Gravitational Waves from Cosmological Phase Transitions

安海鹏（清华大学）

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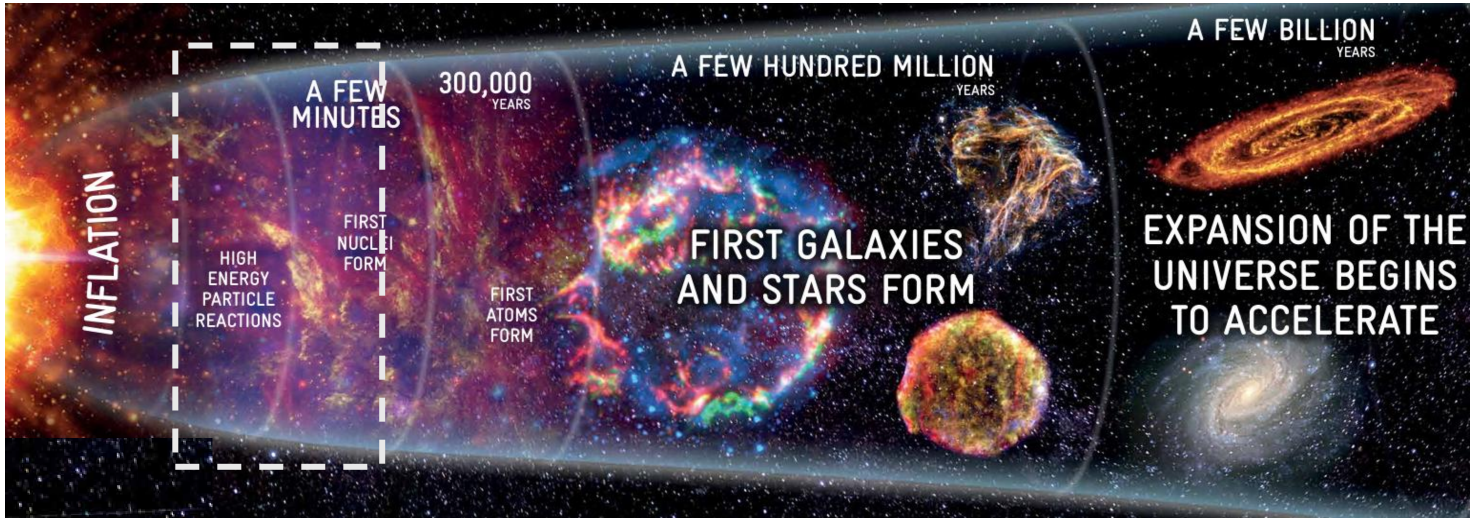
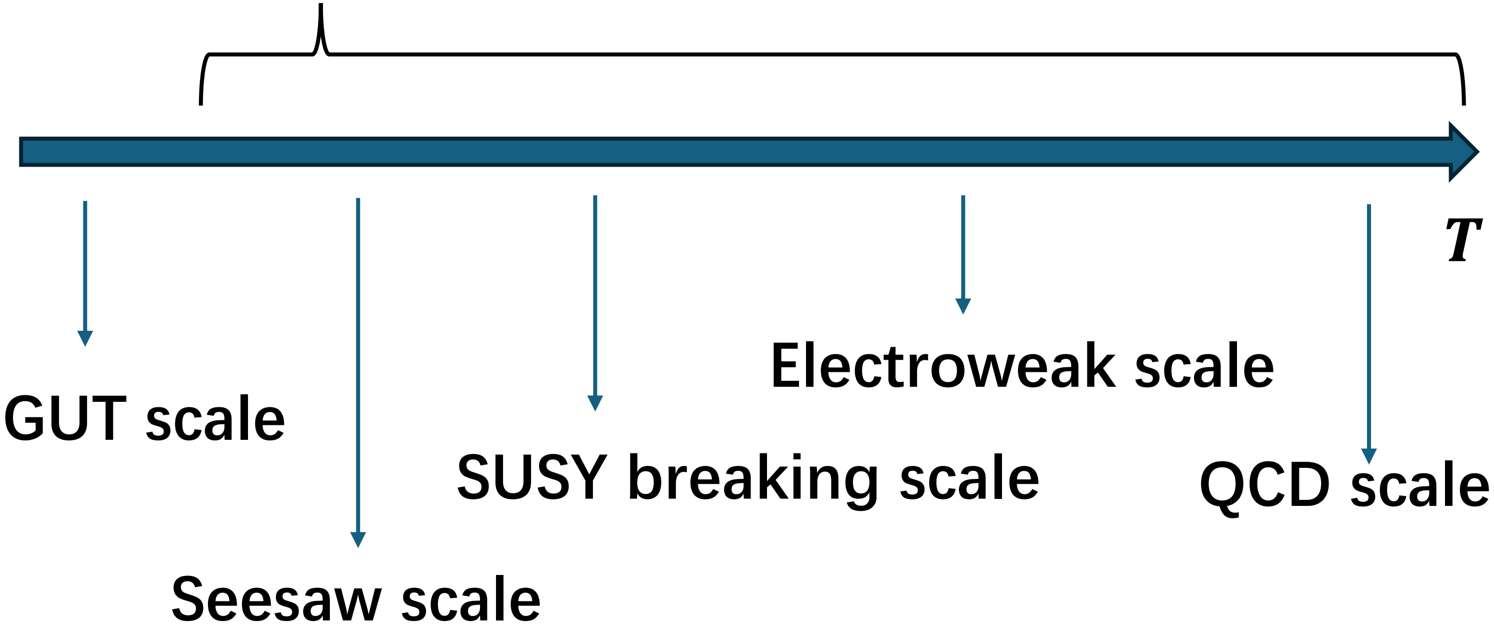
The evolution of the Universe

- The temperature evolves.
- The Hubble expansion rate

  $H \sim T^2/M_{pl}$

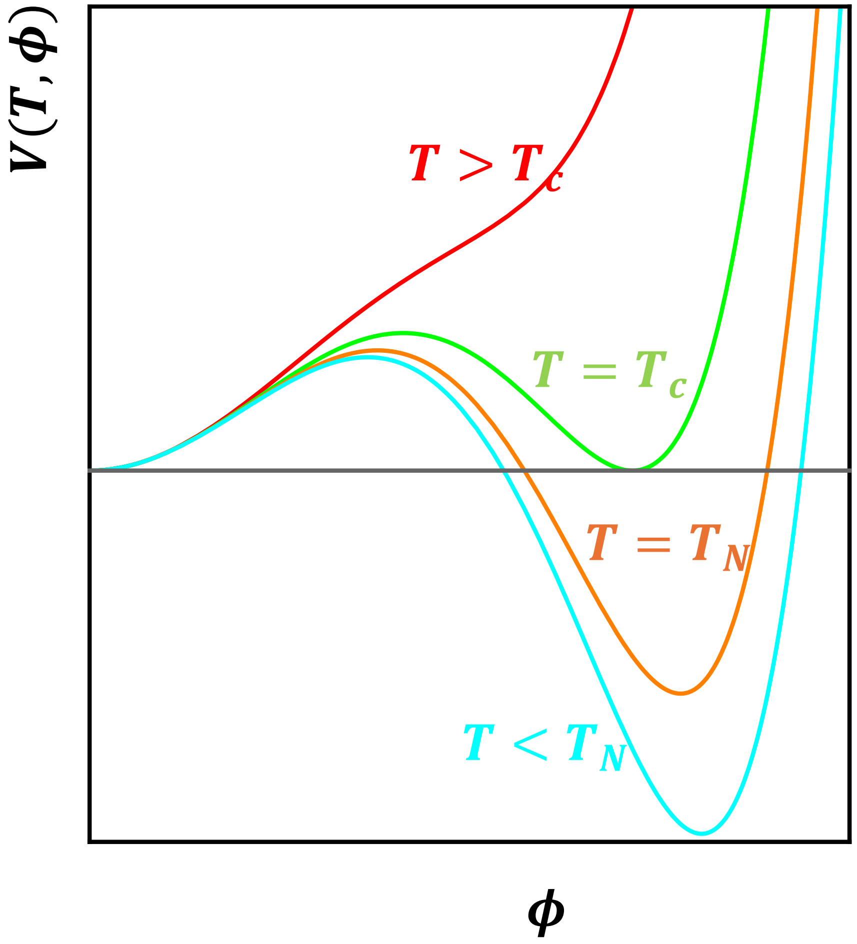
  is small!
- The evolution is mostly adiabatic.
- Phase transition happens when T goes across some typical energy scales.





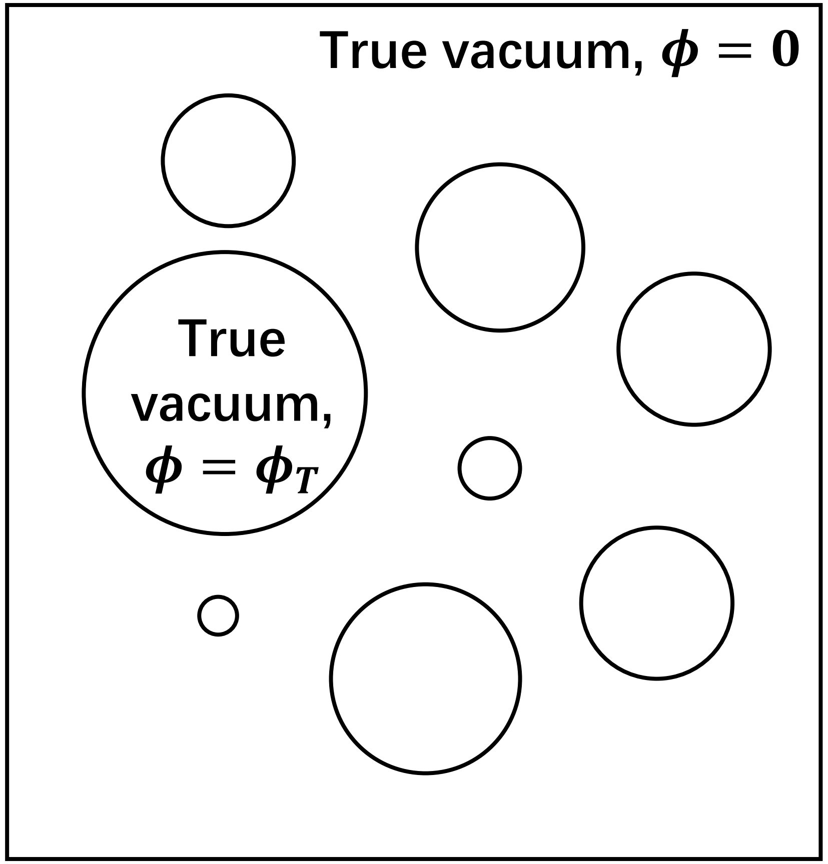
Gravitational waves from First-order phase transitions

- In cosmological phase transitions an (effective) scalar field transitions from false vacuum to true vacuum.



Phases separated by potential barrier

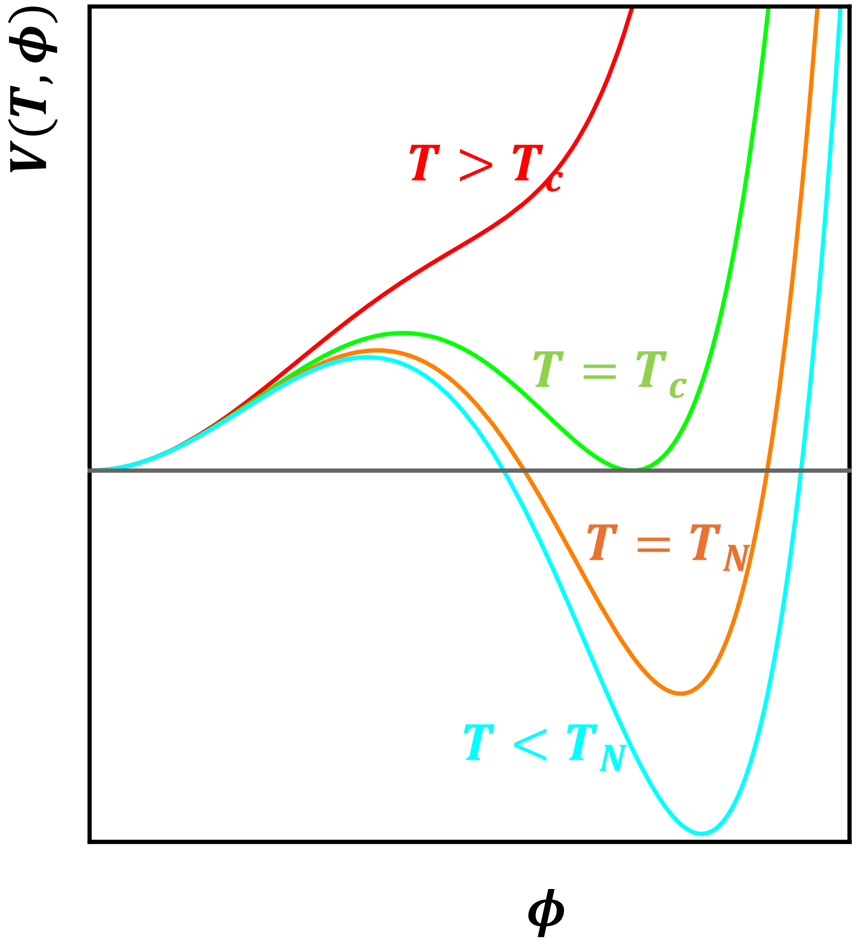
Bubbles of the true vacuum nucleate, expand, and collide.



Gravitational waves from First-order phase transitions

- Phase transition starts to complete when the Hubble expansion rate equals to the nucleation rate per Hubble volume.

Coleman 1977, Callan and Coleman 1977, Linde 1981



$$\Gamma/V = H^4 \text{ at } T = T_N$$

$$m^4 e^{-S_3/T}$$

$m \sim T_N$: energy scale of the system

S_3: 3D bounce action

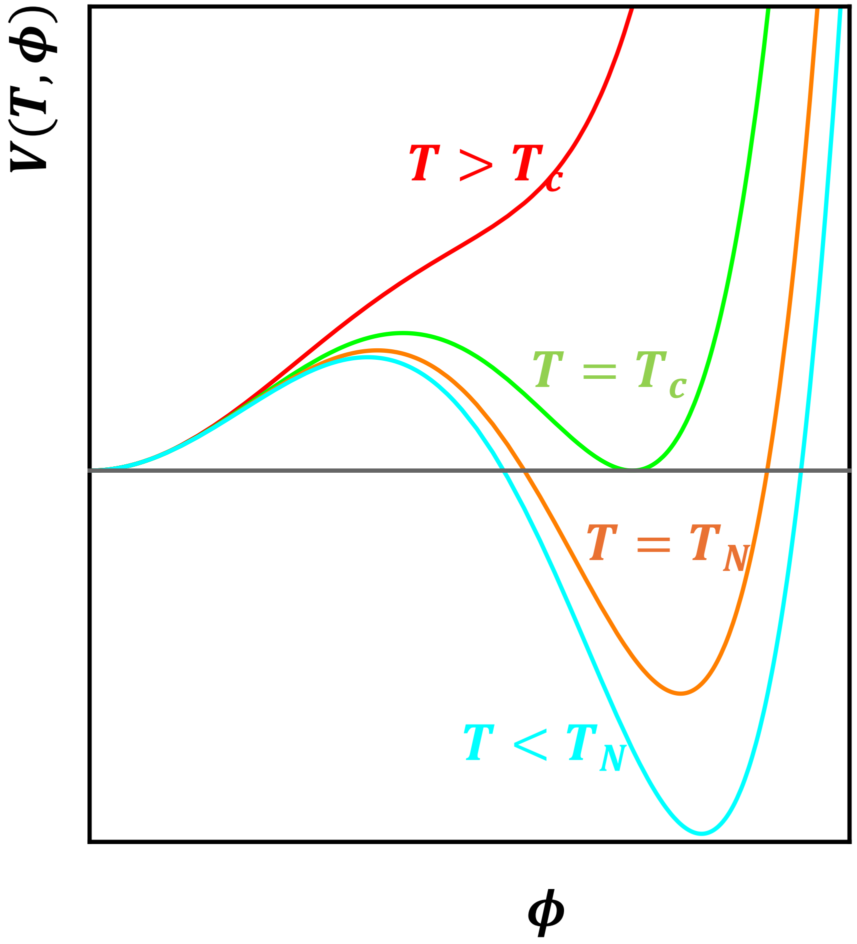
$$\frac{S_3}{T} = 4\ln\left(\frac{m}{H}\right)$$

$$\sim 4\ln\left(\frac{M_{pl}}{m}\right) \sim 150$$

for electroweak phase transitions

First-order phase transitions in the RD era

- Phase transition starts to complete at $T = T_N$, when the Hubble expansion rate equals to the nucleation rate per Hubble volume.



$$\frac{\Gamma}{V} = m^4 e^{-S_3/T}$$

$$\frac{S_3}{T} \approx \frac{S_3(T_N)}{T_N} + \beta(t - t_N)$$

$$\beta = -\frac{d}{dt}\left(\frac{S_3}{T}\right)_{T=T_N}$$

β determines scales of the phase transition:

(1) Phase transition completes $t \sim t_N + \beta^{-1}$.

(2) Bubble radius ~ β^{-1}.

(3) GW wavelength ~ β^{-1}.

First-order phase transitions in the RD era

- $\beta = -\frac{d}{dt}\left(\frac{S_3}{T}\right) = -\frac{dT}{Tdt}\frac{d}{d\log T}\left(\frac{S_3}{T}\right) = H\frac{d}{d\log T}\left(\frac{S_3}{T}\right) \sim H\left(\frac{S_3}{T}\right) \sim 150H$
- For first-order phase transitions happened in RD era: for EWPT

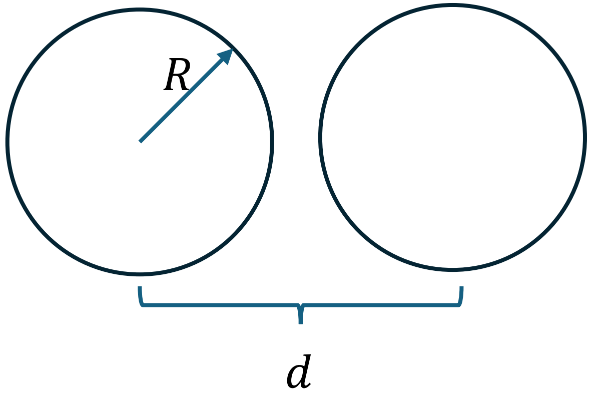
  β determines:
  - The duration of the phase transition
  - the typical value of bubble radius
  - the physical wavelength of the gravitational waves when it is produced.
- Today's wavelength
- $\lambda_{today} \sim \lambda_{phys}\left(\frac{T_N}{T_{CMB}}\right) \sim \frac{M_{pl}}{T_N^2} \times \left(\frac{S_3(T_N)}{T_N}\right)^{-1} \times \frac{T_N}{T_{CMB}} \sim 10^7\ \mathrm{km} \times \left(\frac{1\ \mathrm{TeV}}{T_N}\right)$

LISA, TAIJI, TIANQIN

GWs from first-order phase transiton in RD

- The metric around flat space-time $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$
- The gravitational waves are in h_{ij}^{TT}, the traceless and transverse part of h_{ij}.
- The gravitational potential are in h_{00}.
- In a relativistic system, we can use h_{00} to estimate h_{ij}^{TT}.
- The GW strength is controlled by:

$$\beta \sim H\left(\frac{S_3}{T}\right) \text{ and } \alpha = \frac{\Delta\rho}{\rho_R}$$



$R \sim d \sim \beta^{-1}$

$M \sim R^3 \Delta\rho$

Latent heat density

$V \sim R^3$

$\Phi \sim GM/R$

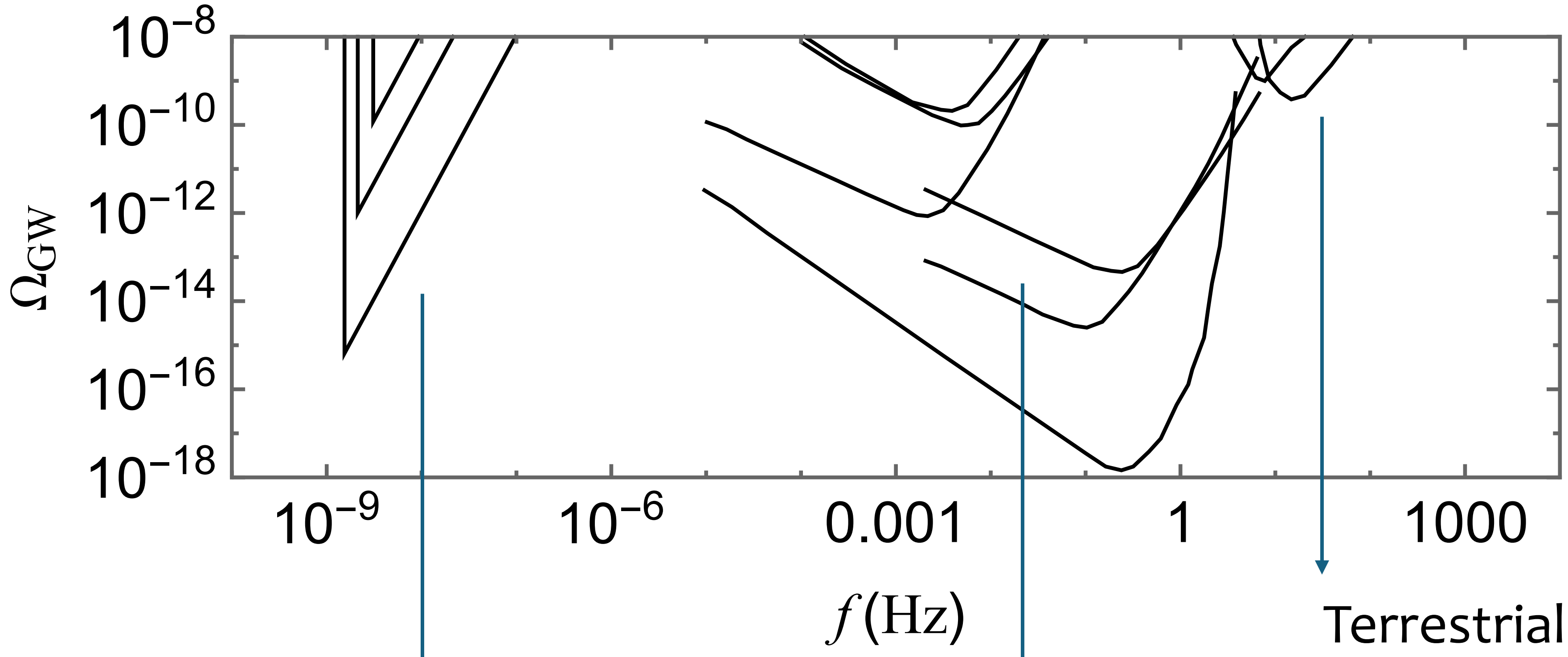
$$\rho_{GW} \sim \rho_0 \sim \frac{\Phi M}{V} \sim \frac{GM^2}{R^4} \sim G\Delta\rho^2 R^2 \sim \frac{G\Delta\rho^2}{\beta^2}$$

$$\Omega_{GW} = \Omega_R \times \frac{\rho_{GW}}{\rho_R} \sim \Omega_R \times \frac{G\Delta\rho^2}{\beta^2 \rho_R} \sim \Omega_R \times \frac{H^2}{\beta^2} \times \frac{\Delta\rho^2}{\rho_R^2}$$

$\Omega_R \approx 10^{-5}$

In RD: $H^2 \sim G\rho_R$

GW detectors



Pulsar timing arrays:
IPTA, NanoGrav, CPTA
…

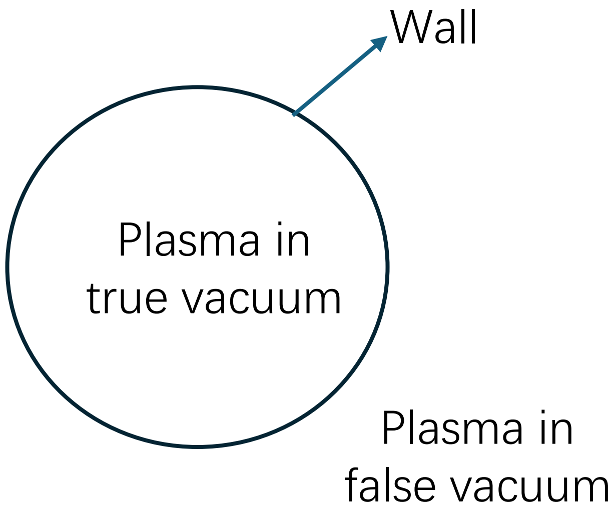
Space GW detectors:
LISA, TAIJI, TIANQIN,
DECIGO, BBO …

Terrestrial detectors:
Cosmic explorer,
Einstein Telescope
…

Sources for GWs

- Wall collision

$$h^2\Omega_{\rm env} = 1.67\times 10^{-5}\left(\frac{100}{g_*}\right)^{\frac{1}{3}}\left(\frac{H_*}{\beta}\right)^2\left(\frac{\kappa_\phi\alpha}{1+\alpha}\right)^2\frac{0.48v_w^3}{1+5.3v_w^2+5v_w^4}$$
$$\times\left[c_l\left(\frac{f}{f_{\rm env}}\right)^{-3}+c_m\left(\frac{f}{f_{\rm env}}\right)^{-1}+c_h\left(\frac{f}{f_{\rm env}}\right)\right]^{-1},$$



Jinno and Takimoto, 1707.03111

- Sound waves

$$\Omega_{\rm sw}h^2 \simeq 2.65\times 10^{-6}\Upsilon_{\rm sw}\left(\frac{H_*}{\beta}\right)\left(\frac{\kappa_v\alpha}{1+\alpha}\right)^2\left(\frac{100}{g_*}\right)^{1/3}v_w\,(f/f_{\rm sw})^3\left(\frac{7}{4+3(f/f_{\rm sw})^2}\right)^{7/2}$$

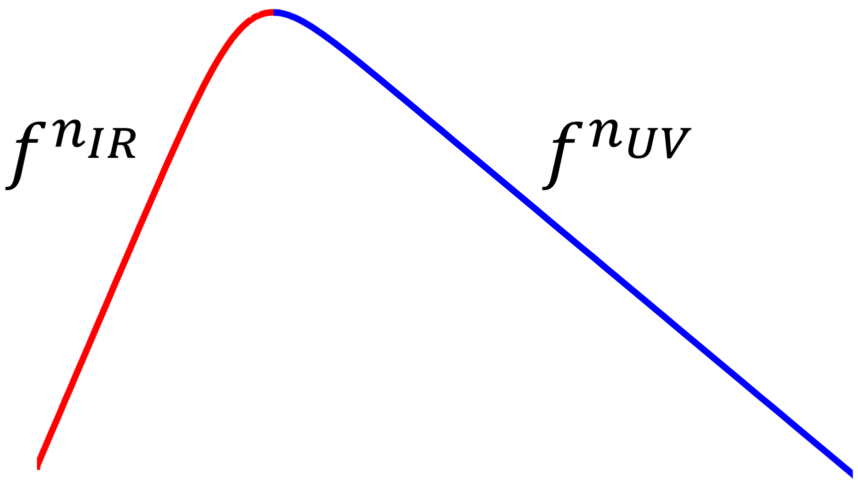
Hindmarsh, Huber, Rummukainen, Weir, 1704.05871

- Turbulence

$$\Omega_{\rm turb}h^2 \simeq 3.35\times 10^{-4}\left(\frac{H_*v_w}{\beta}\right)\left(\frac{\kappa_{\rm turb}\alpha}{1+\alpha}\right)^{3/2}\left(\frac{100}{g_*}\right)^{1/3}\frac{(f/f_{\rm turb})^3}{(1+f/f_{\rm turb})^{11/3}(1+8\pi f/H_*)}$$

Binetruy, Bohe, Caprini and Dufaux, 1201.0983

Features of the GW spectrum



Sources	$\boldsymbol{n_{UV}}$
Wall collision	-1
Sound waves	-7
Turbulence	-5/3

$n_{IR} = 3$

Due to causality argument

Cai, Pi, Sasaki, 1909.13728

v_w, κ_ϕ, κ_v, κ_{turb} are model dependent, and are very hard to calculate.

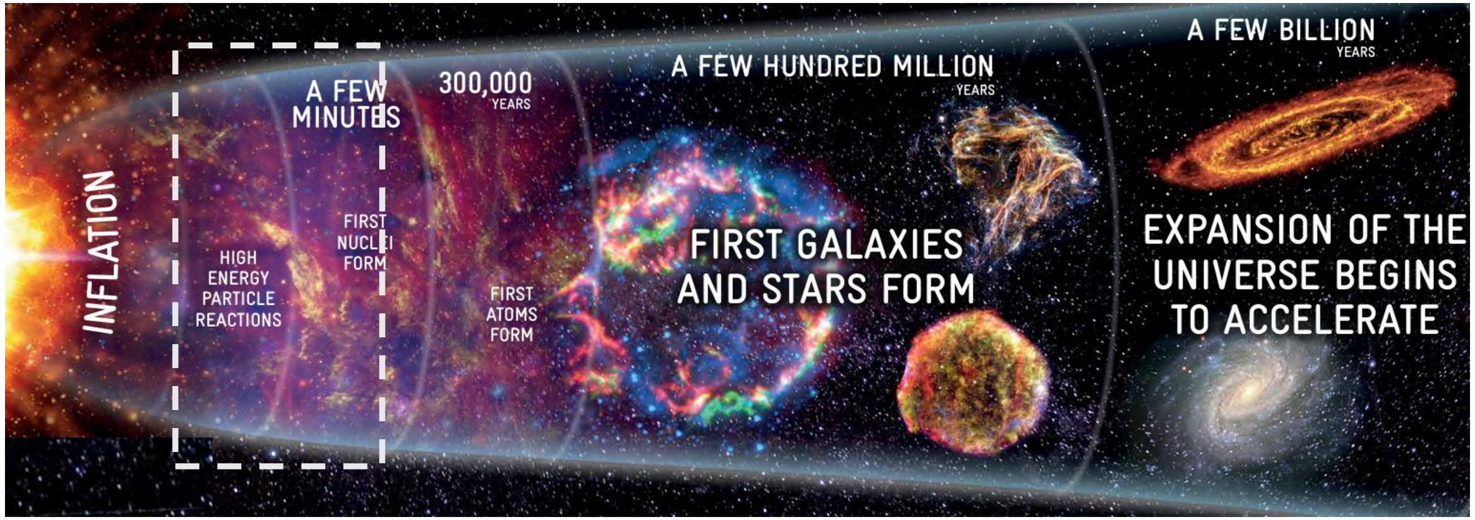
What is a phase transition?

- In cosmology phase transition usually means:

  Non-adiabatic (non-quasi-static) processes induced by adiabatic (quasi-static) evolution.

Cosmological phase transitions

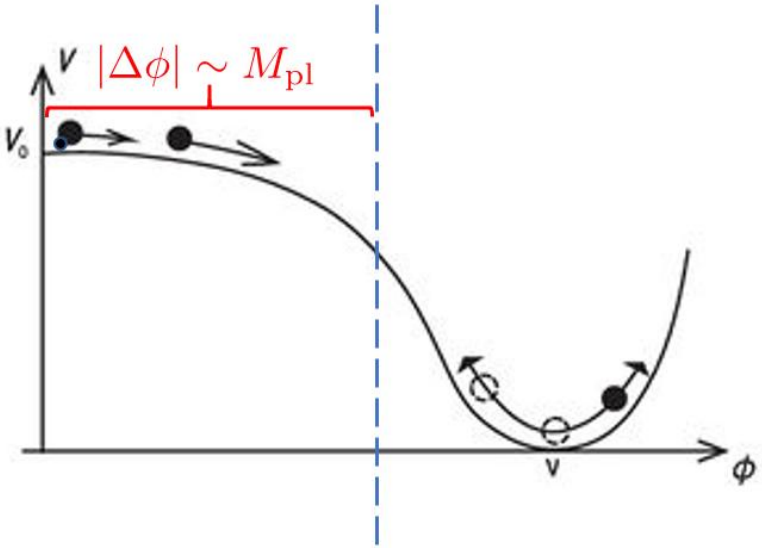
- Phase transitions happened in the thermal expansion.

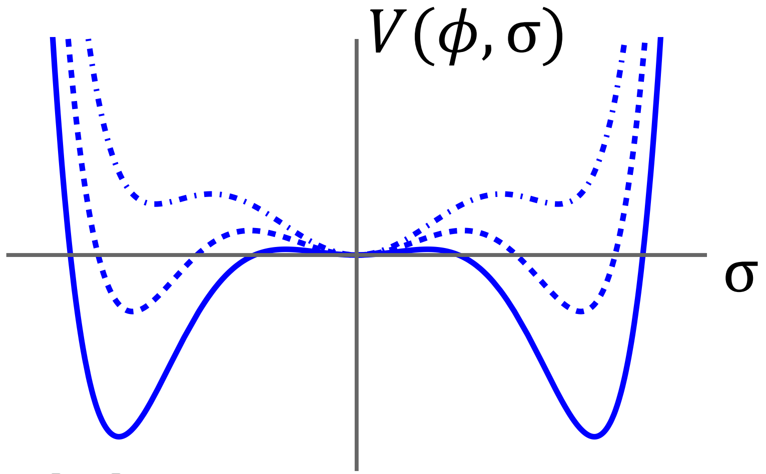


Phase transitions during inflation

- ϕ: inflaton field σ: spectator field

Example 1: $$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$





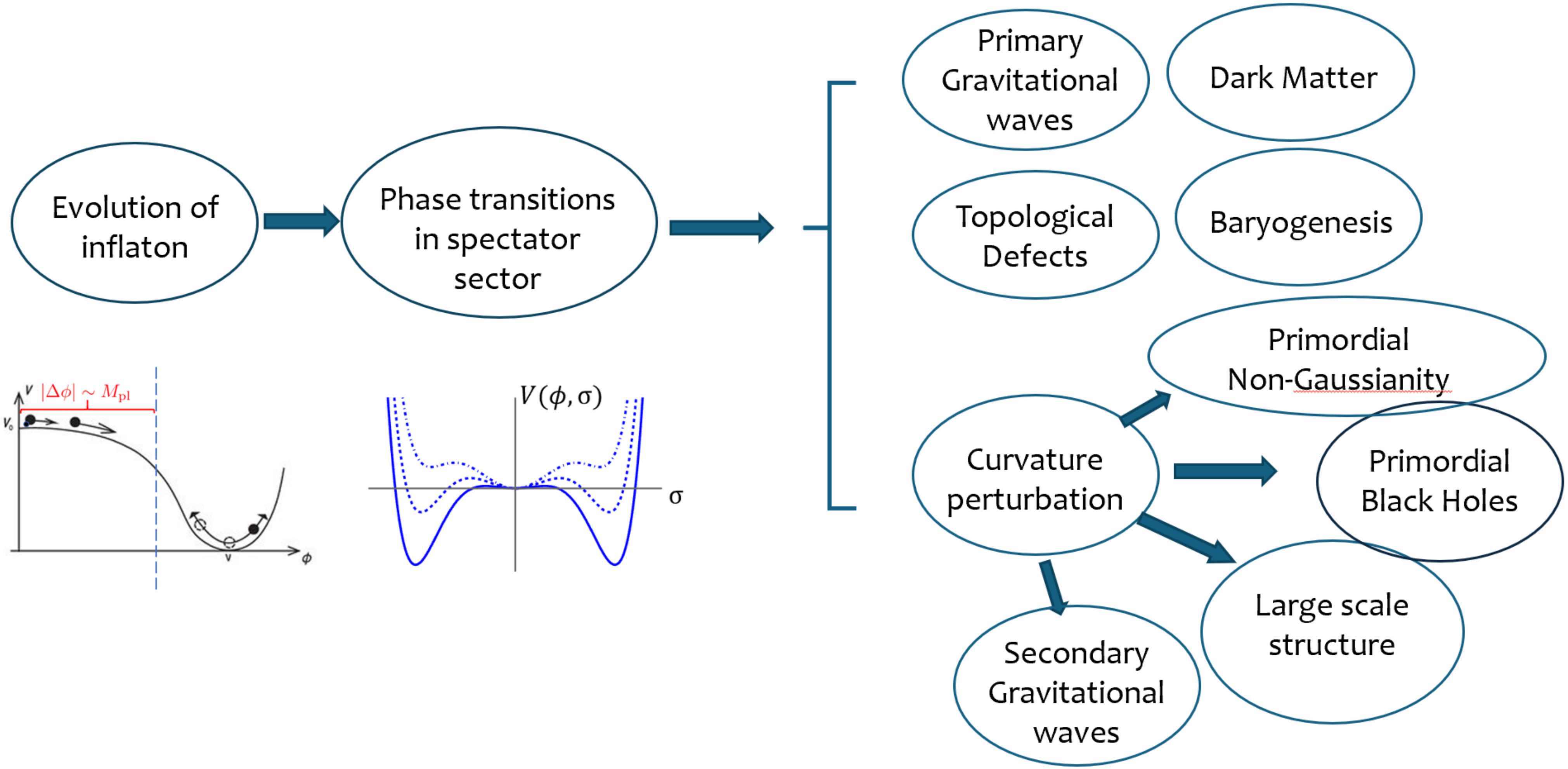
Example 2: $$\mathcal{L}_\sigma = -(1 - \frac{c^2\phi^2}{\Lambda^2})\frac{1}{4g^2}G^a_{\mu\nu}G^{a\mu\nu}$$

$f(\phi)$

Grand Unification | Baryogenesis | SUSY Breaking | Dark sector Phase transitions | …

Cosmological phase transitions

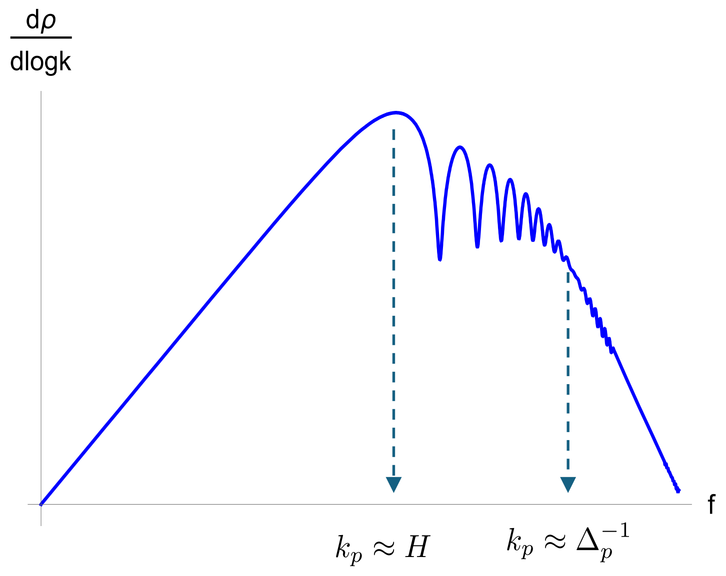
- Phase transitions during inflation



Primary GW vs secondar GW

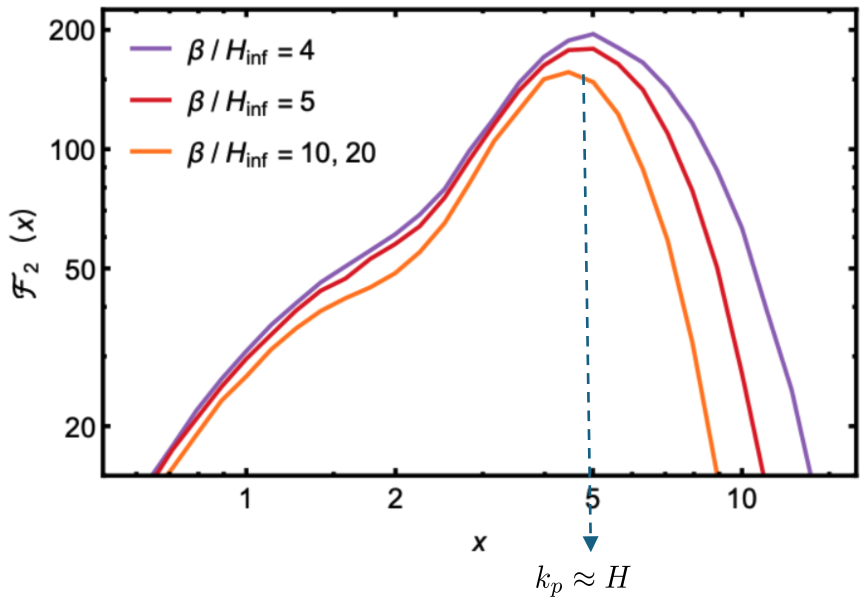
- Primary GW

$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$



- Secondary GW

$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{\mathcal{A}}{\epsilon}\right)^2 \left(\frac{M_{\rm pl}}{\phi_0}\right)^4 \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^4$$

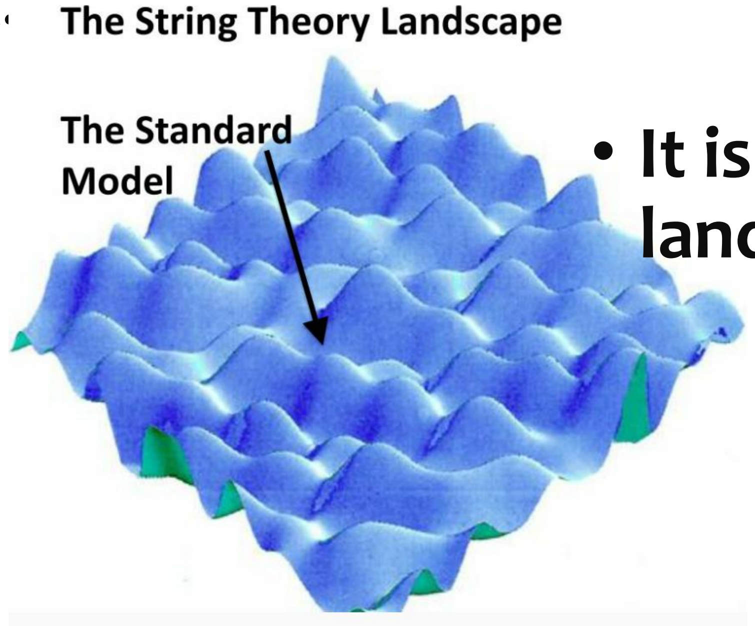


Dark sector meta stable vacuum decay

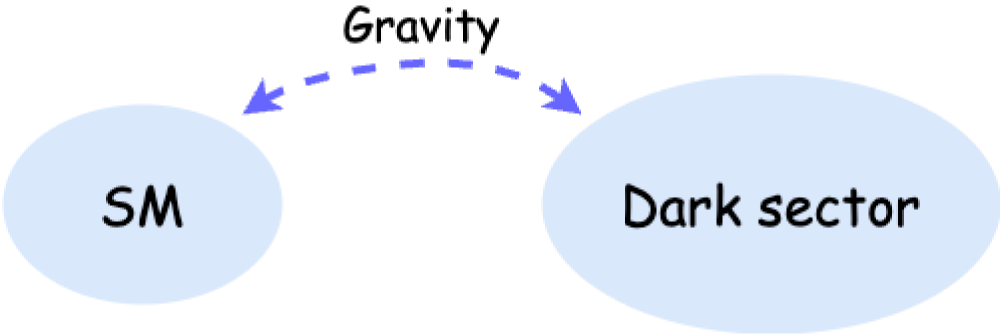
- The Hubble parameter decreases during the expansion of the Universe.
- It can influence all the sectors of the Universe.

Dark sector meta stable vacuum decay

- The **c**osmological **c**onstant problem is still the deepest mystery in theoretical physics!
- The **anthropic principle** may be the only solution.
- Landscape is required for anthropic principle to work.
- **String landscape** naturally provides the condition for anthropic principle.



- The dark matter may only couple to the SM sector via gravity.

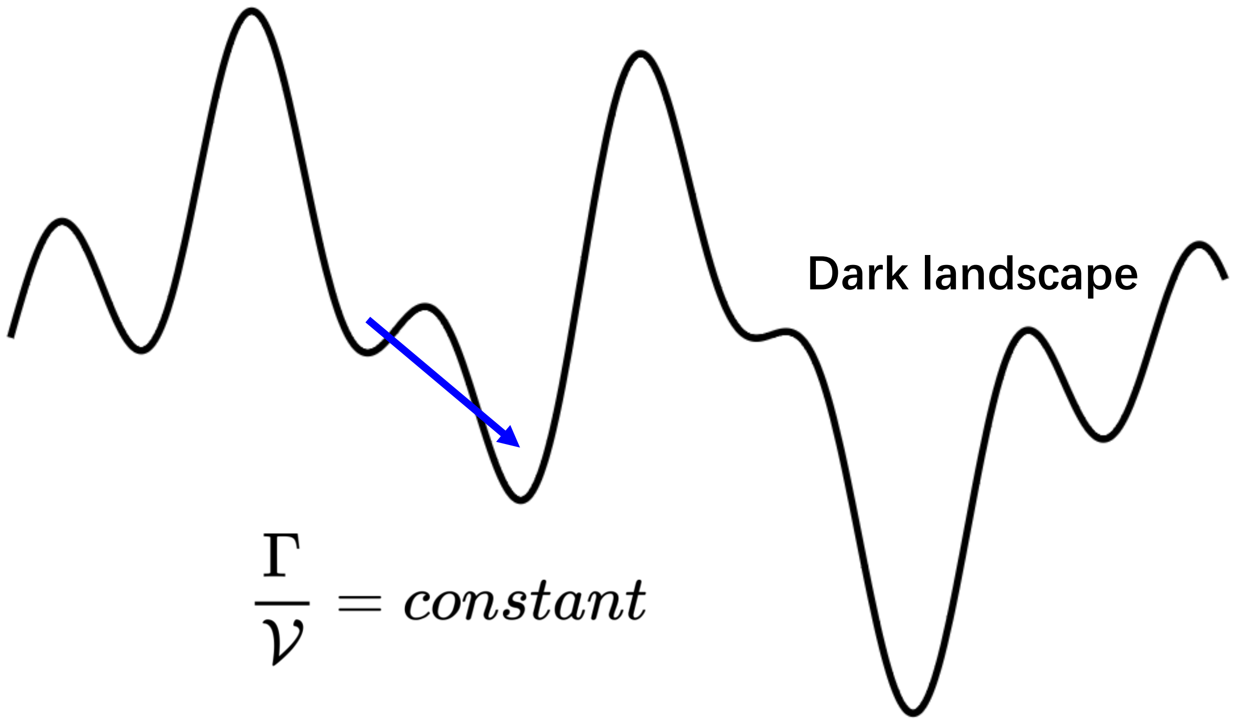


- **It is possible to have a dark landscape!**

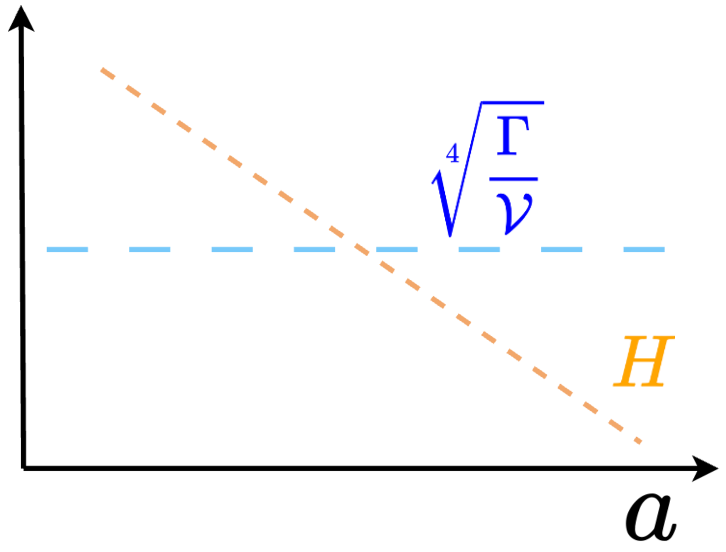
Signals from dark landscape

- Dark phase transitions

Phase transition condition: $\frac{\Gamma}{\mathcal{V}} \sim H^4$



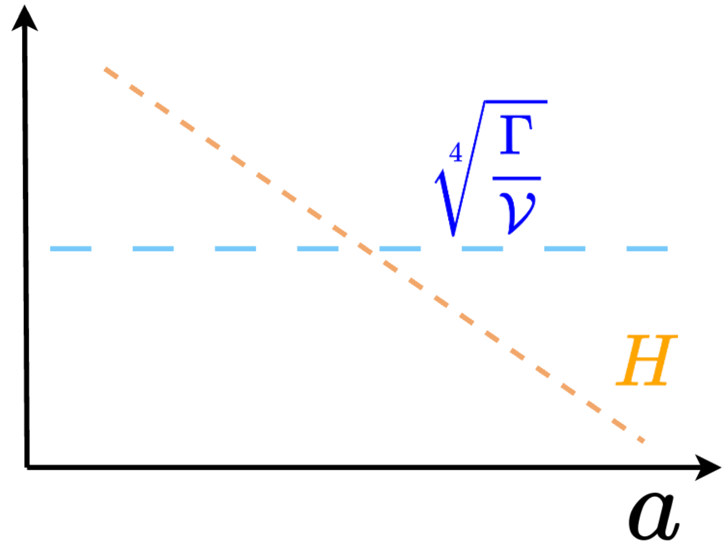
$$\frac{\Gamma}{\mathcal{V}} = constant$$



$$H \sim \frac{T^2}{M_{pl}} \sim a^{-2}$$

Properties of the GWs from DS vacuum decay

- No concept of β
- Bubble radius ~ H^{-1}
- $\Omega_{GW} \sim \Omega_R \times \frac{\Delta\rho^2}{\rho_R^2}$



Dark sector PT:

$$\frac{\Gamma}{\mathcal{V}} = constant$$

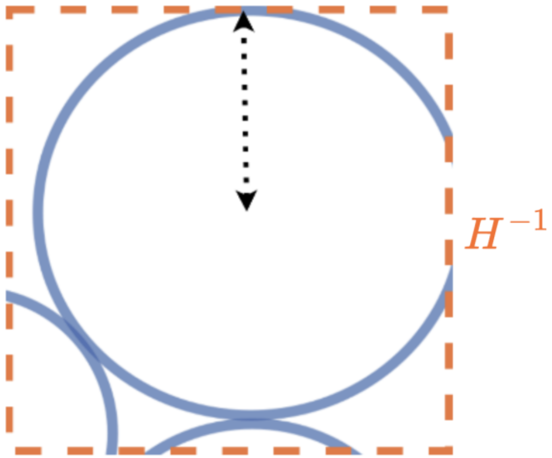
(No fluid, no plasma)

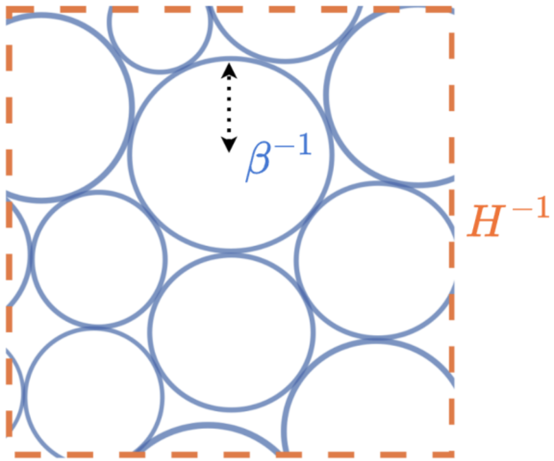
Thermal PT:

$$\frac{\Gamma}{\mathcal{V}} \sim \exp(\beta t)$$

Phase transition condition:

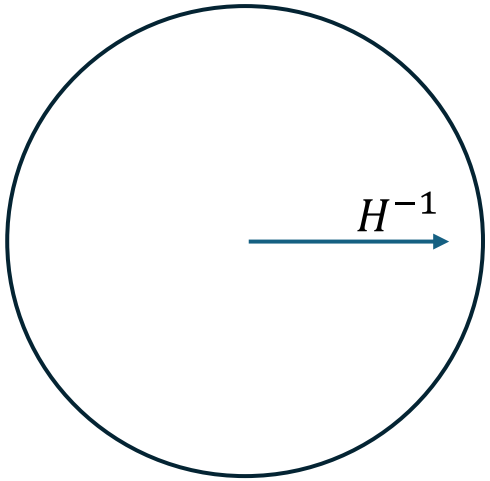
$$\frac{\Gamma}{\mathcal{V}} \sim H^4$$





Properties of the GWs from DS vacuum decay

- No plasma or fluid
- No frictions to bubble expansion
- The boost can be very large
- The walls are very thin.



Energy release: $E \sim \frac{4\pi}{3} H^{-3} \Delta\rho$

Rest energy of the wall: $4\pi H^{-2} m^3 \sim 4\pi H^{-2} \Delta\rho^{3/4}$

Boost: $\gamma \sim \frac{\Delta\rho^{\frac{1}{4}}}{H} \sim \left(\frac{\Delta\rho}{\rho}\right)^{\frac{1}{4}} \frac{M_{pl}}{T} \sim 10^{15} \left(\frac{\Delta\rho}{\rho}\right)^{\frac{1}{4}}$ (EW)

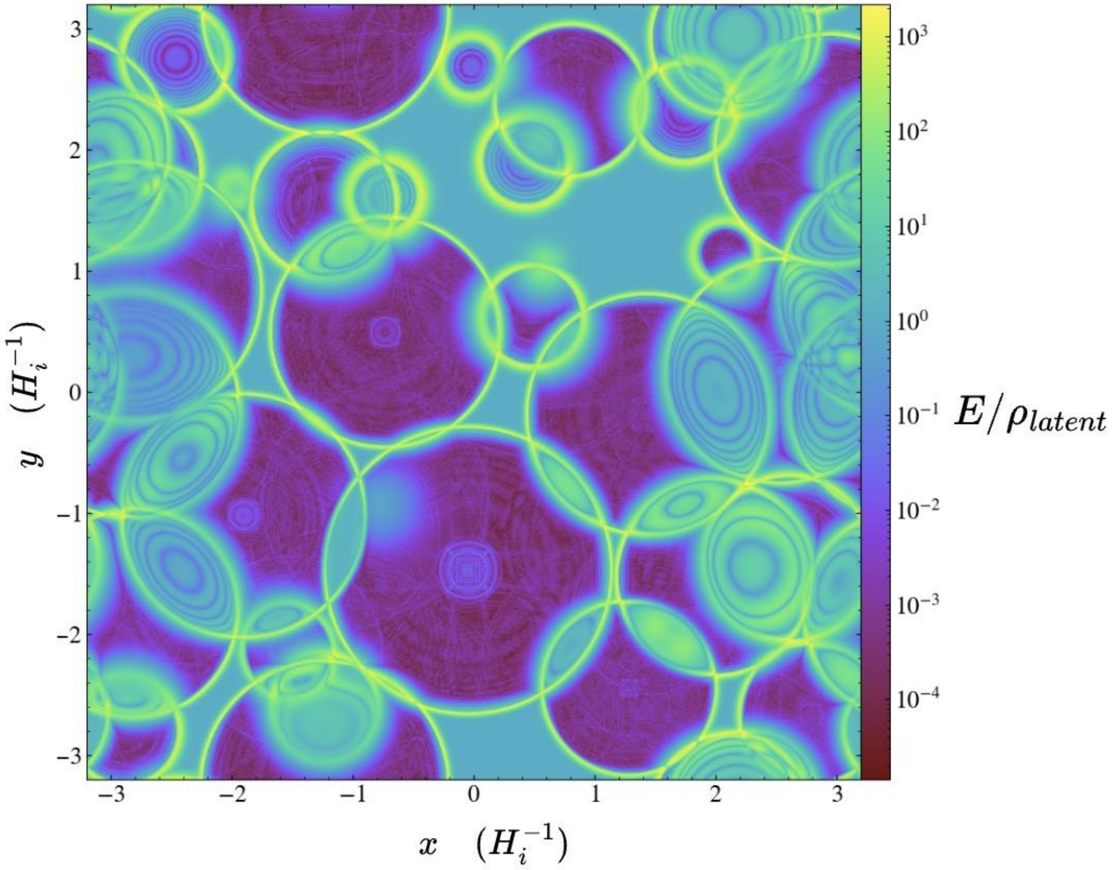
Properties of the GWs from DS vacuum decay

- Long lived shell structure after collision
- EoM of the scalar field:

$$\frac{\partial^2 \phi}{\partial \eta^2} + 2\mathcal{H}\frac{\partial \phi}{\partial \eta} - \frac{1}{r}\frac{\partial^2}{\partial r^2}(r\phi) + a^2\frac{\mathrm{d}V}{\mathrm{d}\phi} = 0$$

γ^2 γ^1 γ^2 γ^0

- The shells still form spherical structures.
- Potential terms are negligible.
- Subsequent collisions are not important.



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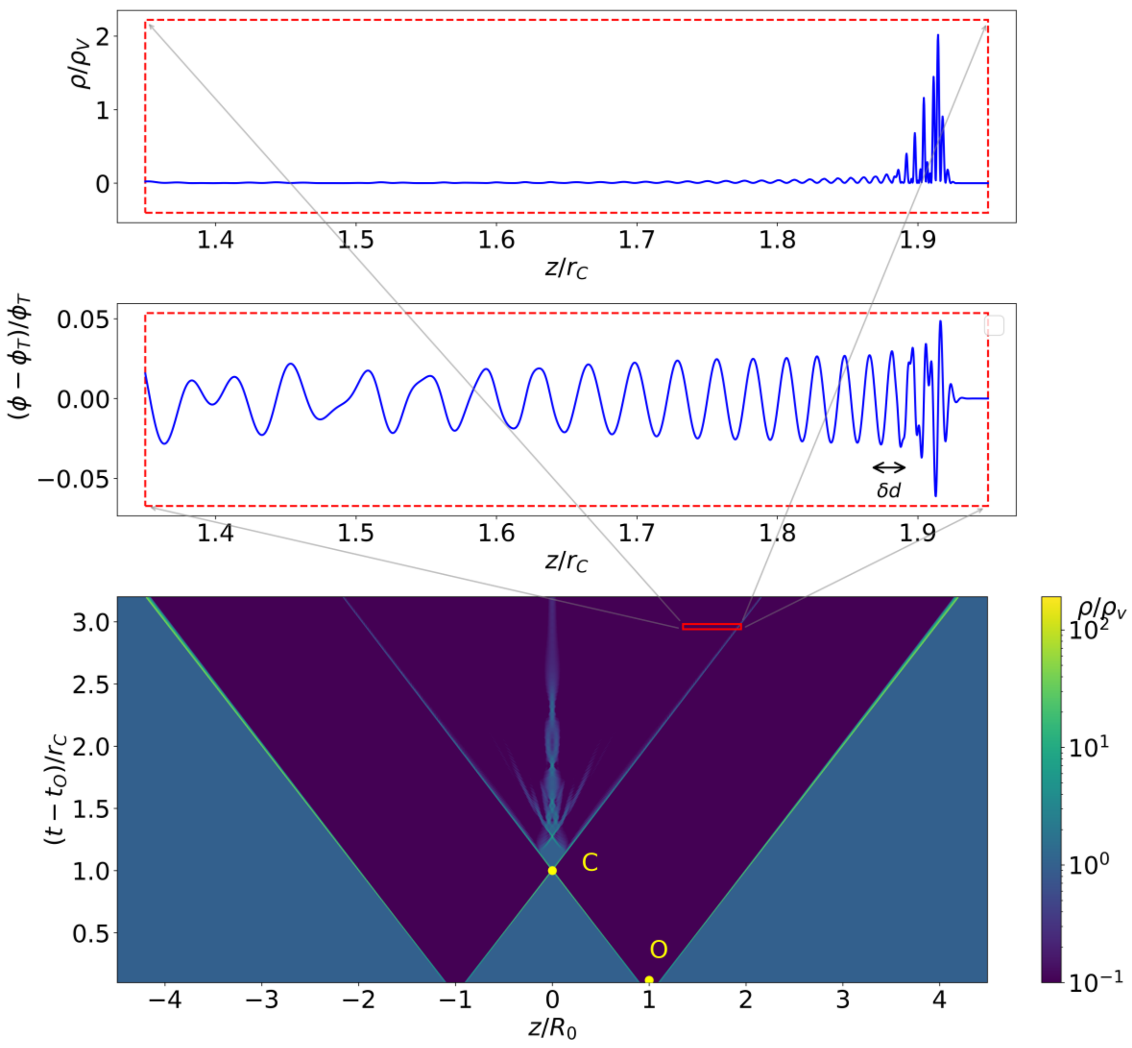
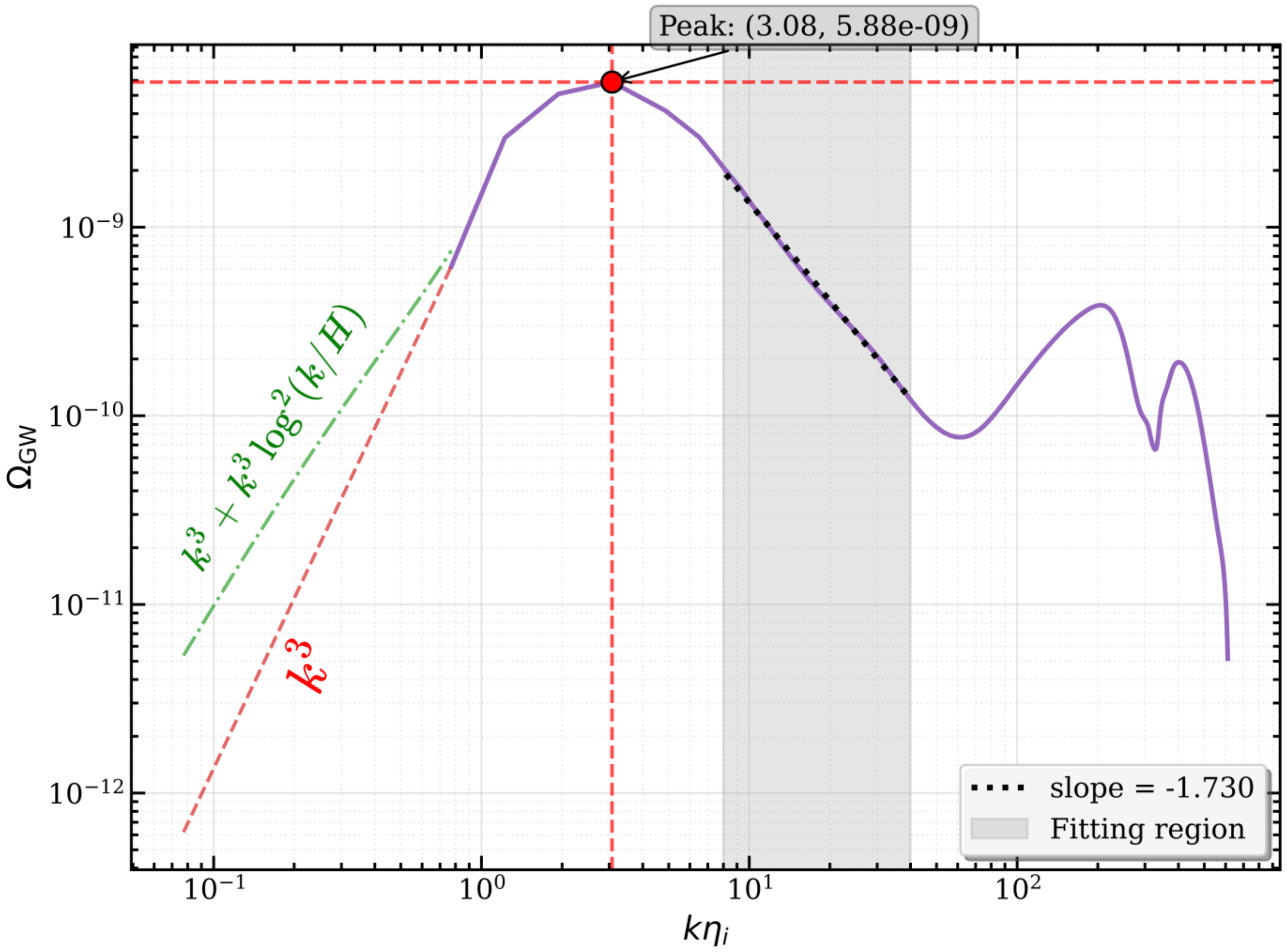


Properties of the GWs from DS vacuum decay

- Results:
- IR: $$\frac{d\rho_{\mathrm{GW}}^{\mathrm{IR}}}{d\ln k} \propto k^3 \log^2 \frac{k}{H_0} + O\left(k^3\right)$$
- UV: $$\Omega_{GW} \propto k^{-1.73}$$
- Peak: $$k_{peak}/H = 3.08$$
- $$\Omega_{\mathrm{GW}}^{\mathrm{peak}} = 1.5\Omega_R \left(\frac{L}{\rho}\right)^2$$



GW power spectrum from 2048^3 simulation

HA, Tingyu Li, Chen Yang, 2601.xxxxx

Summary

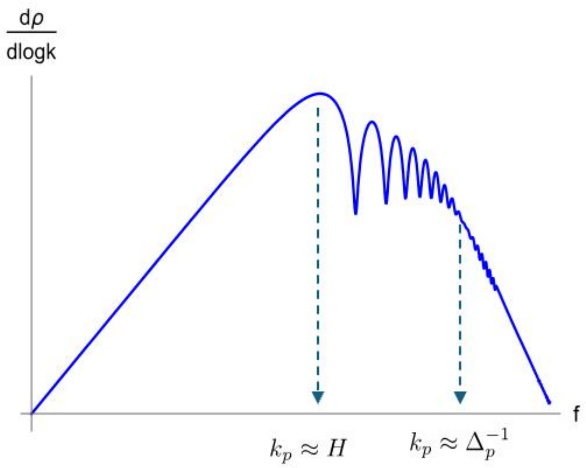
- Thermal first-order phase transition in RD:

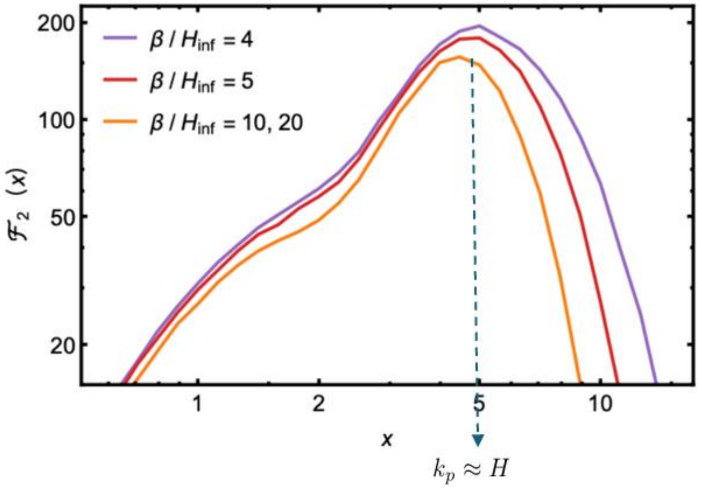
  IR spectrum: k^3, UV spectrum: model dependent, $\Omega_{\rm GW}^{\rm peak} \sim \Omega_R \left(\frac{H}{\beta}\right)^2 \left(\frac{\Delta\rho}{\rho}\right)^2$

- First-order phase transition during inflation:

$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{\mathcal{A}}{\epsilon}\right)^2 \left(\frac{M_{\rm pl}}{\phi_0}\right)^4 \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^4$$





Summary

- Dark sector meta stable vacuum decay:

No concept of β

Bubble radius ~ H^{-1}

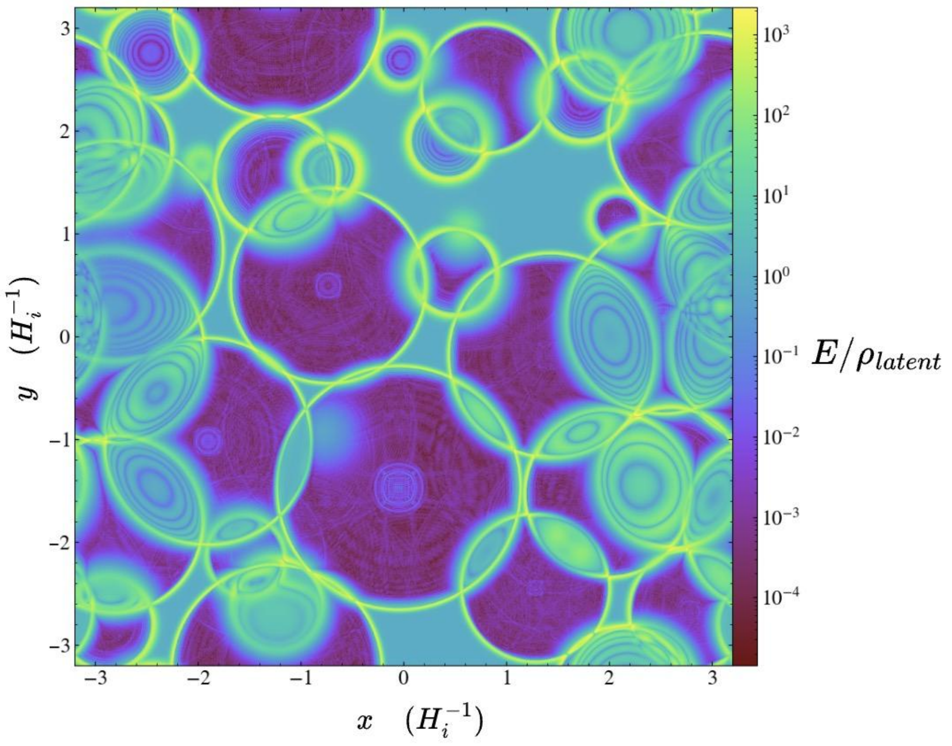
$$\Omega_{GW} \sim \Omega_R \times \frac{\Delta\rho^2}{\rho_R^2}$$

Very large boost bubble wall: $\gamma \sim \frac{\Delta\rho^{\frac{1}{4}}}{H} \sim \left(\frac{\Delta\rho}{\rho}\right)^{\frac{1}{4}} \frac{M_{pl}}{T} \sim 10^{15} \left(\frac{\Delta\rho}{\rho}\right)^{\frac{1}{4}}$ （EW）

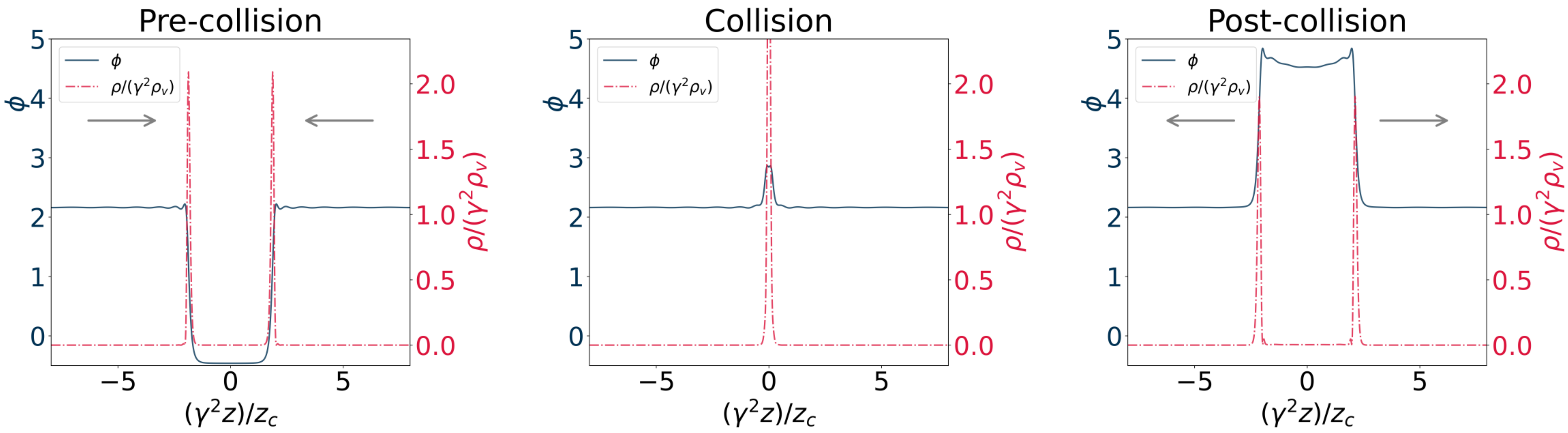
Peak frequency: $k_{\text{peak}} \approx 3.1\, H$

GW IR spectrum:

$$\frac{d\rho_{\text{GW}}^{\text{IR}}}{d\ln k} \propto k^3 \log^2 \frac{k}{H_0} + O\left(k^3\right)$$



Simulation of two bubble collision



$\boldsymbol{H}$

- The Hubble parameter evolves during the thermal expansion of the Universe.
- It can influence all the sectors of the Universe.

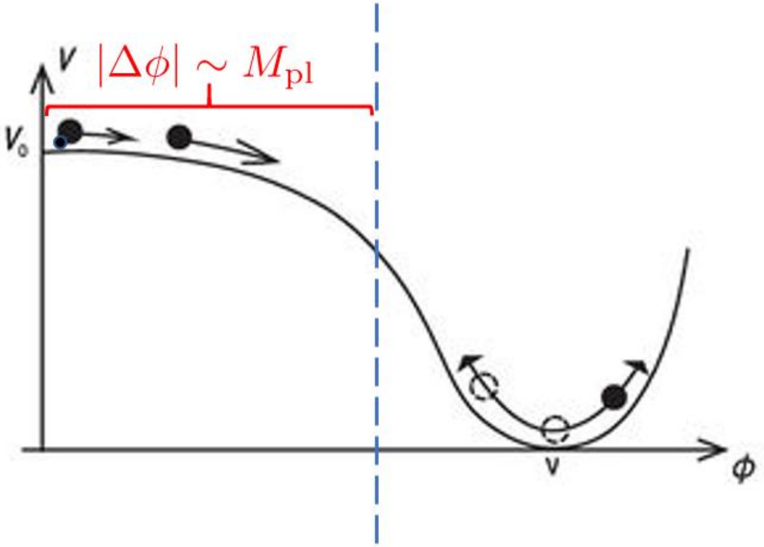
$f(\phi)$

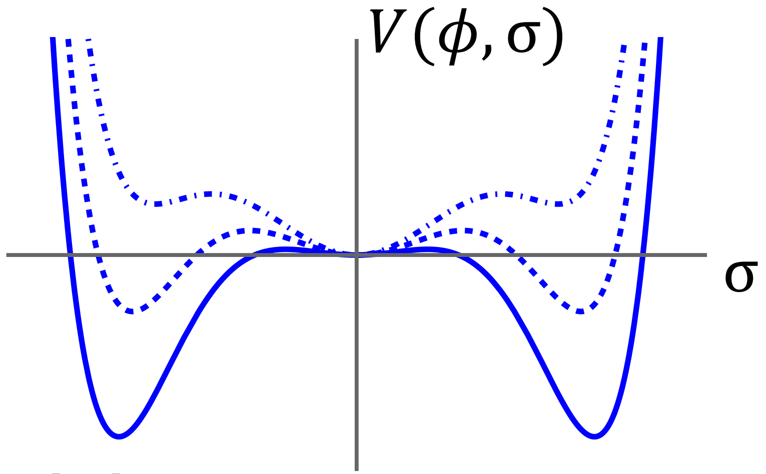
Grand Unification | Baryogenesis | SUSY Breaking | Dark sector Phase transitions | …

Phase transitions during inflation

- ϕ: inflaton field $\qquad$ σ: spectator field

Example 1: $$V_1(\phi,\sigma) = -\frac{1}{2}(\mu^2 - c^2\phi^2)\sigma^2 + \frac{\lambda}{4}\sigma^4 + \frac{1}{8\Lambda^2}\sigma^6$$



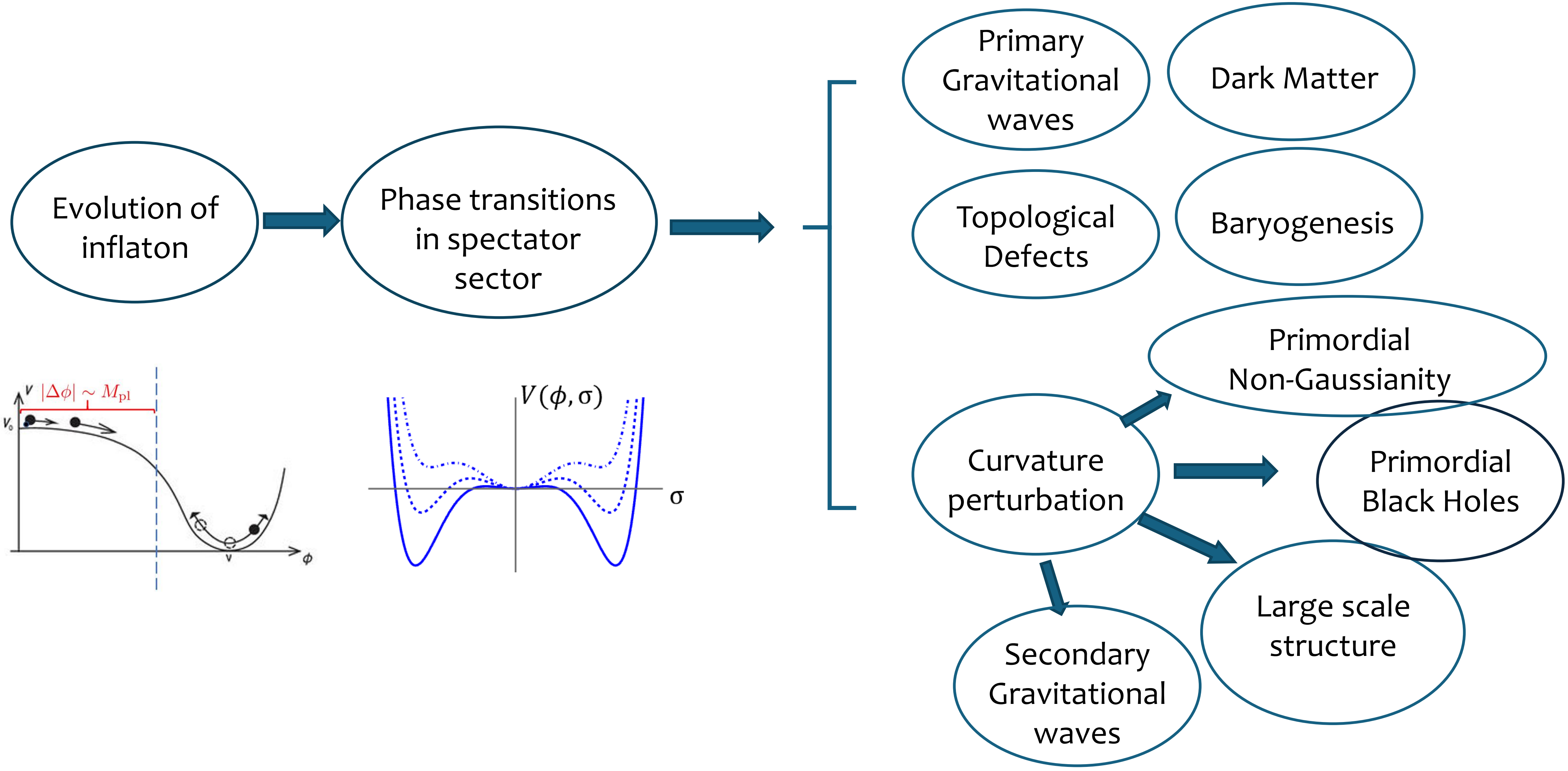


Example 2: $$\mathcal{L}_\sigma = -(1 - \frac{c^2\phi^2}{\Lambda^2})\frac{1}{4g^2}G^a_{\mu\nu}G^{a\mu\nu}$$

$f(\phi)$

Grand Unification | Baryogenesis | SUSY Breaking | Dark sector Phase transitions | …

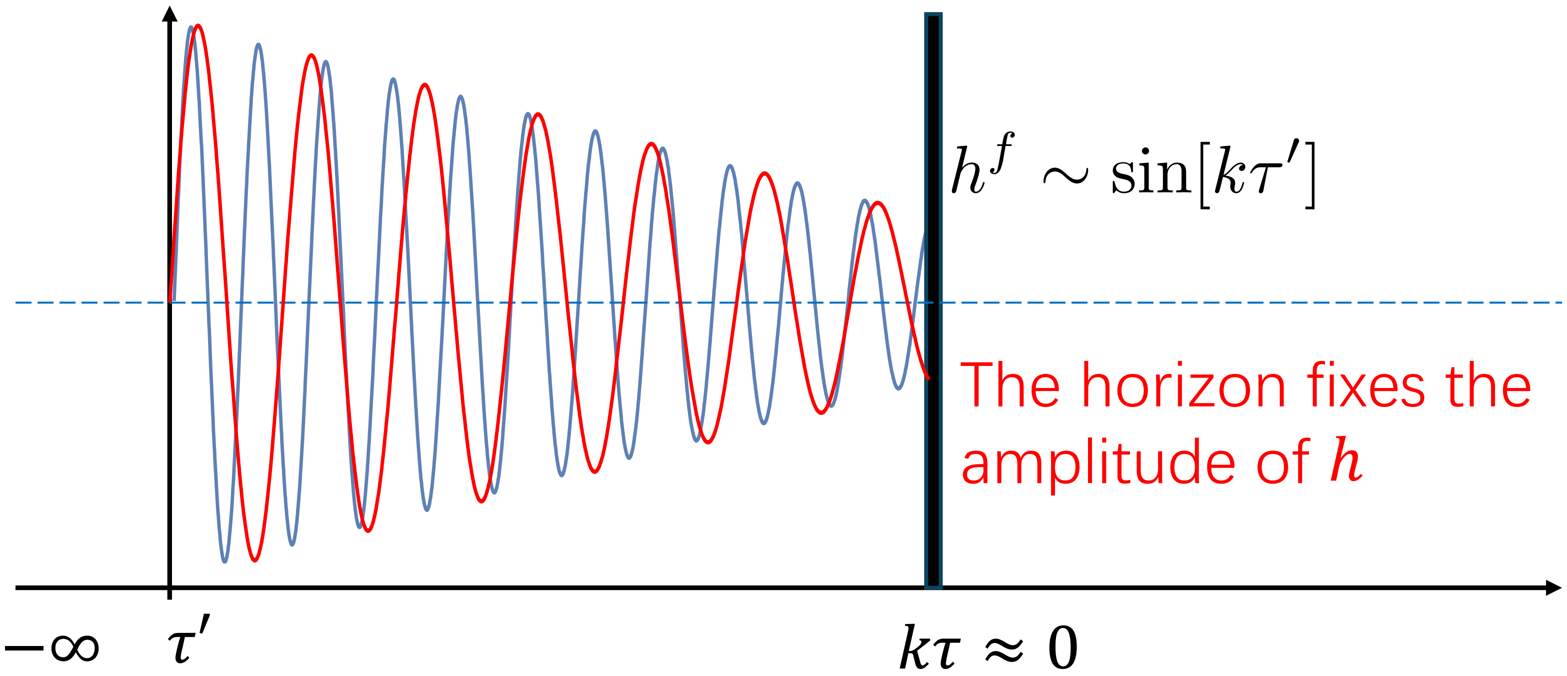
Consequences of the phase transitions



GWs from first-order phase transitions during inflation

- Linearized Einstein Equation:

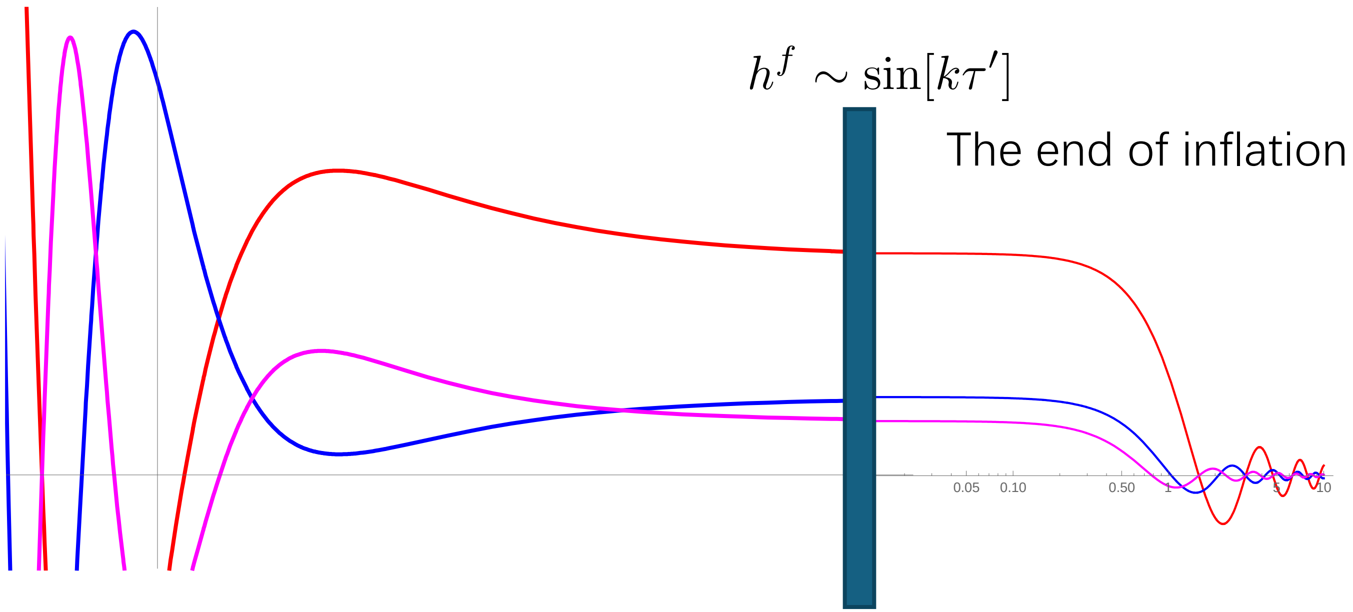
$$h''(\tau,\mathbf{k}) + \frac{2a'}{a}h'(\tau,\mathbf{k}) + k^2 h(\tau,\mathbf{k}) = 16\pi G_N a^{-1} T\delta(\tau-\tau')$$

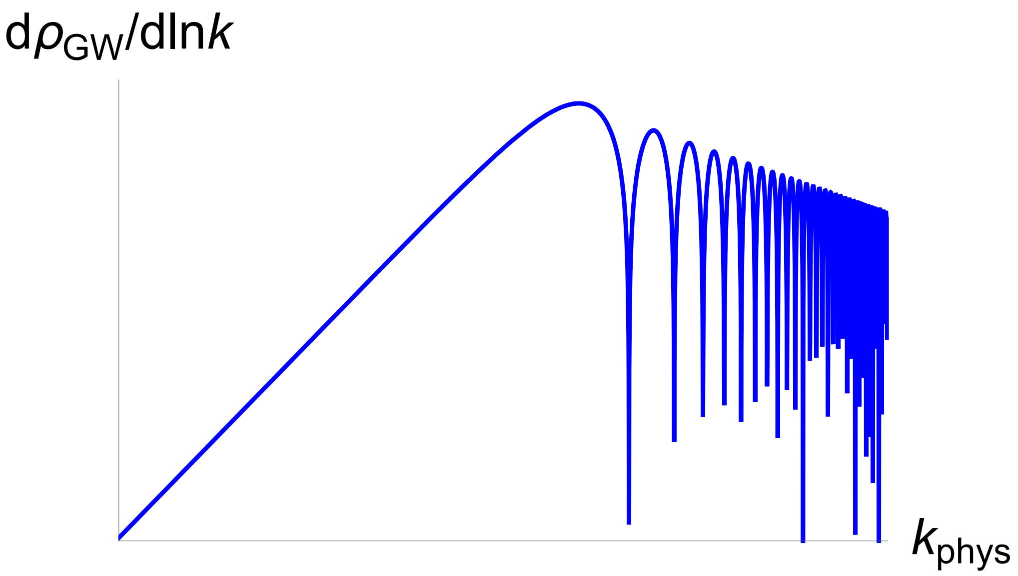


$g_{\mu\nu} = g_{\mu\nu}^{(0)} + h_{\mu\nu}$
GW is h_{ij}^{TT}.

GW spectrum from instantaneous source

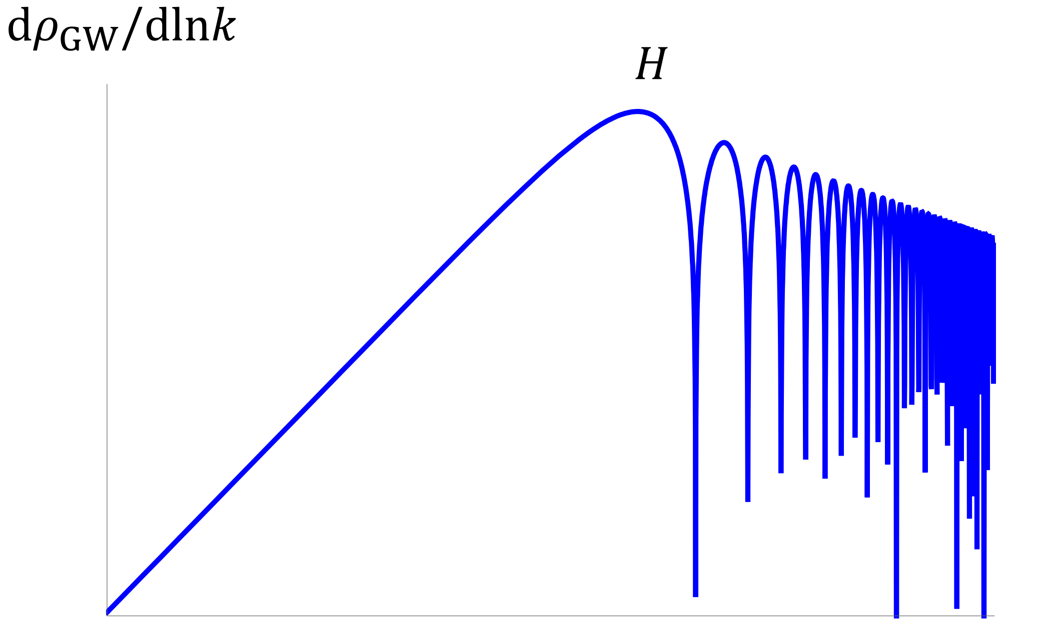
- $h^f(k)$ is the initial amplitude for the GW oscillation after inflation.
- All the modes start to oscillate with the same phase.
- Example, in RD, the oscillation is $\sin k\tau / k\tau$.



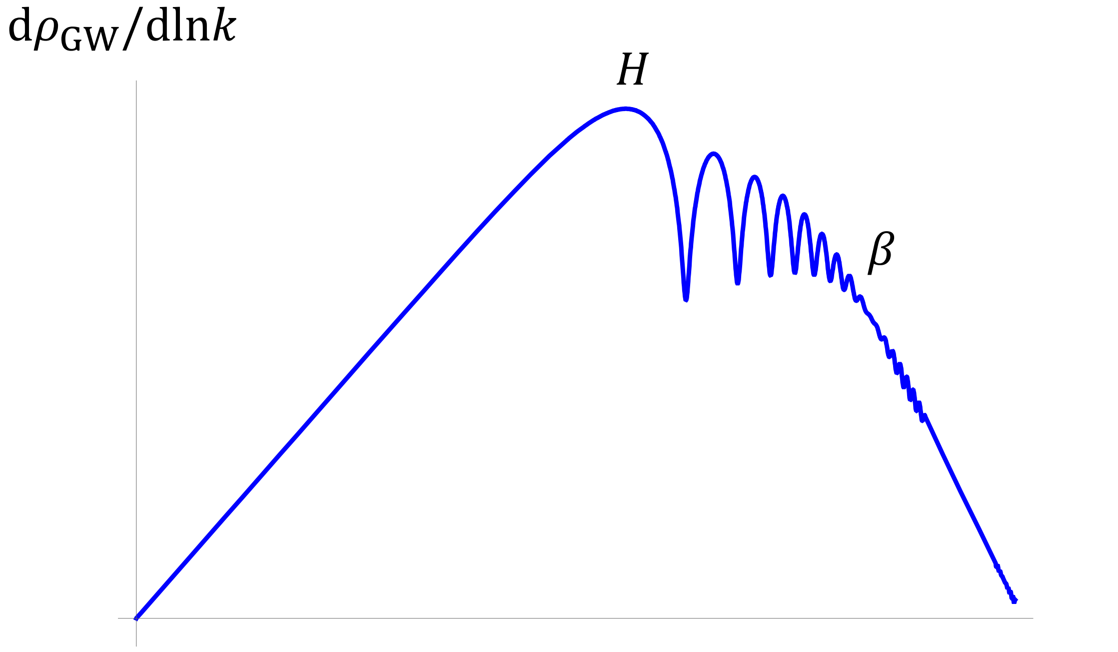


GW spectrum from first-order phase transitions

- For phase transition to complete, $\beta = -\frac{dS_b}{dt} \gg H$.



Instantaneous source



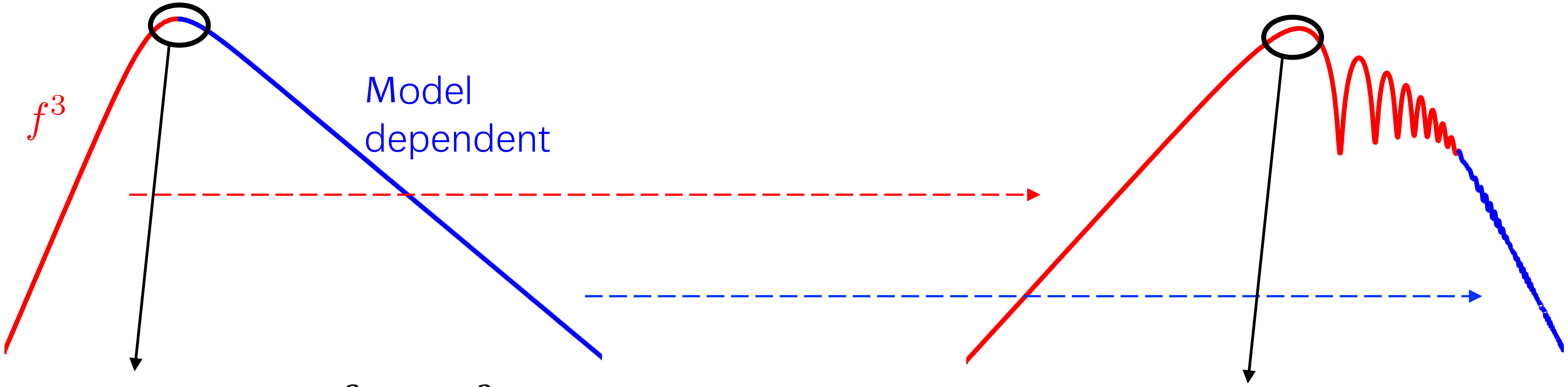
First-order phase transition

Features of GW spectrum from FOPT during inflation

- The oscillatory feature

GW produced during RD

GW produced during inflation



$$\Omega_{\mathrm{GW}} \approx \Omega_R \left(\frac{H}{\beta}\right)^2 \left(\frac{L}{\rho_{\mathrm{inf}}}\right)^2 \approx 10^{-9} \left(\frac{H}{0.01\beta}\right)^2 \left(\frac{L}{\rho_{\mathrm{inf}}}\right)^2$$

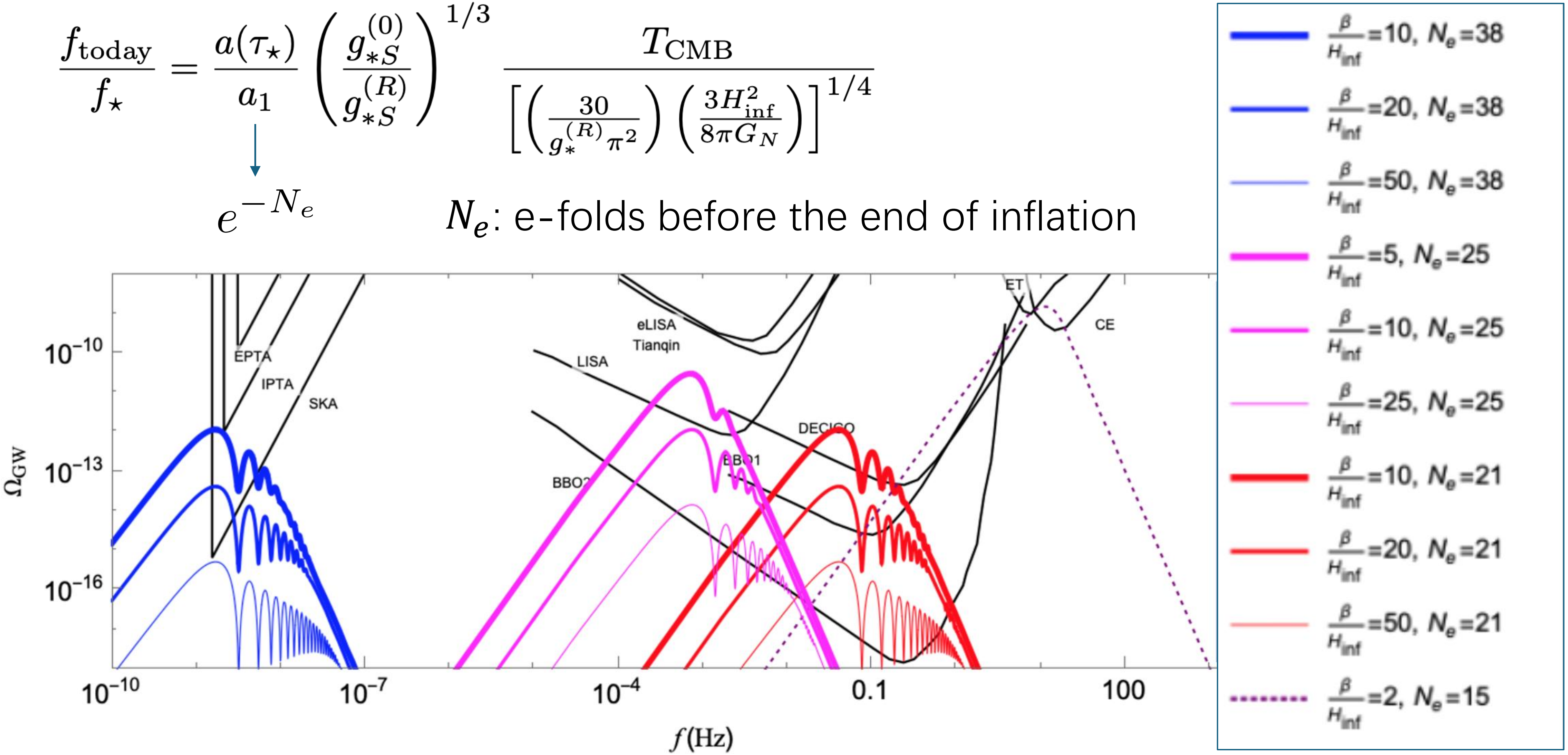
$$\Omega_{\mathrm{GW}} \approx \Omega_R \left(\frac{H}{\beta}\right)^5 \left(\frac{L}{\rho_{\mathrm{inf}}}\right)^2 \approx 10^{-12} \left(\frac{H}{0.1\beta}\right)^5 \left(\frac{L}{\rho_{\mathrm{inf}}}\right)^2$$

Frequency of GW from FOPT during inflation

$$\frac{f_{\rm today}}{f_\star} = \frac{a(\tau_\star)}{a_1}\left(\frac{g_{*S}^{(0)}}{g_{*S}^{(R)}}\right)^{1/3} \frac{T_{\rm CMB}}{\left[\left(\frac{30}{g_*^{(R)}\pi^2}\right)\left(\frac{3H_{\rm inf}^2}{8\pi G_N}\right)\right]^{1/4}}$$

$\frac{a(\tau_\star)}{a_1} \to e^{-N_e}$

N_e: e-folds before the end of inflation



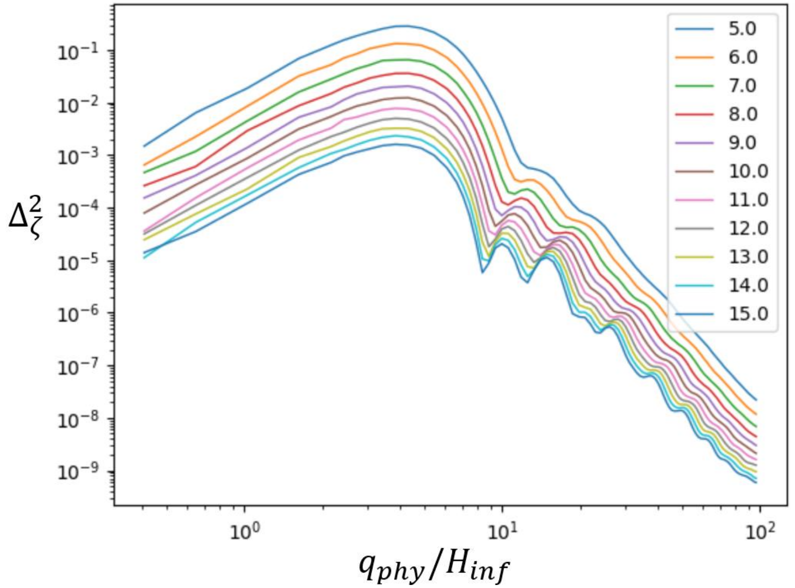
HA, Kun-Feng Lyu, Lian-Tao Wang, Siyi Zhou, 2009.12381, 2201.05171

Induced curvature perturbations

- The phase transition sector induces a back reaction to the inflaton sector

$$\mathcal{L} = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V(\phi,\sigma)$$

$$V(\phi,\sigma) = V_0(\phi) + V_1(\phi,\sigma) \xrightarrow{\phi = \phi_0 + \delta\phi} \frac{\partial V_1}{\partial \phi_0}\delta\phi$$



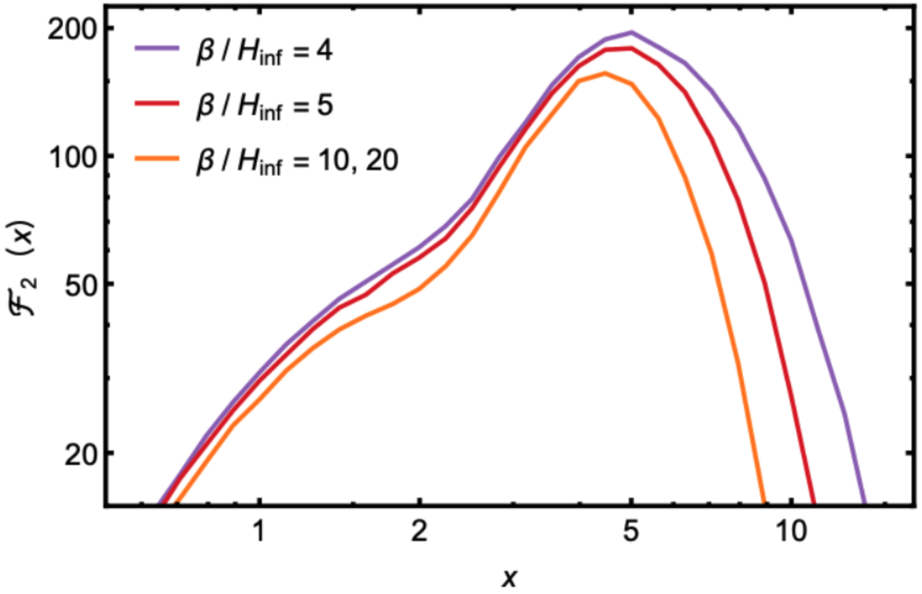
- Curavature perturbation

$$\zeta_{\mathbf{q}} = -\tilde{\Psi}_{\mathbf{q}} - \frac{H_{\rm inf}\delta\tilde{\phi}_{\mathbf{q}}}{\dot{\phi}_0}$$

- Scalar induced GWs

Expand the Einstein Equation to second order

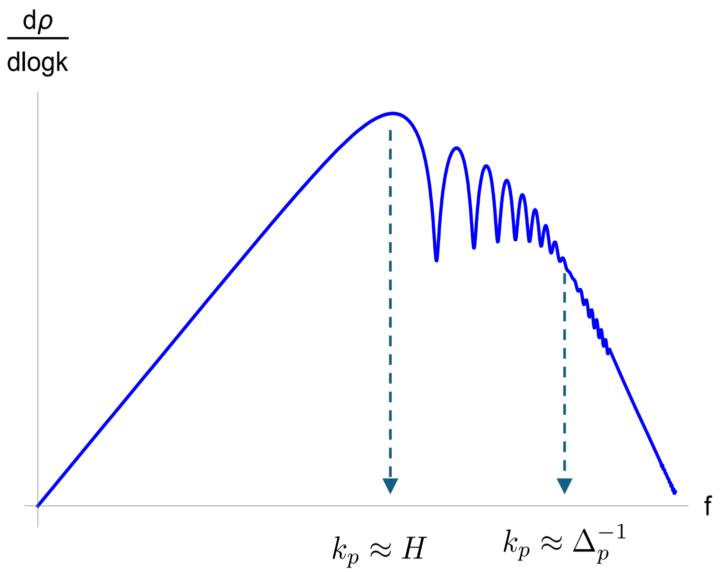
$$h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = -4\hat{\mathcal{T}}_{ij}{}^{lm}\mathcal{S}_{lm}\ ,$$



Primary GW vs secondar GW

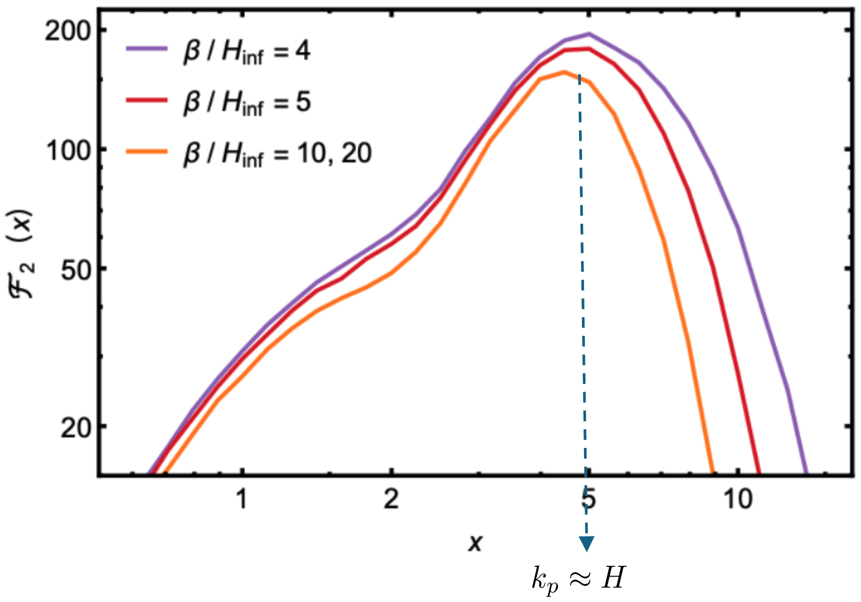
- Primary GW

$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{H_{\rm inf}}{\beta}\right)^5 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^2$$

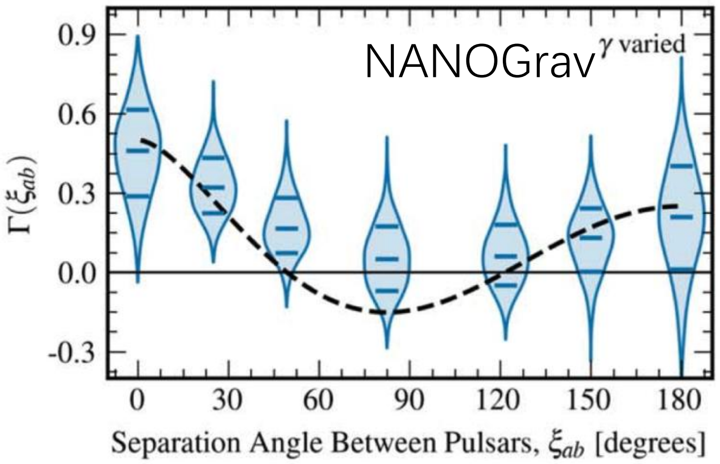


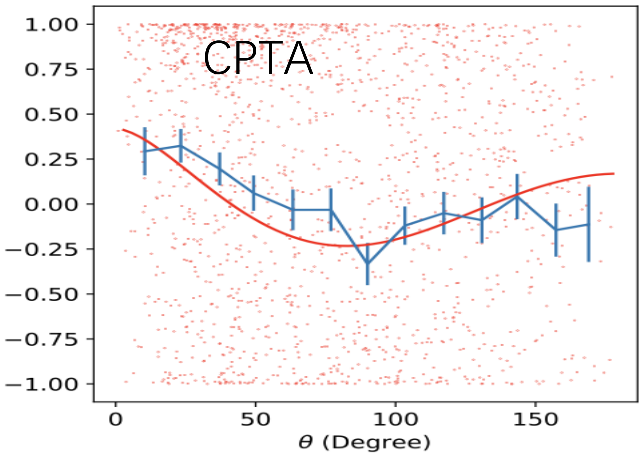
- Secondary GW

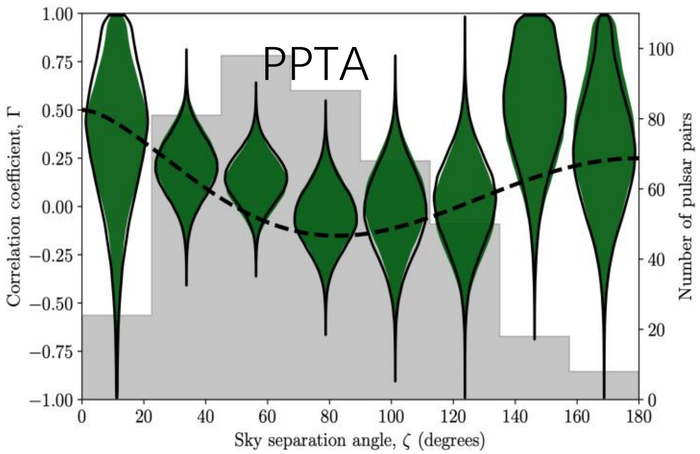
$$\Omega_{\rm GW} \approx \Omega_R \left(\frac{\mathcal{A}}{\epsilon}\right)^2 \left(\frac{M_{\rm pl}}{\phi_0}\right)^4 \left(\frac{H_{\rm inf}}{\beta}\right)^6 \left(\frac{\Delta\rho_{\rm vac}}{\rho_{\rm inf}}\right)^4$$

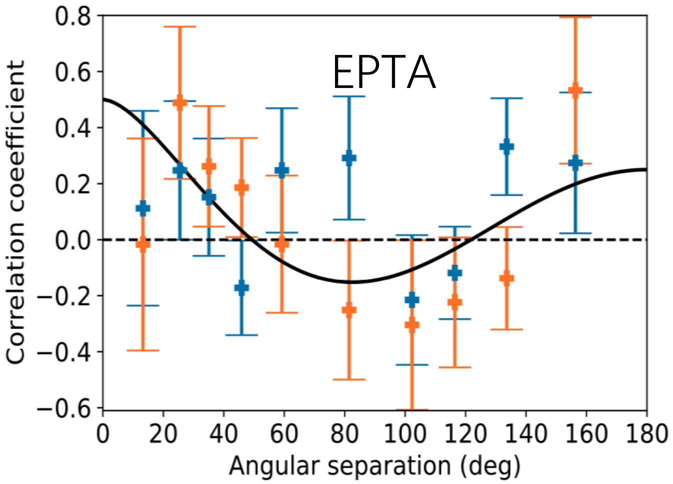


Pulsar timing array observations



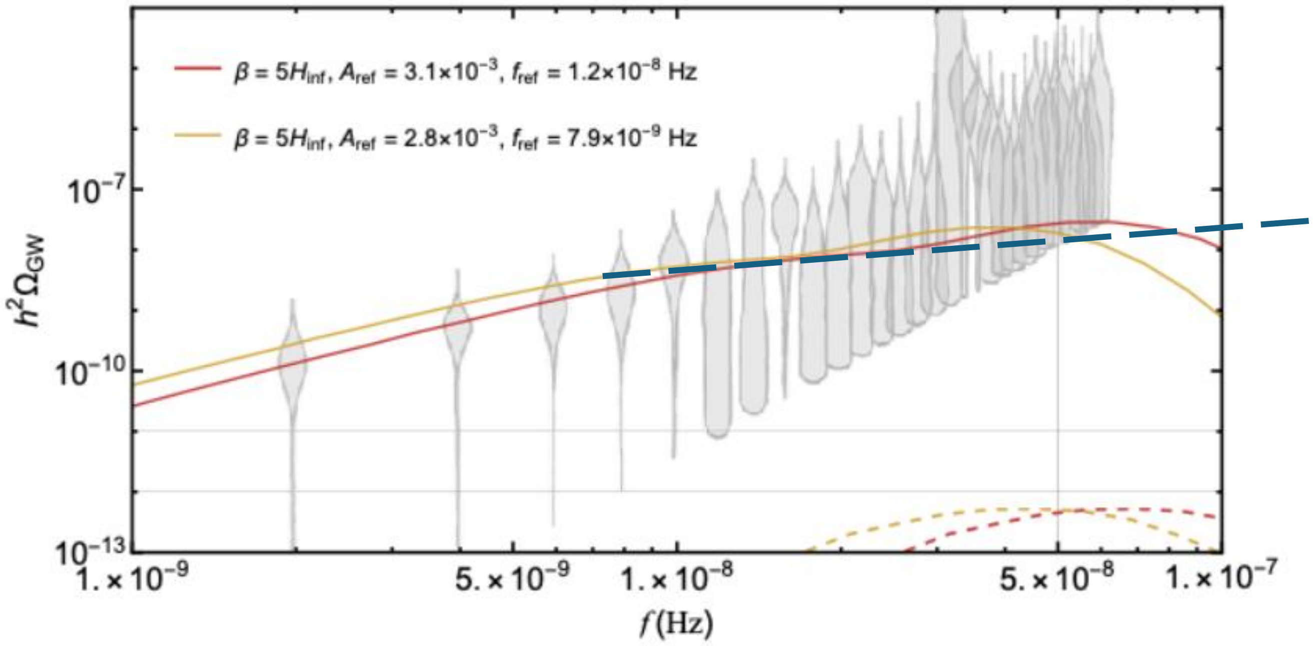


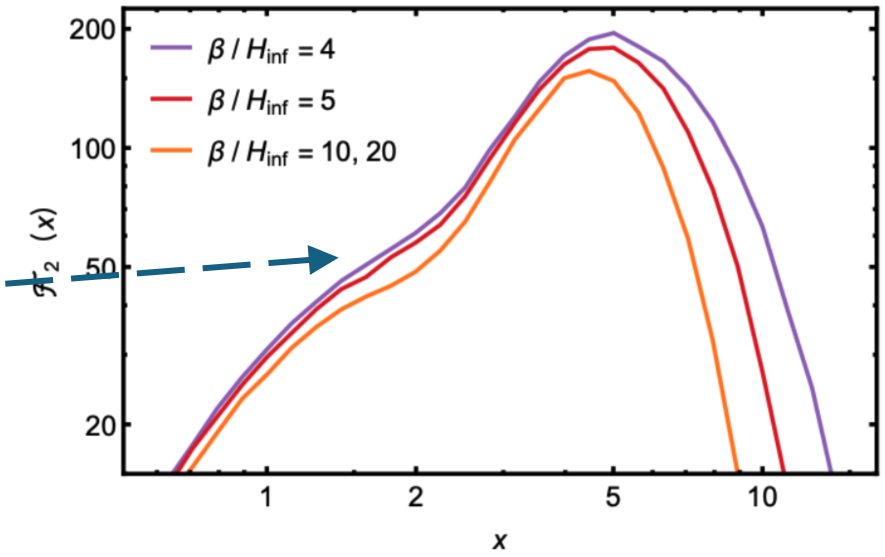




Fitting the spectrum of NanoGrav

- The transfer function softens the GW spectrum in the IR region





$$\Omega_{\rm GW}^{(2)}(f) = \Omega_R A_{\rm ref}^2 \mathcal{F}_2\left(\frac{q_{\rm phys}}{H_{\rm inf}}\right)$$

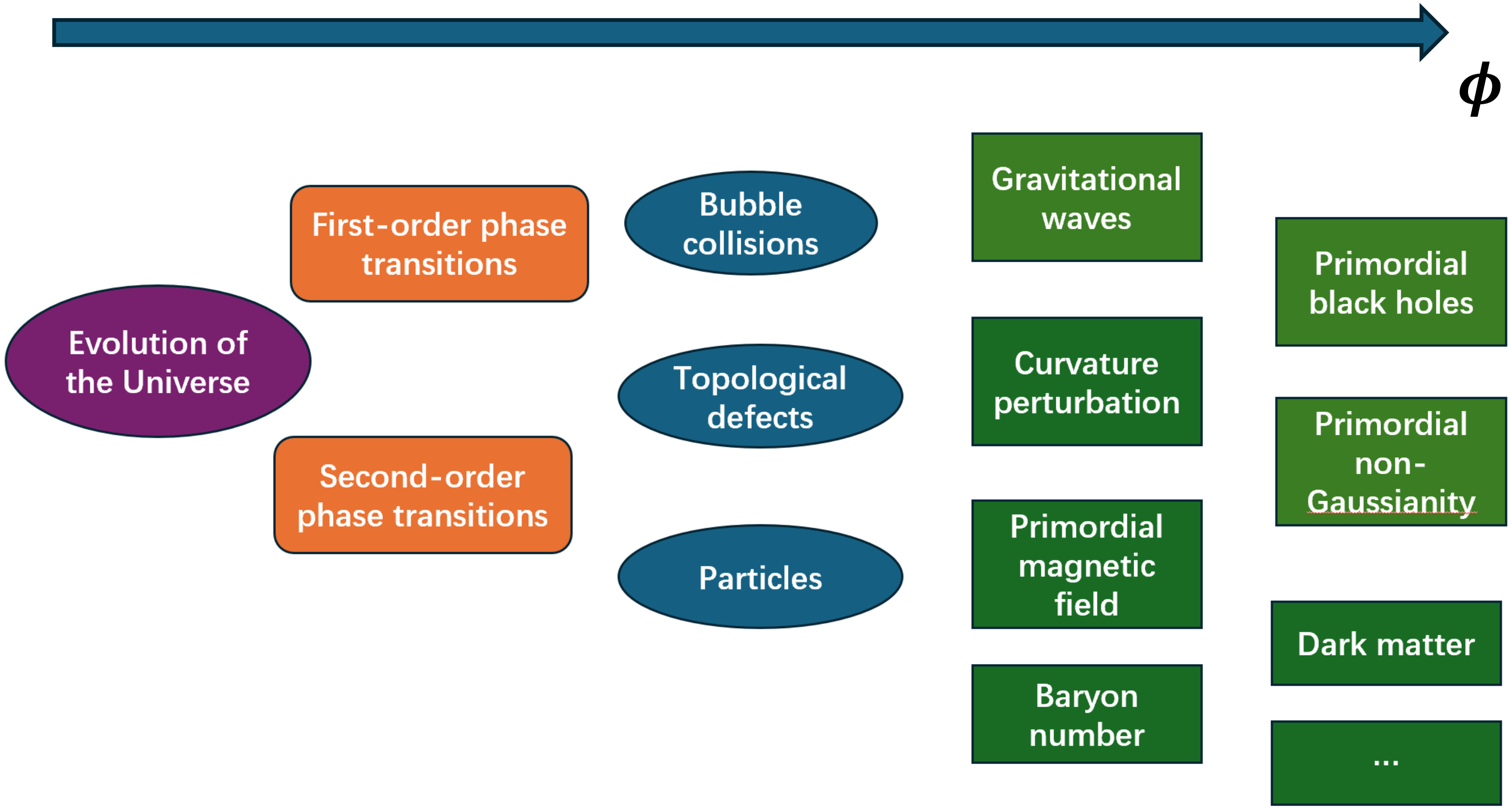
HA, Boye Su, Hanwen Tai, Lian-Tao Wang, Chen Yang, 2308.00070

$$\mathcal{F}_2^{\rm IR}(x) \approx x^3\left(\frac{6}{5}\log^2 x + \frac{16}{25}\log x + \frac{28}{125}\right)$$

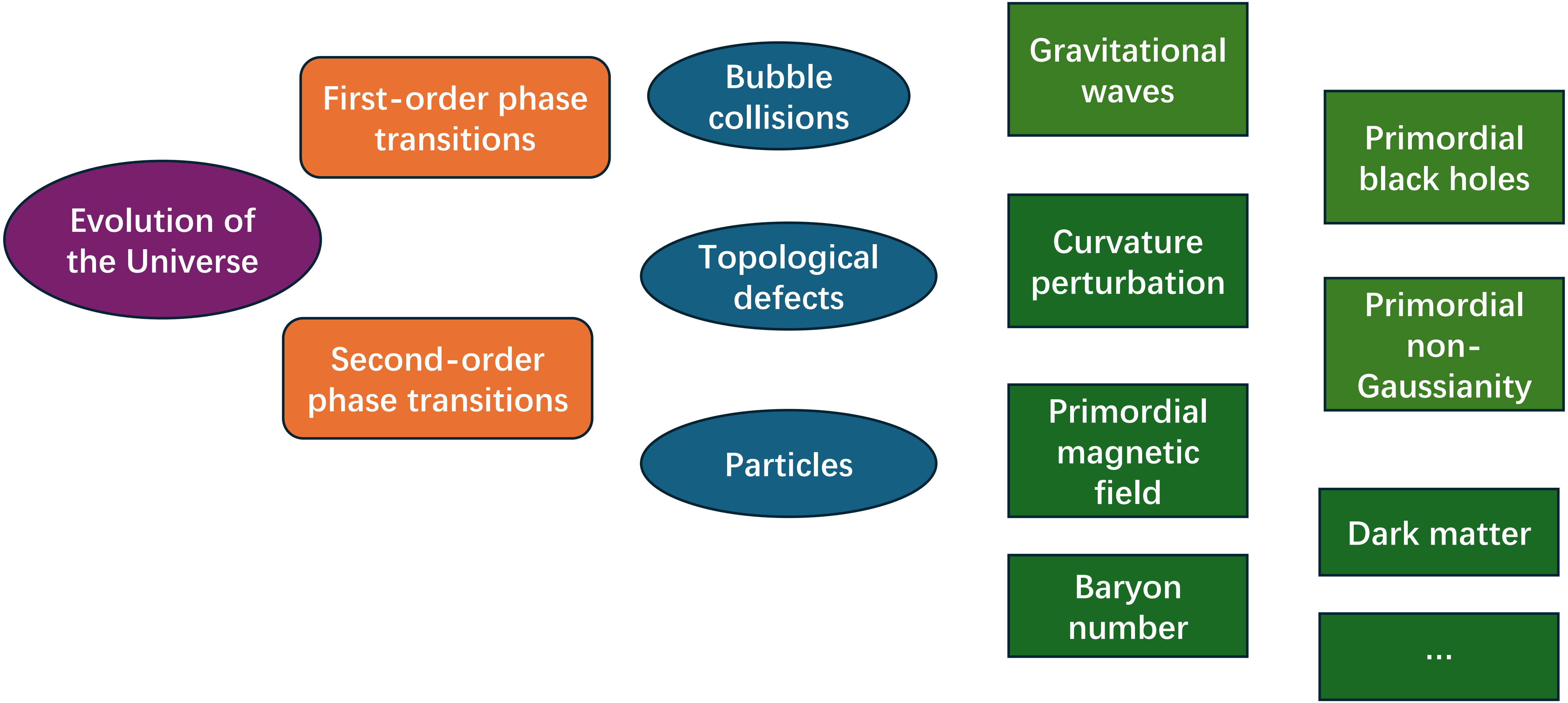
Summary

- During the thermal expansion of the universe:

- During inflation



Cosmological phase transitions



The evolution of the Universe

- The universe is always evolving even in the inflationary era.
- In the simplest inflation model.