第十屆 海峽兩岸粒子物理和宇宙學研討會
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A unified Dark-Matter-Driven Relativistic Bondi Route to Black-Hole Growth from Stellar to Supermassive Scales

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緣起

兩岸粒子物理與宇宙學會議起源於1996年9月在北京由中國科學院高能物理研究所主辦的第一屆兩岸中高能物理研討會。之後，每兩年在大陸及台灣地區輪流舉辦。 由於中高能物理之領域過於偏大，在2005年由中央大學主辦之中壢會議後停辦。

但為了建立及提升兩岸高能物理之學術交流和發展，有必要繼續本類型之兩岸研討會。為了使研討會更有成效以及配合兩岸在中高能物理之實質發展，兩岸中高能物理研討會改為兩岸粒子物理與宇宙學研討會。

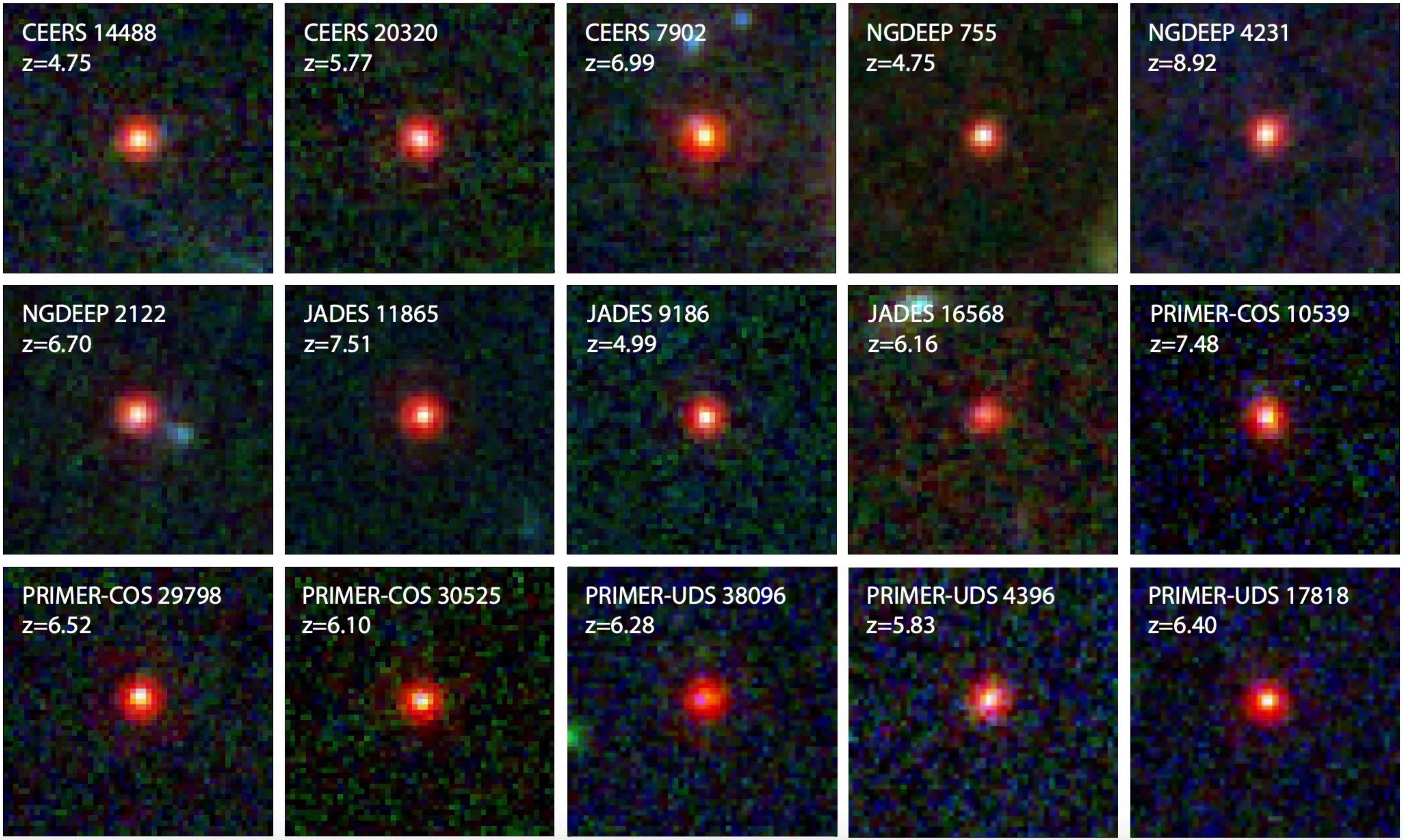
2010年7月，李靈峰教授和耿朝強教授在大陸山東省威海市參加由中國科學院理論物理研究所主辦的暗能量與暗物質之創新及973計畫。期間在吳岳良等教授的推動下重新啟動及維持兩岸交流的傳統，決定自2011年開始每年在兩岸輪流舉辦粒子物理與宇宙學研討會。

2011年在新竹清華大學首次舉辦，
2012在重慶郵電大學、
2013中壢中央大學、
2014中國科學技術大學、
2015香港中文大學、
2016花蓮東華大學、
2017濟南山東大學、
2018年台北淡江大學、
2019年新疆烏魯木齊.....
2026年廣州暨南大學

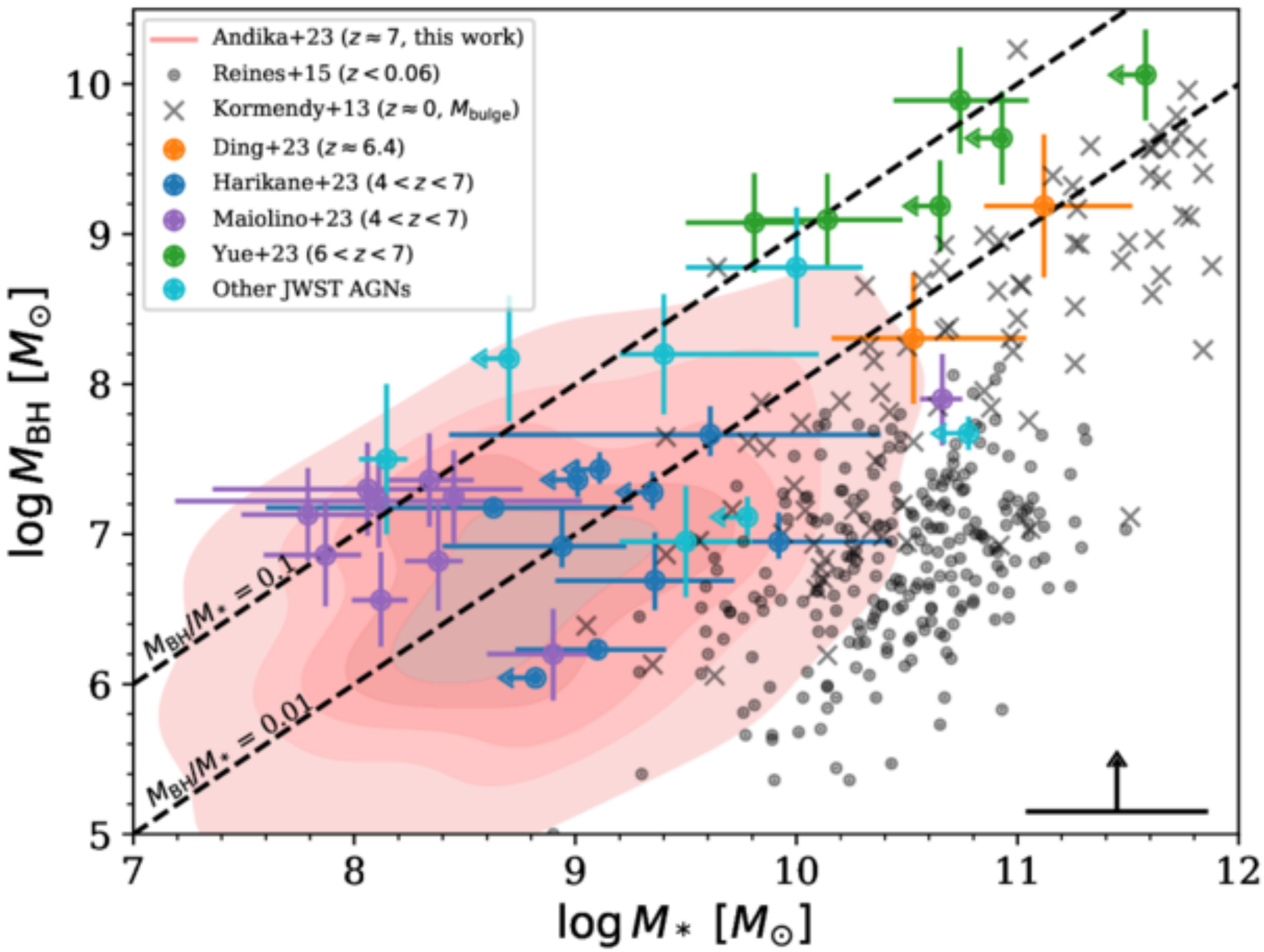
Outline

- Introduction of little red dots (LRTs)
- Basic accretion mechanisms : Eddington accretion and Bondi accretion
- Role played by dark matter
- Relativistic Bondi accretion of self-interacting dark matter
- Primordial black holes as the seeds and the prediction of black hole mass function
- Conclusion

JWST reveals the existence of abundant supermassive quasars in early stage of universe



Mass ratios between supermassive BH (M_{BH}) v.s. host halo (M_*)



- MBH ~$10^9 M_\odot$ by ~0.8 Gyr

Puzzle:
How can a black hole grow this massive so early?

A Simple Cosmic Timeline for Stellar Growth

- **0.38 Myr: Recombination (z ~ 1100)**

- **100–200 Myr: First stars (Pop III)**

  Typical mass scale ~ 10-1000 $M_\odot$, consisting H and He, forming in dark matter minihalos, H_2 cooling (gas/cloud gravitational collapse and fragmentation, protostellar disks and feedback)

- **200–400 Myr: First BH seeds form**

  Two classes : 1. Light seeds (~10-1000 $M_\odot$),

  2. Heavy seeds (~10^4-10^6 $M_\odot$), Direct Collapse Black Holes (DCBHs).

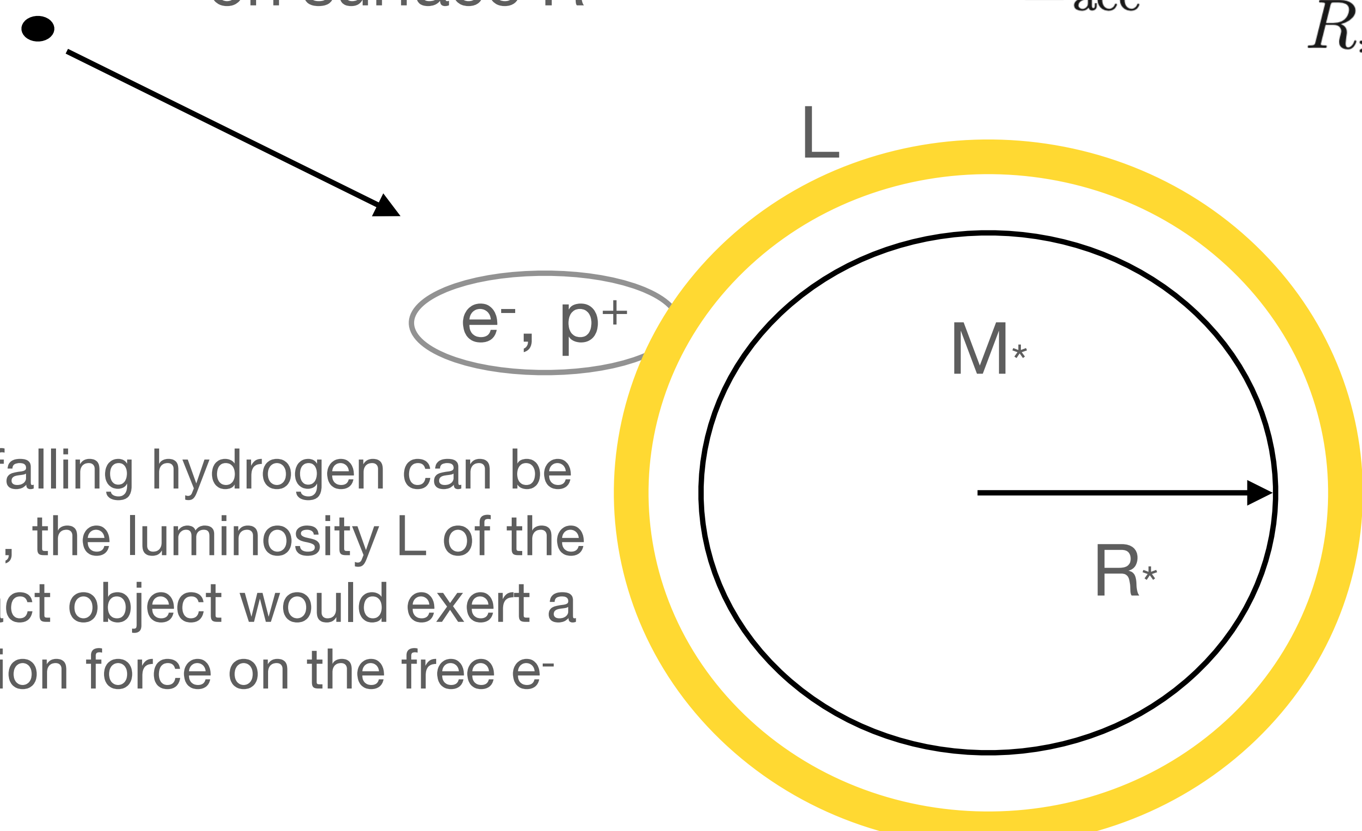
- **700–900 Myr: Bright quasars (10^9 $M_\odot$ at z ~ 6–7)**

Two classical Pathways to Black Hole Growth

- Eddington accretion : **Radiation-gravity balance**

Hydrogen mass m in falling on surface R_*

$$E_{\rm acc} = \frac{GM_*m}{R_*} = \frac{R_{\rm Sch}}{2R_*}mc^2$$



The infalling hydrogen can be ionized, the luminosity L of the compact object would exert a radiation force on the free e^-

Outward radial force on electron

$$f_{\rm out} = \sigma_T L/(4\pi r^2 c)$$

$$\sigma_T = 6.7 \times 10^{-25}\,{\rm cm}^2$$

Thomson cross section of the e^-

The balance between the inward gravitational force on the e-p pair and the outward radial force due to luminosity

$$f_{\rm in} = GM_*m_p/r^2 = f_{\rm out} = \sigma_T L/(4\pi r^2 c) \;\Rightarrow\; L_{\rm Edd} = \frac{4\pi GM_*m_pc}{\sigma_T} = 3.4\times10^4\,{\rm L}_\odot\left(\frac{M_*}{{\rm M}_\odot}\right)$$

Eddington luminosity

The existence of the Eddington luminosity implies the maximum accretion rate

$$\dot{M}_{\rm Edd} = \frac{L_{\rm Edd}}{\eta c^2} = \frac{4\pi G M m_p}{\eta c \sigma_T} \propto M.$$

η : radiation efficiency, typically~0.1

~ 2.2×10^{-5} M$_\odot$ for 1000 M$_\odot$ BH

Also quasars and AGNs have luminosities ~ 10^{14} L$_\odot$ referring to M~3×10^{9} M$_\odot$

From Eddington accretion we have $M(t) \sim e^{t/t_{\rm Edd}}$ with $t_{\rm Edd} \simeq 4.5 \times 10^8$ yrs.

~450 Myr

Reminder

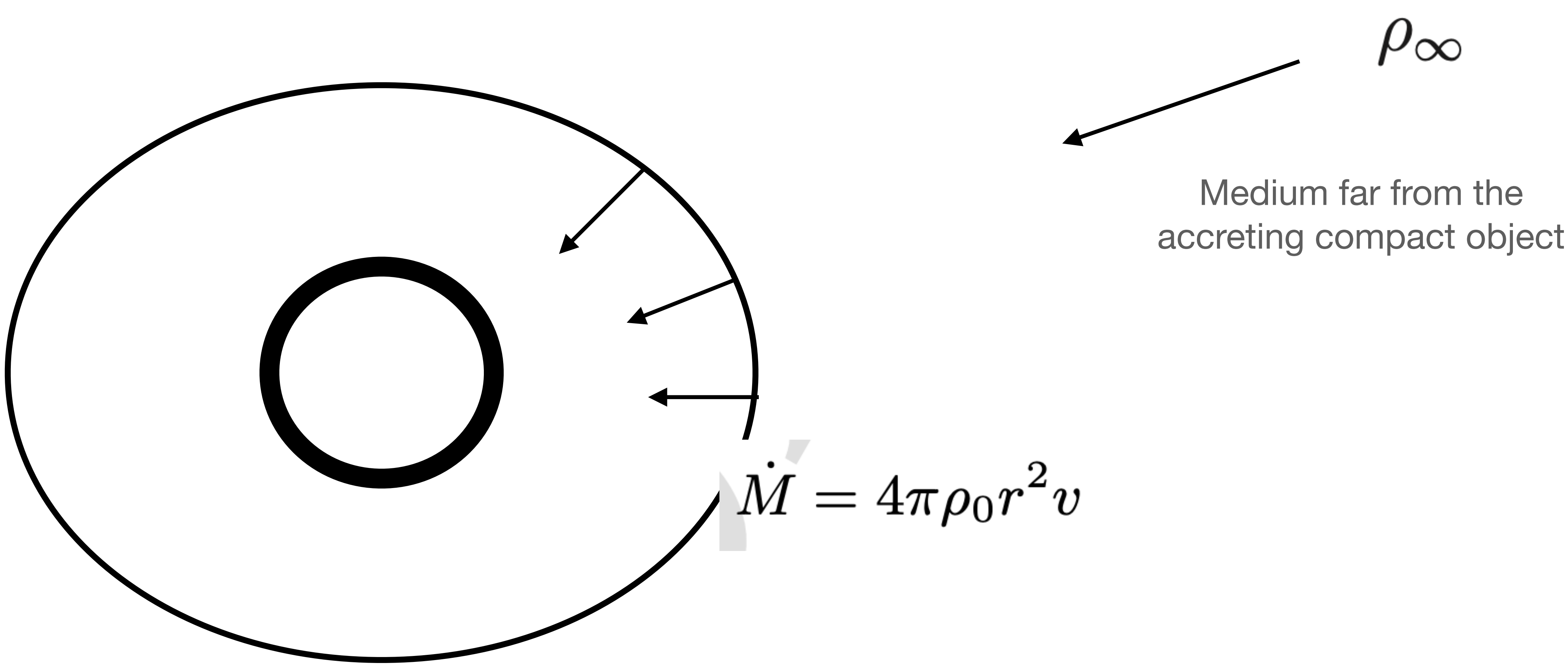
- **200–400 Myr: First BH seeds form**

Two classes : 1. Light seeds (~10-1000 M$_\odot$),

2. Heavy seeds (~10^{4}-10^{6} M$_\odot$), Direct Collapse Black Holes (DCBHs

- **700–900 Myr: Bright quasars (10^{9} M$_\odot$ at z ~ 6–7)**

- Bondi accretion : Spherically symmetric, steady-state, and adiabatic flow
  Gravity-thermal pressure balance



Euler equation $$v\frac{dv}{dr}+\frac{1}{\rho_0}\frac{dp}{dr}+\frac{G_N M}{r^2}=0$$

Combining continuity eq. (Mass conservation), one has

$$\frac{1}{2}\left(1-\frac{c_s^2}{v^2}\right)\frac{dv^2}{dr}=-\frac{G_N M}{r^2}\left(1-\frac{2c_s^2 r}{G_N M}\right)$$: Bondi eq.

sound speed $c_s^2=\frac{dp}{d\rho_0}$

Sonic horizon $r_s=\frac{G_N M}{2c_s^2(r_s)}$, at which $v(r_s)=c_s(r_s)$

If one assume the equation of state of the fluid is the polytropic form $p=\kappa\rho_0^\gamma$

Integrating Euler eq. with EoS, imposing boundary condition at infinity

$$\frac{v^2}{2}+\frac{c_s^2}{\gamma-1}-\frac{G_N M}{r}=\text{constant}=\frac{c_s^2(\infty)}{\gamma-1}$$

$$c_s(r_s)=c_s(\infty)\left(\frac{2}{5-3\gamma}\right)^{1/2}$$

Applying the continuity eq. : $\dot{M} = -4\pi r^2 \rho_0 v = 4\pi r_s^2 \rho_0(r_s) c_s(r_s)$

$$\rho_0(r_s) = \rho_0(\infty)\left[\frac{c_s(r_s)}{c_s(\infty)}\right]^{2/(\gamma-1)}$$

And the Bondi accretion rate (non-relativistic)

$$\dot{M} = \pi G_N^2 M^2 \frac{\rho_0(\infty)}{c_s^3(\infty)}\left[\frac{2}{5-3\gamma}\right]^{\frac{(5-3\gamma)}{2(\gamma-1)}}$$

M^2 dependent

Environment dependent

EoS dependent

Low sound speed seems to enhance the rate —> redundant

$\sim 3.6 \times 10^{-9}\, M_\odot/\mathrm{yr}$ For 1000 $M_\odot$ BH

Comparison between Eddington / Bondi accretions

$$\dot{M}_{\rm Edd} = \frac{L_{\rm Edd}}{\eta c^2} = \frac{4\pi G M m_p}{\eta c \sigma_T} \propto M.$$

$$\dot{M} = \pi G_N^2 M^2 \frac{\rho_0(\infty)}{c_s^3(\infty)} \left[\frac{2}{5-3\gamma}\right]^{\frac{(5-3\gamma)}{2(\gamma-1)}}$$

Naively the Bondi accretion rate appears larger as it is proportional to M^2 comparing to Eddington accretion is proportional to M. In reality the Bondi accretion is far suppressed in comparison to the Eddington accretion.

1. The Eddington is the balance between the radiation pressure and gravity : the heat due to the collision can be depleted by the radiation to prevent the heating-up feedback.

2. Bondi depends on energy density ρ of the accreting fluid, which is usually insufficient to supply enough inflow of matter to form SMBHs.

3. If one considers the accretion of CDM, the local density of CDM in the inner region is estimated to be far smaller than the baryonic matter in the halos with NFW structure.

4. If considering the Bondi accretion of relativistic DM, the Bondi rate is suppressed by large sound speed c_s.

The usual consideration of forming SMBH by accretion mechanism, the Eddington is usually the dominant mechanism.

Small sound speed c_s enhancement on Bondi accretion ?

$$\dot{M} = \pi G_N^2 M^2 \frac{\rho_0(\infty)}{c_s^3(\infty)} \left[\frac{2}{5-3\gamma}\right]^{\frac{(5-3\gamma)}{2(\gamma-1)}}$$

1. The zero sound speed c_s implies no pressure support —> no Bondi inflow. Thus c_s —> 0 is a critical limit, so the Bondi accretion is replaced by ballistic free fall of dust with the accretion rate

$$\dot{M}_{\rm dust} = 4\pi r_{\rm cap}^2 \rho_\infty v_\infty = 16\pi \rho_\infty \frac{(GM)^2}{v_\infty^3}$$

$$r_{\rm cap} = \frac{2GM}{v_\infty^2}$$

r_{cap} is the gravitational capture radius and v_∞ is the asymptotic ballistic velocity.

2. Physically, the vanishing c_s implies the CDM is almost collisionless, so the capture by the central BH simply relies on gravity. It needs a small impact parameter, resulting in a smaller effective accretion rate.

3. The tiny scattering cross-section of CDM limits the formation of “Bondi radius”. Thus, it lacks a mechanism to realize huge “Bondi focusing” even with a tiny sound speed.

Numerical comparison
Take 3M$_\odot$ BH as an example

$$\dot{M}_{\rm B} \;=\; 4\pi\lambda(\gamma)\,\frac{(GM)^2\rho_\infty}{c_{s,\infty}^3} \qquad C = 4\pi\,\lambda(\gamma)\,G^2\,\frac{\rho}{c_s^3}$$

Environment	n [cm^{-3}]	c_s [km/s]	C [kg^{-1}s^{-1}]	$\dot{M}$ [$M_\odot$/yr]
Post-reion. IGM	10^{-6}	12	1.4×10^{-59}	10^{-20}
Pre-reion. IGM ($z\sim50$)	2.7×10^{-2}	1	7.4×10^{-52}	10^{-14}
Warm neutral medium	0.5	8	1.7×10^{-54}	10^{-16}
Molecular cloud	100	0.3	1.0×10^{-46}	10^{-11}
Dense core (optimistic)	10^4	0.2	3.5×10^{-44}	10^{-9}
Cluster ICM (hot)	10^{-3}	1000	2.8×10^{-62}	10^{-22}
Relativistically hot gas†	$\rho=10^{-19}\,$kg/m^3	$c/\sqrt{3}$	3×10^{-64}	$\ll 10^{-20}$
Eddington limit	–	–	–	6×10^{-8}

- BH accretion process can be “relativistic”
- Bondi accretion needs certain thermal pressure (interaction) —> we consider the self-interacting dark matter (SIDM)

Relativistic Bondi accretion on SIDM

$$\mathcal{L} = -\frac{1}{2} g^{\mu\nu} \phi^*_{;\mu} \phi_{;\nu} - V(|\phi|) \qquad V(|\phi|) = \frac{1}{2} m^2 |\phi|^2 + \frac{\lambda}{4!} |\phi|^4 .$$

$$\mathrm{d}s^2 = -\left(1 - \frac{2M}{r}\right) \mathrm{d}t^2 + \left(1 - \frac{2M}{r}\right)^{-1} \mathrm{d}r^2 + r^2 (\mathrm{d}\theta^2 + \sin^2\theta \mathrm{d}\phi^2) \, .$$

EoS: $$\frac{\rho}{\rho_B} = \frac{3p}{\rho_B} + 4\sqrt{\frac{p}{\rho_B}}$$

$$\rho_B := \frac{3m^4}{2\lambda} = \frac{3.48}{\lambda} \left(\frac{m}{\mathrm{GeV}}\right)^4 \times 10^{20} \ \mathrm{kg\ m^{-3}} .$$

Sound speed : $$a^2 := \left(\frac{\partial p}{\partial \rho}\right)_{\mathrm{adiabatic}} = \frac{1}{3}\left(1 - \frac{1}{\sqrt{1 + 3\rho/4\rho_B}}\right)$$

$$a^2 \leq 1/3 \, .$$

For relativistic fluid (mass-energy conservation): $\left(\frac{\partial\rho}{\partial\rho_0}\right)_{\text{adiabtic}} = \frac{p+\rho}{\rho_0}\ .$

Relativistic Euler eq.

$$\frac{u'}{u} + \frac{\rho_0'}{\rho_0} = -\frac{2}{r}\,,$$

$$uu' + \left(1 - \frac{2M}{r} + u^2\right)\frac{a^2}{\rho_0}\rho_0' = -\frac{M}{r^2},$$

Relativistic Bernoulli eq.

$$\left(\frac{P+\rho}{\rho_0}\right)^2\left(1+u^2-\frac{2M}{r}\right) = \left(\frac{p_\infty+\rho_\infty}{\rho_{0,\infty}}\right)^2$$

Or

$$\left(\frac{1-a^2}{1-3a^2}\right)^2\left(1+u^2-\frac{2M}{r}\right) = \left(\frac{1-a_\infty^2}{1-3a_\infty^2}\right)^2$$

$$\frac{\rho_0}{\rho_{0,\infty}} = \frac{a^2}{a_\infty^2}\sqrt{\frac{(1-a^2)(1-3a_\infty^2)^3}{(1-a_\infty^2)(1-3a^2)^3}}$$

One can also define the “Bondi radius, r_B” which characterize the effective capture radius for the Bondi accretion

$$r_B := \frac{2M}{a_\infty^2} ,$$

The accretion rate is given by $\dot{M}_{\rm Bondi} = 4\pi r_s^2 \rho_{0,s} u_s$

By imposing critical condition and Euler eq. , we have

$$u_s^2 = \frac{a_s^2}{1+3a_s^2} = \frac{2M}{r_s}$$

$$a_\infty^2 = \frac{a_s^2(9a_s^2-1)}{2-3a_s^2+9a_s^4}$$

$$1/9 \le a_s^2 \le 1/3 , \qquad 0 \le a_\infty^2 \le 1/3 .$$

Bondi accretion rate : $\dot{M}_{\rm Bondi} = 2\pi \frac{(1+a_s^2)^3}{a_s^3(9a_s^2-1)} M^2 \rho_{0,\infty} .$

$$\epsilon := a_s^2 - \frac{1}{9} \ll 1 \quad \longrightarrow \quad \dot{M}_{\rm Bondi} = \frac{128\pi}{9}\frac{1}{\epsilon}M^2\rho_{0,\infty} \ .$$

In this critical regime, the sonic horizon radius is $r_s \simeq 24M$.

Consistently, we can also express ϵ as $\epsilon \simeq \frac{2}{9}\frac{\rho_\infty}{\rho_B}$

At infinity, one can also have $p_\infty \simeq \frac{81}{64}\rho_B\epsilon^2$ and $\rho_\infty = \frac{9}{2}\rho_B\epsilon$,

This means we can safely treat the fluid as pressureless at spatial infinity, we then have the Bondi accretion in this critical regime to be expressed as

$$\dot{M}_{\rm Bondi} = 64\pi\rho_B\frac{G^2M^2}{c^3}$$

Environmental independent !

$$\rho_B := \frac{3m^4}{2\lambda} = \frac{3.48}{\lambda}\left(\frac{m}{\rm GeV}\right)^4 \times 10^{20}\ {\rm kg\ m^{-3}}.$$

Furthermore, it is well known the SIDM mass/cross section to be constrained

$$0.1\ \mathrm{cm}^2\,\mathrm{g}^{-1} \lesssim \sigma_{\mathrm{DM}}/\mathrm{m} \lesssim \mathcal{O}(1)\ \mathrm{cm}^2\,\mathrm{g}^{-1}.$$

$$\sigma_{\phi^4} = \frac{\lambda^2}{64\pi m^2}\,.$$

$$30\Big(\frac{m}{\mathrm{GeV}}\Big)^{3/2} \lesssim \lambda \lesssim 90\Big(\frac{m}{\mathrm{GeV}}\Big)^{3/2}\,.$$

$$\rho_B := \frac{3m^4}{2\lambda} = \frac{3.48}{\lambda}\left(\frac{m}{\mathrm{GeV}}\right)^4 \times 10^{20}\ \mathrm{kg\ m}^{-3}.$$

$$\rho_B = 6.96\times 10^{18}\left(\frac{m}{\mathrm{GeV}}\right)^{5/2}\mathrm{kg\ m}^{-3} = 2.2\times 10^{-4}\left(\frac{m}{\mathrm{eV}}\right)^{5/2}\mathrm{kg\ m}^{-3}$$

$$\dot{M}_{\mathrm{Bondi}} = 1.45\times 10^{13}\left(\frac{M}{M_\odot}\right)^2\left(\frac{m}{\mathrm{GeV}}\right)^{5/2}\frac{M_\odot}{\mathrm{yr}}\,.$$

Depend on DM mass only !

To illustrate the result, we take a stellar mass scale black hole to grow by simultaneously Eddington accretion and our relativistic SIDM Bondi accretion

$$\dot{M} = \dot{M}_{\rm Edd} + \dot{M}_{\rm Bondi} = \Gamma M + K M^2$$

$$M(z) = \frac{M_0\, e^{\Gamma\tau(z)}}{1 - \frac{K}{\Gamma} M_0 \left(e^{\Gamma\tau(z)} - 1\right)} \qquad \tau(z) \;=\; \int_z^{z_i} \frac{dz'}{(1+z')\,H(z')}$$

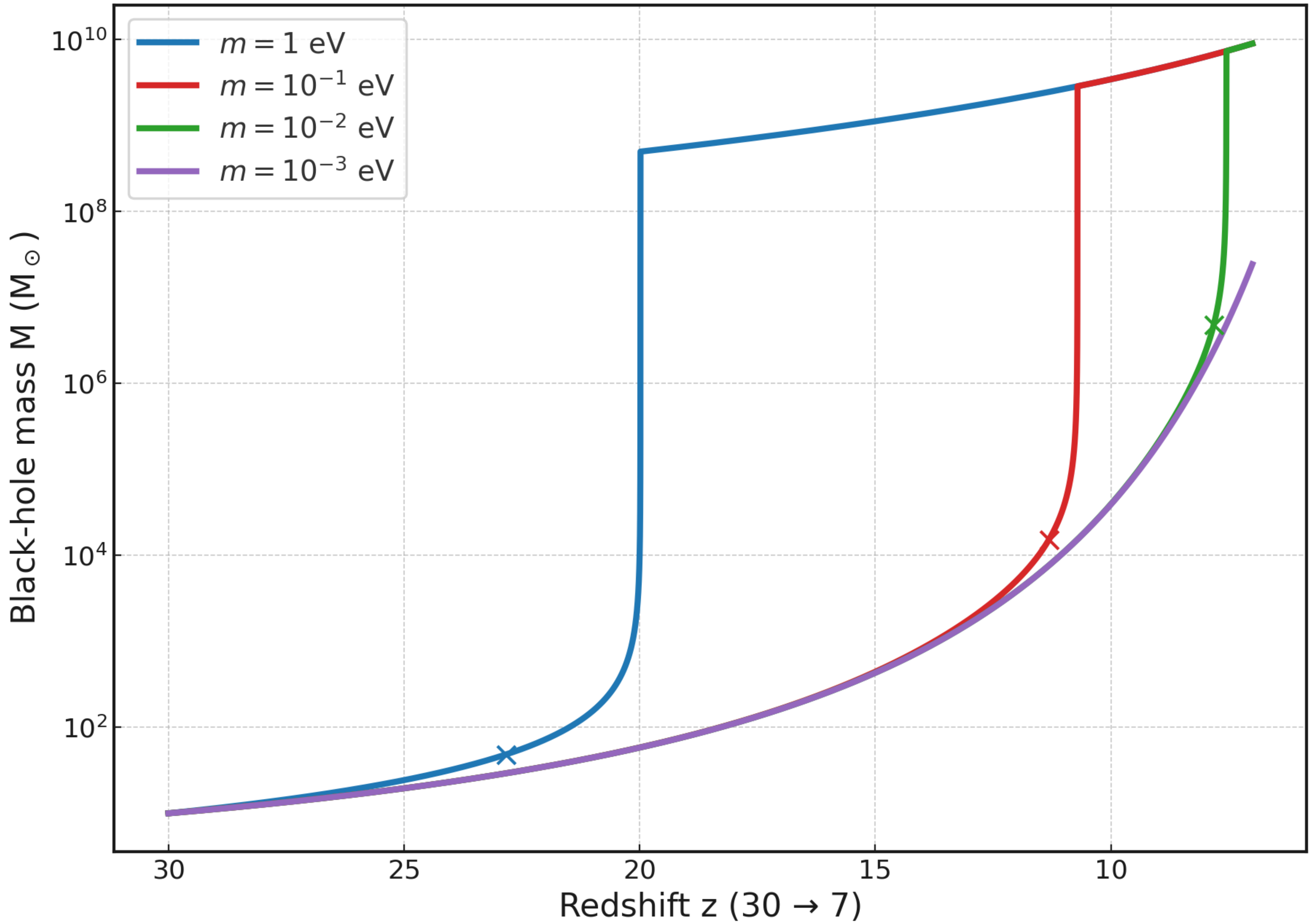
The ratio Γ/K = $M_\star$ represents the transition mass that determines the domination of Bondi over Eddington

$$\dot{M}_{\rm Edd} = \frac{L_{\rm Edd}}{\eta c^2} = \frac{4\pi G M m_p}{\eta c \sigma_T} \propto M.$$

$$\dot{M}_{\rm Bondi} = 1.45 \times 10^{13} \left(\frac{M}{M_\odot}\right)^2 \left(\frac{m}{\rm GeV}\right)^{5/2} \frac{M_\odot}{\rm yr}\ .$$

We take $M_0=10M_\odot$ to study the BH growth

m (eV)	$M_\star$ ($M_\odot$)	Growth regime
10^{-3}	1.5307×10^9	Bondi never dominates
10^{-2}	4.8403×10^6	Bondi dominates around $z \sim 11$
10^{-1}	1.5307×10^4	Early Bondi onset
1	4.8403×10^1	Bondi dominates almost immediately

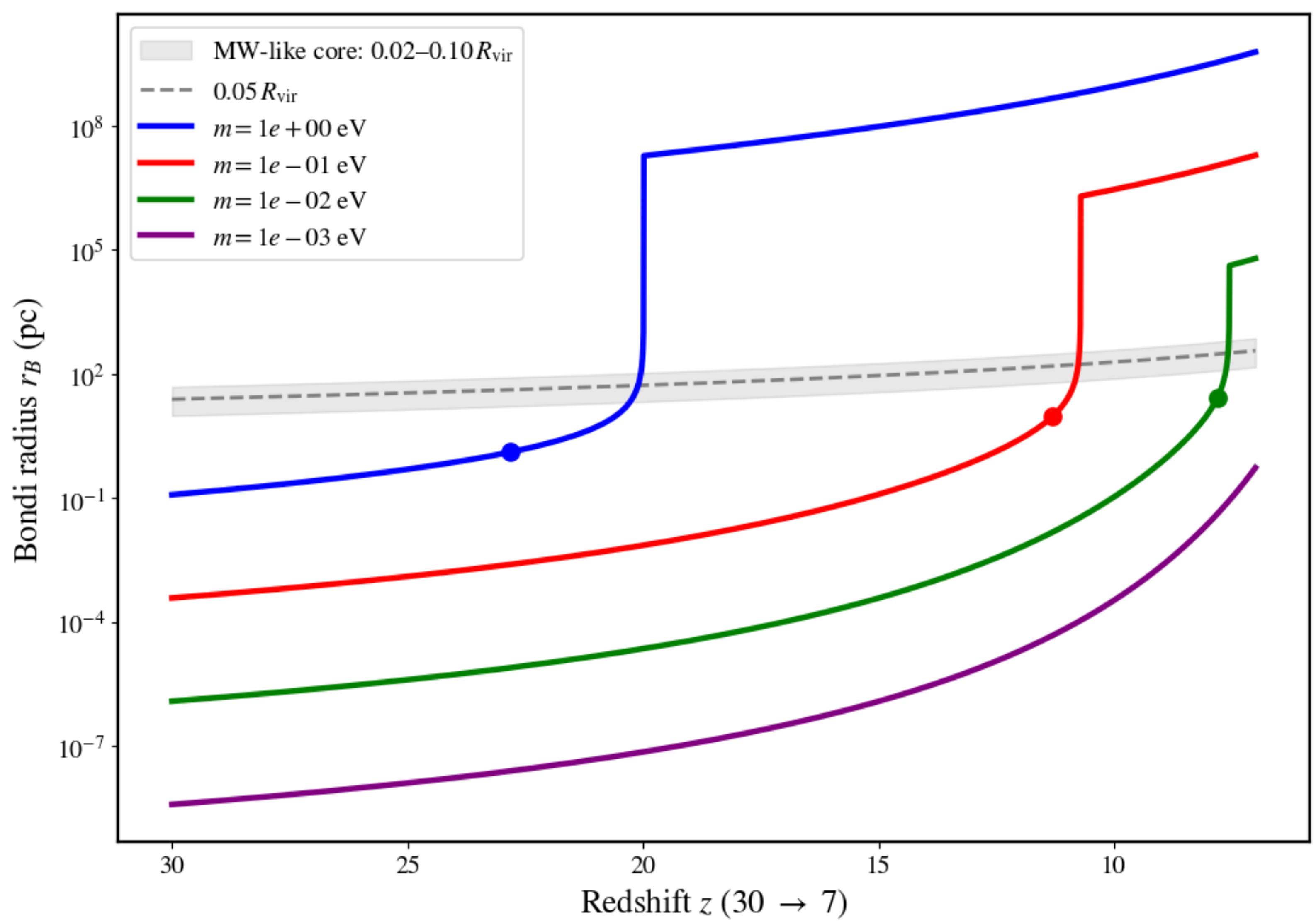


For Bondi radius $$r_B := \frac{2M}{a_\infty^2} \simeq \frac{81}{16}\frac{\rho_B}{\rho_{\rm DM}} M$$

We adopt a conservative supply model with the ambient DM density around the galaxy halos to be

$$\rho_{\rm DM}(z) = 5 \times 10^{-22}(1+z)^3\, \Delta_{\rm vir}\ {\rm kg\,m^{-3}}$$

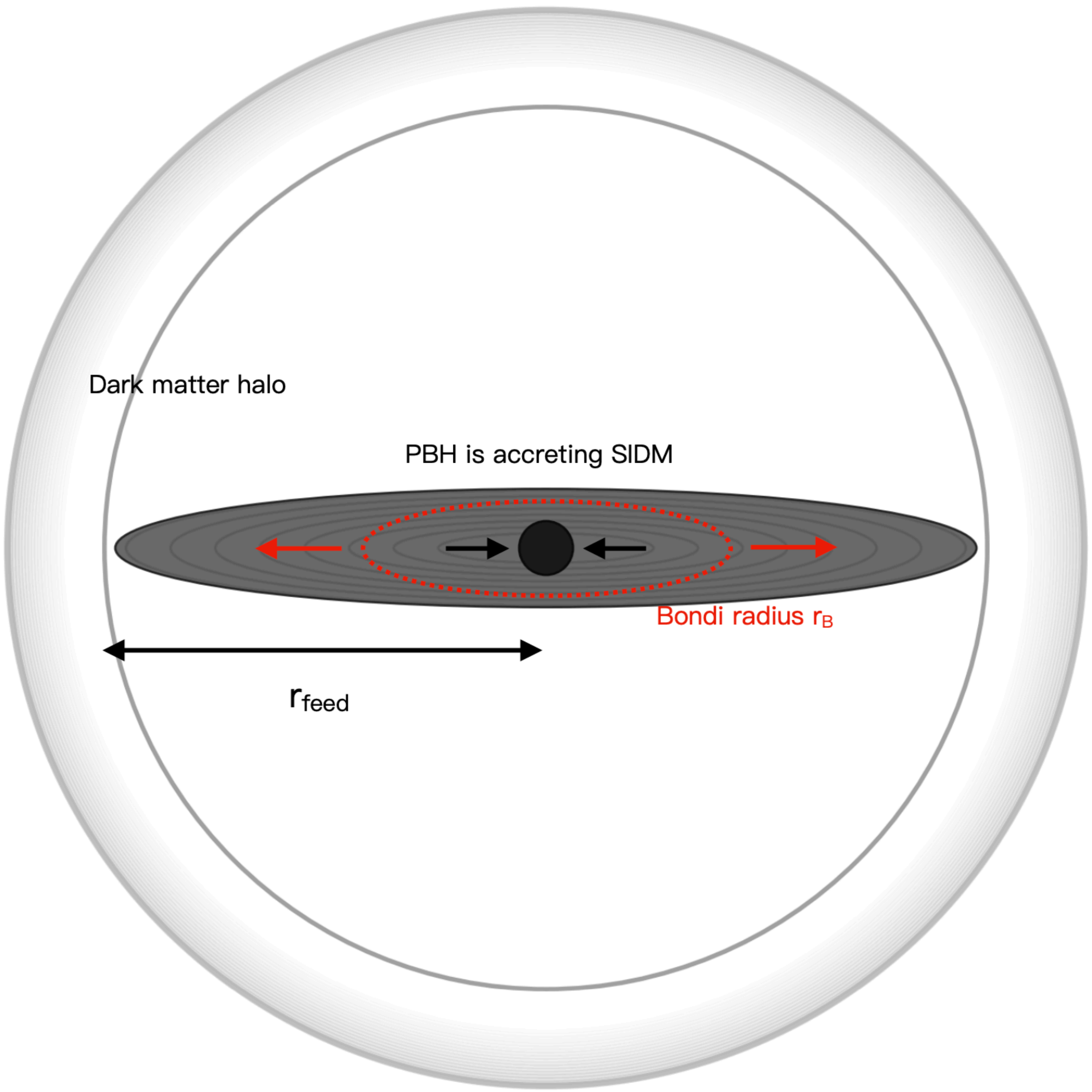
virial overdensity $\Delta_{\rm vir} \simeq 300$



$$\epsilon \simeq \frac{2}{9}\frac{\rho_{\rm DM}}{\rho_B} \quad \longrightarrow \quad \epsilon(z,m) = 1.52\times 10^{-16}\,\frac{(1+z)^3}{(m/{\rm eV})^{5/2}}\ .$$

We require ϵ to be less than 10^{-3} at z~7

$$m \gtrsim 10^{-3.34}{\rm eV}\ .$$



Schematic accretion

Next we try to explain the abundance of Supermassive BHs

Idea of primordial BHs —

- 1967, Y. Zel'dovich and I. Novikov proposed that BHs can be formed if density perturbations were large enough in early Universe.
- 1971, S. Hawking and Carr noticed at horizon entry the large overdensities could be formed —> the critical overdensity threshold for PBH collapse during radiation ear is about $\delta \geq 0.3$-0.5.
- By including the Hawking radiation effect, a light PBH, $M_{PBH} < 10^{15}$ g would evaporate at the lifetimes shorter than the age of Universe. —> footprint on CMB

PBH mass scale is essentially the horizon mass at its formation time:

$$M_H(T) \sim \gamma \frac{c^3 t}{G} \sim 30 M_\odot \left(\frac{T}{200\text{MeV}}\right)^{-2}$$

γ : efficiency factor

This work : QCD phase transition, $t \sim 10^{-5}$ s or $T \sim 100$ MeV

Some issues have been addressed, simply mention in this talk

$$\beta(M) \approx \int_{\delta_c}^{\infty} \frac{1}{\sqrt{2\pi}\sigma(M)} \exp\left[-\frac{\delta^2}{2\sigma^2(M)}\right] d\delta.$$

PBH formation probability with variance of density perturbations σ

$$\sigma^2(M) \sim \int W^2(kR) P_\delta(k) \frac{dk}{k}$$

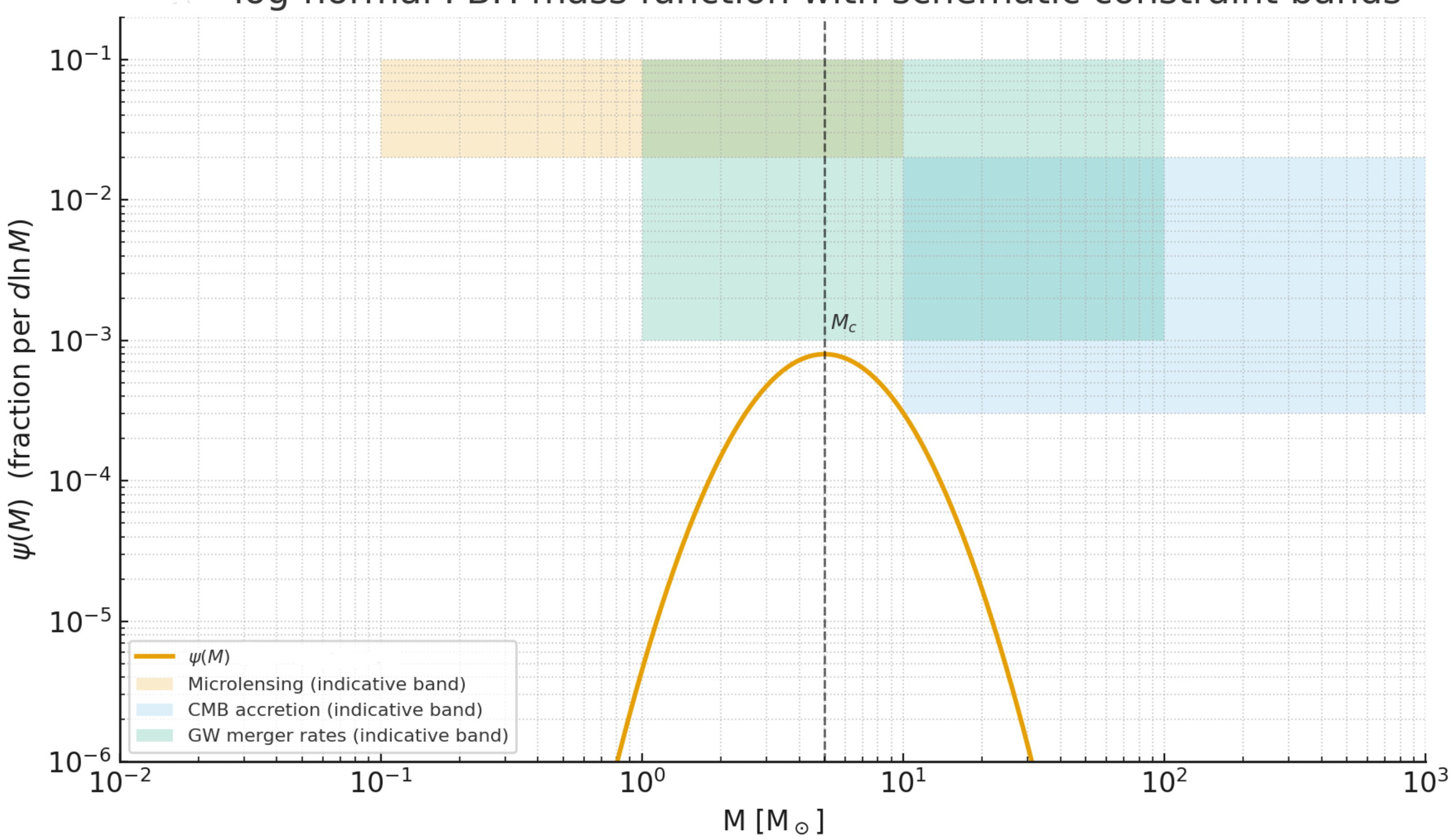
σ can be related to the power spectrum with certain window function (filter function), R refers to the horizon scale.

$$\beta(M) = \frac{\rho_{\rm PBH}(t_{\rm form})}{\rho_{tot}(t_{\rm form})}$$

$$\boxed{\frac{\rho_{\rm PBH}}{\rho_{rad}} \propto a \propto \frac{1}{T}}$$

Present day fraction: $\Omega_{\rm PBH}(M) \simeq \beta(M) \frac{a_{eq}}{a_{form}} \Omega_m$

log-normal PBH mass function with schematic constraint bands



Current constraint : $f_{\rm PBH} \leq 10^{-3}\, \Omega_{\rm DM}$

We study three typical formation function distributions

Log-normal :

$$\psi_0^{\rm LN}(M_0) = \frac{f_{\rm PBH}}{\sqrt{2\pi}\sigma M_0} \exp\left[-\frac{(\ln(M_0/M_c))^2}{2\sigma^2}\right],$$

$M_c = 5M_\odot$ and $\sigma = 0.5$;

Power law

$$\psi_0^{\rm PL}(M_0) = \frac{(1-\alpha) f_{\rm PBH}}{M_{\rm max}^{1-\alpha} - M_{\rm min}^{1-\alpha}} M_0^{-\alpha} ,$$

by choosing $\alpha = 2$, and $M_0 \in [M_{\rm min}, M_{\rm max}]$ with $M_{\rm min} = 0.1M_\odot$ and $M_{\rm max} = 100M_\odot$.

Critical-collapse

$$\psi_0^{\rm CC}(M_0) = \tilde{A} M_0^{\frac{1}{\gamma}-1} \exp\left[- \beta (M_0)/M_c)^{1/\gamma}\right],$$

$\gamma \simeq 0.36$, $\beta \simeq 1$, and $M_c = 5M_\odot$.

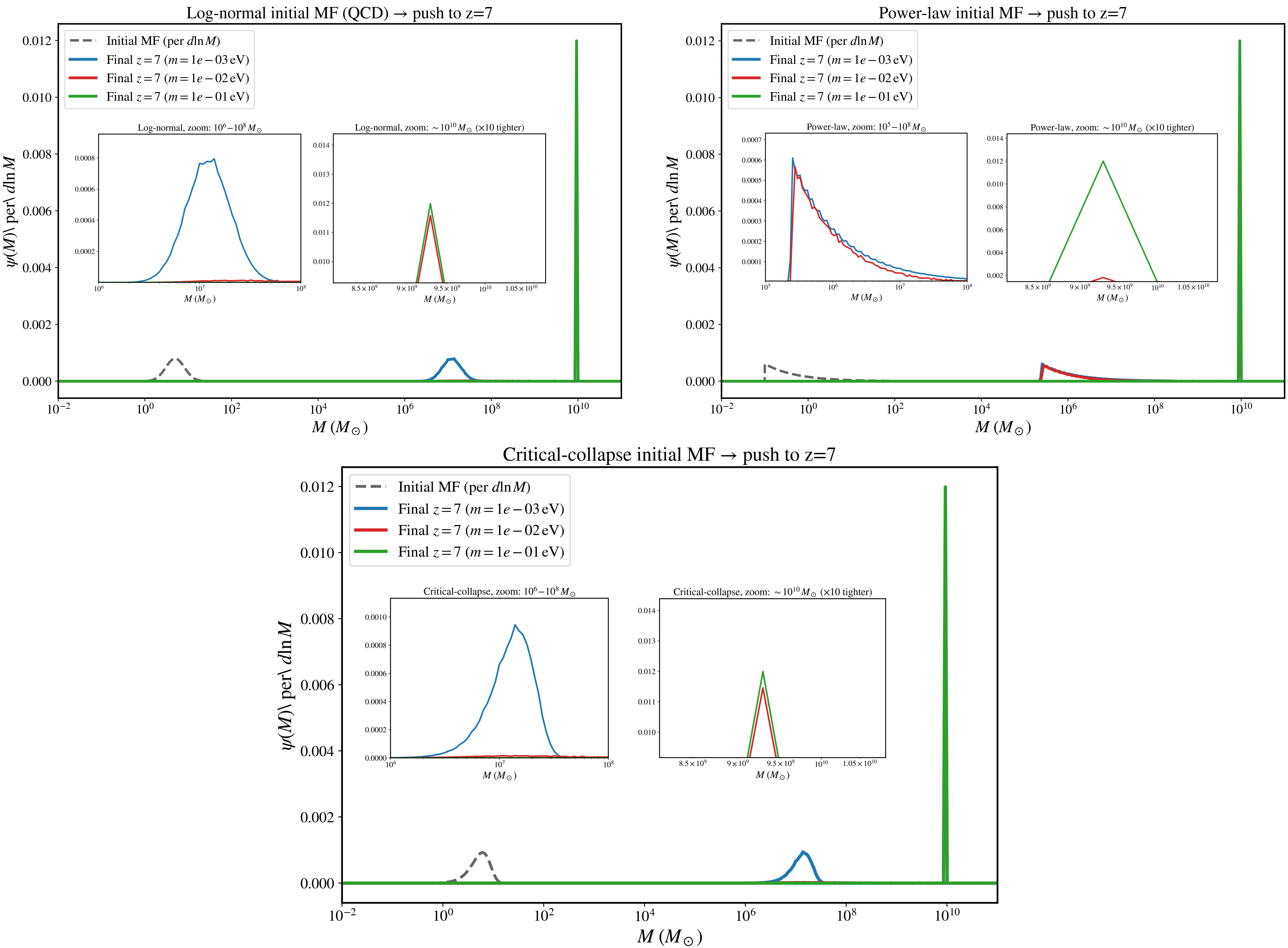
Normalization condition $\int \psi_0(M_0) dM_0 = f_{\rm PBH}$:

Assuming no mergers —> number conservation,
we are able to obtain the analytical mass function of PBH at z=7 for each initial distribution

$$\psi_f(M, z)dM = \psi_0(M_0)dM_0$$

We have

$$\psi_f(M, z) = \frac{\psi_0(M_0(M, z))e^{\Gamma\tau z}}{[e^{\Gamma\tau z} + \frac{K(m)}{\Gamma}M(z)(e^{\Gamma\tau z} - 1)]^2},$$


Log-normal initial MF (QCD) → push to z=7
Power-law initial MF → push to z=7
Critical-collapse initial MF → push to z=7
Initial MF (per $d\ln M$)
Final $z=7$ ($m=1e-03$ eV)
Final $z=7$ ($m=1e-02$ eV)
Final $z=7$ ($m=1e-01$ eV)
$\psi(M)$\ per\ $d\ln M$
M ($M_\odot$)
Log-normal, zoom: 10^6–$10^8\,M_\odot$
Log-normal, zoom: $\sim 10^{10}\,M_\odot$ (×10 tighter)
Power-law, zoom: 10^5–$10^8\,M_\odot$
Power-law, zoom: $\sim 10^{10}\,M_\odot$ (×10 tighter)
Critical-collapse, zoom: 10^6–$10^8\,M_\odot$
Critical-collapse, zoom: $\sim 10^{10}\,M_\odot$ (×10 tighter)

Conclusion

- We proposed a new mechanism, relativistic SIDM Bondi accretion, for the BH growth.
- At critical regime, the accretion rate is independent on the environmental condition but only depends on dark matter mass.
- We illustrate PBHs as the initial BH seeds, we demonstrate even $M_0 \sim 10\ M_\odot$, a supermassive BH can be achieved in first 800 millions years
- For stellar scale initial BHs, we prefer $m_{DM} > 10^{-3}$ eV.
- The mechanism can apply to any potential initial BHs.
- Mass function of BHs could be revealed.