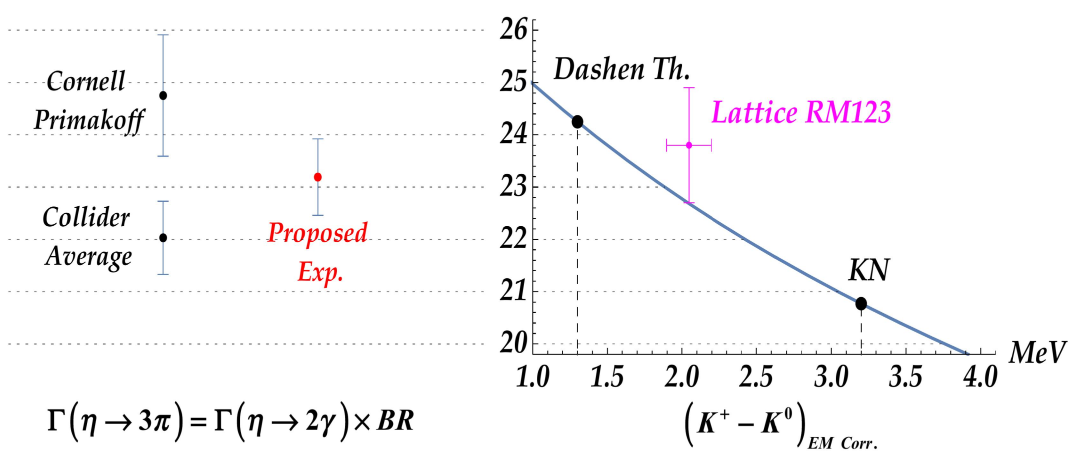
(4) Anomalous decays(odd intrinsic parity): $\pi^0/\eta/\eta' \to \gamma^{(*)}\gamma^{(*)}$, $\eta/\eta' \to \gamma\pi^+\pi^-(\pi^0\pi^0)$, $\eta' \to \pi^+\pi^-\pi^+\pi^-(\pi^0\pi^0)$

- $\Gamma_{\eta\to\gamma\gamma}$ crucial for other partial widths of η: $\Gamma_{\eta\to X} = \Gamma_{\eta\to\gamma\gamma} BR_{\eta\to X} / BR_{\eta\to\gamma\gamma}$



Requires further Exp clarification for the discrepancy!

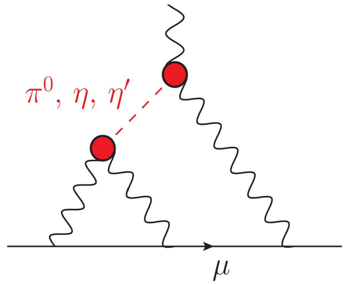
Reflection in the determination of Q via $\Gamma_{\eta\to3\pi}$



$$\Gamma(\eta\to3\pi) = \Gamma(\eta\to2\gamma)\times BR$$

- **Important inputs for $(g-2)_\mu$ via the hadronic light-by-light process**

Transition Form Factors (TFFs) of $\pi^0/\eta/\eta'$, not only their two-photon widths, are required !



$F_{\eta/\eta'\gamma^*\gamma^*}(q_1^2, q_2^2)$: double virtual (both $q_i^2 \neq 0$), single virual ($q_1^2=0$ or $q_2^2=0$)

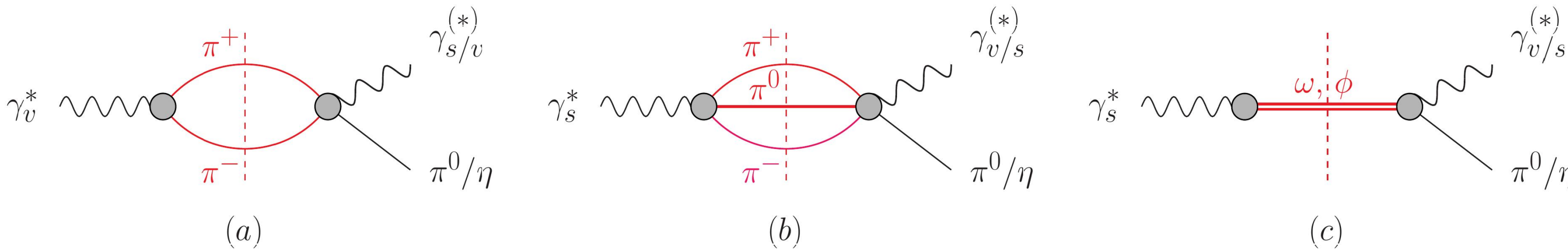
- **Double virtual: challenge in Exp (tiny BRs)**
  - **Time-like region ($q_i^2>0$)**

    η: $\eta\to e^+e^-e^+e^-$ (direct, tiny BRs), $V\to\eta e^+e^-$ (indirect), ...

    η': $\eta'\to Ve^+e^-$ (direct, single-virtual TFF), $e^+e^-\to V\eta'$ (indirect), ...
  - **Space-like region ($q_i^2<0$)**

    $e^+e^-\to\gamma^*\gamma^*e^+e^-\to P\, e^+e^-$
- **Single virtual TFFs: important progresses in Exp**
  - **Time-like region: $P\to l^+l^-\gamma$, $e^+e^-\to\gamma^*\to P\gamma$, ...**
  - **Space-like region: $e^+e^-\to\gamma^*\gamma^*e^+e^-\to P\, e^+e^-$ (one lepton tagged)**

- **Leading hadronic intermediate states**



$\eta/\eta' \to \gamma\pi^+\pi^-$ **$\eta/\eta' \to \gamma\pi^0\pi^+\pi^-$** **$V \to P\gamma^{(*)}$**

❑ **Comprehensive RχT studies of VPγ^(*), Pγγ^(*) and J/ψ→VP, Pγ^(*)**

$$\begin{pmatrix} \hat{\eta} \\ \hat{\eta}' \end{pmatrix} = \frac{1}{F} \begin{pmatrix} F_8 \cos\theta_8 & -F_0 \sin\theta_0 \\ F_8 \sin\theta_8 & F_0 \cos\theta_0 \end{pmatrix} \begin{pmatrix} \eta_8 \\ \eta_0 \end{pmatrix}$$

$\eta \to \gamma\gamma^*$, $\eta' \to \gamma\gamma^*$ $\phi \to \eta\gamma^*$

$P \to V\gamma$, $V \to P\gamma$, $P \to \gamma\gamma$, $P \to \gamma l^+ l^-$, $V \to P l^+ l^-$,

$J/\psi \to P\gamma$, $\psi' \to P\gamma$, $J/\psi \to PV$, $\psi' \to PV$, $J/\psi \to P l^+ l^-$, $\psi' \to P l^+ l^-$

$P = \pi, K, \eta, \eta'$, $V = \rho, K^*, \omega, \phi$

[Chen, ZHG, Zheng, PRD'12 '14]

[Chen, ZHG, Zou, PRD'15]

[Yan, ZHG, et al., PRD'23]

(5) axion/axion-like particle production in η decay: $\eta\to a\pi^+\pi^-$ ($\pi^0\pi^0$)

Why focus on axion in η decay:

- ✓ $\eta\to\pi\pi\pi$ (IB suppressed), $\eta\to\pi\pi a$ (no IB suppression)
- ✓ $\eta\to\pi\pi a$: theoretically easier to handel than $\eta'\to\pi\pi a$ (next step)

Previous works:

- ❖ Most of them rely on leading-order χPT
- ❖ Possible issue: bulk contributions@LO χPT are constant terms, and potential large corrections from higher orders may result.
- ❖ Hadron resonance effects may lead to enhancements.

Advances in our work :

- ➢ Study of renormalization of $\eta\to\pi\pi a$ @1-loop level in SU(3) χPT
- ➢ To implement unitarization to the $\eta\to\pi\pi a$ χPT amplitude
- ➢ Uncertainty analyes in the phenomenological discussions

LO χPT Lagrangian

$$\mathcal{L}_2 = \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + \chi_a U^\dagger + U \chi_a^\dagger \rangle + \frac{\partial_\mu a}{2 f_a} J_A^\mu|_{\rm LO} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2$$

$$\chi_a = 2B_0 M(a) \qquad M(a) \equiv \exp\left(-i\frac{a}{2f_a}Q_a\right) M \exp\left(-i\frac{a}{2f_a}Q_a\right) \qquad J_A^\mu|_{\rm LO} = -i\frac{F^2}{2}\langle Q_a \left\{\partial^\mu U, U^\dagger\right\}\rangle$$

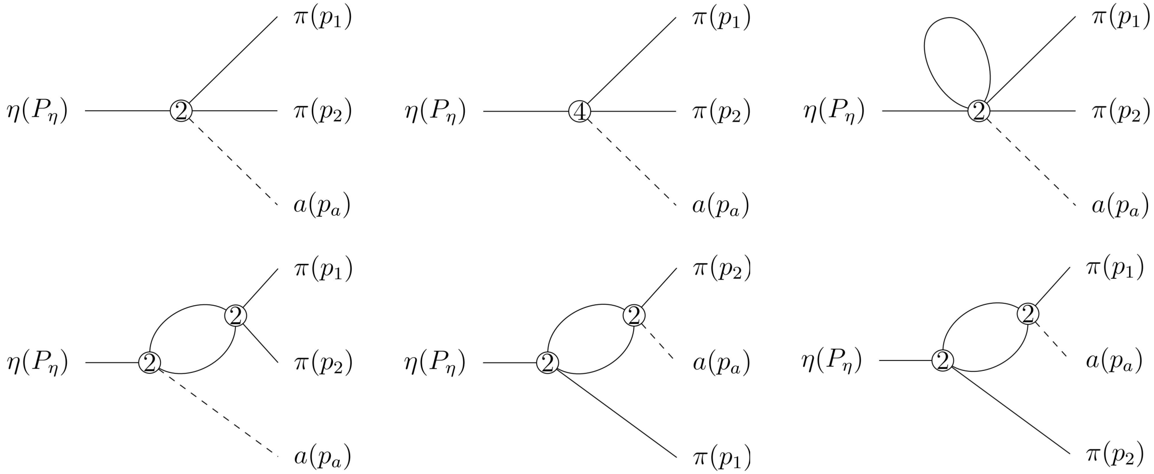
Note: we consider the octet part ($\bar{Q}_a$) of Q_a in SU(3) χPT

NLO χPT Lagrangian

$$\mathcal{L}_4 = L_1 \langle \partial_\mu U \partial^\mu U^\dagger \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \cdots + \frac{\partial_\mu a}{2 f_a} J_A^\mu|_{\rm NLO},$$

$$J_A^\mu|_{\rm NLO} = -4iL_1 \langle \bar{Q}_a \{U^\dagger, \partial^\mu U\}\rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \cdots .$$

Feynman diagrams up to NLO



Parameters

Masses and F_π [MeV]				LECs $L_i^r(\mu)$ at $\mu = 770$ MeV (in unit of 10^{-3})							
m_π	m_K	m_η	F_π	L_1^r	L_2^r	L_3^r	L_4^r	L_5^r	L_6^r	L_7^r	L_8^r
137	496	548	92.1	1.0(1)	1.6(2)	−3.8(3)	0.0(3)	1.2(1)	0.0(4)	−0.3(2)	0.5(2)

[J. Bijnens and G. Ecker, Ann. Rev. Nucl. Part. Sci. 64, 149 (2014)]

✓ **Renomarlization condition is verified to be consistent with conventional ChPT.**

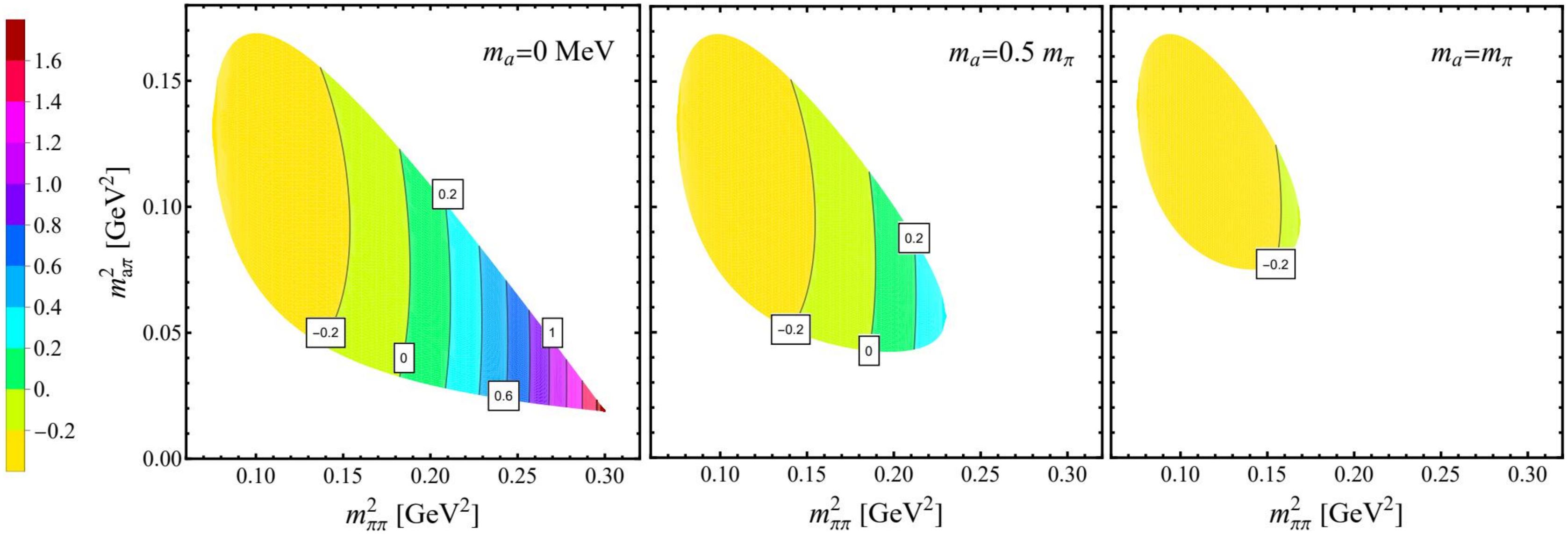
Observations:

- Strong isospin breaking effects enter the $\eta \to \pi\pi a$ amplitudes at the order of $(m_u - m_d)^2$
- In the isospin limit ($m_u = m_d$), the amplitudes with $\pi^+\pi^-$ and $\pi^0\pi^0$ in $\eta \to \pi\pi a$ processes are identical.

- **Dalitz plots to show the NLO/LO convergence**

$$\left(2\mathcal{M}^{(2)}_{\eta;\pi\pi a}\mathrm{Re}\left(\mathcal{M}^{(4)}_{\eta;\pi\pi a}\right) + \left|\mathcal{M}^{(4)}_{\eta;\pi\pi a}\right|^2\right) / \left|\mathcal{M}^{(2)}_{\eta;\pi\pi a}\right|^2$$

[Wang,ZHG,Lu,Zhou, JHEP'24]



Important lessons:

- Non-perturbative effect in the $\pi\pi$ subsystem can be important.
- Perturbative treatment of the $a\pi$ subsystem is justified.

- **Unitarization of the partial-wave η→ππa amplitude**

$$\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s) = \frac{\mathcal{M}^{00,\mathrm{L}}_{\eta;\pi\pi a}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}\,,$$

$$G_{\pi\pi}(s) = -\frac{1}{(4\pi)^2}\left(\log\frac{m_\pi^2}{\mu^2} - \sigma_\pi(s)\log\frac{\sigma_\pi(s)-1}{\sigma_\pi(s)+1} - 1\right)\,,$$

$$\mathcal{M}^{00,\mathrm{L}}_{\eta;\pi\pi a}(s) = \mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s) + \mathcal{M}^{00,(4)}_{\eta;\pi\pi a}(s) - G_{\pi\pi}(s)\,\mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s)\,T^{00,(2)}_{\pi\pi\to\pi\pi}(s)\,.$$

The unitarized amplitude satisfies the relation

$$\mathrm{Im}\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s) = \rho_{\pi\pi}(s)\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s)\left(T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s)\right)^*\,, \qquad (2m_\pi < \sqrt{s} < 2m_K)$$

with the unitarized PW ππ amplitude $$T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s) = \frac{T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}$$

- **Unitarized PW amplitude based on LO η→ππa amplitude**

$$\mathcal{M}^{00,\mathrm{Uni-LO}}_{\eta;\pi\pi a}(s) = \frac{\mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}\,.$$

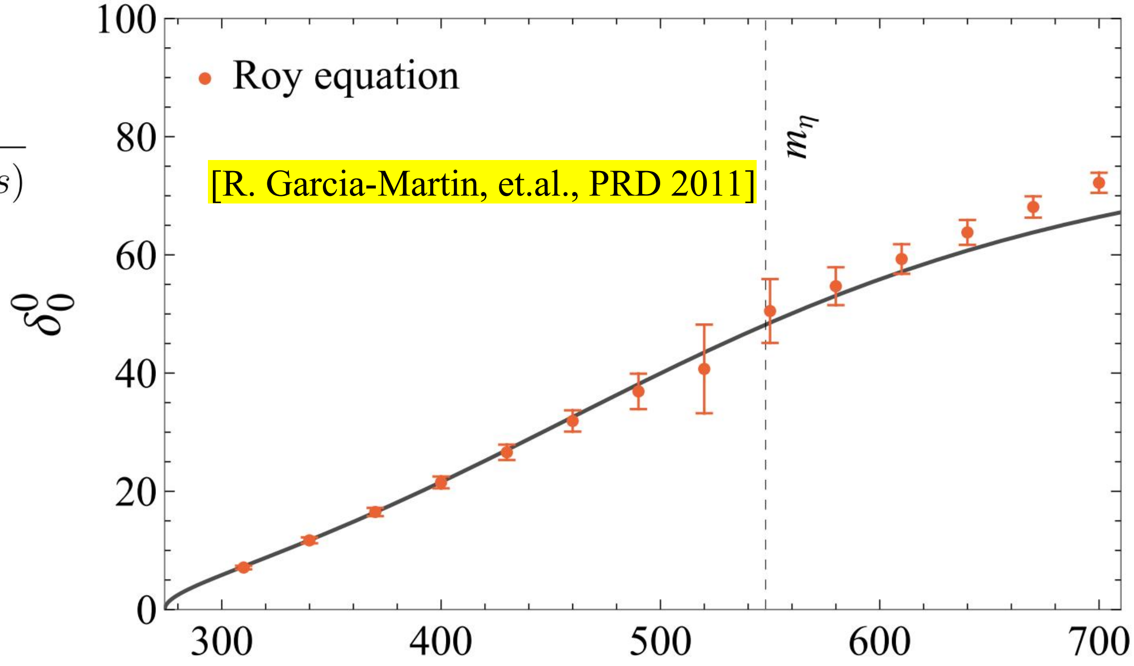
Resemble the method:

[Alves,Gonzalez-Solis,JHEP'24]

$$M_0(s) = P(s)\Omega_0^0(s)$$

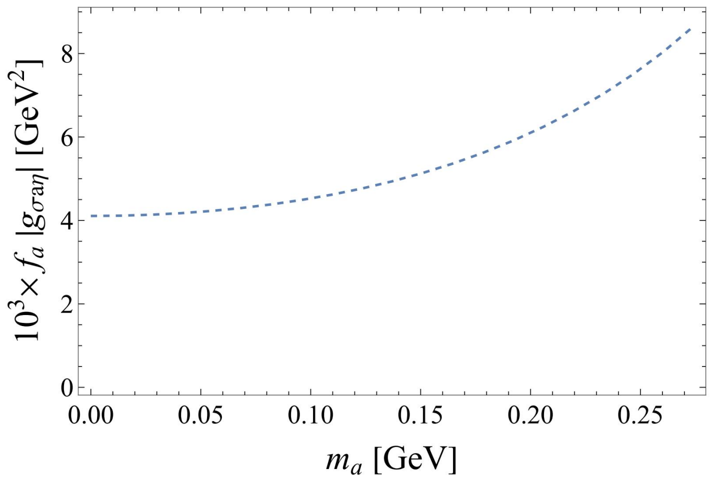
Phase shifts from the unitarized PW $\pi\pi$ amplitude

$$T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s) = \frac{T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}{1 - G_{\pi\pi}(s)T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}$$



- **Pole position of $f_0(500)/\sigma$:** $\sqrt{s_\sigma} = 457 \pm i251$ MeV

$$\mathcal{M}^{00,\mathrm{Uni,II}}_{\eta;\pi\pi a}(s)\big|_{s\to s_\sigma} \sim -\frac{g_{\sigma\pi\pi}g_{\sigma a\eta}}{s - s_\sigma}$$



Predictions of the $\eta \to \pi\pi a$ branching ratios by varying m_a

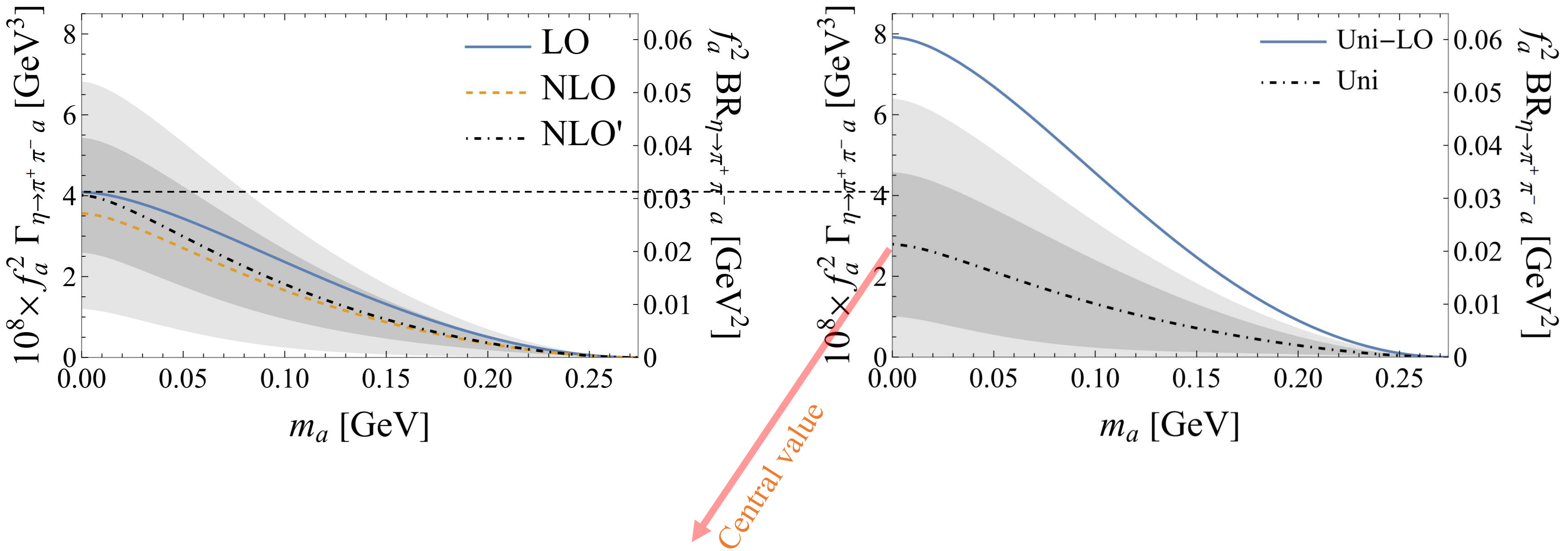
Uncertainty bands:

- **Lighter regions:**

L_1^r	L_2^r	L_3^r	L_4^r	L_5^r	L_6^r	L_7^r	L_8^r
1.0(1)	1.6(2)	−3.8(3)	0.0(3)	1.2(1)	0.0(4)	−0.3(2)	0.5(2)

- **Darker regions: freeze the 1/Nc suppressed ones (L_4, L_6, L_7)**

[Wang,ZHG,Lu,Zhou, JHEP'24]



$$\mathrm{BR}_{\eta\to\pi^+\pi^- a}\Big|_{m_a\to 0} = 2.1\times 10^{-2}\left(\frac{\mathrm{GeV}^2}{f_a^2}\right)$$

Possible detection channels: $a\to\gamma\gamma$, $a\to e^+e^-$, $a\to\mu^+\mu^-$

结语： η/η’ 包含丰富有趣的物理：

- 不仅有诗和远方 --诱人的新物理现象--：

  B-boson, 轴子/类轴子, 新的C/CP破坏， … …

- 也充满了烟火气息 --亟需提升精度/澄清的SM允许的过程--

- 标准模型的精确检验:

  m_u-m_d， $(g\text{-}2)_\mu$ ，… …

- 强相互作用相关的物理:

  强子共振态，手征对称性，形状因子, … …

谢谢大家！