

# Supernova Axion Emissivity with $\Delta(1232)$ Resonance in Heavy Baryon Chiral Perturbation Theory

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PRD 107 (2023) 7, 075002 [arXiv:2212.01155]

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21/Jan/2026

第十届海峡两岸粒子物理和宇宙学研讨会

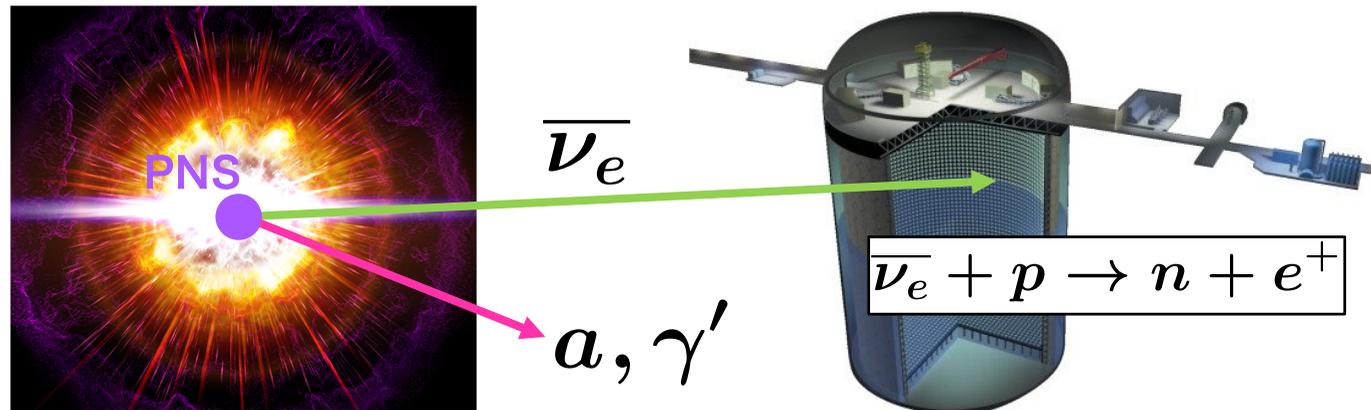
# The QCD axion

- ✦ A dynamic solution for the strong CP problem in QCD.  
(Peccei–Quinn mechanism)
- ✦ A possible candidate of the cold dark matter.  
(Coherently oscillating scalar field)
- ✦ It can couple to the SM particles with strength inversely proportional to the decay constant  $f_a$ .
- ✦ It may form a BEC due to its self-gravitational interaction.

# Axion emission from celestial bodies

★ The axions can be produced copiously from some and hot dense celestial objects such as supernovae (SNe), neutron stars, and white dwarfs.

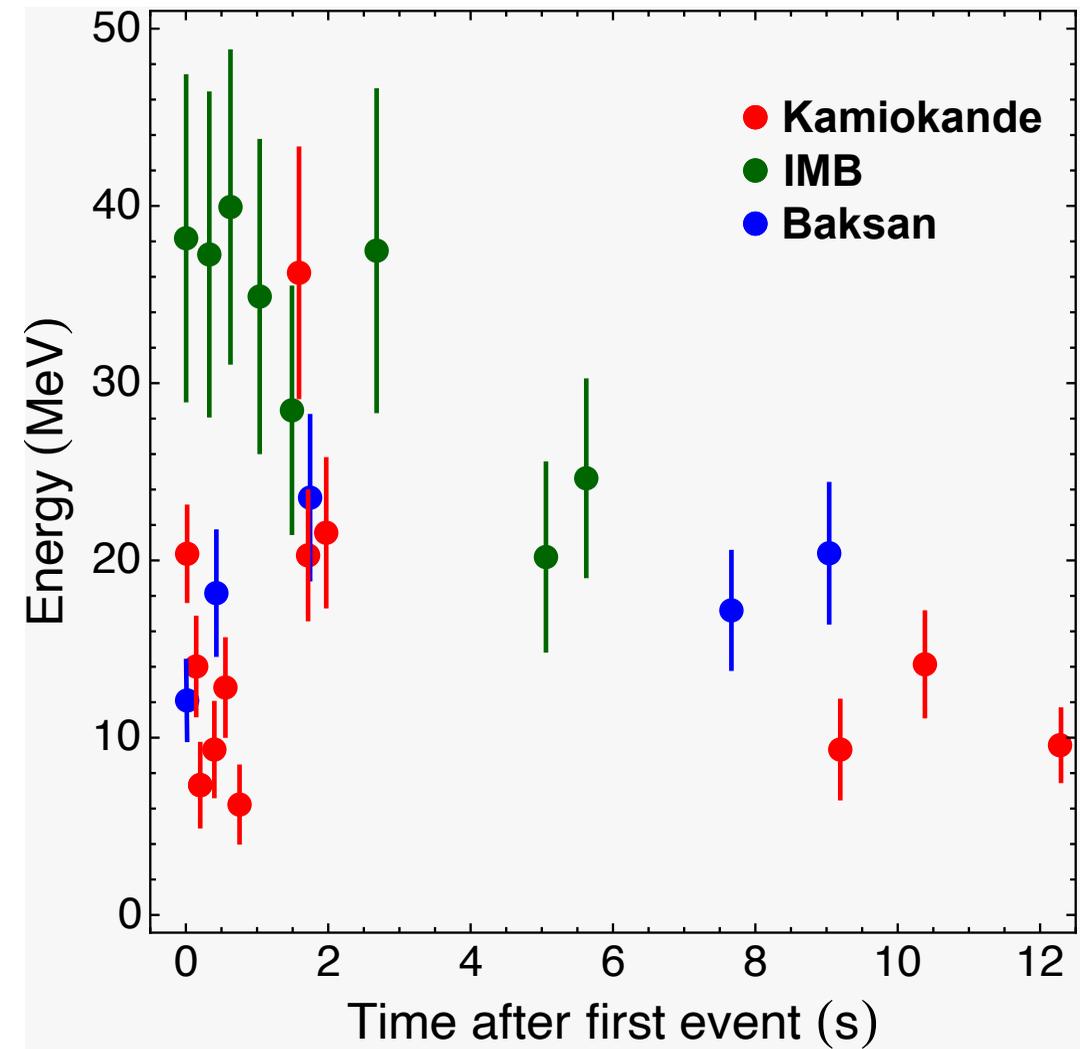
➤ e.g. SN1987A



➤ Raffelt's criteria

$$L_{\text{new particle}} < L_{\nu} \sim 3 \times 10^{52} \text{ erg/s}$$

Raffelt '90

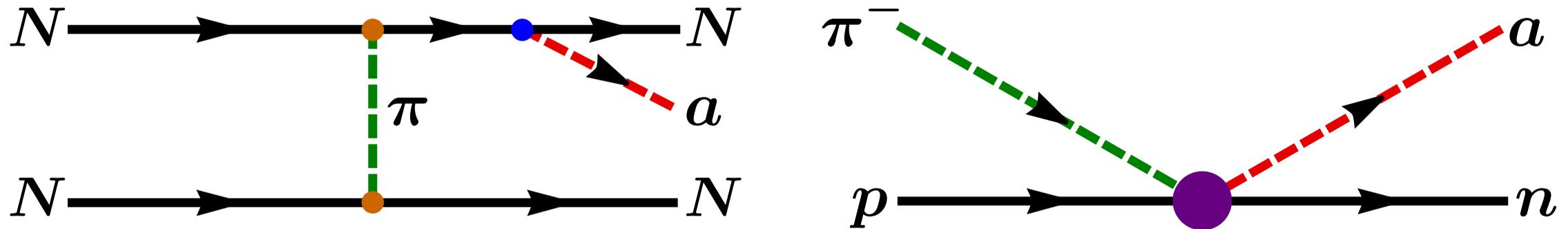


# Axion emission from Supernovae

★ Two hadronic processes that can create axions inside SNe

➤ **Nucleon-nucleon bremsstrahlung (NNB)** :  $NN \rightarrow NN a$

➤ **Pion-induced Compton-like scattering (PCS)** :  $\pi^- p \rightarrow n a$

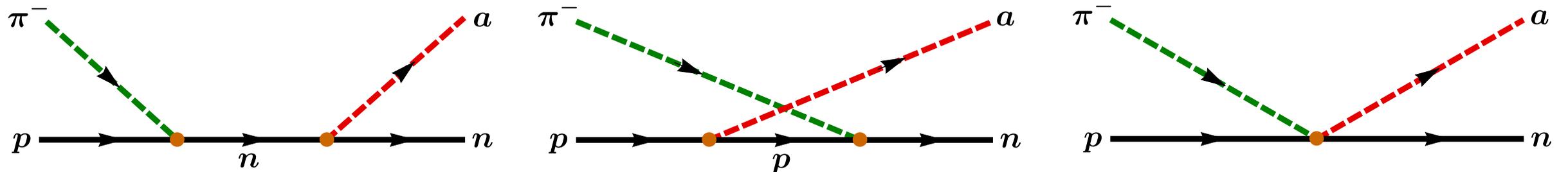


➤ It has been thought the NNB as the dominant axion emission for a while due to the underestimate of the  $n_{\pi^-}$  inside SNe.

➤ Recent studies have shown that the **PCS dominates over the NNB** to be the main source of the axion emission inside SNe.

# What we did

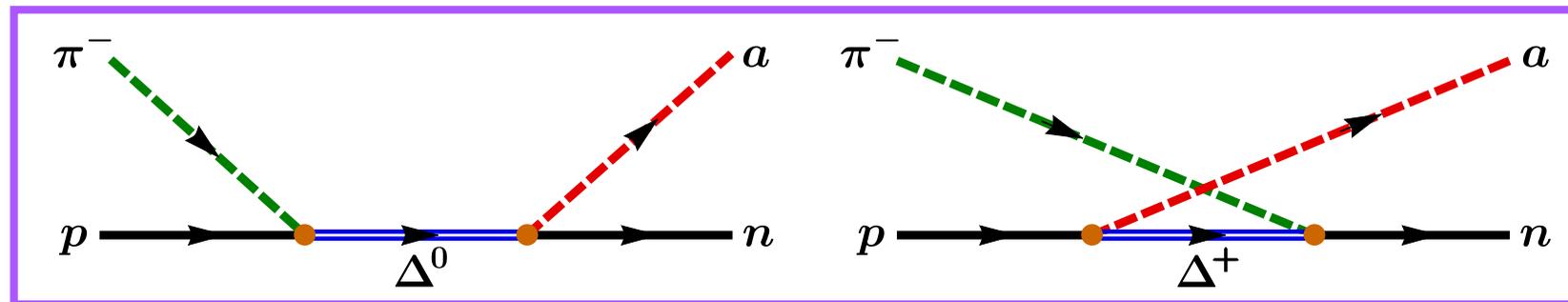
★ We evaluate the supernova axion emission rate including the  $\Delta$  resonance in the heavy baryon chiral perturbation theory



P. Carenza, B. Fore, M. Giannotti, A. Mirizzi and S. Reddy (2021)

K. Choi, H. J. Kim, H. Seong & C. S. Shin (2022)

$$|k_\pi| \simeq m_\pi \ll m_p$$



In our work

➤ For  $T_{\text{SN}} \sim 30 \text{ MeV}$ ,  $|k_\pi| \simeq \sqrt{3m_\pi T_{\text{SN}}} \simeq m_\pi$ ,  $E_\pi \sim 180 \text{ MeV}$

➤ The  $m_{\pi^- p}$  is somewhere in the middle of  $\Delta$  and  $N$  masses.

# Outline

✦ Introduction

✦ Heavy Baryon Chiral Perturbation Theory

✦ Axion Couplings to Baryons and Mesons

✦ Scattering Cross Section of  $\pi^- + p \rightarrow n + a$

✦ Supernova Axion Emissivity with  $\Delta(1232)$  Resonance

✦ Summary

# Effective Chiral Lagrangian

Jenkins & Manohar '91

## Interaction between meson octet and baryon octet

$$\mathcal{L}_{\pi B} = i \langle \bar{\mathcal{B}}_v v^\mu \mathcal{D}_\mu \mathcal{B}_v \rangle + 2D \langle \bar{\mathcal{B}}_v S_v^\mu \{ \mathcal{A}_\mu, \mathcal{B}_v \} \rangle + 2F \langle \bar{\mathcal{B}}_v S_v^\mu [ \mathcal{A}_\mu, \mathcal{B}_v ] \rangle$$

$$+ \frac{1}{4} f_\pi^2 \langle \partial^\mu \mathbf{\Pi} \partial_\mu \mathbf{\Pi}^\dagger \rangle + b \langle \mathcal{M}_q (\mathbf{\Pi} + \mathbf{\Pi}^\dagger) \rangle + \dots, \quad \langle \dots \rangle = \text{tr}(\dots)$$

$f_\pi \simeq 92.4 \text{ MeV}$

➤  $\mathcal{B}_v = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma_v^0 + \frac{1}{\sqrt{6}} \Lambda_v & \Sigma_v^+ & p_v \\ \Sigma_v^- & -\frac{1}{\sqrt{2}} \Sigma_v^0 + \frac{1}{\sqrt{6}} \Lambda_v & n_v \\ \Xi_v^- & \Xi_v^0 & -\frac{2}{\sqrt{6}} \Lambda_v \end{pmatrix}, \quad \mathcal{D}_\mu \mathcal{B}_v = \partial_\mu \mathcal{B}_v + [\mathcal{V}_\mu, \mathcal{B}_v]$

baryon octet

$$\mathcal{V}_\mu = \frac{1}{2} (\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi), \quad \mathcal{A}_\mu = \frac{i}{2} (\xi \partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi),$$

$$\xi = \exp\left(\frac{i\boldsymbol{\pi}}{f_\pi}\right), \quad \mathbf{\Pi} = \xi^2, \quad \boldsymbol{\pi} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^- & \bar{K}_0 & -\frac{2}{\sqrt{6}} \eta \end{pmatrix}$$

meson octet

# Effective Chiral Lagrangian

Jenkins & Manohar '91

★ Interaction between meson octet and baryon octet

$$\mathcal{L}_{\pi B} = i \langle \bar{\mathcal{B}}_v v^\mu \mathcal{D}_\mu \mathcal{B}_v \rangle + 2D \langle \bar{\mathcal{B}}_v S_v^\mu \{ \mathcal{A}_\mu, \mathcal{B}_v \} \rangle + 2F \langle \bar{\mathcal{B}}_v S_v^\mu [ \mathcal{A}_\mu, \mathcal{B}_v ] \rangle \\ + \frac{1}{4} f_\pi^2 \langle \partial^\mu \mathbf{\Pi} \partial_\mu \mathbf{\Pi}^\dagger \rangle + b \langle \mathcal{M}_q (\mathbf{\Pi} + \mathbf{\Pi}^\dagger) \rangle + \dots ,$$

➤ Spin operator :  $S_v^\mu = \gamma^5 [ \psi, \gamma^\mu ] / 4$      $v \cdot S_v = 0$

➤ Quark mass matrix :  $\mathcal{M}_q = \text{diag}(m_u, m_d, m_s)$      $SU(3)_L \otimes SU(3)_R \rightarrow SU(3)_V$

➤ Under the  $SU(3)_L \otimes SU(3)_R$  symmetry

$$\mathcal{B}_v \rightarrow \mathcal{U}_H \mathcal{B}_v \mathcal{U}_H^\dagger , \quad \mathcal{D}_\mu \mathcal{B}_v \rightarrow \mathcal{U}_H (\mathcal{D}_\mu \mathcal{B}_v) \mathcal{U}_H^\dagger , \quad \mathbf{\Pi} \rightarrow \mathcal{U}_L \mathbf{\Pi} \mathcal{U}_R^\dagger ,$$

$$\xi \rightarrow \mathcal{U}_L \xi \mathcal{U}_H^\dagger = \mathcal{U}_H \xi \mathcal{U}_R^\dagger , \quad \mathcal{V}_\mu \rightarrow \mathcal{U}_H \mathcal{V}_\mu \mathcal{U}_H^\dagger + \mathcal{U}_H \partial_\mu \mathcal{U}_H^\dagger , \quad \mathcal{A}_\mu \rightarrow \mathcal{U}_H \mathcal{A}_\mu \mathcal{U}_H^\dagger$$

$$\mathcal{U}_{L,R} \in SU(3)_{L,R} \quad \mathcal{U}_H = \mathcal{U}_H(x) \in SU(3)_H \text{ (local)}$$

# Effective Chiral Lagrangian

Jenkins & Manohar '91

★ Interaction between meson octet and baryon octet

$$\mathcal{L}_{\pi B} = i \langle \bar{\mathcal{B}}_v v^\mu \mathcal{D}_\mu \mathcal{B}_v \rangle + 2D \langle \bar{\mathcal{B}}_v S_v^\mu \{ \mathcal{A}_\mu, \mathcal{B}_v \} \rangle + 2F \langle \bar{\mathcal{B}}_v S_v^\mu [ \mathcal{A}_\mu, \mathcal{B}_v ] \rangle + \frac{1}{4} f_\pi^2 \langle \partial^\mu \mathbf{\Pi} \partial_\mu \mathbf{\Pi}^\dagger \rangle + b \langle \mathcal{M}_q (\mathbf{\Pi} + \mathbf{\Pi}^\dagger) \rangle + \dots ,$$

➤ To the first order in  $\pi / f_\pi$

$$\xi = \mathbb{I}_{3 \times 3} + i\pi / f_\pi \quad \mathcal{A}_\mu = \partial_\mu \pi / f_\pi \quad \mathcal{V}_\mu = 0$$

➔  $\mathcal{L}_{\pi B} \supset \frac{2(D+F)}{f_\pi} \langle \bar{\mathcal{B}}_v S_v^\mu (\partial_\mu \pi) \mathcal{B}_v \rangle + \frac{2(D-F)}{f_\pi} \langle \bar{\mathcal{B}}_v S_v^\mu \mathcal{B}_v (\partial_\mu \pi) \rangle$

$$g_A = D + F \simeq 1.254$$

$$\mathcal{L}_{\pi N} = \frac{\sqrt{2} g_A}{f_\pi} \left( \bar{p}_v S_v^\mu n_v \partial^\mu \pi^+ + \bar{n}_v S_v^\mu p_v \partial^\mu \pi^- \right) \quad \text{pion-nucleon interaction}$$

# Effective Chiral Lagrangian

Jenkins & Manohar '91

✦ Interactions of meson octet, baryon octet & baryon decuplet

$$\mathcal{L}_{\pi BT} = -i \overline{(\mathcal{T}_v^\mu)_{ijk}} v^\rho \mathcal{D}_\rho (\mathcal{T}_{v\mu})_{ijk} + \Delta m_{TB} \overline{(\mathcal{T}_v^\mu)_{ijk}} (\mathcal{T}_{v\mu})_{ijk} \\ + \mathcal{C} \epsilon_{ijk} \left[ \overline{(\mathcal{T}_v^\mu)_{ilm}} (\mathcal{A}_\mu)_{lj} (\mathcal{B}_v)_{mk} + \overline{(\mathcal{B}_v)_{km}} (\mathcal{A}_\mu)_{jl} (\mathcal{T}_{v\mu})_{ilm} \right] + \dots,$$

➤ Spin-3/2 Rarita-Schwinger field :  $(\mathcal{T}_v^\mu)_{ijk}$

$$\Delta m_{TB} = m_T - m_B \\ \mathcal{C} \simeq 3g_A/2$$

➤ Under the  $SU(3)_L \otimes SU(3)_R$  symmetry

$$(\mathcal{T}_v^\mu)_{ijk} \rightarrow (\mathcal{U}_H)_{il} (\mathcal{U}_H)_{jm} (\mathcal{U}_H)_{kn} (\mathcal{T}_v^\mu)_{lmn}$$

➤ Rep. of the Delta baryon :  $(\mathcal{T}_{v\mu})_{112} = \frac{1}{\sqrt{3}} \Delta_{v\mu}^+$ ,  $(\mathcal{T}_{v\mu})_{122} = \frac{1}{\sqrt{3}} \Delta_{v\mu}^0$

➔ 
$$\mathcal{L}_{\pi N \Delta} = \frac{\mathcal{C}}{\sqrt{6} f_\pi} \left( \overline{n}_v \Delta_{v\mu}^+ \partial^\mu \pi^- + \overline{\Delta_{v\mu}^+} n_v \partial^\mu \pi^+ - \overline{p}_v \Delta_{v\mu}^0 \partial^\mu \pi^+ - \overline{\Delta_{v\mu}^0} p_v \partial^\mu \pi^- \right)$$

pion-nucleon-delta interaction

# Hadronic Axial Vector Currents

★ The Lagrangian invariant under the local  $SU(3)_H$  symmetry

$$\mathcal{L}_{\pi B} \supset i \langle \overline{\mathcal{B}}_v v^\mu \mathcal{D}_\mu \mathcal{B}_v \rangle + 2D \langle \overline{\mathcal{B}}_v S_v^\mu \{ \mathcal{A}_\mu, \mathcal{B}_v \} \rangle + 2F \langle \overline{\mathcal{B}}_v S_v^\mu [ \mathcal{A}_\mu, \mathcal{B}_v ] \rangle$$

$$\mathcal{L}_{\pi BT} \supset \mathcal{C} \epsilon_{ijk} \left[ \overline{(\mathcal{T}_v^\mu)_{ilm}} (\mathcal{A}_\mu)_{lj} (\mathcal{B}_v)_{mk} + \overline{(\mathcal{B}_v)_{km}} (\mathcal{A}_\mu)_{jl} (\mathcal{T}_{v\mu})_{ilm} \right]$$

$$\mathcal{B}_v \rightarrow \mathcal{U}_H \mathcal{B}_v \mathcal{U}_H^\dagger, \quad \mathcal{D}_\mu \mathcal{B}_v \rightarrow \mathcal{U}_H (\mathcal{D}_\mu \mathcal{B}_v) \mathcal{U}_H^\dagger, \quad \mathcal{A}_\mu \rightarrow \mathcal{U}_H \mathcal{A}_\mu \mathcal{U}_H^\dagger$$

$$(\mathcal{T}_v^\mu)_{ijk} \rightarrow (\mathcal{U}_H)_{il} (\mathcal{U}_H)_{jm} (\mathcal{U}_H)_{kn} (\mathcal{T}_v^\mu)_{lmn}$$

➤ Noether's theorem :  $\xi \rightarrow \mathcal{U}_H \xi \mathcal{U}_H^\dagger \rightarrow (1 + i\epsilon^A t^A) \xi \quad \epsilon^A \rightarrow 0$

$$\begin{aligned} \mathcal{J}_{\pi B}^{A\mu} &= D \langle \overline{\mathcal{B}}_v S_v^\mu \{ \xi^\dagger t^A \xi + \xi t^A \xi^\dagger, \mathcal{B}_v \} \rangle + F \langle \overline{\mathcal{B}}_v S_v^\mu [ \xi^\dagger t^A \xi + \xi t^A \xi^\dagger, \mathcal{B}_v ] \rangle \\ &\quad + \frac{1}{2} v^\mu \langle \overline{\mathcal{B}}_v [ \xi^\dagger t^A \xi - \xi t^A \xi^\dagger, \mathcal{B}_v ] \rangle, \end{aligned} \quad \begin{array}{l} t^A (A = 1, 2, \dots, 8) \\ \text{Gell-Mann matrices} \end{array}$$

$$\mathcal{J}_{\pi BT}^{A\mu} = \frac{\mathcal{C}}{2} \epsilon_{ijk} \left[ \overline{(\mathcal{T}_v^\mu)_{ilm}} (\xi^\dagger t^A \xi + \xi t^A \xi^\dagger)_{lj} (\mathcal{B}_v)_{mk} + \overline{(\mathcal{B}_v)_{km}} (\xi^\dagger t^A \xi + \xi t^A \xi^\dagger)_{lj} (\mathcal{T}_{v\mu})_{ilm} \right]$$

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# The QCD axion Lagrangian

★  $v_{\text{PQ,EW}} \gg T \gg \Lambda_{\text{QCD}}$  & at leading order in  $a/f_a$

$$\mathcal{L}_{aqq} = \frac{1}{2} \partial_\mu a \partial^\mu a + \frac{g_s^2}{32\pi^2} \frac{a}{f_a} G_{\mu\nu}^c \tilde{G}^{c\mu\nu} + \bar{q} i \gamma^\mu \partial_\mu q$$

$$- (\bar{q}_L \mathcal{M}_q q_R + \text{h.c.}) + \frac{\partial_\mu a}{2f_a} \bar{q} \gamma^\mu \gamma^5 \mathcal{X}_q q$$

axion derivative interactions

➤ Light quark fields :  $q = (u, d, s)^\top$

➤ Axion coupling matrix :  $\mathcal{X}_q = \text{diag}(X_u, X_d, X_s)$

➤ Typically, one introduces an SM-singlet complex scalar field  $\Phi \sim (1, 1)_0$  with a PQ charge in UV models. After the  ~~$v_{\text{PQ}}$~~ , the phase of  $\Phi \propto e^{ia/f_a}$  is then identified as the axion.

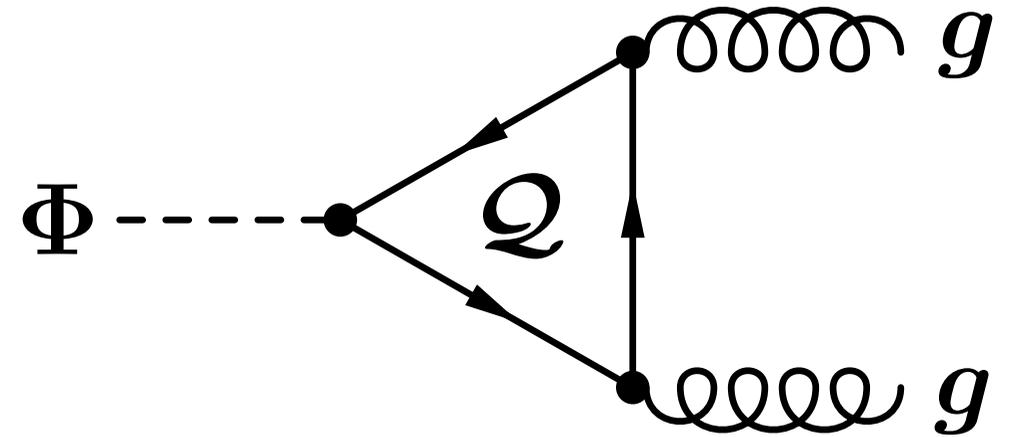
# Axion models

## ★ KSVZ model

Kim '79,  
Shifman, Vainshtein, Zakharov '80

- The QCD anomaly is realized by introducing a **heavy vector-like fermion**.

$$Q = Q_L + Q_R \sim (\mathbf{3}, \mathbf{1})_0$$



- Interactions :  $y_Q \Phi \overline{Q}_L Q_R + \text{h.c.}$

- Under PQ symmetry

$$\Phi \rightarrow e^{iq_{\text{PQ}}} \Phi \quad Q_L \rightarrow e^{iq_{\text{PQ}}/2} Q_L \quad Q_R \rightarrow e^{-iq_{\text{PQ}}/2} Q_R$$

- Only  $\Phi$  and  $Q$  have PQ charges :  $X_u = X_d = X_s = 0$

(at tree level)

# Axion models

Dine, Fischler, Srednicki '81  
Zhitnitsky '80

## ★ DFSZ model

➤ The QCD anomaly is induced by assuming **2HDM**  $H_u$  &  $H_d$  couples to the SM quark fields.

➤ Interactions :  $H_u^\dagger H_d (\Phi^*)^2 \overline{Q}_L (\mathcal{Y}_u \tilde{H}_u U_R + \mathcal{Y}_d H_d D_R) + \text{h.c.}$

➤ Under PQ symmetry

$$\Phi \rightarrow e^{iq_{\text{PQ}}} \Phi \quad H_u \rightarrow e^{-iq_{\text{PQ}}} H_u \quad H_d \rightarrow e^{iq_{\text{PQ}}} H_d$$

$$Q_L \rightarrow Q_L \quad U_R \rightarrow e^{-iq_{\text{PQ}}} U_R \quad D_R \rightarrow e^{-iq_{\text{PQ}}} D_R$$

➤ The axion as a **linear combination of the CP-odd scalars** can

couple to the SM quarks :  $X_u = \frac{\cos^2 \beta}{3}$  ,  $X_{d,s} = \frac{\sin^2 \beta}{3}$   $\tan \beta = \frac{\langle H_u \rangle}{\langle H_d \rangle}$

# Axion couplings to hadrons

✦ Below the QCD chiral S.B. scale, one can eliminate the axion-gluon coupling by the **chiral trans.** on the light quark fields

$$q \rightarrow \mathcal{R}_a q = \exp\left(-i\gamma^5 \frac{a}{2f_a} \mathcal{Q}_a\right) q, \quad \langle \mathcal{Q}_a \rangle = 1$$

act on the quark flavor space

$$\rightarrow \int \mathcal{D}q \mathcal{D}\bar{q} \rightarrow \int \mathcal{D}q \mathcal{D}\bar{q} \exp\left[i \int d^4x \left(-\frac{g_s^2}{32\pi^2} \frac{a}{f_a} G_{\mu\nu}^c \tilde{G}^{c\mu\nu} \langle \mathcal{Q}_a \rangle\right)\right]$$

$\epsilon^{0123} = +1$

✦ To avoid the axion- $\pi^0$  mass mixing, the customary choice is

$$\mathcal{Q}_a = \frac{\mathcal{M}_q^{-1}}{\text{tr}(\mathcal{M}_q^{-1})} = \frac{m_u m_d m_s}{m_u m_d + m_u m_s + m_d m_s} \text{diag}\left(\frac{1}{m_u}, \frac{1}{m_d}, \frac{1}{m_s}\right)$$

# Axion couplings to hadrons

✦ Under the **chiral trans.**, the quark kinetic term is shifted as

$$\bar{q}i\gamma^\mu\partial_\mu q \rightarrow \bar{q}i\gamma^\mu\partial_\mu q + \frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma^5\mathcal{Q}_a q + \mathcal{O}\left(\frac{a^2}{f_a^2}\right)$$

✦ The light quark mass term becomes

$$\bar{q}_L\mathcal{M}_q q_R \rightarrow \bar{q}_L\mathcal{M}_a q_R, \quad \bar{q}_R\mathcal{M}_q q_L \rightarrow \bar{q}_R\mathcal{M}_a^\dagger q_L$$

$$\mathcal{M}_a \equiv \mathcal{R}_a\mathcal{M}_q\mathcal{R}_a$$

Up to the second order in the axion field

$$\mathcal{M}_a = \mathcal{M}_q - i\frac{a}{2f_a}\{\mathcal{M}_q, \mathcal{Q}_a\} - \frac{a^2}{8f_a^2}\{\{\mathcal{M}_q, \mathcal{Q}_a\}, \mathcal{Q}_a\} + \mathcal{O}\left(\frac{a^3}{f_a^3}\right)$$

# Axion couplings to hadrons

✦ The resulting Lagrangian with only the axion and quark fields

$$\mathcal{L}_{aq} = \frac{1}{2} \partial_\mu a \partial^\mu a + \bar{q} i \gamma^\mu \partial_\mu q - (\bar{q}_L \mathcal{M}_a q_R + \bar{q}_R \mathcal{M}_a^\dagger q_L) + \frac{\partial_\mu a}{2f_a} \bar{q} \gamma^\mu \gamma^5 (\mathcal{X}_q + \mathcal{Q}_a) q$$

➤ Use  $M_{3 \times 3} = 2 \langle M_{3 \times 3} \hat{t}^A \rangle \hat{t}^A$  for any 3x3 Hermitian matrix  $M_{3 \times 3}$   
 $\{\hat{t}^A\} = \{t^A\} \cup \{t^0\}$   $t^0 = \mathbb{I}_{3 \times 3} / \sqrt{6}$

$$\mathcal{L}_{aq} = \frac{1}{2} \partial_\mu a \partial^\mu a + \bar{q} i \gamma^\mu \partial_\mu q + \langle \mathcal{M}_a q_R \bar{q}_L + \mathcal{M}_a^\dagger q_L \bar{q}_R \rangle + \frac{\partial_\mu a}{f_a} \langle (\mathcal{X}_q + \mathcal{Q}_a) \hat{t}^A \rangle \mathcal{J}_q^{A\mu}$$

quark axial vector currents  $\mathcal{J}_q^{A\mu} = \bar{q} \gamma^\mu \gamma^5 \hat{t}^A q$

➤ The next step is to replace the light quark fields with the corresponding hadron fields in the HBChPT.

# Axion couplings to hadrons

★ Axion couplings to pions :  $\mathcal{U}_L(q_L \bar{q}_R) \mathcal{U}_R^\dagger \sim \mathcal{U}_L \mathbf{\Pi} \mathcal{U}_R^\dagger$

$$\langle \mathcal{M}_a q_R \bar{q}_L + \mathcal{M}_a^\dagger q_L \bar{q}_R \rangle \longrightarrow \mathcal{L}_{a\pi} = \frac{1}{2} f_\pi^2 B_0 \langle \mathcal{M}_a \mathbf{\Pi}^\dagger + \mathcal{M}_a^\dagger \mathbf{\Pi} \rangle$$

$$\blacktriangleright \mathcal{M}_a = \underbrace{\mathcal{M}_q}_{\text{pion mass}} - i \frac{a}{2f_a} \underbrace{\{\mathcal{M}_q, \mathcal{Q}_a\}}_{\text{axion-}\pi^0 \text{ mass mixing}} - \frac{a^2}{8f_a^2} \underbrace{\{\{\mathcal{M}_q, \mathcal{Q}_a\}, \mathcal{Q}_a\}}_{\text{axion mass}} \quad \mathcal{Q}_a = \frac{\mathcal{M}_q^{-1}}{\text{tr}(\mathcal{M}_q^{-1})}$$

$$\blacktriangleright \text{Axion mass : } m_a = \sqrt{\frac{z}{(1+z)(1+z+w)}} \frac{f_\pi m_\pi}{f_a} \simeq 6 \text{ meV} \left( \frac{10^9 \text{ GeV}}{f_a} \right)$$

$$z \equiv m_u/m_d \simeq 0.485$$

$$w \equiv m_u/m_s \simeq 0.025$$

$$m_\pi = \sqrt{B_0(m_u + m_d)} \simeq 139.57 \text{ MeV}$$

# Axion couplings to hadrons

★ Axion couplings to pions and baryons :  $\mathcal{J}_q^{A\mu} \sim \mathcal{J}_{\text{hadron}}^{A\mu}$

$$\frac{\partial_\mu a}{f_a} \langle (\mathcal{X}_q + \mathcal{Q}_a) \hat{t}^A \rangle \mathcal{J}_q^{A\mu} \longrightarrow \mathcal{L}_{a\pi B} = \frac{\partial_\mu a}{f_a} \left[ \langle (\mathcal{X}_q + \mathcal{Q}_a) t^A \rangle \mathcal{J}_{\pi B}^{A\mu} + \frac{1}{3} S \langle \mathcal{X}_q + \mathcal{Q}_a \rangle \mathcal{J}_{\pi B}^{0\mu} \right]$$

$$\mathcal{J}_{\pi B}^{0\mu} = \langle \bar{\mathcal{B}}_v S_v^\mu \mathcal{B}_v \rangle$$

➤ Axion couplings to pions and nucleons :

$$\mathcal{L}_{a\pi N} = \frac{\partial_\mu a}{f_a} \left[ C_{ap} \bar{p}_v S_v^\mu p_v + C_{an} \bar{n}_v S_v^\mu n_v + \frac{i}{2f_\pi} C_{a\pi N} (\pi^+ \bar{p}_v v^\mu n_v - \pi^- \bar{n}_v v^\mu p_v) \right]$$

$$C_{ap} = X_u \Delta u + X_d \Delta d + X_s \Delta s + \frac{\Delta u + z \Delta d + w \Delta s}{1 + z + w}$$

nucleon matrix element

$$\langle p | \bar{q} S_v^\mu q | p \rangle = s^\mu \Delta q / 2$$

$$C_{an} = X_d \Delta u + X_u \Delta d + X_s \Delta s + \frac{z \Delta u + \Delta d + w \Delta s}{1 + z + w}$$

$$\Delta u = 0.847$$

$$\Delta d = -0.407$$

$$\Delta s = -0.035$$

$$C_{a\pi N} = \frac{1}{\sqrt{2}} \left( X_u - X_d + \frac{1 - z}{1 + z + w} \right) = \frac{C_{ap} - C_{an}}{\sqrt{2} g_A}$$

# Axion couplings to hadrons

★ Axion couplings to pions and baryons :  $\mathcal{J}_q^{A\mu} \sim \mathcal{J}_{\text{hadron}}^{A\mu}$

$$\frac{\partial_\mu a}{f_a} \langle (\mathcal{X}_q + \mathcal{Q}_a) \hat{t}^A \rangle \mathcal{J}_q^{A\mu} \longrightarrow \mathcal{L}_{a\pi BT} = \frac{\partial_\mu a}{f_a} \langle (\mathcal{X}_q + \mathcal{Q}_a) t^A \rangle \mathcal{J}_{\pi BT}^{A\mu} \quad \epsilon_{ijk} \overline{(\mathcal{T}_v^\mu)_{ijm}} (\mathcal{B}_v)_{mk} = 0$$

➤ Axion couplings to pions, nucleons and Delta baryons :

$$\mathcal{L}_{aN\Delta} = \frac{\partial_\mu a}{2f_a} \left[ C_{ap\Delta} (\overline{p}_v \Delta_\mu^+ + \overline{\Delta}_\mu^+ p_v) + C_{an\Delta} (\overline{n}_v \Delta_\mu^0 + \overline{\Delta}_\mu^0 n_v) \right]$$

$$C_{ap\Delta} = C_{an\Delta} \equiv C_{aN\Delta} = -\frac{\mathcal{C}}{\sqrt{3}} \left( X_u - X_d + \frac{1-z}{1+z+w} \right) = -\frac{\sqrt{3}}{2} (C_{ap} - C_{an})$$

➤ Note that  $C_{a\pi N}$  and  $C_{aN\Delta}$  are not independent parameters since they can be expressed in terms of  $C_{ap} - C_{an}$ .

# Axion couplings to hadrons

 Numerically, we obtain

$$C_{ap} = \begin{cases} +0.430 & \text{KSVZ model} \\ +0.712 - 0.430 \sin^2 \beta & \text{DFSZ model} \end{cases}$$
$$C_{an} = \begin{cases} +0.002 & \text{KSVZ model} \\ -0.134 + 0.406 \sin^2 \beta & \text{DFSZ model} \end{cases}$$
$$C_{a\pi N} = \begin{cases} +0.241 & \text{KSVZ model} \\ +0.477 - 0.471 \sin^2 \beta & \text{DFSZ model} \end{cases}$$
$$C_{aN\Delta} = \begin{cases} -0.370 & \text{KSVZ model} \\ -0.732 + 0.724 \sin^2 \beta & \text{DFSZ model} \end{cases}$$

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✦ Introduction

✦ Heavy Baryon Chiral Perturbation Theory

✦ Axion Couplings to Baryons and Mesons

✦ Scattering Cross Section of  $\pi^- + p \rightarrow n + a$

✦ Supernova Axion Emissivity with  $\Delta(1232)$  Resonance

✦ Summary

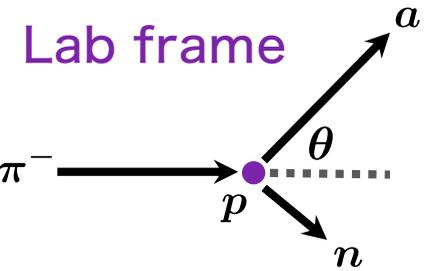
# Squared matrix element



$$|\mathcal{M}_{\pi^- p \rightarrow na}|^2 = \frac{2m_N^2}{f_\pi^2 f_a^2} \langle P_+ \Omega^\dagger P_+ \Omega \rangle$$

$$P_+ = \text{diag}(1, 1, 0, 0)$$

$$\Theta = \text{diag}(e^{+i\theta}, e^{-i\theta}, e^{+i\theta}, e^{-i\theta})$$

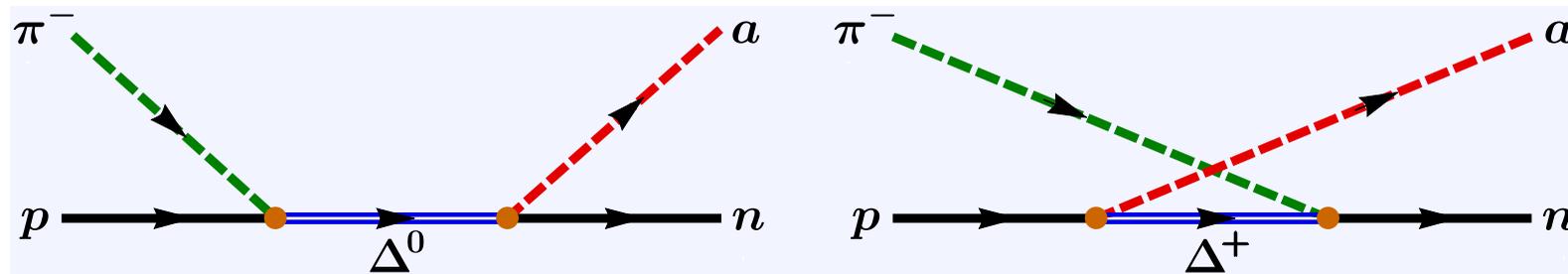
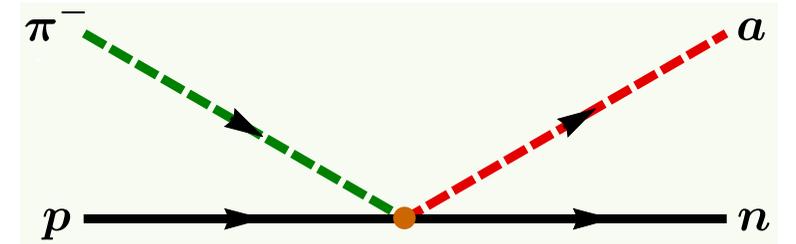
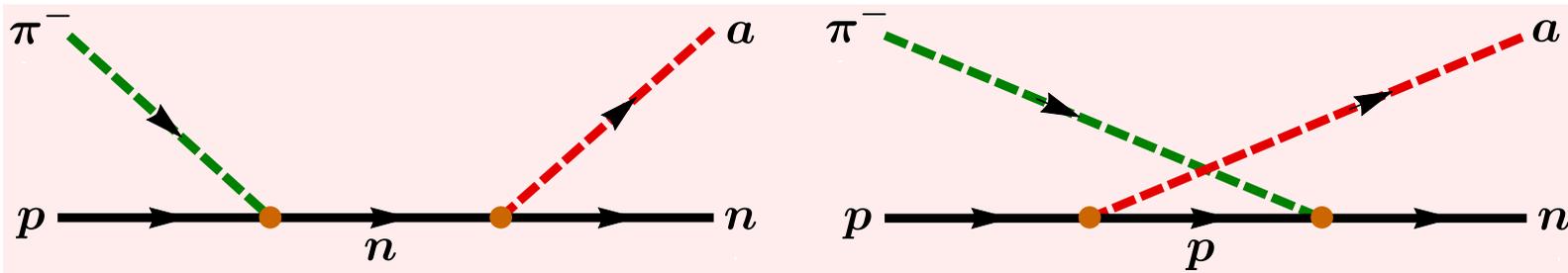


$$\Omega = \frac{\sqrt{2}g_A |\mathbf{k}_\pi| |\mathbf{k}_a|}{4E_\pi} (C_{ap} \Theta - C_{an} \Theta^\dagger) + \frac{C_{a\pi N} |\mathbf{k}_a|}{2} \mathbb{I}_{4 \times 4}$$

$$\Delta m = m_\Delta - m_N \simeq 293 \text{ MeV}$$

$$\Gamma_\Delta \simeq 117 \text{ MeV}$$

$$+ \frac{\mathcal{C} |\mathbf{k}_\pi| |\mathbf{k}_a|}{6\sqrt{6}} \left[ \frac{C_{an\Delta} (3\cos\theta \mathbb{I}_{4 \times 4} - \Theta^\dagger)}{E_\pi - \Delta m + i\Gamma_\Delta/2} + \frac{C_{ap\Delta} (3\cos\theta \mathbb{I}_{4 \times 4} - \Theta)}{E_\pi + \Delta m - i\Gamma_\Delta/2} \right]$$



# Scattering cross section

## ★ Cross section formula

$$\sigma_{\pi^- p \rightarrow na} = \int \frac{d^3 \mathbf{k}_a}{(2\pi)^3 2E_a} \frac{d^3 \mathbf{k}_n}{(2\pi)^3 2E_n} (2\pi)^4 \delta^{(4)}(k_\pi + k_p - k_a - k_n) \frac{|\mathcal{M}_{\pi^- p \rightarrow na}|^2}{4[(k_\pi \cdot k_p)^2 - (m_\pi m_N)^2]^{1/2}}$$

$$\Rightarrow \sigma_{\pi^- p \rightarrow na} = \frac{E_\pi m_N^2}{16\pi f_\pi^2 f_a^2 |\mathbf{k}_\pi|} \mathcal{G}_a(|\mathbf{k}_\pi|) \quad C_\pm \equiv (C_{ap} \pm C_{an})/2 \quad \bar{\Gamma}_\Delta = \Gamma_\Delta/2$$

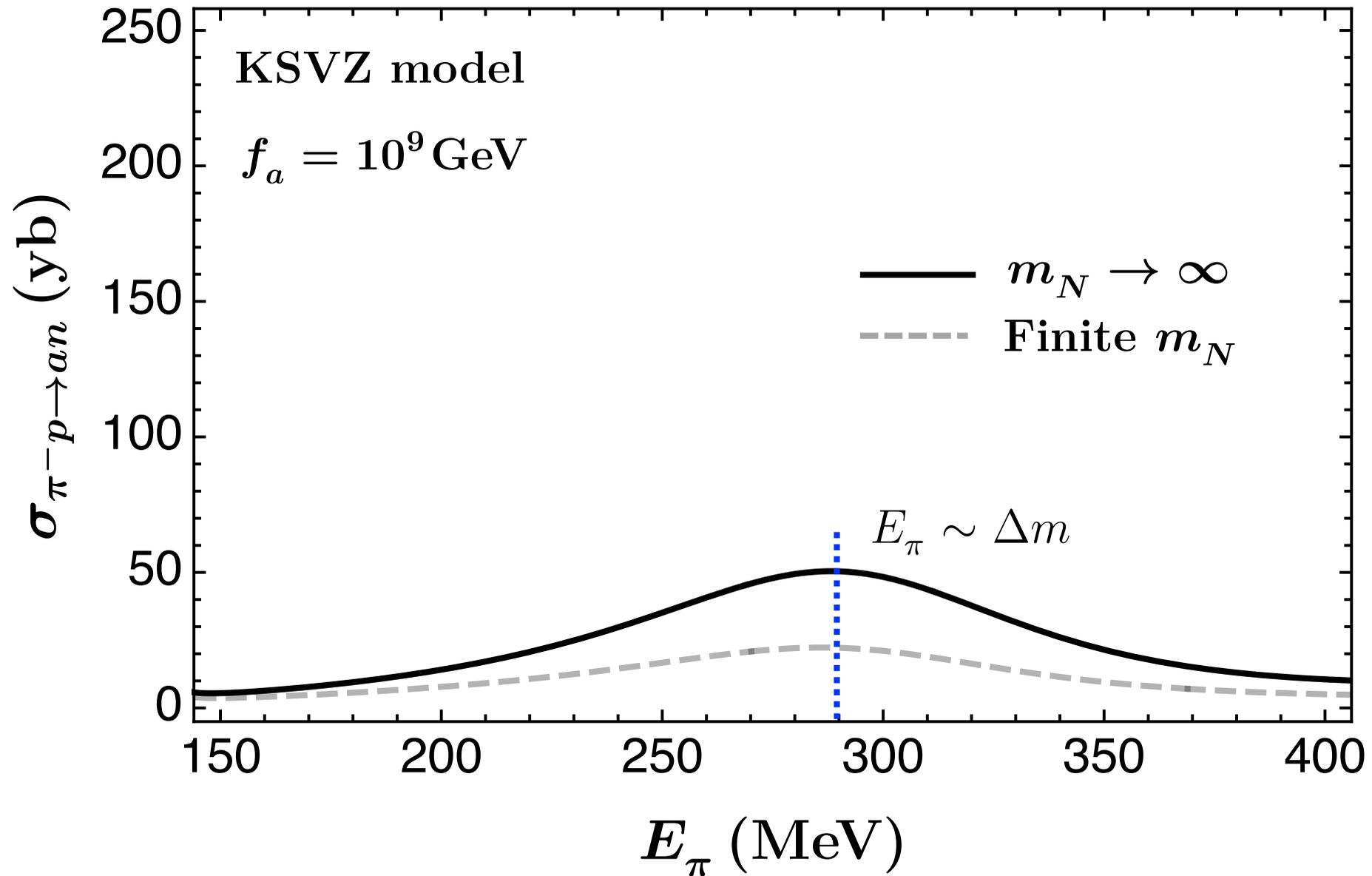
$$\begin{aligned} \mathcal{G}_a(|\mathbf{k}_\pi|) = & \frac{2g_A^2(2C_+^2 + C_-^2)}{3} \left(\frac{|\mathbf{k}_\pi|}{m_N}\right)^2 + C_{a\pi N}^2 \left(\frac{E_\pi}{m_N}\right)^2 + \frac{8\sqrt{2}g_A C_{a\pi N} C_-}{3} \left(\frac{|\mathbf{k}_\pi|}{m_N}\right)^2 \left(\frac{E_\pi}{m_N}\right) \\ & + \frac{4C_{aN\Delta}^2 C^2}{81} \frac{E_\pi^2 (\Delta m^2 + 2E_\pi^2 + \bar{\Gamma}_\Delta^2)}{[(\Delta m - E_\pi)^2 + \bar{\Gamma}_\Delta^2][(\Delta m + E_\pi)^2 + \bar{\Gamma}_\Delta^2]} \left(\frac{|\mathbf{k}_\pi|}{m_N}\right)^2 \\ & - \frac{8\sqrt{3}g_A C_{aN\Delta} C}{27} \frac{E_\pi [(\Delta m^2 - E_\pi^2)(C_+ \Delta m + C_- E_\pi) + \bar{\Gamma}_\Delta^2 (C_+ \Delta m - C_- E_\pi)]}{[(\Delta m - E_\pi)^2 + \bar{\Gamma}_\Delta^2][(\Delta m + E_\pi)^2 + \bar{\Gamma}_\Delta^2]} \left(\frac{|\mathbf{k}_\pi|}{m_N}\right)^2 \\ & - \frac{16\sqrt{6}C_{a\pi N} C_{aN\Delta} C}{27} \frac{E_\pi^2 (\Delta m^2 - E_\pi^2 - \bar{\Gamma}_\Delta^2)}{[(\Delta m - E_\pi)^2 + \bar{\Gamma}_\Delta^2][(\Delta m + E_\pi)^2 + \bar{\Gamma}_\Delta^2]} \left(\frac{|\mathbf{k}_\pi|}{m_N}\right)^2 \left(\frac{E_\pi}{m_N}\right) \end{aligned}$$

Delta resonance

$m_N \gg |\mathbf{k}_\pi|, E_\pi$

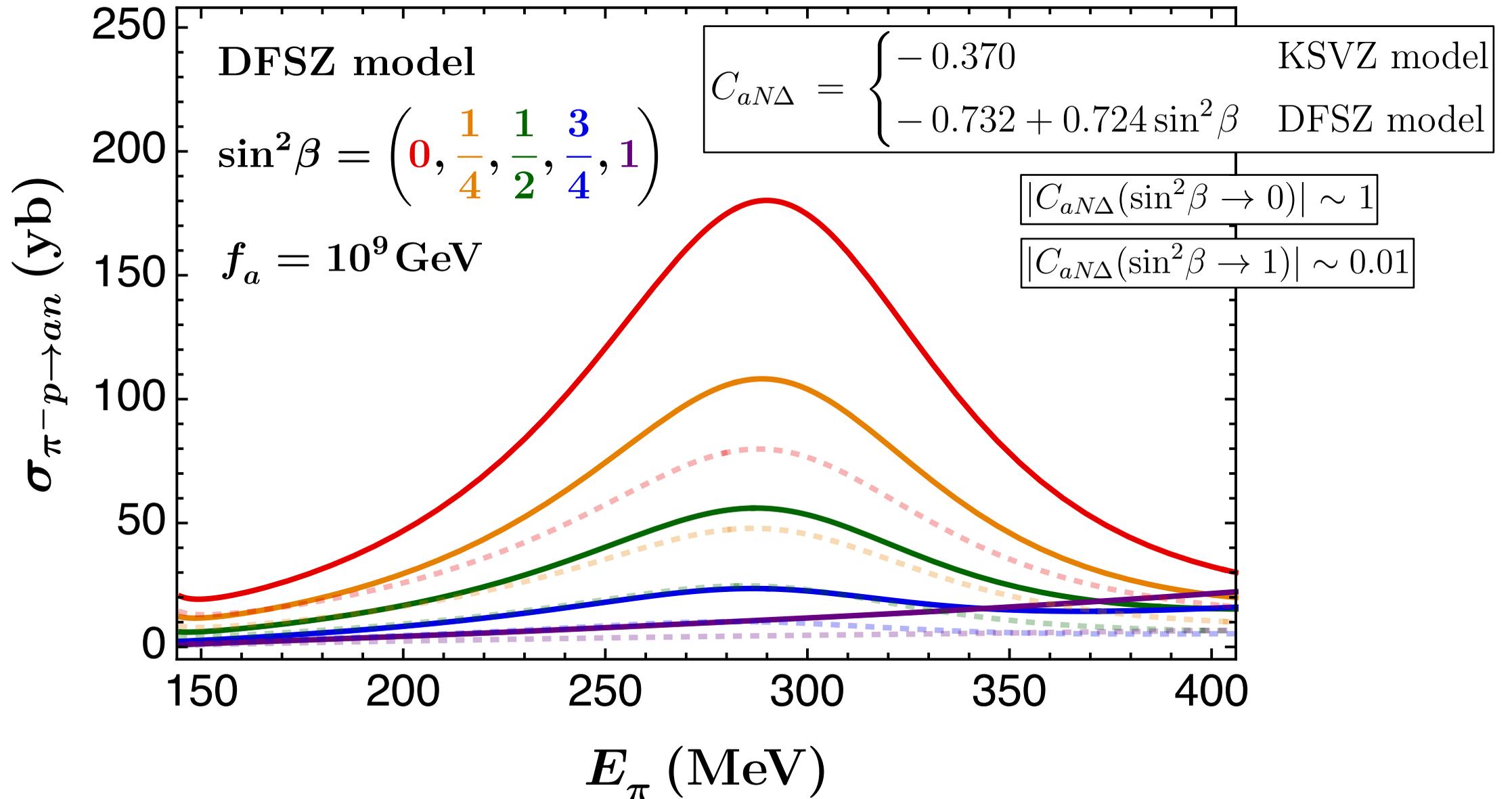
# Scattering cross section v.s. $E_\pi$

★ DFSZ model



# Scattering cross section v.s. $E_\pi$

 DFSZ model



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# Supernova Axion Emissivity

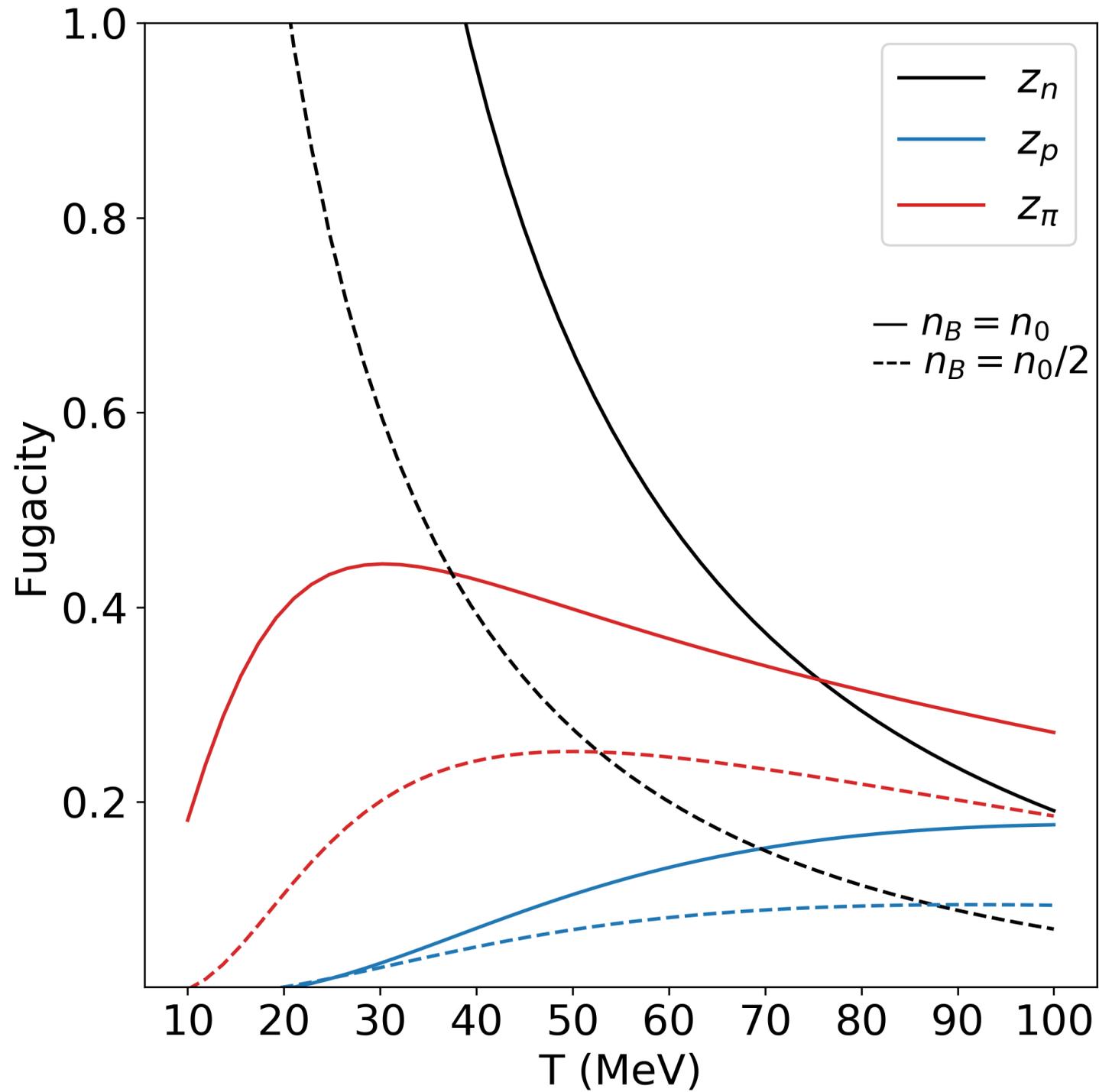
★ Emission rate (the energy loss per unit volume and time)

$$\dot{\mathcal{E}}_a = \int \frac{d^3\mathbf{k}_\pi}{(2\pi)^3 2E_\pi} \frac{d^3\mathbf{k}_p}{(2\pi)^3 2E_p} \frac{d^3\mathbf{k}_a}{(2\pi)^3 2E_a} \frac{d^3\mathbf{k}_n}{(2\pi)^3 2E_n} (2\pi)^4 \delta^{(4)}(k_\pi + k_p - k_a - k_n) \\ \times f_\pi(|\mathbf{k}_\pi|) f_p(|\mathbf{k}_p|) [1 - f_n(|\mathbf{k}_n|)] |\mathcal{M}_{\pi-p \rightarrow na}|^2 E_a, \quad f_j(|\mathbf{k}_j|) = 1/[e^{(E_j - \mu_j)/T} \pm 1]$$

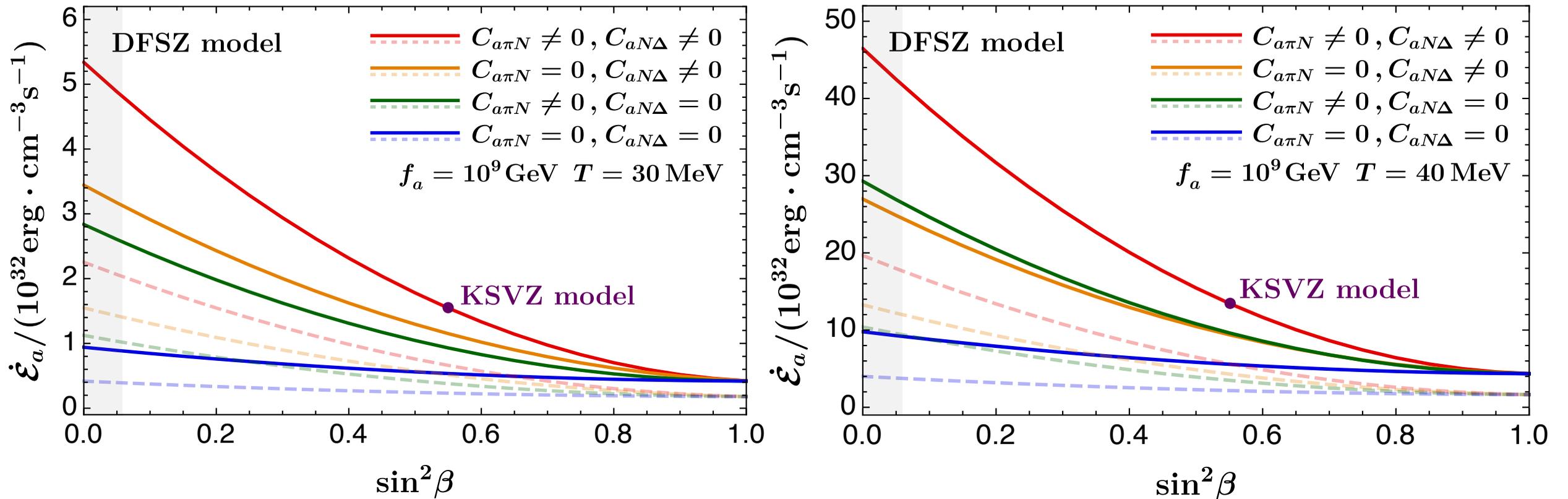
$$\blacktriangleright \dot{\mathcal{E}}_a = \frac{z_\pi z_p}{f_\pi^2 f_a^2} \sqrt{\frac{m_N^7 T^{11}}{128\pi^{10}}} \int_0^\infty dx_p \frac{x_p^2 e^{x_p^2}}{(e^{x_p^2} + z_n)(e^{x_p^2} + z_p)} \int_0^\infty dx_\pi \frac{x_\pi^2 \epsilon_\pi [\mathcal{G}_a(x_\pi) + \Delta\mathcal{G}_a(x_\pi)]}{e^{\epsilon_\pi - y_\pi} - z_\pi}$$

$$x_p = \frac{|\mathbf{k}_p|}{\sqrt{2m_N T}}, \quad x_\pi = \frac{|\mathbf{k}_\pi|}{T}, \quad \epsilon_\pi = \frac{E_\pi}{T}, \quad y_\pi = \frac{m_\pi}{T}, \quad z_j = e^{(\mu_j - m_j)/T}$$

$$\Delta\mathcal{G}_a(x_\pi) = \frac{\sqrt{2} g_A C_{a\pi N} C_-}{3} \frac{E_\pi^4 - 3(\Delta m^2 - \bar{\Gamma}_\Delta^2) E_\pi^2 + 2(\Delta m^2 + \bar{\Gamma}_\Delta^2)^2}{[(\Delta m - E_\pi)^2 + \bar{\Gamma}_\Delta^2][(\Delta m + E_\pi)^2 + \bar{\Gamma}_\Delta^2]} \left(\frac{|\mathbf{k}_\pi|}{m_N}\right)^2 \left(\frac{E_\pi}{m_N}\right)$$



# Supernova Axion Emissivity v.s. $\sin^2 \beta$

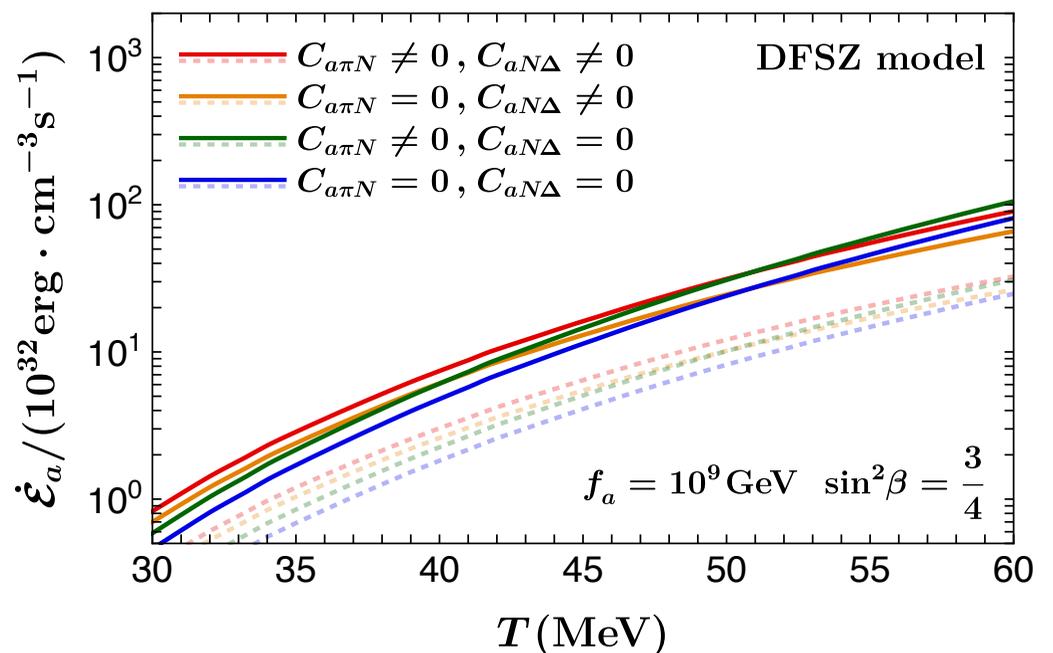
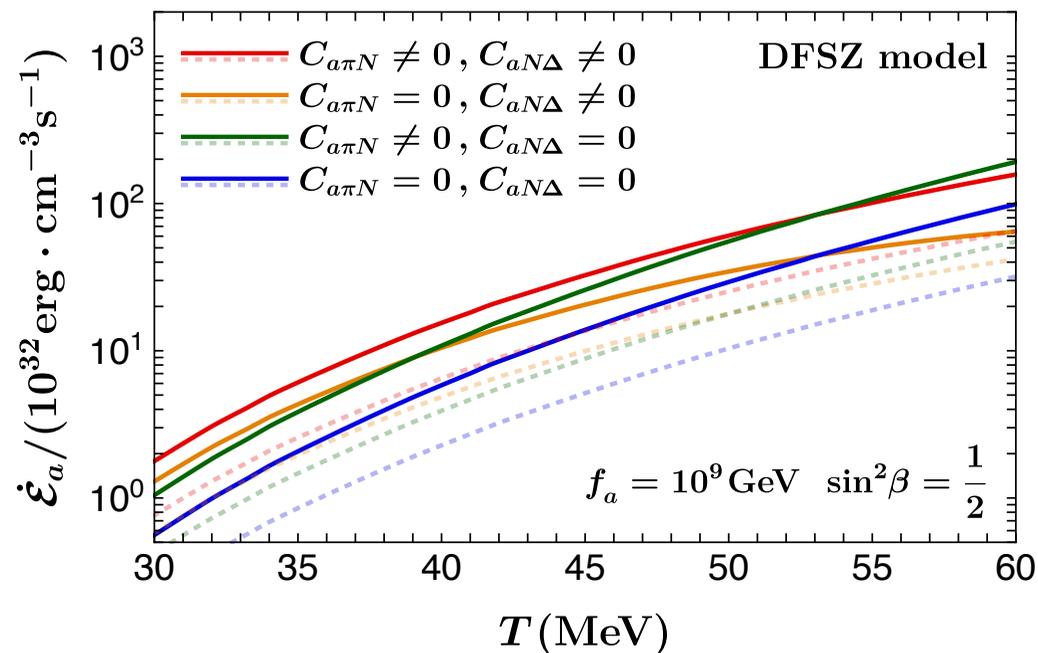
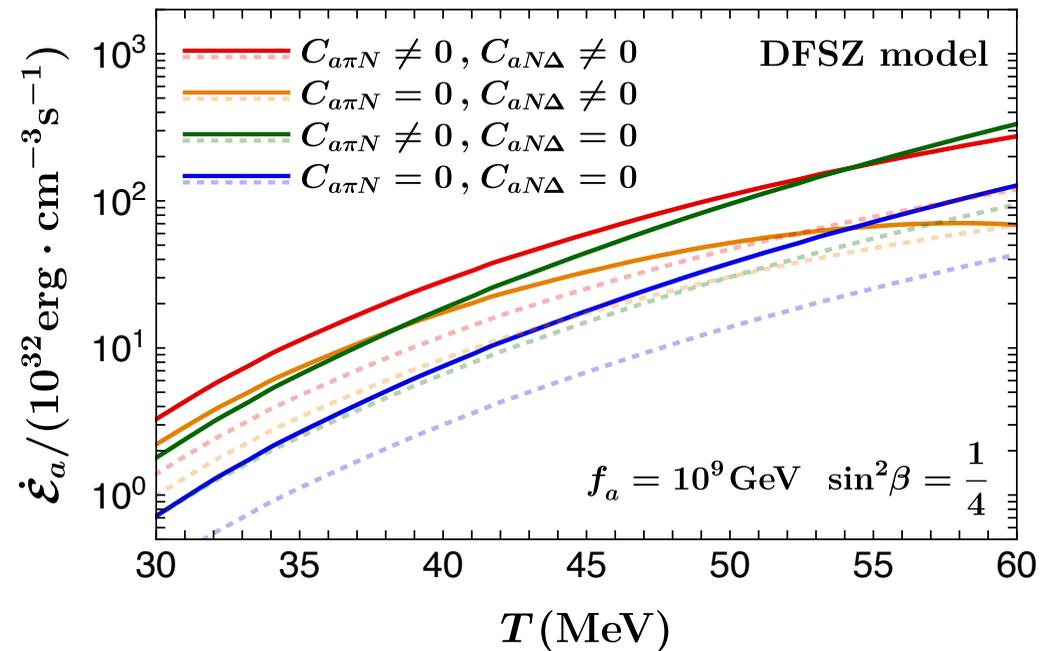
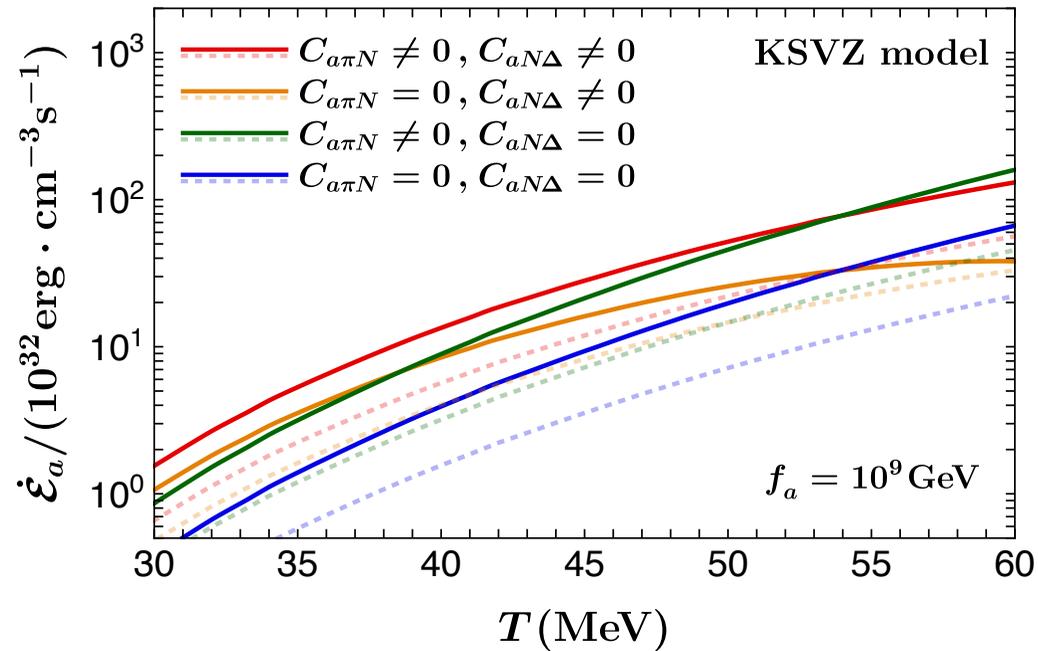


✨ The gray band is excluded by tree-level unitarity of fermion scattering :  $0.25 \lesssim \tan \beta \lesssim 170$

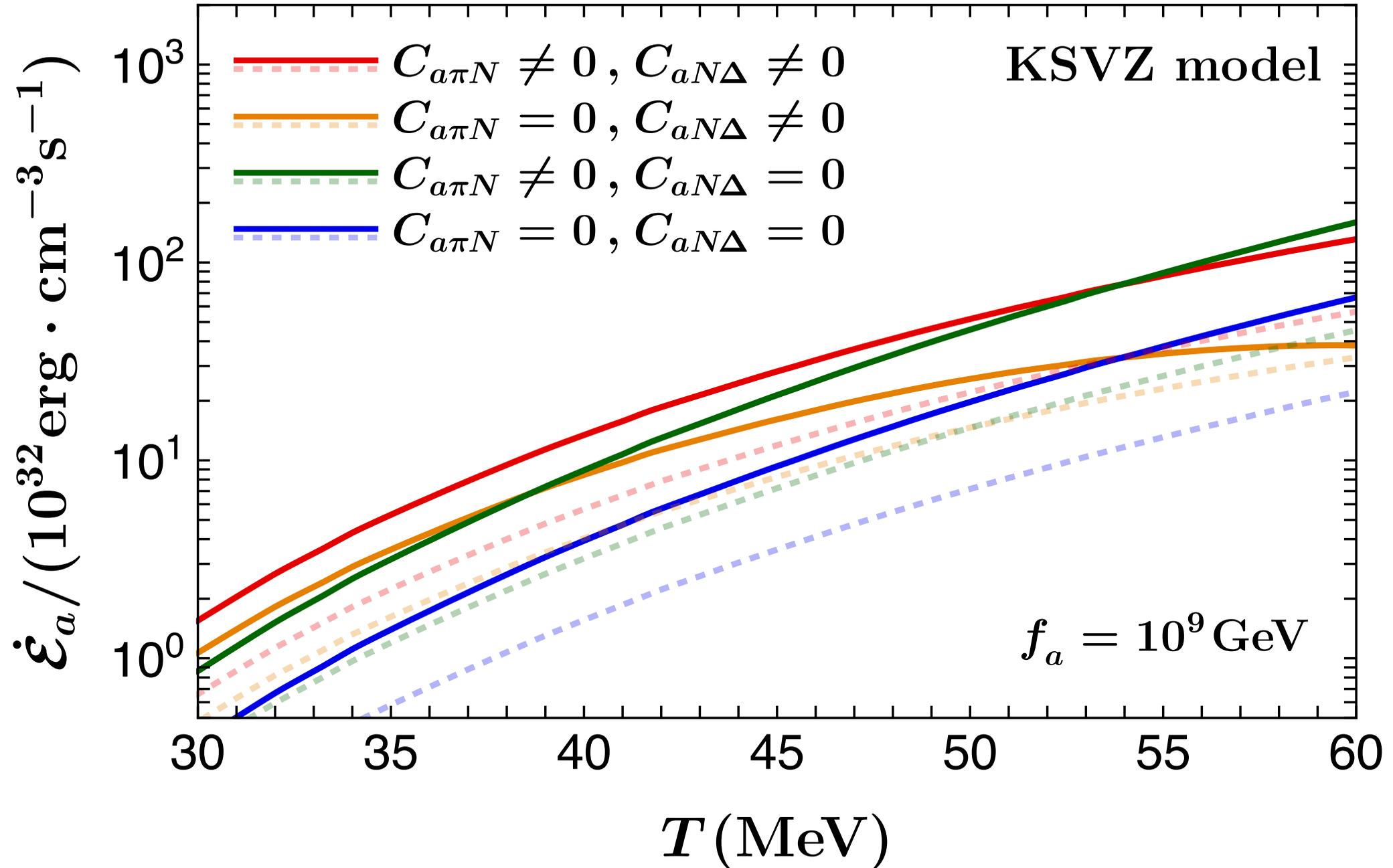
Luzio, et al. 2021

✨ Supernova axion emissivity can be enhanced at most by a factor of  $\sim 5$  for  $\beta \rightarrow 0$  compared to the earlier studies.

# Supernova Axion Emissivity v.s. $T$



# Supernova Axion Emissivity v.s. $T$



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# Summary

- ✦ We have estimated the supernova axion emissivity with the  $\Delta(1232)$  resonance in the HBChPT.
- ✦ We have noticed that the supernova axion emissivity was overestimated by  $m_N \rightarrow \infty$  in DFSZ and KSVZ models.
- ✦ We have shown that the supernova axion emissivity can be enhanced by a factor of  $\sim 4$  in the KSVZ model and up to a factor of  $\sim 5$  in the DFSZ model with  $\tan \beta \rightarrow 0$  compared to the case without the  $C_{a\pi N}$  and  $C_{aN\Delta}$ .
- ✦ We have found that the  $\Delta$  resonance can give a destructive contribution to the supernova axion emissivity at high  $T_{\text{SN}}$ .

**Back up**

# The Strong CP Problem in QCD

★ The CP-violating term in QCD

$$\mathcal{L}_\theta = \theta \frac{g_s^2}{32\pi} G^{c\mu\nu} \tilde{G}_{\mu\nu}^c$$

strong CP phase

★ Experimental bound from neutron EDM :  $|\theta| < 10^{-10}$

★ Theoretically, this problem even more puzzling

$$\theta = \theta_0 + \arg \det(M_u M_d)$$

theta vacuum

chiral transformation

Why  $\theta$  is so small is the strong CP problem.

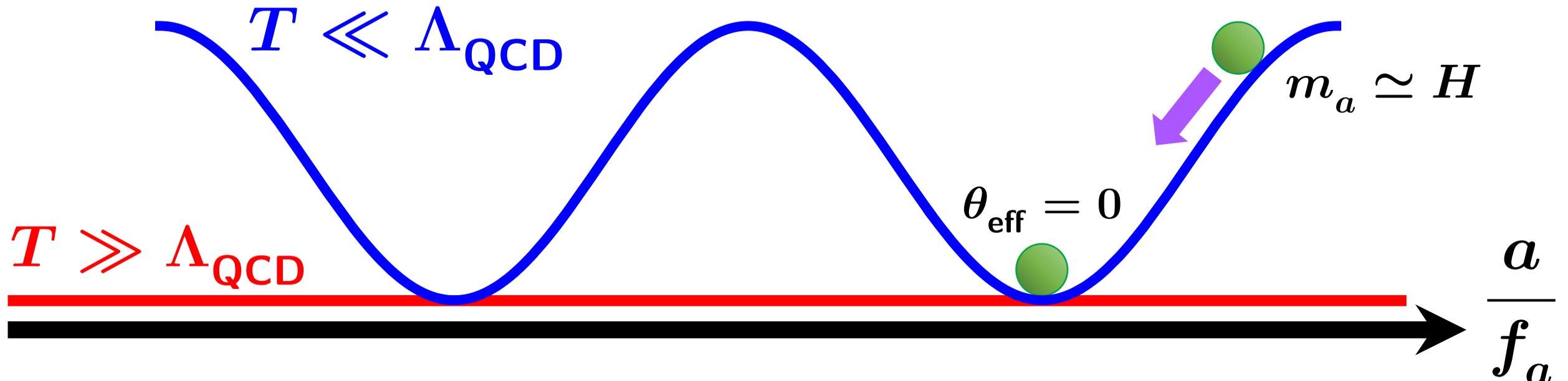
# The QCD axion

★ Peccei-Quinn (PQ) mechanism : Strong CP phase is promoted to a dynamical variable :

Peccei, Quinn '77, Weinberg '78, Wilczek '78

$$\mathcal{L}_\theta = \underbrace{\left[ \theta + \frac{a(x)}{f_a} \right]}_{\theta_{\text{eff}}(x)} \frac{g_s^2}{32\pi} G^{c\mu\nu} \tilde{G}_{\mu\nu}^c$$

$f_a$  : decay constant



# The **QCD** axion window

★ The traditional range of the QCD axion decay constant

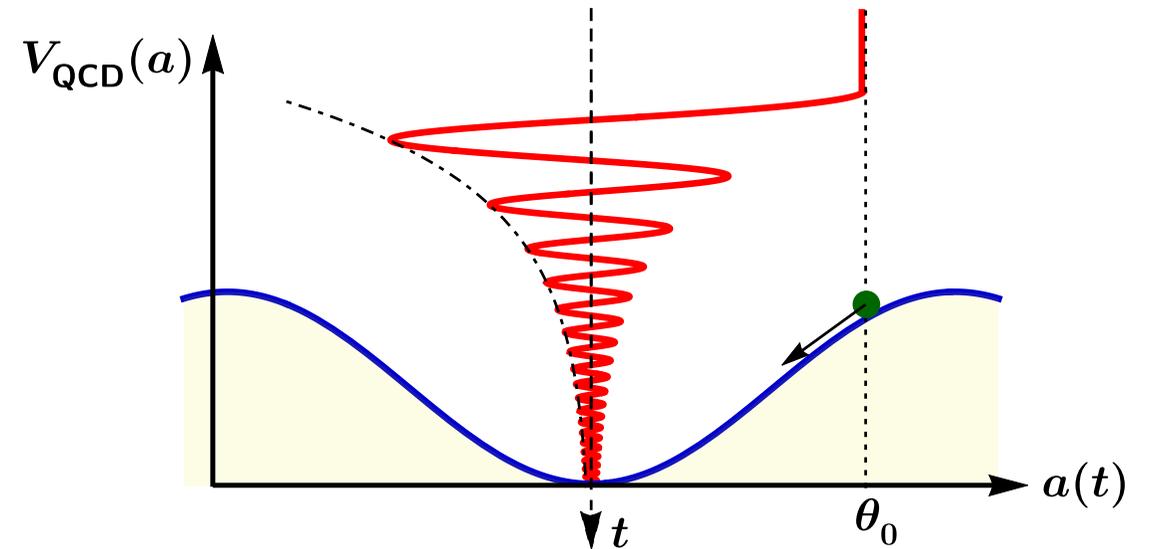
$$10^9 \text{ GeV} \lesssim f_a \lesssim 10^{12} \text{ GeV}$$



SN1987A neutrino burst duration

$$\dot{\mathcal{E}}_a \lesssim 10^{(32-33)} \text{ erg cm}^{-3} \text{ s}^{-1}$$

Raffelt '90



$$\Omega_a h^2 \simeq 0.14 \theta_0^2 \left( \frac{f_a}{10^{12} \text{ GeV}} \right)^{1.17}$$

# Axion Interactions with matter

## ★ Axion-electron interaction

$$\mathcal{L}_{aee} = -iC_{ae} \frac{m_e}{f_a} a \overline{\psi}_e \gamma^5 \psi_e \stackrel{\text{E.O.M. \& I.P.}}{=} C_{ae} \frac{\partial_\mu a}{2f_a} \overline{\psi}_e \gamma^\mu \gamma^5 \psi_e$$

$C_{ae}$  : model-dependent coefficient

## ★ Axion-nucleons interaction

$$\mathcal{L}_{aNN} = \sum_{N=p,n} C_{aN} \frac{\partial_\mu a}{2f_a} \overline{\psi}_N \gamma^\mu \gamma^5 \psi_N \quad (\text{related to our work})$$

★ The axion couples to the SM particles with strength inversely proportional to the decay constant. Hence, the axion feebly couples to the SM particles due to the large decay constant.

# Heavy Baryon Formalism

Jenkins & Manohar '91

✦ In this formalism, the nucleon is **almost on-shell** with a nearly **unchanged velocity** when it exchanges some tiny momentum with the pion

$$p_N^\mu = m_N v^\mu + \delta k_\pi^\mu \quad v^2 = 1 \quad \begin{array}{l} m_N / \Lambda_\chi \sim 1 \\ \delta k_\pi^\mu / \Lambda_\chi \ll 1 \end{array}$$

➤ Velocity-dependence baryon field

$$\Lambda_\chi \sim 1 \text{ GeV}$$

$$\mathcal{B}_v(x) = e^{im_B v \cdot x} \mathcal{B}(x) \longrightarrow \overline{\mathcal{B}}(i\cancel{\partial} - m_B) \mathcal{B} \longrightarrow \overline{\mathcal{B}}_v i\cancel{\partial} \mathcal{B}_v$$

- The power counting expansion of the effective field theory for pions and baryons can be systematic and well-behaved.
- The algebra of the **spin operator formalism** can be much simpler than that of the gamma matrix formalism.