第119期强子物理在线论坛

Quark Models in the Multiquark Era: From the Cornell Potential to Deep Neural Networks

Lu Meng (孟璐)

Southeast University

Sep. 5th, 2025


Shi-Lin Zhu


Wei-Lin Wu


Yao Ma


Yan-Ke Chen

Based on: PRD108, 114016 (2023), PRD110, 034030 (2024), PRD109, 054034 (2024), arXiv:2506.20555 …

History of the multiquark states

Phys.Lett. 8 (1964) 214-215

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

M. GELL-MANN
California Institute of Technology, Pasadena, California

Received 4 January 1964

. . .

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" 6) q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations (qqq), $(qqqq\bar{q})$, etc., while mesons are made out of $(q\bar{q})$, $(qq\bar{q}\bar{q})$, etc. It is assuming that the lowest baryon configuration (qqq) gives just the represen-

8419/TH.412
21 February 1964

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

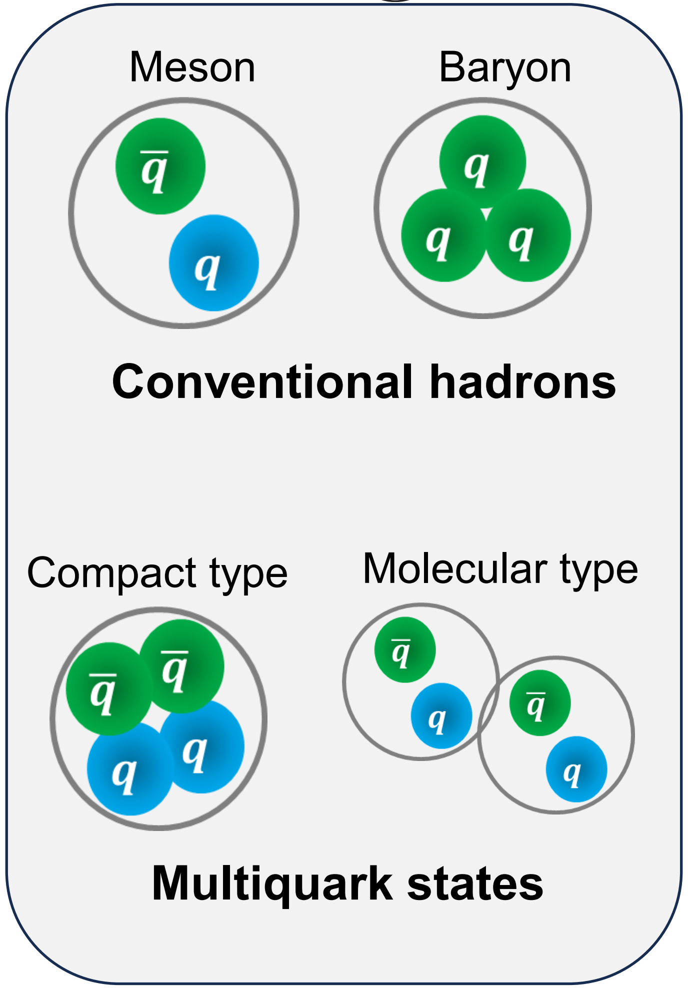
II *)

G. Zweig
CERN---Geneva

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

. . .

6) In general, we would expect that baryons are built not only from the product of three aces, AAA, but also from $\bar{A}AAAA$, $\bar{A}\bar{A}AAAAA$, etc., where $\bar{A}$ denotes an anti-ace. Similarly, mesons could be formed from $\bar{A}A$, $\bar{A}\bar{A}AA$ etc. For the low mass mesons and baryons we will assume the simplest possibilities, $\bar{A}A$ and AAA, that is, "deuces and treys".



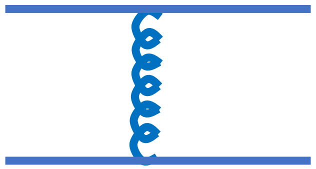
The multiquark states were predicted at the birth of quark model

Quark potential models

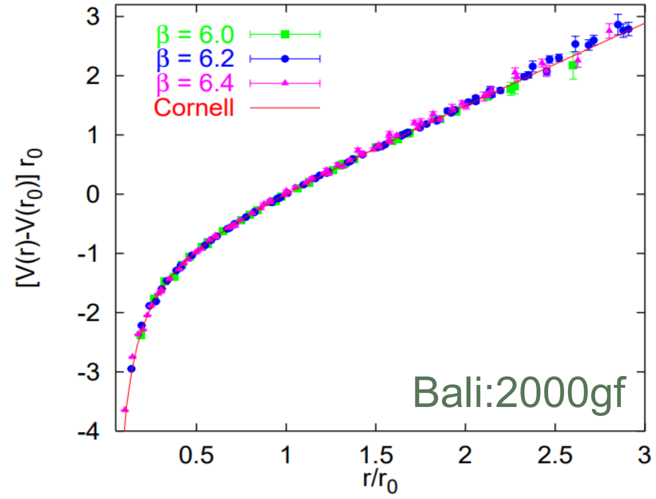
- A minimal model: one-gluon-exchange+Confinement

Cornell model: Eichten:1978tg,
BGS model: Barnes:2005pb
…

$$V_{ij}(r) = \left[\underbrace{\frac{\alpha_s}{r} - \frac{8\pi\alpha_s}{3m_i m_j}\frac{\tau^3}{\pi^{3/2}}e^{-\tau^2 r^2}\boldsymbol{s}_i\cdot\boldsymbol{s}_j}_{\text{OGE}} + \underbrace{\left(-\frac{3b}{4}r + V_c\right)}_{\text{Confinement}} \right]\frac{\lambda_i\cdot\lambda_j}{4}$$

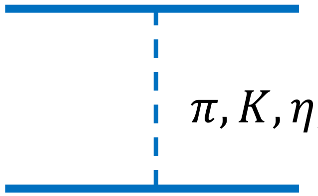


One-gluon-exchange



- Chiral quark models

Manohar:1983md, Zhang:1997ny, Vijande:2004he, Gonzalez:2012gka…



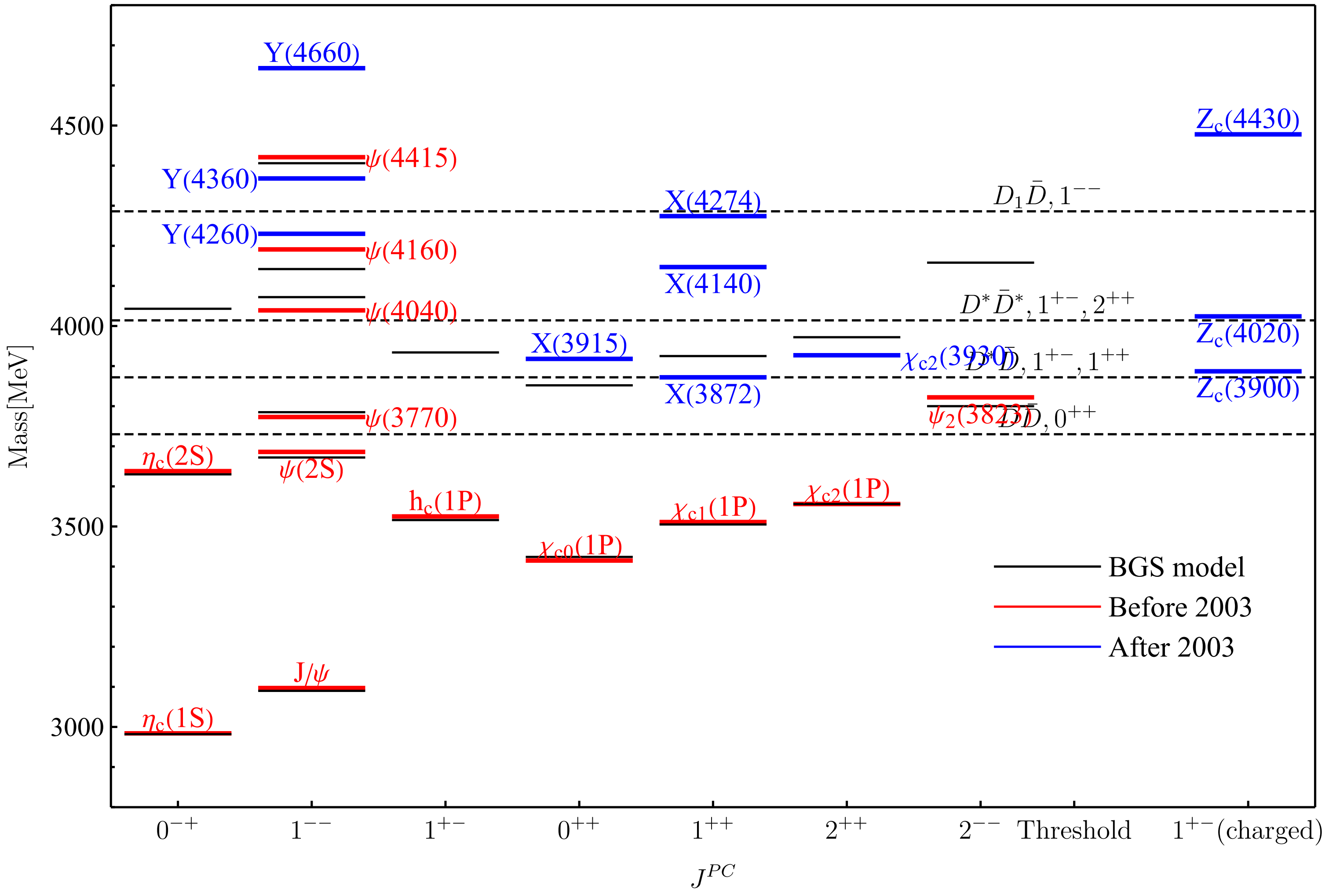
- Relativized Godfrey-Isgur model

Godfrey:1985xj…

Heavy quarkonium

BGS model

- Mass
- EM transitions
- Open flavor strong decays



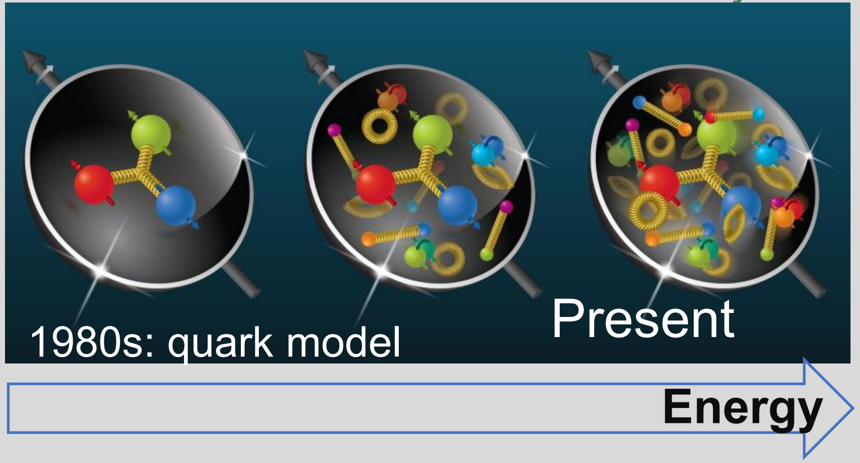
- NRQM with only 4 paras.
- Work well below open flavor thresholds
- Work better for bottomonium

T. Barnes, S. Godfrey, and E. S. Swanson,, Phys. Rev. D 72, 054026 (2005).

Do we still need quark models?

东南大学 SOUTHEAST UNIVERSITY

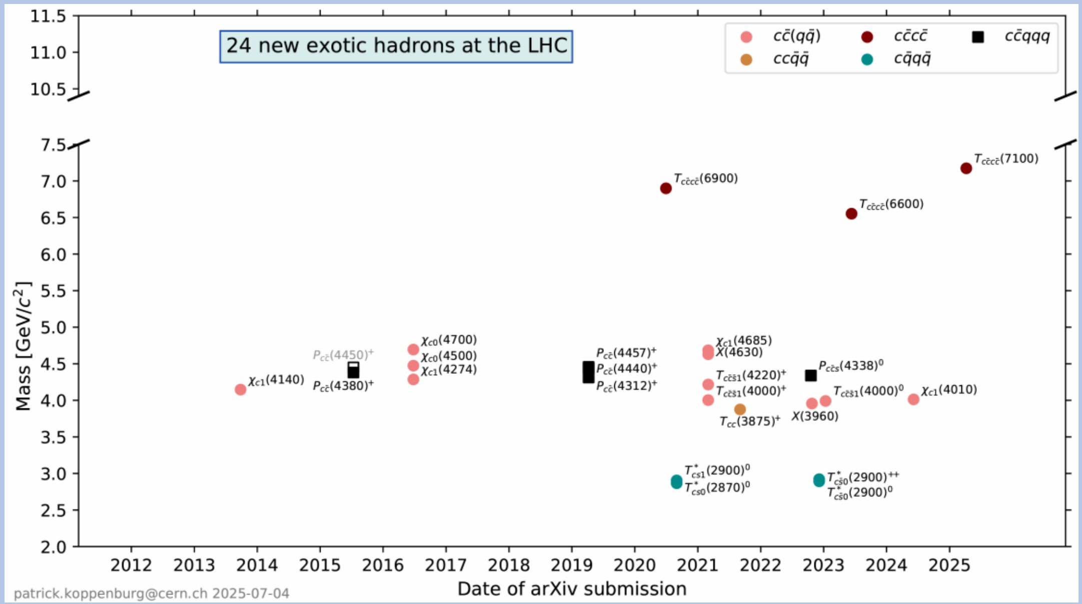
- Evolving view of the proton Courtesy of BNL



- Alternative methods:
  - ▶ Lattice QCD
  - ▶ Low energy EFTs
  - ▶ Dyson-Schwinger equations
  - ▶ …

Against

- We even do not know the **patterns**



For

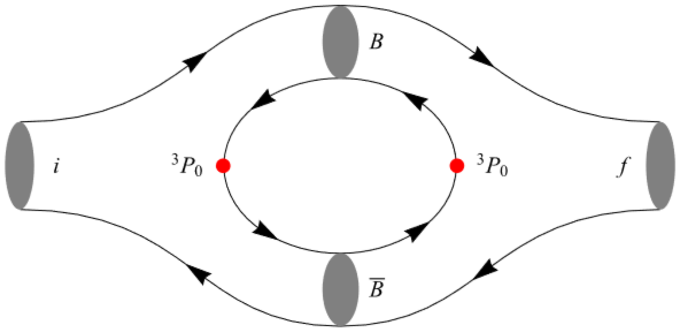
- Quark models
  - ▶ A clear picture to uncover patterns
  - ▶ Lower computational costs
  - ▶ Prediction power
  - ▶ Also decay models
  - ▶ …

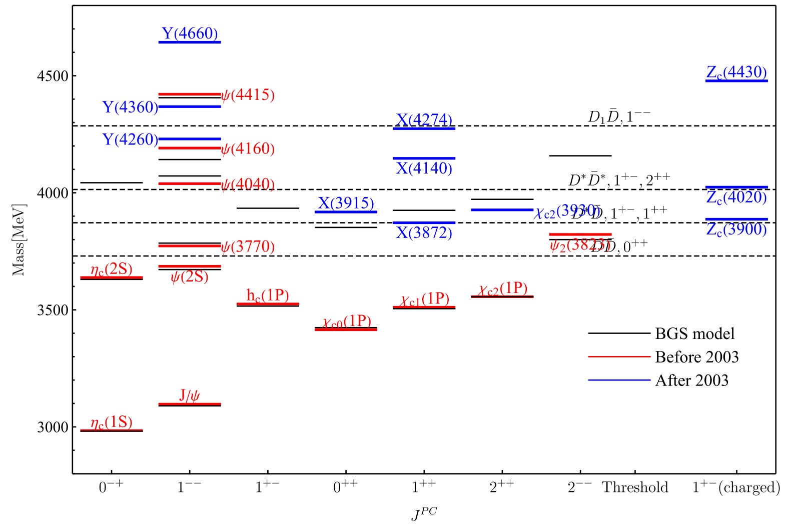
We still need quark model !

The right question: How to **adapt the quark models to meet the new demands**?

- Heavy-quarkonium-like states: unquenched quark model

B-S Zou's talks
https://indico.cern.ch/event/1457095/contributions/6563231/





- **Multiquark systems**

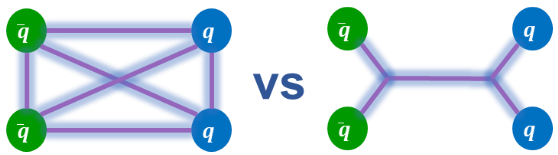
$[c\bar{c}qqq]$ P_c	$[cc\bar{c}\bar{c}]$ $X(6900)$ $X(6600)$ $X(7100)$	$[cs\bar{u}\bar{d}]$ $T_{cs1}(2900)$ $T_{cs0}(2900)$	$[cs\bar{u}\bar{d}]$ $Z_{cs}(3985)$ $Z_{cs}(4000)$	$[cc\bar{u}\bar{d}]$ $T_{cc}(3875)^+$	$[c\bar{s}u\bar{d}][c\bar{s}\bar{u}d]$ $T_{c\bar{s}0}(2900)^{++}$ $T_{c\bar{s}0}(2900)^0$	$[c\bar{c}qqq]$ $P_{cs}(4338)$ $P_{cs}(4459)$
	2006.16957 2306.07164 2304.08962 2506.07944	2009.00025 2009.00026	2011.07855 2103.01803	2109.01038 2109.01056	2212.02716 2212.02717 2411.19781	2210.10346 2012.10380 2502.09951

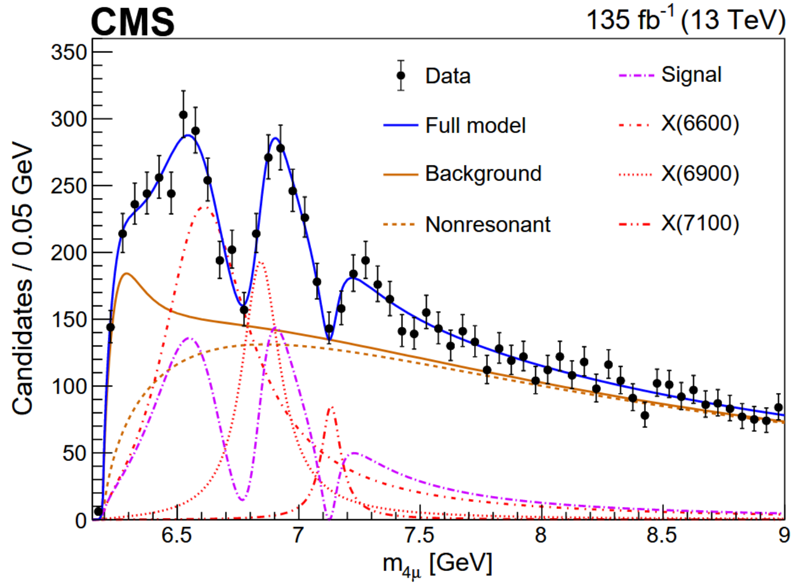
Particle Zoo 2.0

Chances and difficulties

- **Opportunities:** all-charm tetraquark **family**
  - ▶ Great experimental advances: LHCb, CMS, ATLAS
  - ▶ Simple systems
    - **Constituent quark is almost the current quark**
    - **Small relativistic effect**
    - **Unlikely exchange light mesons between (anti)quarks**
  - ▶ Different confinements would leave imprints on the mass spectrum

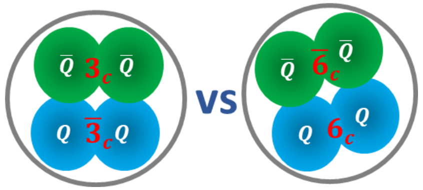
Alexandrou:2004ak, Okiharu:2004ve, Bicudo:2017usw



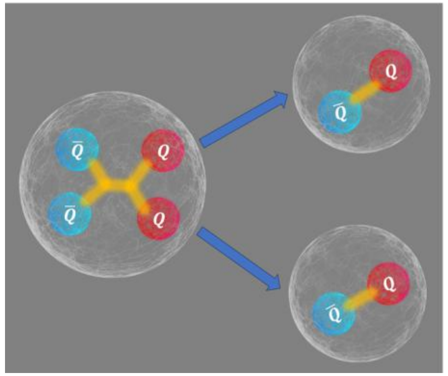


- **Challenges**
  - ▶ Color structures: e.g. $3-\bar{3}$ and $6-\bar{6}$ tetraquark
  - ▶ Matrix element of double Y-type potential
  - ▶ Four/five body problem
  - ▶ Resonance above the di-hadron thresholds
  - ▶ ….

Interactions?
few-body (resonance) problem?



Color structures

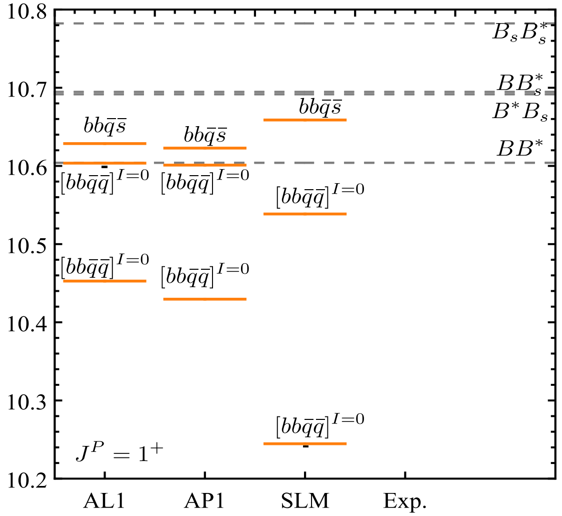


quark rearrangement

Contents

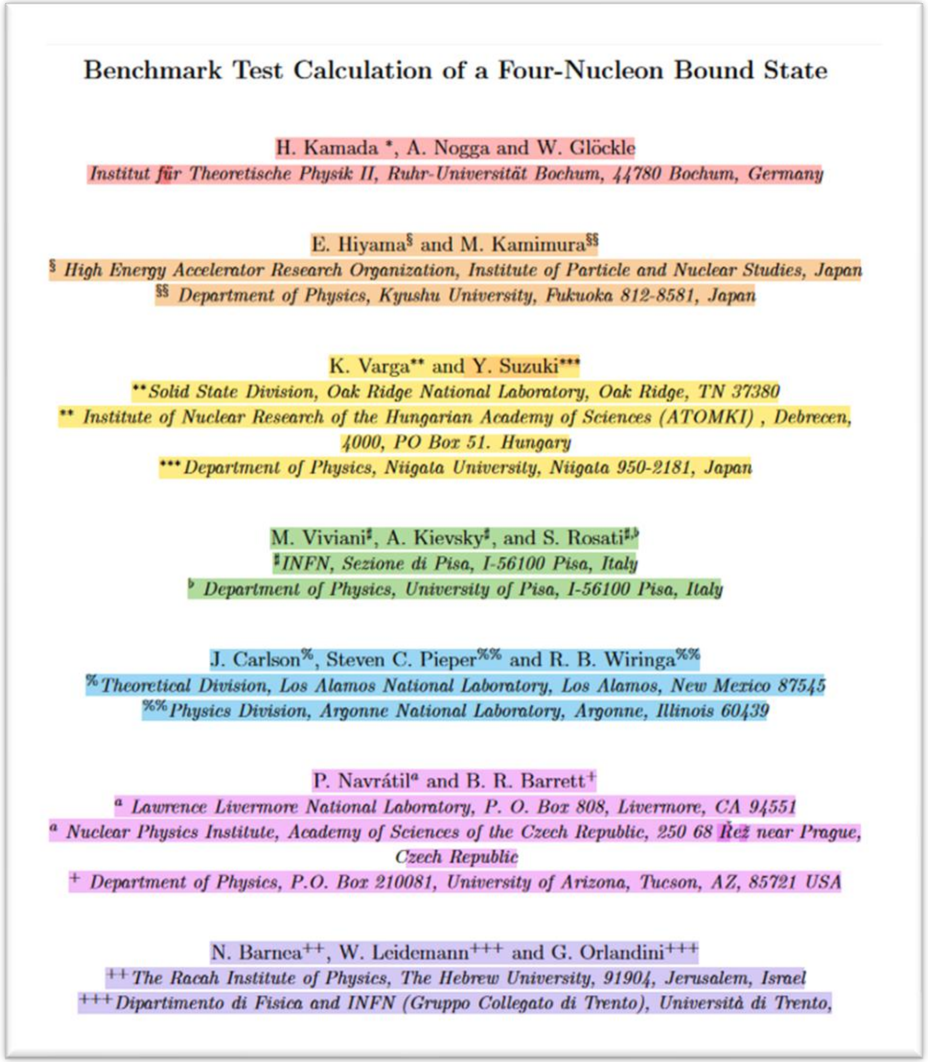
- **Bound** state: benchmark test
- **Resonance:** complex scaling method
- **DNNs**: towards tetraquark **confinement**

Bound state: benchmark tests



Benchmark test calculation of a four-nucleon bound state

東南大學
SOUTHEAST UNIVERSITY


Benchmark Test Calculation of a Four-Nucleon Bound State

H. Kamada [*], A. Nogga and W. Glöckle
Institut für Theoretische Physik II, Ruhr-Universität Bochum, 44780 Bochum, Germany

E. Hiyama[§] and M. Kamimura[§§]
[§] High Energy Accelerator Research Organization, Institute of Particle and Nuclear Studies, Japan
[§§] Department of Physics, Kyushu University, Fukuoka 812-8581, Japan

K. Varga[**] and Y. Suzuki[***]
[**] Solid State Division, Oak Ridge National Laboratory, Oak Ridge, TN 37380
[**] Institute of Nuclear Research of the Hungarian Academy of Sciences (ATOMKI), Debrecen, 4000, PO Box 51. Hungary
[***] Department of Physics, Niigata University, Niigata 950-2181, Japan

M. Viviani[♯], A. Kievsky[♯], and S. Rosati[♯,♭]
[♯] INFN, Sezione di Pisa, I-56100 Pisa, Italy
[♭] Department of Physics, University of Pisa, I-56100 Pisa, Italy

J. Carlson[%], Steven C. Pieper[%%] and R. B. Wiringa[%%]
[%] Theoretical Division, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
[%%] Physics Division, Argonne National Laboratory, Argonne, Illinois 60439

P. Navrátil[a] and B. R. Barrett[+]
[a] Lawrence Livermore National Laboratory, P. O. Box 808, Livermore, CA 94551
[a] Nuclear Physics Institute, Academy of Sciences of the Czech Republic, 250 68 Řež near Prague, Czech Republic
[+] Department of Physics, P.O. Box 210081, University of Arizona, Tucson, AZ, 85721 USA

N. Barnea[++], W. Leidemann[+++] and G. Orlandini[+++]
[++] The Racah Institute of Physics, The Hebrew University, 91904, Jerusalem, Israel
[+++] Dipartimento di Fisica and INFN (Gruppo Collegato di Trento), Università di Trento,

Kamada:2001tv

	E_b
Faddeev-Yakubovsky Eq.	-25.94(5)
Gaussian basis expansion	-25.90
stochastic variational method	-25.92
Hyperspherical variational	-25.90(1)
Green’s function MC/Diffusion MC	-25.93(2)
No-core shell model	-25.80(20)
Hyperspherical harmonic methods	-25.944(10)

Benchmark test of tetraquark bound state?

Gaussian expansion method (GEM)

Resonating group method (RGM)

Diffusion Monte Carlo (DMC)

Quark potential models

- Semay-Silvestre-Brac Models

Semay:1994ht, Silvestre-Brac:1996myf

$$V_{ij}(r) = \left[-\frac{\kappa}{r} + \lambda r^p - \Lambda + \frac{2\pi}{3m_i m_j}\kappa' \frac{1}{\pi^{3/2} r_0^3} e^{(-r^2/r_0^2)} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right] \lambda_i \cdot \lambda_j$$

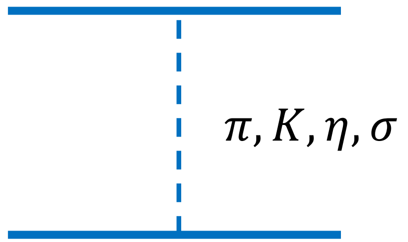
AL1: $p = 1$ and AP1: $p = 2/3$

- Chiral quark models [e.g Salamanca model (SLM)]

Vijande:2004he, Gonzalez:2012gka

$$V_{ij}(r) = \left[\frac{\alpha_s}{4} \left(\frac{1}{r} - \frac{1}{6m_i m_j} \frac{e^{-r/r_0}}{r_0^2 r} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right) + \underbrace{\left(-a_c(1 - e^{-\mu_c r}) + \Delta\right)}_{\text{Screened confinement}} \right] \lambda_i \cdot \lambda_j$$

$$+V_\pi + V_K + V_\eta + V_\sigma$$



- In this work, we use **AL1, AP1 and SLM** $\otimes$ **GEM, DMC and RGM**

[GeV]	π	K	D	D_s	B	B_s	B_c	η_c	η_b
Exp.	0.139	0.494	1.870	1.968	5.279	5.367	6.274	2.984	9.399
AL1	0.138	0.491	1.862	1.962	5.293	5.361	6.292	3.005	9.424
AP1	0.139	0.498	1.881	1.955	5.311	5.356	6.269	2.982	9.401
SLM	0.140	0.469	1.896	1.983	5.275	5.348	6.275	2.990	9.451

- **Semay-Silvestre-Brac Models**

Semay:1994ht, Silvestre-Brac:1996myf

$$V_{ij}(r) = \left[-\frac{\kappa}{r} + \lambda r^p - \Lambda + \frac{2\pi}{3m_i m_j}\kappa' \frac{1}{\pi^{3/2} r_0^3} e^{(-r^2/r_0^2)} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right] \lambda_i \cdot \lambda_j$$

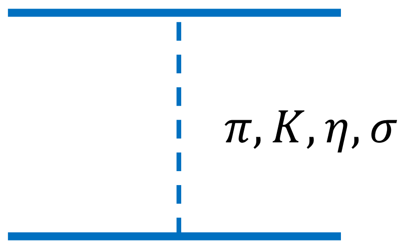
AL1: $p = 1$ and AP1: $p = 2/3$

- **Chiral quark models [e.g Salamanca model (SLM)]**

Vijande:2004he, Gonzalez:2012gka

$$V_{ij}(r) = \left[\frac{\alpha_s}{4} \left(\frac{1}{r} - \frac{1}{6m_i m_j} \frac{e^{-r/r_0}}{r_0^2 r} \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j \right) + \underbrace{\left(-a_c (1 - e^{-\mu_c r}) + \Delta \right)}_{\text{Screened confinement}} \right] \lambda_i \cdot \lambda_j$$

$$+V_\pi + V_K + V_\eta + V_\sigma$$



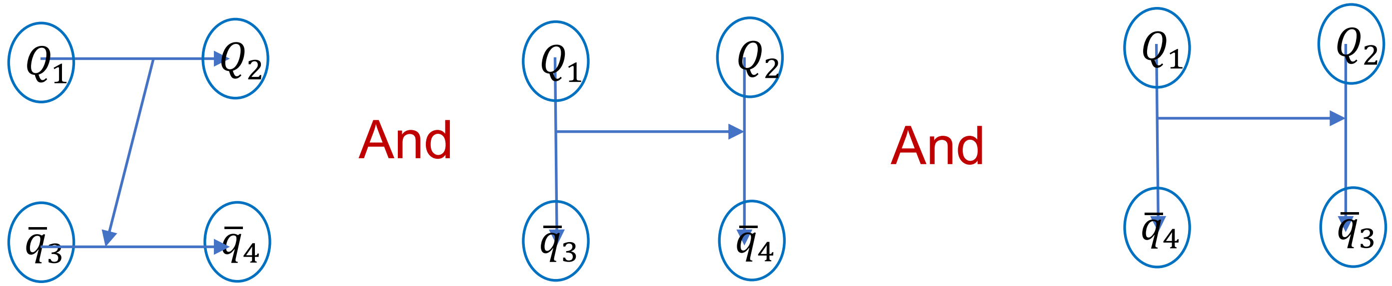
- In this work, we use **AL1, AP1 and SLM** $\otimes$ **GEM, DMC and RGM**

[GeV]	ρ	ω	ϕ	K^*	D^*	D_s^*	B^*	B_s^*	B_c^*	J/ψ	Υ
Exp.	0.775	0.783	1.019	0.892	2.010	2.112	5.325	5.415	6.329	3.097	9.460
AL1	0.770	0.770	1.021	0.903	2.016	2.102	5.350	5.417	6.343	3.101	9.461
AP1	0.770	0.770	1.021	0.908	2.033	2.107	5.367	5.418	6.338	3.102	9.461
SLM	0.773	0.693	1.000	0.902	2.018	2.111	5.317	5.393	6.329	3.097	9.501

Method 1: Gaussian Expansion Method

SOUTHEAST UNIVERSITY

- Spatial wave functions



Diquark-antidiquark **Di-meson** **Di-meson** …

$$\phi_{nlm}(\boldsymbol{r}) = N_{lm} r^l e^{-\frac{r^2}{r_n^2}} Y_{lm}(\hat{r})$$

- ▶ Geometric progression: $r_n = r_0 a^{n-1}$
- ▶ Embed both long- and short-range correlations

Hiyama:2003cu

- Antisymmetrization (e.g. $Q_1 = Q_2$ and $q_3 = q_4$) :

$$\psi = \mathcal{A}\left[\psi_{color} \otimes \psi_{spin} \otimes \psi_{spatial} \otimes \psi_{flavor}\right], \qquad \mathcal{A} = (1 - P_{12})(1 - P_{34})$$

- Based on variational principle: **upper limit** of the energy

Tetraquark systems

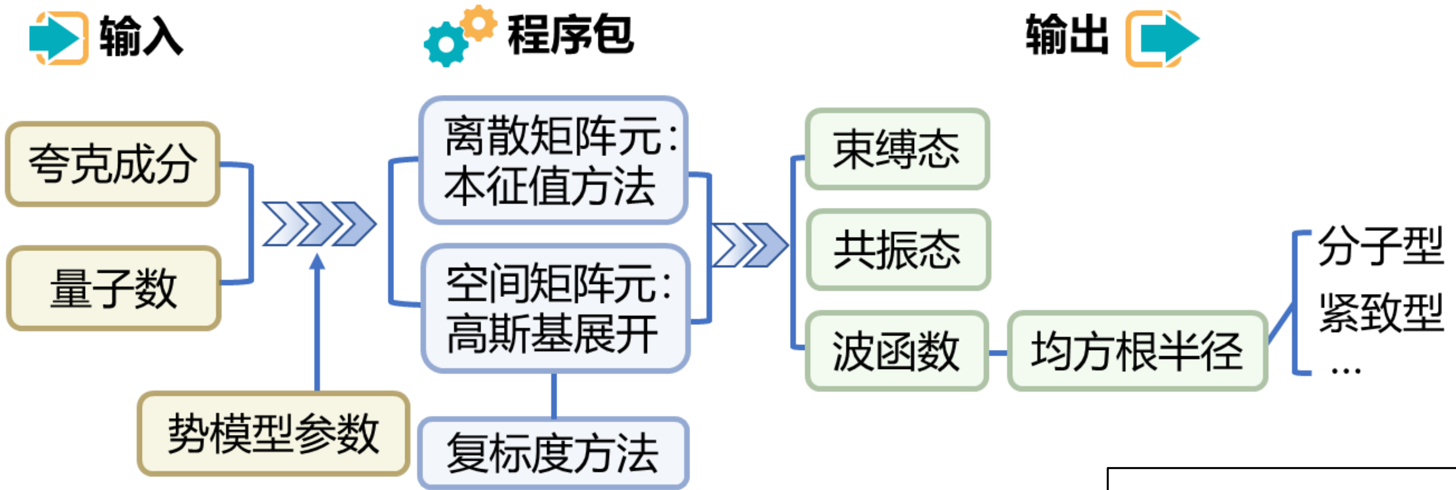
- Fully heavy tetraquark states ($QQ\bar{Q}\bar{Q}$)
- Triply heavy tetraquark states ($QQ\bar{Q}\bar{q}$)
- Doubly heavy tetraquarks states ($QQ\bar{q}\bar{q}$)
- Single heavy strange states ($Qs\bar{q}\bar{q}\,,\ Q\bar{s}q\bar{q}$)

$q = u, d, s;\ \ Q = b, c$

$J^P = 0^+, 1^+, 2^+$, Only S-wave

Only bound states

Over 150 states



Total runtime:
2.5h on my laptop

Fully heavy tetraquark states

- Our conclusions:

No bound state for any reasonable **pairwise** interaction

J. P. Ader, J. M. Richard, and P. Taxil, PRD**25**, 2370 (1982).
M.-S. Liu, Q.-F. Lü, X.-H. Zhong, and Q. Zhao, , PRD100, 016006 (2019).
X. Jin, Y. Xue, H. Huang, and J. Ping, EPJC**80**, 1083 (2020).
…

- Ref [Ader:1981db]

> tial models already used in the study of heavy mesons and baryons. We first consider the situation where the quarks have the same mass and interact through a two-body potential due to color-octet exchange. In this case, we show that for any reasonable confining potential there is no state below the threshold corresponding to the spontaneous dissociation into two mesons. We investigate in detail different possibilities of modifying this negative

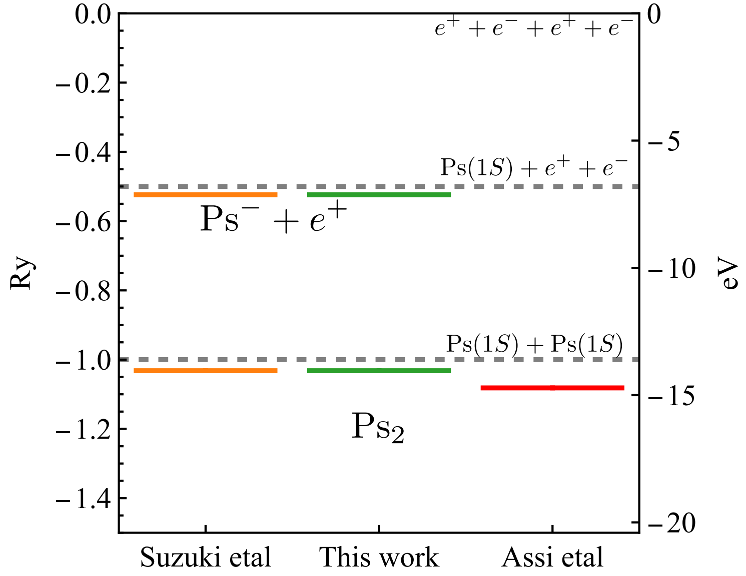
- Ref [Assi:2023dlu]: claimed the existence of fully heavy tetraquark bound state in version 1

> FERMILAB-PUB-23-660-T
>
> Tetraquarks made of sufficiently heavy quarks are bound in QCD
>
> Benoît Assi[1] and Michael L. Wagman[1]
>
> [1] Fermi National Accelerator Laboratory, Batavia, IL, 60510

version 1

> FERMILAB-PUB-23-660-T
>
> Tetraquarks made of sufficiently unequal-mass heavy quarks are bound in QCD
>
> Benoît Assi[1] and Michael L. Wagman[1]
>
> [1] Fermi National Accelerator Laboratory, Batavia, IL, 60510

version 2

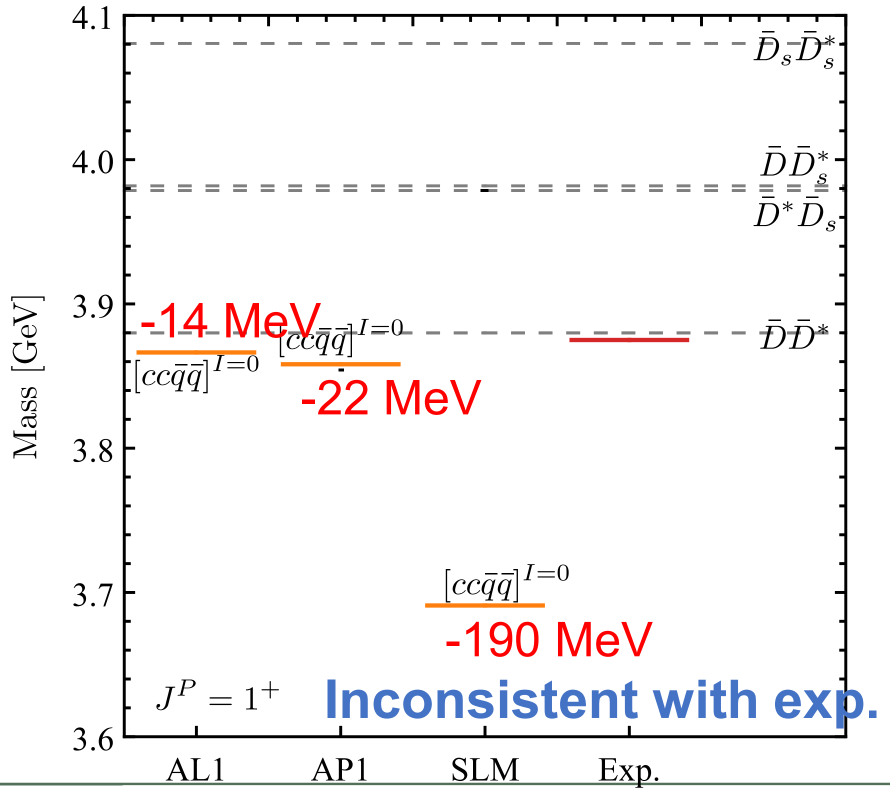


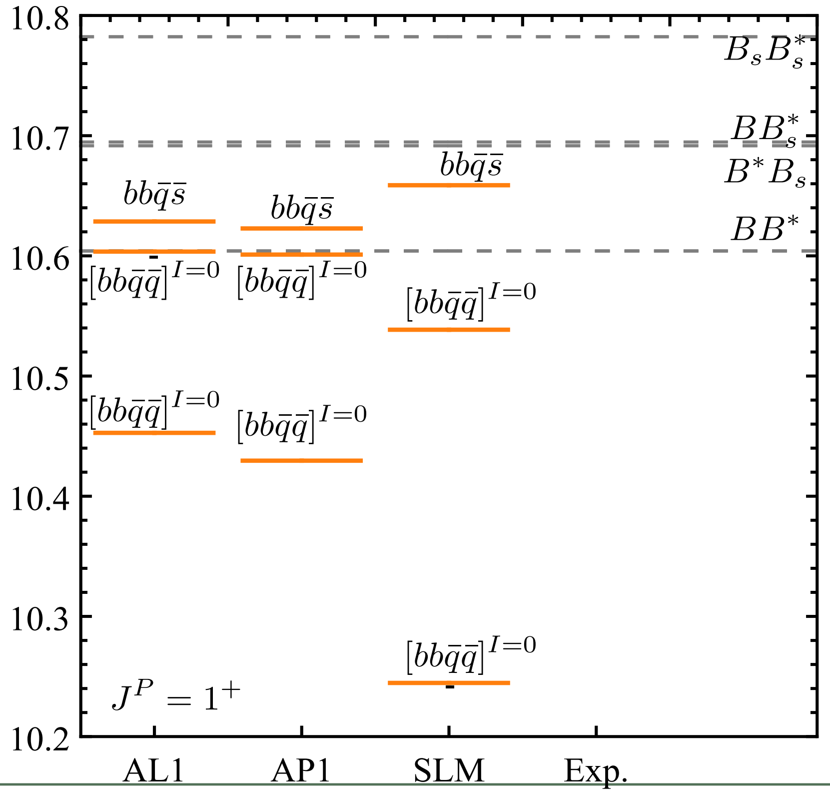
东南大学 SOUTHEAST UNIVERSITY

"Predictions"

- Points of agreement
  - ▶ $[cc\bar{q}\,\bar{q}]^{I=0}$, $[bb\bar{q}\,\bar{q}]^{I=0}$, $[bb\bar{q}\bar{s}]$ bound states ;
  - ▶ 1st excited state $[bb\bar{q}\,\bar{q}]^{I=0}$ is bound state
  - ▶ No $[bb\bar{q}\,\bar{q}]^{I=1}$ states
- SLM: deeper states

  (1) $[cc\bar{q}\bar{q}]^{I=0}$ and $[bb\bar{q}\bar{q}]^{I=0}$ are deeper than AL1 and AP1



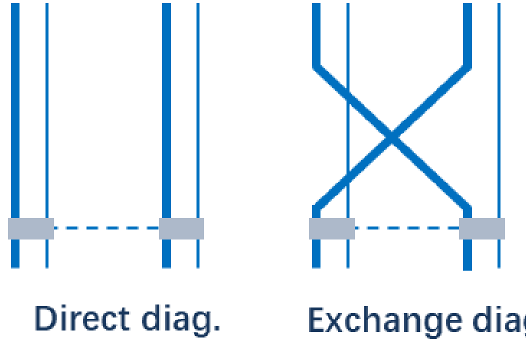


Method 2: Resonating Group Method

- Dimeson-wave function

$$\psi_{AB}(\boldsymbol{P}) = \mathcal{A}[\phi_A(\boldsymbol{p}_A)\phi_B(\boldsymbol{p}_B)\chi(\boldsymbol{P})\chi_{AB}^{CST}]$$

  - ϕ_A and ϕ_B are meson wave functions: from GEM



Entem:2000mq, Ortega:2022efc

- Schrödinger equation of RGM

$$\left(\frac{\boldsymbol{P}'^2}{2\mu} - E\right)\chi(\boldsymbol{P}') + \int d^3\boldsymbol{P}\left(V_D(\boldsymbol{P}',\boldsymbol{P}) + K_{Ex}(\boldsymbol{P}',\boldsymbol{P})\right)\chi(\boldsymbol{P}) = 0$$

V_D direct interaction, K_{Ex} the exchange kernel

- Only the di-meson-type spatial wave are included, not as general as GEM

$$E_{RGM} \gtrsim E_{GEM}$$

  - Valid only when clustering behavior is assumed

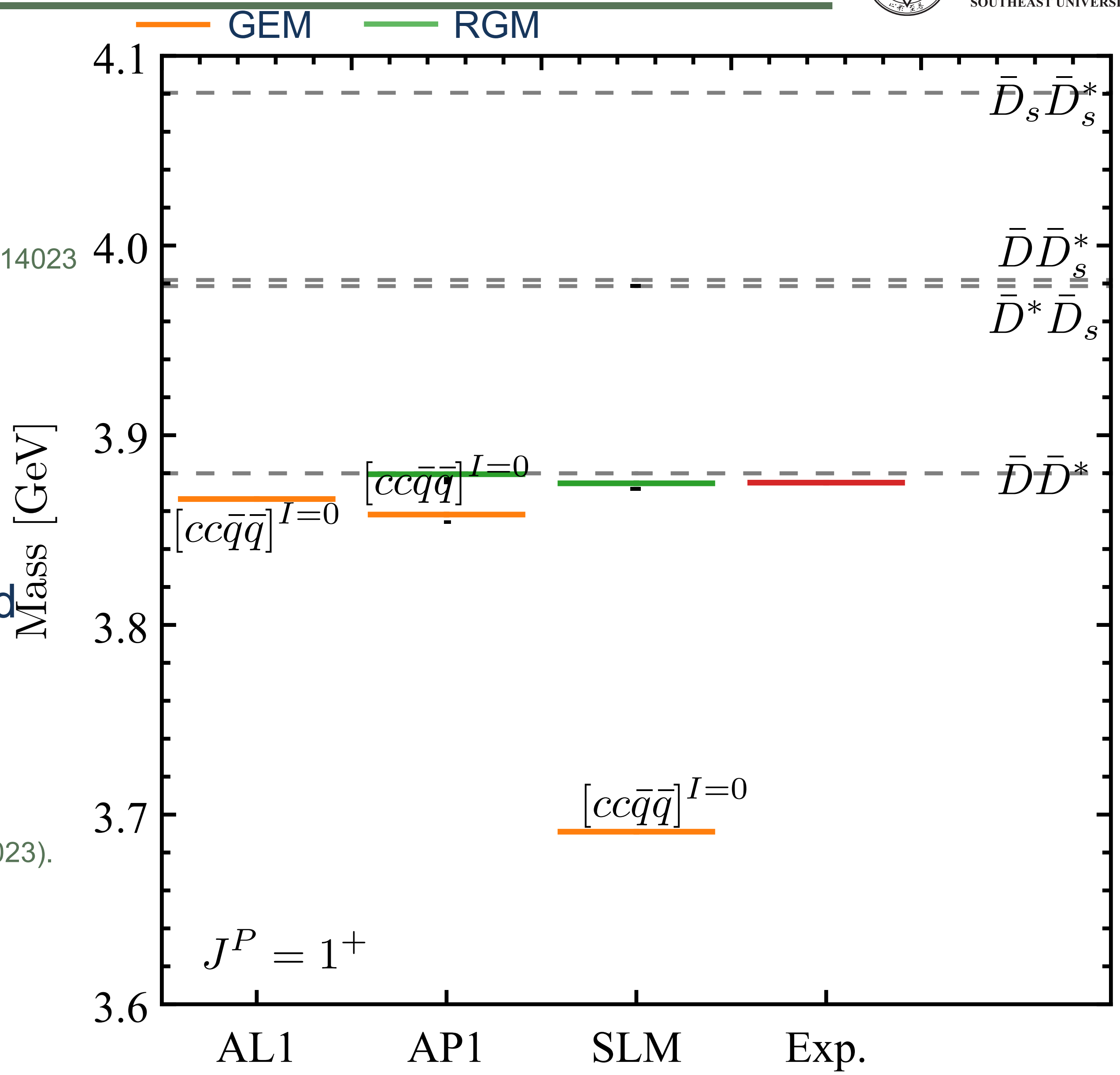
RGM results

- The RGM results agree with the GEM neglecting diquark-antidiquark correlation
  - ▶ Similar conclusion in literature

Y. Yang, C. Deng, J. Ping and T. Goldman, RRD80 (2009), 114023

- SLM group: SLM interaction+ RGM method
  - ▶ Loosely bound T_{cc} states
  - ▶ A correct result coincidentally from two wrong premises

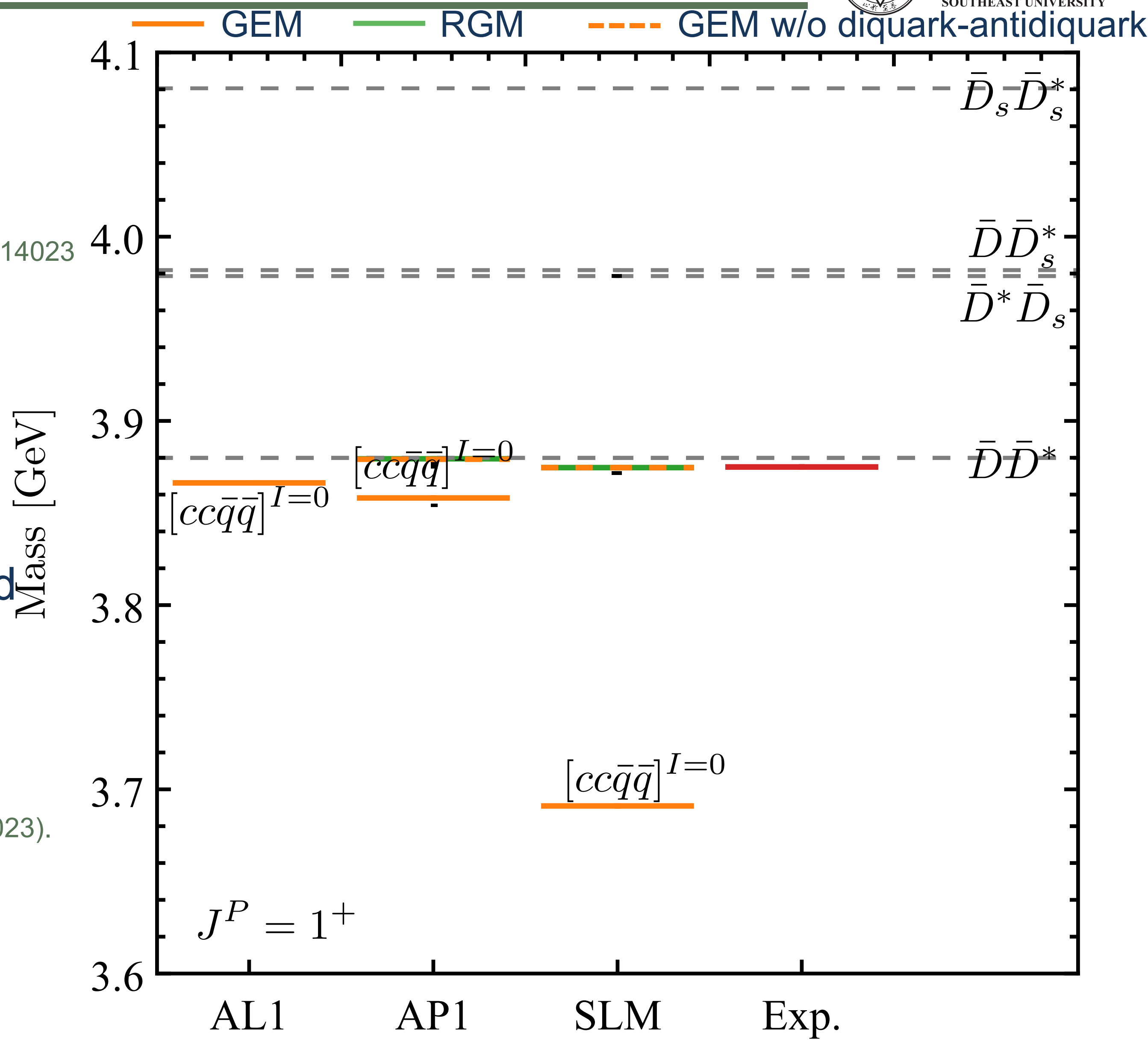
P. G. Ortega, J. Segovia, D. R. Entem, and F. Fernández, PLB 841, 137918 (2023).



RGM results

- The RGM results agree with the GEM neglecting diquark-antidiquark correlation
  - ▶ Similar conclusion in literature

Y. Yang, C. Deng, J. Ping and T. Goldman, RRD80 (2009), 114023



- SLM group: SLM interaction+ RGM method
  - ▶ Loosely bound T_{cc} states
  - ▶ A correct result coincidentally from two wrong premises

P. G. Ortega, J. Segovia, D. R. Entem, and F. Fernández, PLB 841, 137918 (2023).

Method 3: Diffusion Monte Carlo

东南大學 SOUTHEAST UNIVERSITY

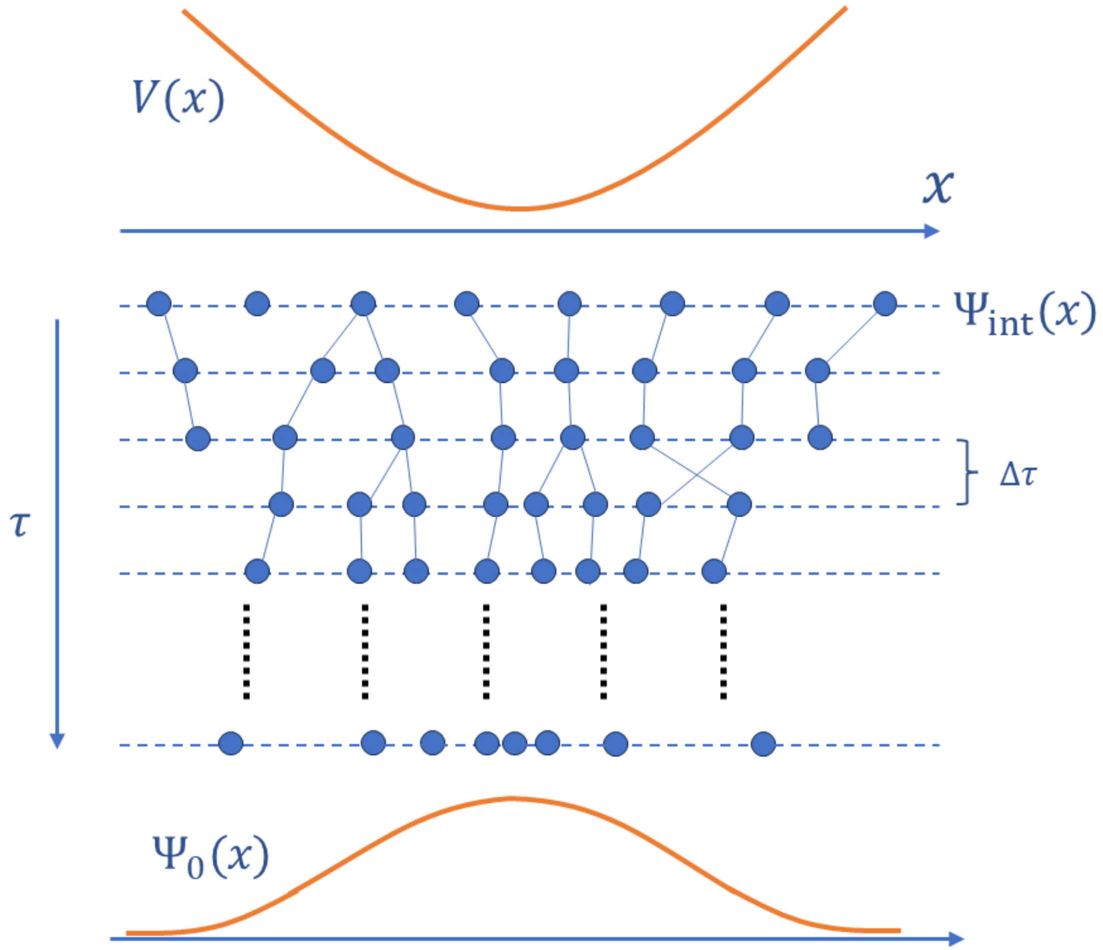
- $t \to i\tau$; Schrödinger equation→Diffusion equation

$$-\frac{\partial \Psi(\boldsymbol{R},\tau)}{\partial \tau} = \Big[\underbrace{-\frac{\nabla^2}{2m}}_{\text{Diffusion}} + \underbrace{V(\boldsymbol{R}) - E_R}_{\text{Source or Sink}}\Big]\Psi(\boldsymbol{R},\tau),$$

- $E_R \to E_0$, the $\Psi(\boldsymbol{R},t)$ →ground state when $t \to \infty$

$$\Psi(\boldsymbol{R},\tau) = \sum_i c_i \Phi_i(\boldsymbol{R}) e^{-[E_i - E_R]\tau},$$

- In practices: importance sampling
- Milder increase computational cost as particles numbers
- DMC results consistent with the GEM results basically



Y. Ma, LM, Y.-K. Chen, and S.-L. Zhu, PRD**107**, 054035 (2023), PRD**109**, 074001 (2024).

All results

	$QQ\bar{Q}\bar{Q}$	$QQ\bar{Q}\bar{q}$	$QQ\bar{q}\bar{q}$	$Qs\bar{q}\bar{q}$	$Q\bar{s}q\bar{q}$
$J^P = 0^+$	No bound	No bound	😊	😊	No bound
$J^P = 1^+$	No bound	No bound	😊	😊	No bound
$J^P = 2^+$	No bound	No bound	😊	😊	No bound

- Recommended tetraquark states below di-meson thresholds (consistent predictions of 3 models)

$J^P = 1^+$	$[cc\bar{q}\bar{q}]^{I=0}$	$[bb\bar{q}\bar{q}]^{I=0}$	$[bc\bar{q}\bar{q}]^{I=0}$	$bb\bar{q}\bar{s}$	$[bs\bar{q}\bar{q}]^{I=0}$
$J^P = 0^+$	$[cb\bar{q}\bar{q}]^{I=0}$	$[cs\bar{q}\bar{q}]^{I=0}$	$[bs\bar{q}\bar{q}]^{I=0}$		
$J^P = 2^+$	$[cb\bar{q}\bar{q}]^{I=0}$				

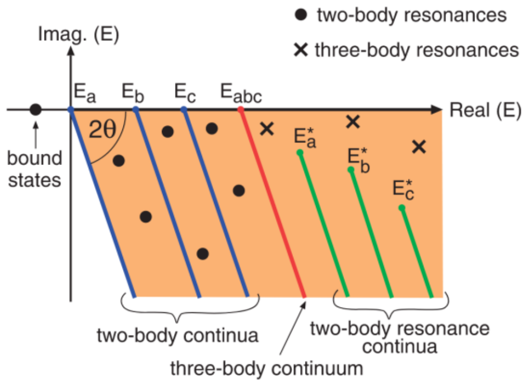
- SLM model: overestimate the binding energy or predict extra states

Not for all the chiral QMs

B-R He, M. Harada, B-S Zou, PRD108 (2023) , 054025

- Resonating group method: only applicable for clustered systems

Resonance: complex scaling method



Benchmark calculations for 4 nucleon resonant states are still missing.

Complex scaling method

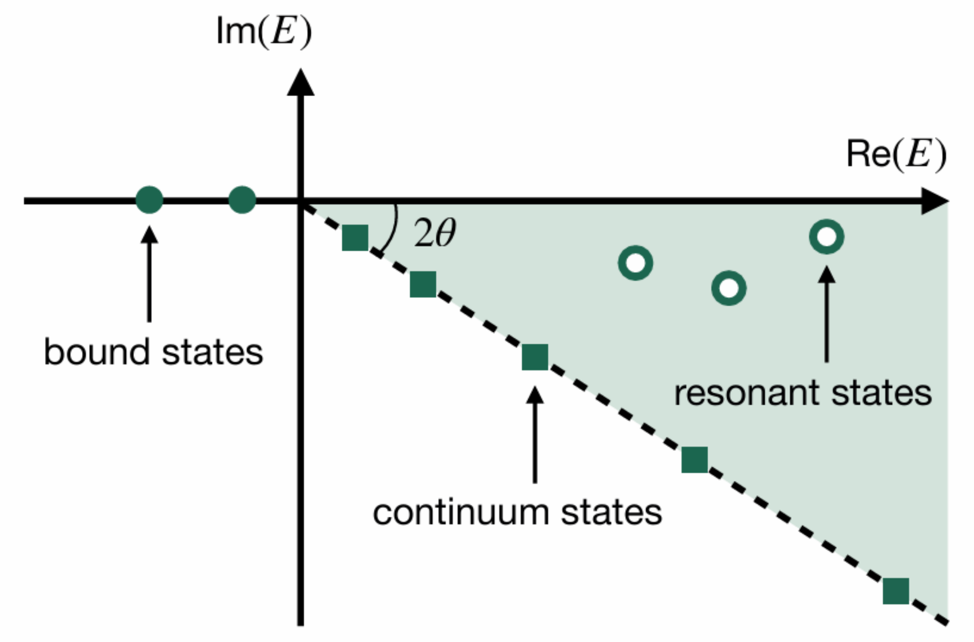
- Original Hamiltonian

$$H = \sum_i^n \left(m_i + \frac{\boldsymbol{p}_i^2}{2m_i} \right) + \sum_{i<j=1}^n V_{ij} \, ,$$

- Complex scaling

$$U(\theta)\boldsymbol{r} = \boldsymbol{r}e^{i\theta}, \quad U(\theta)\boldsymbol{p} = \boldsymbol{p}e^{-i\theta}.$$

$$H(\theta) = \sum_{i=1}^n \left(m_i + \frac{p_i^2 e^{-2i\theta}}{2m_i} \right) + \sum_{i<j=1}^n V_{ij}\left(r_{ij}e^{i\theta}\right) .$$

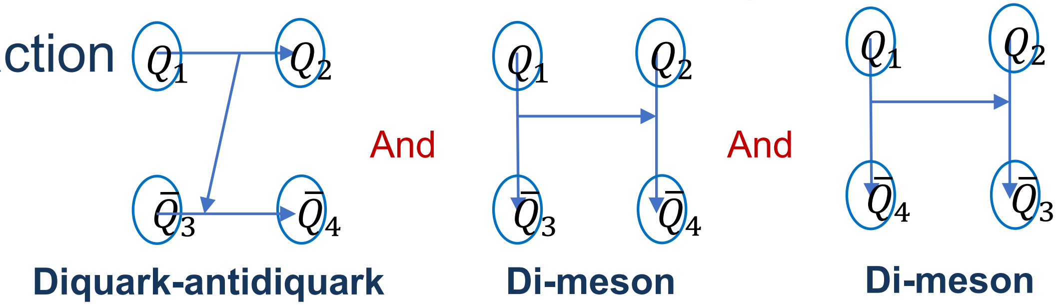


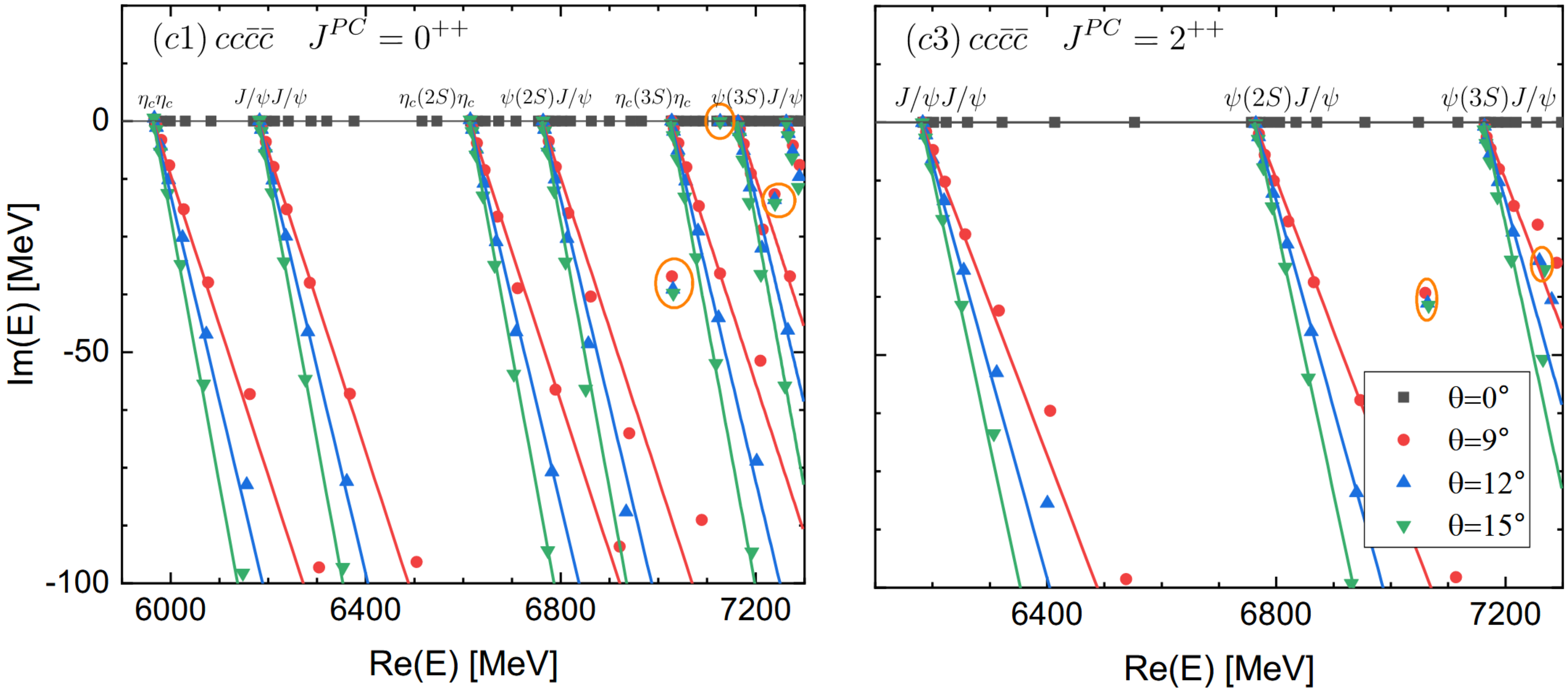
- Resonance appearing as the eigenvalue of $H(\theta)$
- Equivalence of the CSM and **contour deformation method** in 2-body systems
- Resonances from the GEM+CSM
  - ▶ $QQ\bar{Q}\bar{Q}$
  - ▶ $\bar{Q}\bar{Q}qq$
  - ▶ $Qs\bar{q}\bar{q}$
  - ▶ $ss\bar{s}\bar{s}$
  - ▶ …

Aguilar:1971ve, Balslev:1971vb, Aoyama:2006hrz…

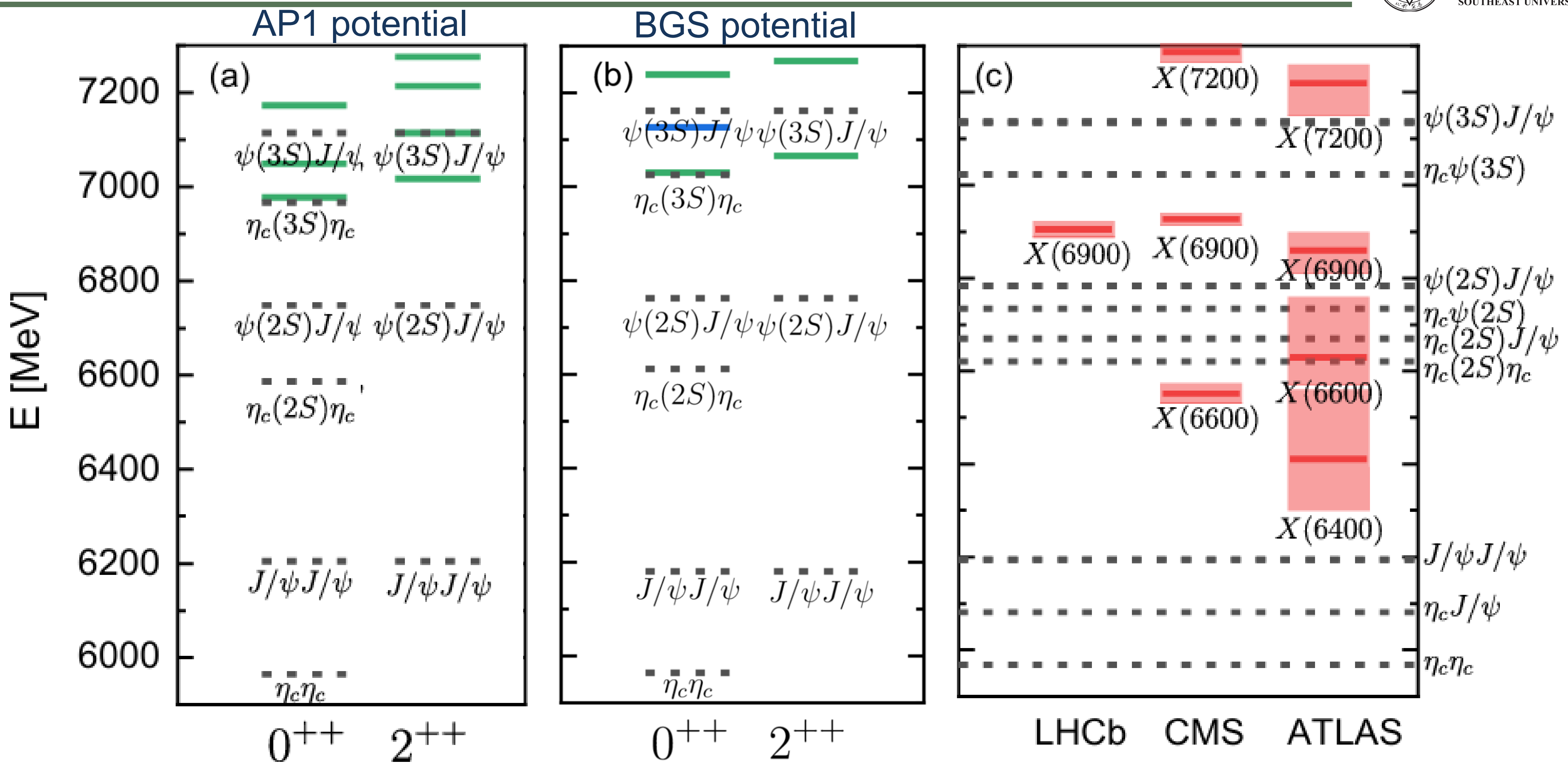
All-charm tetraquarks

- BGS, AL1, AP1 models: **pairwise** confinement interaction
- Configurations: diquark-antidiquark, di-meson× 2
- Tetraquark as resonances





All-charm tetraquarks

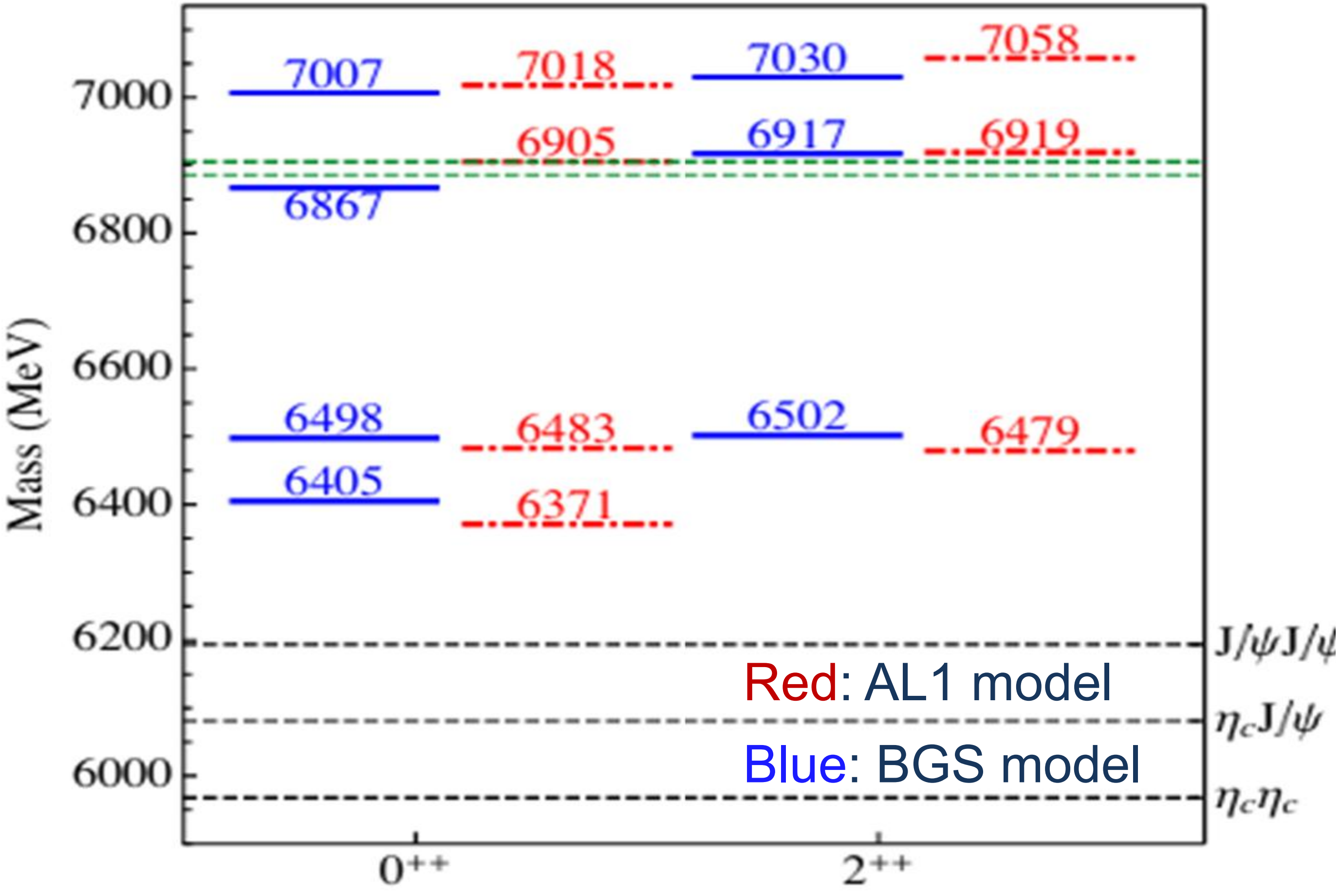


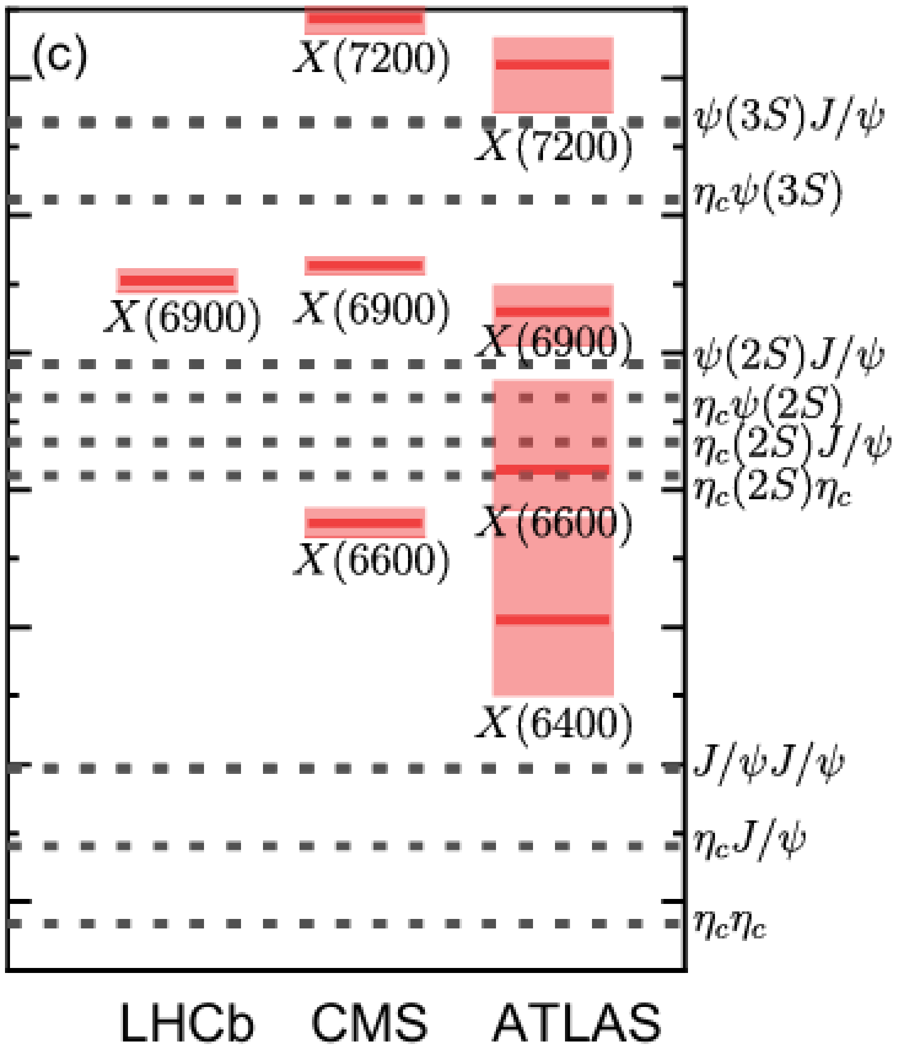
X(6600) is missing in pairwise confinement models

See also G.-J. Wang, Q. Meng, and M. Oka, PRD106, 096005 (2022).

All-charm tetraquarks

- Earlier calculation neglecting di-meson configurations

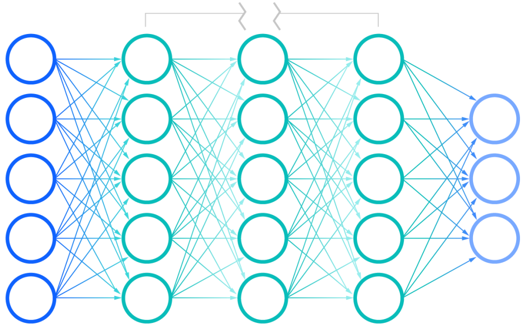


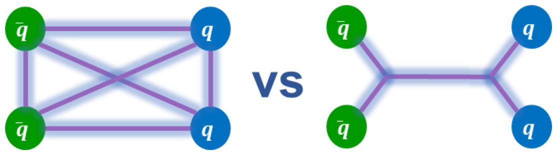


G-J Wang, LM, M Oka, S-L Zhu, PRD 104, 036016 (2021)

- Pairwise model: overestimate the transition between diquark-antidiquark and dimeson??

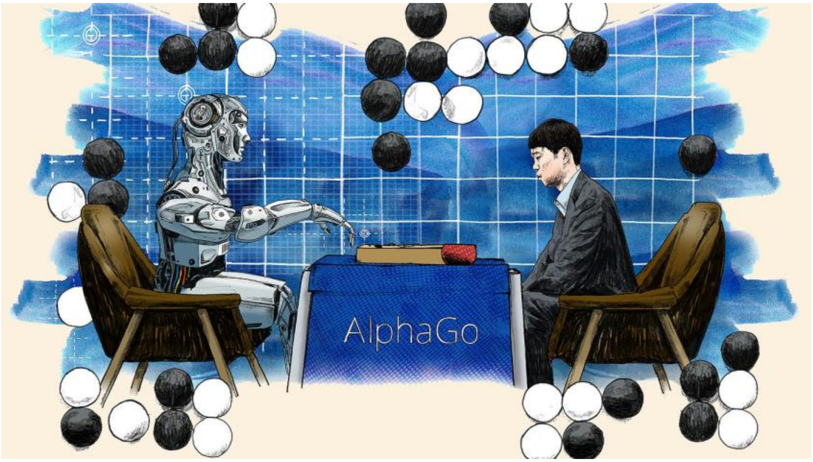
DNNs: towards tetraquark confinement





Turning point of AI

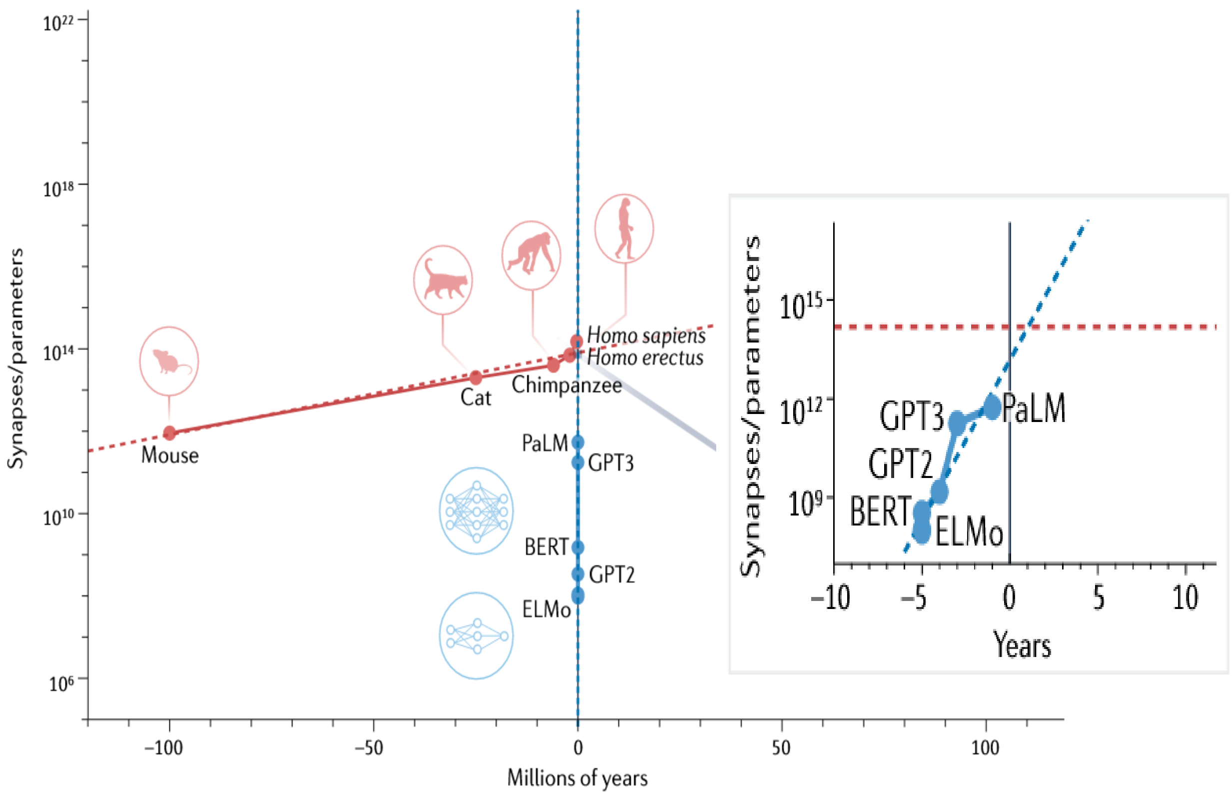
Year	Milestone	Who	Significance
2010	ImageNet	Fei-Fei Li	Large-scale dataset that enabled the deep learning revolution.
2012	AlexNet	Krizhevsky et al.	Proven power of deep CNNs and GPU acceleration for AI.
2014	GANs	Ian Goodfellow	Introduced framework for generative AI to create realistic data.
2015/16	TensorFlow/ Pytorch	Google/Meta	Critical open-source software that democratized AI research globally.
2016	AlphaGo	DeepMind	First AI to defeat a world champion in the game of Go.
2010s	AI Chips	NVIDIA, Google, Apple	Specialized GPUs/TPUs enabled training of massive AI models.
2017	Transformer	Google	Revolutionary architecture that made modern large language models possible.
2018	BERT	Google	Bidirectional language model that set new standards for NLP.
2020	AlphaFold 2	DeepMind	Solved the decades-old scientific challenge of protein folding.
2022	ChatGPT	OpenAI	Popularized large language models through accessible public interaction.



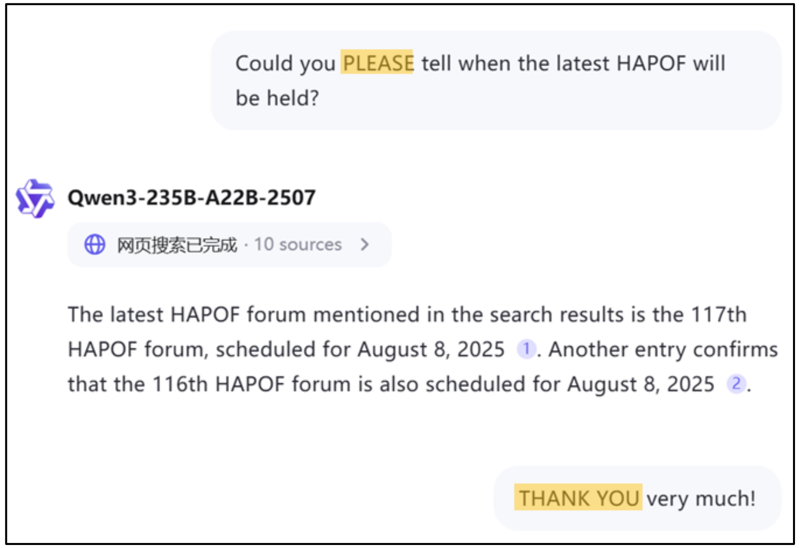
10 milestones of AI selected by DeepSeek

deepseek

Biological intelligence vs AI



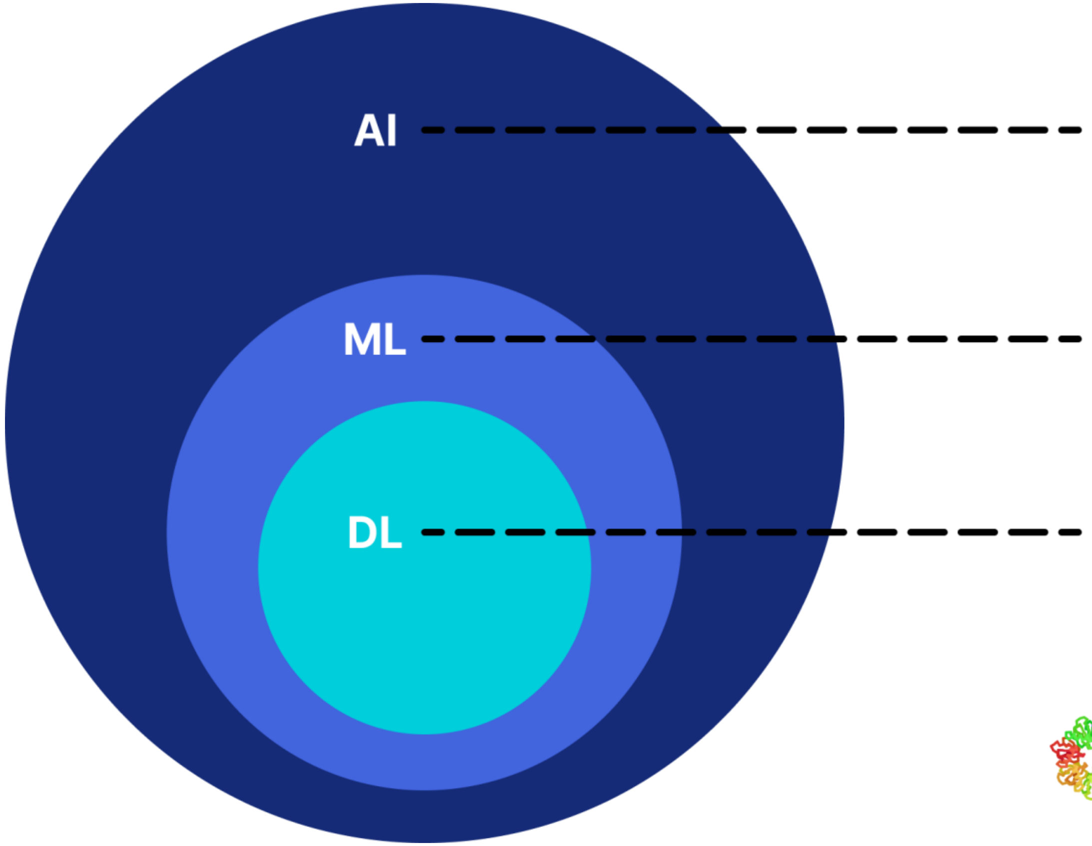
The evolution of biological and artificial intelligence



Being more polite to chatbots may increase your chances of survival when robots win

Schwartz, M. D. (2022). *Nature Reviews Physics*, *4*(12), 741–742.

AI vs ML vs DL



Artificial Intelligence
Engineering of making intelligent machines and programs.

Machine Learning
Ability to learn without being explicitly programmed.

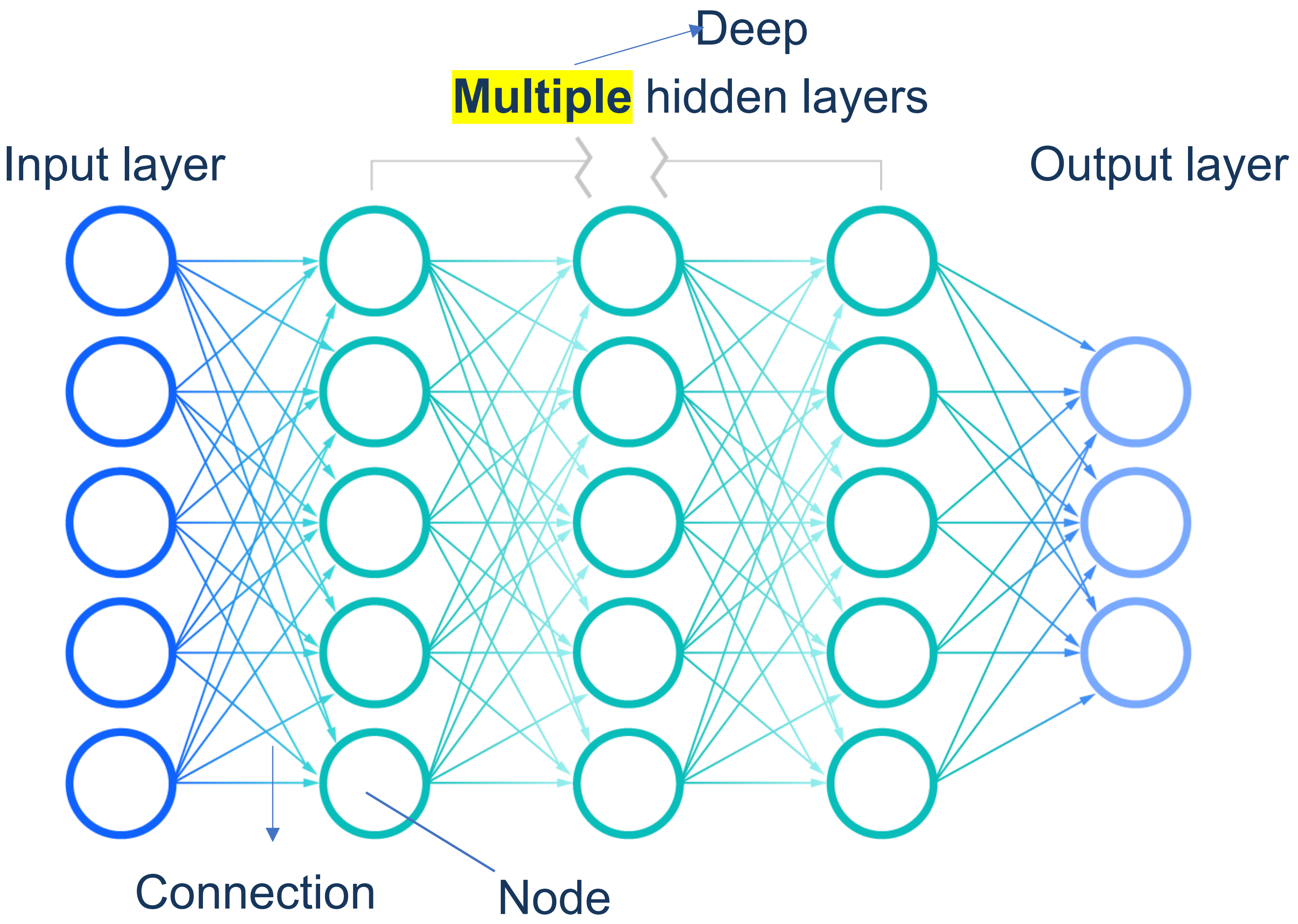
Deep Learning
Learning based on deep neural networks.

neurosnap.ai

https://neurosnap.ai/blog/post/64279cadfeb3e5ca5ba0904a

AI boom from 2010s was powered by **deep learning**

Deep neutral network



Neural network: simple units (neurons) connected to form complex network

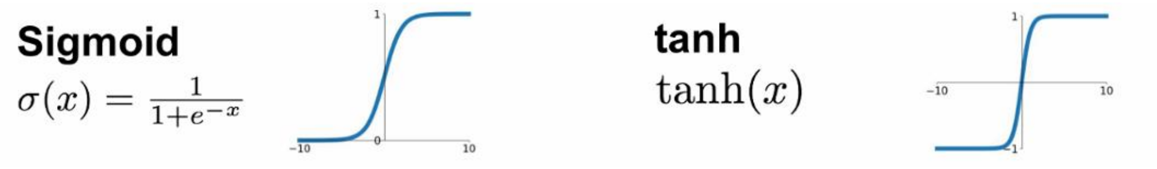
Deep： Enable learn complex relation in data

Connections between layers

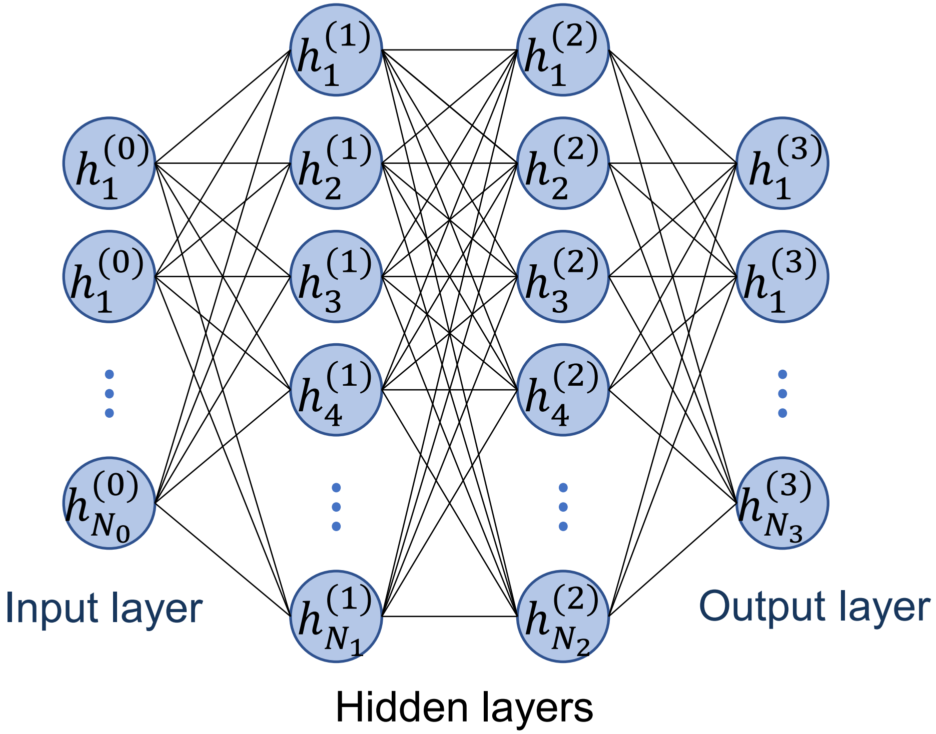
$$\vec{h}^{(i+1)} = \sigma\left[W^{(i)}\vec{h}^{(i)} + \vec{b}^{(i)}\right]$$

Nonlinear   Linear

- Linear part: Weight matrices $W^{(i)}$ and biases $\vec{b}^{(i)}$
- Nonlinear part: Activation functions σ

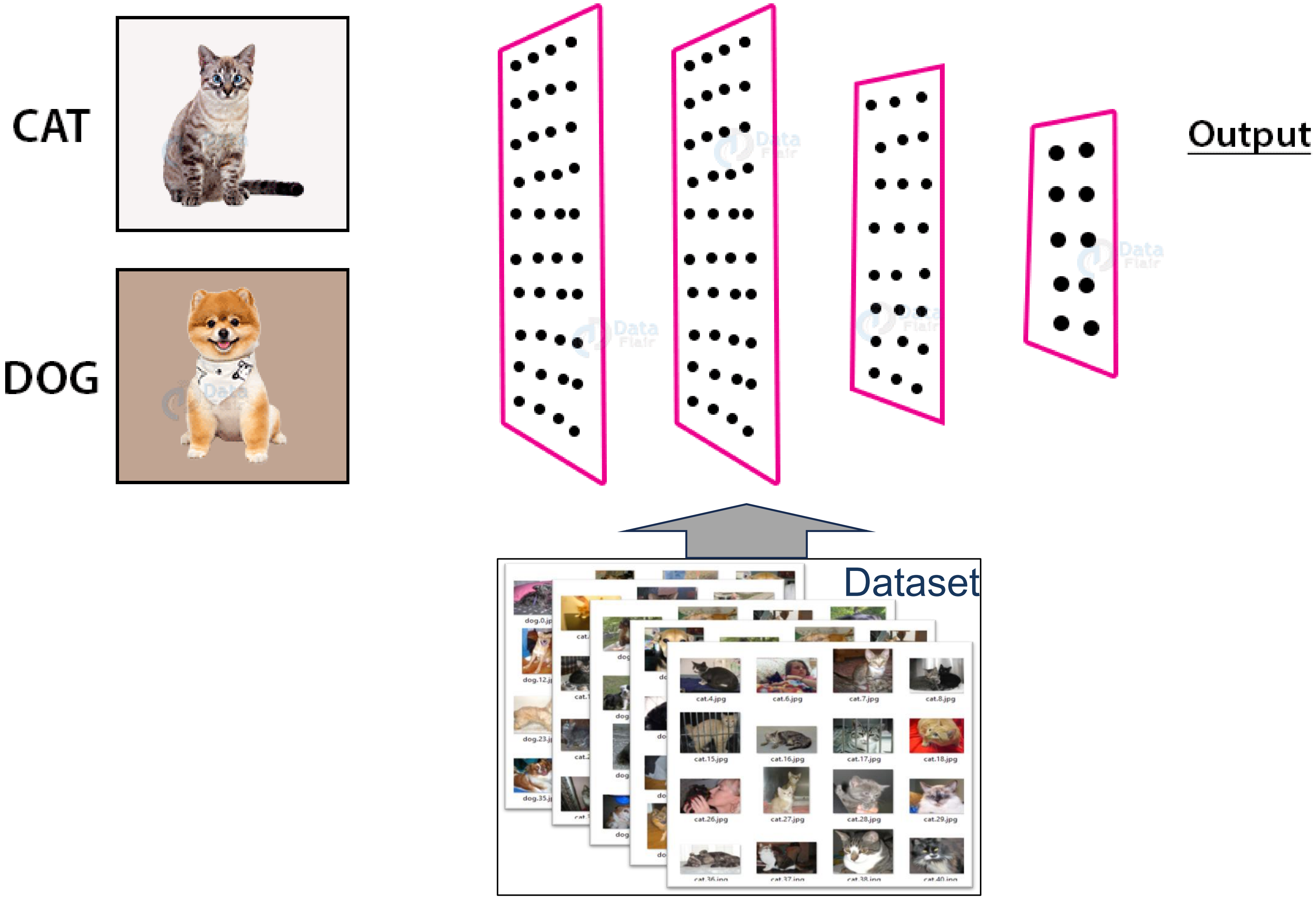


…



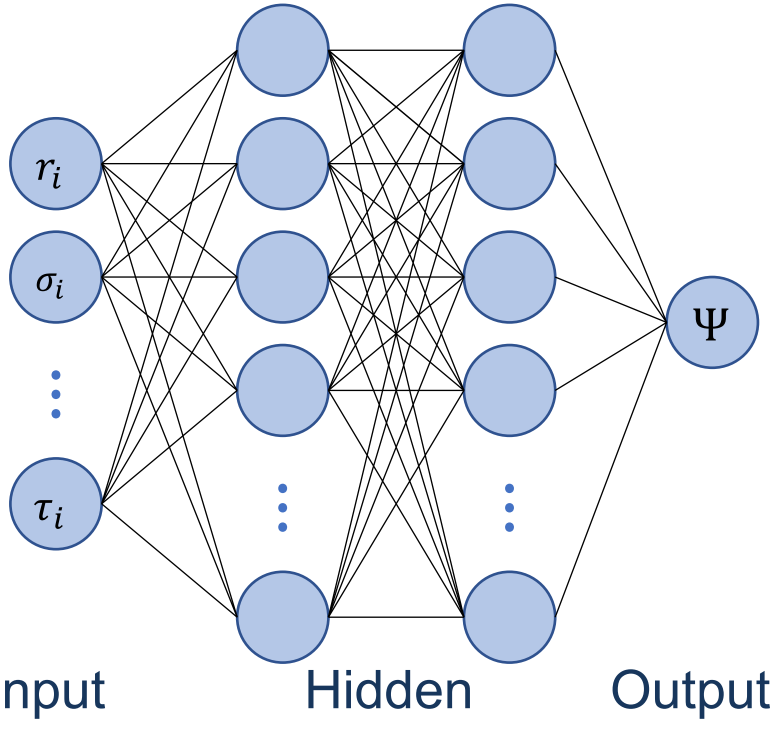
- Training: determine $W^{(i)}$ and biases $\vec{b}^{(i)}$ to minimize loss functions
  - ▶ Loss function： quantifies the difference between the expected and actual output.
- Highly efficient optimization
  - ▶ Backpropagation: automatic differentiation (AD)
  - ▶ GPUs
  - ▶ …

DNNs: Data-driven application



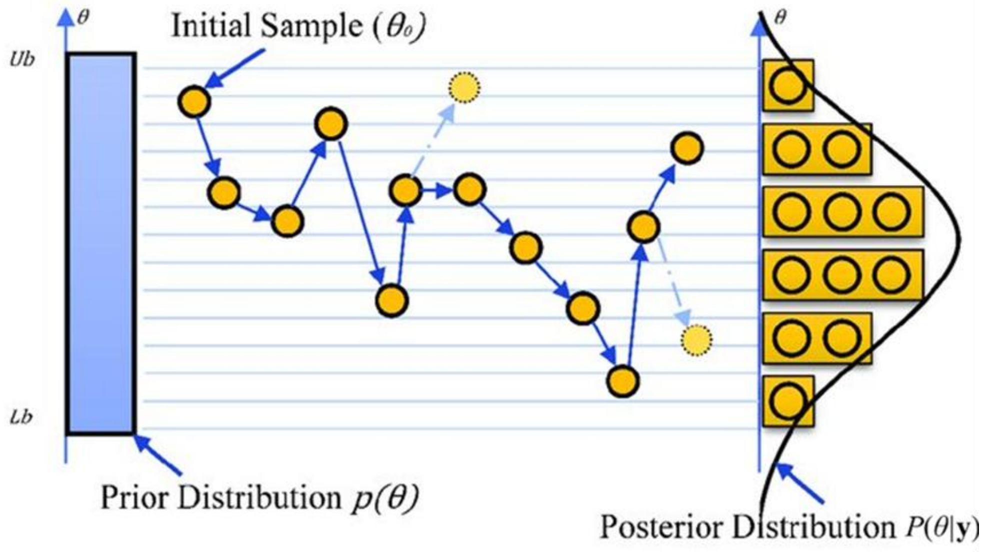
DNN-based VMC method

Neural Network Quantum States (NQSs):



Loss function: $E_{\boldsymbol{\theta}} = \frac{\langle \psi_{\boldsymbol{\theta}} | H | \psi_{\boldsymbol{\theta}} \rangle}{\langle \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle} \geq E_0$

Variational Monte Carlo (VMC)



Sample using Metropolis-Hasting algorithm

$$E_{\boldsymbol{\theta}} = \frac{\int |\psi_{\boldsymbol{\theta}}(r)|^2 \frac{H\psi(r)}{\psi(r)} dr}{\int |\psi_{\boldsymbol{\theta}}(r)|^2 dr}$$

Train the DNNs not by the data but by the physics equation: **physics-informed**

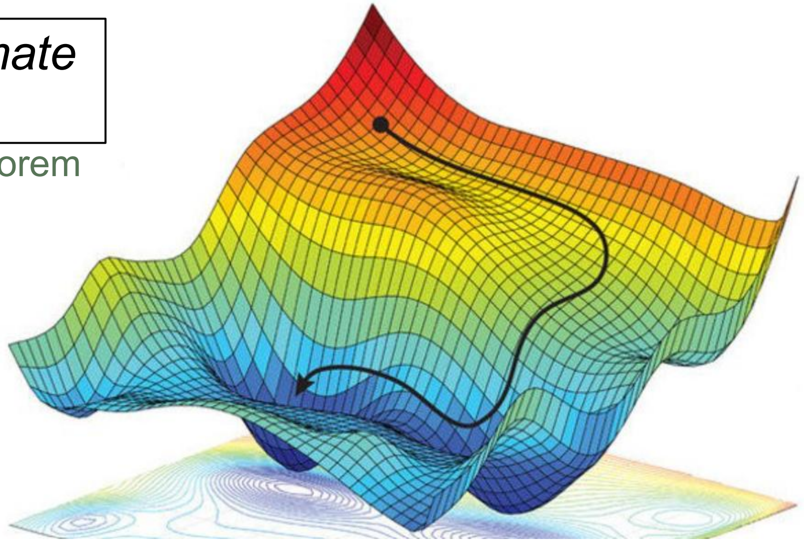
Advantages

- DNNs: approximate high-dimensional functions
  - ▶ **Universal approximation theorems**

sufficiently large or deep

> *Neural networks with a certain structure can, in principle, approximate* ***any continuous function*** *to* ***any desired accuracy***

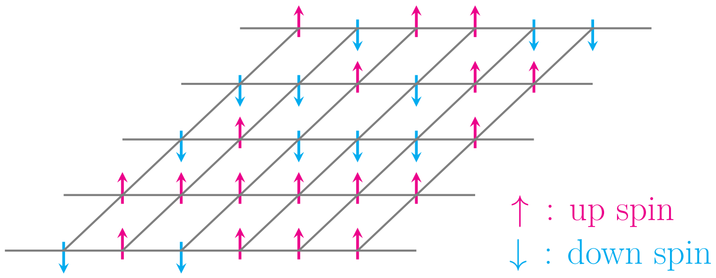
https://en.wikipedia.org/wiki/Universal_approximation_theorem



  - ▶ Variational principle: the more general the trial function, the more accurate the solution
  - ▶ Unbiasedly distinguish molecular and compact tetraquark states
- VMC: applicable for few-body potential
  - ▶ Flux-tube confinement potential
- Circumvents the sign problem in imaginary time evolution of DMC
- Fast optimization: AD+GPUs
- Easy-to-use open-source frameworks

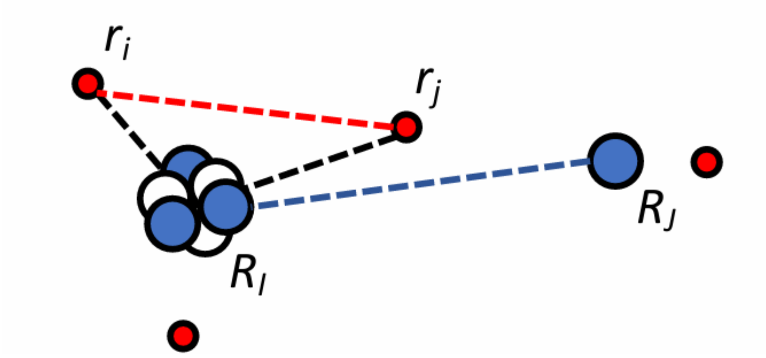
DNNs in many-body problems

Spin models



G. Carleo and M. Troyer, Science 355, (2017).

Electronic systems



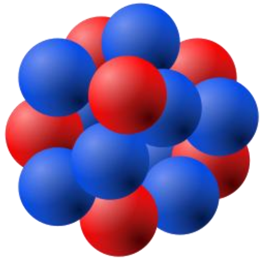
PauliNet: J. Hermann, Z. Schätzle, and F. Noé, Nat. Chem. 12, 891 (2020).

FermiNet: D. Pfau et al., Phys. Rev. Res. 2, 033429 (2020).

…

Review: J. Hermann et al., Nat. Rev. Chem.7, 692–709 (2023)

Nuclear structure



Deuteron: J. W. T. Keeble and A. Rios, Phys. Lett. B 809, 135743 (2020).

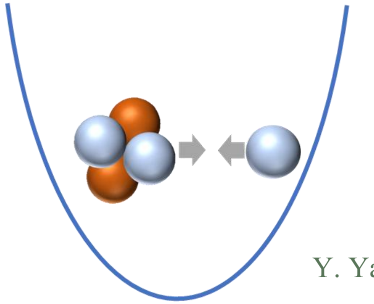
***A* ≤ 4:** C. Adams *et al.*, Phys. Rev. Lett. **127**, 022502 (2021).

FeynmanNet: Y. Yang and P. Zhao, Phys. Rev. C **107**, 034320 (2023).

Hypernuclei: Z.-X. Zhang *et al.*, arXiv:2508.03575

…

Neutron-α scattering



Chiral Nuclear force
Trapped five-body problems
(DNNs+VMC)

Y. Yang, E. Epelbaum, J. Meng, **LM**, and P. Zhao, arXiv:2502.09961

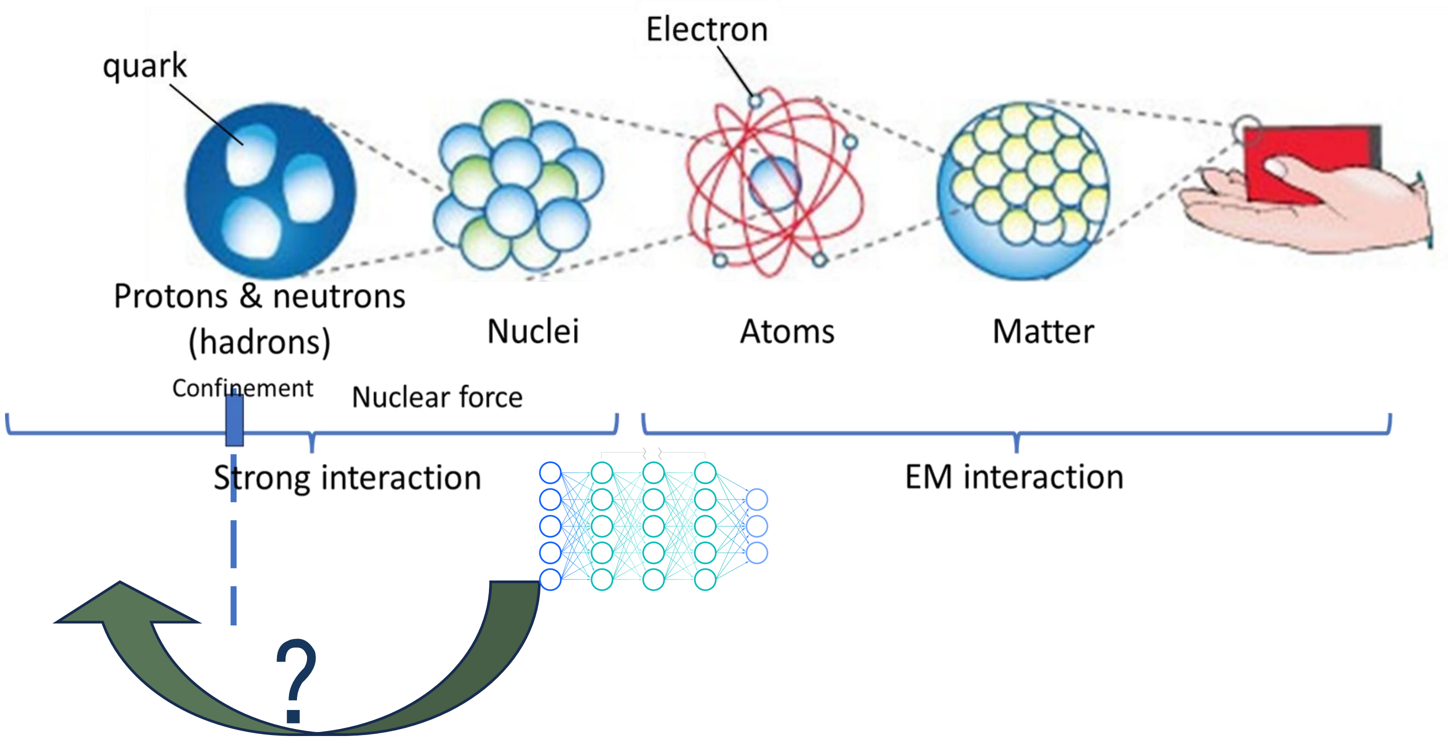
Quark potential models

$$V_{ij}(r) = V_{ij}^{OGE} + V_{ij}^{conf.}$$
$$= \left[\frac{\alpha_s}{r} - \frac{8\pi\alpha_s}{3m_Q^2}\tilde{\delta}(\sigma;r)\boldsymbol{S_i}\cdot\boldsymbol{S_j} - \frac{3}{4}br\right]\frac{\lambda_i\cdot\lambda_j}{4}$$

Quark potential model

QCD-inspired model

- quarks as the degree of freedom
- SU(3) color symmetry
- Confinement
- ….



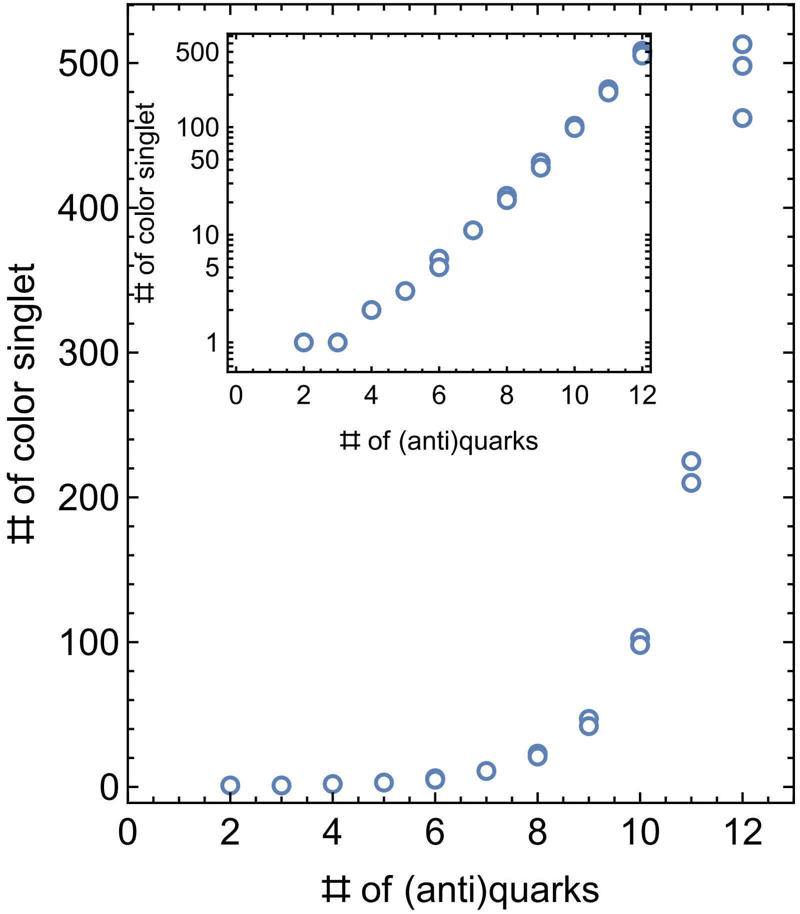
Challenge 1: Spin and color projection

- Conventional spin projection
  - ▶ Option 1： no projection, calculate the ground state
  - ▶ Option 2： penalty terms

$$f_{loss}(\theta) = \langle E\rangle_\theta + \langle S^2\rangle_\theta$$

However, for multiquark systems

- State with higher spin could more interesting
  - ▶ $[cc\bar{q}\,\bar{q}]_{S=0}$: DD threshold; $[cc\bar{q}\,\bar{q}]_{S=1}$: T_{cc} state
- Color projection is needed
  - ▶ color singlet
  - ▶ SU(3) symmetry

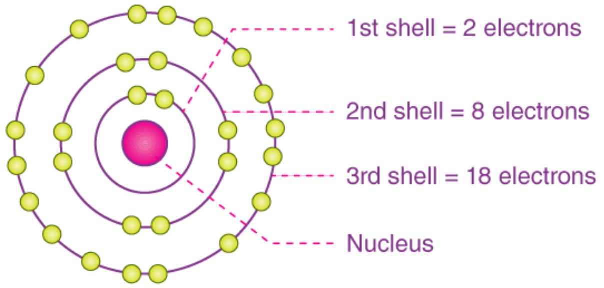


Challenge 2: Strong correlation

- Shell model: good guidance or initial point

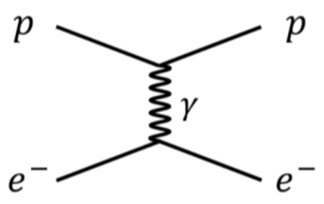
$$\Psi(\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N) = \frac{1}{\sqrt{N!}} \begin{vmatrix} \chi_1(\mathbf{x}_1) & \chi_2(\mathbf{x}_1) & \cdots & \chi_N(\mathbf{x}_1) \\ \chi_1(\mathbf{x}_2) & \chi_2(\mathbf{x}_2) & \cdots & \chi_N(\mathbf{x}_2) \\ \vdots & \vdots & \ddots & \vdots \\ \chi_1(\mathbf{x}_N) & \chi_2(\mathbf{x}_N) & \cdots & \chi_N(\mathbf{x}_N) \end{vmatrix}$$

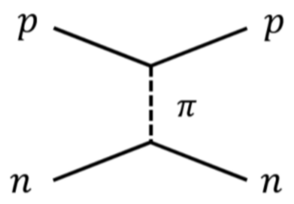
For electron and nucleon systems

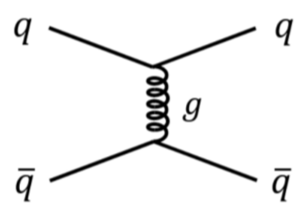


Electron Shell

- Much stronger interaction among quarks
  - ▶ No evidence of a multiquark shell structure
  - ▶ Few-body correlation could be important







$e^2 = 0.1$

$m_e \approx 0.5\ \text{MeV}$

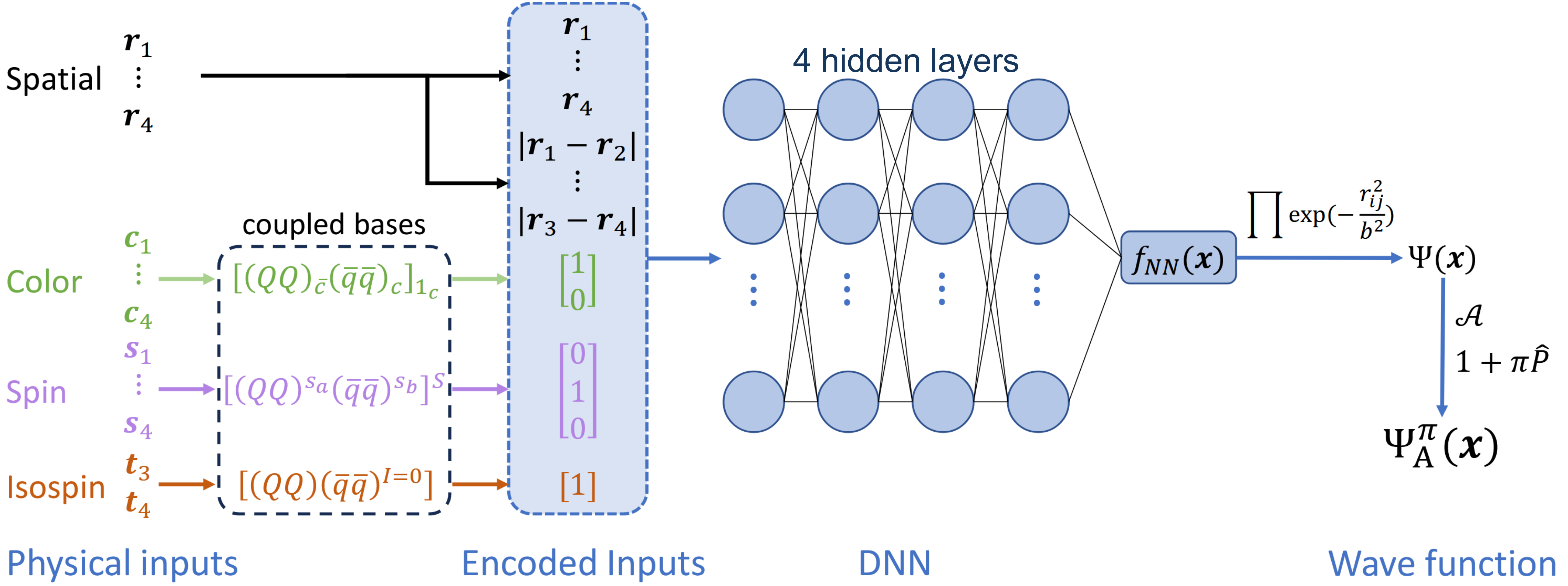
$\frac{m_\pi^2 g_A^2}{16\pi f_\pi^2} \sim 0.1$

$m_N = 938\ \text{MeV}$

$\frac{4}{3}\alpha_s(m_c) \sim 0.4$

$m_c \approx 1273\ \text{MeV}$

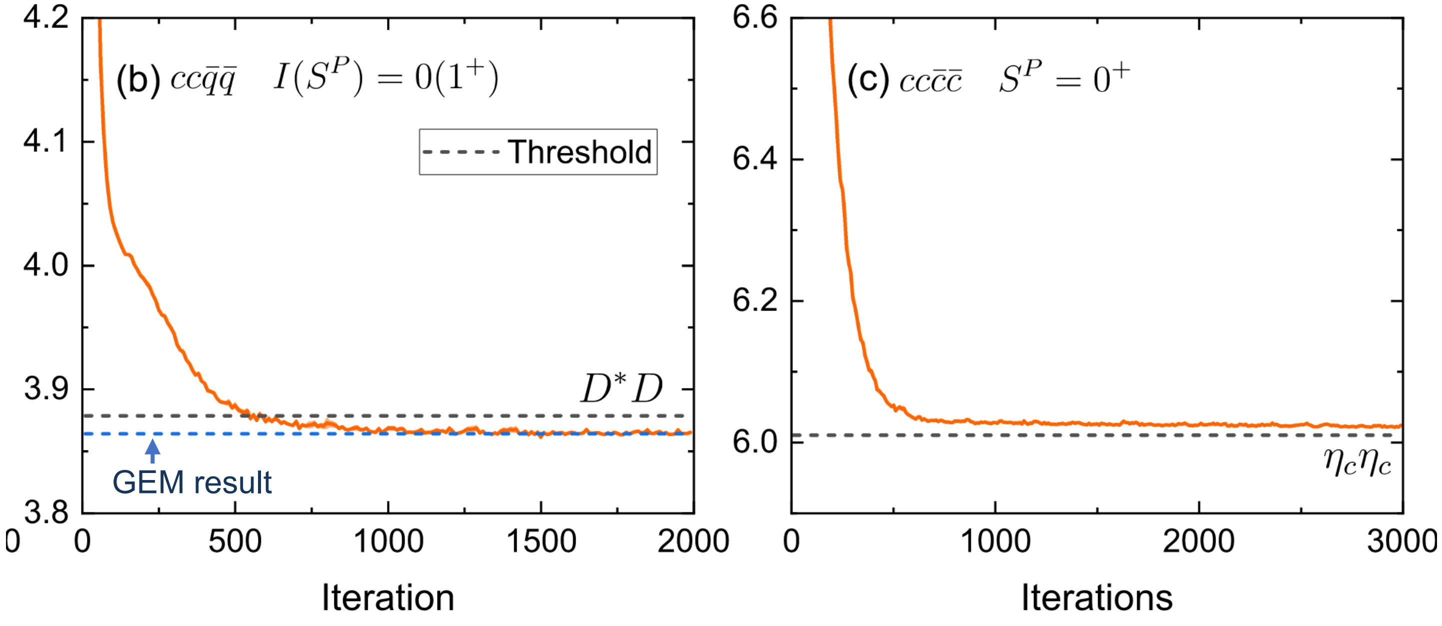
DeepQuark



Powered by NetKet
https://github.com/netket

Tetraquark states

- Convergence after 1000 iterations
- Statistical errors less than 0.1 MeV
- Competitive performance with GEM
- $cc\bar{q}\bar{q}$: molecular state
- $bb\bar{q}\bar{q}$: compact heavy diquark
- $QQ\bar{Q}\bar{Q}$: no bound states



	$I(S^P)$	Thresh.	ΔE	$P_{\bar{3}_c \otimes 3_c}$	$P_{6 \otimes 6_c}$	r_{QQ}	$r_{\bar{q}\bar{q}}$	$r_{Q\bar{q}}$
$cc\bar{q}\bar{q}$	$\mathbf{0}(\mathbf{1}^+)$	DD^*	-15	55%	45%	1.24	1.41	1.06
$bb\bar{q}\bar{q}$	$\mathbf{0}(\mathbf{1}^+)$	$\bar{B}\bar{B}^*$	-153	97%	3%	0.33	0.78	0.69
$QQ\bar{Q}\bar{Q}$	$\mathbf{0}(\mathbf{0}^+)$ $\mathbf{0}(\mathbf{1}^+)$ $\mathbf{0}(\mathbf{2}^+)$	$\eta_c\eta_c$ $\eta_c J/\psi$ $J/\psi J/\psi$	No bound					

ΔE in MeV, r in fm

Pentaquark states

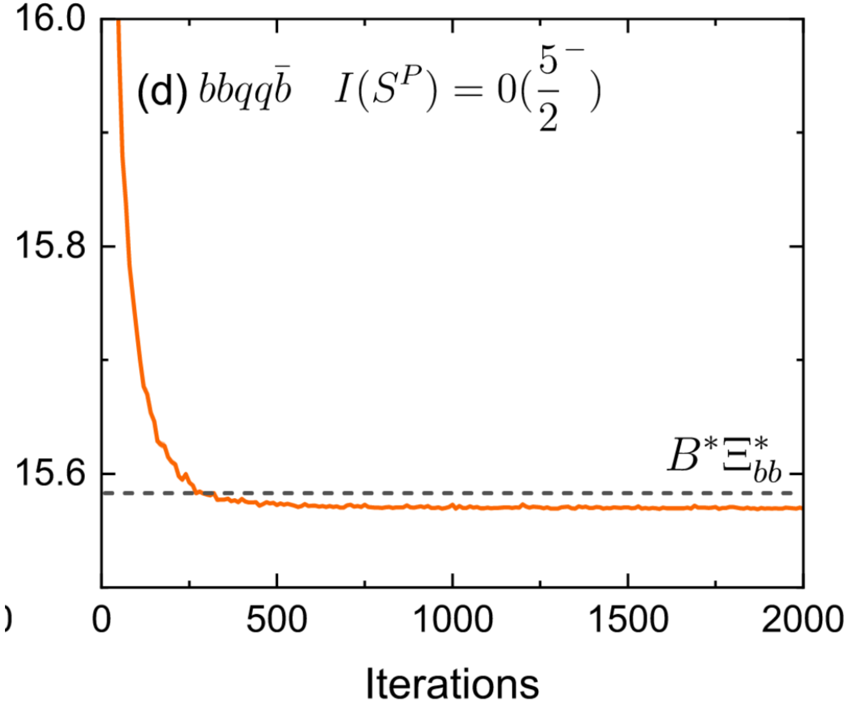
- Exact pentaquark calculations in are computationally prohibitive
  - ▶ Approximations in the spatial or color configurations
- Possible bound pentaquark systems
  - ▶ Heavy-diquark-antiquark-symmetry: $(QQ)_{\bar{3}_c} \to \bar{Q}$
  - ▶ $\bar{Q}\bar{Q}qq \to QQ\bar{Q}qq$
  - ▶ Given $\mathrm{M}_{\bar{Q}\bar{Q}qq} < M_{\bar{Q}q} + M_{\bar{Q}q}$, $\mathrm{M}_{QQ\bar{Q}qq} < M_{QQq} + M_{\bar{Q}q}$?
- Exact calculation via DeepQuark

$$\chi_{\bar{3}_c\otimes\bar{3}_c} = \left\{\left[(QQ)_{\bar{3}_c}(qq)_{\bar{3}_c}\right]_{3_c}\bar{Q}\right\}_{1_c},$$

$$\chi_{\bar{3}_c\otimes 6_c} = \left\{\left[(QQ)_{\bar{3}_c}(qq)_{6_c}\right]_{3_c}\bar{Q}\right\}_{1_c},$$

$$\chi_{6_c\otimes\bar{3}_c} = \left\{\left[(QQ)_{6_c}(qq)_{\bar{3}_c}\right]_{3_c}\bar{Q}\right\}_{1_c}.$$

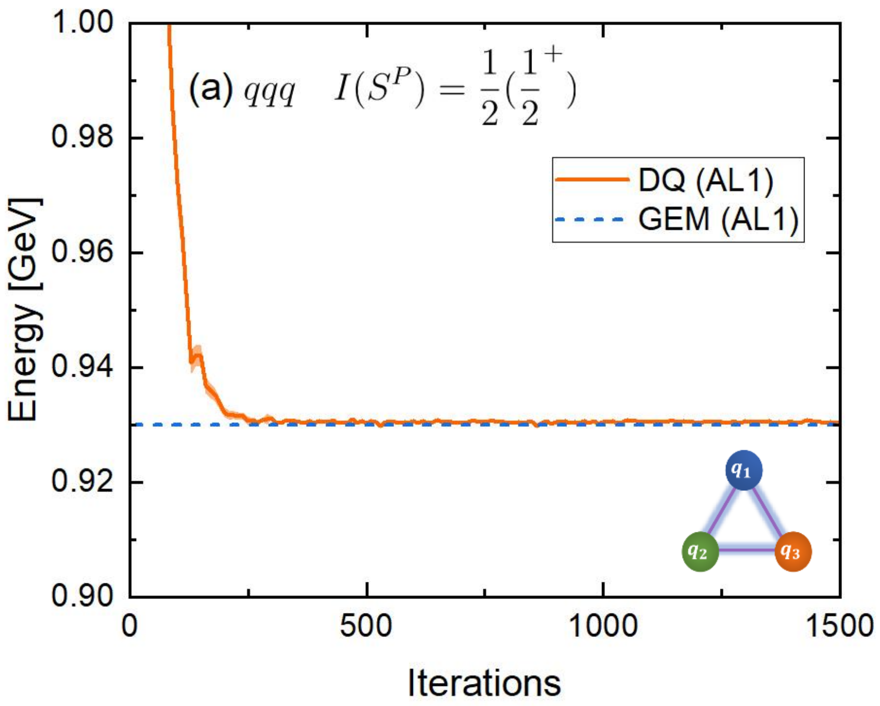
  - ▶ Moderate increase in comput. cost relative to the tetraquark

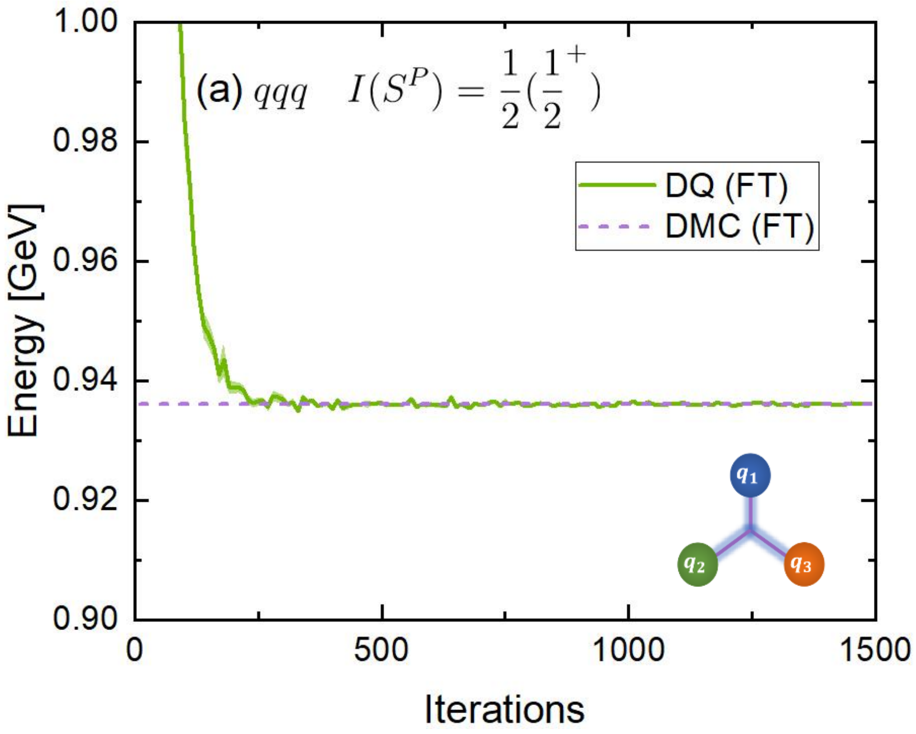


	S^P	Thresholds	ΔE	$\chi_{\bar{3}_c\otimes\bar{3}_c}$	$\chi_{\bar{3}_c\otimes 6_c}$	$\chi_{6_c\otimes\bar{3}_c}$	r_{QQ}	r_{Qq}	r_{qq}	$r_{Q\bar{Q}}$	$r_{q\bar{Q}}$
$ccqq\bar{c}$	$\frac{1}{2}^-, \frac{3}{2}^-$	$\eta_c\Lambda_c, J/\psi\Lambda_c$	NB	~35%	0%	~65%					
	$\frac{5}{2}^-$	$\bar{D}^*\Xi^*_{cc}$	-3	27%	73%	0%	0.50	1.39	1.90	1.73	1.38
$bbqq\bar{b}$	$\frac{1}{2}^-, \frac{3}{2}^-$	$\eta_b\Lambda_b, \Upsilon\Lambda_b$	NB	~35%	0%	~65%					
	$\frac{5}{2}^-$	$B^*\Xi^*_{bb}$	-14	19%	80%	1%	0.30	0.89	1.22	0.88	0.88

Flux-tube confinements

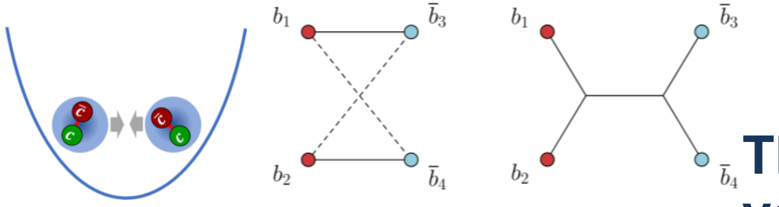
- Lattice QCD supports the flux-tube potential for qqq baryons and tetraquarks
- Baryons: Y-type interaction, junction position to minimize the total string length
  - ► Small impact on baryon spectrum, but complicate dramatically matrix element calculations
- DeepQuark: incorporate Y-type interaction without additional cost, thanks to the VMC
- Ready for tetraquark systems, where different confinement patterns lead to distinct signatures





Summary and outlook

- We still need quark models to understander the **pattens** of multiquark states
  - ▶ **Difficulties:** interactions (confinement) and solving few-body (resonant) problem
  - ▶ **Chances:** All-charm tetraquark family ⇒ confinement pictures
- Bound states: benchmark tests AL1, AP1, SLM ⊗ GEM, DMC, RGM
  - ▶ No bound $QQ\bar{Q}\bar{Q}$ for **pairwise** interaction
  - ▶ SLM: too deep bound states or extra states
  - ▶ Resonating group method: only works well for clustered system
- Resonances: complex scaling method
  - ▶ Exp. $cc\bar{c}\bar{c}$ results: disfavor the pairwise confinement interaction
  - ▶ Too strong coupling between dimeson and diquark-antidiquark (?)
- DeepQuark: DNN-based framework
  - ▶ Surpass traditional methods starting from pentaquark states
  - ▶ Ready for flux-tube confinement
- Outlook:
  - ▶ DNNs: excited states, scattering, resonances
  - ▶ All-charm tetraquark with flux-tube confinement



Thanks for your attention!

Backup

Color-electric interaction for multiquark states

- For color-singlet multiquark states $\{Q_1, Q_2, ..., Q_n\}$, $Q_i = Q$ or $\bar{Q}$, if two-body interaction $V_{Q_iQ_j} = V_8(r_{ij})\lambda_i \cdot \lambda_j$, then

$$V_{\{Q_1,Q_2,\dots Q_n\}} = \sum_{i<j} a_{ij} V_8(r_{ij}), \quad \sum_{i<j} a_{ij} = -\frac{8}{3}n$$

Proof: $2\langle \sum_{i<j} \lambda_i \cdot \lambda_j \rangle = \langle \sum_i \lambda_i \rangle^2 - \sum_i \langle \lambda_i \rangle^2$

- A general problem: For fixed $\sum_{i<j} a_{ij}$, what distribution of a_{ij} give the lowest mass?

Walter Thirring, E.M. Harrell, Quantum mathematical physics: atoms, molecules and large systems

$\Rightarrow$ the symmetric case gives the worse result: $M_n(a_{ij}) \leq M_n^{(S)}$

Proof:

$$H = H^S + \Delta V = H^S + aV_8(r_{12}) - aV_8(r_{34}), \quad \langle \psi^S | \Delta V | \psi^S \rangle = 0, \quad (32)$$

$$\langle \psi^S | H^S | \psi^S \rangle = \langle \psi^S | H | \psi^S \rangle \geq \min_{\psi \in \mathcal{H}} \langle \psi | H | \psi \rangle \quad (33)$$

- Intuitively, less symmetric a_{ij}, more deeply bound ground states

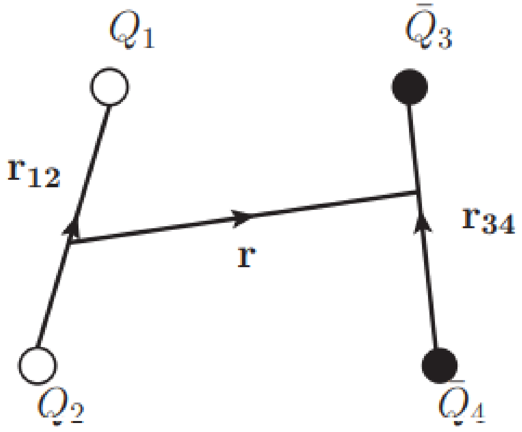
Phys.Rev. D25 (1982) 2370

Color-electric interaction for $QQ\bar{Q}\bar{Q}$

- $\{a_{ij}\}$ distribution for $QQ\bar{Q}\bar{Q}$

a_{ij}	$a_{12} = a_{34}$	$a_{13} = a_{24}$	$a_{14} = a_{24}$
Di-meson	0	$-\frac{16}{3}$	0
$\bar{3}_c - 3_c$	$-\frac{8}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$
$\bar{6}_c - 6_c$	$\frac{4}{3}$	$-\frac{10}{3}$	$-\frac{10}{3}$

$$2M(Q\bar{Q}) < M(6-\bar{6}) < M(3-\bar{3})$$



- Things become different, when

  $\Rightarrow$ unequal quark masses e.g. $QQ\bar{q}\bar{q}$ PRL118 142001; PRL119 202001; PRL119 202002

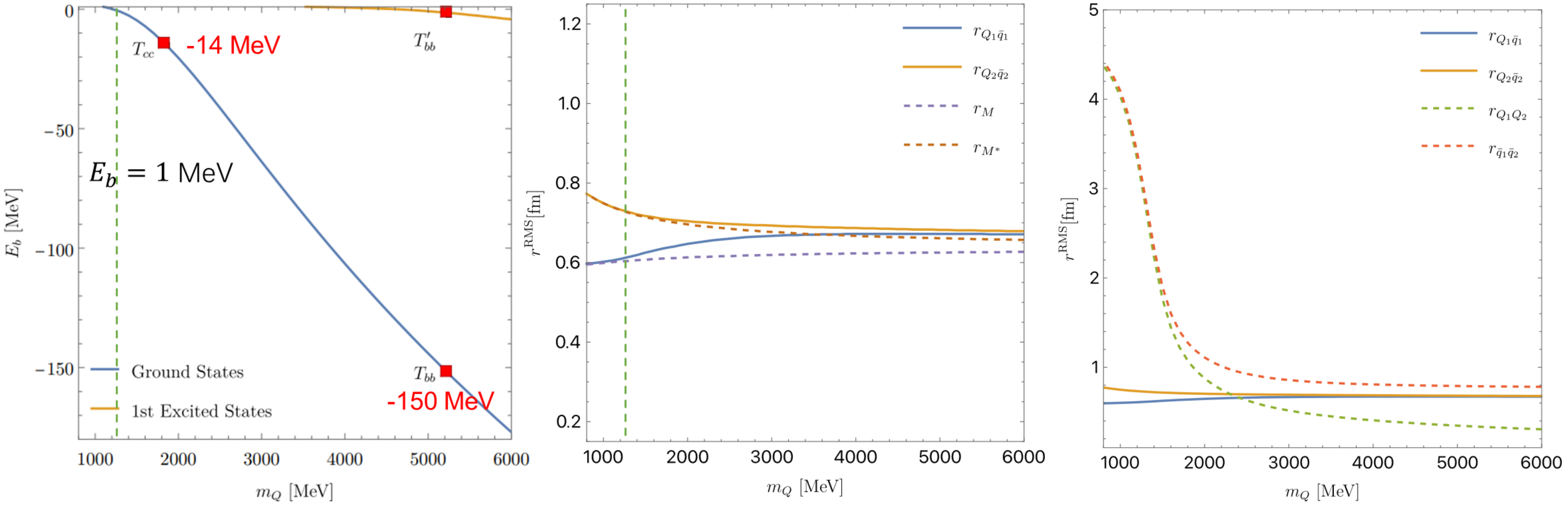
  $\Rightarrow$ hyperfine correction, e.g. color-magnetic interaction $S_i \cdot S_j \lambda_i \cdot \lambda_j$

  $\Rightarrow$ multibody interaction, e.g. doubly-Y interaction

  $\Rightarrow$...

Phys.Rev. D25 (1982) 2370

Molecular or compact ?



- Tuning the m_Q to m_b: (bb) compact diquark
- Tuning the m_Q to make $E_b < 1$ MeV: molecular states

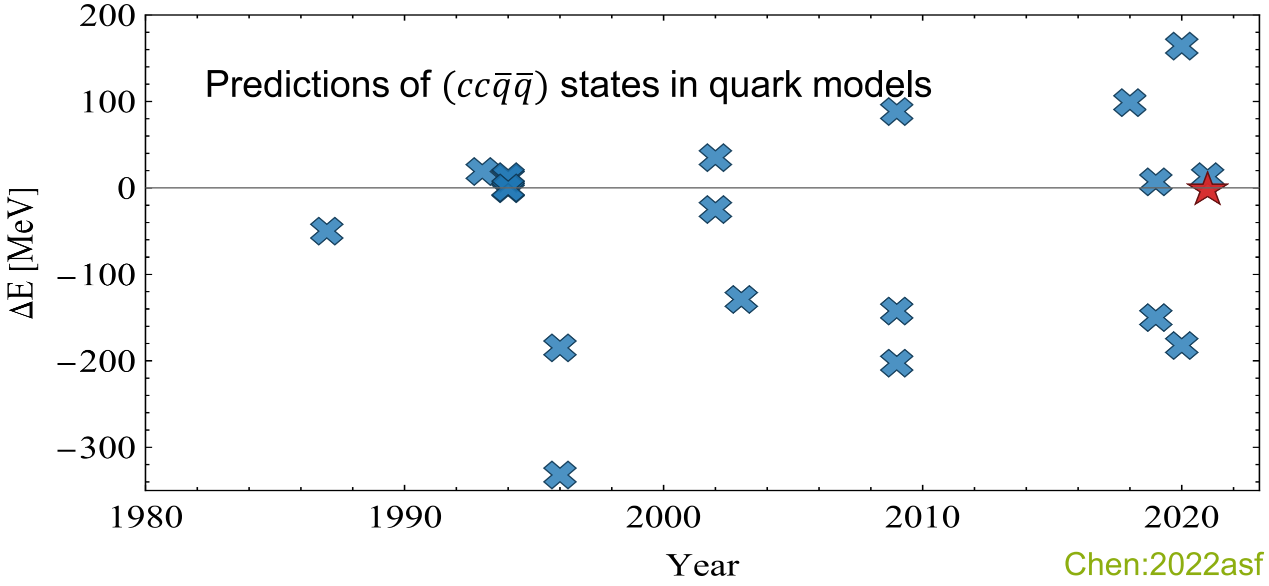
Computational error in literature

- Ref.[Ortega:2022efc]: Same SLM interactions
  - $[bb\bar{q}\bar{q}]^{I=1}$ bound state! Our results: there is no isospin vector states
  - Get a $J^P(I) = 0^+(0)$ state dominated by S-wave BB states!

Violating Boson principle

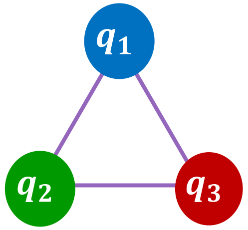
Mass	E_B	$\mathcal{P}_{B^0B^{*+}}$	$\mathcal{P}_{B^+B^{*0}}$	$\mathcal{P}_{B^{*+}B^{*0}}$	$\mathcal{P}_{I=0}$	$\mathcal{P}_{I=1}$
10582.2	21.9	47.8	50.0	2.2	99.99	0.01
10593.5	10.5	51.0	48.6	0.4	0.02	99.98

J^P	I	Mass	Width	E_B	$\mathcal{P}_{BB}$	$\mathcal{P}_{B^*B^*}$	Γ_{BB}	$\Gamma_{B^*B^*}$
0^+	0	10553.0	0	6.0	92%	8%	0	0
		10640.7	2.8	8.7	76%	24%	2.8	0
	1	10545.9	0	13.1	93%	7%	0	0
		10672.6	72.0	−23.2	39%	61%	30.7	41.3
2^+	1	10642.3	0	7.1	-	100%	-	0

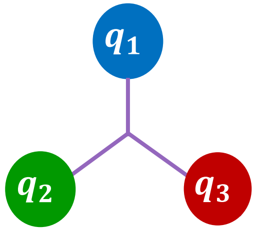


Chen:2022asf

Baryons



$$V_{\text{conf}}^{\Delta} = \sigma_{\Delta} \sum_{i<j} r_{ij}$$



$$V_{\text{conf}}^{Y} = \sigma_{Y} L_{min}$$

$$0.5 \leq \frac{L_{min}}{L_{\Delta}} \leq 0.58$$

Other set

- Activation function: $\sigma(x) = \tanh(x)$
- Optimizer

$$\boldsymbol{\theta}^{i+1} = \boldsymbol{\theta}^{i} - \eta (S + \epsilon I)^{-1} \nabla_{\boldsymbol{\theta}^i} E_{\boldsymbol{\theta}^i}\,, \tag{8}$$

where i is the iteration step, η is the learning rate, $\epsilon = 10^{-3}$ is taken for numerical stability, and S is the Quantum Fisher information matrix,

$$S_{ab} = \frac{\langle \partial_{\theta_a} \psi_{\boldsymbol{\theta}} | \partial_{\theta_b} \psi_{\boldsymbol{\theta}} \rangle}{\langle \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle} - \frac{\langle \partial_{\theta_a} \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle}{\langle \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle} \frac{\langle \psi_{\boldsymbol{\theta}} | \partial_{\theta_b} \psi_{\boldsymbol{\theta}} \rangle}{\langle \psi_{\boldsymbol{\theta}} | \psi_{\boldsymbol{\theta}} \rangle}\,. \tag{9}$$

TABLE V. The number of nodes in each hidden layer and the total number of variational parameters in the DNN for different systems.

Systems	S^P	Nodes	Parameters
e^+e^-	0^+	(16, 16, 16, 16)	961
$e^+e^-e^-$	0^+	(16, 16, 16, 16)	1041
$e^+e^+e^-e^-$	0^+	(16, 16, 16, 16)	1137
qqq	$\frac{1}{2}^+$	(16, 16, 16, 16)	1105
$QQ\bar{q}\bar{q}$	1^+	(32, 16, 16, 16)	1889
$QQ\bar{Q}\bar{Q}$	0^+	(32, 16, 16, 16)	1825
$QQ\bar{Q}\bar{Q}$	1^+	(32, 16, 16, 16)	1857
$QQ\bar{Q}\bar{Q}$	2^+	(32, 16, 16, 16)	1793
$QQqq\bar{Q}$	$\frac{1}{2}^-$	(40, 20, 20, 20)	3081
$QQqq\bar{Q}$	$\frac{3}{2}^-$	(40, 20, 20, 20)	3041
$QQqq\bar{Q}$	$\frac{5}{2}^-$	(40, 20, 20, 20)	2921

Related work

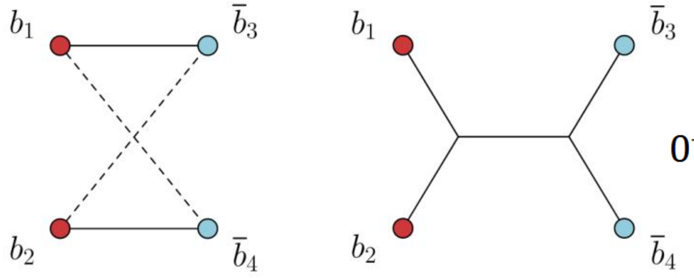


Fig. 1. Left panel: the flip-flop configuration of disconnected di-mesons. Right panel: the butterfly configuration with two connected diquarks. The two middle connecting points are chosen to minimize the total path.

$V^{4Q} \equiv \min\left(V^{\text{flip-flop}}, V^{\text{butterfly}}\right)$ + DMC

0^{++} state has a mass of 18.69 ± 0.03 GeV, which is around 100 MeV below twice the η_b mass.

Y. Bai, S. Lu, and J. Osborne, Beauty-full tetraquarks, Phys. Lett. B **798**, 134930 (2019).

$$
\begin{aligned}
V = {} & \Theta(V_{MM} - V_{YY})\Theta(V_{MM'} - V_{YY})\left(V_{YY}|\bar{\mathbf{3}}_{12}\mathbf{3}_{34}\rangle\langle\bar{\mathbf{3}}_{12}\mathbf{3}_{34}| + \min(V_{MM}, V_{MM'})|\mathbf{6}_{12}\bar{\mathbf{6}}_{34}\rangle\langle\mathbf{6}_{12}\bar{\mathbf{6}}_{34}|\right) \\
& + \Theta(V_{YY} - V_{MM})\Theta(V_{MM'} - V_{MM})\left(V_{MM}|\mathbf{1}_{13}\mathbf{1}_{24}\rangle\langle\mathbf{1}_{13}\mathbf{1}_{24}| + \min(V_{YY}, V_{MM'})|\mathbf{8}_{13}\mathbf{8}_{24}\rangle\langle\mathbf{8}_{13}\mathbf{8}_{24}|\right) \\
& + \Theta(V_{YY} - V_{MM'})\Theta(V_{MM} - V_{MM'})\left(V_{MM'}|\mathbf{1}_{14}\mathbf{1}_{23}\rangle\langle\mathbf{1}_{14}\mathbf{1}_{23}| + \min(V_{YY}, V_{MM})|\mathbf{8}_{14}\mathbf{8}_{23}\rangle\langle\mathbf{8}_{14}\mathbf{8}_{23}|\right),
\end{aligned}
$$

+ multidimensional numerical integration

+ analytical continuation

$qq\bar{Q}\bar{Q}$ systems

P. Bicudo and M. Cardoso, Phys. Rev. D 94, 094032 (2016).