Realization of EW baryogenesis in 4$^{\text{th}}$ generation model

Hsiang-nan Li

Presented at XS 2026

Jan. 19, 2026

2502.16099
2601.XXXXX

Analyticity and SM flavor structure

- Physical observables, being analytical, respect dispersion relations

- Dispersion relation connects various dynamics at different scales; heavy meson lifetimes link EW and strong interactions; Higgs decays into b quark pairs link Yukawa coupling and strong interactions,…

- Numerous observables imply numerous links --- nontrivial constraints

- SM parameters may not be completely free

- SM flavor structure governed by analyticity

- Echo ``S-matrix bootstrap conjecture'' (Geoffrey Chew, 1960s)

- A well-defined but infinite set of self-consistency conditions determines uniquely aspects of particles in nature

Why 4G model?

- Dispersion relations for heavy quark lifetimes, neutral meson mixing muonium mixing,.., fix fermion masses

- Predict neutrino mass normal ordering

- Distinct mixing patterns of quarks and leptons attributed to different mass hierarchies in the two sectors
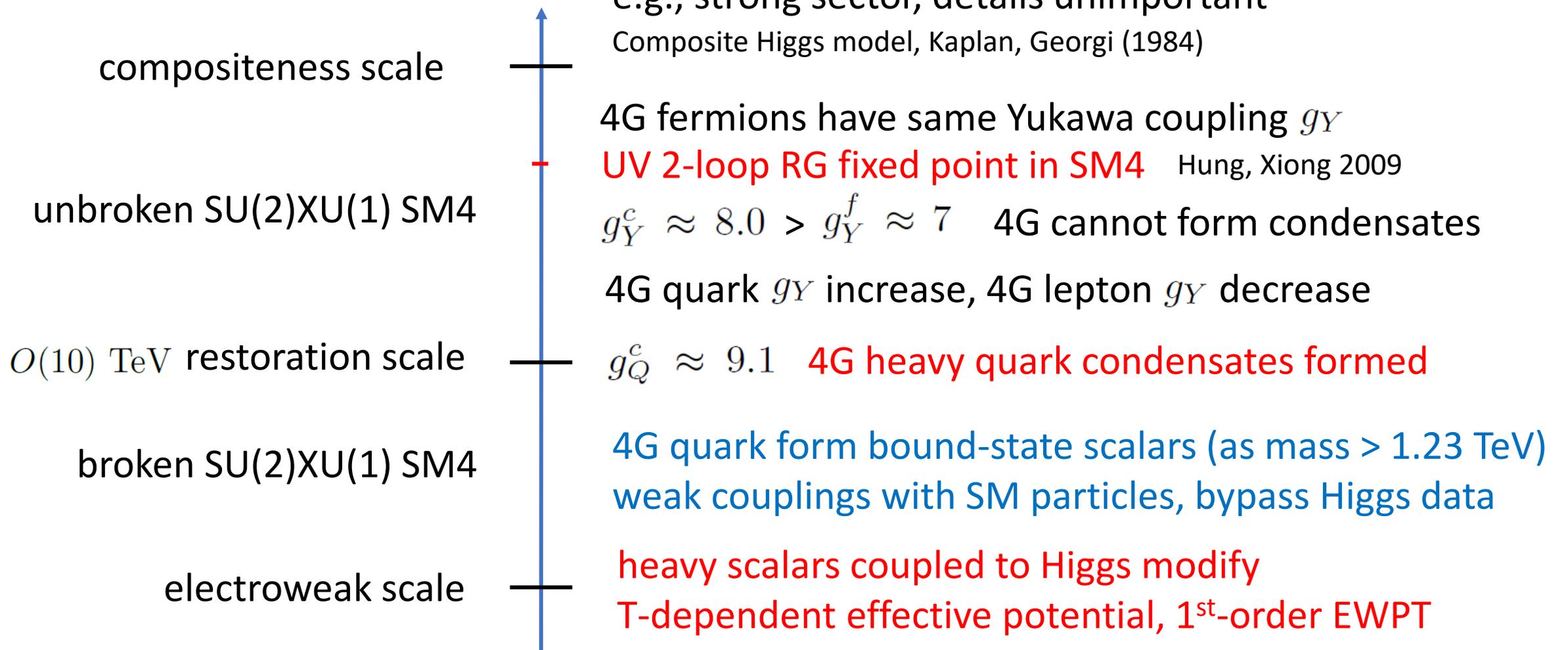
$$sin\theta_{12} \sim \left| \frac{m_2^2}{m_3^2} \approx 3.1 \times 10^{-2} \gg \frac{m_s^2}{m_b^2} \approx 9.0 \times 10^{-4} \right|$$

- Speculate that only three gauge couplings are fundamental

- 4th generation model is then simple and natural extension of SM

- Predict masses t' quark b' quark, charged lepton L neutrino ν_4

       200 TeV   2.7 TeV, 270 GeV, 170 GeV

# Our SM4 scenario

Li, 2309.15602
Li, 2407.07813

e.g., strong sector, details unimportant

Composite Higgs model, Kaplan, Georgi (1984)

compositeness scale ——

4G fermions have same Yukawa coupling g_Y

UV 2-loop RG fixed point in SM4   Hung, Xiong 2009

unbroken SU(2)XU(1) SM4

$g_Y^c \approx 8.0 > g_Y^f \approx 7$ 4G cannot form condensates

4G quark g_Y increase, 4G lepton g_Y decrease

$O(10)\ \mathrm{TeV}$ restoration scale ——

$g_Q^c \approx 9.1$ 4G heavy quark condensates formed

broken SU(2)XU(1) SM4

4G quark form bound-state scalars (as mass > 1.23 TeV)
weak couplings with SM particles, bypass Higgs data

electroweak scale ——

heavy scalars coupled to Higgs modify
T-dependent effective potential, 1st-order EWPT

4G lepton-sourced baryon asymmetry explains baryon-over-entropy ratio $\eta_B \equiv \dfrac{n_B - n_{\bar{B}}}{n_\gamma} \sim 10^{-10}$

EW symmetry breaking

Dynamical mass generation

- Dyson-Schwinger equation
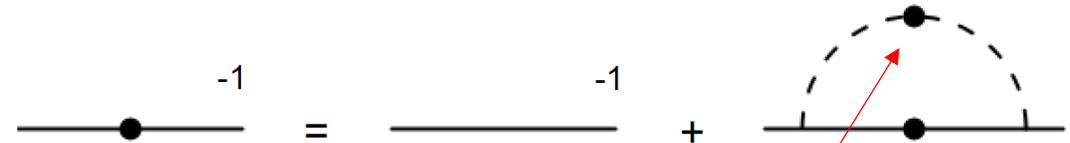
bare mass = 0

$$-i[\not{p} - m(p^2)] = -i(\not{p} - m_0) - 2(-ig_Y)^2 \int \frac{d^4q}{(2\pi)^4} \frac{i}{(p-q)^2 - \mu^2(q^2)} \frac{i[\not{q} + m(q)]}{q^2 - m^2(q^2)}$$

$$t' \to t'\phi^0 \text{ or } t' \to b'\phi^+$$

large enough Yukawa coupling ➡ dynamically generated masses

- Rainbow approximation and leading-order vertices have been applied
- Scalar mass arises from fermion condensates
- In view of effective theory
  dynamical symmetry breaking
  critical coupling $g_Q^c \approx 9.1$

Hung, Xiong 2011
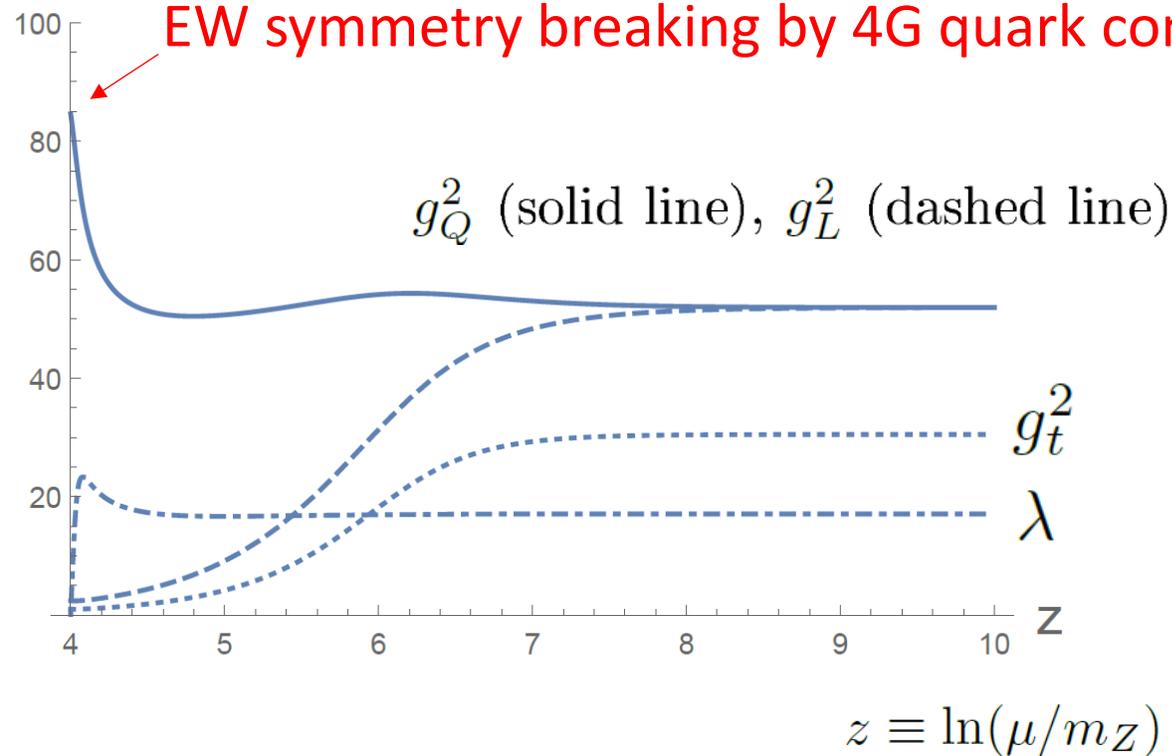
$\langle \bar{f}f \rangle < 0$

➡ $\frac{\mu^2}{2}\phi^2$
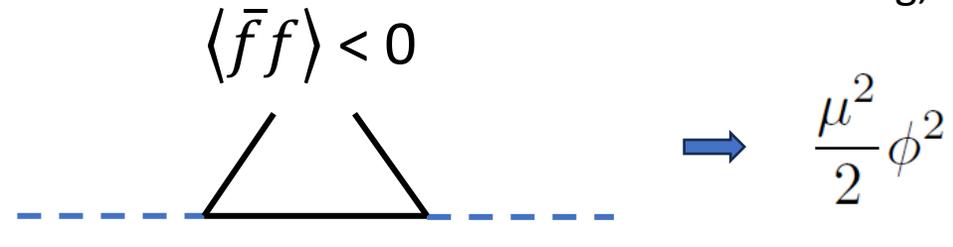
2-loop RG evolution in SM4

- **UV fixed point appears**

common Yukawa coupling $g_Y^f \approx 7$ as switching off small gauge interactions

EW symmetry breaking by 4G quark condensates

g_Q^2 (solid line), g_L^2 (dashed line)

g_t^2

λ

z

$z \equiv \ln(\mu/m_Z)$

g_3^2

g_2^2

g_1^2

$t = \log(E/91.2 \text{Gev})$
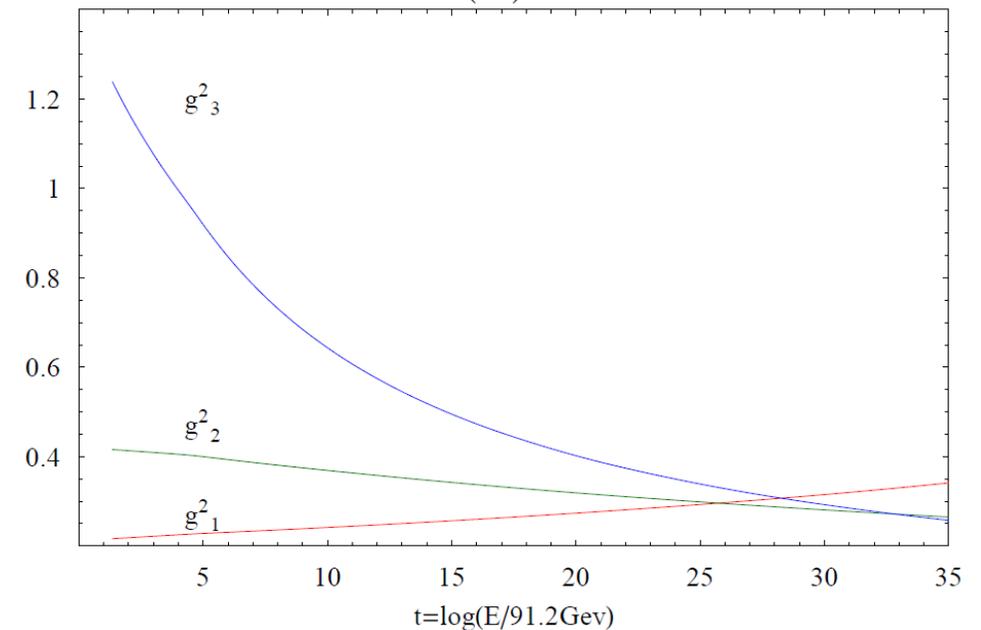
EW phase transition

Effective Higgs potential

- Following standard formalism, consider

$$V_{\text{eff}}(\phi, T, \mu_R) = V_0(\phi) + V_1(\phi, \mu_R) + V_T(\phi, T)$$

- Tree-level potential, result of RG evolution from restoration scale

$$V_0(\phi) = \frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 \quad \mu^2 = -m_H^2/2 \text{ and } \lambda = -\mu^2/v^2$$
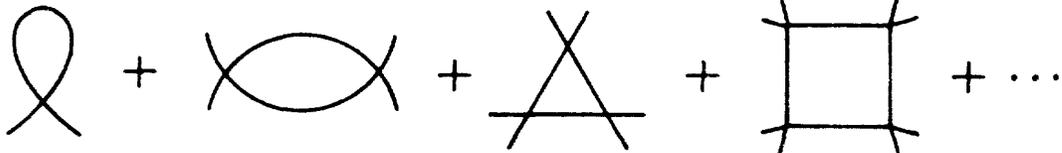
- 1-loop Coleman-Weinberg potential $V_1(\phi, \mu_R)$

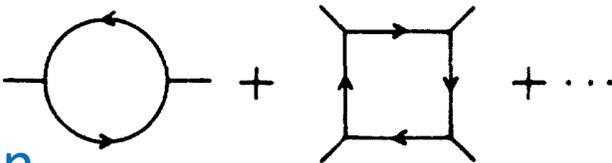- All particles coupled to Higgs contribute
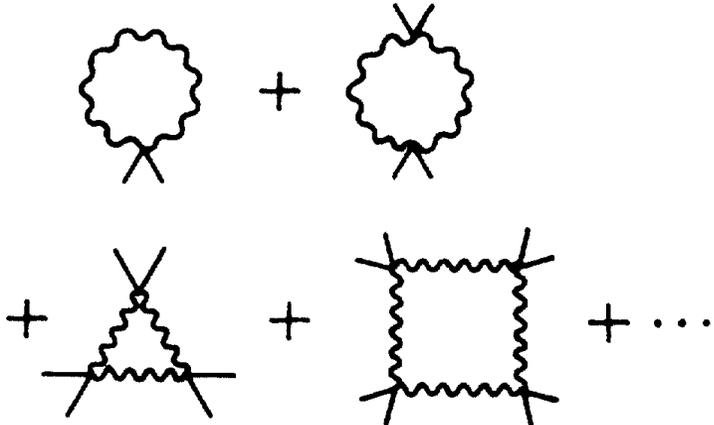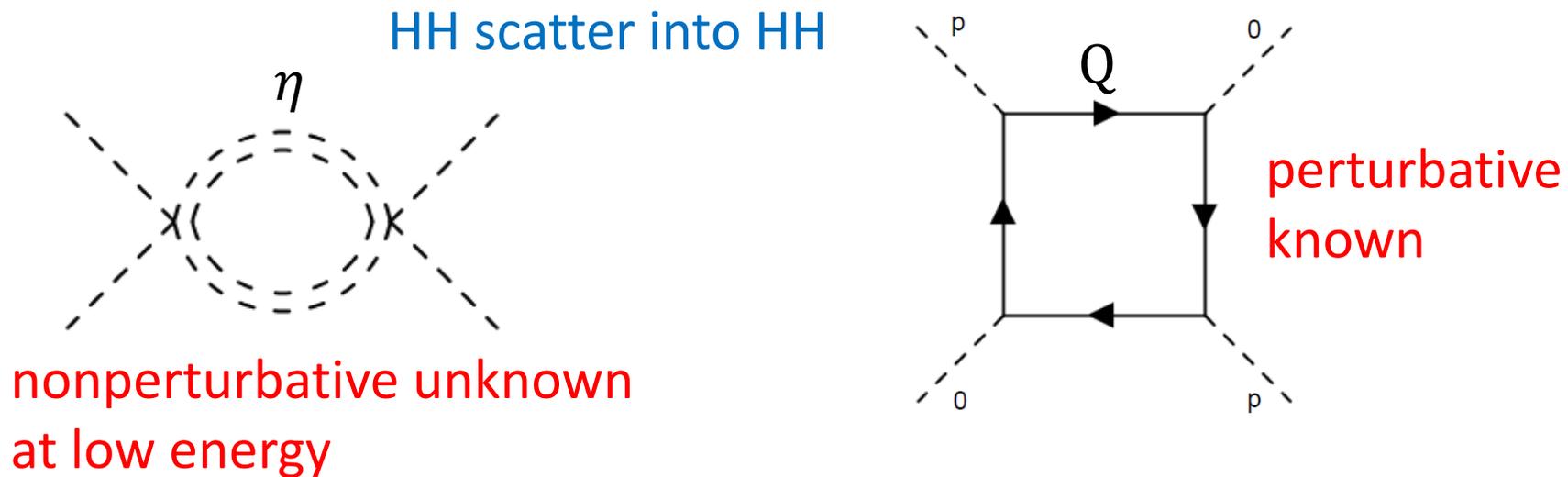

gauge boson

scalar contribution

fermion

Heavy scalar-Higgs coupling

Hung, Xiong 2011

- Heavy fermions with mass above 1.23 TeV form bound states
- t' and b' quarks form bound states, but 4G leptons do not
- Estimate heavy scalar-Higgs coupling using effective theory

HH scatter into HH

η

nonperturbative unknown
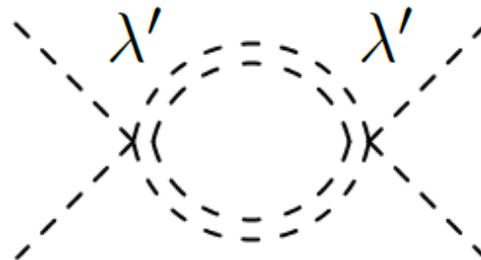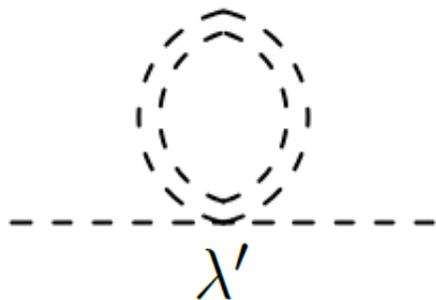at low energy

perturbative
known

match the two theories at large $p^2 \approx s$, solve for
scale-dependent effective coupling, run it to $p^2 \approx v^2$

Heavy scalar contribution

- Derive effective coupling $\lambda'{=}3.8$

- Compute one-loop corrections to $V_1(\phi, \mu_R)$ in effective theory
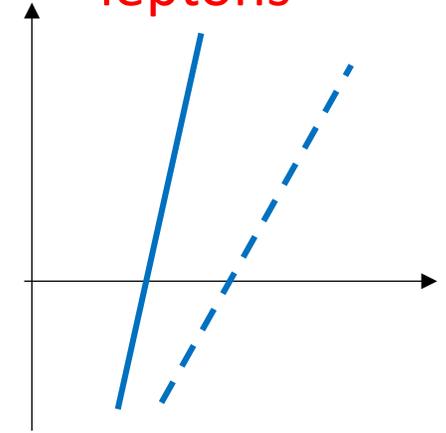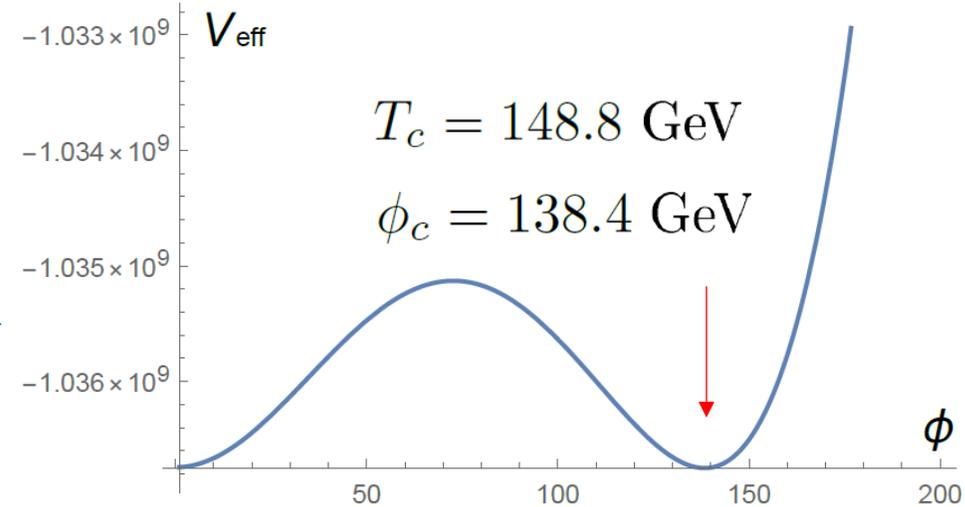
- Diagrams similar to those from Higgs

… to all orders

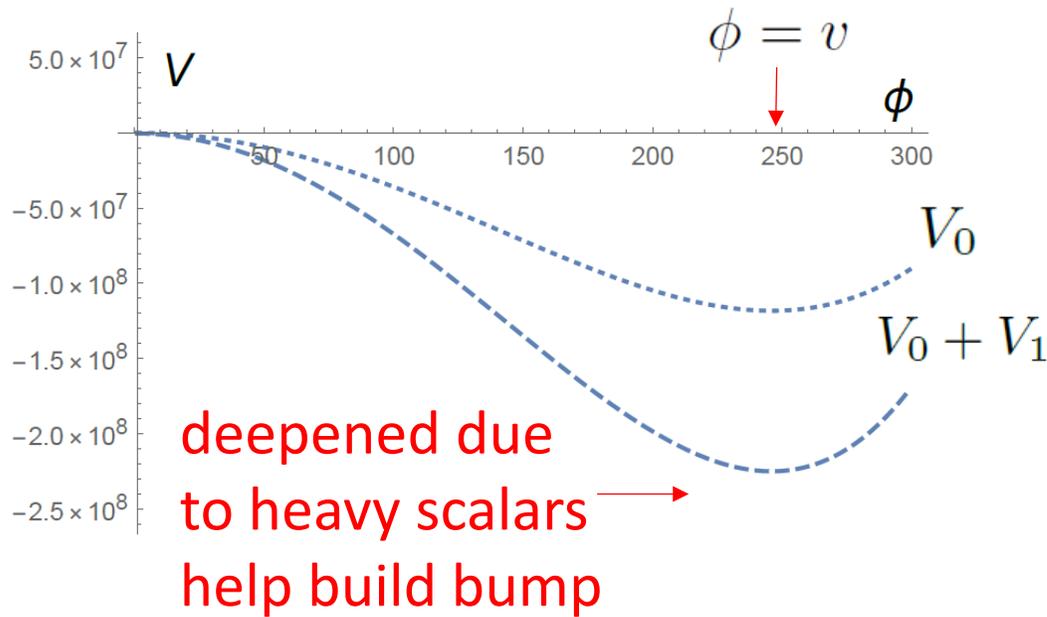# Effective potential

- Heavy scalars and 4G leptons have different impacts
- 4G leptons make T-dependent potential increase faster, and lower Tc

with 4G leptons

$$\phi_c/T_c \approx 0.9$$

$T_c = 148.8 \text{ GeV}$

$\phi_c = 138.4 \text{ GeV}$

deepened due to heavy scalars help build bump

meets criterion $\phi_c/T_c \gtrsim 1$ roughly, Implying strong 1st-order EWPT

EW baryogenesis

Sakharov's conditions

- To explain baryon asymmetry in the Universe (BAU)

  require three conditions

- B violation

- Departure from thermal equilibrium

- C, CP violations

- Whether they are sufficient depends on their strength; need quantitative analysis

- SM contains all three sources, but 2nd and 3rd not strong enough

$$\eta_B \equiv \frac{n_B - n_{\bar{B}}}{s} \sim 10^{-10}$$

entropy

temperature-independent

pedagogical review: Cline 2006

Bubbles formed and expand in 1st order PT

1eV ~ 11000 K
EW phase transition
critical T ~ 100 GeV
extremely high T
➡ plasma of elementary
particles
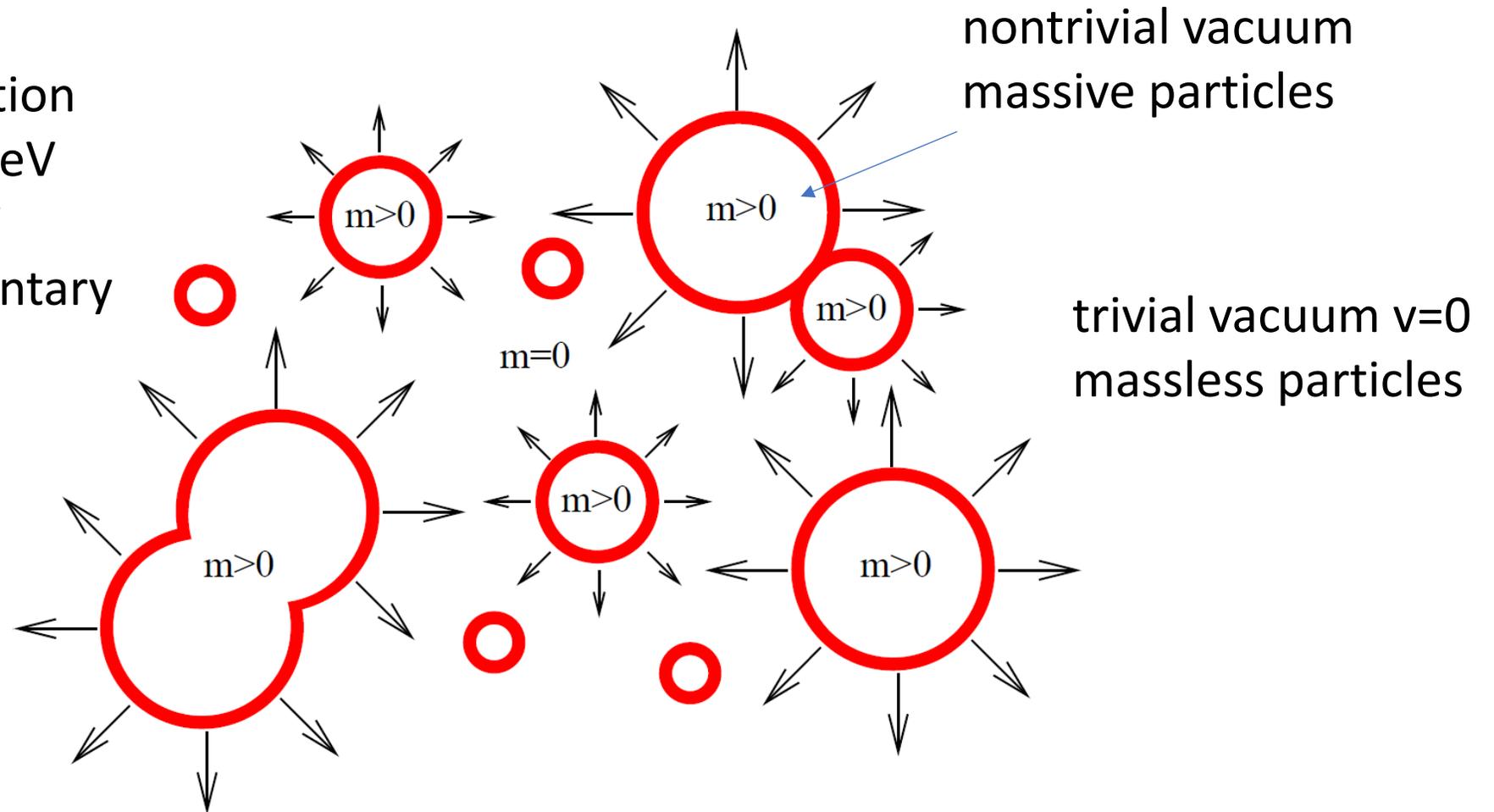
nontrivial vacuum
massive particles

trivial vacuum v=0
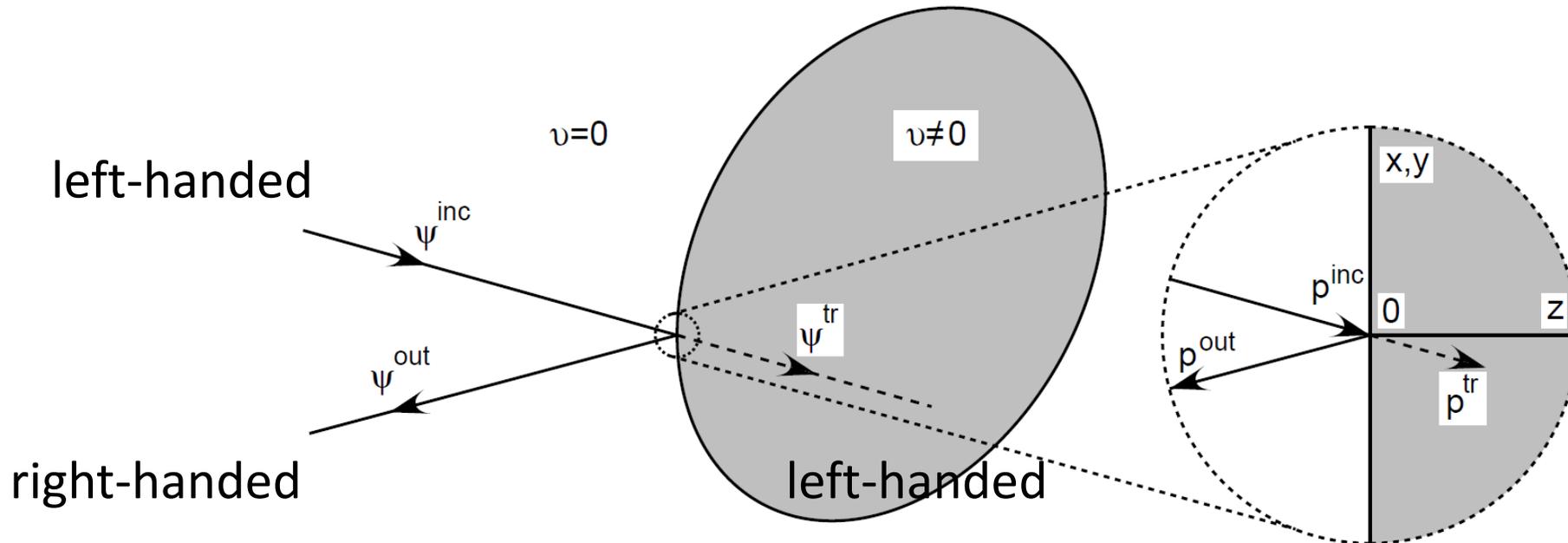massless particles

Quark scattering off bubble wall

- Quarks (and leptons) collide with bubble walls; some reflect and some transmit

- Left-handed and right-handed, quarks and antiquarks may have different reflection rates due to CPV

CPV & CPT invariance

- Need CP-odd phase, CP-even phase and interference of different channels

- CPV gives $R(d_L \to s_R) > R(\overline{d_R} \to \overline{s_L})$

- Fewer d quarks, $n(d_L) < n(\overline{d_R})$

- But CPV invariance

$$R(d_L \to s_R) = R(\overline{s_L} \to \overline{d_R})$$
$$R(\overline{d_R} \to \overline{s_L}) = R(s_R \to d_L)$$

- fewer anti-s quarks due to
$$R(\overline{s_L} \to \overline{d_R}) > R(s_R \to d_L)$$

- Compensation, no baryon asymmetry

wall

m=0

CP-odd phase

d

L

$\phi^{\pm}$

R

R

u,c,t

R

L

s

on-shell internal particles give CP-even phases different quarks different masses

⟹ different CP-even phases

like light reflected by medium can have phase different from incident one, depending on refractive indices
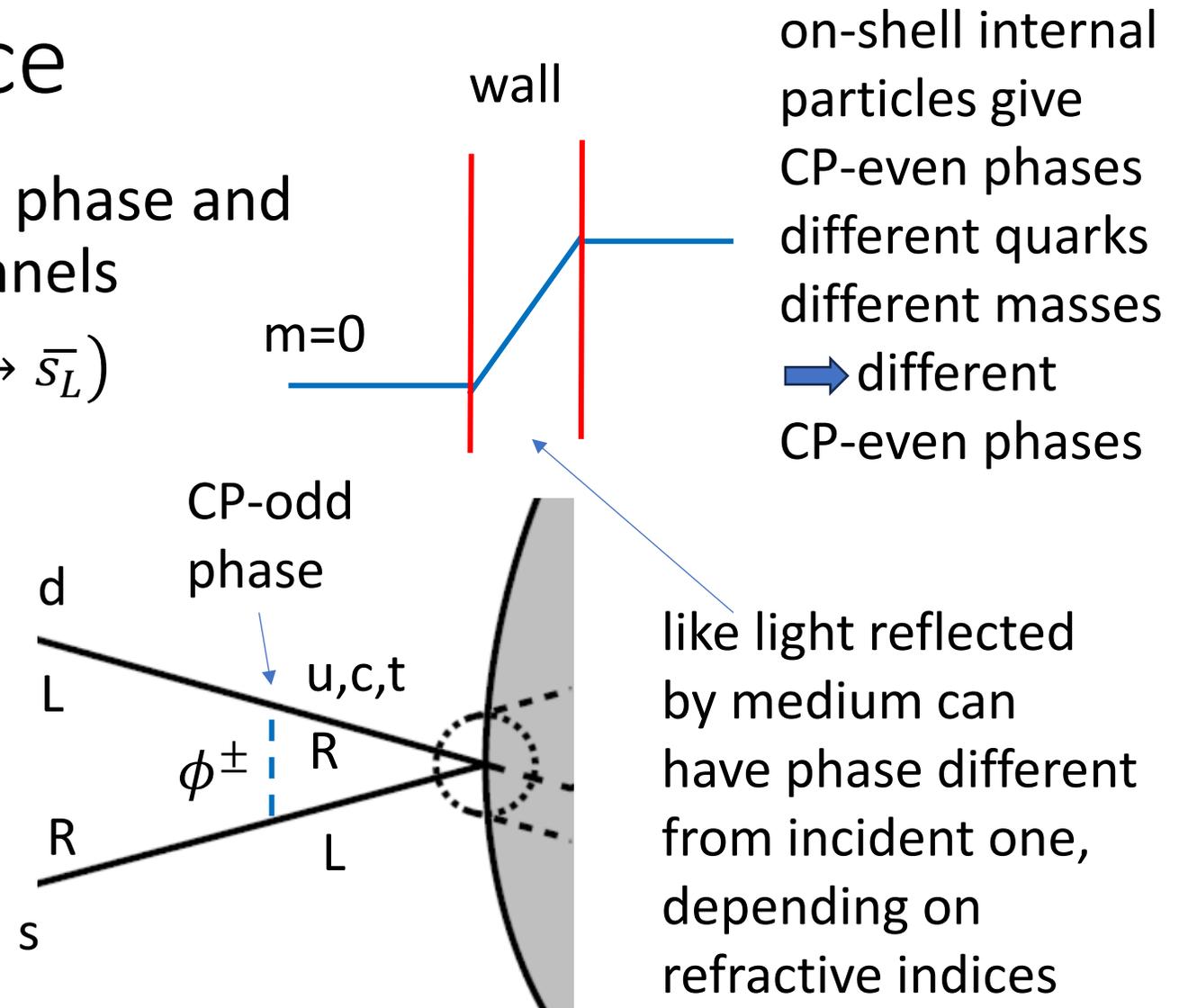
Weak sphaleron (due to axial anomaly)

- Now weak sphalerons (B violating interaction) begin to play

- Hopping over bump between vacua of different baryon numbers is allowed at high temperature

- Hopping, changing B number, occurs only for left-handed fermions

- Due to $n(d_L) < n(\overline{d_R})$
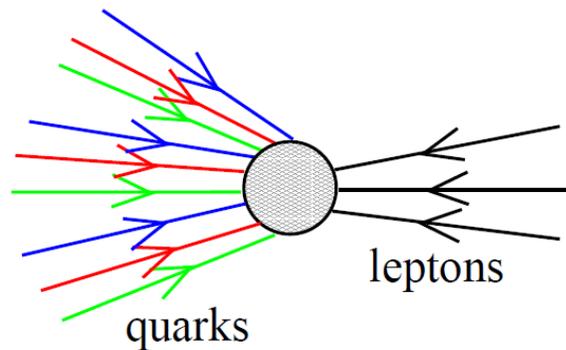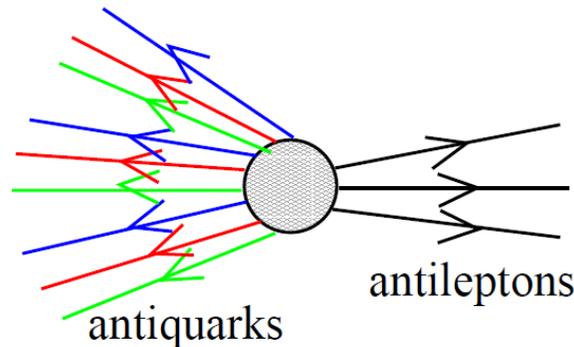
but recall

$$n(L) + n(R) = n(\bar{L}) + n(\bar{R})$$

chiral asymmetry

is slower than

B-L conserved

leptons

quarks

antileptons

antiquarks

- Left-handed quark asymmetry washed out, matter asymmetry created

- Lepton asymmetry can also contribute due to B-L conservation

Strong sphalerons (also due to axial anomaly)

- Chiral asymmetry (more R than L) created in front of bubbles

- Strong sphalerons with higher rates are faster than weak sphalerons

- Strong sphalerons, interacting with both chirality, wash out chiral asymmetry of quarks (not leptons)

- No chiral asymmetry to be converted into matter asymmetry

- Turn out that quark-sourced BAU is tiny (inefficient source)

- Lepton source plays more crucial role

- 4G lepton source explains BAU
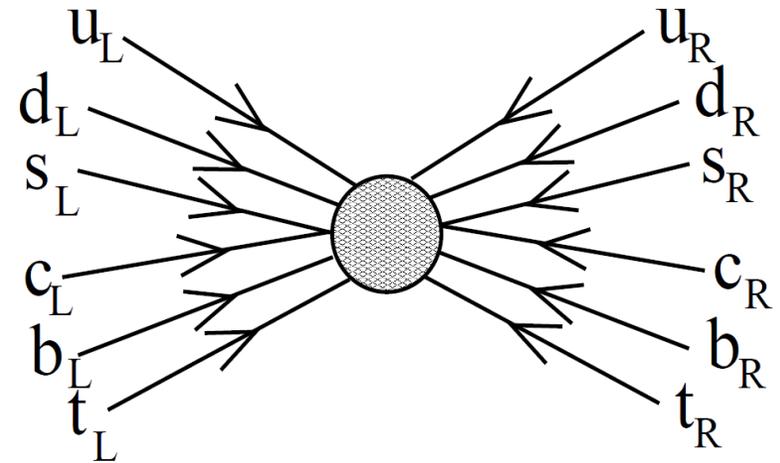
Baryon asymmetry

Effective operators with CPV source

- Instead of engaging cumbersome exercise on individual models, model-independent effective theory approach developed

- Dimention-6 operators

de Vires et al, 1811.11104

$$\mathcal{L}_6 = -i \left[\bar{Q}_L \tilde{Y}_U \tilde{H} u_R \ + \ \bar{Q}_L \tilde{Y}_D H \, d_R \ + \ \bar{L}_L \tilde{Y}_L H \, e_R \right] (H^\dagger H) + \text{h.c.}$$

- After EW symmetry breaking

proportional to Yukawa coupling of fermion f

discretionary sign, chosen to explain the observation

CPV source in collisions with bubbles

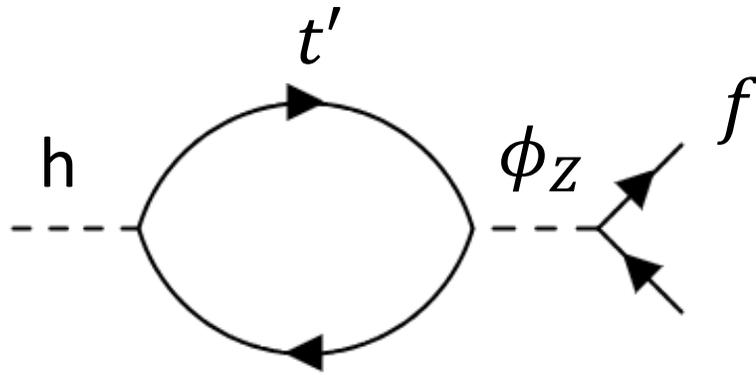$$- \frac{s_f m_f}{\Lambda_f^2} \, \bar{f} \, i\gamma^5 f \, vh$$

unknown new physics scale, bounded by EDM

CP-odd phase

- Strategy: translate SM4 to effective theory, derive Wilson coefficient, implement effective operators into formalism for EW baryogenesis

Matching to full theory

- Construct dim-6 operators from 4G heavy quarks, no free parameters
- One-loop diagram in R_ξ gauge, no CP-odd phase

when shrink heavy quarks to point,
get dim-6 operator with pseudoscalar current

- Three-loop diagram, add charged scalars, CP-odd phase shows up

$t' \to b' \to t \to b \to t'$

CKM matrix element

$\mathrm{Im}(V_{t'b'}^* V_{tb'} V_{tb}^* V_{t'b})$

Jarlskog invariant

this arrangement gives leading CPV source

pseudoscalar penguin

$1/q^2$ cancelled, 4-fermion operator
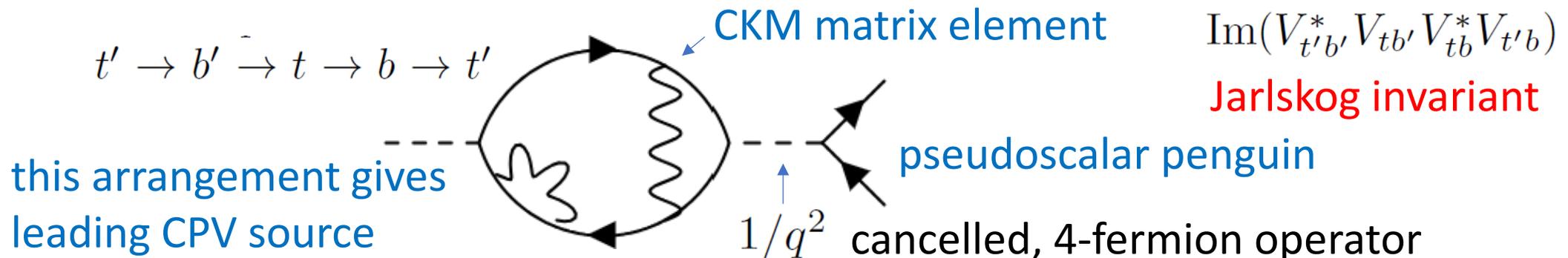
Transport equations

- All the above mechanism can be implemented into coupled transport equations for particle number densities (left-handed quarks, right-handed leptons, Higgs, ...), which look like

de Vires et al, 1811.11104

left-handed
t+b density   Relaxation rates   Yukawa rates   strong sphalerons   CPV sources
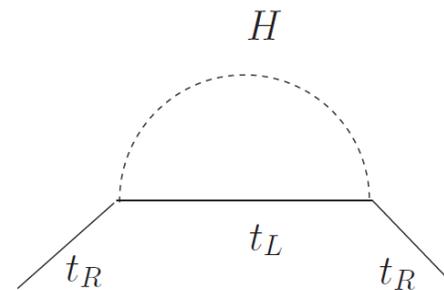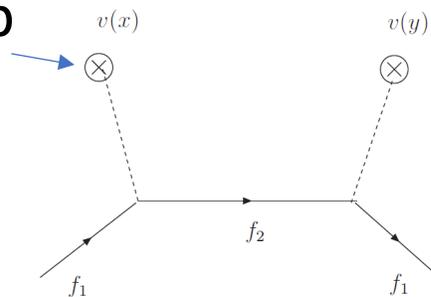
$$\partial_\mu q^\mu = +\Gamma_M^{(t)} \mu_M^{(t)} + \Gamma_M^{(b)} \mu_M^{(b)} + \Gamma_Y^{(t)} \mu_Y^{(t)} + \Gamma_Y^{(b)} \mu_Y^{(b)} - 2\Gamma_{\rm ss}\,\mu_{\rm ss} + \Gamma_{\rm QL}\,\mu_{\rm QL} - S_t - S_b$$

space-dep
vev

quark-lepton coupling

- Then implement weak sphalerons, solve another transport equation, get $\eta_B = (7.3\text{-}8.3) \times 10^{-11}$

Summary

- SM flavor structure governed by analyticity (bootstrap)
- Only three gauge couplings are fundamental, motivating 4G model
- EW symmetry broken dynamically by 4G quark condensates
- Heavy scalars formed by 4G quarks couple to gluons weakly
- Irrelevant dim-5 operators in IR region bypass Higgs data constraints
- Heavy scalars important for 1st-order EW phase transition
- 4G leptons sourced by 4X4 CKM (also fixed by dispersion relations) important for making BAU
- EW baryogenesis can be realized in SM4 (contrary to literatures)
- Likely to detect b' by end of LHC Run4 (sensitivity ~ 5σ), if 4G exists

thank XB Yuan, and LG Bian, ZG Si, JH Yu

Back-up slides

Basic features

- Wick rotation into Euclidean space

$$m(p^2) = -2g_Y^2 \int \frac{d^4q}{(2\pi)^4} \frac{1}{(p-q)^2 + \mu^2(q^2)} \frac{m(q^2)}{q^2 + m^2(q^2)} \qquad (1)$$

$$\mu^2(p^2) = -(2N_C + 2)\frac{2g_Y^2 m(p^2)}{p^2 + m^2(p^2)} \int \frac{d^4q}{(2\pi)^4} \frac{m(q^2)}{q^2 + m^2(q^2)}$$

$$t' \to t'\phi^0 \text{ or } t' \to b'\phi^+$$

in symmetric phase

all 4th generations contribute, same Yukawa coupling at fixed point

- Physical solution $m(p^2) \geq 0$ implies $\mu^2(p^2) \leq 0$

- If $|\mu^2(q^2)|$ is tiny due to small g_Y, RHS of (1) < 0, contradiction

- Critical coupling g_Y^c exists

- If $m(q^2)$ extends to infinity, can choose sufficiently large p^2 to overcome $\mu^2(q^2)$, RHS of (1) < 0, contradiction

- Solution of $m(q^2)$ looks like step function, UV cutoff exists automatically

Supplement

- Dimensionless version of DS equations

$$M(x) = -\frac{g_Y^2}{8\pi^2} \int_0^\infty y\,dy \frac{f(x,y)}{\sqrt{xy}} \frac{M(y)}{y + M^2(y)}$$

$$f(p^2, q^2) \equiv \frac{1}{\pi} \int_0^\pi \frac{\sin^2 \psi\, d\psi}{A - \cos \psi} \approx \begin{cases} A - \sqrt{A^2 - 1}, & A > 1 \\ A, & -1 < A < 1, \\ A + \sqrt{A^2 - 1}, & A < -1, \end{cases}$$

$$\tilde{\mu}^2 \equiv \frac{\mu^2(0)}{m^2(0)} = -\frac{g_Y^2}{\pi^2} \int_0^\infty dy \frac{M(y)y}{y + M^2(y)}$$

$$A = [p^2 + q^2 + \mu^2(q^2)]/(2pq)$$

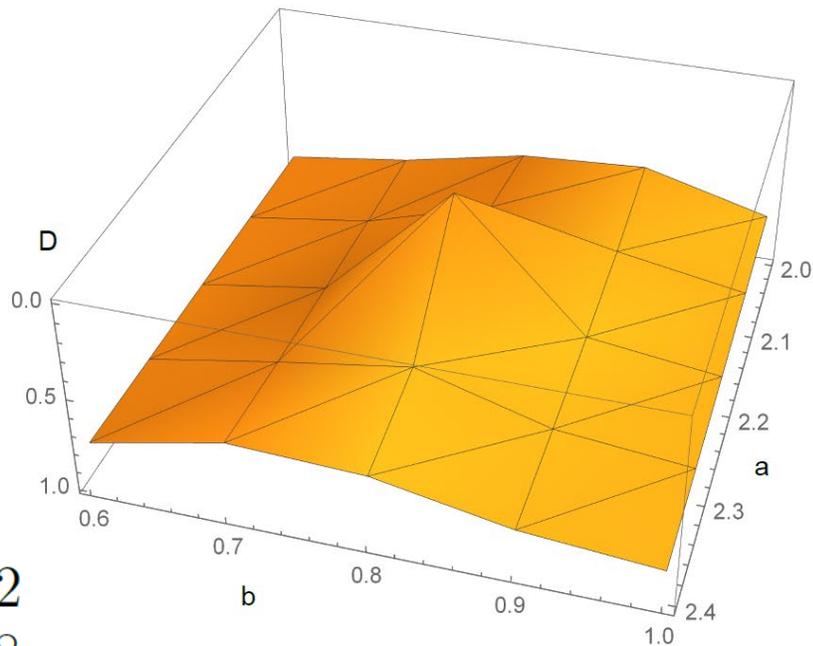- Definition of deviation $\epsilon = 10^{-5}$ $M(\epsilon) = 1$

$$D = \left| \frac{g_Y^2}{8\pi^2} \int_0^\beta y\,dy \frac{f(\epsilon, y)}{\sqrt{\epsilon y}} \frac{M(y)}{y + M^2(y)} + M(\epsilon) \right| + \left| \frac{g_Y^2}{8\pi^2} \int_0^\beta y\,dy \frac{f(0.9\beta, y)}{\sqrt{0.9\beta y}} \frac{M(y)}{y + M^2(y)} + M(0.9\beta) \right|$$

- Deviation mainly from high y region; smaller g_Y, smaller a

Solution

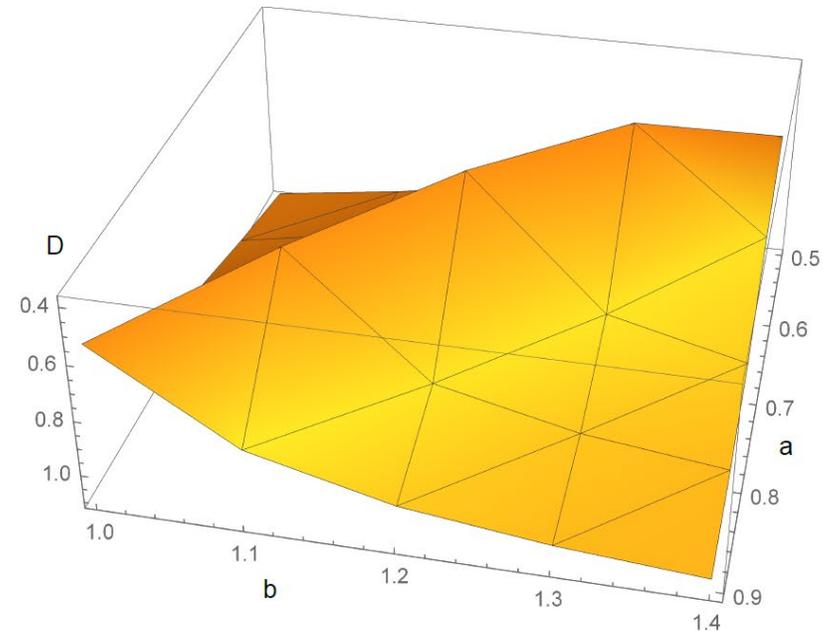- Dimensionless version in terms of $p^2 = m^2(0)x$, $q^2 = m^2(0)y$, $M(y) = m(y)/m(0)$

- Can show $M'(0) < 0$

- Propose parametrization $M(x) = \exp(-ax)\theta(b-x)$, $\quad a > 0$

- Best fit two sides of (1) by tuning parameters

$g_Y = 15$
deviation
$D \approx 0$

$a = 2.2$
$b = 0.8$

$g_Y = 10$
$D \approx 0.4$

$a = 0.70$
$b = 1.2$

Critical couplings

$$g_Y v/\sqrt{2} \approx 1.4 \text{ TeV}$$

- Critical coupling around fixed point $g_Y^c \approx 8.0$ larger than $g_Y^f \approx 7$

- 4$^{\text{th}}$ generations cannot form condensates

- Electroweak symmetric phase exists below compositeness scale

- Provide high-energy inputs for dispersion relations that constrain fermion masses and mixing angles

  Li, 2306.03463

- As energy lowered more, 4G quark Yukawa couplings increase (like strong coupling) and 4G lepton Yukawa couplings decrease

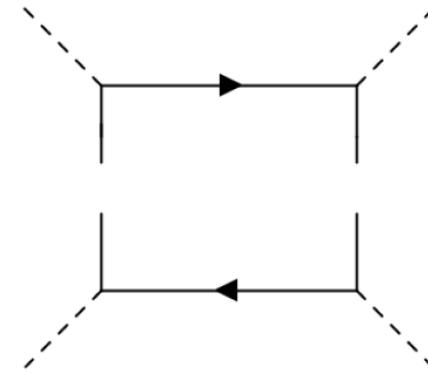| t' quark	b' quark, charged lepton L neutrino ν_4
200 TeV	2.7 TeV, 270 GeV, 170 GeV

- At critical coupling $g_Q^c \approx 9.1$, $\langle \bar{t}'t' + \bar{b}'b' \rangle$ formed, symmetry broken

Quartic coupling

- 4G quark condensates also induce Higgs quartic coupling

$$-i\lambda = -2\left(\frac{g_Q^c}{\sqrt{2}}\right)^4 \frac{4N_c}{m^2(0)} \int \frac{d^4q}{(2\pi)^4} \left[\frac{m(q^2)}{q^2 - m^2(q^2)}\right]^2$$

4 quark condensate

- Solution to DS equations leads to

$$\lambda = \frac{N_c g_Q^{c4}}{8\pi^2} \int_0^\infty y\,dy \left[\frac{M(y)}{y + M^2(y)}\right]^2 \approx 19$$

- Definition for VEV $v = \sqrt{-\mu^2/\lambda}$ is satisfied

Coleman-Weinberg potential

Coleman, Weinberg 1973

- One-loop Coleman-Weinberg potential

light- quarks, gauge bosons neglected

$$V_1(\phi, \mu_R) = \frac{1}{64\pi^2} \sum_{i=h,\eta,t,L,\nu_4} n_i m_i^4(\phi) \left[\ln \frac{m_i^2(\phi)}{\mu_R^2} - \frac{3}{2} \right] + \frac{1}{2} A(\mu_R)\phi^2$$

degeneracies per particle $\quad n_h = 1, \ n_\eta = 1, \ n_t = -12, \ n_L = -4 \text{ and } n_{\nu_4} = -4$

field-dependent masses $\quad m_h^2(\phi) = \mu^2 + a_h\phi^2, \ m_\eta^2(\phi) = \mu^2 + a_\eta\phi^2, \ m_t^2(\phi) = a_t\phi^2, \ m_L^2(\phi) = a_L\phi^2$

$$m_{\nu_4}^2(\phi) = a_{\nu_4}\phi^2$$

coefficients $\quad a_h = 3\lambda, \ a_\eta = \lambda', \ a_t = g_t^2/2, \ a_L = g_L^2/2 \text{ and } a_{\nu_4} = g_{\nu_4}^2/2$

- Renormalization condition on quadratic term enforces minimum of

$$V_{\text{eff}}(\phi, T, \mu_R) \text{ to be at } \phi = v$$

$$A(\mu_R) = -\frac{1}{16\pi^2} \sum_{i=h,\eta,t,L,\nu_4} n_i a_i m_i^2(v) \left[\ln \frac{m_i^2(v)}{\mu_R^2} - 1 \right]$$

Temperature-dependent potential

- One-loop T-dependent potential

Dolan, Jackiw 1974

$$V_T(\phi, T) = \frac{T^4}{2\pi^2} \left[\sum_{i=h,\eta} n_i J_B(m_i^2(\phi)/T^2) + \sum_{i=t,L,\nu_4} n_i J_F(m_i^2(\phi)/T^2) \right]$$

- Thermal function

$$J_{B,F}(x) = \int_0^\infty dy\, y^2 \ln\left[1 \mp \exp\left(-\sqrt{y^2+x}\right)\right]$$

- Derivation similar to one-loop potential, but with temperature-dependent propagator

$$D_\beta(k) = \frac{i}{k^2 - m^2}$$

~ 1/T

$$= \frac{-i}{(4\pi^2 n^2/\beta^2) + \vec{k}^2 + m^2}$$

1st-order EWPT

- The ratio $\phi_c/T_c \approx 0.9$ meets criterion $\phi_c/T_c \gtrsim 1$ roughly, implying strong 1st-order EWPT

- 4G lepton contribution increases slope in ϕ

- Switching it off leads to higher T_c, and weakens EWPT strength

$$\phi_c = 148.2 \text{ GeV and } T_c = 177.2 \text{ GeV, i.e., } \phi_c/T_c \approx 0.8$$

- Heavy scalar contribution crucial for building bump

- Switching it off $(\lambda' = 0)$ turns EWPT into 2nd order or crossover

- The above explain how 4G generations makes 1st-order EWPT

- Conclusion insensitive to λ', 20% variation causes 4% change of ratio

Quark scattering off bubble wall

- Quarks (and leptons) collide with bubble walls; some reflect and some transmit

- Left-handed and right-handed, quarks and antiquarks may have different reflection rates due to CPV
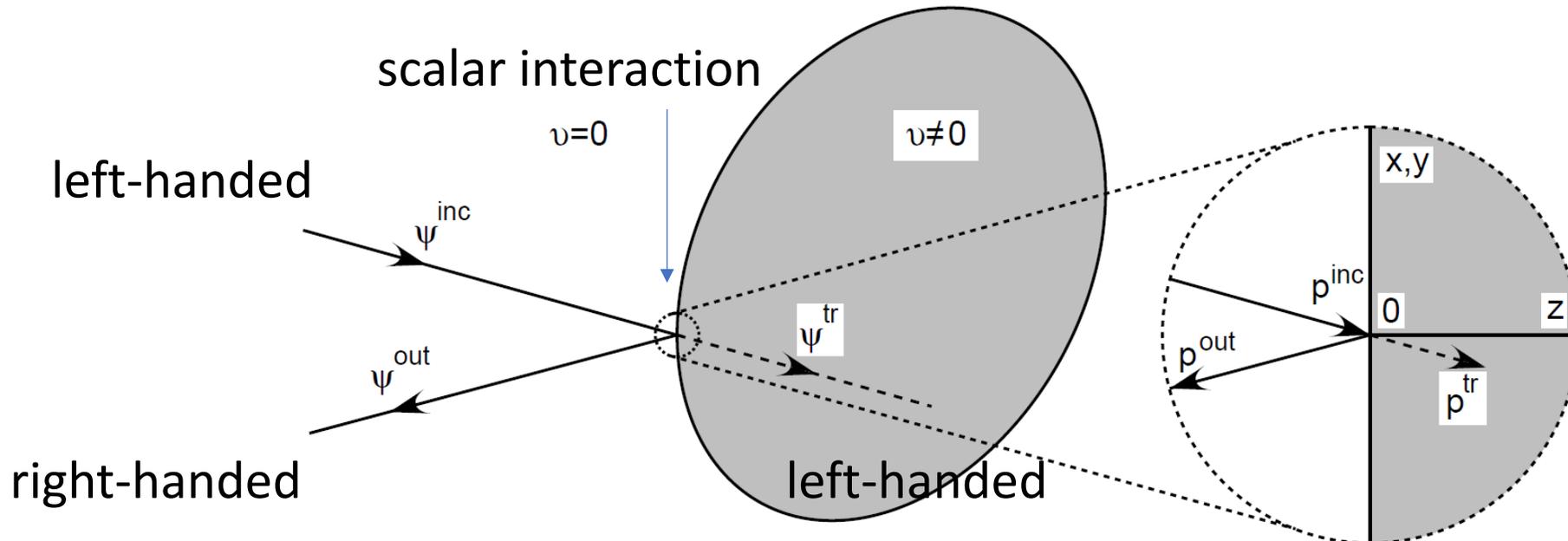
4X4 CKM matrix

- To compute CPV source, need to know 4X4 CKM matrix elements
- Consider dispersive constraints from $c\bar{u}\text{-}\bar{c}u, \ t\bar{u}\text{-}\bar{t}u \ \text{and} \ t\bar{c}\text{-}\bar{t}c$ mixings
- $t'\bar{u}\text{-}\bar{t'}u$ mixing not physical, $m_{t'} \approx 200 \ \text{TeV}$ above restoration scale
- Will not go to details, but quote result

$$\lambda_d \frac{m_{b'}^2 - m_d^2}{m_W^2 - m_d^2} + \lambda_s \frac{m_{b'}^2 - m_s^2}{m_W^2 - m_s^2} + \lambda_b \frac{m_{b'}^2 - m_b^2}{m_W^2 - m_b^2} \approx 0 \qquad \lambda_i = V_{Qi}^* V_{qi}, \ i = d, \ s \ \text{and} \ b.$$

- Given $m_s = 0.094 \ \text{GeV}$, $m_b = 4.18 \ \text{GeV} \ \text{and} \ m_W = 80.4 \ \text{GeV}$

  $\lambda \approx 0.225, \ A \approx 0.811, \ C \approx 0.403, \ \delta \approx 1.15 \quad C = \sqrt{\rho^2 + \eta^2}$ (Wolfenstein parameters)

$V_{ub'} = 2.54^{+0.02}_{-0.05} \times 10^{-4} \exp[-(1.15 \pm 0.03)i], \ \ V_{cb'} = 3.23^{-0.26}_{+0.43} \times 10^{-3} \exp[-(0.489^{-0.137}_{+0.122})i]$

$V_{tb'} = 5.20^{+0.34}_{-0.45} \times 10^{-2}.$

$\qquad\qquad\qquad\qquad J = -(3.30^{-0.89}_{+0.85}) \times 10^{-6}$

Sign of CPV source

- Matching dim-6 operator to full theory

Hou 2008: CPV enhanced by 4G quark masses

3rd component of isospin of f

$$\frac{s_f g_f v^2}{\sqrt{2}\Lambda^2} = \frac{I_{3b'} I_{3f} g_f g_{t'}^4 g_t^2}{16^3 \pi^4 \sqrt{2}} \left[1 + Li_2 \left(-\frac{\Lambda_s^2}{m_{t'}^2} \right) \right] J \bar{f} \gamma_5 f$$

-1/2

restoration scale from upper bound of loop momentum

- To ensure positive Λ^2 , must choose $s_f = 2 I_{3f}$

- Sign is not discretionary in SM4, but related to isospin component

- It was chosen to fit observed BAU in literature

New physics scale

- Matched at the scale with $m_{t'} = \Lambda_s$, restoration scale (t' mass runs with scale)

- if $m_{t'} > \Lambda_s$, internal particles in pseudoscalar penguin become massless, not full theory;

- if $m_{t'} < \Lambda_s$, b' form bound states, incorrect full theory

- New physics scale fixed unambiguously, flavor independent

$$\Lambda = \frac{64\pi^2 v^3}{g_t \sqrt{-cJ}\Lambda_s^2} \qquad c = 1 - \pi^2/12$$

- It was constrained by data of electron EDM in literature, only upper bounds were obtained and depend on flavors

EW symmetry restoration scale

- Determination of restoration scale requires RG of Yukawa couplings

- Separate evolution variable $x \equiv \ln(\mu/m_Z)$ into two regions

- Region 1: $0 \leq x < x_s$, 4G quarks form bound states, weak coupling with Higgs, not contribute to RG

- Region 2: $x > x_s$ up to UV fixed point, 4G quarks are physical degrees of freedom, contribute to RG

- Tune x_s , such that known top Yukawa couplings from low end and from fixed point matches smoothly

$$\Lambda_s = m_Z \exp(z_s) \approx 22 \text{ TeV}$$

critical coupling
for forming
condensates

g_L^2

g_t^2

x_s

BAU
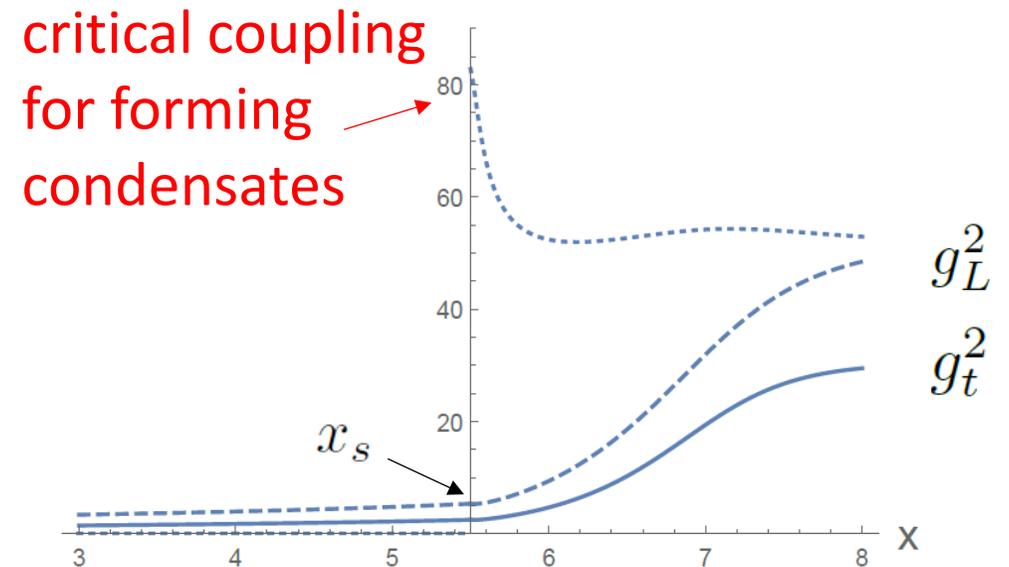
- New physics scale $\Lambda \approx 16 \text{ TeV}$, much higher than $\Lambda_\tau = 1 \text{ TeV}$

- $s_t = +1$ and $s_\tau = -1$, were chosen to explain observation in literature $\quad \eta_B = (7.3\text{-}8.3) \times 10^{-11}$ de Vires et al, 1811.11104

- top-sourced operator unimportant due to strong sphalerons

- τ-sourced BAU overestimated; its role replaced by 4G leptons

- Similar transport equations; BAU proportional to CPV source $\quad \bar{S} \propto \dfrac{y_f}{\Lambda^2}$,

- Both charged and neutral 4G leptons contribute via weak sphalerons

- Our estimate, based on $\dfrac{(g_{\tau'} + g_{\nu'})/\Lambda^2}{g_\tau/\Lambda_\tau^2} \approx 1$ and same benchmark parameters, $\eta_B \approx 10^{-10}$

- Observed BAU can be accommodated in SM4 with reasonable inputs

Axial anomaly

- Axial current not conserved

$$\partial_\mu j_5^\mu = \frac{N_f g^2}{8\pi^2} Tr(G^{\mu\nu}\widetilde{G_{\mu\nu}})$$

$$j_5^\mu = \bar{\psi}\gamma^\mu\gamma^5\psi$$

quantum effect $\longrightarrow$

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f_{bc}^a A_\mu^b A_\nu^c .$$

$$\tilde{G}_{\mu\nu}^a = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta a},$$

apply to both weak and strong gauge fields
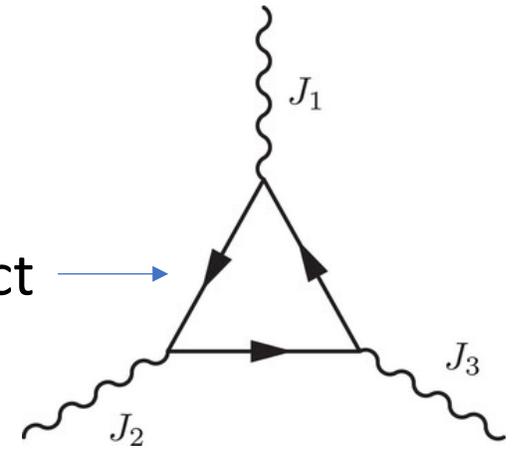
- Left-handed current is not conserved

$$j_L^\mu = \overline{\psi_L}\gamma^\mu\psi_L \qquad \psi_L = P_L\psi \qquad P_L = (1-\gamma^5)/2$$

Number density of left-handed particles

- Change of particle number --- Gauss law

$$\int d^4x\, \partial_\mu j_L^\mu = \int d\sigma_\mu j_L^\mu = \frac{N_f g^2}{8\pi^2}\int d^4x\, Tr(G^{\mu\nu}\widetilde{G_{\mu\nu}}) \neq 0$$

Baryon number

- Each set of gauge configuration has minimal energy---vacuum of nonabelian gauge theory characterized by baryon number B

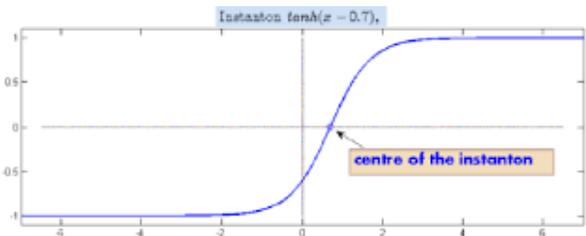$$\int d^4x \, \partial_\mu j_L^\mu = \int d\sigma_\mu j_L^\mu = \frac{N_f g^2}{8\pi^2} \int d^4x \, Tr(G^{\mu\nu}\widetilde{G_{\mu\nu}}) = B$$
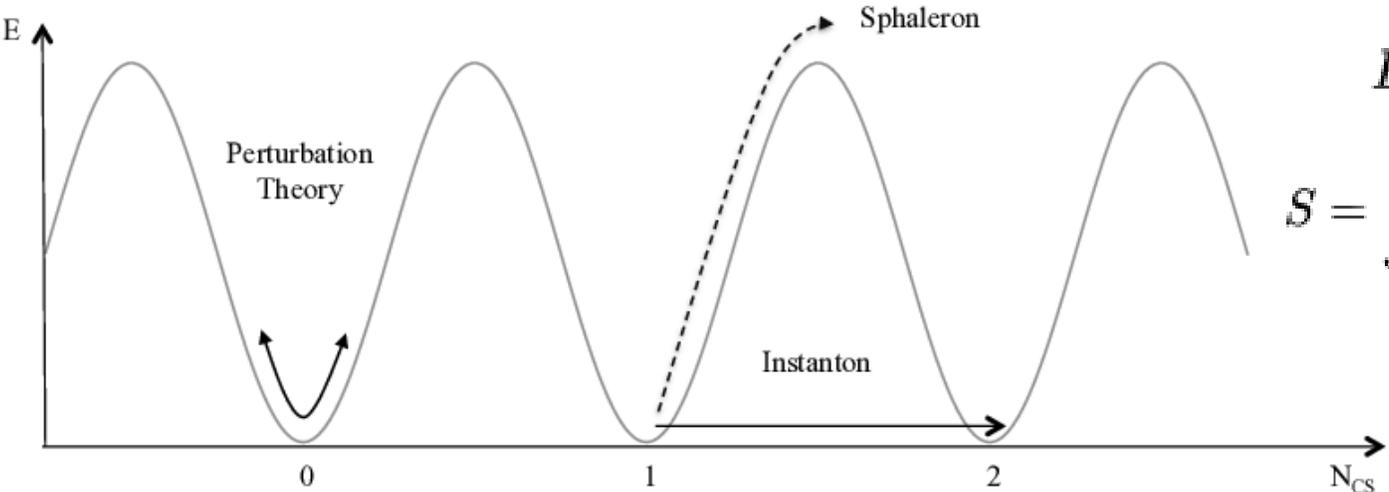
$$\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} = F_{\mu\nu}F^{\mu\nu}$$

$$S = \int dx^4 \frac{1}{4}F^2 = \int dx^4 \frac{1}{8}(F \pm \tilde{F})^2 \mp \int dx^4 \frac{1}{4}F\tilde{F}$$

minimum occurs for self-dual or anti-self-dual configurations

B violation breaks CPT

Moving bubble walls

extremely complicated even package "WallGo" developed, 2510.27691

- Non-equilibrium (1st order PT) begins to play
- Walls need to move, so that bubbles eat baryon asymmetry
- But wall velocity cannot be too big. Otherwise, particles in symmetric phase eaten, before weak sphalerons work
- 0 < v(wall)/c < 1; calculation prefers v/c ~ O(0.1)
- Temperature continues to drop inside bubbles, hopping difficult and weak and strong sphalerons stop working, BAU kept in our world
- All involved rates, collision, weak sphaleron, bubble expansion, need to cooperate to produce observed BAU
- If 2nd order or crossover, whole space change uniformly; no bubbles, no boundary, no moving walls