HENAN NORMAL UNIVERSITY

第十届海峡两岸
粒子物理和宇宙学研讨会

基于非可逆融合规则的辐射轻子模型

A Radiative Lepton Model in a Non-invertible Fusion Rule

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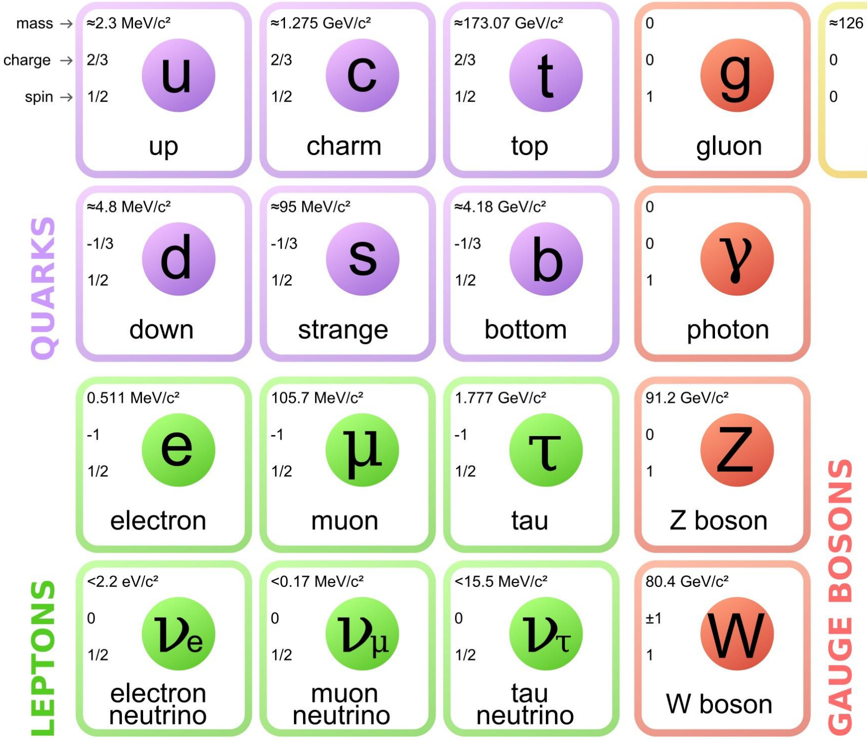
广州 · 2026年1月21日

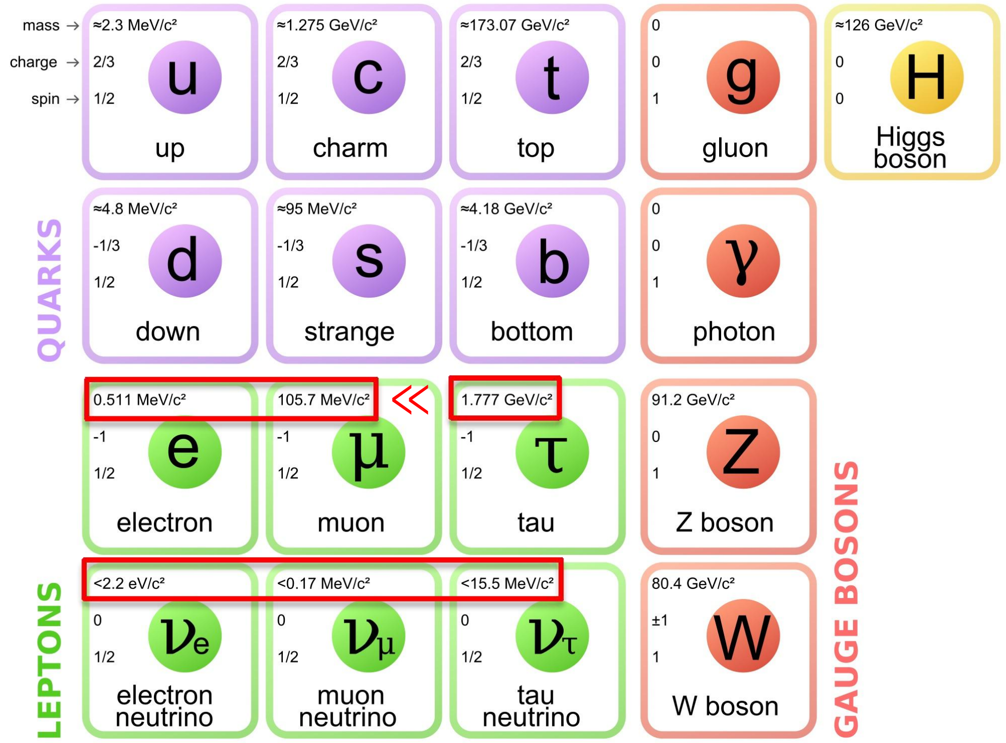
- Jingqian Chen, Chao-Qiang Geng, Hiroshi Okada, and Jia-Jun Wu, [arXiv:2507.11951[hep-ph]].

The Standard Model and Beyond



- The Origin of Neutrino Mass
- The Nature of Dark Matter
- Muon *g*-2 Anomaly
- … … …

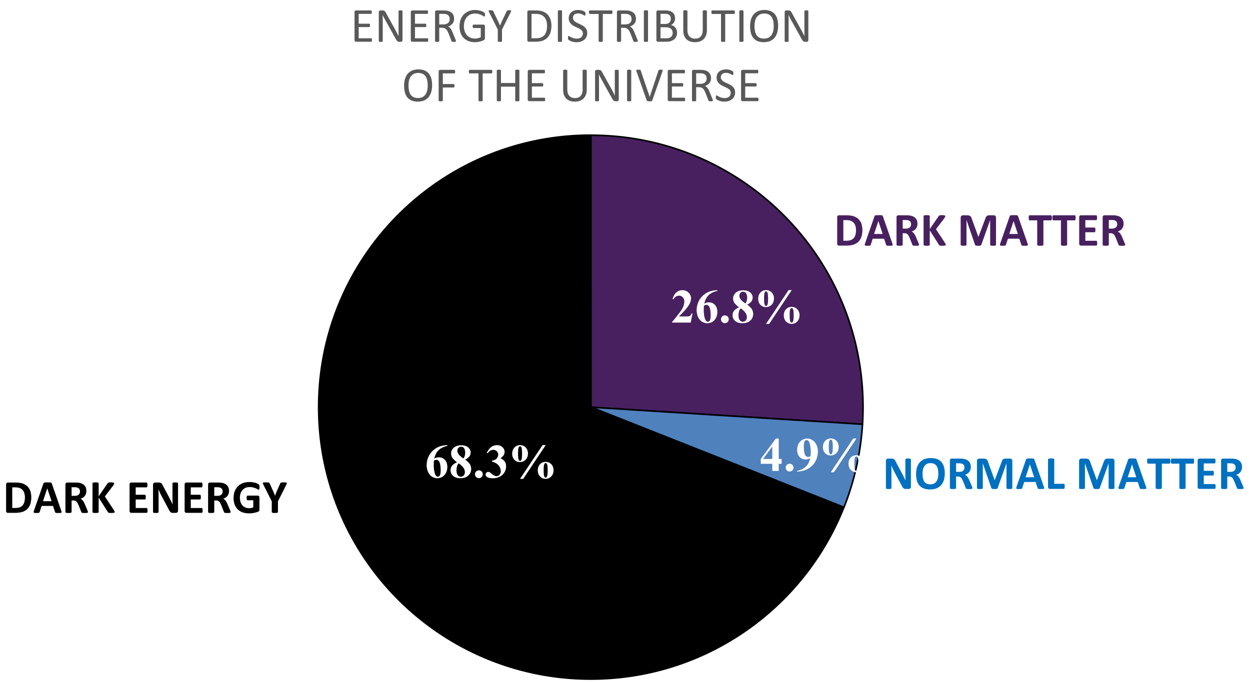
Lepton Mass： Neutrino Mass & Charged Lepton Mass Hierarchy Problem



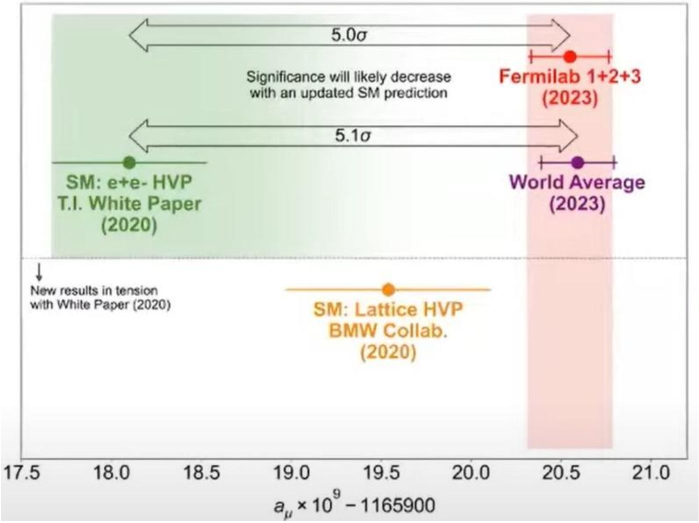
- The τ lepton has a significantly larger mass than other charged leptons.

- In the Standard Model, neutrinos are massless, whereas experimental observations have revealed non-zero neutrino masses.

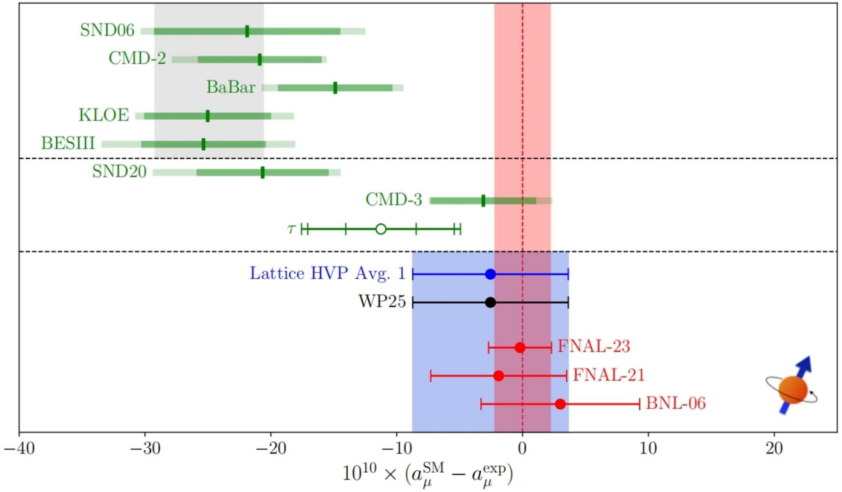
Dark Matter


ENERGY DISTRIBUTION
OF THE UNIVERSE
DARK MATTER
26.8%
4.9%
NORMAL MATTER
68.3%
DARK ENERGY

Muon g-2 Anomaly



August 2023



June 2025

- Aguillard D P, Albahri T, Allspach D, et al. Measurement of the Positive Muon Anomalous Magnetic Moment to 0.20 ppm[J]. arXiv:2308.06230(2023).
- D.~P.~Aguillard, et al. Measurement of the Positive Muon Anomalous Magnetic Moment to 127 ppb. arXiv:2506.03069 [hep-ex].
- Aliberti, R., et al. "The anomalous magnetic moment of the muon in the Standard Model: an update." arXiv:2505.21476 (2025).

Non-invertible Fusion Rule

A Group： $g_1 g_2 = g_3,$

The product on the right-hand side is a single element, and every element has an inverse.

Non-invertible fusion rule: $U_i U_j = \sum_k c_{ij}^k U_k,$

The product on the right-hand side can be a linear combination of several elements, and the elements may not have inverses.

Ising fusion rules (IFR): $\epsilon \otimes \epsilon = \mathbb{I}, \quad \sigma \otimes \sigma = \mathbb{I} \oplus \epsilon, \quad \sigma \otimes \epsilon = \epsilon \otimes \sigma = \sigma,$

$$\{\epsilon, \sigma\} \otimes \mathbb{I} = \mathbb{I} \otimes \{\epsilon, \sigma\} = \{\epsilon, \sigma\}.$$

⇓

σ is an element without an inverse

Non-invertible Fusion Rule

Physical realization and origin: fusion rules in 2D CFT and developments in generalized symmetries

- S. H. Shao, [arXiv:2308.00747 [hep-th]].
- C. M. Chang, Y. H. Lin, S. H. Shao, Y. Wang and X. Yin, [arXiv:1802.04445 [hep-th]].
- J. Kaidi, Y. Tachikawa and H. Y. Zhang, [arXiv:2402.00105 [hep-th]].
- R. Thorngren and Y. Wang, [arXiv:2106.12577 [hep-th]].
- … … …

Recently introduced into particle physics and applied to phenomenology：

- T. Kobayashi, H. Otsuka, M. Tanimoto and H. Uchida, [arXiv:2505.07262 [hep-ph]].
- M. Suzuki and L. X. Xu, [arXiv:2503.19964 [hep-ph]].
- T. Kobayashi, H. Otsuka and T. T. Yanagida, [arXiv:2508.12287 [hep-ph]].
- H. Okada and Y. Shigekami, [arXiv:2507.16198 [hep-ph]].
- … … …

The Model

2 Generations (over E_R, E_L, N_R)

	$L_{L_{e,\mu}}$	L_{L_τ}	ℓ_R	E_R	E_L	N_R	H	η	S
$SU(2)_L$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$
$U(1)_Y$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	-1	0	$\frac{1}{2}$	$\frac{1}{2}$	0
IFR	ϵ	$\mathbb{I}$	$\mathbb{I}$	σ	σ	σ	$\mathbb{I}$	σ	σ

$$(\epsilon \otimes \epsilon = \mathbb{I}, \quad \sigma \otimes \sigma = \mathbb{I} \oplus \epsilon, \quad \sigma \otimes \epsilon = \epsilon \otimes \sigma = \sigma)$$

Lagrangian for the Lepton Sector:

$$-\mathcal{L}_\ell = \sum_{\ell=e,\mu}\sum_{a=1,2} y_{\ell a}\overline{L_{L_\ell}}\eta E_{R_a} + \sum_{a=1,2}\sum_{i=1}^{3} y_{E_{ai}}\overline{E_{L_a}}\ell_{R_i} S + \sum_{a=1}^{3} y_{\tau a}\overline{L_{L_\tau}} H \ell_{R_a} + \sum_{a=1,2} h_{\tau a}\overline{L_{L_\tau}}\eta E_{R_a}$$
$$+ \sum_{\ell=e,\mu}\sum_{a=1,2} y_{\eta_{\ell a}}\overline{L_{L_\ell}}\tilde{\eta} N_{R_a} + \sum_{a=1,2} y_{\eta_{\tau a}}\overline{L_{L_\tau}}\tilde{\eta} N_{R_a} + M_{E_a}\overline{E_{L_a}}E_{R_a} + M_{N_a}\overline{N^C_{R_a}}N_{R_a} + \text{h.c.},$$

The Model

The Scotogenic Model

	$L_{L_{e,\mu}}$	L_{L_τ}	ℓ_R	E_R	E_L	N_R	H	η	S
$SU(2)_L$	**2**	**2**	**1**	**1**	**1**	**1**	**2**	**2**	**1**
$U(1)_Y$	$-\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	-1	0	$\frac{1}{2}$	$\frac{1}{2}$	0
IFR	ϵ	$\mathbb{I}$	$\mathbb{I}$	σ	σ	σ	$\mathbb{I}$	σ	σ

$$(\epsilon \otimes \epsilon = \mathbb{I}, \quad \sigma \otimes \sigma = \mathbb{I} \oplus \epsilon, \quad \sigma \otimes \epsilon = \epsilon \otimes \sigma = \sigma)$$

Lagrangian for the Lepton Sector:

$$-\mathcal{L}_\ell = \sum_{\ell=e,\mu} \sum_{a=1,2} y_{\ell a} \overline{L_{L_\ell}} \eta E_{R_a} + \sum_{a=1,2} \sum_{i=1}^{3} y_{E_{ai}} \overline{E_{L_a}} \ell_{R_i} S + \sum_{a=1}^{3} y_{\tau a} \overline{L_{L_\tau}} H \ell_{R_a} + \sum_{a=1,2} y_{\tau a} \overline{L_{L_\tau}} \eta E_{R_a}$$
$$+ \sum_{\ell=e,\mu} \sum_{a=1,2} y_{\eta_{\ell a}} \overline{L_{L_\ell}} \tilde{\eta} N_{R_a} + \sum_{a=1,2} y_{\eta_{\tau a}} \overline{L_{L_\tau}} \tilde{\eta} N_{R_a} + M_{E_a} \overline{E_{L_a}} E_{R_a} + M_{N_a} \overline{N_{R_a}^C} N_{R_a} + \text{h.c.},$$

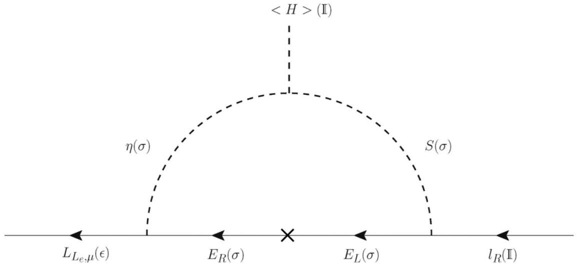
The $\overline{L_{L_{e,\mu}}} \ell_R H$ term is forbidden by the IFR, and therefore no tree-level mass can be generated:

$$\epsilon \otimes \mathbb{I} \otimes \mathbb{I} = \epsilon \neq \mathbb{I}$$

The electron and muon masses are generated at one loop, analogously to the neutrino mass generation mechanism.

Lepton Masses

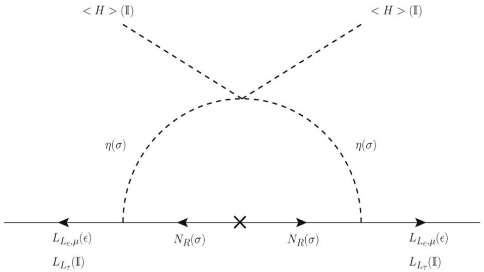
Generation of electron and muon Masses:



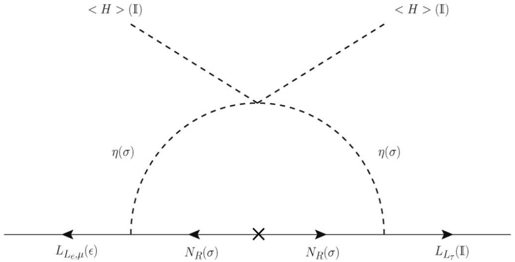
$$\epsilon \otimes \mathbb{I} \otimes \mathbb{I} = \epsilon \neq \mathbb{I}$$

IFR dynamically broken at the loop

Generation of neutrino Masses:



IFR Invariant

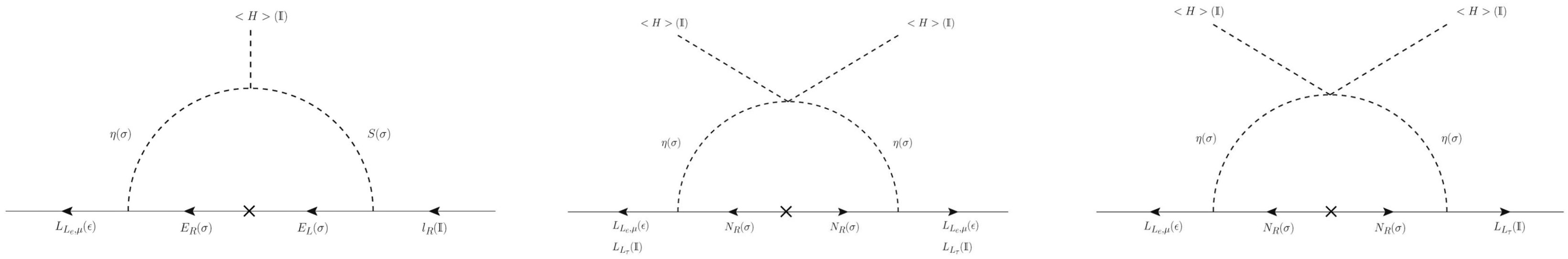


IFR broken

Lepton Masses

Mixing Terms in the Scalar Potential:

$$\mathcal{V}_{\text{mix}} = \lambda_3 \left(H^\dagger H\right)\left(\eta^\dagger \eta\right) + \lambda_4 \left(H^\dagger \eta\right)\left(\eta^\dagger H\right) + \frac{1}{2}\lambda_5 \left[\left(H^\dagger \eta\right)^2 + \text{h.c.}\right]$$
$$+ \lambda_{HS}\, H^\dagger H S^* S + \lambda_{\eta S}\, \eta^\dagger \eta\, S^* S + \boxed{\left(\mu H^\dagger \eta S + \text{h.c.}\right)}.$$



$$\sigma \otimes \sigma \sim \mathbb{I}$$

In these loop diagrams, the "groupification" of the Ising fusion rules gives rise to an effective Z_2 symmetry associated with a conserved σ-parity, which stabilizes the lightest σ particle so that it can serve as a dark matter candidate.

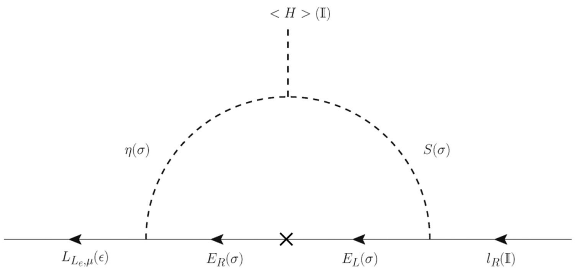
- J. Kaidi, Y. Tachikawa and H. Y. Zhang, [arXiv:2402.00105 [hep-th]].
- M. Suzuki and L. X. Xu, [arXiv:2503.19964 [hep-ph]].

Lepton Masses

Charged Lepton Masses (tree level):

$$\mathcal{M}_\ell^{\rm tree} = \frac{v_H}{\sqrt{2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ y_{\tau 1} & y_{\tau 2} & y_{\tau 3} \end{pmatrix},$$

Charged Lepton Masses (one-loop level):



$$\left(\mu H^\dagger \eta S + \text{h.c.}\right)$$

$$\eta \equiv [\eta^+, (\eta_R + i\eta_I)/\sqrt{2}]^T$$

$$S \equiv (S_R + iS_I)/\sqrt{2}$$

⇒

$$\eta_R = c_\theta H_1 + s_\theta H_2, \quad \eta_I = c_\theta A_1 + s_\theta A_2,$$

$$S_R = -s_\theta H_1 + c_\theta H_2, \quad S_I = -s_\theta A_1 + c_\theta A_2,$$

⇓

$$\sum_{\ell=e,\mu} \sum_{a=1,2} \frac{y_{\ell a}}{\sqrt{2}} \overline{\ell'_{L_\ell}} E_{R_a} (c_\theta H_1 + s_\theta H_2) + i \sum_{\ell=e,\mu} \sum_{a=1,2} \frac{y_{\ell a}}{\sqrt{2}} \overline{\ell'_{L_\ell}} E_{R_a} (c_\theta A_1 + s_\theta A_2)$$

$$+ \sum_{a=1,2} \sum_{i=1}^{3} \frac{y_{E_{ai}}}{\sqrt{2}} \overline{E_{L_a}} \ell_{R_i} (-s_\theta H_1 + c_\theta H_2) + i \sum_{a=1,2} \sum_{i=1}^{3} \frac{y_{E_{ai}}}{\sqrt{2}} \overline{E_{L_a}} \ell_{R_i} (-s_\theta A_1 + c_\theta A_2) + \text{h.c.},$$

Lepton Masses

Charged Lepton Masses (one-loop level)：

$$(\delta m)_{\ell i} = \sum_{a=1,2} \frac{y_{\ell a} M_{E_a} y_{E_{ai}}}{2(4\pi)^2} s_\theta c_\theta \left[F_I(H_1, H_2, E_a) - F_I(A_1, A_2, E_a)\right], \quad \text{其中，} F_I(b_1, b_2, f) = \left[\frac{m_{b_1}^2}{m_f^2 - m_{b_1}^2} \ln\left(\frac{m_{b_1}^2}{m_f^2}\right) - \frac{m_{b_2}^2}{m_f^2 - m_{b_2}^2} \ln\left(\frac{m_{b_2}^2}{m_f^2}\right)\right],$$

Complete charged-lepton mass matrix:

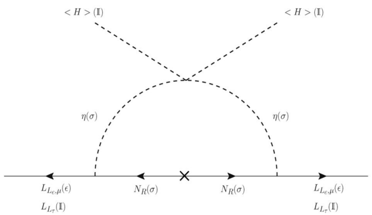
$$\mathcal{M}_\ell \equiv \mathcal{M}_\ell^{\text{tree}} + \delta m \quad \Rightarrow \quad \frac{v_H}{\sqrt{2}} \begin{pmatrix} \sqrt{2}\frac{\delta m_{e1}}{v_H} & \sqrt{2}\frac{\delta m_{e2}}{v_H} & \sqrt{2}\frac{\delta m_{e3}}{v_H} \\ \sqrt{2}\frac{\delta m_{\mu 1}}{v_H} & \sqrt{2}\frac{\delta m_{\mu 2}}{v_H} & \sqrt{2}\frac{\delta m_{\mu 3}}{v_H} \\ y_{\tau 1} & y_{\tau 2} & y_{\tau 3} \end{pmatrix}$$

$$\Downarrow$$

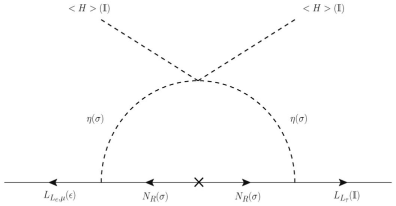
$$\mathcal{M}_\ell \mathcal{M}_\ell^\dagger \sim \begin{pmatrix} (\delta m \delta m^\dagger)_{2\times 2} & (\delta m \mathcal{M}_\ell^{\text{tree},\dagger})_{2\times 1} \\ (\mathcal{M}_\ell^{\text{tree}} \delta m^\dagger)_{1\times 2} & (\mathcal{M}_\ell^{\text{tree}} \mathcal{M}_\ell^{\text{tree},\dagger})_{1\times 1} \end{pmatrix} \quad \Rightarrow \quad m_e, m_\mu \ll m_\tau$$

Lepton Masses

Neutrino Masses:



IFR Invariant



IFR Broken

$$(m_\nu)_{ij} = \sum_{a=1}^{2} \frac{(y_\eta)_{ia} M_{N_a} (y_\eta^T)_{aj}}{2(4\pi)^2} \left[c_\theta^2 F_I(H_1, A_1, N_a) + s_\theta^2 F_I(H_2, A_2, N_a)\right] \equiv \sum_{a=1}^{2} (y_\eta)_{ia} D_{N_a} (y_\eta^T)_{aj}.$$

Casas–Ibarra parameterization： $y_\eta = U_\nu^* \sqrt{D_\nu} O_N \sqrt{D_N^{-1}}$ $\quad \mathrm{NH}: \; O_N = \begin{bmatrix} 0 & 0 \\ \cos z & -\sin z \\ \sin z & \cos z \end{bmatrix}, \quad \mathrm{IH}: \; O_N = \begin{bmatrix} \cos z & -\sin z \\ \sin z & \cos z \\ 0 & 0 \end{bmatrix}$

$(\mathrm{NH}): \sum D_\nu \sim \sqrt{\Delta m_{\mathrm{atm}}^2} \sim \mathcal{O}(50)$ meV $\quad \Delta m_{\mathrm{atm}}^2 \equiv |D_{\nu_3}^2 - D_{\nu_1}^2|$

$(\mathrm{IH}): \sum D_\nu \sim 2\sqrt{\Delta m_{\mathrm{atm}}^2} \sim \mathcal{O}(100)$ meV $\quad \Delta m_{\mathrm{atm}}^2 \equiv |D_{\nu_2}^2 - D_{\nu_3}^2|$

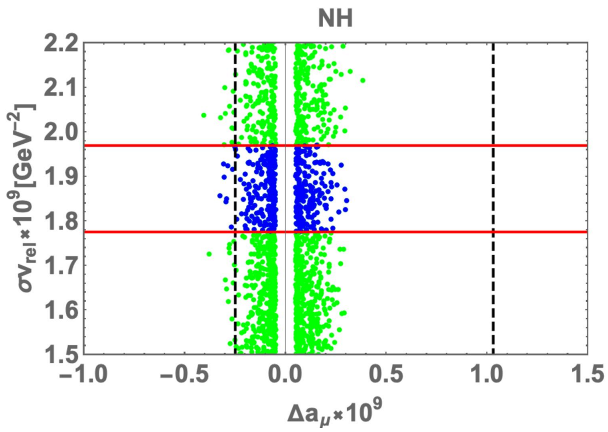
Cosmological： $\sum D_\nu \leq 120$ meV

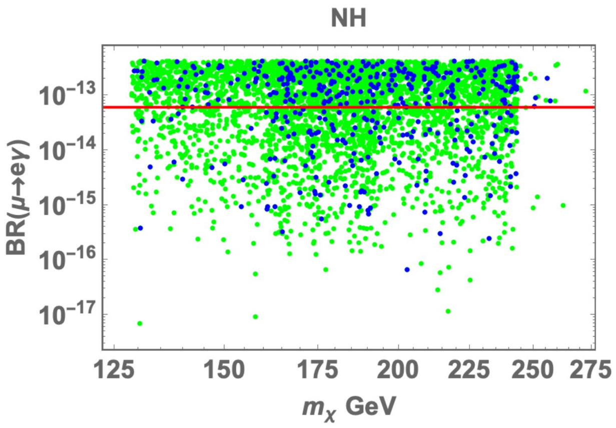
DESI + CMB： $\sum D_\nu \leq 72$ meV

Numerical Results

Taking into account the LFV constraints and requiring the muon g-2 deviation to be around 1σ, we take H_2 as the dark matter candidate:

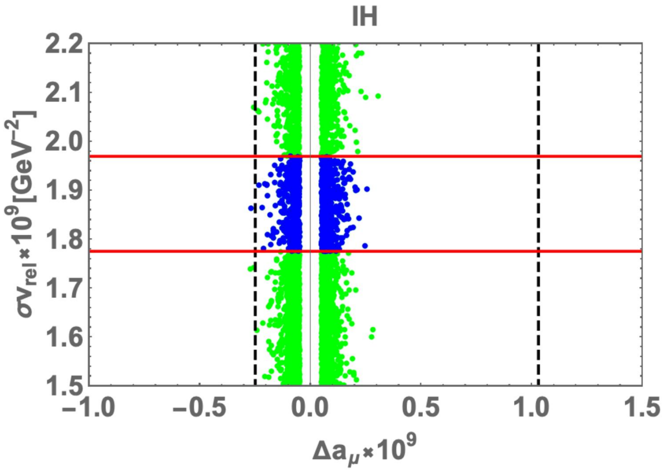
The NH case：

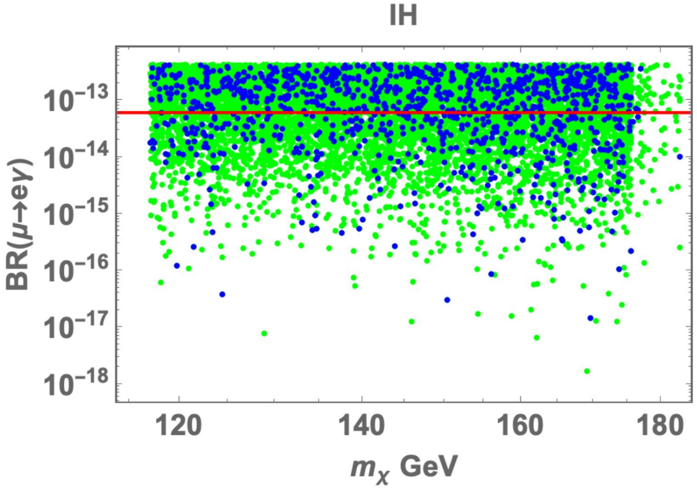




Numerical Results

The IH case：





Summary:

- **We proposed a mechanism where the electron and muon masses are radiatively generated at the one-loop level through the dynamical breaking of the Ising fusion rule.**

- **Neutrino masses are also generated at one loop, the IFR remains unbroken in some components of the neutrino mass matrix.**

- **the IFR plays a crucial role in stabilizing the particles inside the loop, it effectively acts as an exact Z_2 symmetry that persists unbroken at loop orders.**

- **There exists a parameter region where neutrino oscillation data, LFV bounds, the observed dark-matter relic density, and the muon g−2 deviation can be simultaneously satisfied.**

THANKS!