A gauge horizontal model of charged fermion masses

We-Fu Chang 張維甫

National Tsing Hua Univ.
(Taiwan)

10th X-strait
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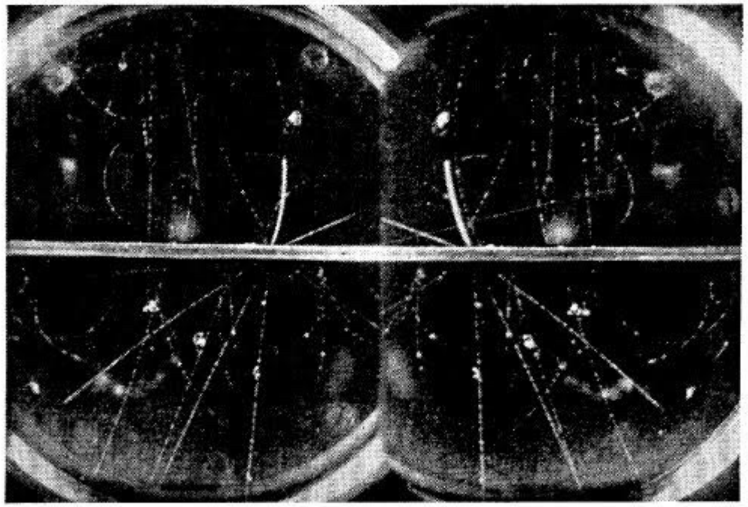
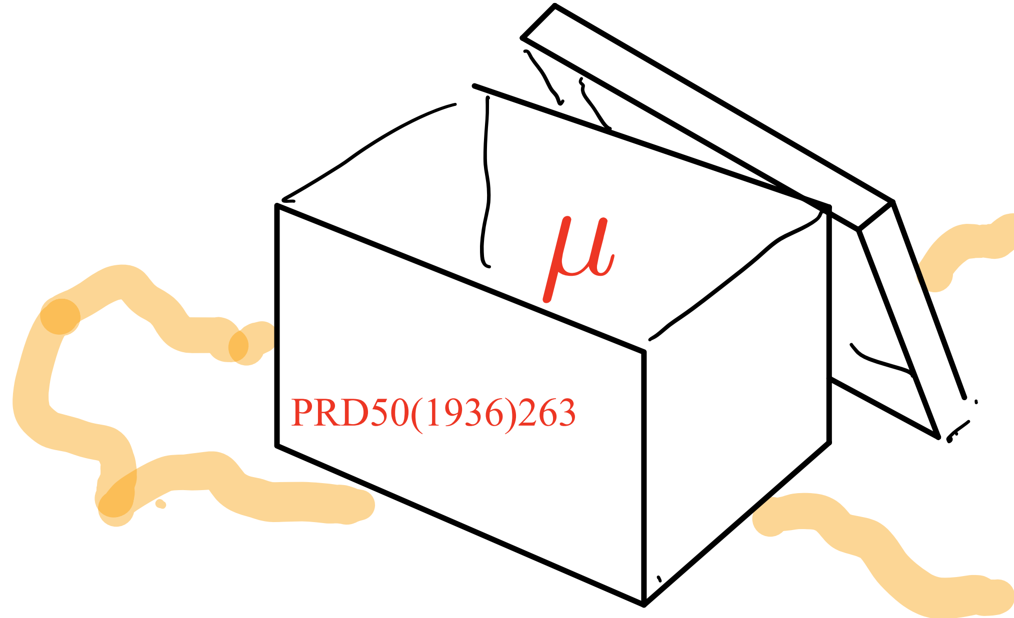
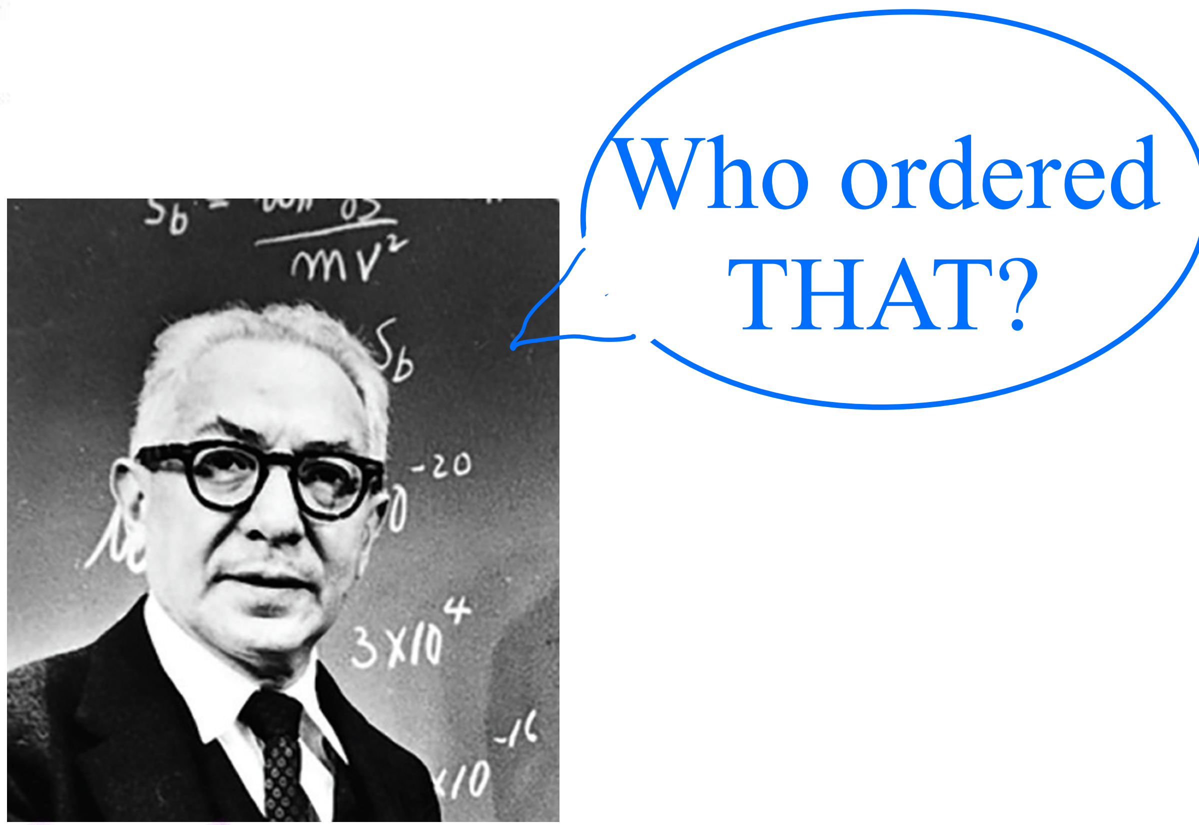
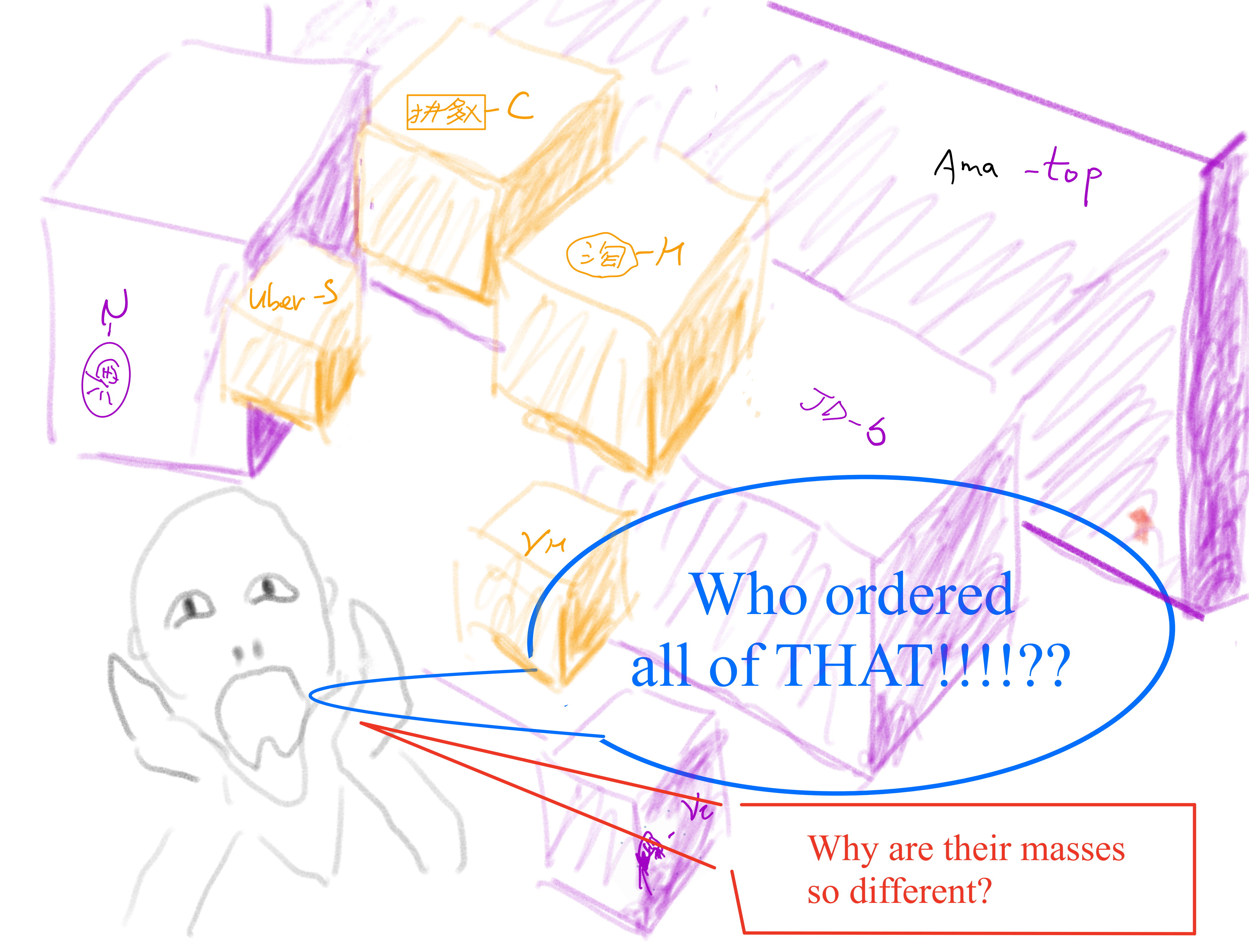


FIG. 12. Pike's Peak, 7900 gauss. A disintegration produced by a nonionizing ray occurs at a point in the 0.35 cm lead plate, from which six particles are ejected. One of the particles (strongly ionizing) ejected nearly vertically upward has the range of a 1.5 MEV proton. Its energy (given by its range) corresponds to an $H\rho = 1.7 \times 10^5$, or a radius of 20 cm, which is three times the observed value. If the observed curvature were produced entirely by magnetic deflection it would be necessary to conclude that this track represents a massive particle with an e/m much greater than that of a proton or any other known nucleus. As there are no experimental data available on the multiple scattering of low energy protons in argon it is difficult to estimate to what extent scattering may have modified the curvature in this case. The particle is therefore tentatively interpreted as a proton. The other particle ejected upward to the right may be either an electron or a fast proton. The four particles ejected downward are positively charged and do not ionize sufficiently strongly to represent protons of the curvatures shown. If they are positrons their energies are respectively 105, 250, ~500 and 60 MEV.







Ama -top
淘 -M
Uber -S
JD - b
ν_μ
Who ordered all of THAT!!!!??
ν_e
Why are their masses so different?

[11] In obtaining the expression (11) the mass difference between the charged and neutral has been ignored.

[12] M. Ademollo and R. Gatto, Nuovo Cimento 44A, 282 (1966); see also J. Pasupathy and R. E. Marshak, Phys. Rev. Letters 17, 888 (1966).

[13] The predicted ratio [eq. (12)] from the current algebra is slightly larger than that (0.23%) obtained from the ρ-dominance model of Ref. 2. This seems to be true also in the other case of the ratio $\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\gamma\gamma)$ calculated in Refs. 12 and 14.

[14] L. M. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962).

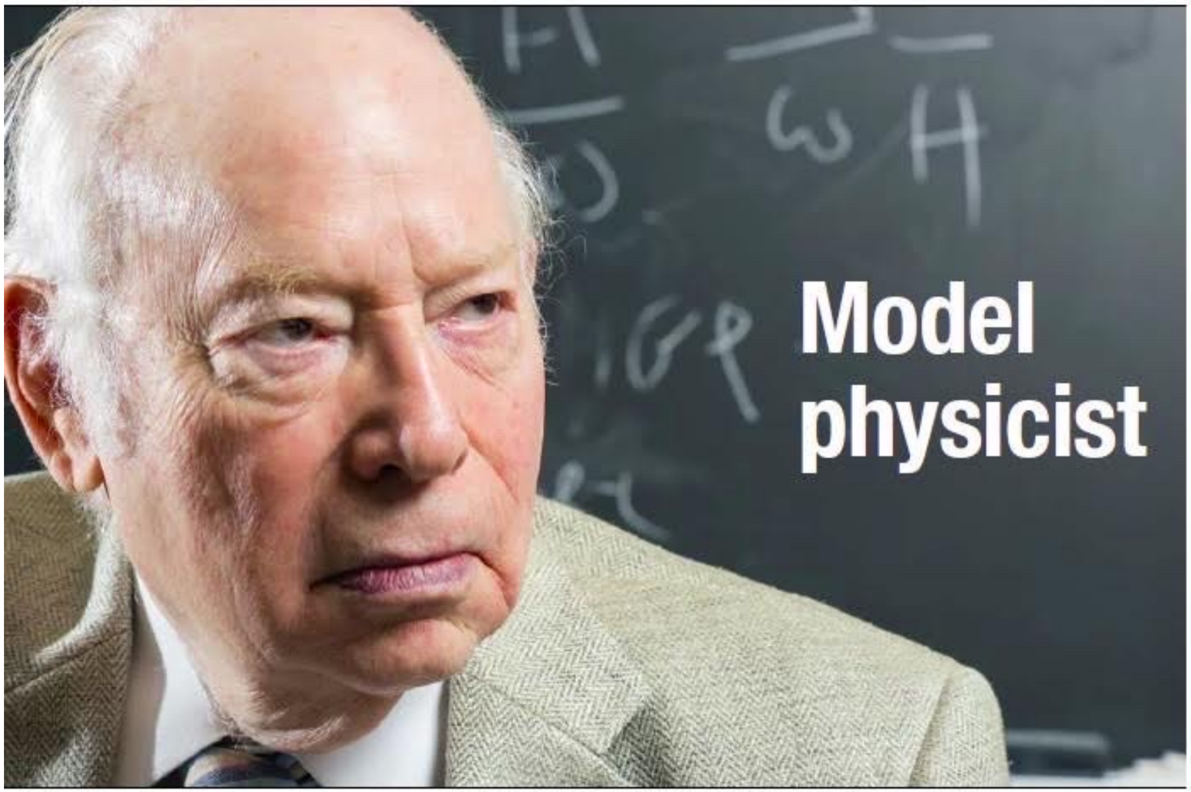
A MODEL OF LEPTONS*

Steven Weinberg†
Laboratory for Nuclear Science and Physics Department, Massachusetts Institute of Technology, Cambridge, Massachusetts
(Received 17 October 1967)

CERN Courier November 2017

Interview: Steven Weinberg

2017



1933 – 2021

Asked what single mystery, if he could choose, he would like to see solved in his lifetime, Weinberg doesn't have to think for long: he wants to be able to explain the observed pattern of quark and lepton masses. In the summer of 1972, when the SM was coming together, he set himself the task of figuring it out but couldn't come up with anything. "It was the worst summer of my life! I mean, obviously there are broader questions such as: why is there something rather than nothing? But if you ask for a very specific question, that's the one. And I'm no closer now to answering it than I was in the summer of 1972," he says, still audibly irritated.

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Models of lepton and quark masses

Steven Weinberg*

Theory Group, Department of Physics, University of Texas, Austin, Texas 78712, USA

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A class of models is considered in which the masses only of the third generation of quarks and leptons arise in the tree approximation, while masses for the second and first generations are produced respectively by one-loop and two-loop radiative corrections. So far, for various reasons, these models are not realistic.

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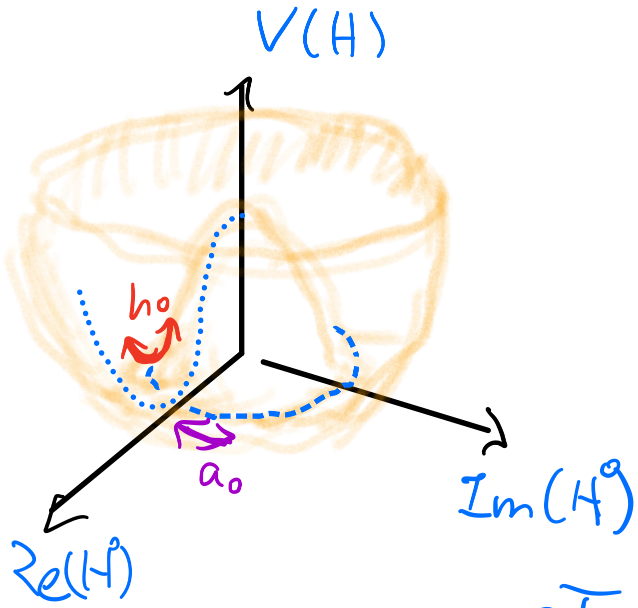
Yukawa coupling

- $\mathcal{L} \supset -y_{ij}\, \bar{\psi}_{L_i} \psi_{R_j} H^{(2)}$



$H \to \begin{pmatrix} h^{\pm} \\ \frac{v_0 + h_0 + i a_0}{\sqrt{2}} \end{pmatrix}$ $\quad \langle H \rangle = \frac{v_0}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$v_0 \simeq 246\ \text{GeV}$

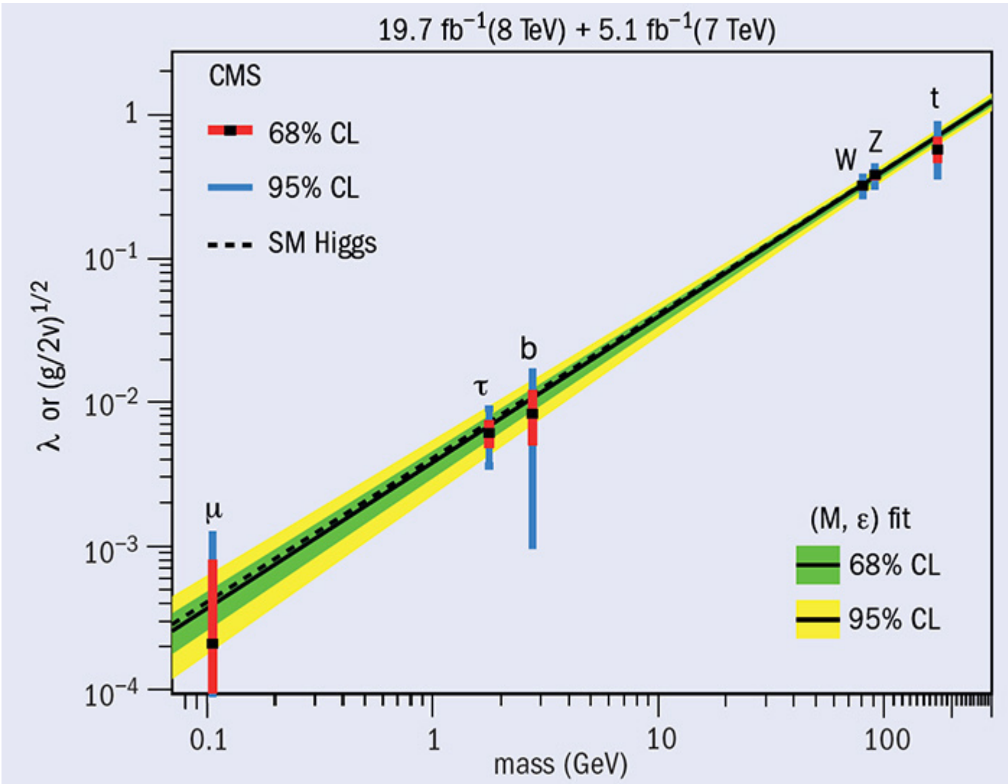
$h^{\pm}, a_0 \to W^{\pm}, Z^0$
longitudinal component

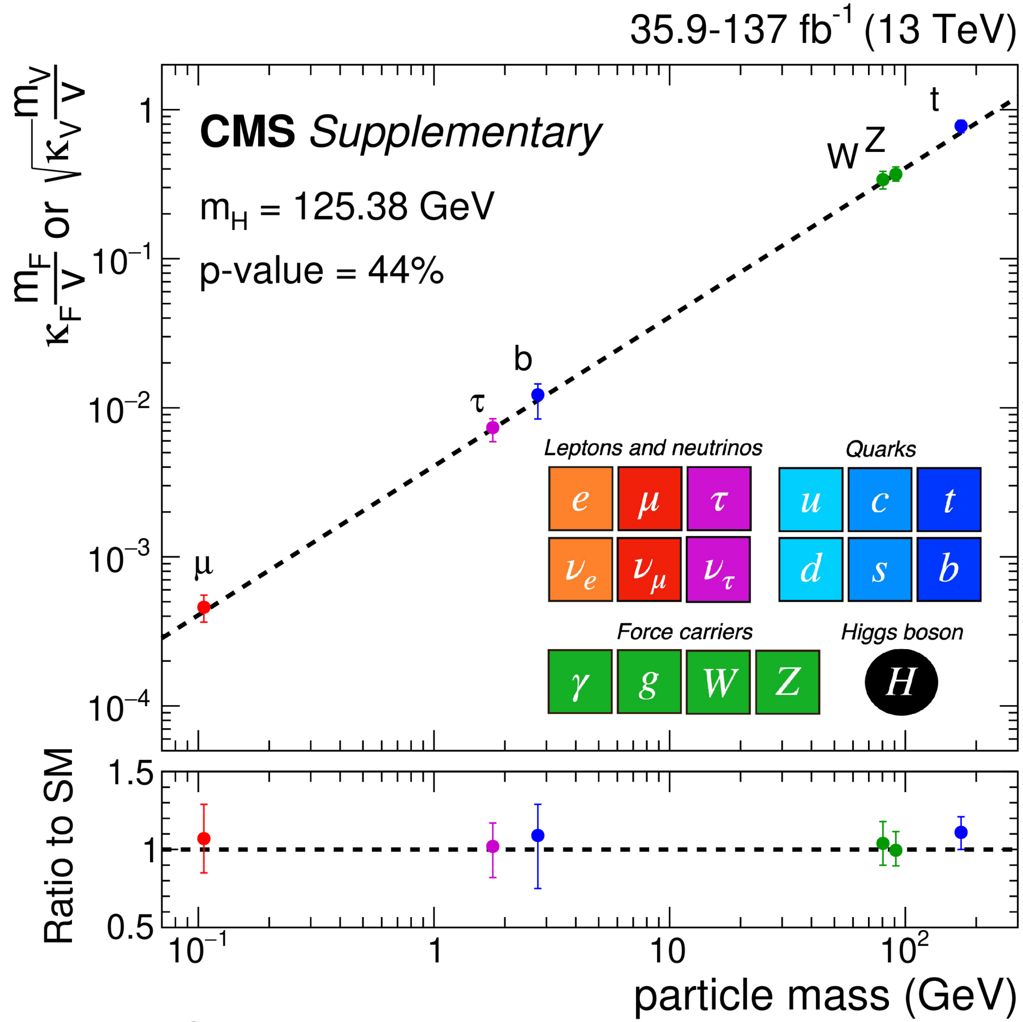
$\Rightarrow \bar{\psi}_{L_i} M_{ij} \psi_{R_j}$ and $M_{ij} \simeq \frac{y_{ij}}{\sqrt{2}} v_0$,

- $U_L M U_R^{\dagger} = M_{diag}$

- $\bar{\psi}_f \psi_f H \to y_f \frac{v_0 + h_0}{\sqrt{2}} \bar{\psi}_f \psi_f$ $\quad$ Yukawa $\simeq \frac{m_f}{v_0}$

$y_f = \frac{m_f}{V_0}$





- CMS RUN-1 / future
- ATLAS has similar results and projection
- Agree with the SM prediction ~few10%

- Robert Wadlow

  1918 – 1940

  272 cm 199kg

-

$$\text{mass} \sim \rho \cdot L^3$$

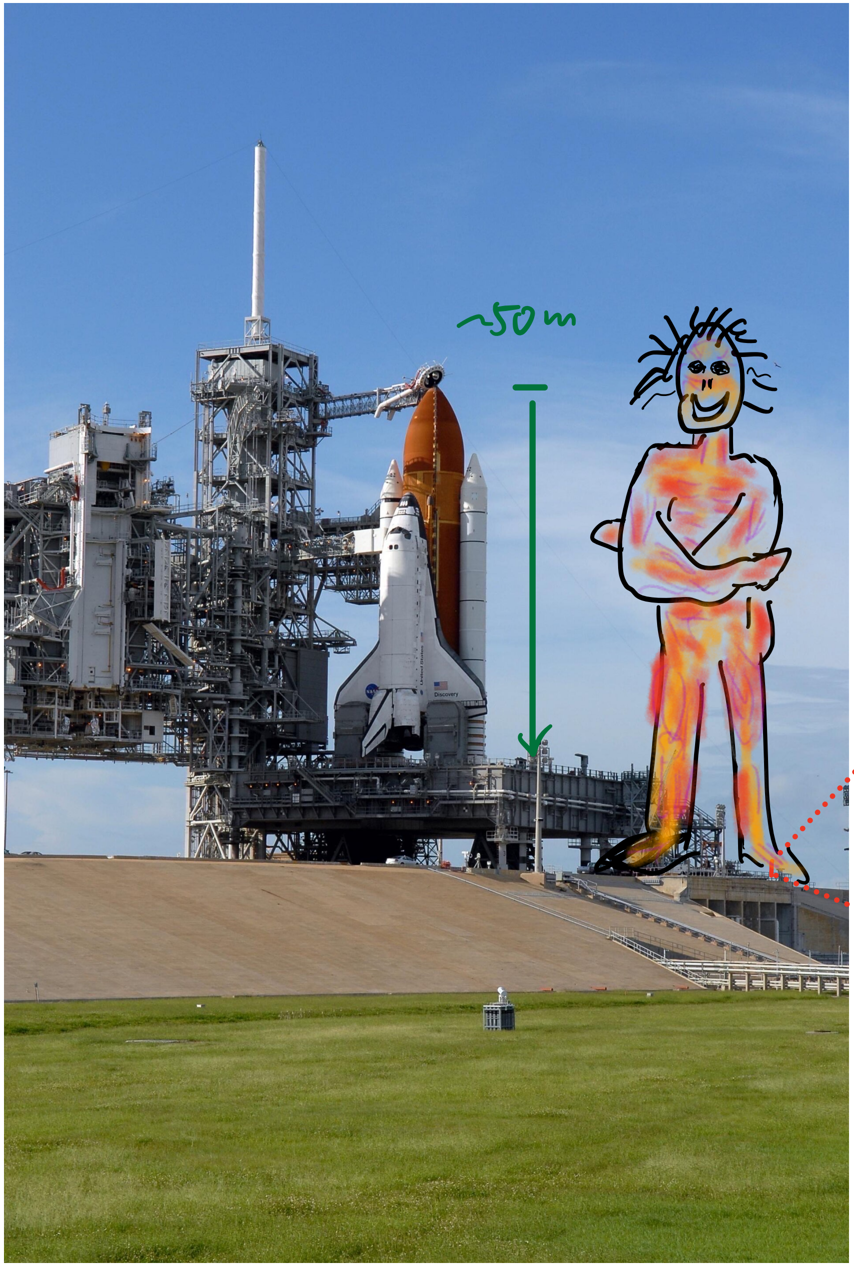
$$\text{heigh} \propto (\text{mass})^{\frac{1}{3}}$$

$$\left(\frac{200\text{kg}}{50\text{kg}}\right)^{\frac{1}{3}} \sim 1.58$$

$$\frac{272\text{ cm}}{1.58} \sim 170 \text{ cm}$$

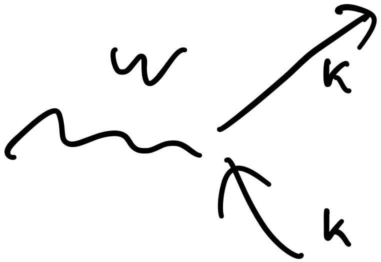
- $\left(\frac{m_t}{m_u}\right)^{\frac{1}{3}} \sim 10^{\frac{5}{3}} = 46.4$ $\qquad 1.7\,m \times 46.4 \sim 80\ m$

$(10^5)^{\frac{1}{3}}$




Original Godzilla
Argentinosaurus
Brachiosaurus
Diplodocus
King Kong
Trex
Jurassic world raptor
進撃の巨人

$\boxed{\text{CKM}}$



$$\mathcal{L} \supset \sum_{L=1}^{3} \bar{\Psi}_{L,k}\, \gamma^{\mu} \frac{\sigma^{\alpha}}{2} \Psi_{L,k} W^{\alpha}_{\mu}$$

- in the weak basis

$$\Rightarrow \bar{\Psi}_{L i}\, U_{ik}\, \gamma^{\mu} \frac{\sigma^{\alpha}}{2} W^{\alpha}_{\mu}\, U^{\dagger}_{kj}\, \Psi_{L j}$$

- C.C.int. in the mass basis

$$\Rightarrow \quad V_{CKM} = U^{u}_{L}\, U^{d\,\dagger}_{L}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \cdot \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \cdot \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

- $|V_{CKM}| \sim \begin{pmatrix} 0.974 & 0.225 & 0.004 \\ 0.224 & 0.974 & 0.042 \\ 0.009 & 0.041 & 0.999 \end{pmatrix}$

$|V_{cd}| \sim |V_{us}|$

$|V_{ub}| \sim |V_{td}|$

$|V_{cb}| \sim |V_{ts}|$

$\theta_{13} = 0.20^\circ \ll \theta_{23} = 2.38^\circ \ll \theta_{12} \doteq 13.04^\circ \ll 1 \quad \delta \simeq 1.2$

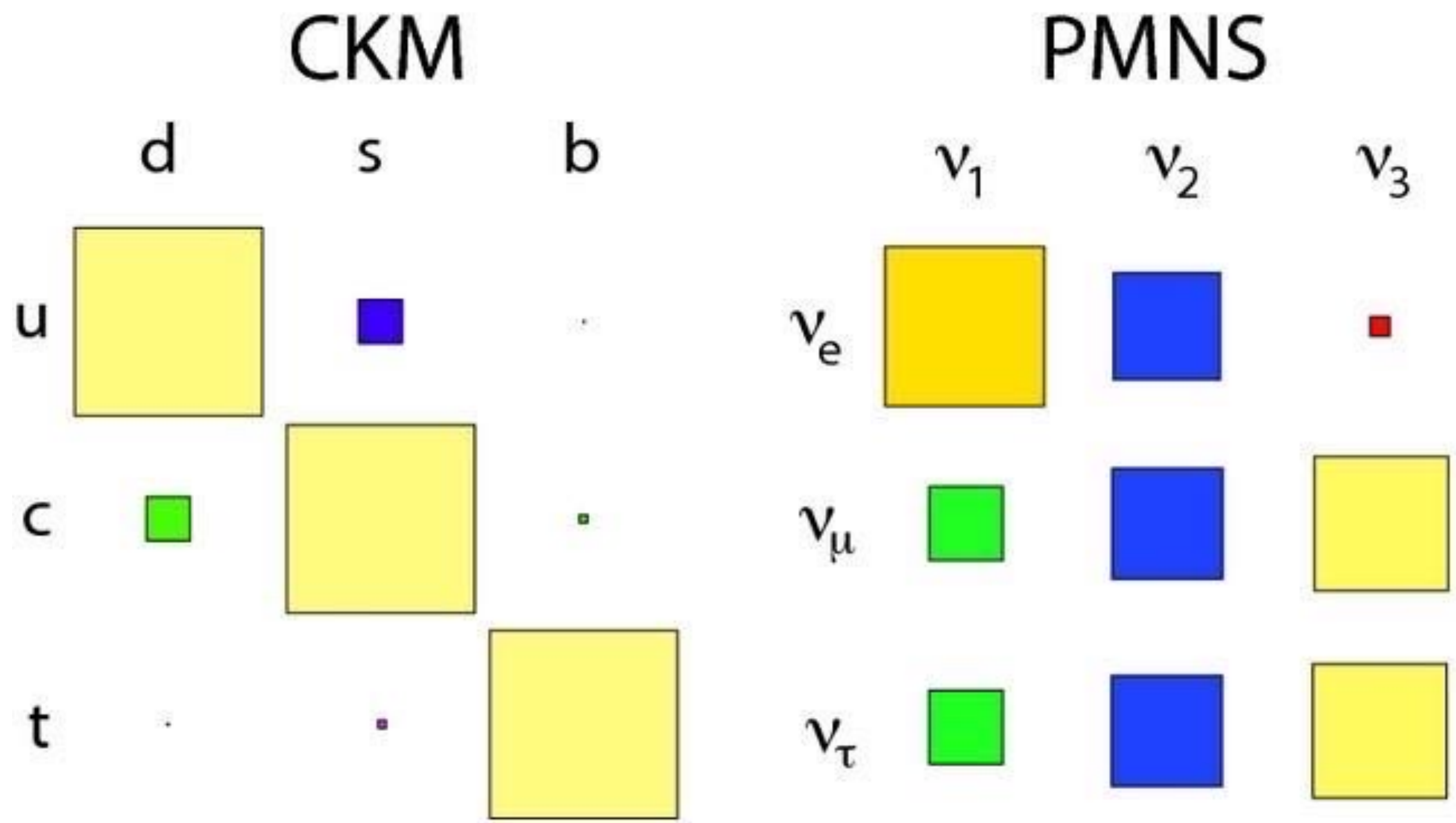
$$\boxed{V_{CKM} \sim \mathbb{1}_{3\times 3}}$$

- Jarlskog invariance

$$J = c_{12}\, c_{13}^2\, c_{23}\, s_{12}\, s_{13}\, s_{23} \sin\delta \doteq 3.2 \times 10^{-5}$$

- Small weak $\cancel{CP}$

(can't explain the mater-antimatter asymmetry)



$J_Q \sim 3.2 \times 10^{-5}$

Small mixing

$\theta_{12} \sim 13^\circ$

$\theta_{23} \sim 2.4^\circ$ $\delta = 1.2$

$\theta_{13} \sim 0.2^\circ$

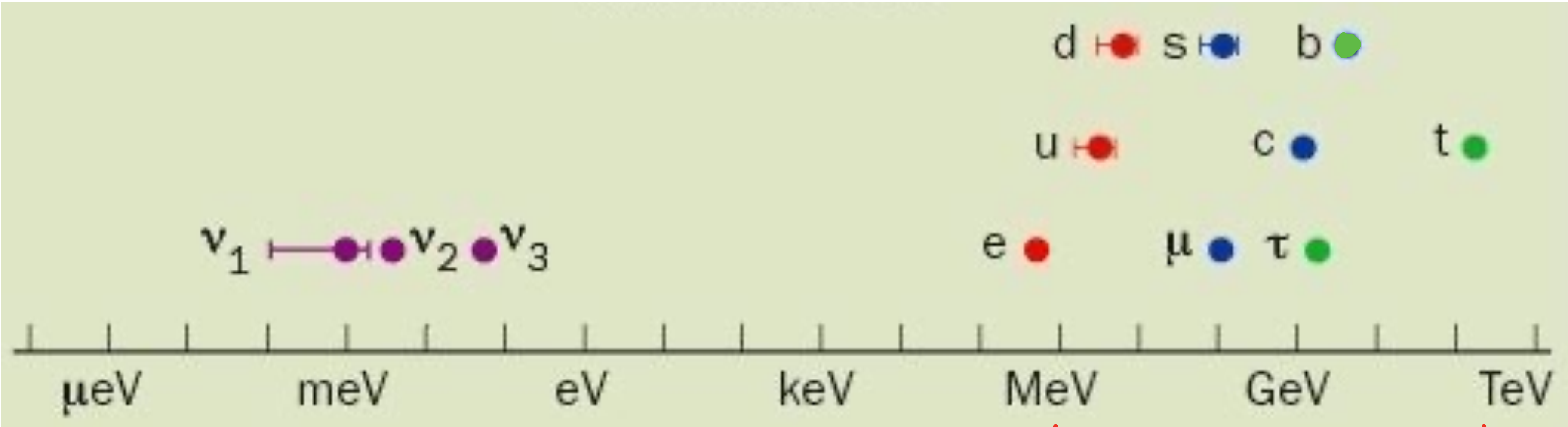
$J_L \sim 2.8 \times 10^{-2}$

bi-large

$\theta_{12} \sim 33^\circ$

$\theta_{23} \sim 49^\circ$ $\delta \sim O(1)$

$\theta_{13} \sim 8.6^\circ$

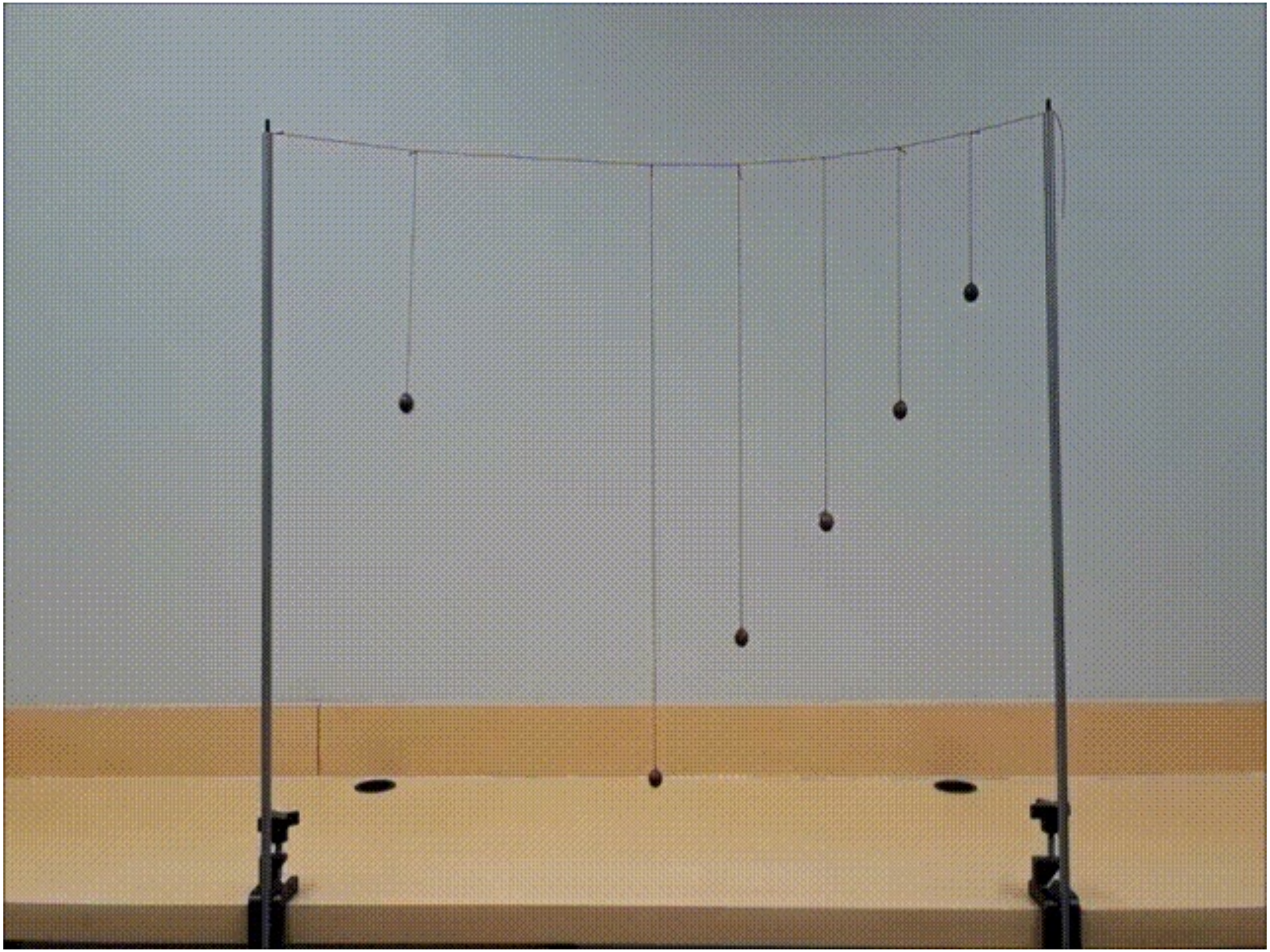


- Large mixings
- May or may not couple to SM Higgs

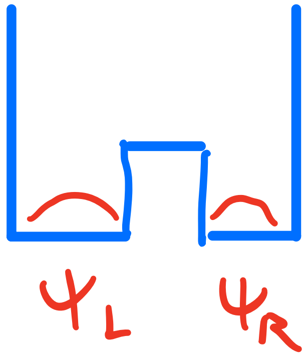
- Small mixings
- ~ 5 O.M. Diff
- mass $\longleftrightarrow$ Higgs Yukawa

  agree with SM
  $\sim O(30\%)$

- mass generation of m_ν & m_f are different
- No attempt # of Generations
- focus on the charged fermion sector



- Same quantum # $\to$ large mixings



$$\longrightarrow \quad \psi_\pm = \frac{1}{\sqrt{2}}(\psi_L \pm \psi_R)$$

- Mass hierarchy and CKM $\sim \mathbb{1}$ (small mixings)

  $\Rightarrow$ diff. species!

  $\Rightarrow$ diff. Q# in QFT

- flavor problem
  ~ origin of the pattern of Yukawa

- Other flavor limits to keep in mind:

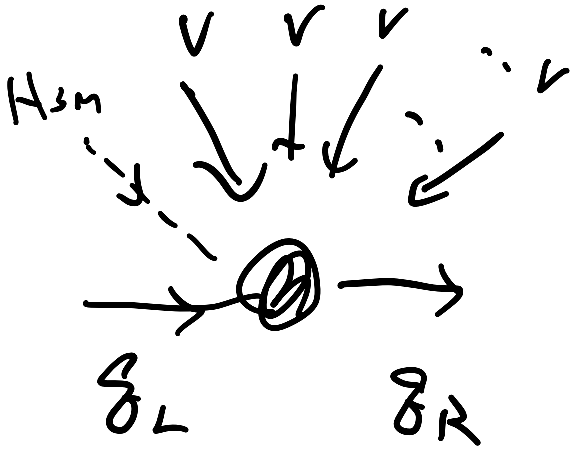
$Br(\mu \to e\gamma) < 4.2\times10^{-13}$ (MEG) SM (-54)

$Br(\mu \to 3e) < 1.0\times10^{-12}$ (SINDRUM)

$|d_e| < 1.1\times10^{-29}$ e-cm (ACME-II), (-38)

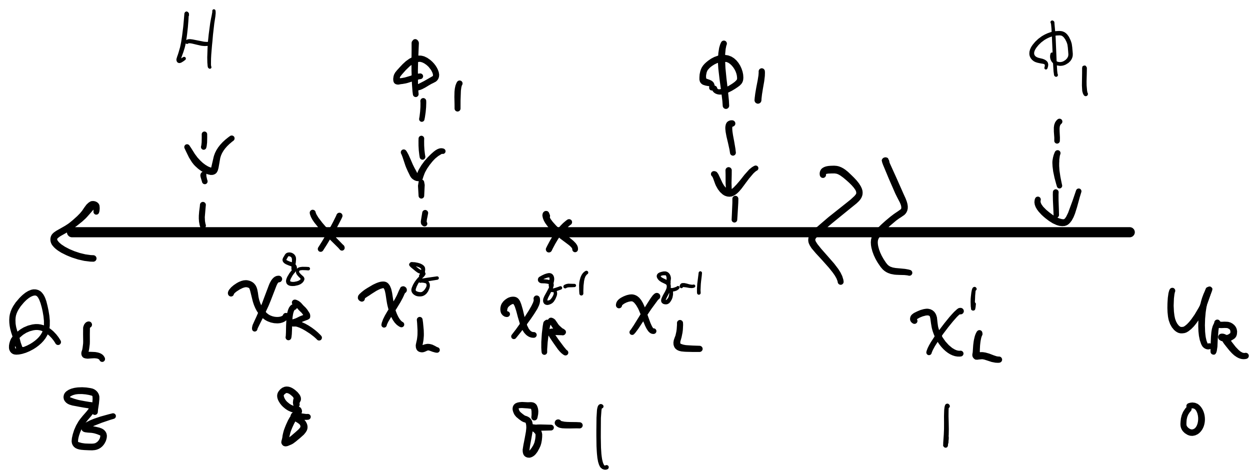
$\Rightarrow 0.41\times10^{-29}$ (JILA)

Froggatt & Nielsen (1979)



$$m_l \sim v_0 \, \epsilon^{q_R - q_L - q_H}$$

$$\epsilon = \left(\frac{v}{\Lambda}\right) \ll 1$$



- too many color χ (for quark sector)
- diff χ for quark / lepton sector

structure zeros

H. Fritzsch

- M_u, M_d, and M_e are Hermitian.
- some $m_{ij} = $ "0"

Z.-z. Xing / Physics Reports 854 (2020) 1–147

Table 14
The five phenomenologically viable five-zero textures of Hermitian quark mass matrices.

	I	II	III	IV	V
$M_u =$	$\begin{pmatrix} 0 & C_u & 0 \\ C_u^* & B'_u & 0 \\ 0 & 0 & A_u \end{pmatrix}$	$\begin{pmatrix} 0 & C_u & 0 \\ C_u^* & 0 & B_u \\ 0 & B_u^* & A_u \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & D_u \\ 0 & B'_u & 0 \\ D_u^* & 0 & A_u \end{pmatrix}$	$\begin{pmatrix} 0 & C_u & 0 \\ C_u^* & B'_u & B_u \\ 0 & B_u^* & A_u \end{pmatrix}$	$\begin{pmatrix} 0 & 0 & D_u \\ 0 & B'_u & B_u \\ D_u^* & B_u^* & A_u \end{pmatrix}$
$M_d =$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & B_d \\ 0 & B_d^* & A_d \end{pmatrix}$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & B_d \\ 0 & B_d^* & A_d \end{pmatrix}$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & B_d \\ 0 & B_d^* & A_d \end{pmatrix}$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & 0 \\ 0 & 0 & A_d \end{pmatrix}$	$\begin{pmatrix} 0 & C_d & 0 \\ C_d^* & B'_d & 0 \\ 0 & 0 & A_d \end{pmatrix}$

- $|A| \gg |B| \gg |C|, |D|$ (by hand)

Horizontal symmetry

- continuous : $U(1)$, $SU(2)$ . . .
- discrete : Δ_N S_N A_4 Z_N . . .

flavor problem $\Rightarrow$

- How to assign the representations?
- How to break the Symmetry?

Extra-dimension

- mass ~ LEFT × RIGHT
- 

Small m Large M

- Simple but NOT simpler

  $\Rightarrow U(1)$

- global ⇒ gauged

  - Anomaly-free as a guide

    (note. in FN, $q_i \to q_i + c$ is still a solution)

  - Avoid the massless Goldstone

  - Moreover, new gauge boson could have testable phenomenology.

- Without introducing new chiral fermions

SM fermions

$Q = T_3 + Y$

	LH: $Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix}$	LH: $L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix}$	RH: $u_{R\,i}$	RH: $d_{R\,i}$	RH: $e_{R\,i}$
$SU(3)_c$	3	1	3	3	1
$SU(2)_L$	2	2	1	1	1
$U(1)_Y$	$\frac{1}{6}$	$-\frac{1}{2}$	$+\frac{2}{3}$	$-\frac{1}{3}$	-1
$U(1)_X$	q_X^i	ℓ_X^i	u_X^i	d_X^i	e_X^i

(× 3 generations)

Cancellation of the Chiral anomalies

- If $\begin{cases} \pm q_X & \pm u_X & \pm d_X & \pm \ell_X & \pm e_X \\ 0 & 0 & 0 & 0 & 0 \end{cases}$

then NO $\quad SU^2(3)\,U(1)_X,\ SU^2(2)\,U(1)_X,\ U^3(1)_X,\ U^2(1)_Y\,U(1)_X$

For $U(1)_Y U(1)_X^2$ to cancel

$$q_X^2 + d_X^2 + e_X^2 = 2u_X^2 + e_X^2$$

- Yukana $\sim \bar{\Psi}_1 \Psi_2 H$

$$-Q_1 + Q_2 + Q_4 = 0$$

$$\neq 0$$

SM Yukawa is allowed

structure zero

or new doublet scalar

Requirements

- $m_t \sim 173\,\mathrm{GeV} \Rightarrow y_t \sim 1.0$

  ⇒ only 3rd generation has the SM Yukawa

- The mass of 2nd G ~ FN + Structure zeros

- $m_u, m_d, m_e = 0$ at tree level

  ⇒ radiactive generation

- We have explored all possible integer solutions

  for $1 \leq |Q_X| \leq 9$

⇒ only a few are realistic

One of the few realistic Sol.

$z_x = 2, \quad u_x = 1 \quad d_x = 3, \quad \ell_x = 6, \quad E_x = 5$

$\bar{Q}$	u: 1	-1	0	d: 3	-3	0
-2	-1	-3	-2	1	-5	-2
$+2$	3	$\boxed{1}$	2	5	$\boxed{-1}$	2
0	$\boxed{1}$	-1	0	3	-3	0

$\bar{L}$	e: -5	5	0
6	1	11	6
-6	-11	$\boxed{-1}$	-6
0	-5	5	0

- m_t, m_b, m_τ from SM Yukawa At leading order

$$M_{u/d} = \mathrm{diag}\{0, \; 0, \; m_t/m_b\}$$

$$M_e = \mathrm{diag}\{0, 0, m_\tau\}$$

- Scalar fields for FN

$$H_{SM} : (2, \tfrac{1}{2}, 0) \qquad S_1 : (1, 0, +1)$$

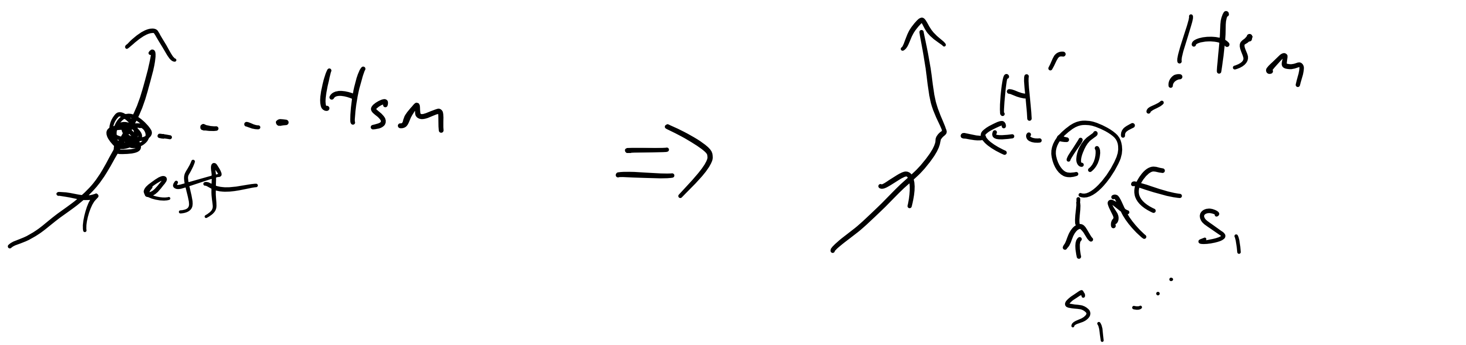
$$\boxed{H_1} : (2, \tfrac{1}{2}, +1) \qquad H_2 : (2, \tfrac{1}{2}, +2)$$

$$H_3 : (2, \tfrac{1}{2}, -3) \qquad H_2 : (2, \tfrac{1}{2}, -2)$$

- SM H_{SM} and S_1 get VEV

$$\langle H_{SM} \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_0 \end{pmatrix}, \qquad \langle S_1 \rangle = \frac{v_1}{\sqrt{2}}$$

- Scalar see-saw (~ FN), others are suppressed



$$\mathcal{M}_u \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_t \end{pmatrix} + v_0 \begin{pmatrix} 0 & y^u_{-3} & y^u_{-2} \\ 0 & y^u_{1} & y^u_{+2} \\ y^{u\prime}_{1} & 0 & 0 \end{pmatrix}_{\mathrm{eff}}$$

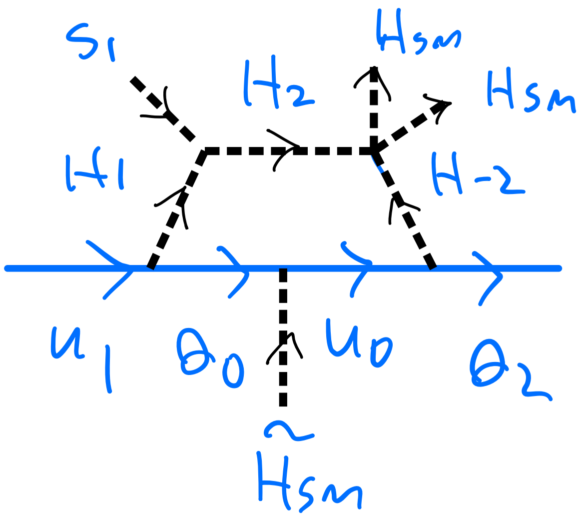
$$\mathcal{M}_d \simeq \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & m_b \end{pmatrix} + v_0 \begin{pmatrix} 0 & 0 & y^d_{2} \\ 0 & y^d_{1} & y^d_{-2} \\ y^d_{-3} & 0 & 0 \end{pmatrix}_{\mathrm{eff}}$$

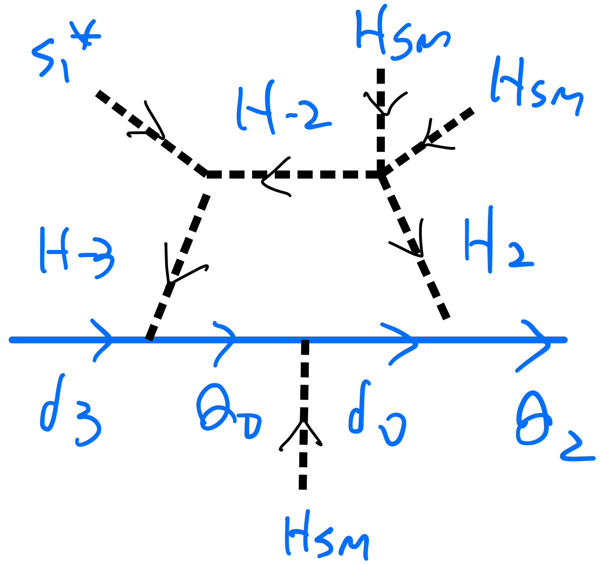
- FN + structure zeros (3+4)

- At this level, $m_u = m_d = 0$

- However, with the new scalars, $m^{1\text{-loop}}_{u/d} \neq 0$

(And all the structure zeros disappear.)

- 



- $$\delta m_u \sim \frac{y^u_{-2}\, y^u_1}{16\pi^2}\, y_t\, \frac{\mu_{12}\lambda_{\pm 2}}{\Lambda^4}\, v_1 v_0^3 \ln \frac{\Lambda^2}{v_0^2}$$

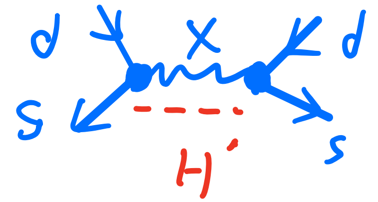
$$\delta m_d \sim \frac{y^d_{-3}\, y^d_2}{16\pi^2}\, y_b\, \frac{\mu_{23}\lambda_{\pm 2}}{\Lambda^4}\, v_1 v_0^3 \ln \frac{\Lambda^2}{v_0^2}$$

Some Numerical highlights

- 6 masses, 3 angles, 1 δ_{CP} (at $\mu = 1$ TeV by RGE)

  Can be easily fitted

- FCNC X/H couplings ($K\bar{K}$, $B\bar{B}$, $D\bar{D}$ ...)



e.g.

$$g_X \bar{\psi}_i \gamma^\mu Q_X^i \psi_i X_\mu \longrightarrow g_X \bar{\psi}_a \gamma^\mu \bar{Q}^X_{ab} \psi_b X_\mu$$

$$\bar{Q}^X_{ab} = U^\dagger_{ai} Q^i_X U_{ib} \neq \delta_{ab} \quad \text{if} \quad Q_X \neq \alpha \mathbb{1}$$

- $\Lambda \sim 10^3 - 10^4$ TeV, ($\rightarrow$ 100 TeV in some parameter space)

- 5 OMD in SM Yukawa $\Rightarrow$ 2 OMD

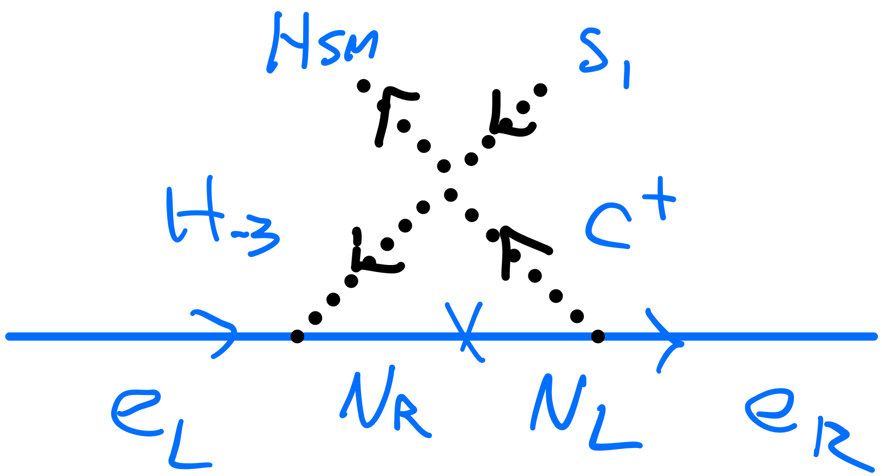
  ($\sim m_t$ vs. m_b and m_c)

  m_t: 174, m_b: 4, m_c: 1.7

- For electron mass, we introduce $M_N \bar{N} N$

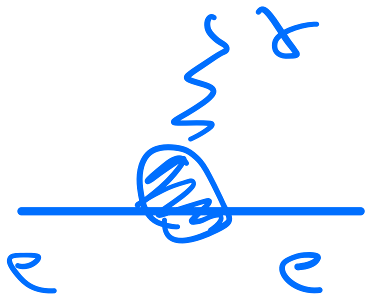
  a vector fermion $N_{L/R}$: $(1, 0, -9)$

  and a charged singlet c^+ : $(1, 1, -4)$

- 

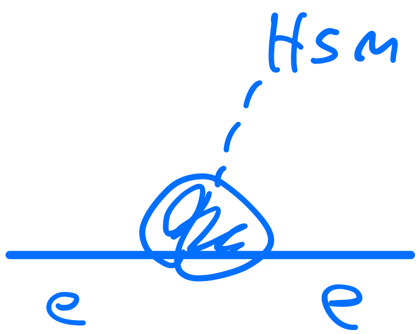
- $$\delta m_e \sim \frac{y_3^{e*} y_c^e}{16\pi^2} \lambda_e \frac{M_N v_1 v_0}{\Lambda^2} \ln\frac{\Lambda^2}{v_0^2}$$

- Since $m_e \simeq \delta m_e \simeq 511\,\mathrm{keV}$ is real, the phase of $y^{e*}_{3} y^{e}_{c}$ must be removed by field redefinition.

- 

$\Delta a_e > 0$

and $d_e = 0$ at 1-loop
and 2-loop !!

- 

eff. Higgs Yukawa could be very diff from the SM one.

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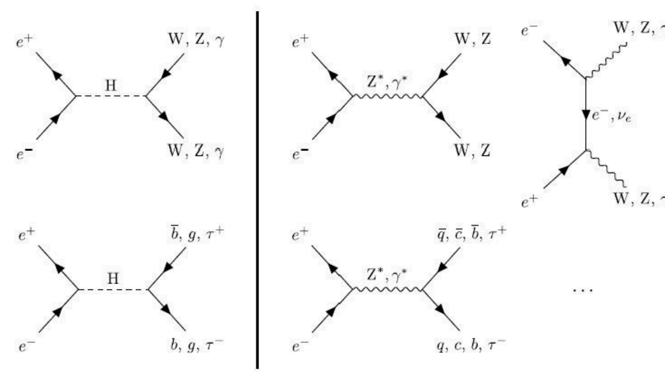

$H \to WW^* \to \ell\nu + 2\,$jets. For a benchmark monochromatization with 4.1-MeV c.m. energy spread (leading to $\sigma_{ee\to H} = 0.28$ fb) and $10\,\mathrm{ab}^{-1}$ of integrated luminosity, a 1.3σ signal significance can be reached, corresponding to an upper limit on the $e^{\pm}$ Yukawa coupling at 1.6 times the SM value: $|y_e| < 1.6|y_e^{\mathrm{SM}}|$ at 95% confidence level, per FCC-ee interaction point per year. Directions for future improvements of the study are outlined.

- Moreover, accidental global U(1)'s:

$$\mathcal{L} \supset -y^e_{-2}\, \overline{L_{-6}}\, N_R\, \tilde{H}_{-3} - y^e_c\, \overline{N_L}\, e_{-5}\, C^+ \quad - M_N \bar{N} N$$

L_{-6}, e_{-5}, $N_{L/R}$: one electron-#

L_{+6}, e_{+5} : one μ-#

L_0, e_0 : one τ-#

- Holds even after the EWSB

- No CLFV, such as $\mu \to e\gamma$, $\tau \to e\gamma$

$\mu \to 3e$ ... μ-e conv.

Conclusion

- Economic/simple gauge U(1) without exotic chiral fermions
  (doublet × 4   singlet × 1   vector N × 1)
- anomaly-free —> only a few realistic sol.
- All charged fermion masses, CKM, and J accommodated.

  3rd G → SM   with $\Lambda \sim 10^3 - 10^4$ TeV
  2nd G → SFN
  1st G → 1-loop

- $\Delta a_e > 0$, $d_e = 0$ at 2-loop, $\left| \frac{y^e_{eff}}{y^e_{SM}} \right| \gg 1$
- No CLFV
- Hierarchy in Yukawa $\underset{(SM)}{10^{5-6}} \Rightarrow \underset{(U(1)_X)}{10^2}$