

The Fate of Chiral Gauge Theories

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Why Chiral Gauge theories (CGTs)

➤ High Energy Physics:

- Asymptotic free, no mass scale, one parameter in the theory
- Original SU(5) GUT is a strongly coupled chiral gauge theory
- Composite Higgs models for solving the Hierarchy problem

➤ Condense Matter Physics:

- Quantum Hall Effects, Chiral Magnetic effects, Luttinger liquid...
- Symmetric Mass generation → Helps put chiral gauge theory on lattice by gapping the fermion-doubler [D. Tong 2104.03997], and see [Wang, You, 2204.14271] for a review

➤ Crucial to study the phase of CGTs:

Conformal? Confinement? Dynamical Symmetry Breaking?

Tools for vector-like theories

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 - vector-like symmetry cannot be broken

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- **Lattice Experiment**

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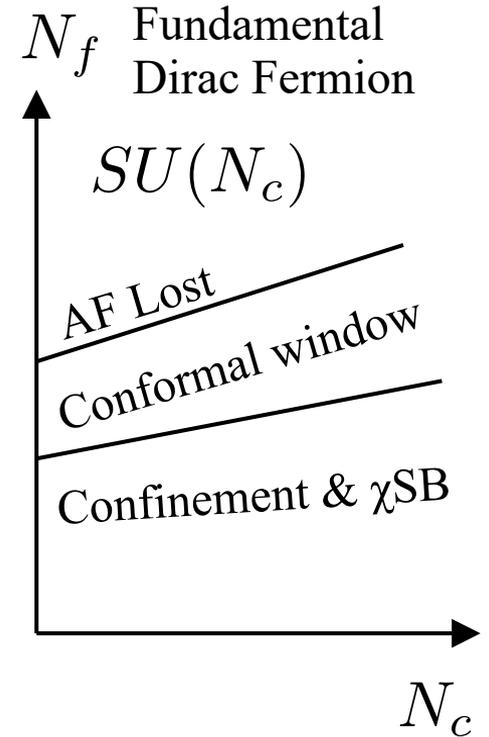
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-- Different gauge representations, different N_c dependence
- Lattice Experiment

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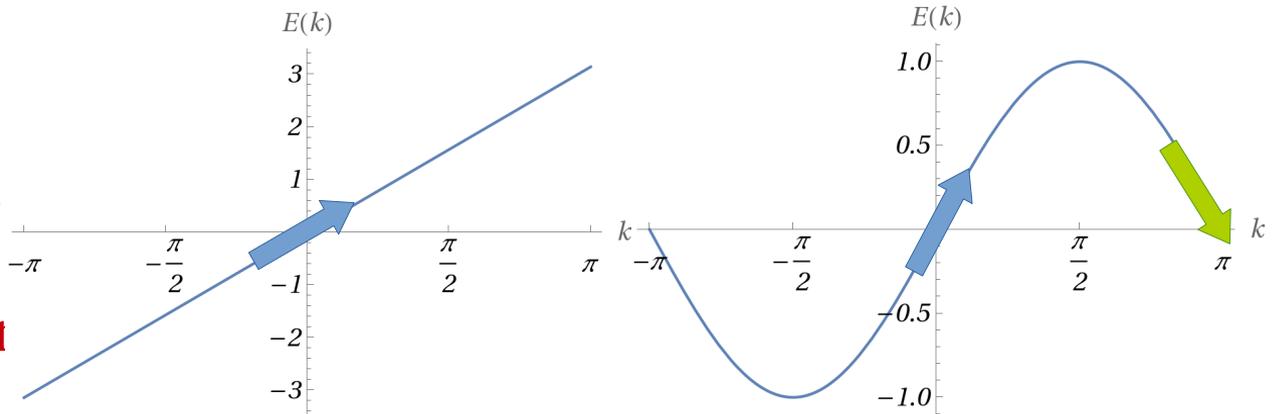
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-- invoke persistence mass

➤ ~~Large-N limit~~ [Banks, Zaks]

-- Different gauge representation

➤ ~~Lattice Experiment~~ fermion-doubler, Nielsen-Ninomiya theorem



The toy model

Georgi-Glashow model:

	$SU(N_c)$	Flavor
χ	\square	1
ψ	$\bar{\square}$	$N_c - 4$

The Global symmetry: $SU(N_c - 4) \times U(1)$

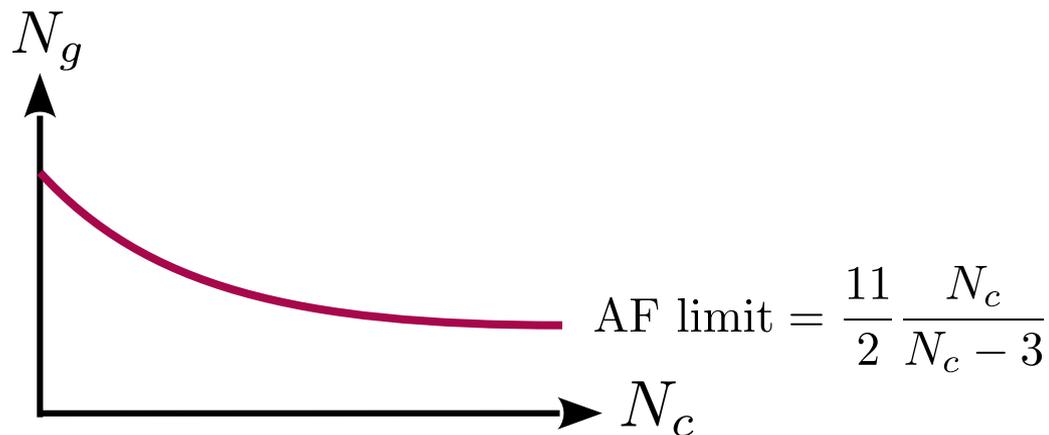
Baryon: $\mathcal{B} = A \{ \bar{F} \bar{F} \}$ Match the t'Hooft anomaly

The toy model

General Georgi-Glashow model:

	$SU(N_c)$	Flavor	
χ	\square	1	$\times N_g$
ψ	$\bar{\square}$	$N_c - 4$	

The Global symmetry: $SU(N_g) \times SU(N_g(N_c - 4)) \times U(1)$



The tool: Functional renormalization group method

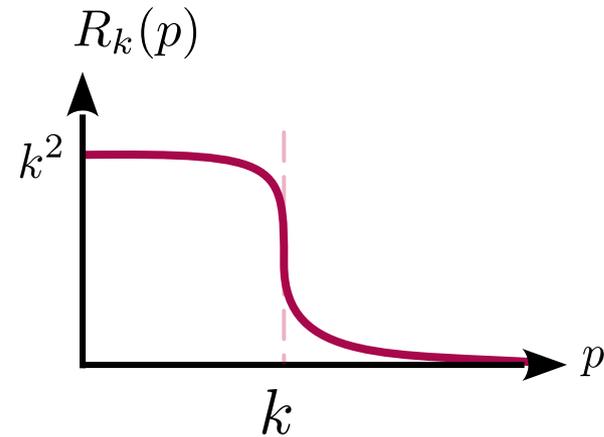
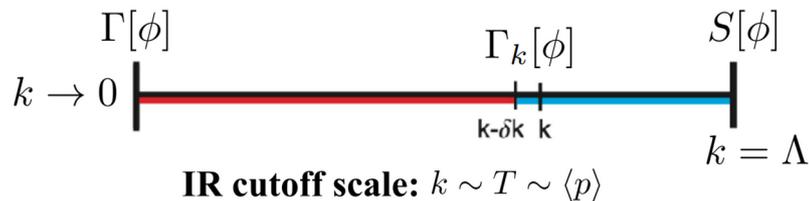
- Functional Renormalization Group: a non-perturbative method widely used in QCD

See Dupuis, et.al [2006.04853] for a reievew

- Basic idea: effective average action: $\Gamma_k[\phi]$

$$\int [\mathcal{D}\phi]_{p>k} = \int \mathcal{D}\phi \exp(-\Delta S_k[\phi]) \quad \Delta S_k[\phi] = \int_p \phi(p) R_k \phi(-p)$$

$$\Gamma_k[\phi] = \int_x J(x)\phi(x) - \mathcal{W}_k[J] - \Delta S_k[\phi]$$



The tool: Functional renormalization group method

Master formula:

Wetterich '93

$$\partial_t \Gamma_k[\phi] = \frac{1}{2} \text{Tr} \left[\frac{1}{\Gamma_k^{(2)} + R_k} \partial_t R_k \right]$$

$$\partial_t \equiv k \partial_k$$

$$\partial_t \Gamma_k^{(1)} = -\frac{1}{2} \text{STr} \Gamma_k^{(3)} \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k} \quad \text{Both UV and IR finite}$$

$$\partial_t \Gamma_k^{(2)} = -\frac{1}{2} \text{STr} \left[\Gamma_k^{(4)} - 2\Gamma_k^{(3)} \Gamma_k^{(2)} \Gamma_k^{(3)} \right] \frac{\partial_t R_k}{\Gamma_k^{(2)} + R_k}$$



Infinite tower of flow equation, needs to truncate $\Gamma_k^{(i > n_{\max})} = 0$

We choose $n_{\max} = 4$, enough to capture the fermion condensates

The tool: Functional renormalization group method

Truncated effective action:

$$\Gamma_k = \int \mathcal{L}_{kin} + \mathcal{L}_{gh} + \mathcal{L}_{gf} + Z_\psi^2 \sum_{i=1}^2 \lambda_i \mathcal{O}_i + Z_\chi^2 \sum_{i=3}^5 \lambda_i \mathcal{O}_i + Z_\psi Z_\chi \sum_{i=6}^7 \lambda_i \mathcal{O}_i$$

$$\lambda_i = \lambda_i(p_m) |_{p_m \rightarrow 0}$$

$$\mathcal{O}_1 = (\psi^\dagger \bar{\sigma}^\mu \psi) (\psi^\dagger \bar{\sigma}^\mu \psi)$$

$$\mathcal{O}_2 = (\psi^\dagger f_1 \bar{\sigma}^\mu \psi_{f_2}) (\psi^\dagger f_2 \bar{\sigma}^\mu \psi_{f_1})$$

$$\mathcal{O}_3 = (\chi^\dagger f_1 \bar{\sigma}^\mu \chi_{f_2}) (\chi^\dagger f_2 \bar{\sigma}^\mu \chi_{f_1})$$

$$\mathcal{O}_4 = (\chi^\dagger \bar{\sigma}^\mu \chi) (\chi^\dagger \bar{\sigma}^\mu \chi)$$

$$\mathcal{O}_5 = (\chi^\dagger \bar{\sigma}^\mu T_{\text{anti}} \chi) (\chi^\dagger \bar{\sigma}^\mu T_{\text{anti}} \chi)$$

$$\mathcal{O}_6 = (\psi^\dagger \bar{\sigma}^\mu \psi) (\chi^\dagger \bar{\sigma}^\mu \chi)$$

$$\mathcal{O}_7 = (\psi^\dagger \bar{\sigma}^\mu T_{\text{a-fund}} \psi) (\chi^\dagger \bar{\sigma}^\mu T_{\text{anti}} \chi)$$

Complete four fermion basis is derived for arbitrary N_g and N_c . Constructed with **Young Tensor Method** [HLL, et.al. 2005.00008, 2201.04639]

Except for $N_g=1$ and $N_c=5$, where one additional operator is needed:

$$\mathcal{O}_8 = \epsilon_{i_1 i_2 i_3 i_4 i_5} (\chi^{i_1 i_2} \chi^{i_3 i_4}) (\chi^{i_5 j} \psi_j)$$

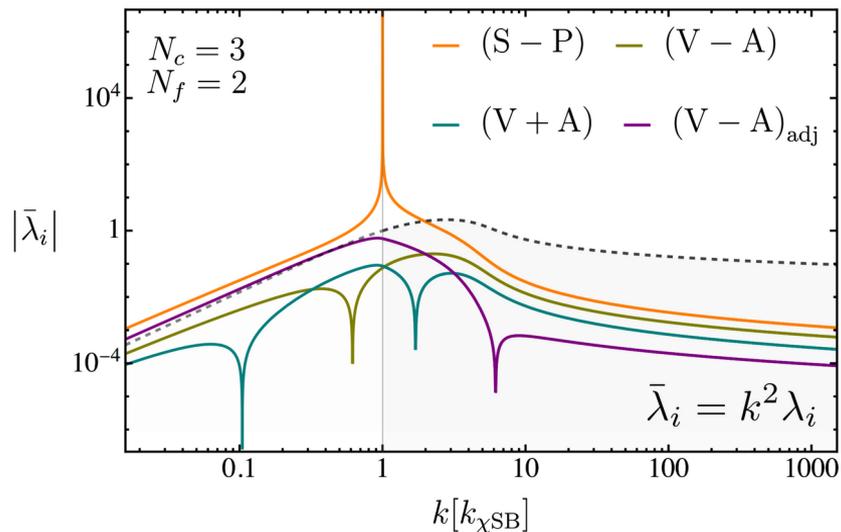
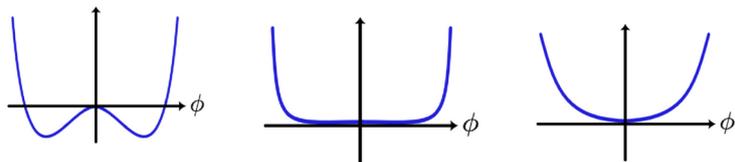
The tool: Functional renormalization group method

How to diagnose Symmetry breaking?

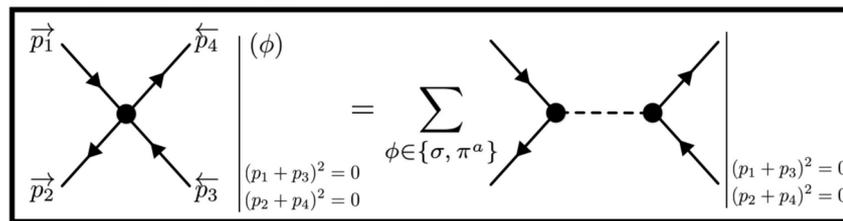
Divergent 4-fermi flow
 $(\psi_1\psi_2)(\psi_1\psi_2)$



Fermion condensate
 $\langle\psi_1\psi_2\rangle \neq 0$



Exact field redefinition Stratonovich'57 Hubbard'59



$$\lambda_i \propto 1/m_{\phi_i}^2 \qquad m_{\phi_i}^2 = \partial_{\phi_i^2} V(\phi_1)$$

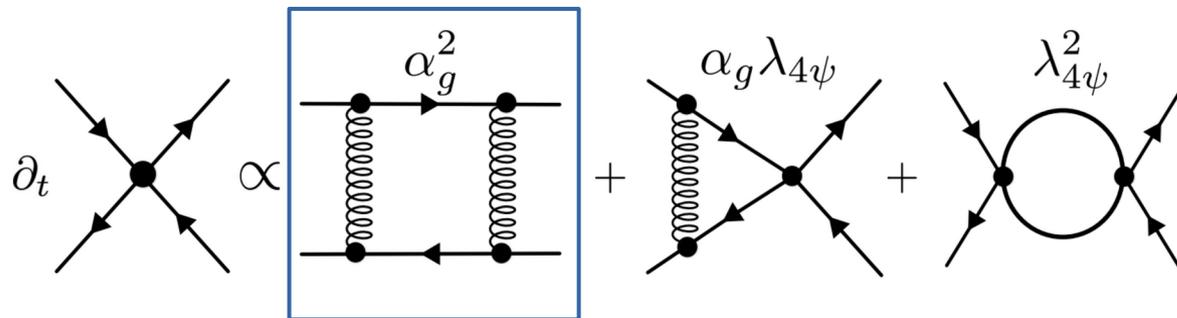
$$\lambda_i \rightarrow \infty \qquad m_{\phi_i}^2 \rightarrow 0 \qquad \xi \rightarrow \infty$$

$$\langle\bar{\psi}\psi\rangle \neq 0$$

See [2006.04853] for detailed discussion

The tool: Functional renormalization group method

4-fermion flows:



$$\bar{\lambda} = k^2 \lambda$$

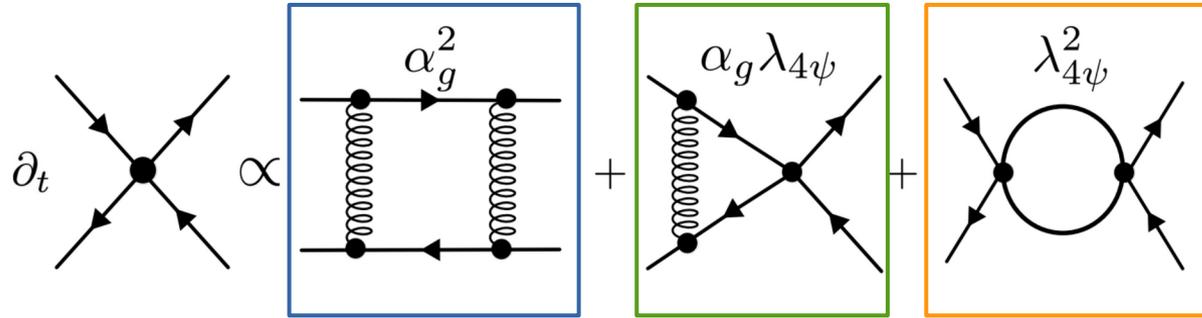
$$\partial_t \bar{\lambda}_i \propto 2 \bar{\lambda}_i + \mathbf{c}_{A,i} \cdot \alpha_g^2 + \mathbf{c}_{B,ij} \cdot \alpha_g \bar{\lambda}_j + \mathbf{c}_{C,ijk} \cdot \bar{\lambda}_j \bar{\lambda}_k + \dots$$



Canonical scaling

The tool: Functional renormalization group method

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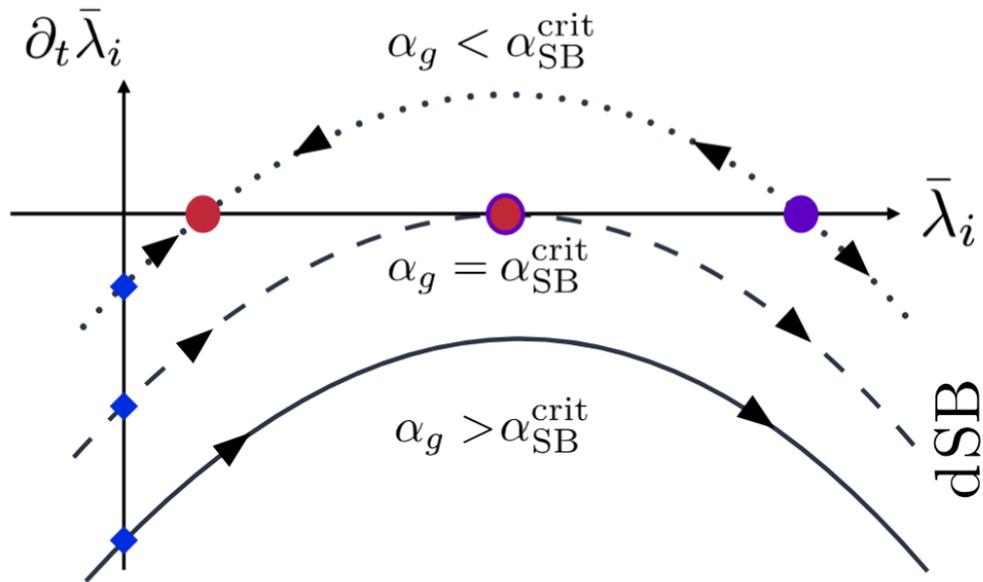


Canonical scaling

The tool: Functional renormalization group method

Focus on a $\bar{\lambda}_i$, the flow is a quadratic function of $\bar{\lambda}_i$:

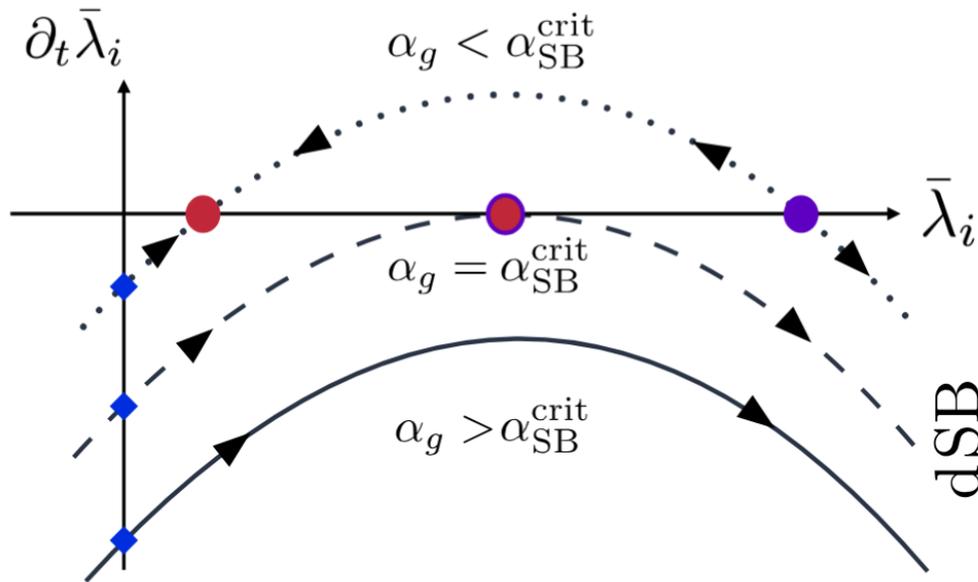
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Necessary condition for dSB:

1. resonant structure (naive)

$$\frac{\mathbf{c}_{A,i}}{\mathbf{c}_{C,iii}} > 0$$

2. α_{SB}^{crit} is reached

The tool: Functional renormalization group method

Technical side

FeynRules:
4-fermion
Feynman Rules

DoFun:
Generate Diagram and
Flow equation

FormTracer
& adapted to:
SU(N) trace algebra
for anti-sym rep

4ψ For example

$$\partial_t \bar{\lambda}_1 = (2 + 2\eta_\psi) \bar{\lambda}_1 + \frac{k^2}{Z_\psi^2} \frac{\partial_t \left[\mathcal{P}_{1,R}^{abcd} \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} \right] (N_c N_\psi + 1) - \partial_t \left[\mathcal{P}_{2,R}^{abcd} \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} \right] (N_c + N_\psi)}{128 N_c (N_c^2 - 1) N_\psi (N_\psi^2 - 1)},$$

$$\partial_t \bar{\lambda}_2 = (2 + 2\eta_\psi) \bar{\lambda}_2 + \frac{k^2}{Z_\psi^2} \frac{\partial_t \left[\mathcal{P}_{2,R}^{abcd} \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} \right] (N_c N_\psi + 1) - \partial_t \left[\mathcal{P}_{1,R}^{abcd} \Gamma_k^{(\bar{\psi}_a \psi_b \bar{\psi}_c \psi_d)} \right] (N_c + N_\psi)}{128 N_c (N_c^2 - 1) N_\psi (N_\psi^2 - 1)}.$$

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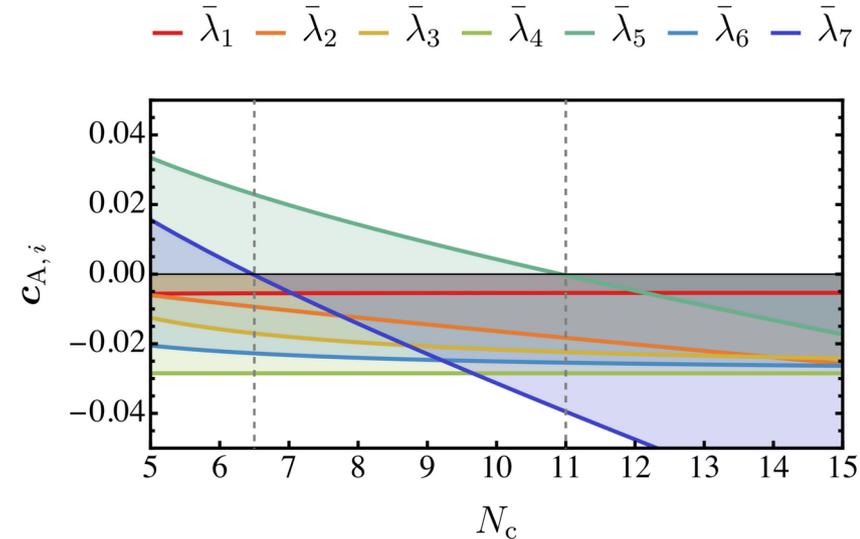
$$\partial_t \bar{\lambda}_i \propto 2 \bar{\lambda}_i + \mathbf{c}_{A,i} \cdot \alpha_g^2 + \mathbf{c}_{B,ij} \cdot \alpha_g \bar{\lambda}_j + \mathbf{c}_{C,ijk} \cdot \bar{\lambda}_j \bar{\lambda}_k + \dots$$

In General GG model: $\mathbf{c}_{C,iii} > 0 \quad \forall i$

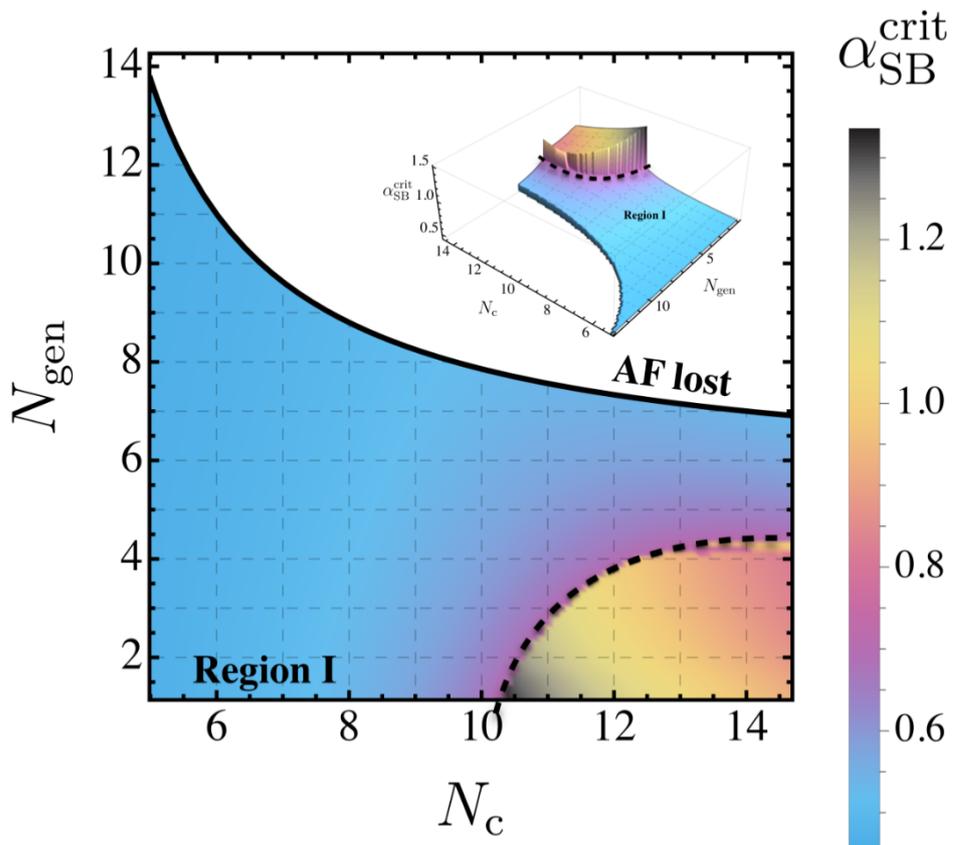
$$\mathbf{c}_{A,i} = \frac{9}{2} \left\{ -\frac{1}{4N_c^2} - \frac{3}{16}, -\frac{N_c^2 - 8}{16N_c}, -1, \boxed{-1 + \frac{4 + 2N_c}{N_c^2}}, \right. \\ \left. 1 - \frac{N_c}{8} + \frac{4}{N_c}, \boxed{-1 + \frac{N_c + 2}{N_c^2}}, 1 - \frac{N_c}{4} + \frac{4}{N_c} \right\}.$$

$$\mathcal{O}_7 = (\psi^\dagger \bar{\sigma}^\mu T_{\text{a-fund}} \psi) (\chi^\dagger \bar{\sigma}^\mu T_{\text{anti}} \chi)$$

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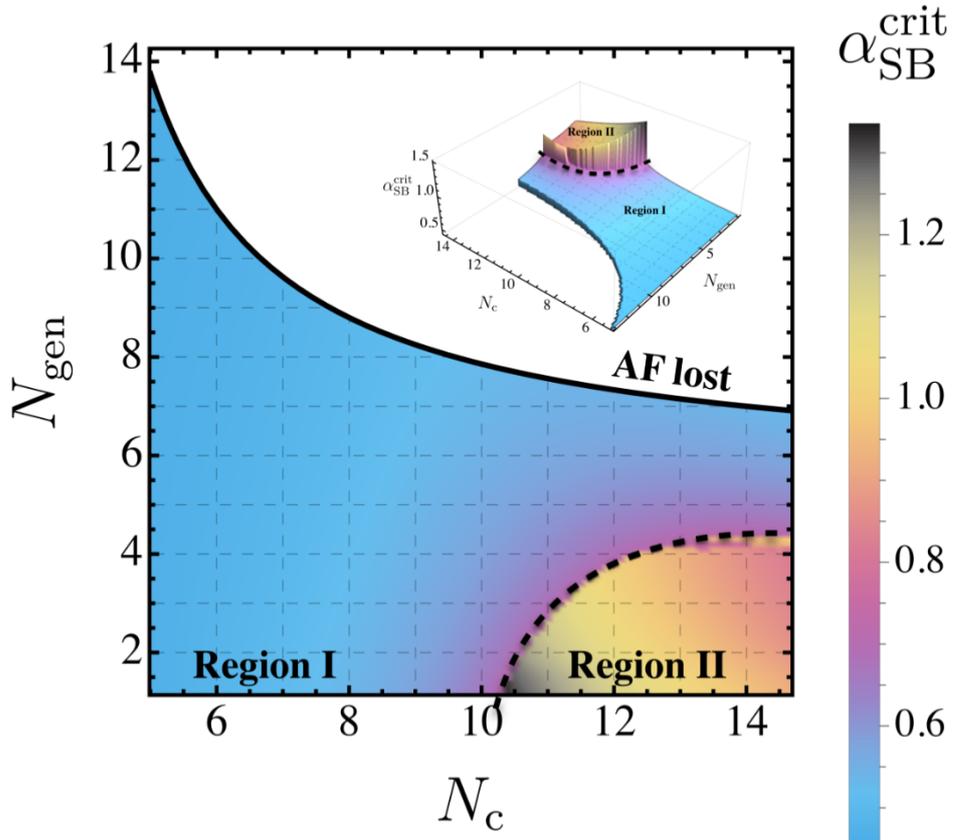
Results



- **Region I:**

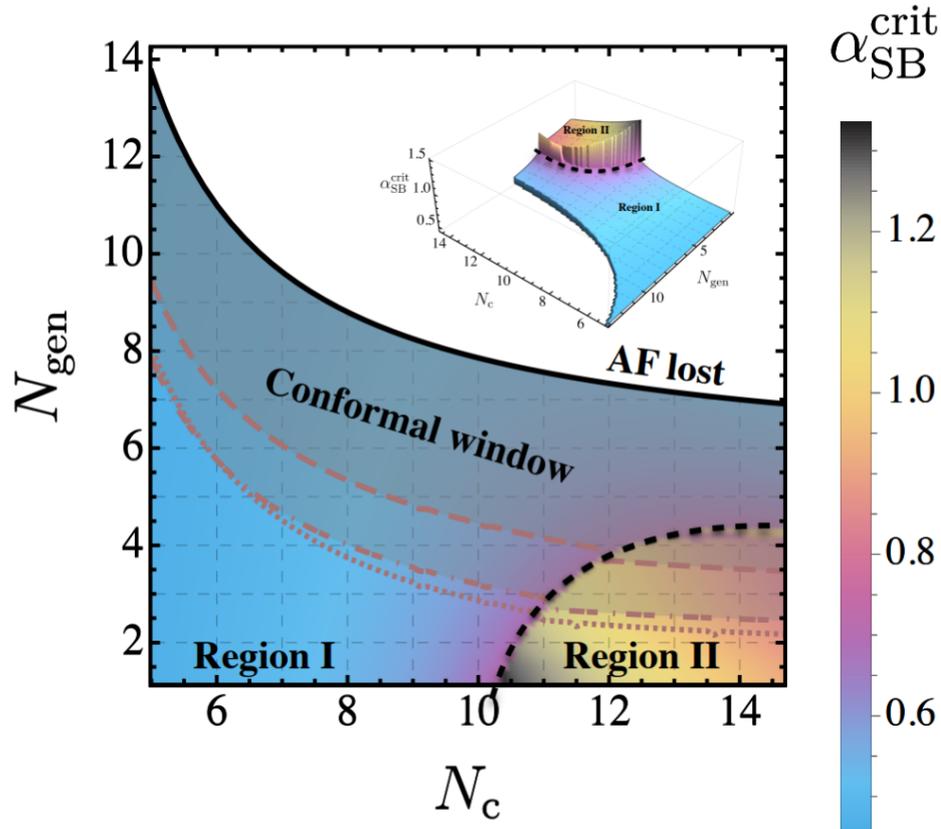
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- Clear dominance of $\mathcal{O}_5 = (\chi^\dagger \bar{\sigma}^\mu T_{\text{anti}} \chi) (\chi^\dagger \bar{\sigma}^\mu T_{\text{anti}} \chi)$
- Condensate $\langle \chi \chi \rangle \neq 0$

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 - strong α_g^{crit} non-perturbative, higher-order effects relevant
 - cannot resolve a clear single resonant channel.

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- **Region II:**

- strong α_g^{crit} non-perturbative, higher-order effects relevant
- cannot resolve a clear single resonant channel.

- **Conformal window:**

$$\alpha_g^{\text{crit}} = \alpha_g^*$$

$$\begin{cases} \alpha_g^* \Big|_{\overline{\text{MS}} \text{ 2-loop}} & \text{---} \\ \alpha_g^* \Big|_{\overline{\text{MS}} \text{ 3-loop}} & \text{---} \\ \alpha_g^* \Big|_{\overline{\text{MS}} \text{ 4-loop}} & \text{---} \text{ New!} \end{cases}$$

Summary

- We find strong evidence for the dSB in General Georgi-Glashow models
- In particular, for $N_c \leq 10$, naive resonance with fixed point merging occurs indicates condensate in the anti-symmetric fermion.
- For $N_c > 10$, α^{crit} is large so might need higher order effects.

Outlook

- Bosonize the theory to find the exact direction of the condensate, i.e. finding the residual symmetry.
- Needs to study the confinement effects, which may suppress the gauge fermion coupling after photon developed a mass gap.

Comparison Ngen=1

MAC: $\langle \chi\psi \rangle$ $N = 5, N \geq 7$ $\langle \chi\chi \rangle$ $\langle \chi\psi \rangle$ $N = 6$

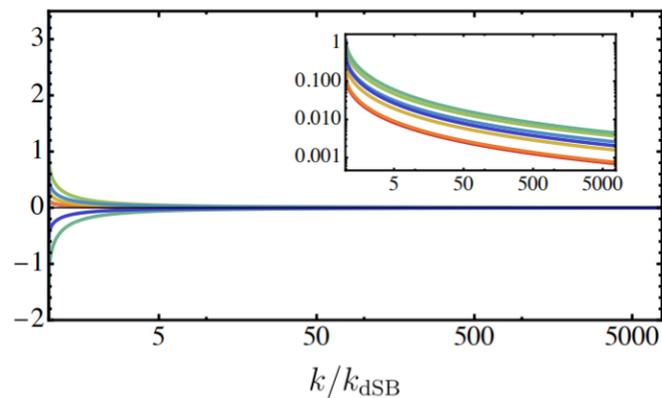
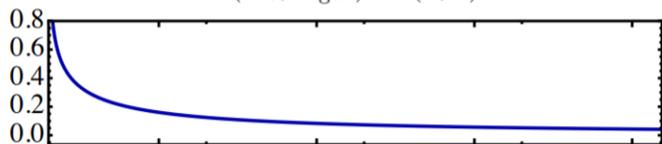
Anomaly
matching: $\langle \chi\chi \rangle$ $\langle \chi\psi \rangle$

Anomaly
mediated SUSY
breaking $\langle \chi\psi \rangle$ $\langle \psi\psi \rangle$

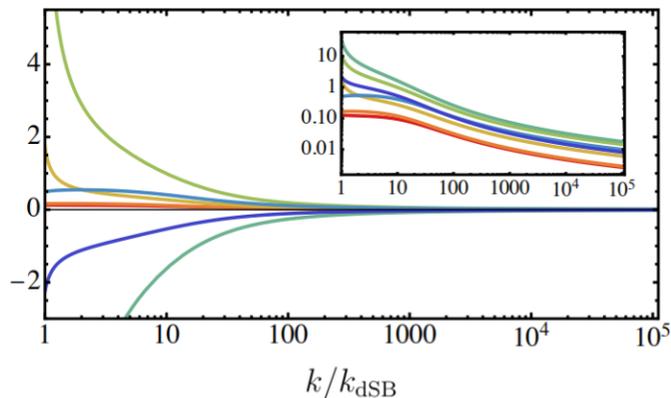
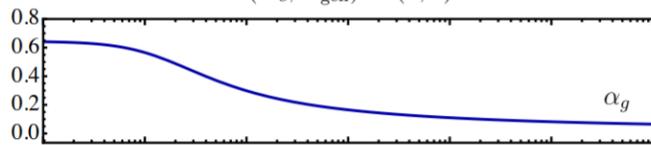
Some result

— λ_1 — λ_2 — λ_3 — λ_4 — λ_5 — λ_6 — λ_7

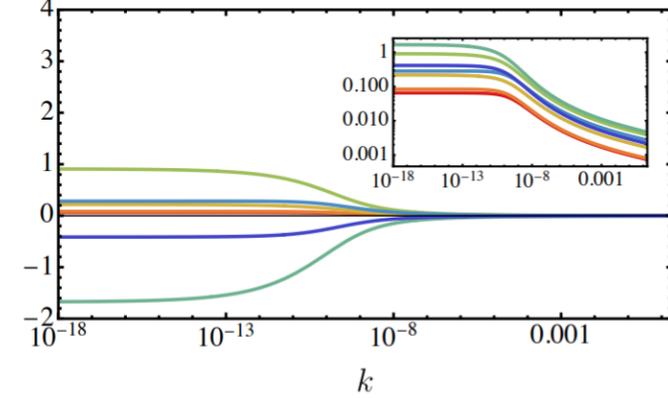
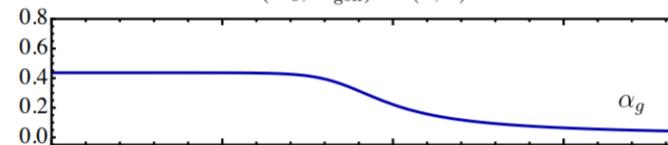
$(N_c, N_{\text{gen}}) = (5, 3)$

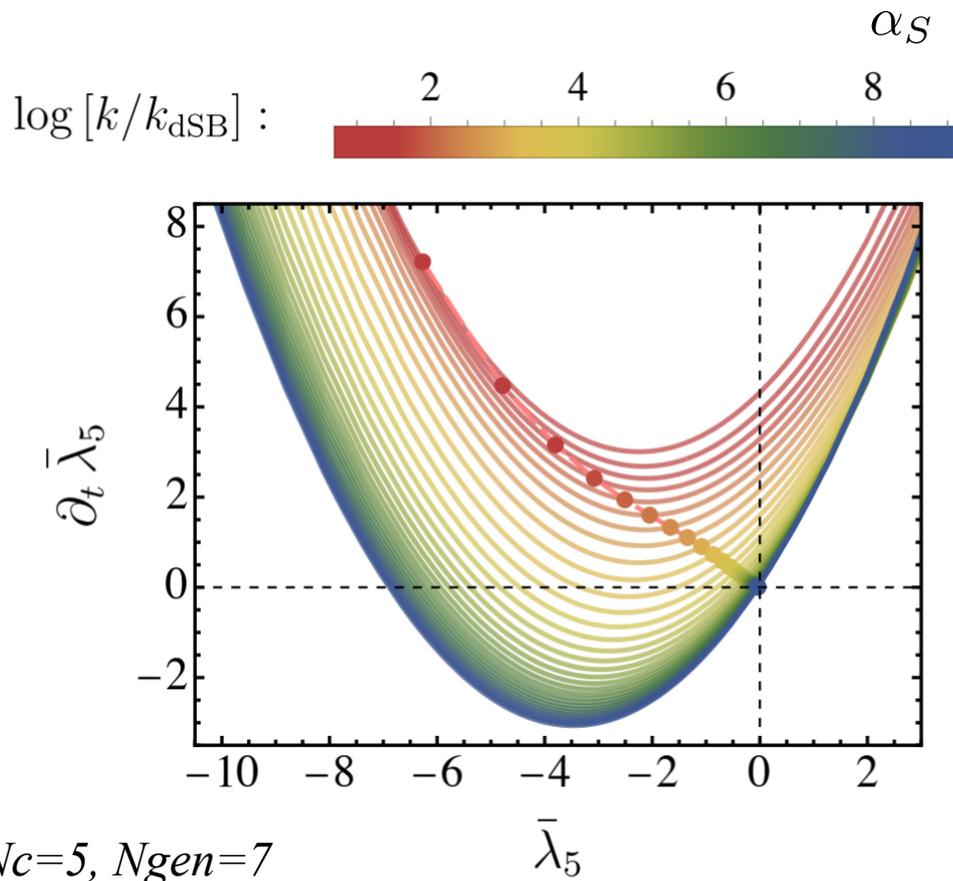


$(N_c, N_{\text{gen}}) = (5, 7)$



$(N_c, N_{\text{gen}}) = (5, 8)$





$$R_{A,c}(p^2) = Z_\phi p^2 r_{A,c}(x),$$

$$R_\psi(p^2) = iZ_\psi \gamma_\mu p_\mu r_\psi(x),$$

$$x = p^2/k^2$$

$$r_{A,c}(x) = (1/x - 1)\theta(1-x),$$

$$r_\psi(x) = (1/\sqrt{x} - 1)\theta(1-x),$$

$$S = \int \bar{\psi} \not{\partial} \psi - \frac{\lambda}{2} \left((\bar{\psi} \psi)^2 - (\bar{\psi} \gamma^5 \psi)^2 \right)$$

$$Z \propto \int \mathcal{D}\bar{\psi} \mathcal{D}\psi e^S = \mathcal{N} \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}\phi e^{\frac{m^2}{2} \phi^2} e^S \quad \phi = (\sigma, \pi)$$

$$\sigma \rightarrow \sigma + y \frac{\bar{\psi} \psi}{\sqrt{2m^2}} \quad \pi \rightarrow \pi + i y \frac{\bar{\psi} \gamma^5 \psi}{\sqrt{2m^2}}$$

$$Z \propto \mathcal{N} \int \mathcal{D}\bar{\psi} \mathcal{D}\psi \mathcal{D}\phi e^{S_{BF}} \quad S_{BF} = \int \bar{\psi} \not{\partial} \psi + \frac{y}{\sqrt{2}} \bar{\psi} (\sigma + i\pi \gamma^5) \psi + \frac{m^2}{2} (\sigma^2 + \pi^2) \quad \lambda \equiv \frac{y^2}{2m^2}$$

$$\sigma = \frac{-y}{\sqrt{2m^2}} \bar{\psi} \psi \quad \pi = \frac{-iy}{\sqrt{2m^2}} \bar{\psi} \gamma^5 \psi \quad S_B = \int d^4x \left\{ m^2 \phi^* \phi - \ln \det [i\not{\partial} + h (P_L \phi - P_R \phi^*)] \right\}$$

$$U(1)_A \quad \begin{pmatrix} \sigma \\ \pi \end{pmatrix} = \begin{pmatrix} \cos 2\alpha & \sin 2\alpha \\ -\sin 2\alpha & \cos 2\alpha \end{pmatrix} \begin{pmatrix} \sigma \\ \pi \end{pmatrix}$$

$$\partial_t \Gamma[\bar{\phi}] = \frac{1}{2} \text{Tr} G_k \partial_t R_k,$$

$$\partial_t \Gamma^{(1)}[\bar{\phi}] = -\frac{1}{2} \text{Tr} \Gamma_k^{(3)} \left(G_k \partial_t R_k G_k \right),$$

$$\partial_t \Gamma^{(2)}[\bar{\phi}] = -\frac{1}{2} \text{Tr} \left[\Gamma_k^{(4)} - 2 \Gamma_k^{(3)} G_k \Gamma_k^{(3)} \right] \left(G_k \partial_t R_k G_k \right),$$

$$\partial_t \Gamma^{(3)}[\bar{\phi}] = -\frac{1}{2} \text{Tr} \left[\Gamma_k^{(5)} - 6 \Gamma_k^{(4)} G_k \Gamma_k^{(3)} + 6 \Gamma_k^{(3)} G_k \Gamma_k^{(3)} G_k \Gamma_k^{(3)} \right] \left(G_k \partial_t R_k G_k \right),$$

$$\begin{aligned} \partial_t \Gamma^{(4)}[\bar{\phi}] = & -\frac{1}{2} \text{Tr} \left[\Gamma_k^{(6)} - 8 \Gamma_k^{(5)} G_k \Gamma_k^{(3)} - 6 \Gamma_k^{(4)} G_k \Gamma_k^{(4)} + 18 \Gamma_k^{(4)} G_k \Gamma_k^{(3)} G_k \Gamma_k^{(3)} \right. \\ & \left. + 12 \Gamma_k^{(3)} G_k \Gamma_k^{(4)} G_k \Gamma_k^{(3)} - 24 G_k \Gamma_k^{(3)} G_k \Gamma_k^{(3)} G_k \Gamma_k^{(3)} \cdot G_k \Gamma_k^{(3)} \right] \left(G_k \partial_t R_k G_k \right), \end{aligned}$$

$$\begin{aligned}
\Gamma_{\text{glue},k}[A, c, \bar{c}] = & \frac{1}{2} \int_p A_\mu^a(p) \left[Z_{A,k}(p^2 + m_{\text{gap},k}^2) \Pi_{\mu\nu}^\perp(p) + \frac{1}{\xi} Z_{A,k}^\parallel(p^2 + m_{\text{mSTI},k}^2) \frac{p_\mu p_\nu}{p^2} \right] A_\nu^a(-p) \\
& + \frac{1}{3!} \int_{p_1, p_2} Z_{A,k}^{3/2} \lambda_{A^3,k} \left[\mathcal{T}_{A^3}^{(1)}(p_1, p_2) \right]_{\mu_1 \mu_2 \mu_3}^{a_1 a_2 a_3} \prod_{i=1}^3 A_{\mu_i}^{a_i}(p_i) \\
& + \frac{1}{4!} \int_{p_1, p_2, p_3} Z_{A,k}^2 \lambda_{A^4,k} \left[\mathcal{T}_{A^4}^{(1)}(p_1, p_2, p_3) \right]_{\mu_1 \mu_2 \mu_3 \mu_4}^{a_1 a_2 a_3 a_4} \prod_{i=1}^4 A_{\mu_i}^{a_i}(p_i) + \int_p Z_{c,k} \bar{c}^a(p) p^2 \delta^{ab} c^b(-p) \\
& + \int_{p_1, p_2} Z_{c,k} Z_{A,k}^{1/2} \lambda_{c\bar{c}A,k} \left[\mathcal{T}_{c\bar{c}A}^{(1)}(p_1, p_2) \right]_\mu^{a_1 a_2 a_3} \bar{c}^{a_2}(p_2) c^{a_1}(p_1) A_\mu^{a_3}(-p_1 - p_2).
\end{aligned}$$

