# Generalized distribution amplitudes and gravitational form factors of hadrons

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References:
C. Lorce, B. Pire and Qin-Tao Song, PRD106 (2022), 094030.
B. Pire and Qin-Tao Song, PRD 107 (2023), 114014.
B. Pire and Qin-Tao Song, PRD 109 (2024), 074016.
Qin-Tao Song, O. V. Teryaev and S. Yoshida, PLB 868 (2025), 139797.
Jing Han, B. Pire and Qin-Tao Song, PRD 112 (2025), 014048.
Jing Han, B. Pire and Qin-Tao Song,  Research in Progress.

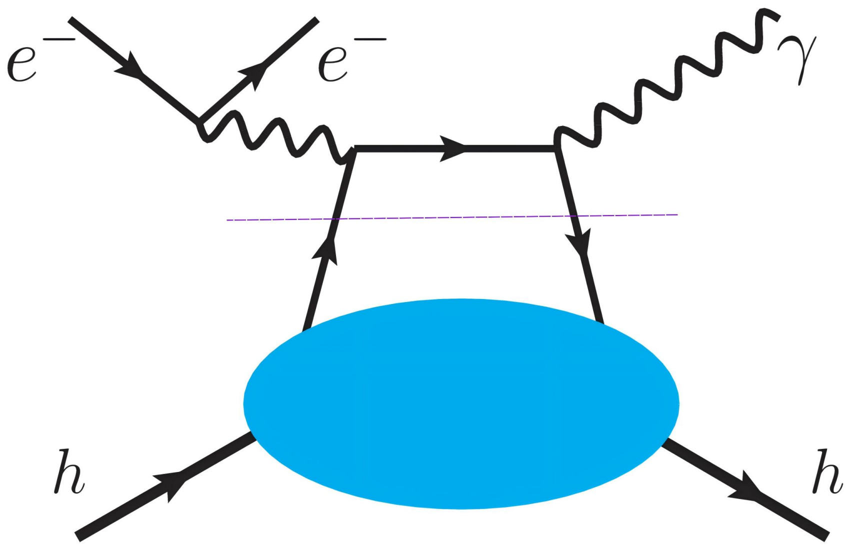
Outline:

- GPDs, GDAs and gravitational FFs

- Production of a scalar meson pair $\begin{cases} \gamma^* \to M_1 M_2 \gamma \\ \gamma^* \gamma \to M_1 M_2 \end{cases}$

- Production of a spin-1/2 baryon pair $\begin{cases} \gamma^* \to \mathrm{B}\overline{B}\gamma \\ \gamma^* \gamma \to \mathrm{B}\overline{B} \end{cases}$

# GPDs (广义部分子分布函数)




$\gamma^* + h \to \gamma + \mathrm{h}$

Deeply Virtual Compton Scattering
(DVCS)

QCD collinear factorization
Hard part: $\gamma^* q \to \gamma q$
Soft part: $qh \to qh$, quark GPDs.

The GPDs are 3-D structure functions that offer opportunities to study a new aspect of nucleon structure: a nucleon tomography.

GPDs $\xrightarrow{\text{forward limit}}$ PDFs (部分子分布函数)

# GPDs and EMT

GPDs can help us to access the hadronic matrix elements of energy momentum tensor (EMT, 能动张量) indirectly.

Second moments of GPDs → Hadronic matrix elements of EMT →

- Proton spin puzzle
- EMT FFs of hadrons

Hadronic matrix elements of EMT:

$$\begin{aligned}
&\langle p', \vec{s}' | T_a^{\mu\nu} | p, \vec{s} \rangle \\
&= \bar{u}(p', \vec{s}') \left[ A_a(t) \frac{P^\mu P^\nu}{M_N} \right. \\
&\quad + D_a(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{4M_N} + \bar{C}_a(t) M_N g^{\mu\nu} \\
&\quad \left. + J_a(t) \frac{P^{\{\mu} i\sigma^{\nu\}\lambda} \Delta_\lambda}{M_N} - S_a(t) \frac{P^{[\mu} i\sigma^{\nu]\lambda} \Delta_\lambda}{M_N} \right] u(p, \vec{s}),
\end{aligned}$$

$J_q(0)$: Quark Augular Momentum
$S_q(0)$: Quark Helicity contribution
$D_q(0)$: D-term (last global unknown charge)

X.D. Ji, PRL 78(1997), 610.
M. V. Polyakov and C. Weiss, PRD 60, 114017 (1999).
X. H. Cao, F. K. Guo, Q. Z. Li and D.L. Yao, Nature Commun. 16 (2025) no.1, 6979

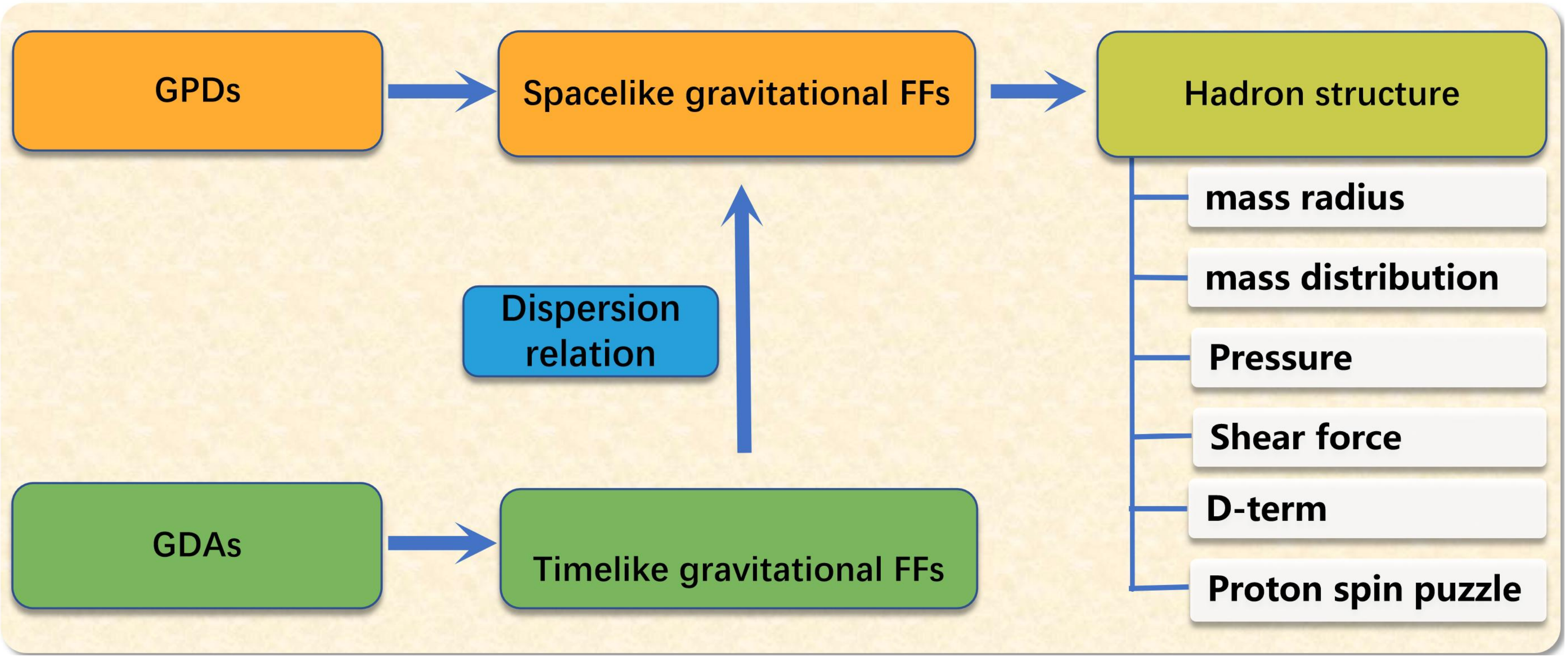
Most hadrons are not stable, we can not use DVCS to study their GPDs. How to obtain EMT FFs for these unstable hadrons?

# EMT FFs and unstable hadrons?

Generalized distribution amplitudes (GDAs, 广义分布振幅) are the s-t crossed quantities of GPDs, the second moments of GDAs lead to timelike EMF FFs.

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL **81** (1998) 1782.
M. V. Polyakov, NPB **555** (1999) 231.

From GPDs and GDAs to hadron gravitational FFs:




M. V. Polyakov and P. Schweitzer, Int. J. Mod. Phys. A 33 (2018) no.26, 1830025.
V. D. Burkert, L. Elouadrhiri, F. Girod, C. Lorce, P. Schweitzer and P. Shanahan, Rev. Mod. Phys. 95 (2023), 041002.

# GDAs are accessed in two-photon reactions

GDAs are also important inputs for decays of B mesons.

W. F. Wang, H. N. Li, W. Wang and C. D. Lu, PRD 91 (2015), 094024.
Y. Li, A. J. Ma, W. F. Wang and Z. J. Xiao, PRD 95 (2017), 056008.
S. Cheng, PRD 99(2019), 053005.
M. K. Jia, C. Q. Zhang, J. M. Li and Z. Rui, PRD 104 (2021), 073001.
J. W. Zhang, B. Y. Cui, X. G. Wu, H. B. Fu and Y. H. Chen, PRD 110(2024), 036015.
……

GDAs are accessed in $\gamma^*\gamma \to h\bar{h}$, which can be measured in $e^+e^-$ collisions.

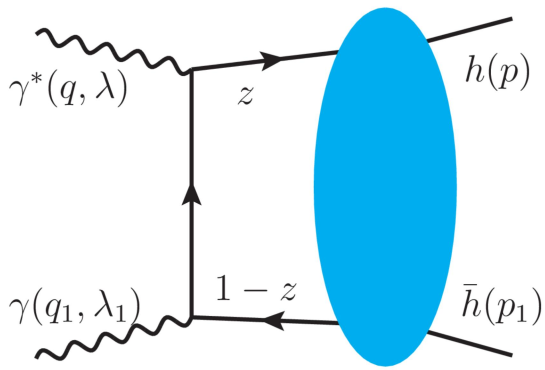
QCD collinear factorization

$q^2 = -Q^2, \quad (q_1)^2 = 0,$

$Q^2 \gg s, \Lambda^2_{\rm QCD}$

Hard part: $\gamma^*\gamma \to q\bar{q}$

Soft part: $q\bar{q} \to h\bar{h}$, quark GDAs.

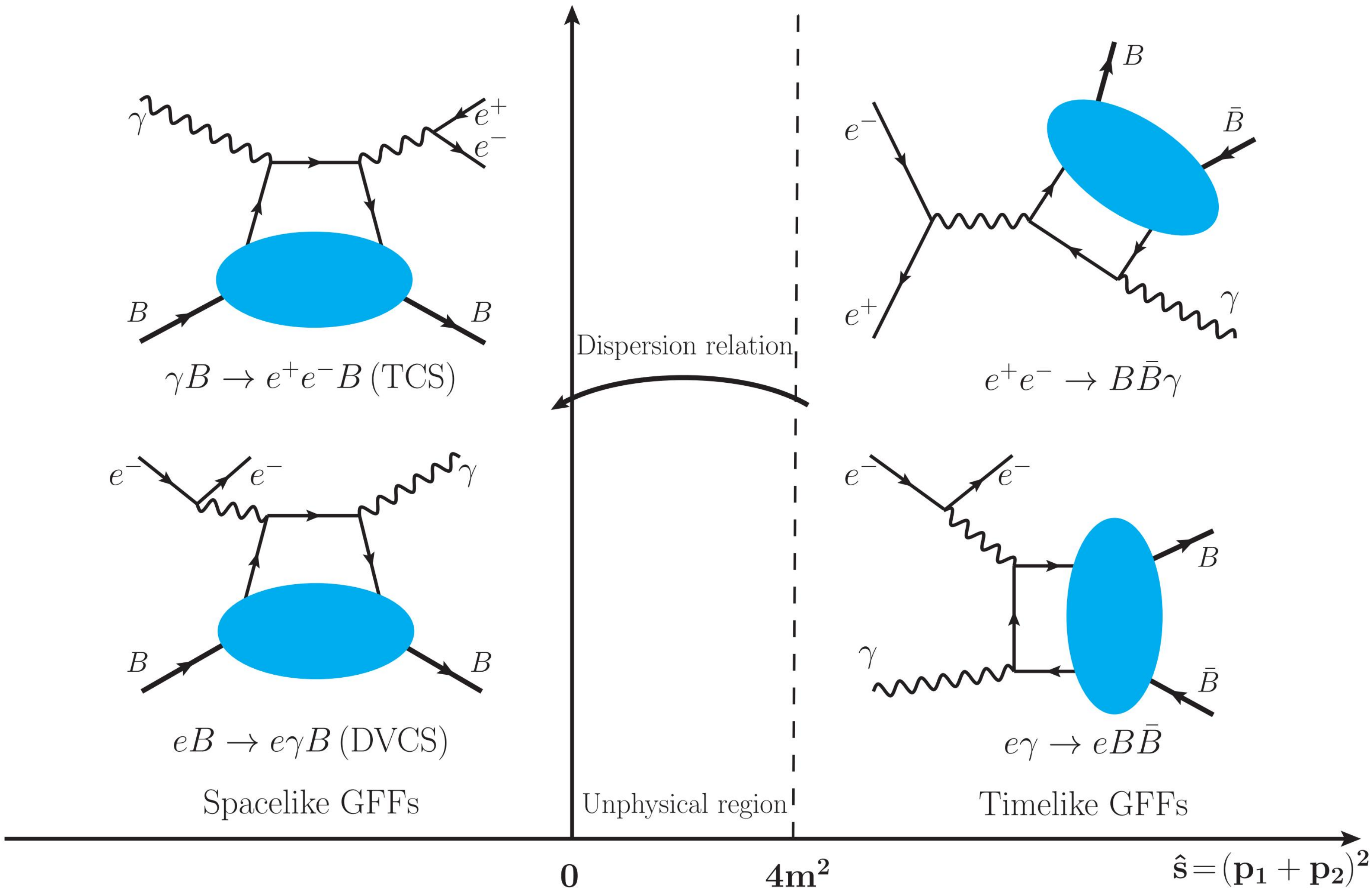



Quark GDA of a scalar hadron pair is defined as:

$$\Phi(z, \xi, s) = \int \frac{dx^-}{2\pi} e^{-izP^+x^-} \langle h(p)\bar{h}(p_1)|\bar{q}(x^-)\gamma^+ q(0)|0\rangle$$

M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.
C. Lorce, B. Pire and Qin-Tao Song, PRD 106 (2022) , 094030.

# Hadron GDAs and GPDs are accessed in two-photon reactions




Spacelike and timelike EMT FFs are probed by different reactions, and the former ones can be obtained from the latter ones using dispersion relation.

# Production of a scalar meson pair: $\gamma^* \to M_1 M_2 \gamma$ and $\gamma^* \gamma \to M_1 M_2$

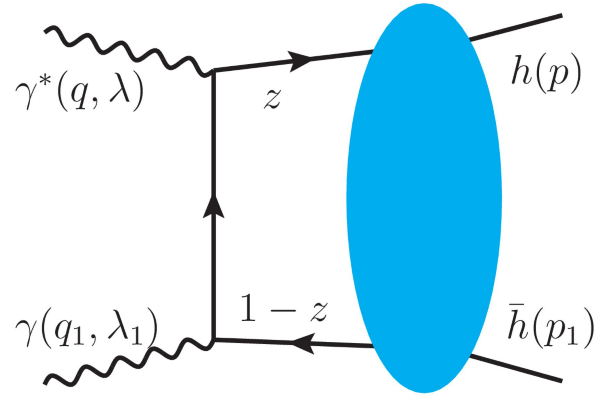
# GDAs in $\gamma^*\gamma \to h\bar{h}$

QCD collinear factorization

Hard part: $\gamma^*\gamma \to q\bar{q}$

Soft part: $q\bar{q} \to h\bar{h}$, quark GDAs.

M. Diehl, T. Gousset, B. Pire and O. Teryaev, PRL **81** (1998) 1782.
M. Diehl, T. Gousset and B. Pire, PRD **62** (2000) 07301.
M. V. Polyakov, NPB **555** (1999) 231.




Helicity amplitudes of a scalar meson pair:

$$A_{\lambda\lambda_1} = T_{\mu\nu}\epsilon^{\mu}(\lambda)\epsilon^{\nu}(\lambda_1)$$

There are three independent helicity amplitudes: $A_{++}$, $A_{0+}$ and $A_{+-}$.

Leading twist (扭度) amplitude: $A_{++}$

Higher twist amplitudes: $A_{0+}$ and $A_{+-}$

At leading twist, only $A_{++}$ appears in the cross section.

Twist expansion:

Cross section=Leading-twist contribution + Higher-twist contribution

Supressed by $1/Q$ or $1/Q^2$

# The extraction of pion GDAs and EMT FFs

If we neglect the higher-twist contribution, the cross section is given by $A_{++}$.

$$A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \, \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z,\zeta,W^2)$$

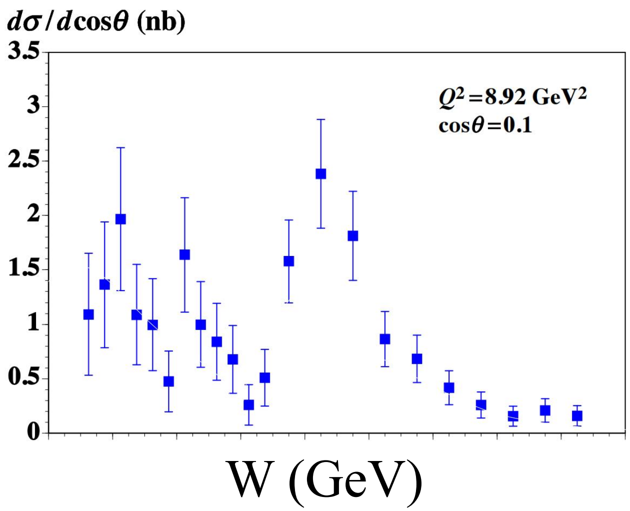
→ Pion GDA, leading twist.

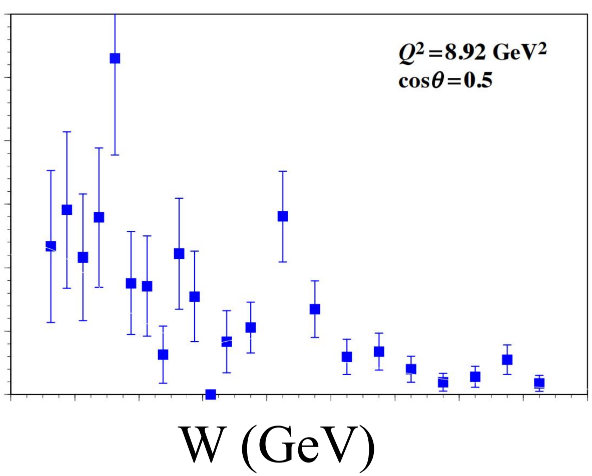
Similar to the meson distribution amplitudes (DAs), we have the simple asymptotic form of pion GDAs at $Q^2 \to \infty$:

$$\Phi(z, \cos\theta, s) = 18z(1-z)(2z-1)[\widetilde{B}_{10}(s) + \widetilde{B}_{12}(s) P_2(\cos\theta)]$$

The 3-D GDA is written as two 1-D functions, S-wave and D-wave.

In 2016, the cross sections of $\gamma^*\gamma \to \pi\pi$ were released by Belle.




Single-tag two-photon collisions at Belle

M. Masuda et al. [Belle Collaboration], PRD 93 (2016), 032003.

# The extraction of pion GDAs and EMT FFs

We extracted the pion GDA and EMT FFs from Belle data using the asymptotic GDA.

From GDA to the gravitational FFs:

$$\int_0^1 dz(2z-1)\,\Phi_q^{\pi^0\pi^0}(z,\zeta,W^2) = \frac{2}{(P^+)^2}\langle\,\pi^0(p_1)\,\pi^0(p_2)\,|\,T_q^{++}(0)\,|\,0\,\rangle.$$

$$\langle\pi(p_2)\pi(p_1)|T_q^{\mu\nu}|0\rangle = \frac{1}{2}[\Theta_1(s)(sg^{\mu\nu} - P^\mu P^\nu) + \Theta_2(s)\Delta^\mu\Delta^\nu]$$

Two timelike EMT FFs for pions!

From the gravitational FFs to mass radius:

$$\sqrt{\langle r^2\rangle_{\rm mass}} = 0.32\text{–}0.39\ \text{fm},$$

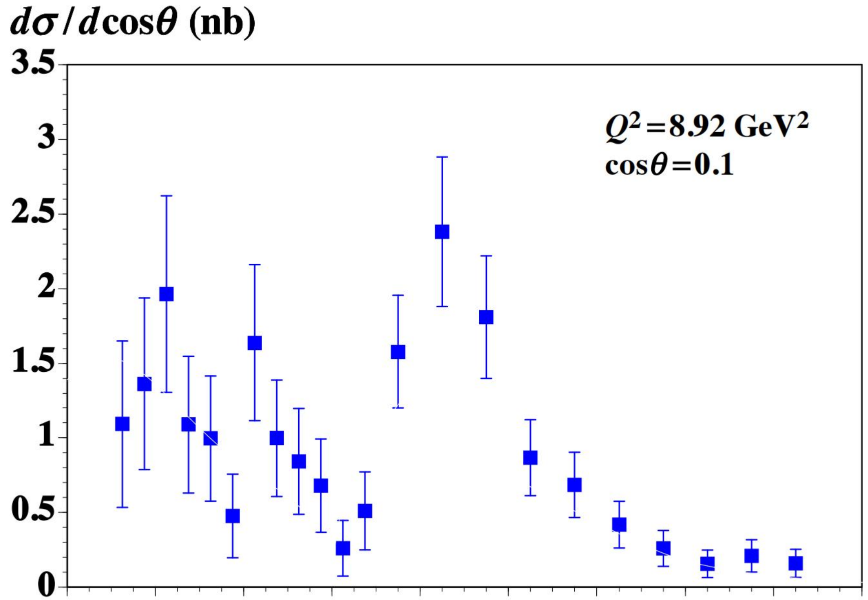
S. Kumano, Qin-Tao Song and O. Teryaev, PRD **97** (2018) 014020.

The mass radius is smaller than the pion charge radius: $\sqrt{\langle r^2\rangle} = 0.67\text{fm}$

Mass radius: $\sqrt{\langle r^2\rangle} = 0.33\text{fm}$ by NJL model A. Freese and I. C. Cloet, PRC **100** (2019), 015201

# Future measurements of $\gamma^*\gamma \to M\bar{M}$ at Belle II

Belle measurements on $\gamma^*\gamma\to\pi^0\pi^0$ in 2016




The errors are very large, and statistical errors are dominant, however, this situation can be improved by Belle II.

Better Luminosity: $2\times 10^{34}\,\mathrm{cm}^{-2}s^{-1} \to 8\times 10^{35}\,\mathrm{cm}^{-2}s^{-1}$

Previous measurements at Belle focused on EM FFs, however, the study of EMT FFs will be the main physical target for measurements of two-photon reactions at Belle II.

See talk of Dr. Masuda at Joint Meeting the APS and JPS 2023.

# Higher-twist corrections

As precise measurements are expected at Belle II, the leading-twist analysis is not enough for the meson GDAs.

Belle measurements:

$$8\ \mathrm{GeV}^2 < Q^2 < 24\ \mathrm{GeV}^2$$
$$0.2\ \mathrm{GeV}^2 < s < 4\ \mathrm{GeV}^2$$

Twist expansion:

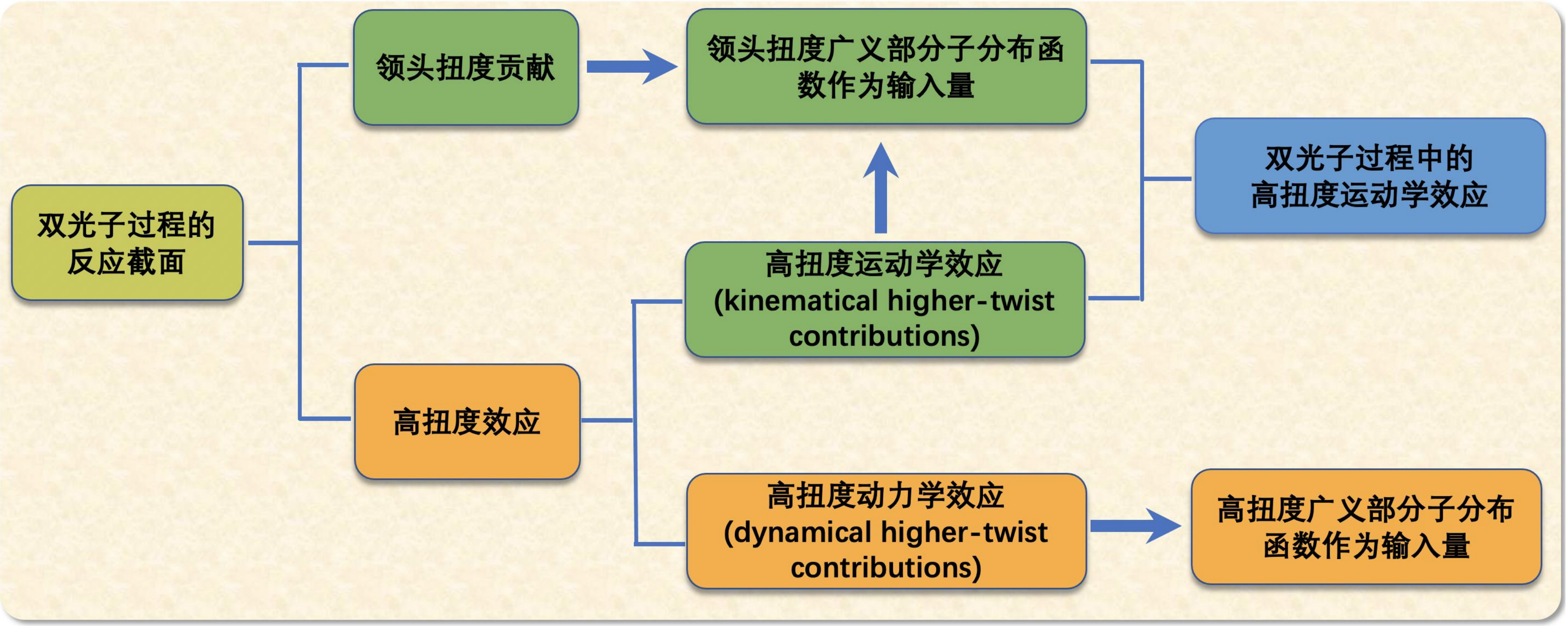
Cross section=Leading-twist contribution + Higher-twist contribution

Supressed by $1/Q$ (twist 3) or $1/Q^2$ (twist 4), it could account for ~20% in the cross section.

Three more higher-twist GDAs are needed for twist-4 contribution!

More new GDAs (less known compared with leading-twist one) will introduce more parameters, which make the extraction of GDAs diffcult!

# Kinematical higher-twist contributions




- Higher-twist corrections: leading and higher-twist GPDs (GDAs).
- Kinematical higher-twist corrections: leading-twist GPDs (GDAs)!
- Higher-order corrections of $\alpha_s$: leading-twist quark and gluon GPDs (GDAs).

V. M. Braun and A. N. Manashov, PRL 107(2011), 202001;JHEP 01 (2012), 085; PPNP 67 (2012), 162–167.

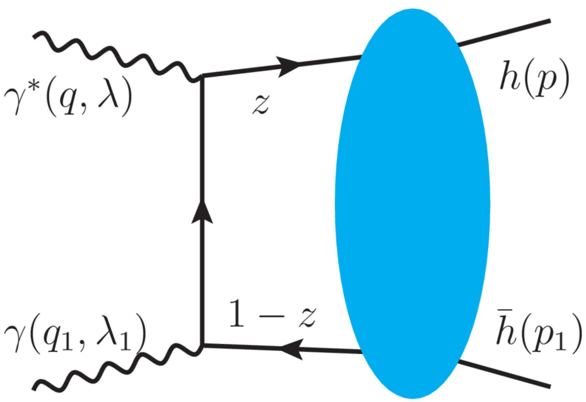
The kinematical corrections are included in recent DVCS measurements.

F. Georges et al. [Jefferson Lab Hall A], PRL. 128 (2022), 252002.
M. Defurne et al., Nature Communication 8(2017), 1408.
M. Defurne et al., Hall A collaboration, PRC92 (2015) no.5, 055202

# Kinematical higher-twist contributions in $\gamma^*\gamma \to M\bar{M}$

Helicity amplitudes of a scalar meson pair:

$$A_{\lambda\lambda_1} = T_{\mu\nu}\epsilon^{\mu}(\lambda)\epsilon^{\nu}(\lambda_1)$$




There are three independent helicity amplitudes: $A_{++}$ ,$A_{0+}$ and $A_{+-}$.

Leading twist (扭度) amplitude: $A_{++}$
Higher twist amplitudes: $A_{0+}$ and $A_{+-}$

We also calculted the kinematical higher-twist contributions for these helicity amplitudes in $\gamma^*\gamma \to M\bar{M}$

# Helicity amplitudes (up to twist 4):

$$A^{(0)} = \chi \Bigg\{ \left(1 - \frac{s}{2Q^2}\right) \int_0^1 dz \, \frac{\Phi(z,\eta,s)}{1-z} - \frac{s}{Q^2} \int_0^1 dz \, \frac{\Phi(z,\eta,s)}{z} \ln(1-z)$$

$$- \left( \frac{2s}{Q^2}\eta + \frac{\Delta_T^2}{\beta_0^2 Q^2} \frac{\partial}{\partial\eta} \right) \frac{\partial}{\partial\eta} \int_0^1 dz \, \frac{\Phi(z,\eta,s)}{z} \left[ \frac{\ln(1-z)}{2} + \mathrm{Li}_2(1-z) - \mathrm{Li}_2(1) \right] \Bigg\},$$

$$A^{(1)} = \frac{2\chi}{\beta_0 Q} \frac{\partial}{\partial\eta} \int_0^1 dz \, \Phi(z,\eta,s) \, \frac{\ln(1-z)}{z},$$

$$A^{(2)} = -\frac{2\chi}{\beta_0^2 Q^2} \frac{\partial^2}{\partial\eta^2} \int_0^1 dz \, \Phi(z,\eta,s) \, \frac{2z-1}{z} \ln(1-z),$$

$\eta = \cos\theta$

C. Lorce, B. Pire and Qin-Tao Song, PRD 106 (2022) , 094030

$A_{++} = A^{(0)}$

$A_{0+} = - A^{(1)} \Delta \cdot \epsilon(-)$ ⟶ $\propto \Delta_T$

$A_{-+} = - A^{(2)} [\Delta \cdot \epsilon(-)]^2$ ⟶ $\propto (\Delta_T)^2$

$\Delta$ is the relative momentum of final meson pair.

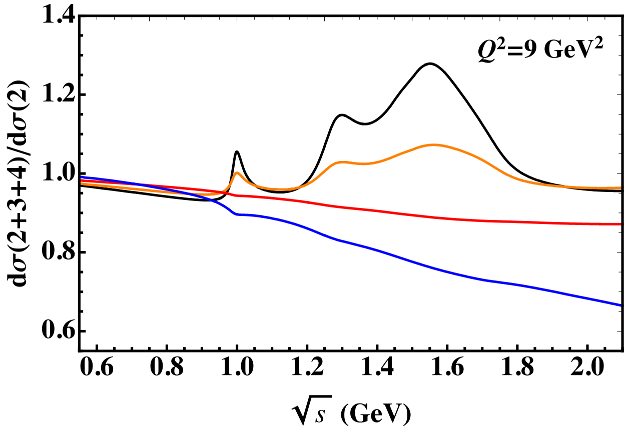
Asymptotic form of pion GDAs:

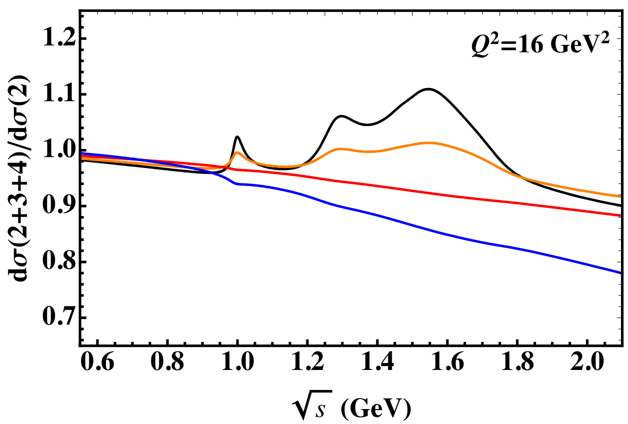
$$\Phi(z, \cos\theta, s) = 18z(1-z)(2z-1)[\widetilde{B}_{10}(s) + \widetilde{B}_{12}(s)\, P_2(\cos\theta)]$$

The nonvanishing helicity-flip amplitudes $A_{0+}$ and $A_{+-}$ indicate the existence of the D-wave GDAs.

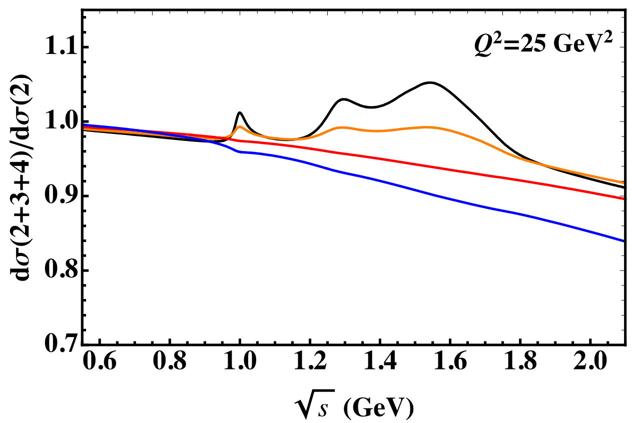
# Ratios are estimated with the asymptotic π π GDA

Ratio=(twist 2+twist 3+twist 4)/ twist 2




Kinematics are chosen according to Belle(II)




Both types of π π GDAs indicate that the higher-twist kinematical contributions cannot be neglected if s > 1 GeV$^2$

GDAs ⟶ Timelike EMT form factors

$\Lambda \geq 3$ GeV$^2$ is necessary for pion EMT form factor, PRD 97 (2018) 014020.

Dispersion relation: Spacelike form factor t<0

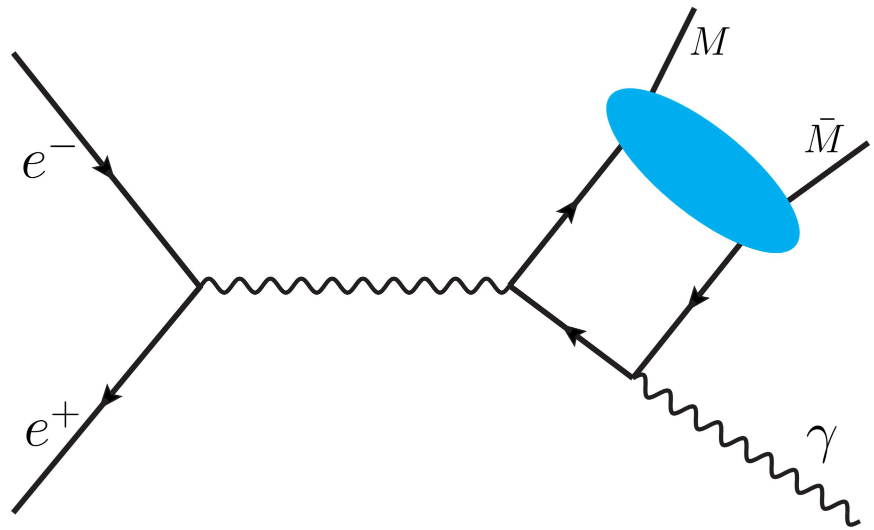
$$F(t) = \int_{4m^2}^{\Lambda} \frac{ds}{\pi} \frac{\mathrm{Im}[F(s)]}{s - t - i\varepsilon}$$

Timelike form factor s>0

# Kinematical higher-twist contributions in $\gamma^* \to M\bar{M}\gamma$

Helicity amplitudes of a scalar meson pair:

$$A_{\lambda\lambda_1} = T_{\mu\nu}\epsilon^{\mu}(\lambda)\epsilon^{\nu}(\lambda_1)$$




There are three independent helicity amplitudes: $A_{++}$ ,$A_{0+}$ and $A_{+-}$.

Leading twist (扭度) amplitude: $A_{++}$
Higher twist amplitudes: $A_{0+}$ and $A_{+-}$

We also calculted the kinematical higher-twist contributions for the helicity amplitudes in $\gamma^* \to M\bar{M}\gamma$

# Helicity amplitudes (up to twist 4):

The leading-twist amplitude: Z. Lu and I. Schmidt, PRD 73 (2006), 094021

Higher-twist helicity amplitudes (up to twist 4): B. Pire and Q. T. Song, PRD 107 (2023), 114014

$$A^{(0)} = \chi \left\{ \left(1 + \frac{\hat{s}}{2s}\right) \int_0^1 dz \frac{\Phi(z,\eta,\hat{s})}{1-z} + \frac{\hat{s}}{s} \int_0^1 dz \frac{\Phi(z,\eta,\hat{s})}{z} \ln(1-z) + \left(\frac{2\hat{s}}{s}\eta + \frac{\Delta_T^2}{\beta_0^2 s}\frac{\partial}{\partial\eta}\right)\frac{\partial}{\partial\eta}\int_0^1 dz \frac{\Phi(z,\eta,\hat{s})}{z}\left[\frac{\ln(1-z)}{2} + \mathrm{Li}_2(1-z) - \mathrm{Li}_2(1)\right]\right\},$$

$$A^{(1)} = -\frac{2\chi}{\beta_0\sqrt{s}}\frac{\partial}{\partial\eta}\int_0^1 dz\, \Phi(z,\eta,\hat{s})\frac{\ln(1-z)}{z},$$

$\eta = \cos\theta$

$$A^{(2)} = \frac{2\chi}{\beta_0^2 s}\frac{\partial^2}{\partial\eta^2}\int_0^1 dz\, \Phi(z,\eta,\hat{s})\frac{2z-1}{z}\ln(1-z),$$

$A_{++} = A^{(0)}$

$A_{0+} = -A^{(1)}\Delta\cdot\varepsilon(-)$ $\longrightarrow$ $\propto \Delta_T$

$A_{-+} = -A^{(2)}[\Delta\cdot\epsilon(-)]^2$ $\longrightarrow$ $\propto (\Delta_T)^2$

$\Delta$ is the relative momentum of final meson pair.

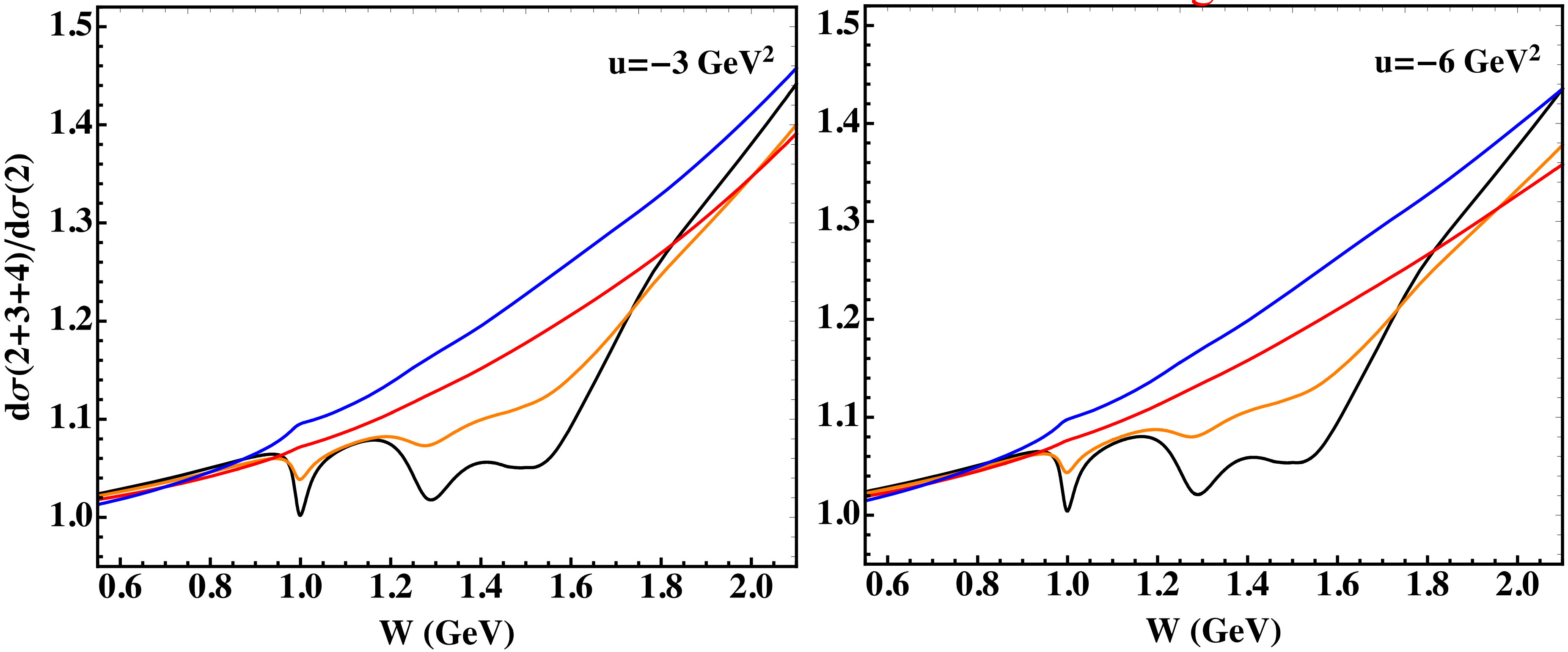
Asymptotic form of pion GDAs:

$$\Phi(z, \cos\theta, s) = 18z(1-z)(2z-1)[\widetilde{B}_{10}(s) + \widetilde{B}_{12}(s)\, P_2(\cos\theta)]$$

# Ratios are estimated with the asymptotic π π GDA

Ratio=(twist 2+twist 3+twist 4)/ twist 2

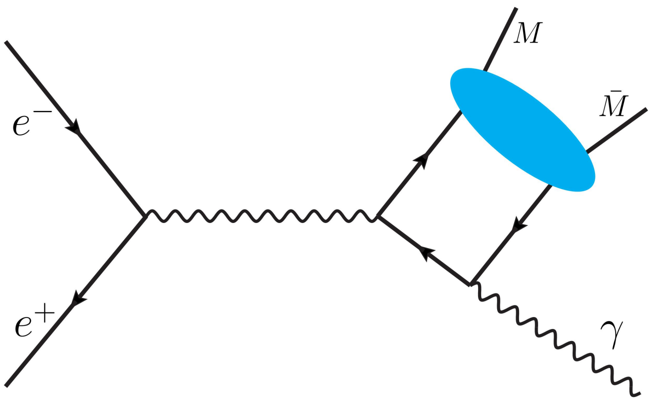
Kinematics are chosen according to BESIII



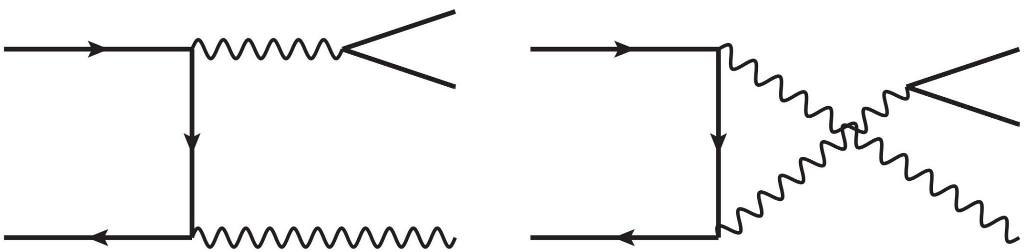

Both types of π π GDAs indicate that the higher-twist kinematical contributions cannot be neglected if W > 1 GeV.

GDAs ⟶ Timelike EMT form factors ↘ Dispersion relation: the region of W > 1 GeV is necessary. ↗ Spacelike EMT form factors

# Charged meson pair: $\pi^+\pi^-$ and $K^+K^-$




GDA process
C even meson pair



ISR process: meson EM FFs
C odd meson pair

Three types of contribution in the cross section:

Interference term

$$d\sigma_{\mathrm{G}} : d\sigma_{\mathrm{I}} : d\sigma_{\mathrm{ISR}} \sim \hat{s} : \sqrt{\hat{s}s} : s$$

GDA process, same as neutral meson pair

ISR process, largest, no GDAs

# Interference contribution in charged meson production

- Larger cross section

- Extraction of the complete information of GDAs

$$d\sigma_{\mathrm{I}} \propto \mathrm{Re}(A_{ij} F_M^*(\hat{s}))$$

$$d\sigma_{\mathrm{G}} \propto \mathrm{Re}(A_{ij} A_{kl}^*) \quad \longrightarrow$$

Imaginary phases of GDAs cannot be extracted.

$$\frac{d\sigma_{\mathrm{I}}}{d\hat{s}\, du\, d(\cos\theta)\, d\varphi} = \frac{\alpha_{\mathrm{em}}^3 \beta_0}{8\pi s^2} \frac{\sqrt{2}\beta_0}{\sqrt{\hat{s} s \epsilon(1+\epsilon)}} \left[C_0 + C_1 \cos\varphi + C_2 \cos(2\varphi) + C_3 \cos(3\varphi)\right]$$

$$C_0 = -\,\mathrm{sgn}(\rho)\,\sqrt{\epsilon(1-\epsilon)}\sqrt{2x(x-1)}\mathrm{Re}(A_{++}F_M^*)\cos\theta + \mathrm{sgn}(\rho)\,(x-1)\sqrt{\epsilon(1-\epsilon)}\mathrm{Re}(A_{0+}F_M^*)\sin\theta,$$
$$C_1 = -\,[1-(1-x)(1-\epsilon)]\,\mathrm{Re}(A_{++}F_M^*)\sin\theta + 2\epsilon\sqrt{2x(x-1)}\mathrm{Re}(A_{0+}F_M^*)\cos\theta + (x-1)\mathrm{Re}(A_{-+}F_M^*)\sin\theta,$$
$$C_2 = \mathrm{sgn}(\rho)\,\sqrt{\epsilon(1-\epsilon)}x\mathrm{Re}(A_{0+}F_M^*)\sin\theta + \mathrm{sgn}(\rho)\,\sqrt{\epsilon(1-\epsilon)}\sqrt{2x(x-1)}\mathrm{Re}(A_{-+}F_M^*)\cos\theta,$$
$$C_3 = -\,\epsilon x\mathrm{Re}(A_{-+}F_M^*)\sin\theta.$$

B. Pire and Qin-Tao Song, PRD 109 (2024), 074016

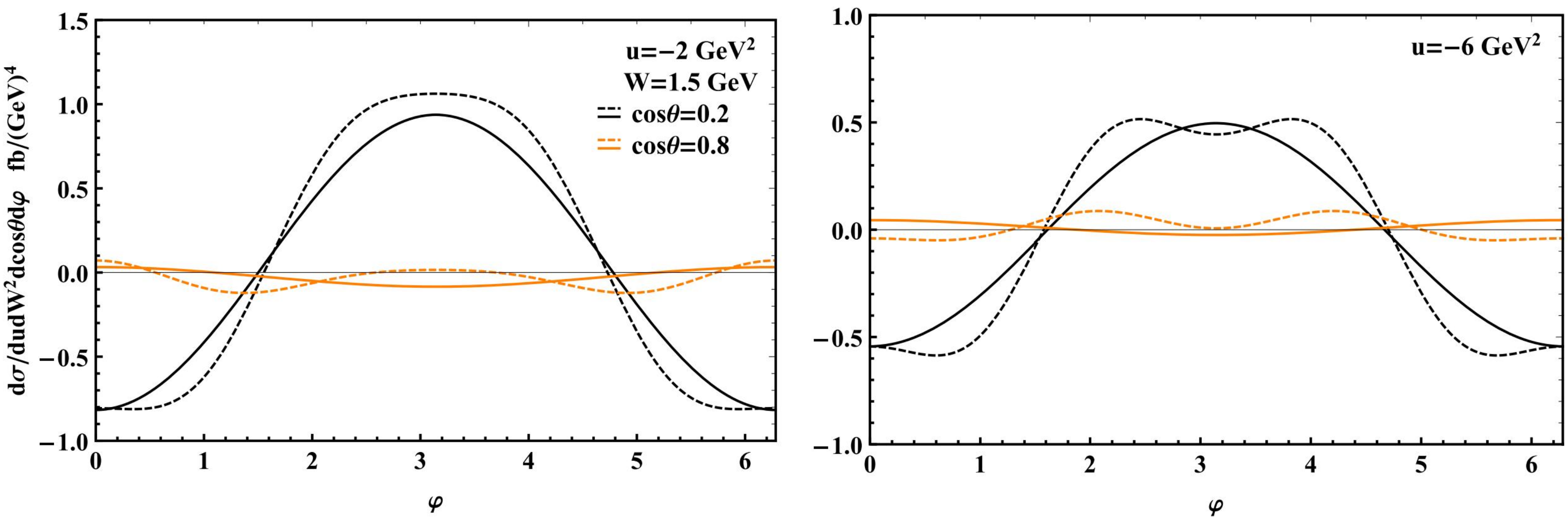
Only the interference term remains if one interchanges meson pair

$$d\sigma(M\bar{M}) - d\sigma(\bar{M}M) = 2d\sigma_{\mathrm{I}}$$

BaBar measurement of pion meson pair : PRD 92 (2015), 072015.

# Numerical estimate of interference term

The dashed curves denote the twist-2 cross sections, and the solid ones include the kinematical higher-twist contributions, $s=12$ GeV$^2$ for BESIII.




The higher-twist kinematical contributions cannot be neglected.

B. Pire and Qin-Tao Song, PRD 109 (2024), 074016

# Exotic hadrons in production of a different scalar meson pair

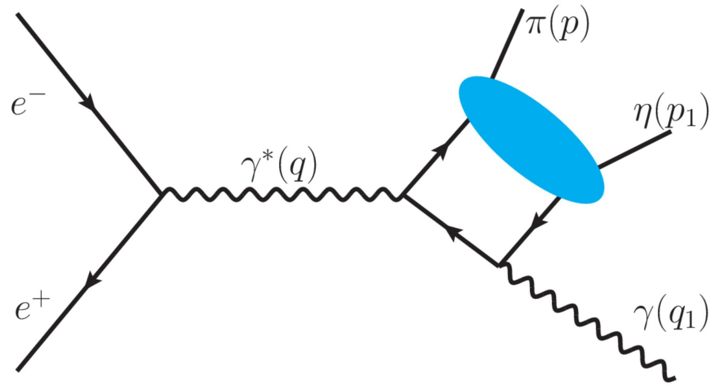
One can search for the candidates of the hybrid mesons from the P-wave of $M_1M_2$ in $\gamma^* \to M_1M_2\gamma$ and $\gamma^*\gamma \to M_1M_2$.

$M_1M_2$: $\pi\eta$, $\pi\eta'$

Isovector hybrid mesons
$I^G(J^{PC}) = 1^-(1^{-+})$
$\pi_1(1400)$, $\pi_1(1600)$

$M_1M_2$: $\eta\eta'$

Isoscalar hybrid mesons
$I^G(J^{PC}) = 0^+(1^{-+})$
$\eta_1(1855)$




$\gamma^* \to \pi\eta\gamma$ at BESIII

The exotic quantum number( $J^{PC} = 1^{-+}$) does not exit in quark model.

$\eta_1(1855)$ was observed by BESIII in $J/\psi \to \eta\eta'\gamma$ recently.

M. Ablikim et al. [BESIII], PRL 129 (2022), 192002.
M. Ablikim et al. [BESIII], PRD 106 (2022), 072012.

$J/\psi \to \gamma^*$: $\gamma^* \to \eta\eta'\gamma$ can be also measured by BESIII.

B. Pire and Q. T. Song, PRD 107 (2023), 114014.

# Shear viscosity term (a new gravitational FF)

If the hybrid mesons are observed in $\gamma^* \to M_1 M_2 \gamma$ and $\gamma^* \gamma \to M_1 M_2$, it will indicate the existence of a new EMT FF.

$$\langle M_2(p_2) M_1(p_1) | T_q^{\mu\nu} | 0 \rangle = \frac{1}{2}[\Theta_1(s)(s g^{\mu\nu} - P^\mu P^\nu) + \Theta_2(s) \Delta^\mu \Delta^\nu + \Theta_3(s) P^{\{\mu} \Delta^{\nu\}}]$$

$$p_1 + p_2 = P, p_2 - p_1 = \Delta$$

O. Teryaev, JPS Conf. Proc. 37(2022), 020406.

The shear viscosity term could exist in matrix element of EMT.

Its sum over quarks and gluons should be zero which is a consequence of the conserved EMT, however, it will exist for a single flavor q on condition that there is P-wave GDA.

The study of shear viscosity term and hybrid mesons by GDAs at BESIII and Belle II.

$\gamma^* \to M_1 M_2 \gamma$
$\gamma^* \gamma \to M_1 M_2$

P wave $M_1 M_2$ →

Shear viscosity term exists

Resonances in P wave →

Exotic (hybrid) mesons observed

Extracted from cross sections ↑

GDAs and EMT FFs

# How to investigate the $\Theta_3$ term theoretically?

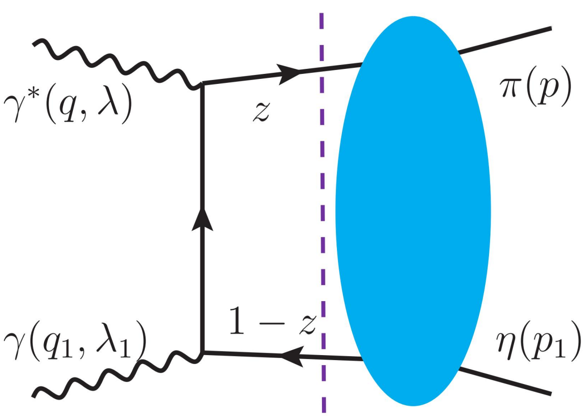
$\gamma^*(q) + \gamma(q_1) \to \pi^+(p) + \pi^-(p_1)$ $\quad q^2 = -Q^2, \quad (q_1)^2 = 0,$

In the perturbative limit: $Q^2 \gg s \gg \Lambda^2_{\rm QCD}$

The two-pion GDA can be expressed in terms of pion DAs, and meson DAs are relatively well-known quantites.

M. Diehl, T. Feldmann, P. Kroll and C. Vogt, PRD 61 (2000), 074029

We use $\pi\eta$ as an example to investigate the $\Theta_3$ term, and try to express the pion-eta GDA in terms of meson DAs in the perturbative limit.




The amplitude of $\gamma^*\gamma \to \pi\eta$ is given by the pion-eta GDA.

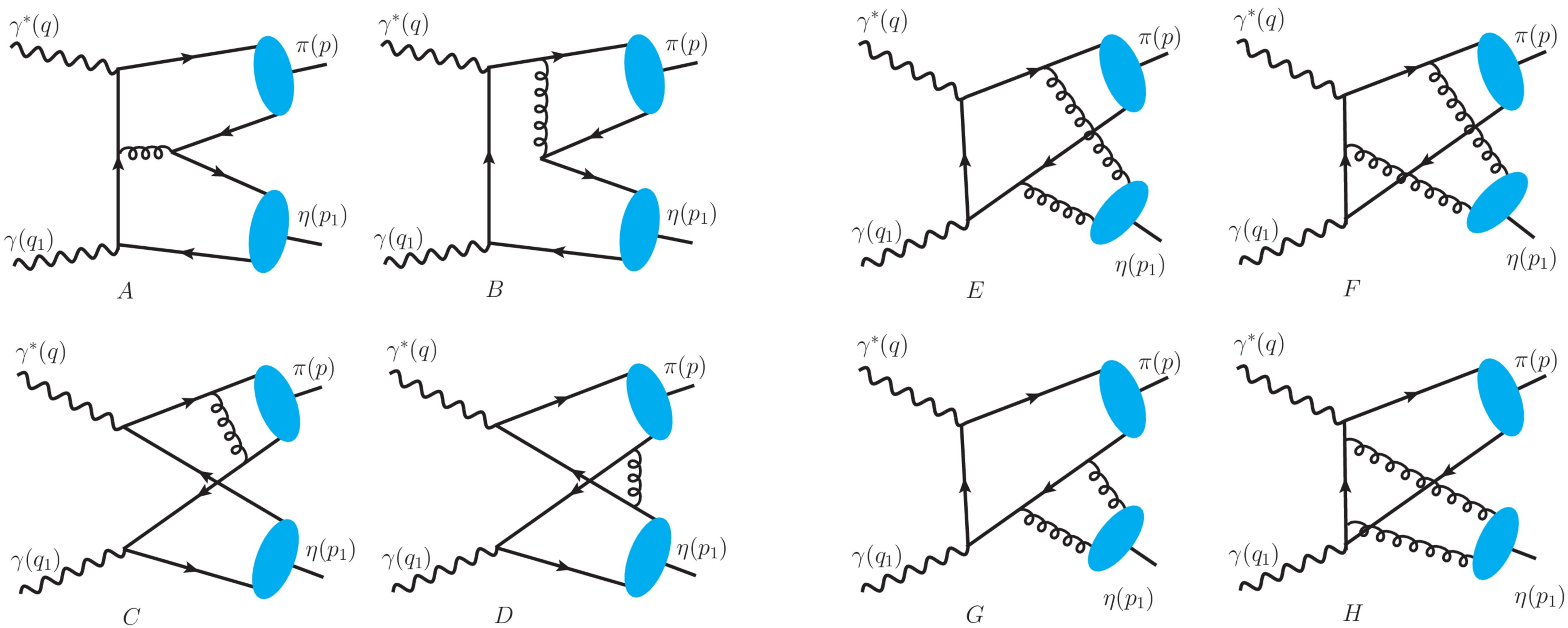
The QCD collinear factorization:

$$Q^2 \gg s, \Lambda^2_{\rm QCD}$$

The pion-eta GDA describes the amplitude from quark-antiquark pair to $\pi\eta$.

# Helicity amplitudes in the perturbative limit

Feynman diagrams for $\gamma^*\gamma \to \pi\eta$ in perturbative limit:




Part 1: the amplitude is expressed in term of quark DAs of mesons, simliar as $\gamma^*\gamma \to \pi^+\pi^-$.

Part 2: the amplitude is expressed in term of pion quark DA and eta gluon DA, which does not exist for $\gamma^*\gamma \to \pi^+\pi^-$.

# Universality (普适性) of GDAs

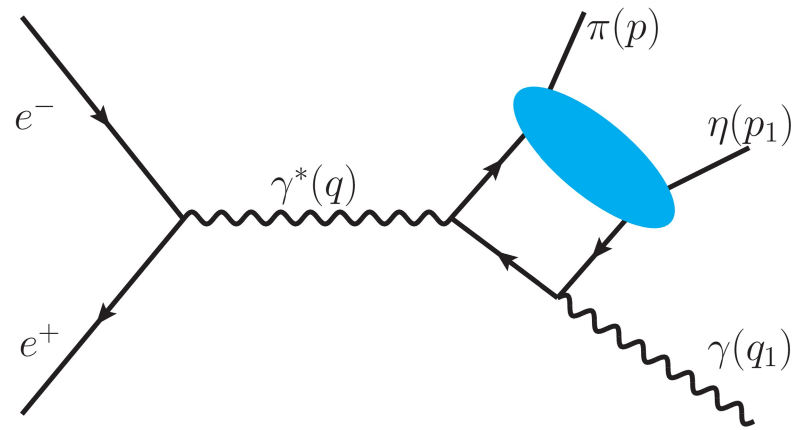
GDAs in the perturbative limit:

$$\hat{\Phi}^{q}_{\pi\eta}(z,\xi,s) = \hat{\Phi}^{q}_{\pi\eta}(z,\xi,s)\Big|_{\text{quark DAs}} + \hat{\Phi}^{q}_{\pi\eta}(z,\xi,s)\Big|_{\text{quark-gluon DAs}}$$

At current stage, there are no experimental facilities to measure $\gamma^*\gamma \to \pi\eta$ in the perturbative limit, and the obtained GDAs cannot be tested by experiment.

The process $\gamma^* \to \pi\eta\gamma$ can be measured at Belle II in the perturbative limit

$$Q^2 \gg \hat{s} \gg \Lambda^2_{\text{QCD}}$$




$\gamma^* \to \pi\eta\gamma$: timelike photon
$\gamma^*\gamma \to \pi\eta$: spacelike photon

→ Test the universality of GDAs

D. Mueller, B. Pire, L. Szymanowski and J. Wagner, PRD 86 (2012), 031502.

After a similar calculation, we find the GDAs are identical for these two processes, which verifies the universality of GDAs

Qin-Tao Song, O. V. Teryaev and S. Yoshida, PLB 868 (2025), 139797.

# From the obtained GDAs to gravitational FFs

$$\int_0^1 dz(2z-1)\Phi^q_{\pi\eta}(z,\xi,s)$$
$$=\frac{2}{(P^+)^2}\left\langle \eta(p_1)\pi(p)\left|T_q^{++}(0)\right|0\right\rangle$$

The first moment of the GDA

$$\left\langle \eta(p_1)\pi(p)\left|T_q^{\mu\nu}(0)\right|0\right\rangle$$
$$=\frac{1}{2}\Big[\Theta_1^q(s)(sg^{\mu\nu}-P^\mu P^\nu)+\Theta_2^q(s)\Delta^\mu\Delta^\nu$$
$$+\Theta_3^q(s)P^{\{\mu}\Delta^{\nu\}}\Big].$$

The $\Theta_3$ term does not exist for $\pi\pi$, which breaks the conservation law of EMT for each quark flavor in its hadronic matrix element.

The gravitation FFs are expressed in terms of meson DAs

$$\Theta_1^q=-\frac{c}{s}\int dxdy\left[\frac{1+\bar{x}+y}{\bar{x}y}\phi_\eta^q(y)+\frac{\tilde{c}y}{\bar{x}x}\phi_\eta^g(y)\right]\phi_\pi^q(x),$$
$$\Theta_2^q=-\frac{c}{s}\int dxdy\left[\frac{1+x+\bar{y}}{\bar{x}y}\phi_\eta^q(y)-\frac{\tilde{c}y}{\bar{x}x}\phi_\eta^g(y)\right]\phi_\pi^q(x),$$
$$\Theta_3^q=\frac{2c}{s}\int dxdy\left[\frac{x-\bar{y}}{\bar{x}y}\phi_\eta^q(y)+\frac{\tilde{c}y}{\bar{x}x}\phi_\eta^g(y)\right]\phi_\pi^q(x),$$

# Gravitational FFs in the perturbative limit

Isospin symmetry: $\Theta_3^u(s) = -\Theta_3^d(s)$

The $\Theta_3$ term vanishes when summing over quark flavors, and the conserved hadronic matrix elements of EMT is recovered.

We need to use the meson DAs for a single flavor q.

$$\phi_M^{u,d}(z) = 6 f_M^{u,d}\, z\bar{z} \sum_{i=0} a_{2i}^M C_{2i}^{3/2}(2z-1), \qquad M = \pi, \eta, \eta'$$

$$\phi_{\eta^{(\prime)}}^{s}(z) = 6 f_{\eta^{(\prime)}}^{s}\, z\bar{z} \sum_{i=0} \tilde{a}_{2i}^{\eta^{(\prime)}} C_{2i}^{3/2}(2z-1), \qquad \text{for } \eta \text{ and } \eta'$$

Gegenbauer coefficients

Flavor $u$ and $d$ in $\pi$: $a_{2i}^{\pi}$ does not mix with gluon DA due to isospin.

Flavor $u$, $d$ and $s$ in $\eta$: $a_{2i}^{\eta}$ and $\tilde{a}_{2i}^{\eta}$ contain both SU(3) flavor-singlet and flavor-octet components.

# Existence of the $\Theta_3$ term

We substitute the meson DAs into these EMT FFs.

$$\Theta_1^q = -\frac{c f_\pi^q}{2s}\left\{6\left[5+4(a_2^\pi + a_2^\eta)+3a_2^\pi a_2^\eta\right] f_\eta^q + \tilde{c}(1+a_2^\pi) b_2 f_\eta^1\right\},$$

$$\Theta_2^q = -\frac{c f_\pi^q}{2s}\left\{6\left[7+8(a_2^\pi + a_2^\eta)+9a_2^\pi a_2^\eta\right] f_\eta^q - \tilde{c}(1+a_2^\pi) b_2 f_\eta^1\right\},$$

$$\Theta_3^q = \frac{c f_\pi^q}{s}\sum_{i=1}\left[6(a_{2i}^\pi - a_{2i}^\eta) f_\eta^q + \tilde{c}(1+\sum_{j=1} a_{2j}^\pi) b_{2i} f_\eta^1\right],$$

The first term arises only when the quark DA of the pion meson differs from that of the eta meson.

The second term will be nonzero provided that the gluon DA does not vanish

$$b_{2i}^\eta \neq 0$$

The evolution equations are different for $a_{2i}^\pi$ and $a_{2i}^\eta$, so their DAs can not be same.Thus, the existence of the $\Theta_3$ term seems quite plausible for the $\pi\eta$ and $\pi\eta'$ pairs

Qin-Tao Song, O. V. Teryaev and S. Yoshida, PLB 868 (2025), 139797.

# Numerical estimate of Gegenbauer coefficients

$$\Theta_3^q = \frac{c f_\pi^q}{s} \sum_{i=1} \left[ 6(a_{2i}^\pi - a_{2i}^\eta) f_\eta^q + \tilde{c}(1 + \sum_{j=1} a_{2j}^\pi) b_{2i}^\eta f_\eta^1 \right]$$

$$a_2^\pi \sim 0.16$$

I. Cloet, L. Chang, C.D.Roberts,et. al.,PRL111(2013), 092001.
J. Hua et. al., [Lattice Parton], PRL 129(2022) no.13, 132001.
T. Zhong, Z.H.Zhu, H.B.Fu et. al., PRD 104(2021), 016021.
C. Shi, M. Li, X. Chen and W.Jia,PRD 104(2021), 094016.
X. Gao, et. al., PRD 106(2022), 074505.
…

$$\mu_F = \sqrt{30}\ \text{GeV}$$

$$a_2^\eta \sim -0.03$$

P. Kroll and K. Passek-Kumericki, J. Phys. G 40 (2013), 075005.

The gluon DA: $b_2^\eta \neq 0$

Yeo-Yie Charng, T. Kurimoto, and Hsiang-nan Li,PRD 74(2006), 074024.
S. Agaev, V. Braun, N. Offen, A. Porkert and A. Schäfer, PRD 90(2014), 074019.
P. Kroll and K. Passek-Kumericki,PRD 67(2003), 054017.

For the $\eta\eta'$ pair

$$\Theta_1^q|_{\eta'\eta} = -\frac{c}{s}\int dxdy \frac{1+\bar{x}+y}{\bar{x}y}\phi_\eta^q(y)\phi_{\eta'}^q(x) - \frac{c\tilde{c}}{s}\int dxdy \left[\frac{y}{\bar{x}x}\phi_{\eta'}^q(x)\phi_\eta^g(y) + \frac{x}{\bar{y}y}\phi_{\eta'}^g(x)\phi_\eta^q(y)\right],$$

$$\Theta_2^q|_{\eta'\eta} = -\frac{c}{s}\int dxdy \frac{1+x+\bar{y}}{\bar{x}y}\phi_\eta^q(y)\phi_{\eta'}^q(x) + \frac{c\tilde{c}}{s}\int dxdy \left[\frac{y}{\bar{x}x}\phi_{\eta'}^q(x)\phi_\eta^g(y) + \frac{x}{\bar{y}y}\phi_{\eta'}^g(x)\phi_\eta^q(y)\right],$$

$$\Theta_3^q|_{\eta'\eta} = \frac{2c}{s}\int dxdy \frac{x-\bar{y}}{\bar{x}y}\phi_\eta^q(y)\phi_{\eta'}^q(x) + \frac{2c\tilde{c}}{s}\int dxdy \left[\frac{y}{\bar{x}x}\phi_{\eta'}^q(x)\phi_\eta^g(y) - \frac{x}{\bar{y}y}\phi_{\eta'}^g(x)\phi_\eta^q(y)\right],$$

The $\Theta_3$ term is related to the **P-wave GDA**. Thus, we can search for the exotic mesons from the P-wave $M_1M_2$ in $\gamma^* \to M_1M_2\gamma$ and $\gamma^*\gamma \to M_1M_2$.

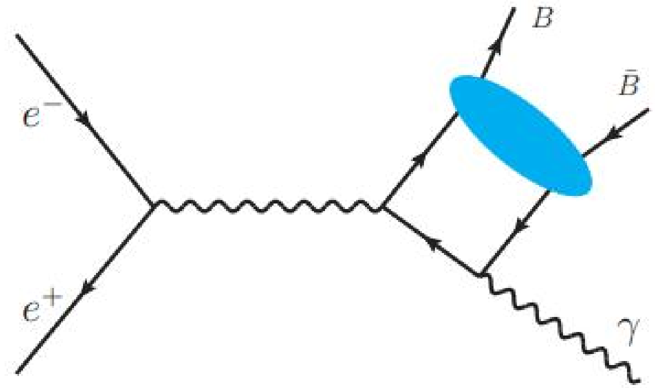
# Production of a spin-1/2 baryon pair: $\gamma^* \to \mathrm{B}\overline{B}\gamma$ and $\gamma^*\gamma \to \mathrm{B}\overline{B}$

# Baryon-antibaryon GDAs in $e^+e^- \to B\bar{B}\gamma$ (BESIII)

There are two subprocesses in $e^-e^+ \to B\bar{B}\gamma$.

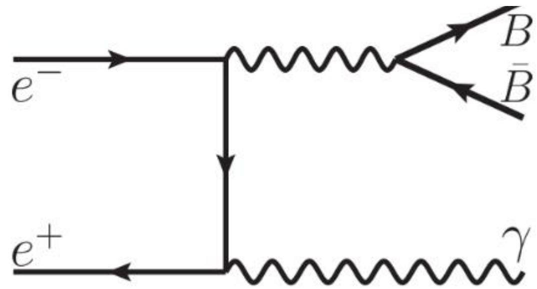
(1) QCD subprocess: $e^-e^+ \to \gamma^* \to B\bar{B}\gamma$:

The $B\bar{B}$ GDAs are involved, QCD collinear factorization.




(2) ISR subprocess: $e^-e^+ \to \gamma^*\gamma \to B\bar{B}$:

The $\gamma^* \to B\bar{B}$ vertex is described by the timelike EM FFs.




The timelike baryon EM FFs will be used to extract the baryon GDAs, and the FFs of the baryon octet family have been extensively studied at BESIII.

BESIII Collaboration, Nat. Phys. 17, 1200 (2021); PLB 817,136328 (2021); PRL130, 151905 (2023); PRL123, 122003 (2019); PRD 107,072005 (2023);PRL132, 081904 (2024); PRD 109,034029 (2024); PLB 820, 136557 (2021); PRD 103, 012005 (2021).......

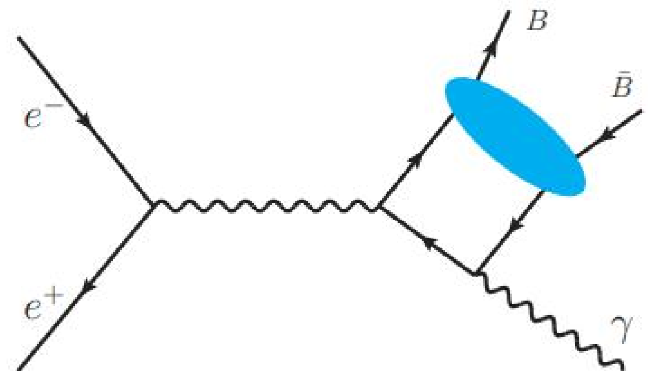
The process $e^-e^+ \to B\bar{B}\gamma$ can be measured at BESIII, Belle II, and STCF. Actually, this process has been used for the recent measurements of baryon EM FFs.

$e^-e^+ \to \Lambda\bar{\Lambda}\gamma$: BESIII Collaboration, PRD 107 (2023), 072005.

# Baryon-antibaryon GDAs

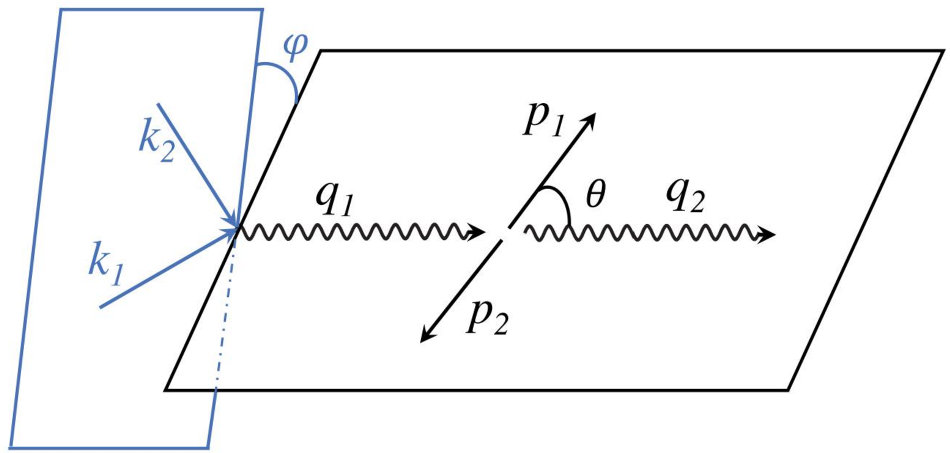
The subprocess $e^- e^+ \to \gamma^* \to B\overline{B}\gamma$, and

the GDAs of a baryon-antibaryon pair:




The center-of-mass frame of the baryon-antibaryon pair:

$e^-(k_1)e^+(k_2) \to \gamma^*(q_1) \to B(p_1)\bar{B}(p_2)\gamma(q_2)$




$$P^+ \int dX^- e^{izP^+ \cdot x^-} \langle \bar{B}(p_2)B(p_1)|\bar{q}(-x^-)\gamma^+\gamma_5 q(0)|0\rangle$$
$$= \Phi_A^q(z,\zeta_0,\hat{s})\bar{u}(p_1)\gamma^+\gamma_5 v(p_2)$$
$$+ \Phi_P^q(z,\zeta_0,\hat{s})\frac{P^+}{2m}\bar{u}(p_1)\gamma_5 v(p_2),$$

$$P^+ \int dX^- e^{izP^+ \cdot x^-} \langle \bar{B}(p_2)B(p_1)|\bar{q}(-x^-)\gamma^+ q(0)|0\rangle$$
$$= \Phi_V^q(z,\zeta_0,\hat{s})\bar{u}(p_1)\gamma^+ v(p_2) + \Phi_S^q(z,\zeta_0,\hat{s})\frac{P^+}{2m}\bar{u}(p_1)v(p_2),$$

Four baryon GDAs

M. Diehl, P. Kroll, and C.Vogt, EPJC26, 567 (2003)

# From Baryon GDAs to timelike FFs:

Four EM FFs:

$$\int_0^1 dz \Phi_i^q(z, \zeta_0, \hat{s}) = F_i^q(\hat{s}) \quad \text{for } i = V, A, P,$$

$$\int_0^1 dz \Phi_S^q(z, \zeta_0, \hat{s}) = \zeta_0 F_S^q(\hat{s}).$$

Four EMT FFs:

$$\int_0^1 dz (2z-1) \Phi_V^q(z, \zeta_0, \hat{s}) = -2\zeta_0 J^q(\hat{s})$$

$$\int_0^1 dz (2z-1) \Phi_S^q(z, \zeta_0, \hat{s}) = D^q(\hat{s}) + [A^q(\hat{s}) - 2J^q(\hat{s})](\zeta_0)^2$$

The quark orbital AM of a Baryon is expressed as

$$L_z^q = J^q(0) - \frac{1}{2} F_A^q(0).$$

The D-terms of baryons: $D = \sum_q D^q(0) + D^g(0)$

X.D. Ji, PRL 78(1997), 610.
M. V. Polyakov and C. Weiss, PRD 60, 114017 (1999).
X. H. Cao, F. K. Guo, Q. Z. Li and D.L. Yao, Nature Commun. 16 (2025) no.1, 6979
Jing Han, Bernard Pire, and Qin-Tao Song, PRD. 112(2025) , 2506.09854

# GDA contribution

We define the timelike Compton FFs:

$$(\zeta_0)\mathcal{F}_i = \sum_q \frac{e_q^2}{2}\int_0^1 dz \frac{2z-1}{z(1-z)}\Phi_i^q(z,\zeta,\hat{s})(i=V,S), \quad \mathcal{F}_{i'} = \sum_q \frac{e_q^2}{2}\int_0^1 dz \frac{1}{z(1-z)}\Phi_{i'}^q(z,\zeta,\hat{s})(i'=A,P)$$

The hardon Tensor (leading twist) is given by

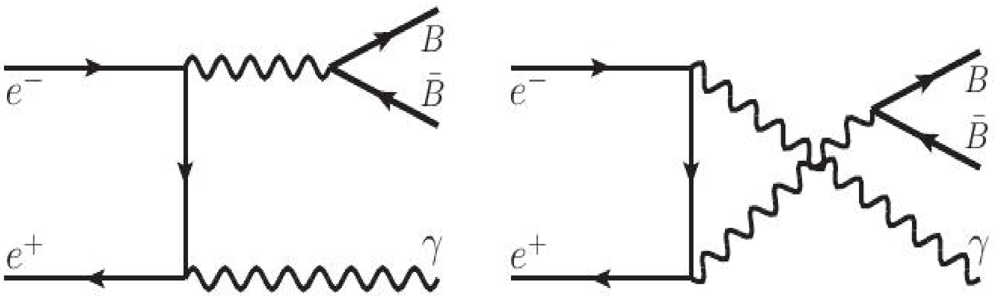
$$T_{\mu\nu} = \frac{-2}{\sqrt{2s}}\left\{g_T^{\mu\nu}\left[\zeta_0\mathcal{F}_V\bar{u}(p_1)\gamma^+ v(p_2) + \mathcal{F}_S\frac{P^+}{2m}\bar{u}(p_1)v(p_2)\right] - i\epsilon_T^{\mu\nu}\left[\mathcal{F}_A\bar{u}(p_1)\gamma^+\gamma^5 v(p_2) + \mathcal{F}_P\frac{P^+}{2m}\bar{u}(p_1)\gamma^5 v(p_2)\right]\right\}$$

Cross section:

$$\frac{d\sigma_{\mathrm{G}}}{d\hat{s}\,du\,d(\cos\theta)\,d\varphi} = \frac{\alpha_{\mathrm{em}}^3\beta_0}{8\pi s^3}\frac{1}{1+\epsilon}\left[|\mathcal{F}_A|^2 - |\mathcal{F}_S|^2 + 2\mathrm{Re}(\mathcal{F}_A\mathcal{F}_P^*) + \frac{\hat{s}\left(|\mathcal{F}_P|^2 + |\mathcal{F}_S|^2\right)}{4m^2}\right.$$
$$\left. + (\beta_0)^2\cos^2\theta\left[|\mathcal{F}_V|^2 + 2\mathrm{Re}(\mathcal{F}_S\mathcal{F}_V^*) - |\mathcal{F}_A|^2\right] - (\beta_0)^4\cos^4\theta|\mathcal{F}_V|^2\right]$$

# ISR contribution

ISR process $e^-e^+ \to \gamma^*\gamma \to B\overline{B}\gamma$




Baryon EM FFs: $\langle \bar{B}(p_2)B(p_1)|\,\bar{q}(0)\gamma^\mu q(0)\,|0\rangle = F_V^q(\hat{s})\bar{u}(p_1)\gamma^\mu v(p_2) + F_S^q(\hat{s})\frac{\Delta^\mu}{2m}\bar{u}(p_1)v(p_2),$

$$G_E(\hat{s}) = F_V^q(\hat{s}) + (\tau - 1)F_S^q(\hat{s}), G_M(\hat{s}) = F_V^q(\hat{s})$$

Cross section:

$$\frac{d\sigma_{\rm ISR}}{d\hat{s}\,du d(\cos\theta) d\varphi} = \frac{\alpha_{\rm em}^3\beta_0^3}{4\pi s^2}\frac{1}{\epsilon\hat{s}}\left[b_0 + b_1\cos^2\theta + b_2\sin^2\theta + b_3\sin(2\theta)\cos\varphi + b_4\sin^2\theta\cos(2\varphi)\right]$$

$$b_0 = [1 - 2x(1-x)(1+\epsilon)](2\lambda - 1)|G_M|^2,$$

$$b_1 = [1 - 2x(1-x)(1-\epsilon)]|G_M|^2 + 4\epsilon x(x-1)(\lambda-1)\left[|G_E|^2 - |G_M|^2\right],$$

$$b_2 = 2\epsilon x(x-1)|G_M|^2 + [1 - 2x(1-x)](\lambda-1)\left[|G_E|^2 - |G_M|^2\right],$$

$$b_3 = \sqrt{\epsilon(1-\epsilon)}\sqrt{2x(x-1)}(2x-1)\mathrm{sgn}(\rho)\left[(\lambda-1)|G_E|^2 - \lambda|G_M|^2\right],$$

$$b_4 = 2\epsilon x(1-x)\left[(\lambda-1)|G_E|^2 - \lambda|G_M|^2\right].$$

# Interference term

The interference term of two subprocesses are also need to included.

$$|\mathcal{M}_{\rm I}|^2 = \mathcal{M}_{\rm ISR}\mathcal{M}_{\rm G}^* + \mathcal{M}_{\rm ISR}^*\mathcal{M}_{\rm G}$$

One can decompose the cross section according to its dependence on angles

$$\frac{d\sigma_{\rm I}}{d\hat{s}\,dud(\cos\theta)d\varphi} = \frac{\alpha_{\rm em}^3\beta_0}{8\pi s^2}\frac{\sqrt{2}\beta_0}{\sqrt{\hat{s}\,s\epsilon(1+\epsilon)}}\left[c_0\cos\theta + c_1\cos^3\theta + c_2\sin\theta\cos\varphi + c_3\sin(2\theta)\cos\theta\cos\varphi\right]$$

where the coefficients read

$$\begin{aligned}
c_0 =& 2\mathrm{sgn}(\rho)\sqrt{\epsilon(1-\epsilon)}\sqrt{2x(x-1)}[\mathrm{Re}(\mathcal{F}_V G_M^*) + \mathrm{Re}(\mathcal{F}_S G_E^*)],\\
c_1 =& 2(\beta_0)^2\mathrm{sgn}(\rho)\sqrt{\epsilon(1-\epsilon)}\sqrt{2x(x-1)}[(\lambda-1)\mathrm{Re}(\mathcal{F}_V G_E^*) - \lambda\mathrm{Re}(\mathcal{F}_V G_M^*)],\\
c_2 =& 2[1-(1-x)(1+\epsilon)]\mathrm{Re}(\mathcal{F}_A G_M^*) + 2[1-(1-x)(1-\epsilon)]\mathrm{Re}(\mathcal{F}_S G_E^*),\\
c_3 =& (\beta_0)^2[1-(1-x)(1-\epsilon)][(\lambda-1)\mathrm{Re}(\mathcal{F}_V G_E^*) - \lambda\mathrm{Re}(\mathcal{F}_V G_M^*)].
\end{aligned}$$

Jing Han, Bernard Pire, and Qin-Tao Song, PRD. 112(2025) , 2506.09854

# Forward-backward asymmetry

The contribution of $d\sigma_I$ is larger than $d\sigma_G$ . To extract baryon GDAs, it is necessary to study the interference contribution.

Consider the exchange of $(\theta, \varphi) \rightarrow (\pi - \theta, \pi + \varphi)$:

$$d\sigma_{\rm ISR} \longrightarrow d\sigma_{\rm ISR} \quad d\sigma_{\rm G} \longrightarrow d\sigma_{\rm G} \quad d\sigma_{\rm I} \longrightarrow -d\sigma_{\rm I}$$

$$d\sigma(B, \bar{B}) - d\sigma(\bar{B}, B) = 2d\sigma_{\rm I}$$

We can also define a new observable, similar to the TCS process, and the forward-backward asymmetry is given by

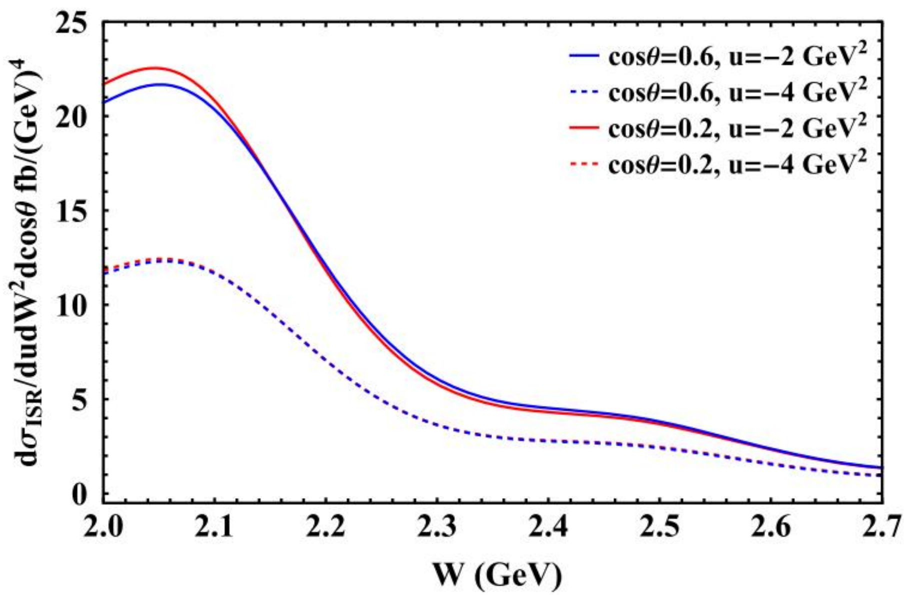
$$A_{\rm FB}(\theta) = \frac{\int_{\pi/2}^{3\pi/2} d\varphi \frac{d\sigma(\theta,\varphi)}{d\cos\theta\, d\varphi} - \int_{3\pi/2}^{2\pi} d\varphi \frac{d\sigma(\pi-\theta,\varphi)}{d\cos\theta\, d\varphi} - \int_{0}^{\pi/2} d\varphi \frac{d\sigma(\pi-\theta,\varphi)}{d\cos\theta\, d\varphi}}{\int_{\pi/2}^{3\pi/2} d\varphi \frac{d\sigma(\theta,\varphi)}{d\cos\theta\, d\varphi} + \int_{3\pi/2}^{2\pi} d\varphi \frac{d\sigma(\pi-\theta,\varphi)}{d\cos\theta\, d\varphi} + \int_{0}^{\pi/2} d\varphi \frac{d\sigma(\pi-\theta,\varphi)}{d\cos\theta\, d\varphi}}$$

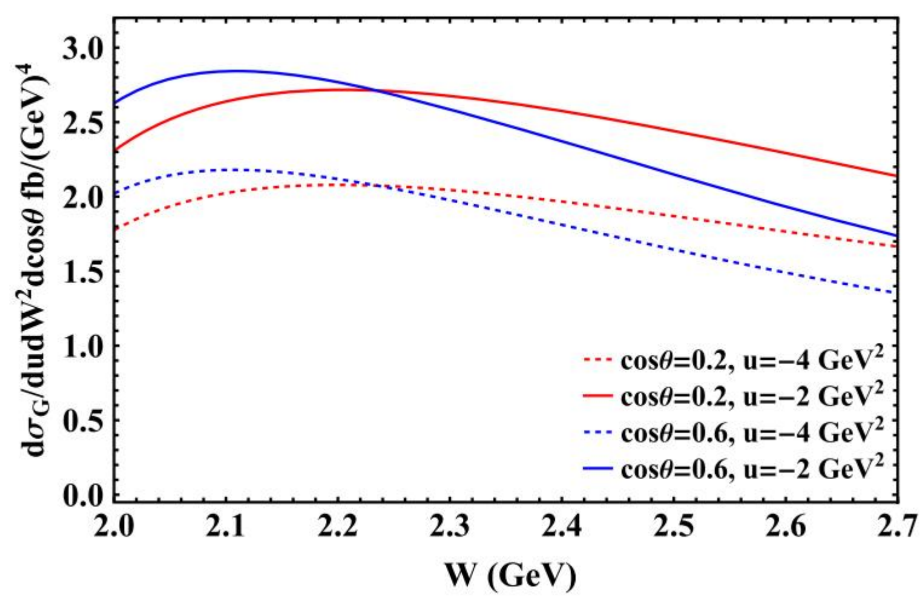
Only the interference term contributes to the numerator, and this provides a way to obtain interference contribution.

E. R. Berger, M. Diehl, and B. Pire, EPJC 23, 675(2002)

# Numerical estimate

We adopted the effective proton EM FF and GDA model for numerical estimate.



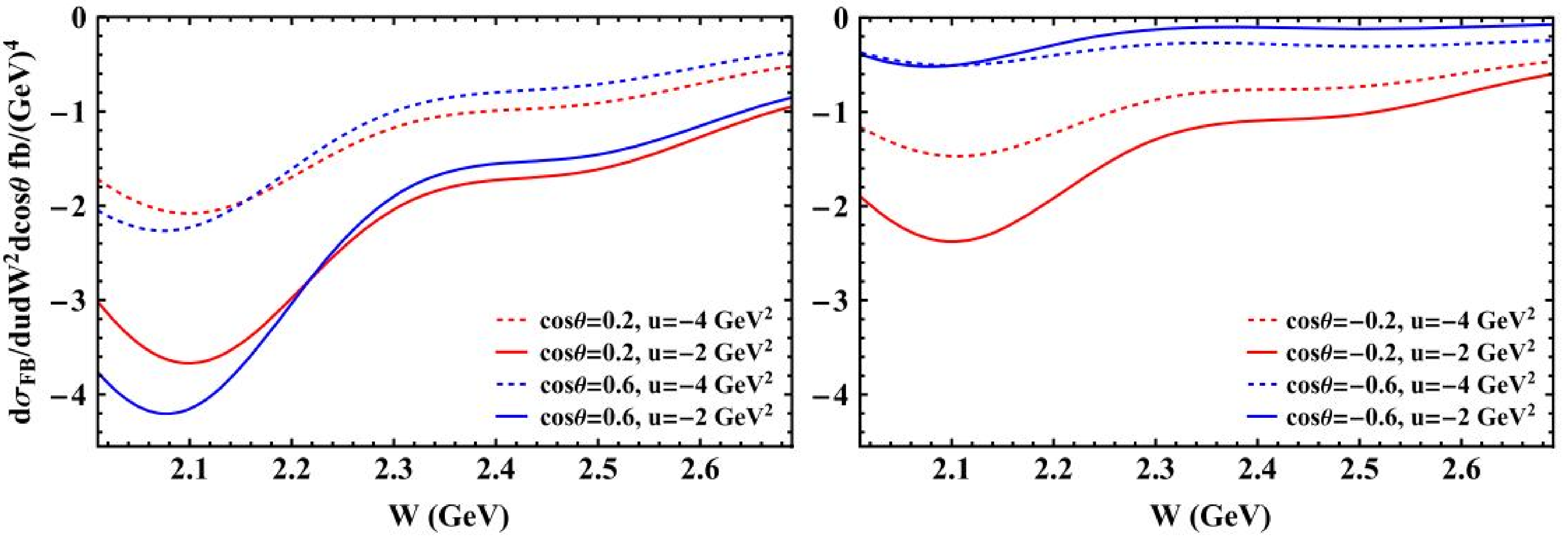




Estimate of ISR contribution

Estimate of $e^+e^- \to \gamma^* \to B\bar{B}\gamma$ contribution

$\sqrt{s} = 4$ GeV is typical for BESIII and the proposed STCF.

# Numerical estimate




Estimate of the forward-backward contribution

The interference term is larger than pure QCD term, it will play an important role in the extraction of baryon GDAs.

Jing Han, Bernard Pire, and Qin-Tao Song, PRD. 112(2025)，2506.09854

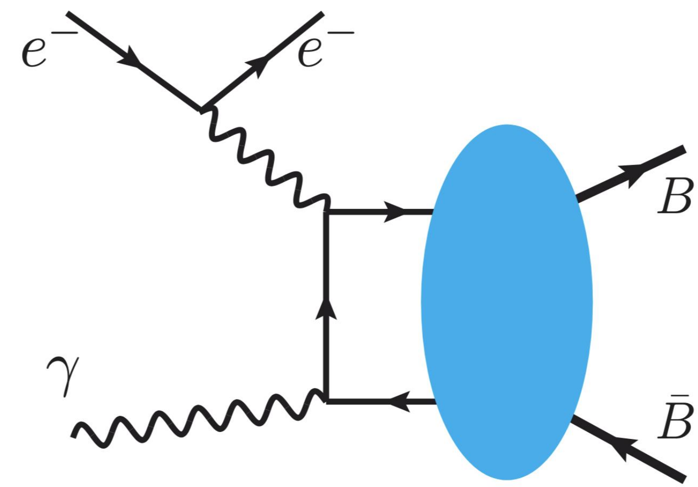
# Baryon-antibaryon GDAs in e$\gamma \to eB\overline{B}$

$$e(k_1)\gamma(q_2) \to e(k_2)B(p_1, S_1)\bar{B}(p_2, S_2)$$

$$s = (k_1 + q_2)^2, \qquad (k_1 - k_2)^2 = (q_1)^2 = -Q^2$$

$$\hat{s} = P^2 = (p_1 + p_2)^2, \qquad (p_1)^2 = (p_2)^2 = m^2$$

The baryon-antibaryon GDAs:



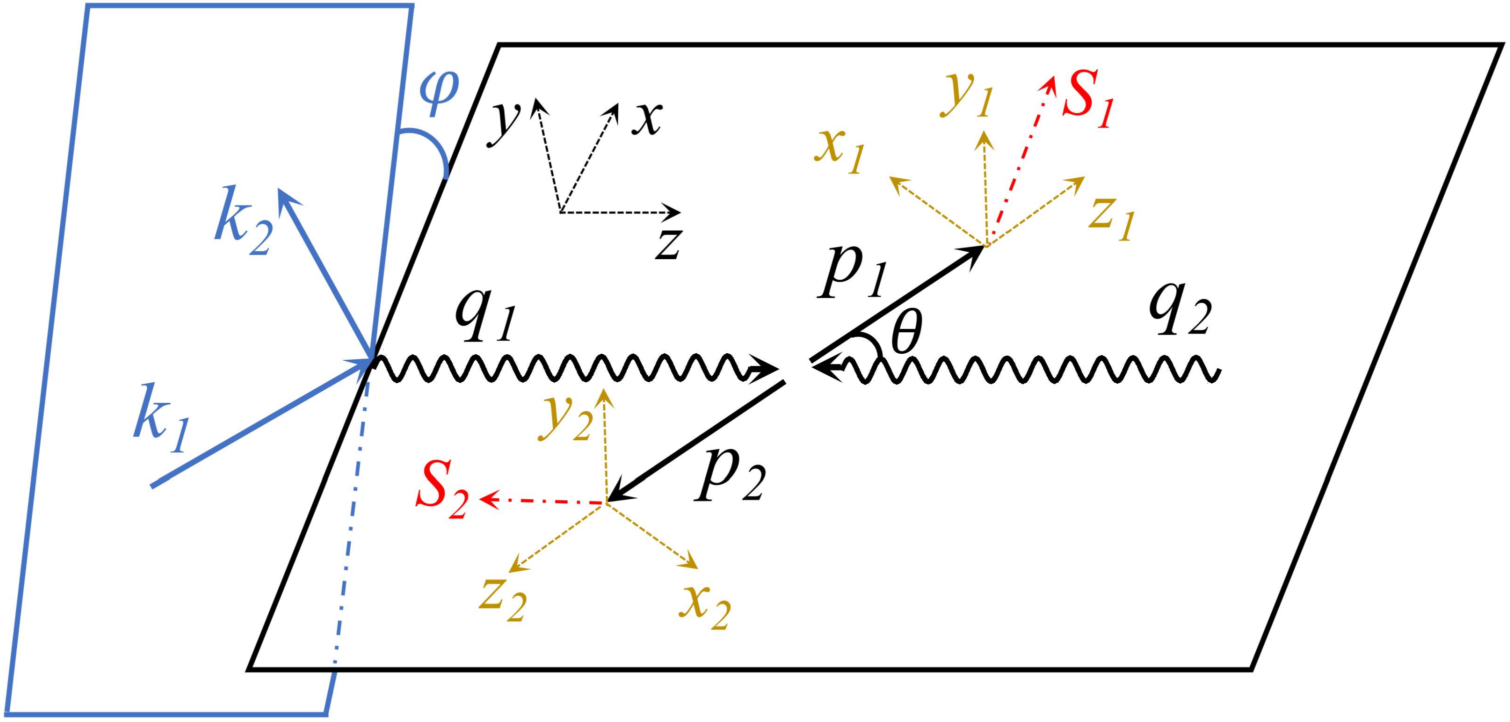
$e\gamma \to eB\bar{B}$

The spin vectors $S_1$ and $S_2$ can be determined from subsequent decays of baryons such as $\Lambda \to p\pi$. The Spin physics has already been a hot topic at BESIII, with significant progress being made recently.

See the talk of Prof. Hai-Bo Li at SPIN2025 for Spin physics at BESIII!

# Baryon-antibaryon GDAs in eγ → $eB\overline{B}$

$$e(k_1)\gamma(q_2) \to e(k_2)B(p_1,S_1)\bar{B}(p_2,S_2)$$




The Center-of-mass frame of the baryon-antibaryon pair

The spin vectors $S_1$ and $S_2$ are defined within their own rest frames.

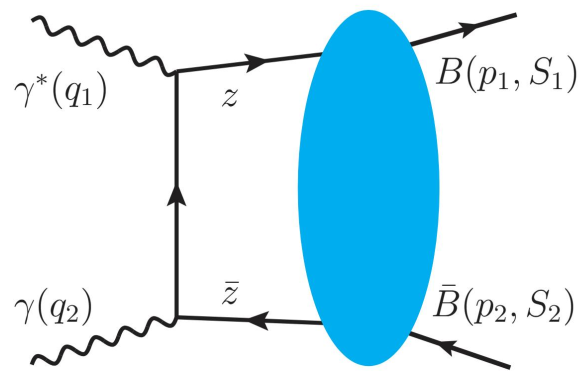
$$S_1^\mu = (0, S_1^x, S_1^y, S_1^z),$$
$$S_2^\mu = (0, S_2^x, S_2^y, S_2^z).$$

# Baryon-antibaryon GDAs in $e^{\pm}\gamma \to e^{\pm}B\overline{B}$

There are two subprocesses in $e^{\pm}\gamma \to e^{\pm}B\overline{B}$.

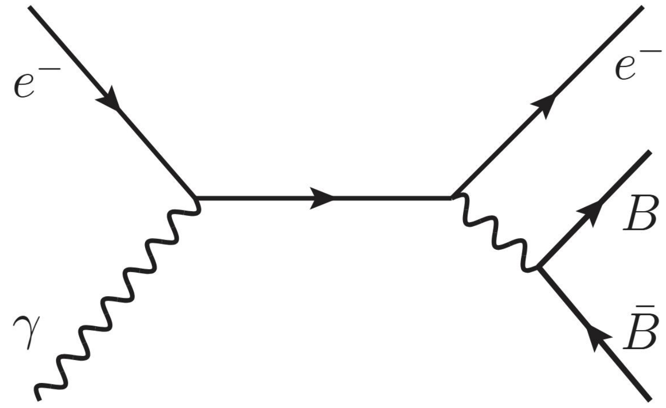
(1) QCD subprocess:$\gamma^*\gamma \to B\overline{B}$:

The $B\overline{B}$ GDAs are involved, QCD collinear factorization.




(2) Bremsstrahlung subprocess

The $\gamma^* \to B\overline{B}$ vertex is described by the timelike EM FFs.



# Spin-independent cross section

The differential cross sections will contain both spin independent and dependent contribution:

$$\frac{d\sigma(S_1,S_2)}{d\hat{s}dQ^2d(\cos\theta)\,d\varphi}=\frac{1}{4}\frac{d\bar{\sigma}}{d\hat{s}dQ^2d(\cos\theta)\,d\varphi}+\frac{d\hat{\sigma}(S_1,S_2)}{d\hat{s}dQ^2d(\cos\theta)\,d\varphi}$$

## GDA contribution:

$$\frac{d\bar{\sigma}_{\mathrm{G}}}{d\hat{s}dQ^2d(\cos\theta)\,d\varphi}=\frac{\alpha_{\mathrm{em}}^3\beta_0}{8\pi s^2Q^2}\frac{1}{1-\epsilon}\Big\{\left|\mathcal{F}_A\right|^2-\left|\mathcal{F}_S\right|^2+2\mathrm{Re}\left(\mathcal{F}_A\mathcal{F}_P^*\right)+\frac{\hat{s}}{4m^2}\left(\left|\mathcal{F}_P\right|^2+\left|\mathcal{F}_S\right|^2\right)$$
$$+\left(\beta_0\right)^2\cos^2\theta\left[\left|\mathcal{F}_V\right|^2+2\mathrm{Re}\left(\mathcal{F}_S\mathcal{F}_V^*\right)-\left|\mathcal{F}_A\right|^2\right]-\left(\beta_0\right)^4\cos^4\theta\left|\mathcal{F}_V\right|^2\Big\},$$

## Bremsstrahlung contribution:

$$\frac{d\bar{\sigma}_{\mathrm{B}}}{d\hat{s}dQ^2d(\cos\theta)\,d\varphi}=\frac{\alpha_{\mathrm{em}}^3\beta_0^3}{4\pi s^2}\frac{1}{\epsilon\hat{s}}\Big\{\omega_1(2\lambda-1)\left|G_M\right|^2+\left[\omega_2\left|G_M\right|^2+2\omega_3(\lambda-1)(\left|G_E\right|^2-\left|G_M\right|^2)\right]\cos^2\theta$$
$$+\left[\omega_3\left|G_M\right|^2+\omega_4(\lambda-1)(\left|G_E\right|^2-\left|G_M\right|^2)\right]\sin^2\theta+\omega_5\left[(\lambda-1)\left|G_E\right|^2-\lambda\left|G_M\right|^2\right]$$
$$\times\sin(2\theta)\cos\varphi-\omega_3\left[(\lambda-1)\left|G_E\right|^2-\lambda\left|G_M\right|^2\right]\sin^2\theta\cos(2\varphi)\Big\},$$

## Interference term:

$$\frac{d\bar{\sigma}_{\mathrm{I}}}{d\hat{s}dQ^2d(\cos\theta)\,d\varphi}=e_l\frac{\alpha_{\mathrm{em}}^3\beta_0}{8\pi s^2}\frac{\sqrt{2}\beta_0}{\sqrt{\hat{s}Q^2\epsilon(1-\epsilon)}}\Big\{2\omega_6\left[\mathrm{Re}\left(\mathcal{F}_VG_M^*\right)+\mathrm{Re}(\mathcal{F}_SG_E^*)\right]\cos\theta-2(\beta_0)^2\omega_6\left[\lambda\mathrm{Re}(\mathcal{F}_VG_M^*)\right.$$
$$\left.-(\lambda-1)\mathrm{Re}\left(\mathcal{F}_VG_E^*\right)\right]\cos^3\theta+2\left[\omega_7\mathrm{Re}(\mathcal{F}_AG_M^*)+\omega_8\mathrm{Re}(\mathcal{F}_SG_E^*)\right]\sin\theta\cos\varphi$$
$$-(\beta_0)^2\omega_8\left[\lambda\mathrm{Re}(\mathcal{F}_VG_M^*)-(\lambda-1)\mathrm{Re}\left(\mathcal{F}_VG_E^*\right)\right]\sin(2\theta)\cos\theta\cos\varphi\Big\},$$

# Spin-dependent cross section

GDA contribution:

Timelike Compton FFs, GDAs are involved.

$$\frac{d\hat{\sigma}_G}{d\hat{s}dQ^2d(\cos\theta)d\varphi} = \frac{\alpha_{\rm em}^3\beta_0^2}{64\pi s^2Q^2(1-\epsilon)}\frac{\sqrt{\hat{s}}}{m}\left[-\beta_0\sin(2\theta)\,\mathrm{Im}\,(\mathcal{F}_V\mathcal{F}_S^*)\,(S_1^y+S_2^y)+2\sin\theta\,\mathrm{Im}\,(\mathcal{F}_A\mathcal{F}_P^*)\,(S_1^y-S_2^y)\right],$$

Only the $S^y$ polarization appears in final baryon pairs!

Bremsstrahlung contribution:

$$\frac{d\hat{\sigma}_B}{d\hat{s}dQ^2d(\cos\theta)d\varphi} = \frac{\alpha_{\rm em}^3\beta_0}{8\pi s^2\epsilon\hat{s}}\frac{m}{\sqrt{\hat{s}}}\mathrm{Im}(G_MG_E^*)\Big\{2\left[\omega_3\sin\theta\sin(2\varphi)-\omega_5\cos\theta\sin\varphi\right](S_1^x-S_2^x)$$
$$+\left[\omega_2\sin(2\theta)+2\omega_5\cos(2\theta)\cos\varphi-2\omega_3\sin(2\theta)\cos^2\varphi\right](S_1^y+S_2^y)\Big\},$$

Both the $S^x$ and $S^y$ polarizations appear in final baryon pairs!

# Spin-dependent cross section

## Interference term:

$$
\begin{aligned}
\frac{d\hat{\sigma}_{\rm I}}{d\hat{s}dQ^2d\left(\cos\theta\right)d\varphi} =& e_l\frac{\alpha_{\rm em}^3\beta_0}{16\pi s^2Q\sqrt{2\hat{s}\epsilon(1-\epsilon)}}\Bigg\{\omega_7{\rm Im}\Big(G_M^*(\mathcal{F}_P+\frac{4m^2}{\hat{s}}\mathcal{F}_A)\Big)\cos\theta\sin\varphi\frac{\sqrt{\hat{s}}}{m}(S_1^x+S_2^x) \\
&+\beta_0\Big[\omega_8{\rm Im}(G_M^*\mathcal{F}_S)\sin\varphi+\omega_8\frac{4m^2}{\hat{s}}{\rm Im}(G_M^*\mathcal{F}_V)\cos^2\theta\sin\varphi+\omega_7\frac{4m^2}{\hat{s}}{\rm Im}(G_E^*\mathcal{F}_A)\sin^2\theta\sin\varphi\Big] \\
&\frac{\sqrt{\hat{s}}}{m}(S_1^x-S_2^x)+\beta_0\Big[\omega_6{\rm Im}(G_M^*\mathcal{F}_S)\sin\theta-\omega_8{\rm Im}\Big(G_M^*(\mathcal{F}_S+\frac{4m^2}{\hat{s}}\mathcal{F}_V)\Big)\cos\theta\cos\varphi \\
&+\omega_6\frac{2m^2}{\hat{s}}{\rm Im}\Big(\mathcal{F}_V(G_M^*-G_E^*)\Big)\sin(2\theta)\cos\theta+\omega_8\frac{2m^2}{\hat{s}}{\rm Im}\Big(\mathcal{F}_V(G_M^*-G_E^*)\Big)\sin(2\theta)\sin\theta\cos\varphi\Big] \\
&\frac{\sqrt{\hat{s}}}{m}(S_1^y+S_2^y)-\omega_7{\rm Im}\Big(G_M^*(\mathcal{F}_P+\frac{4m^2}{\hat{s}}\mathcal{F}_A)\Big)\cos\varphi\frac{\sqrt{\hat{s}}}{m}(S_1^y-S_2^y)+2\omega_7{\rm Im}\Big(G_E^*(\mathcal{F}_P+\frac{4m^2}{\hat{s}}\mathcal{F}_A)\Big) \\
&\sin\theta\sin\varphi(S_1^z+S_2^z)-\beta_0{\rm Im}\Big(G_M^*(\omega_7\mathcal{F}_A-\omega_8\mathcal{F}_V)\Big)\sin(2\theta)\sin\varphi(S_1^z-S_2^z)\Bigg\},
\end{aligned}
$$

We can observe $S^x$, $S^y$, and $S^z$ polarizations in the final baryon pairs.

The numerical estimate of the cross section is in progress.

# Summary

- GDAs can be used to investigate the EMT FFs of unstable hadrons.
- We investigate the processes of $e^- e^+ \to h\bar{h}\gamma$ and e$\gamma \to eh\bar{h}$, from which the hadron GDAs can be extracted.
- The measurements are possible at BESIII, Belle II, and STCF, the study of GDAs at BESIII (STCF) can be a new research direction.

Thank you very much