

第十屆

2026 | 01/18-01/22
中国·广州

海峽兩岸
粒子物理和宇宙學
研討會

Establishing CP violation in b -baryon

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*Phys.Rev.Lett.*134(2025),221801、*Phys.Rev.D.* 112(2025),053007

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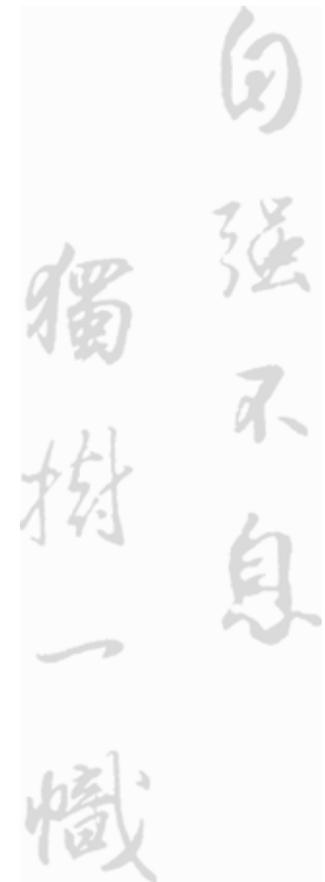


Introduction

Direct CP violation

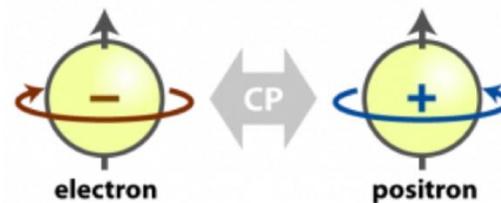
Asymmetry parameters

Summary



CP violation

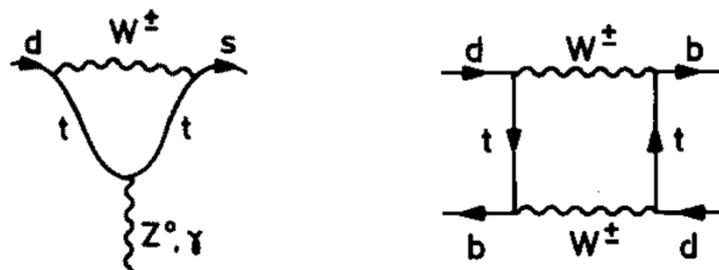
- Matter-antimatter asymmetry in the Universe [Science 109 (2005) 5731]
- Three conditions for Matter-antimatter asymmetry: [Sakharov JETP Lett.(1967)]
 - Interaction to violate baryon number;
 - **C and CP violation**;
 - Deviate thermal equilibrium



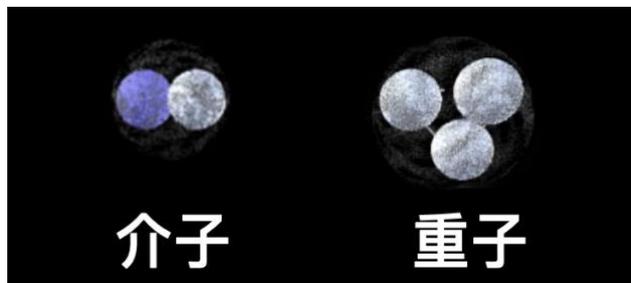
CP: 电荷共轭+宇称变换



- Moreover, CPV relates to parameters in SM, is helpful to search NP indirectly.



- Baryon CPV is more crucial, as visible matter in Universe is made of baryons.





$$\Lambda_b^0 \rightarrow pK^-\pi^+\pi^-$$

$$A_{CP} = (2.45 \pm 0.46 \pm 0.10)\%$$

[LHCb, Nature 643(2025)8074]

Beauty hadron: SM estimates $\sim 10\%$ due to large weak phase difference

[PDG]

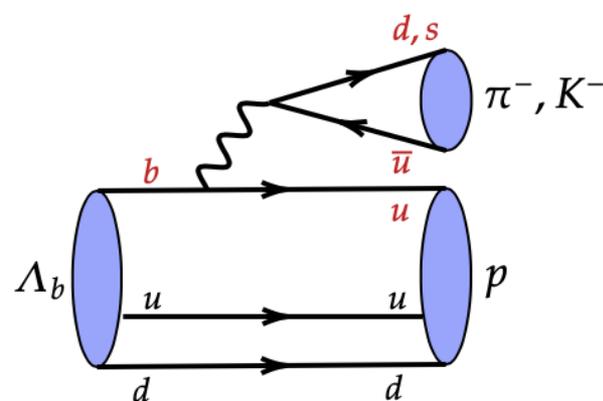
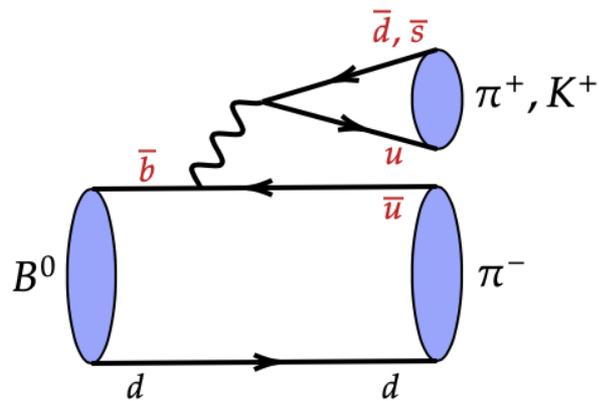
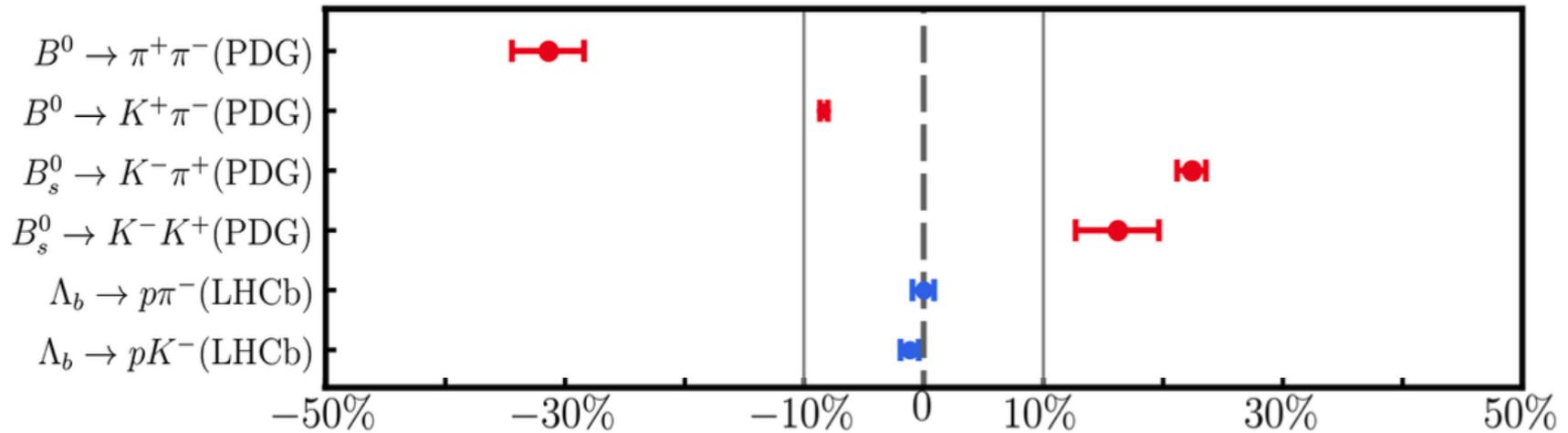
$$A_{CP}(B^0 \rightarrow K^+\pi^-) = (-8.34 \pm 0.32)\% \quad A_{CP}(B_s^0 \rightarrow K^-\pi^+) = (22.4 \pm 1.2)\%$$

	GFA (Hsiao, Yao, Geng, 2017)	QCDF (Zhu, Ke, Wei, 2018)	PQCD(hybrid) (Lü, Wang, et al., 2009)	LFQM (Geng, Liu, Tsai, 2021)
$A_{CP}(\Lambda_b \rightarrow p\pi^-)\%$	-3.9 ± 0.4	-3.4 ± 0.4	-31_{-1}^{+42}	-3.6 ± 0.20
$A_{CP}(\Lambda_b \rightarrow pK^-)\%$	6.7 ± 0.4	10.1 ± 2.0	-5_{-5}^{+26}	6.36 ± 0.28

Puzzle of b -baryon CPVs

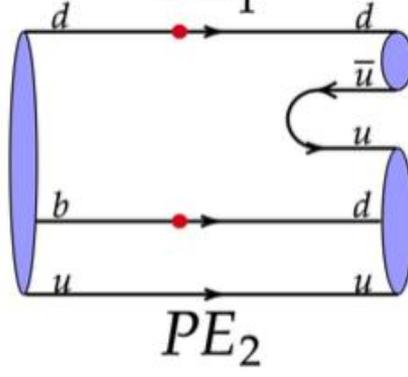
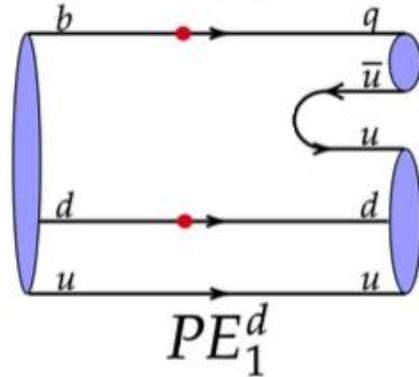
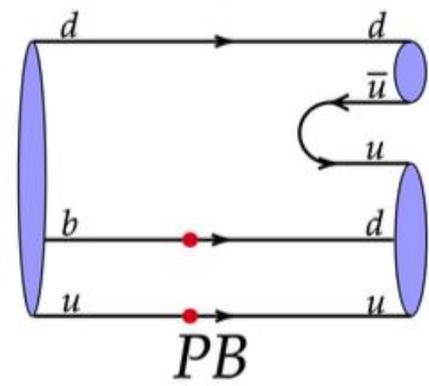
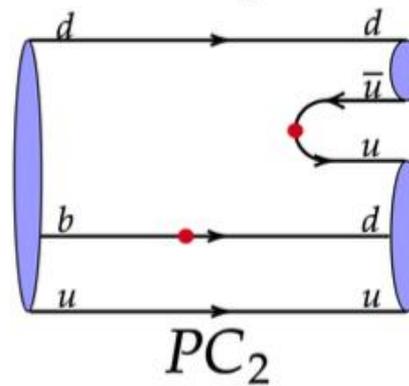
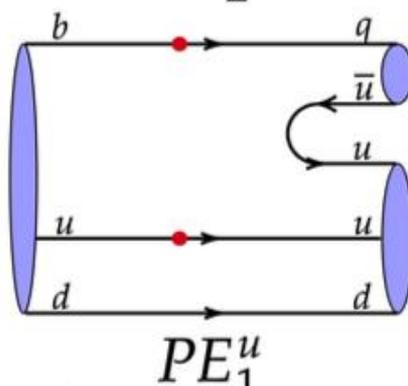
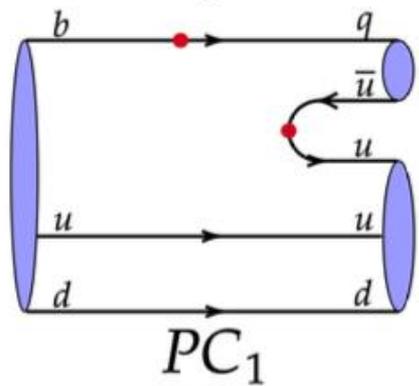
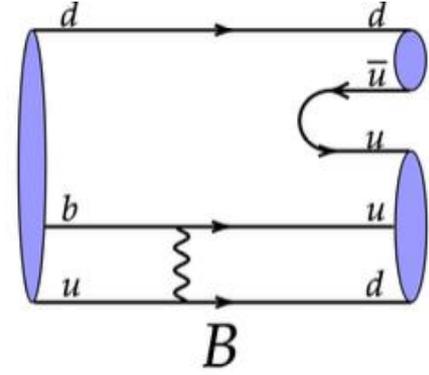
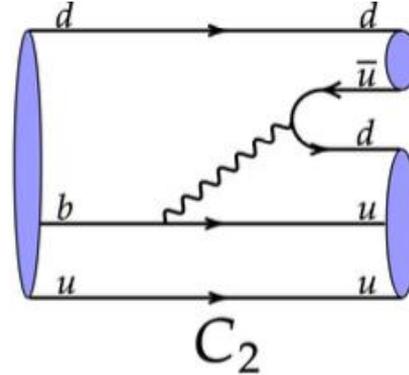
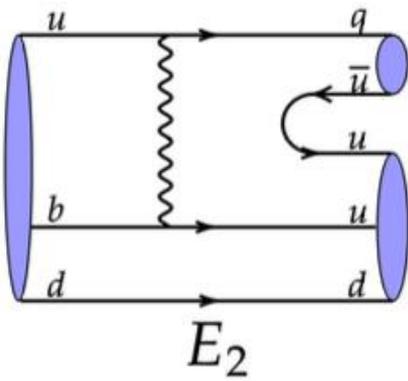
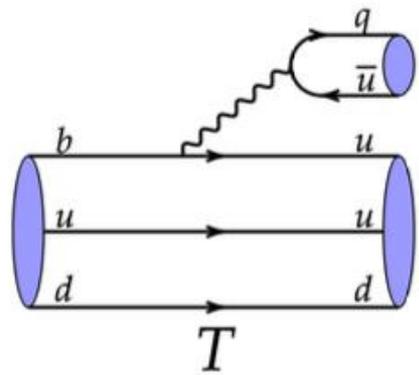
$$A_{CP}(\Lambda_b \rightarrow p\pi^-) = (0.20 \pm 0.83 \pm 0.37)\% \quad [\text{LHCb, 2018,2025}]$$

$$A_{CP}(\Lambda_b \rightarrow pK^-) = (-1.14 \pm 0.67 \pm 0.36)\%$$



only direct CPV
in baryon decays

Topological diagrams of $\Lambda_b \rightarrow p\pi^-, pK^-$



$$\Lambda_b \rightarrow p\pi^-, pK^-$$

- Baryons have half-integer spin, and thus two partial wave amplitudes.

$$\mathcal{A}(\Lambda_b \rightarrow ph) = i\bar{u}_p(S + P\gamma_5)u_{\Lambda_b}$$

$$\begin{array}{l}
 S \text{ wave} \quad S = \underbrace{\lambda_{\mathcal{T}}|S_{\mathcal{T}}|e^{i\delta_{\mathcal{T}}^S}}_{\text{Tree}} + \underbrace{\lambda_{\mathcal{P}}|S_{\mathcal{P}}|e^{i\delta_{\mathcal{P}}^S}}_{\text{Penguin}} \quad \Delta\delta_S = \delta_{\mathcal{P}}^S - \delta_{\mathcal{T}}^S \\
 P \text{ wave} \quad P = \underbrace{\lambda_{\mathcal{T}}|P_{\mathcal{T}}|e^{i\delta_{\mathcal{T}}^P}}_{\text{Tree}} + \underbrace{\lambda_{\mathcal{P}}|P_{\mathcal{P}}|e^{i\delta_{\mathcal{P}}^P}}_{\text{Penguin}} \quad \Delta\delta_P = \delta_{\mathcal{P}}^P - \delta_{\mathcal{T}}^P \\
 \qquad \text{strong phase difference}
 \end{array}$$

$$\Lambda_b \rightarrow p\pi^-, pK^-$$

- Baryons have half-integer spin, and thus two partial wave amplitudes.

$$\mathcal{A}(\Lambda_b \rightarrow ph) = i\bar{u}_p(S + P\gamma_5)u_{\Lambda_b}$$

$$\begin{aligned} A_{CP}^{dir} &\equiv \frac{\Gamma(\Lambda_b \rightarrow ph^-) - \bar{\Gamma}(\bar{\Lambda}_b \rightarrow \bar{p}h^+)}{\Gamma(\Lambda_b \rightarrow ph^-) + \bar{\Gamma}(\bar{\Lambda}_b \rightarrow \bar{p}h^+)} \\ &= \frac{M_+^2(|S|^2 - |\bar{S}|^2) + M_-^2|P|^2 - |\bar{P}|^2}{M_+^2(|S|^2 + |\bar{S}|^2) + M_-^2|P|^2 + |\bar{P}|^2} \\ &= \frac{|S|^2}{|S|^2 + \frac{M_-^2}{M_+^2} \frac{1+A_{CP}^S}{1+A_{CP}^P} |P|^2} A_{CP}^S + \frac{\frac{M_-^2}{M_+^2} |P|^2}{\frac{1+A_{CP}^P}{1+A_{CP}^S} |S|^2 + \frac{M_-^2}{M_+^2} |P|^2} A_{CP}^P \\ &\approx \frac{|S|^2}{|S|^2 + |P|^2} A_{CP}^S + \frac{|P|^2}{|S|^2 + |P|^2} A_{CP}^P \end{aligned}$$

$$A_{CP}^S \equiv \frac{|S|^2 - |\bar{S}|^2}{|S|^2 + |\bar{S}|^2} = \frac{-2r_S \sin\Delta\phi \sin\Delta\delta_S}{1 + r_S^2 + 2r_S \cos\Delta\phi \cos\Delta\delta_S},$$

$$A_{CP}^P \equiv \frac{|P|^2 - |\bar{P}|^2}{|P|^2 + |\bar{P}|^2} = \frac{-2r_P \sin\Delta\phi \sin\Delta\delta_P}{1 + r_P^2 + 2r_P \cos\Delta\phi \cos\Delta\delta_P}$$

$$\Delta\delta_S = \delta_P^S - \delta_T^S$$

$$\Delta\delta_P = \delta_P^P - \delta_T^P$$



strong phase difference

partial-wave CPVs

Explain CPVs of $\Lambda_b \rightarrow p\pi^-, pK^-$

➤ Baryons have half-integer spin, and thus two partial wave amplitudes.

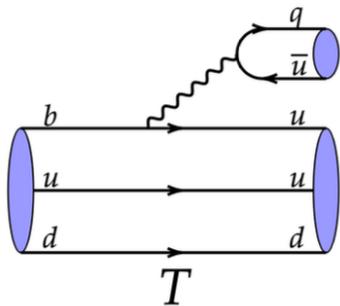
$$\mathcal{A}(\Lambda_b \rightarrow ph) = i\bar{u}_p(S + P\gamma_5)u_{\Lambda_b}$$

$$S \text{ wave } S = \lambda_{\mathcal{T}}|S_{\mathcal{T}}|e^{i\delta_{\mathcal{T}}^S} + \lambda_{\mathcal{P}}|S_{\mathcal{P}}|e^{i\delta_{\mathcal{P}}^S}$$

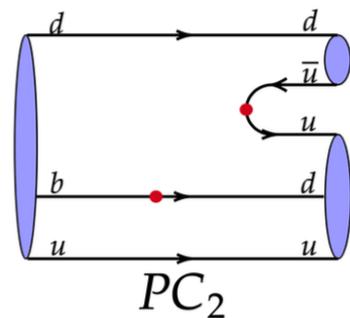
$$P \text{ wave } P = \lambda_{\mathcal{T}}|P_{\mathcal{T}}|e^{i\delta_{\mathcal{T}}^P} + \lambda_{\mathcal{P}}|P_{\mathcal{P}}|e^{i\delta_{\mathcal{P}}^P}$$

Tree

Penguin



$$\sim q^\mu \bar{u}_p \gamma_\mu (1 - \gamma_5) u_{\Lambda_b} \sim \bar{u}_p (1 + \gamma_5) u_{\Lambda_b}$$



$$\sim \bar{u}_p (1 + \gamma_5) (\gamma_5 \not{p}_\pi) (\not{p}_{\Lambda_b} \gamma_5) \not{p}_p (1 - \gamma_5) u_{\Lambda_b} \sim \bar{u}_p (1 - \gamma_5) u_{\Lambda_b}$$

$$\Delta\delta_{S\text{-wave}} = \delta_{PC_2}^{S\text{-wave}} - \delta_T^{S\text{-wave}} \sim 0$$

$$\Delta\delta_{P\text{-wave}} = \delta_{PC_2}^{P\text{-wave}} - \delta_T^{P\text{-wave}} \sim \pi$$

different by π

Strong phases depending on:

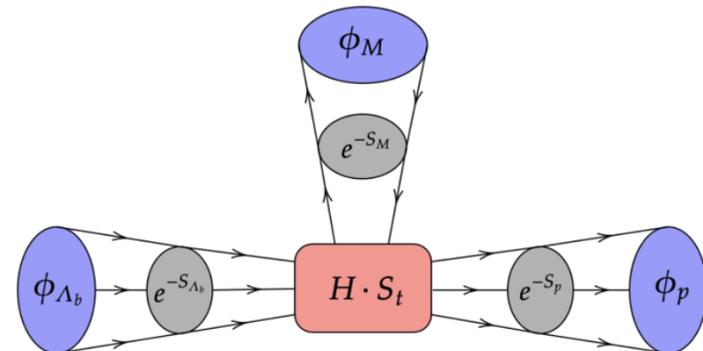
- operators
- diagram
- non-perturbative LCDAs

- Strong phases need to be determined by QCD
- Based on k_T factorization, PQCD approach has successfully predicted B meson CPV

$C_{\pi\pi}(B \rightarrow \pi^+\pi^-)\%$	$A_{CP}(B \rightarrow K^+\pi^-)\%$
~ -40 [Lü,Ukai,Yang,2000]	~ -18 [Keum,Li,Sanda,2000]
$-30 \pm 25 \pm 4$ [BaBar,2002]	$-19 \pm 10 \pm 3$ [BaBar,2001]
$-12.8^{+3.48}_{-3.29}$ [Chai,Cheng,Ju,Yan, Lü,Xiao,2022]	$-5.43^{+2.25}_{-2.34}$ [Chai,Cheng,Ju,Yan, Lü,Xiao,2022]
-31.4 ± 3 [PDG]	-8.31 ± 0.31 [PDG]

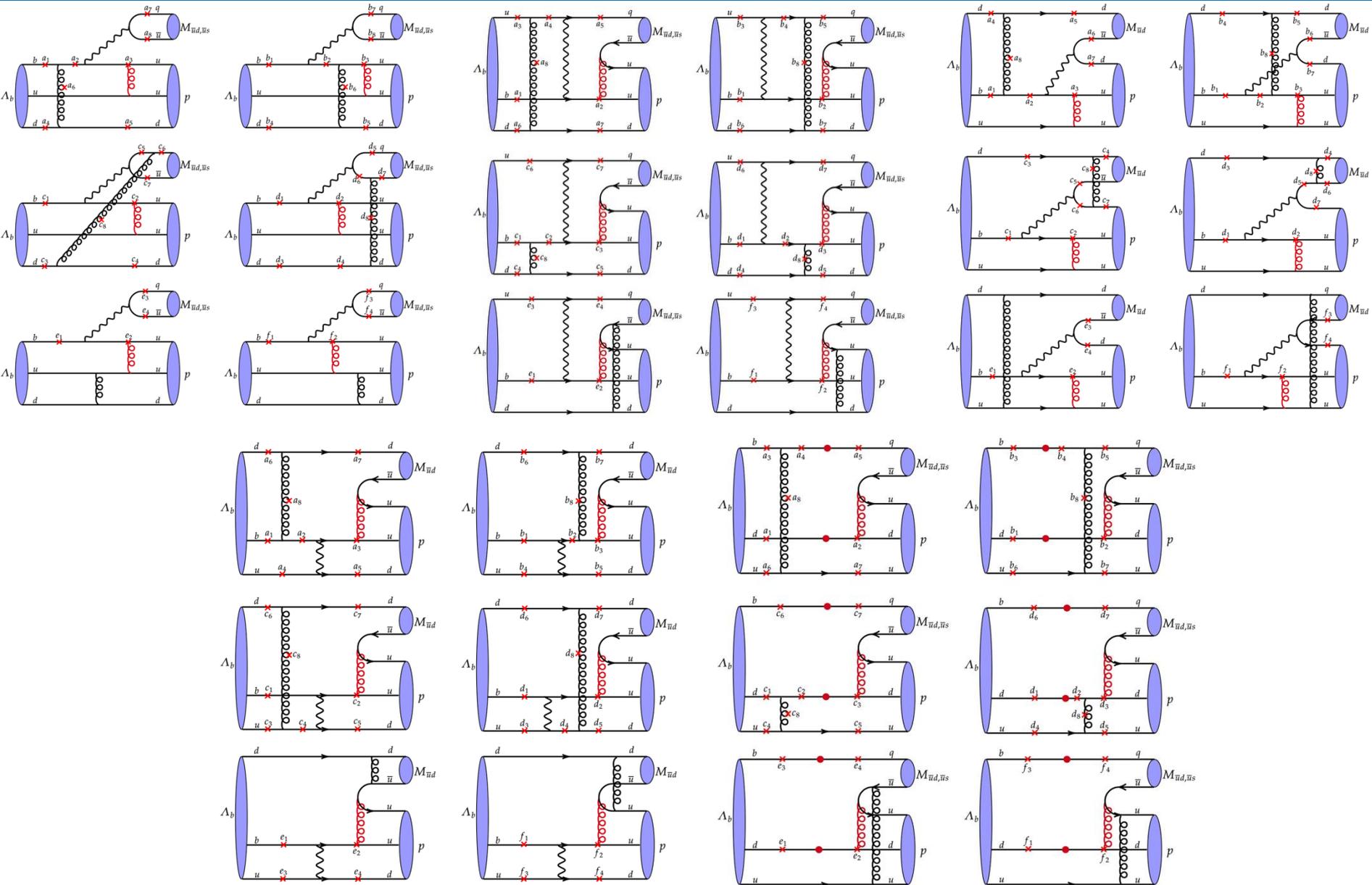
- Amplitudes are expressed as convolution of hard kernels and LCDAs

$$\begin{aligned}
 \mathcal{M} &= \langle pM | H_{eff} | \Lambda_b \rangle && \text{[Sterman,Hsiang-nan Li,1995~2005]} \\
 &\sim \int [d^4k_p][d^4k_M][d^4k_{\Lambda_b}] \Psi_p([k_p], \mu) \Psi_M([k_M], \mu) \Psi_{\Lambda_b}([k_{\Lambda_b}], \mu) \cdot C_i(\mu) H([k_p], [k_M], [k_{\Lambda_b}], \mu) \\
 &\sim \int_0^1 [dx_p][dx_M][dx_{\Lambda_b}] \int [d^2k_p^T][d^2k_M^T][d^2k_{\Lambda_b}^T] \phi_p([x_p], \mu) \phi_M([x_M], \mu) \phi_{\Lambda_b}([x_{\Lambda_b}], \mu) \\
 &\cdot C_i(\mu) H([x_p, k_p^T], [x_M, k_M^T], [x_{\Lambda_b}, k_{\Lambda_b}^T], \mu)
 \end{aligned}$$



$$\mathcal{A}(\Lambda_b \rightarrow pM) \sim \int \phi_{\Lambda_b} e^{-S_{\Lambda_b}} \cdot H S_t \cdot \phi_p e^{-S_p} \phi_M S^{-S_M}$$

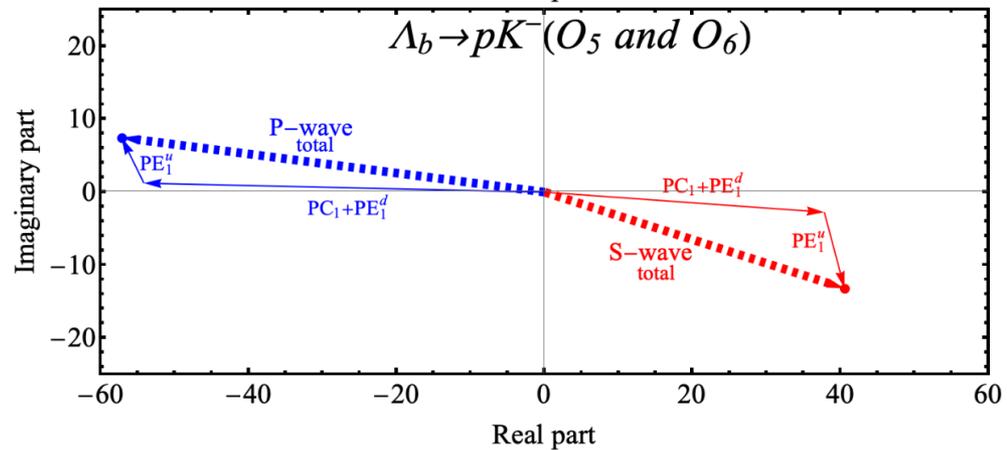
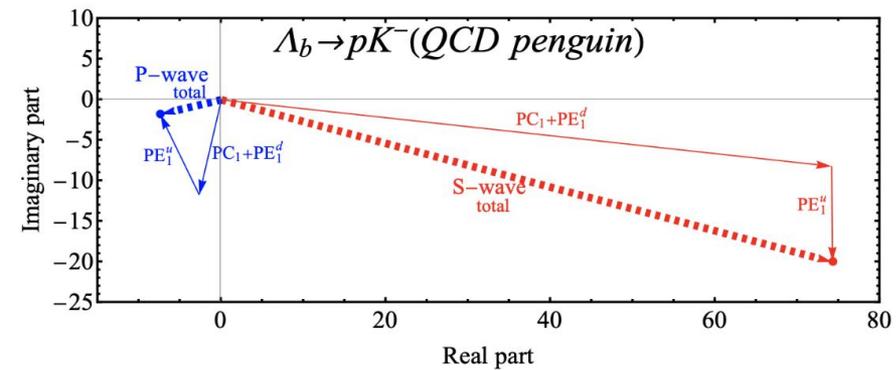
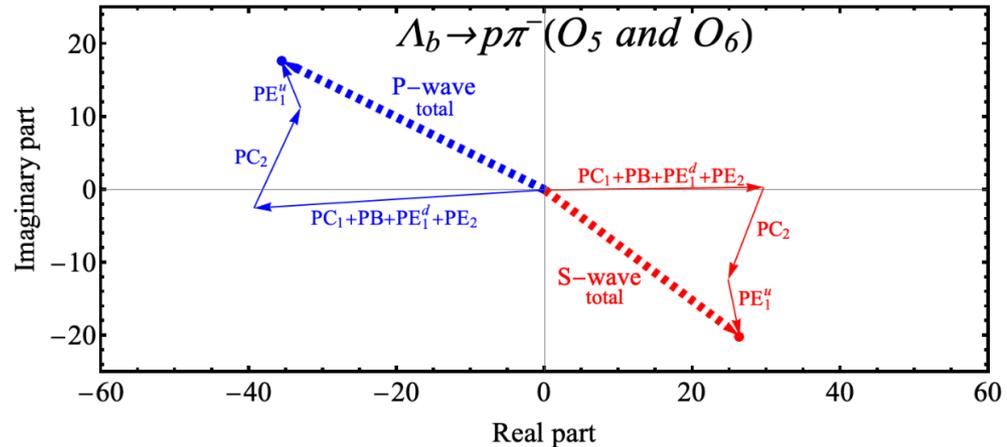
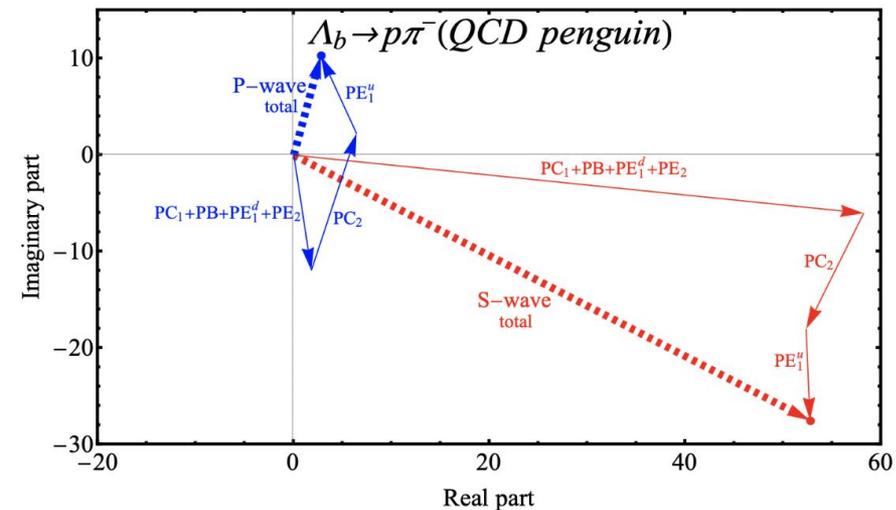
Feynman diagrams



Partial-wave amplitudes of $\Lambda_b \rightarrow p\pi^-$ in PQCD

➤ without the CKM matrix elements, $\times 10^{-9}$

$\Lambda_b \rightarrow p\pi^-$	$ S $	$\delta^S(^{\circ})$	Real(S)	Imag(S)	$ P $	$\delta^P(^{\circ})$	Real(P)	Imag(P)
T_f	707.17	0.00	707.17	0.00	1004.44	0.00	1004.44	0.00
T_{nf}	51.72	-96.64	-5.98	-51.38	267.72	-97.92	-36.90	-265.17
$T_f + T_{nf}$	703.07	-4.19	701.19	-51.38	1003.22	-15.33	967.54	-265.17
$V - A$ \otimes $V - A$ C_2	29.37	154.96	-26.61	12.43	41.51	179.80	-41.51	0.14
E_2	66.94	-145.26	-55.01	-38.14	72.58	119.94	-36.23	62.89
B	10.40	112.64	-4.00	9.60	23.65	-122.56	-12.73	-19.93
Tree	619.26	-6.26	615.57	-67.49	904.75	-14.21	877.08	-222.06
$P_f^{C_1}$	58.44	0.00	58.44	0.00	2.90	0.00	2.90	0.00
$P_{nf}^{C_1}$	1.24	-115.38	-0.53	-1.12	11.16	-95.25	-1.02	-11.11
$P_f^{C_1} + P_{nf}^{C_1}$	57.91	-1.11	57.90	-1.12	11.27	-80.38	1.88	-11.11
$V - A$ \otimes $V + A$ P^{C_2}	13.36	-116.10	-5.88	-12.00	14.93	71.96	4.62	14.20
$P^{E_1^u}$	9.48	-87.62	0.39	-9.47	8.83	114.44	-3.65	8.04
P^B	1.36	-51.30	0.85	-1.06	1.55	-159.86	-1.46	-0.53
$P^{E_1^d} + P^{E_2}$	3.87	-98.18	-0.55	-3.83	1.41	-12.55	1.37	-0.31
Penguin	59.45	-27.54	52.71	-27.49	10.65	74.93	2.77	10.28



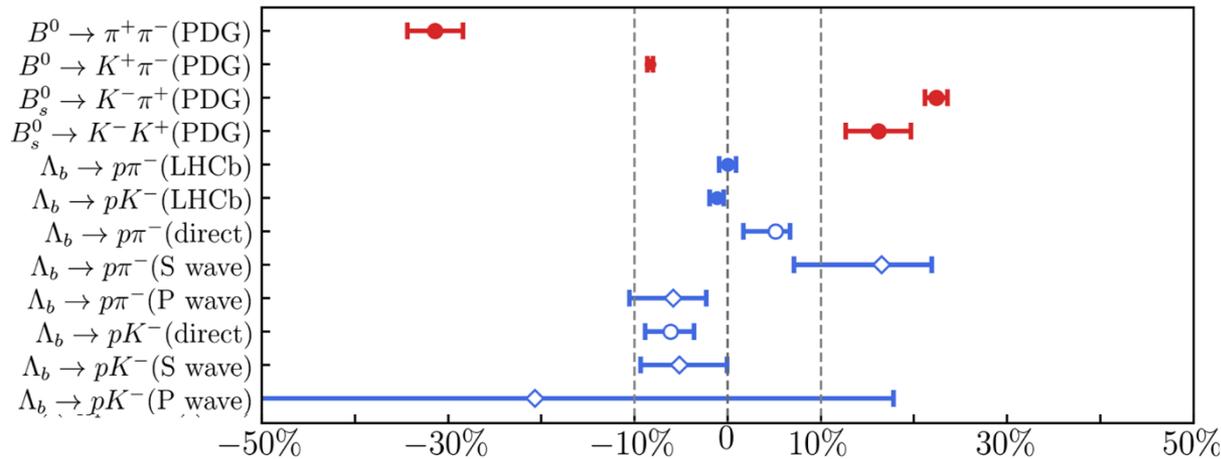
Observables of $\Lambda_b \rightarrow p\pi^-, pK^-$

$$A_{CP}^{dir} \approx \frac{|S|^2}{|S|^2 + |P|^2} A_{CP}^S + \frac{|P|^2}{|S|^2 + |P|^2} A_{CP}^P$$

$$A_{CP}^S \equiv \frac{|S|^2 - |\bar{S}|^2}{|S|^2 + |\bar{S}|^2} = \frac{-2r_S \sin\Delta\phi \sin\Delta\delta_S}{1 + r_S^2 + 2r_S \cos\Delta\phi \cos\Delta\delta_S},$$

$$A_{CP}^P \equiv \frac{|P|^2 - |\bar{P}|^2}{|P|^2 + |\bar{P}|^2} = \frac{-2r_P \sin\Delta\phi \sin\Delta\delta_P}{1 + r_P^2 + 2r_P \cos\Delta\phi \cos\Delta\delta_P}$$

$Br(\times 10^{-6})$		
$\Lambda_b \rightarrow p\pi^-$	$3.34^{+2.53+1.33+0.10+0.47}_{-1.30-1.10-0.11-0.14}$	
$\Lambda_b \rightarrow pK^-$	$2.83^{+2.17+1.17+0.49+2.19}_{-1.05-0.92-0.37-0.66}$	
A_{CP}^{dir}	$A_{CP}^S(\kappa_S)$	$A_{CP}^P(\kappa_P)$
$\Lambda_b \rightarrow p\pi^-$	$0.05^{+0.00+0.00+0.00+0.02}_{-0.02-0.01-0.02-0.01}$ 0.17 $^{+0.01+0.01+0.03+0.04}_{-0.04-0.04-0.07-0.04}$ (49%)	-0.06 $^{+0.01+0.03+0.02+0.00}_{-0.02-0.03-0.03-0.01}$ (51%)
$\Lambda_b \rightarrow pK^-$	$-0.06^{+0.01+0.01+0.02+0.00}_{-0.01-0.01-0.01-0.00}$ $-0.05^{+0.02+0.02+0.04+0.00}_{-0.02-0.01-0.03-0.00}$ (94%)	$-0.21^{+0.07+0.23+0.29+0.04}_{-0.15-0.33-0.27-0.01}$ (6%)



“..... small CP asymmetries in beauty-baryon decays imply that the dynamics in baryon decays are more complicated than in meson decays. For instance, the CP asymmetries for various angular-momentum amplitudes of the same resonance may **cancel**³⁸”

38. Han, J.-J. et al. Establishing CP violation in b -baryon decays. *Phys.Rev.Lett.* **134**, 221801 (2025).

Observables of $\Lambda_b \rightarrow p\pi^-, pK^-$

$$A_{CP}^{dir} \approx \frac{|S|^2}{|S|^2 + |P|^2} A_{CP}^S + \frac{|P|^2}{|S|^2 + |P|^2} A_{CP}^P$$

$$A_{CP}^S \equiv \frac{|S|^2 - |\bar{S}|^2}{|S|^2 + |\bar{S}|^2} = \frac{-2r_S \sin\Delta\phi \sin\Delta\delta_S}{1 + r_S^2 + 2r_S \cos\Delta\phi \cos\Delta\delta_S},$$

$$A_{CP}^P \equiv \frac{|P|^2 - |\bar{P}|^2}{|P|^2 + |\bar{P}|^2} = \frac{-2r_P \sin\Delta\phi \sin\Delta\delta_P}{1 + r_P^2 + 2r_P \cos\Delta\phi \cos\Delta\delta_P}$$

$$\alpha = -\frac{2\kappa_c \text{Re}(S^* P)}{|S|^2 + \kappa_c^2 |P|^2}, \quad \beta = -\frac{2\kappa_c \text{Im}(S^* P)}{|S|^2 + \kappa_c^2 |P|^2}, \quad \gamma = \frac{|S|^2 - \kappa_c^2 |P|^2}{|S|^2 + \kappa_c^2 |P|^2},$$

$$A_{CP}^\alpha = \frac{\alpha + \bar{\alpha}}{2}, \quad A_{CP}^\beta = \frac{\beta + \bar{\beta}}{2}, \quad A_{CP}^\gamma = \frac{\gamma - \bar{\gamma}}{2},$$

$$\langle \alpha \rangle = \frac{\alpha - \bar{\alpha}}{2}, \quad \langle \beta \rangle = \frac{\beta - \bar{\beta}}{2}, \quad \langle \gamma \rangle = \frac{\gamma + \bar{\gamma}}{2}.$$

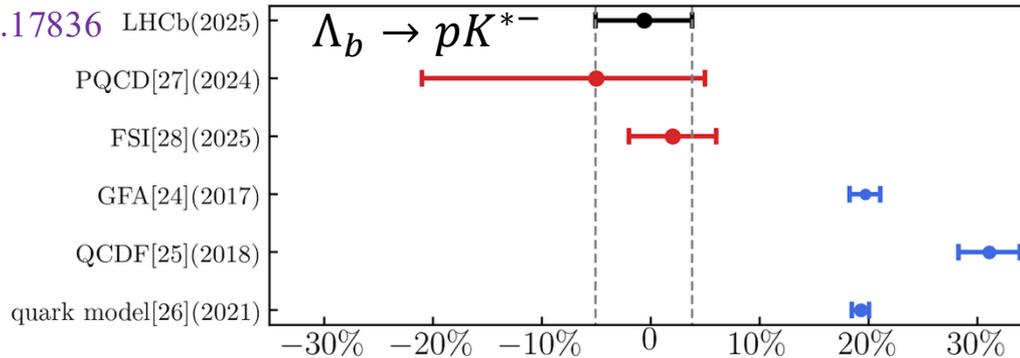
	α	A_{CP}^α	$\langle \alpha \rangle$
$\Lambda_b \rightarrow p\pi^-$	$-0.94^{+0.00+0.02+0.01+0.03}_{-0.02-0.02-0.02-0.02}$	$0.02^{+0.00+0.01+0.00+0.01}_{-0.01-0.01-0.01-0.01}$	$-0.96^{+0.00+0.01+0.01+0.02}_{-0.00-0.01-0.01-0.01}$
$\Lambda_b \rightarrow pK^-$	$0.23^{+0.04+0.02+0.10+0.15}_{-0.03-0.05-0.12-0.07}$	$0.04^{+0.02+0.02+0.01+0.01}_{-0.02-0.03-0.01-0.01}$	$0.20^{+0.02+0.01+0.11+0.14}_{-0.02-0.02-0.12-0.06}$
	β	A_{CP}^β	$\langle \beta \rangle$
$\Lambda_b \rightarrow p\pi^-$	$0.34^{+0.00+0.05+0.01+0.07}_{-0.06-0.06-0.06-0.05}$	$0.22^{+0.00+0.00+0.03+0.07}_{-0.01-0.01-0.04-0.03}$	$0.12^{+0.00+0.05+0.03+0.00}_{-0.05-0.05-0.04-0.02}$
$\Lambda_b \rightarrow pK^-$	$-0.39^{+0.03+0.08+0.08+0.12}_{-0.01-0.04-0.07-0.01}$	$-0.44^{+0.01+0.01+0.02+0.08}_{-0.00-0.00-0.01-0.04}$	$0.05^{+0.03+0.08+0.07+0.04}_{-0.01-0.05-0.07-0.02}$
	γ	A_{CP}^γ	$\langle \gamma \rangle$
$\Lambda_b \rightarrow p\pi^-$	$0.09^{+0.02+0.04+0.04+0.04}_{-0.04-0.06-0.06-0.01}$	$0.11^{+0.01+0.02+0.03+0.03}_{-0.02-0.03-0.04-0.02}$	$-0.02^{+0.01+0.02+0.01+0.01}_{-0.02-0.04-0.01-0.00}$
$\Lambda_b \rightarrow pK^-$	$0.89^{+0.02+0.04+0.04+0.00}_{-0.01-0.02-0.05-0.01}$	$0.02^{+0.02+0.05+0.04+0.00}_{-0.01-0.03-0.04-0.00}$	$0.87^{+0.00+0.01+0.02+0.00}_{-0.00-0.01-0.02-0.01}$

$\Lambda_b \rightarrow p\rho^-, pK^{*-}$

$$A_{CP}^{dir} \approx \kappa_{ST} A_{CP}^{ST} + \kappa_{P_2} A_{CP}^{P_2} + \kappa_{D+SL} A_{CP}^{D+SL} + \kappa_{P_1} A_{CP}^{P_1}$$

	$\text{Br}(\times 10^{-6})$	A_{CP}^{dir}	$A_{CP}^{ST}(\kappa_{ST})$
$\Lambda_b \rightarrow p\rho^-$	$9.66^{+6.23+3.23+0.21+1.89}_{-3.50-3.03-1.20-0.75}$	$0.03^{+0.02+0.01+0.00+0.02}_{-0.02-0.03-0.03-0.02}$	$0.01^{+0.00+0.00+0.00+0.00}_{-0.01-0.02-0.02-0.02}$ (7%)
$\Lambda_b \rightarrow pK^{*-}$	$2.83^{+1.77+0.46+0.37+0.63}_{-1.29-1.23-0.53-0.66}$	$-0.05^{+0.04+0.07+0.01+0.05}_{-0.11-0.07-0.06-0.08}$	$-0.15^{+0.06+0.09+0.02+0.05}_{-0.00-0.04-0.05-0.00}$ (6%)
	$A_{CP}^{SL+D}(\kappa_{SL+D})$	$A_{CP}^{P_1}(\kappa_{P_1})$	$A_{CP}^{P_2}(\kappa_{P_2})$
$\Lambda_b \rightarrow p\rho^-$	$0.02^{+0.03+0.04+0.02+0.05}_{-0.02-0.02-0.00-0.00}$ (44%)	$0.03^{+0.04+0.00+0.00+0.00}_{-0.05-0.04-0.10-0.05}$ (45%)	$0.17^{+0.00+0.00+0.01+0.03}_{-0.02-0.03-0.03-0.04}$ (4%)
$\Lambda_b \rightarrow pK^{*-}$	$0.27^{+0.02+0.06+0.05+0.03}_{-0.17-0.11-0.02-0.18}$ (33%)	$-0.23^{+0.05+0.07+0.02+0.05}_{-0.11-0.11-0.09-0.03}$ (55%)	$-0.14^{+0.01+0.00+0.02+0.01}_{-0.04-0.09-0.02-0.03}$ (6%)

LHCb, arXiv: 2508.17836 LHCb(2025)



“..... The vanishing CP asymmetry observed for the $\Lambda_b^0 \rightarrow pK_S^0\pi^-$ decay in the $K^*(892)^-$ mass region, significantly smaller than the approximately 20% effect predicted by the GFA [24], the QCDF approach [25], and the LFQM approach [26], is in agreement with the hypothesis of a **cancellation** mechanism among the contributing partial waves, as proposed in Refs. [27,28]

[27] J.-J. Han *et al.*, *Establishing CP violation in b-baryon decays*, *Phys. Rev. Lett.* **134** (2025) 221801, arXiv:2409.02821.

$\Lambda_b \rightarrow pa_1, pK_1(1270), pK_1(1400)$

$$A_{CP}^{dir} \approx \kappa_{ST} A_{CP}^{ST} + \kappa_{P_2} A_{CP}^{P_2} + \kappa_{D+S^L} A_{CP}^{D+S^L} + \kappa_{P_1} A_{CP}^{P_1}$$

$$\begin{pmatrix} |K_1(1270)\rangle \\ |K_1(1400)\rangle \end{pmatrix} = \begin{pmatrix} \sin\theta_{K_1} & \cos\theta_{K_1} \\ \cos\theta_{K_1} & -\sin\theta_{K_1} \end{pmatrix} \begin{pmatrix} |K_{1A}\rangle \\ |K_{1B}\rangle \end{pmatrix}$$

$$\theta_K \sim 30^\circ/60^\circ$$

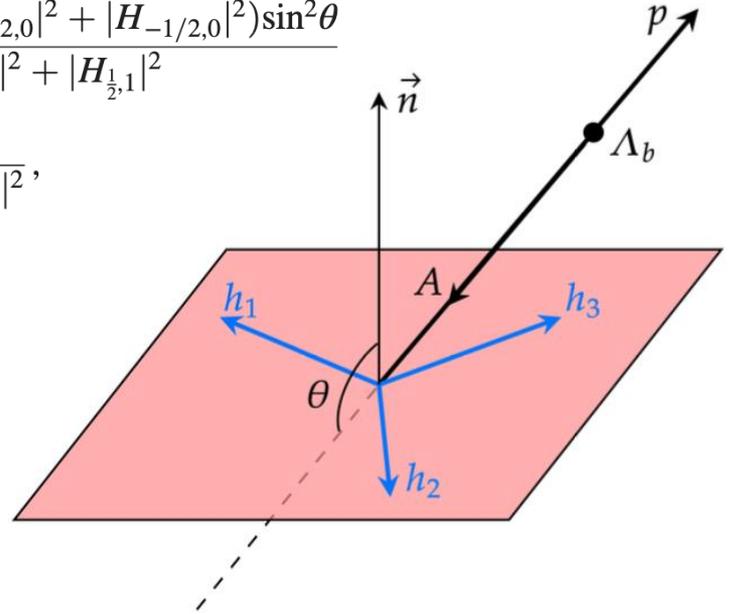
	$Br(\times 10^{-6})$	A_{CP}^{dir}	$A_{CP}^{ST}(\kappa_{ST})$
$\Lambda_b \rightarrow pa_1^-(1260)$	$11.06_{-4.30-3.32-0.46-0.06}^{+8.21+3.88+0.91+1.73}$	$-0.01_{-0.00-0.01-0.02-0.00}^{+0.01+0.03+0.02+0.03}$	$-0.22_{-0.03-0.07-0.07-0.01}^{+0.04+0.07+0.05+0.04}$ (6%)
$\Lambda_b \rightarrow pK_1^-(1270)(30^\circ)$	$5.48_{-1.87-1.55-0.31-1.11}^{+3.63+1.94+0.27+2.49}$	$0.09_{-0.04-0.02-0.02-0.00}^{+0.03+0.07+0.03+0.01}$	$0.34_{-0.02-0.03-0.01-0.05}^{+0.00+0.01+0.01+0.00}$ (8%)
$\Lambda_b \rightarrow pK_1^-(1400)(30^\circ)$	$1.25_{-0.39-0.19-0.19-0.31}^{+0.59+0.33+0.13+0.64}$	$0.06_{-0.03-0.09-0.04-0.01}^{+0.03+0.05+0.03+0.00}$	$0.71_{-0.02-0.16-0.04-0.13}^{+0.05+0.06+0.03+0.03}$ (13%)
$\Lambda_b \rightarrow pK_1^-(1270)(60^\circ)$	$6.28_{-2.13-1.51-0.41-1.32}^{+3.97+1.93+0.18+2.79}$	$0.07_{-0.04-0.04-0.03-0.00}^{+0.01+0.03+0.03+0.01}$	$0.46_{-0.02-0.04-0.02-0.07}^{+0.00+0.00+0.02+0.01}$ (9%)
$\Lambda_b \rightarrow pK_1^-(1400)(60^\circ)$	$0.53_{-0.16-0.19-0.22-0.13}^{+0.33+0.38+0.09+0.36}$	$0.08_{-0.08-0.11-0.04-0.03}^{+0.11+0.09+0.12+0.00}$	$0.07_{-0.12-0.09-0.15-0.10}^{+0.00+0.41+0.08+0.22}$ (3%)
	$A_{CP}^{S^L+D}(\kappa_{S^L+D})$	$A_{CP}^{P_1}(\kappa_{P_1})$	$A_{CP}^{P_2}(\kappa_{P_2})$
$\Lambda_b \rightarrow pa_1^-(1260)$	$-0.11_{-0.00-0.01-0.07-0.03}^{+0.02+0.01+0.02+0.02}$ (46%)	$0.18_{-0.03-0.02-0.03-0.04}^{+0.03+0.02+0.04+0.09}$ (40%)	$-0.24_{-0.02-0.09-0.06-0.06}^{+0.01+0.05+0.04+0.03}$ (8%)
$\Lambda_b \rightarrow pK_1^-(1270)(30^\circ)$	$-0.11_{-0.04-0.06-0.03-0.00}^{+0.01+0.08+0.08+0.03}$ (42%)	$0.19_{-0.06-0.09-0.11-0.01}^{+0.10+0.13+0.05+0.02}$ (42%)	$0.33_{-0.02-0.03-0.02-0.03}^{+0.00+0.04+0.02+0.00}$ (8%)
$\Lambda_b \rightarrow pK_1^-(1400)(30^\circ)$	$0.81_{-0.12-0.14-0.11-0.00}^{+0.09+0.17+0.07+0.04}$ (17%)	$-0.41_{-0.07-0.05-0.11-0.04}^{+0.04+0.05+0.08+0.03}$ (60%)	$0.78_{-0.06-0.20-0.04-0.10}^{+0.04+0.11+0.09+0.05}$ (10%)
$\Lambda_b \rightarrow pK_1^-(1270)(60^\circ)$	$0.06_{-0.03-0.07-0.04-0.00}^{+0.01+0.08+0.07+0.03}$ (37%)	$-0.07_{-0.06-0.05-0.05-0.02}^{+0.05+0.06+0.04+0.01}$ (45%)	$0.46_{-0.01-0.03-0.02-0.06}^{+0.00+0.04+0.04+0.02}$ (9%)
$\Lambda_b \rightarrow pK_1^-(1400)(60^\circ)$	$-0.82_{-0.07-0.09-0.07-0.02}^{+0.14+0.19+0.12+0.21}$ (30%)	$0.52_{-0.01-0.14-0.03-0.07}^{+0.06+0.12+0.37+0.00}$ (64%)	$-0.28_{-0.07-0.36-0.25-0.16}^{+0.27+0.04+0.03+0.03}$ (3%)

Results of $\Lambda_b \rightarrow pa_1, pK_1$

- The angle distribution for $\Lambda_b \rightarrow pA \rightarrow ph_1h_2h_3$: [J.P.Wang,Q.Qin,F.S.Yu,2024]

$$\frac{d\Gamma}{d\cos\theta} \propto \frac{(|H_{1/2,1}|^2 + |H_{-1/2,-1}|^2) \frac{1+\cos^2\theta}{2} + (|H_{1/2,0}|^2 + |H_{-1/2,0}|^2) \sin^2\theta}{|H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},1}|^2} + \frac{R(|H_{1/2,1}|^2 - |H_{-1/2,-1}|^2) \cos\theta}{|H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},1}|^2},$$

$$\frac{d\Gamma}{d\cos\theta} \supset R \operatorname{Re}(S^T P_2^*) \cos\theta$$



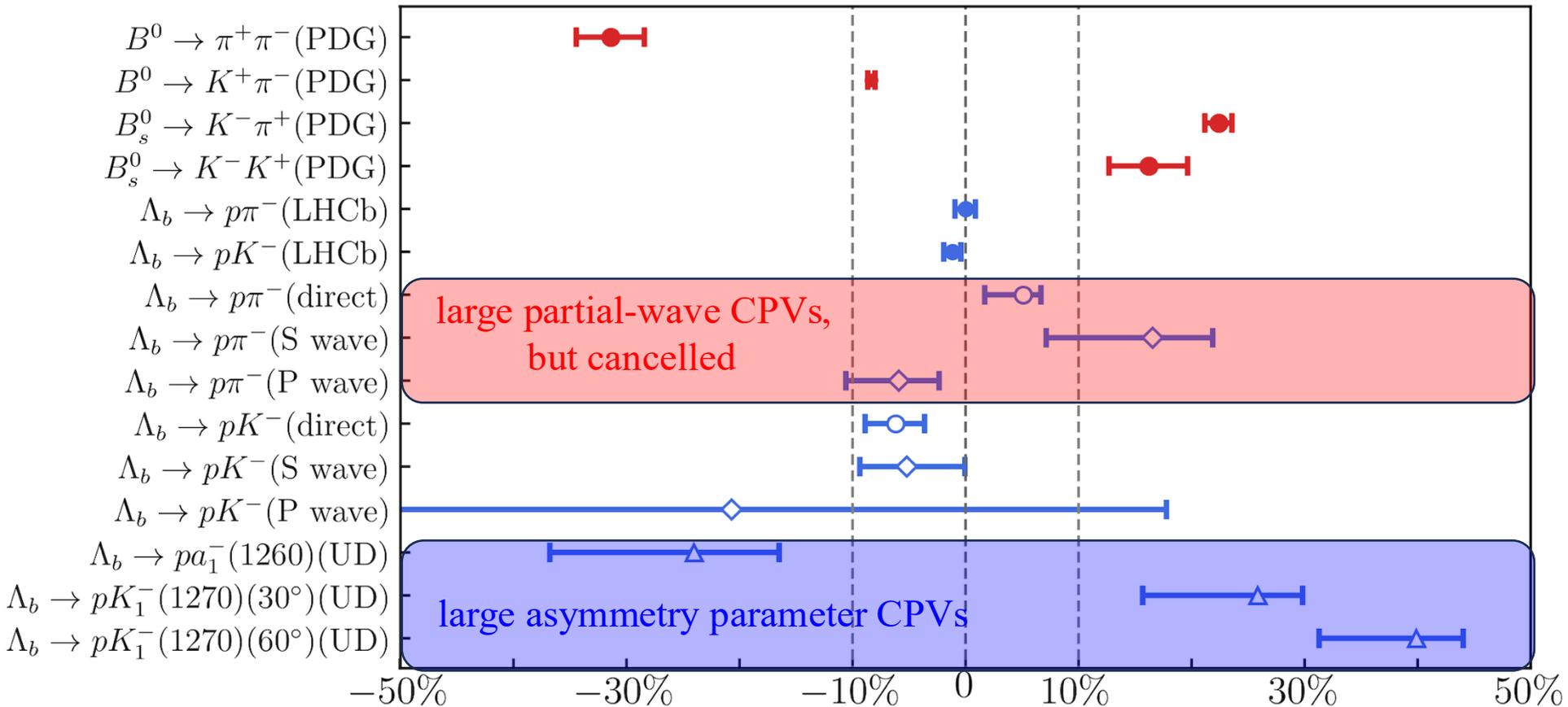
- up-down asymmetry :

$$A_{UD} \equiv \frac{\Gamma(\cos\theta > 0) - \Gamma(\cos\theta < 0)}{\Gamma(\cos\theta > 0) + \Gamma(\cos\theta < 0)} = R \operatorname{Re}(S^T P_2^*)$$

$$A_{CP}^{UD} = \frac{A_{UD} + \bar{A}_{UD}}{A_{UD} - \bar{A}_{UD}}$$

	a_{UD}	A_{CP}^{UD} 
$\Lambda_b \rightarrow pa_1^- (1260)$	$-0.09^{+0.00+0.01+0.02+0.00}_{-0.01-0.01-0.01-0.01}$	$-0.24^{+0.03+0.05+0.05+0.03}_{-0.03-0.09-0.06-0.06}$
$\Lambda_b \rightarrow pK_1^- (1270)(30^\circ)$	$-0.19^{+0.03+0.02+0.01+0.01}_{-0.02-0.02-0.01-0.02}$	$0.26^{+0.02+0.03+0.01+0.00}_{-0.03-0.08-0.04-0.04}$
$\Lambda_b \rightarrow pK_1^- (1400)(30^\circ)$	$-0.38^{+0.06+0.10+0.05+0.00}_{-0.06-0.09-0.07-0.03}$	$0.72^{+0.05+0.13+0.07+0.05}_{-0.05-0.23-0.03-0.12}$
$\Lambda_b \rightarrow pK_1^- (1270)(60^\circ)$	$-0.24^{+0.04+0.04+0.01+0.00}_{-0.02-0.03-0.02-0.03}$	$0.40^{+0.02+0.03+0.02+0.01}_{-0.01-0.04-0.03-0.07}$
$\Lambda_b \rightarrow pK_1^- (1400)(60^\circ)$	$-0.04^{+0.02+0.02+0.01+0.02}_{-0.01-0.05-0.03-0.01}$	$-0.19^{+0.12+0.14+0.00+0.06}_{-0.18-0.19-0.20-0.00}$

First full QCD analysis of baryon decays



Thanks

Backup

Opportunities and puzzle

- LHCb is a baryon factory !

$$\frac{f_{\Lambda_b}}{f_{u,d}} \sim 0.5 \quad \longrightarrow \quad \frac{N_{\Lambda_b}}{N_B^{0,-}} \sim 0.5 \quad [\text{LHCb, 2012}]$$

- Precision of b-baryon CPV measurement reached the order of 1%

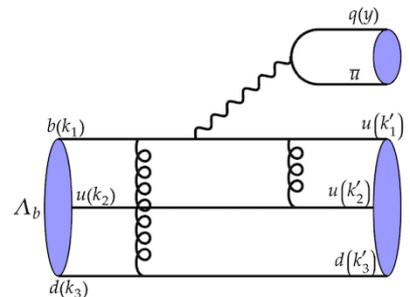
$$A_{CP}(\Lambda_b \rightarrow p\pi^-) = (0.20 \pm 0.83 \pm 0.37)\%$$

$$A_{CP}(\Lambda_b \rightarrow pK^-) = (-1.14 \pm 0.67 \pm 0.36)\% \quad [\text{LHCb, 2024}]$$

- Why CPVs of $\Lambda_b \rightarrow p\pi, pK$ are small ? What difference of dynamics?

- Baryons are very different from mesons!

- non-zero spin/polarization, more information from polarizations and partial waves
- three valence quarks, need at least two hard gluons



- SCET: power counting of baryon is different from meson
 - heavy-to-light form factor is **factorizable at leading power** and **no end-point singularity!**

$$\xi_{\Lambda_b \rightarrow \Lambda} = f_{\Lambda_b} \Phi_{\Lambda_b}(x_i) \otimes J(x_i, y_i) \otimes f_{\Lambda} \Phi_{\Lambda}(y_i)$$

- leading power: $\xi_{\Lambda_b \rightarrow \Lambda}(q^2 = 0) = -0.012$ [W.Wang, 2011]
- Total form factors: $\xi_{\Lambda_b \rightarrow \Lambda}(q^2 = 0) = 0.18$ [Y.L.Shen, Y.M.Wang, 2016]

$$A_{CP}^{dir} \approx \kappa_{ST} A_{CP}^{ST} + \kappa_{P_2} A_{CP}^{P_2} + \kappa_{D+SL} A_{CP}^{D+SL} + \kappa_{P_1} A_{CP}^{P_1}$$

	$Br(\times 10^{-6})$	A_{CP}^{dir}	$A_{CP}^{ST}(\kappa_{ST})$
$\Lambda_b \rightarrow p\rho^-$	$9.66^{+6.23+3.23+0.21+1.89}_{-3.50-3.03-1.20-0.75}$	$0.03^{+0.02+0.01+0.00+0.02}_{-0.02-0.03-0.03-0.02}$	$0.01^{+0.00+0.00+0.00+0.00}_{-0.01-0.02-0.02-0.02}$ (7%)
$\Lambda_b \rightarrow pK^{*-}$	$2.83^{+1.77+0.46+0.37+0.63}_{-1.29-1.23-0.53-0.66}$	$-0.05^{+0.04+0.07+0.01+0.05}_{-0.11-0.07-0.06-0.08}$	$-0.15^{+0.06+0.09+0.02+0.05}_{-0.00-0.04-0.05-0.00}$ (6%)
	$A_{CP}^{SL+D}(\kappa_{SL+D})$	$A_{CP}^{P_1}(\kappa_{P_1})$	$A_{CP}^{P_2}(\kappa_{P_2})$
$\Lambda_b \rightarrow p\rho^-$	$0.02^{+0.03+0.04+0.02+0.05}_{-0.02-0.02-0.00-0.00}$ (44%)	$0.03^{+0.04+0.00+0.00+0.00}_{-0.05-0.04-0.10-0.05}$ (45%)	$0.17^{+0.00+0.00+0.01+0.03}_{-0.02-0.03-0.03-0.04}$ (4%)
$\Lambda_b \rightarrow pK^{*-}$	$0.27^{+0.02+0.06+0.05+0.03}_{-0.17-0.11-0.02-0.18}$ (33%)	$-0.23^{+0.05+0.07+0.02+0.05}_{-0.11-0.11-0.09-0.03}$ (55%)	$-0.14^{+0.01+0.00+0.02+0.01}_{-0.04-0.09-0.02-0.03}$ (6%)
	α	A_{CP}^α	$\langle\alpha\rangle$
$\Lambda_b \rightarrow p\rho^-$	$-0.83^{+0.02+0.01+0.00+0.00}_{-0.02-0.05-0.04-0.01}$	$-0.01^{+0.01+0.01+0.01+0.00}_{-0.00-0.00-0.01-0.00}$	$-0.83^{+0.01+0.01+0.01+0.00}_{-0.02-0.05-0.04-0.01}$
$\Lambda_b \rightarrow pK^{*-}$	$-1.00^{+0.01+0.01+0.00+0.01}_{-0.00-0.00-0.00-0.00}$	$-0.00^{+0.00+0.00+0.00+0.00}_{-0.00-0.00-0.00-0.00}$	$-1.00^{+0.00+0.01+0.00+0.00}_{-0.00-0.00-0.00-0.00}$
	β	A_{CP}^β	$\langle\beta\rangle$
$\Lambda_b \rightarrow p\rho^-$	$-0.98^{+0.05+0.07+0.05+0.06}_{-0.00-0.00-0.00-0.00}$	$0.00^{+0.01+0.02+0.01+0.02}_{-0.00-0.00-0.00-0.00}$	$-0.99^{+0.04+0.05+0.04+0.04}_{-0.00-0.00-0.00-0.00}$
$\Lambda_b \rightarrow pK^{*-}$	$-0.90^{+0.07+0.17+0.11+0.00}_{-0.03-0.03-0.00-0.03}$	$-0.02^{+0.04+0.06+0.04+0.01}_{-0.00-0.04-0.00-0.00}$	$-0.88^{+0.06+0.11+0.08+0.00}_{-0.03-0.06-0.00-0.04}$
	γ	A_{CP}^γ	$\langle\gamma\rangle$
$\Lambda_b \rightarrow p\rho^-$	$-0.11^{+0.01+0.01+0.01+0.01}_{-0.01-0.01-0.02-0.00}$	$-0.01^{+0.00+0.00+0.00+0.00}_{-0.00-0.00-0.00-0.00}$	$-0.10^{+0.01+0.01+0.01+0.00}_{-0.01-0.01-0.02-0.00}$
$\Lambda_b \rightarrow pK^{*-}$	$-0.12^{+0.01+0.00+0.02+0.00}_{-0.06-0.05-0.03-0.05}$	$0.02^{+0.01+0.03+0.01+0.01}_{-0.02-0.02-0.01-0.01}$	$-0.14^{+0.01+0.01+0.02+0.00}_{-0.04-0.07-0.04-0.04}$
	Λ	A_{CP}^Λ	$\langle\Lambda\rangle$
$\Lambda_b \rightarrow p\rho^-$	$-0.96^{+0.05+0.06+0.04+0.05}_{-0.00-0.00-0.00-0.00}$	$0.00^{+0.01+0.02+0.01+0.02}_{-0.00-0.00-0.00-0.00}$	$-0.97^{+0.04+0.04+0.03+0.04}_{-0.00-0.00-0.00-0.00}$
$\Lambda_b \rightarrow pK^{*-}$	$-0.91^{+0.06+0.15+0.09+0.00}_{-0.02-0.02-0.00-0.03}$	$-0.01^{+0.03+0.06+0.03+0.01}_{-0.00-0.03-0.00-0.00}$	$-0.90^{+0.05+0.09+0.07+0.00}_{-0.03-0.05-0.01-0.03}$
	\mathcal{J}	$A_{CP}^{\mathcal{J}}$	$\langle\mathcal{J}\rangle$
$\Lambda_b \rightarrow p\rho^-$	$1.66^{+0.04+0.04+0.02+0.02}_{-0.03-0.03-0.05-0.00}$	$-0.01^{+0.01+0.01+0.01+0.00}_{-0.01-0.01-0.01-0.00}$	$1.67^{+0.03+0.04+0.02+0.02}_{-0.05-0.03-0.05-0.00}$
$\Lambda_b \rightarrow pK^{*-}$	$1.67^{+0.02+0.00+0.04+0.00}_{-0.14-0.12-0.08-0.12}$	$0.04^{+0.02+0.05+0.02+0.01}_{-0.06-0.04-0.02-0.03}$	$1.63^{+0.01+0.03+0.04+0.00}_{-0.08-0.15-0.09-0.09}$

$$A_{CP}(\Lambda_b \rightarrow p\pi^-) = (0.20 \pm 0.83 \pm 0.37)\%$$

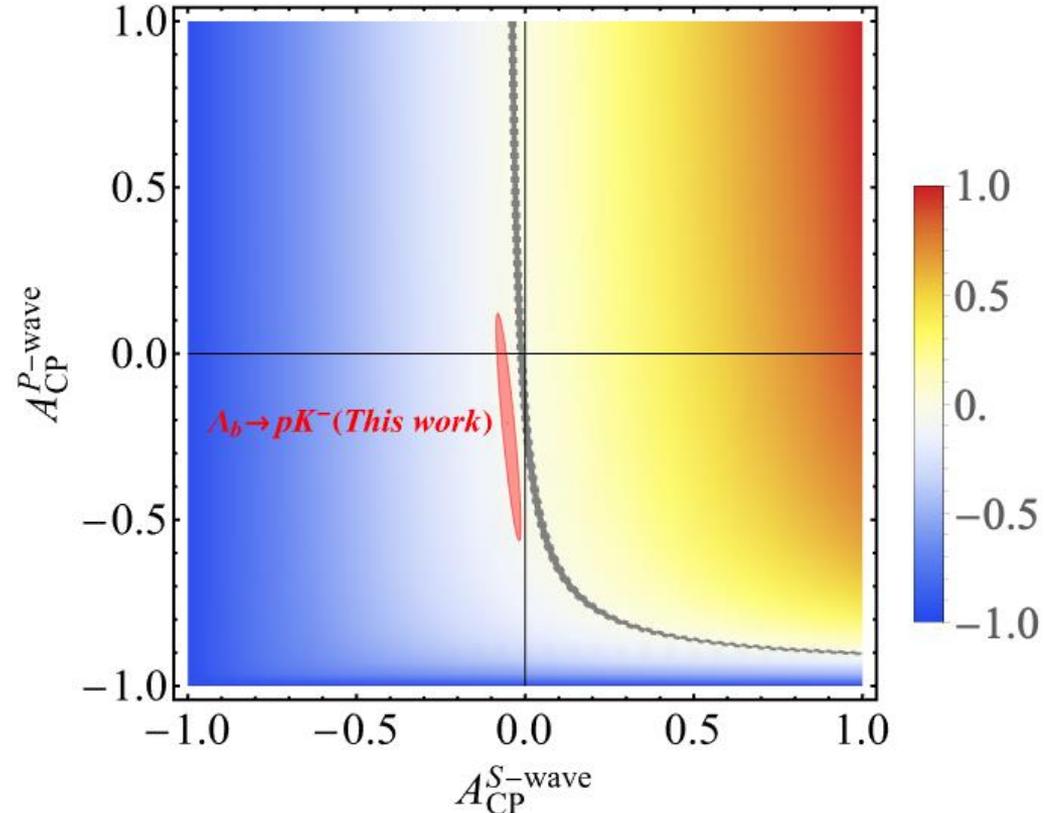
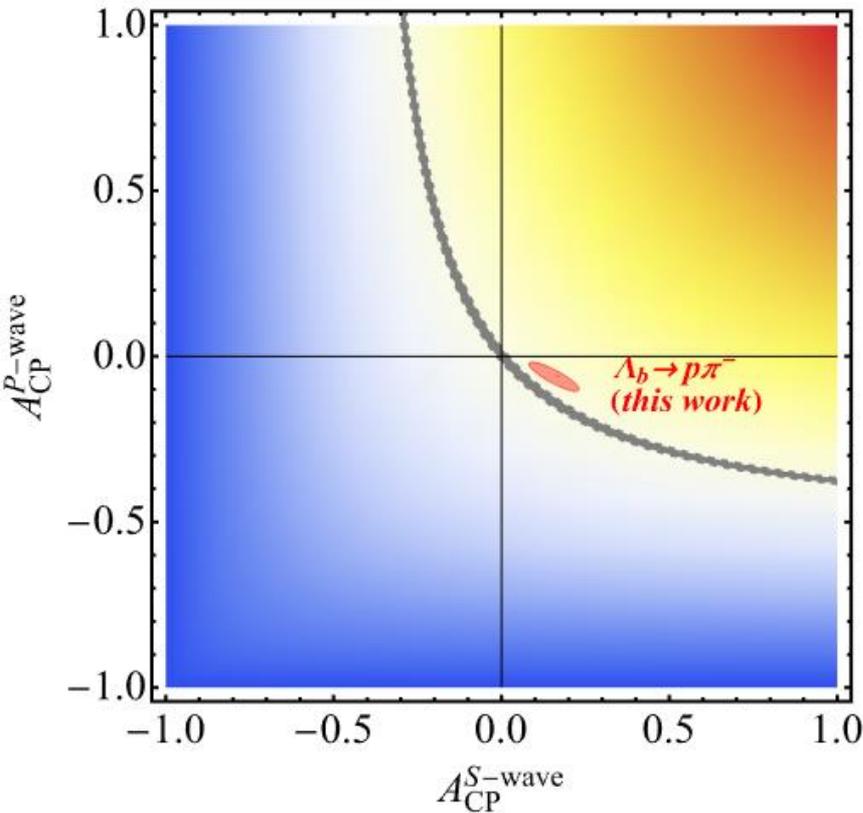
$$A_{CP}(\Lambda_b \rightarrow pK^-) = (-1.14 \pm 0.67 \pm 0.36)\% \text{ [LHCb,2024]}$$

$$A_{CP}^{dir} \equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}}$$

$$= \frac{M_+^2(|S|^2 - |\bar{S}|^2) + M_-^2(|P|^2 - |\bar{P}|^2)}{M_+^2(|S|^2 + |\bar{S}|^2) + M_-^2(|P|^2 + |\bar{P}|^2)}$$

$$= \frac{|S|^2}{|S|^2 + \frac{M_-^2}{M_+^2} \frac{1+A_{CP}^{S-wave}}{1+A_{CP}^{P-wave}} |P|^2} A_{CP}^{S-wave} + \frac{\frac{M_-^2}{M_+^2} |P|^2}{\frac{1+A_{CP}^{P-wave}}{1+A_{CP}^{S-wave}} |S|^2 + \frac{M_-^2}{M_+^2} |P|^2} A_{CP}^{P-wave}$$

$$= \kappa_S A_{CP}^{S-wave} + \kappa_P A_{CP}^{P-wave},$$



Predict CPVs of $\Lambda_b \rightarrow p\rho^-, pK^{*-}$

Invariant amplitudes

$$\left\{ \begin{array}{l} \mathcal{M}^L [B_i(1/2^+) \rightarrow B_f(1/2^+) + V] = \bar{u}_f(p_f) \epsilon_L^{*\mu} \left[A_1^L \gamma_\mu \gamma_5 + A_2^L \frac{(p_f)_\mu}{m_i} \gamma_5 + B_1^L \gamma_\mu + B_2^L \frac{(p_f)_\mu}{m_i} \right] u_i(p_i), \\ \mathcal{M}^T [B_i(1/2^+) \rightarrow B_f(1/2^+) + V] = \bar{u}_f(p_f) \epsilon_T^{*\mu} [A_1^T \gamma_\mu \gamma_5 + B_1^T \gamma_\mu] u_i(p_i). \end{array} \right.$$

Partial wave amplitudes

$$\left\{ \begin{array}{l} S^T = -A_1^T, \\ S^L = -A_1^L, \\ P_1 = -\frac{p_c}{E_V} \left(\frac{m_i + m_f}{E_f + m_f} B_1^L + B_2^L \right), \\ P_2 = \frac{p_c}{E_f + m_f} B_1^T, \\ D = -\frac{p_c^2}{E_V(E_f + m_f)} (A_1^L - A_2^L). \end{array} \right.$$

$$\Gamma(1/2^+ \rightarrow 1/2^+ + V) = \frac{p_c}{4\pi} \frac{E_f + m_f}{m_i} \left\{ 2(|S|^2 + |P_2|^2) + \frac{E_V^2}{m_V^2} (|S + D|^2 + |P_1|^2) \right\}$$

Helicity amplitudes

$$\left\{ \begin{array}{l} H_{1/2,1} = -M_+ A_1^T - M_- B_1^T, \\ H_{-1/2,-1} = M_+ A_1^T - M_- B_1^T, \\ H_{1/2,0} = \frac{1}{\sqrt{2}m_V} [M_+(m_i - m_f)A_1^L - M_- p_c A_2^L + M_-(m_i + m_f)B_1^L + M_+ p_c B_2^L], \\ H_{-1/2,0} = \frac{1}{\sqrt{2}m_V} [-M_+(m_i - m_f)A_1^L + M_- p_c A_2^L + M_-(m_i + m_f)B_1^L + M_+ p_c B_2^L]. \end{array} \right.$$

$$\mathcal{B} = \frac{p_c \tau_{\Lambda_b}}{8\pi m_i^2} (|H_{1/2,1}|^2 + |H_{-1/2,-1}|^2 + |H_{1/2,0}|^2 + |H_{-1/2,0}|^2). \quad [\text{Koener, Kramer, 1992}]$$

$$[\text{Cheng, 1996}]$$

$\Lambda_b \rightarrow p\pi^-$	$ S $	$\delta^S(^{\circ})$	Real(S)	Imag(S)	$ P $	$\delta^P(^{\circ})$	Real(P)	Imag(P)
T_f	707.17	0.00	707.17	0.00	1004.44	0.00	1004.44	0.00
T_{nf}	51.72	-96.64	-5.98	-51.38	267.72	-97.92	-36.90	-265.17
$T_f + T_{nf}$	703.07	-4.19	701.19	-51.38	1003.22	-15.33	967.54	-265.17
C_2	29.37	154.96	-26.61	12.43	41.51	179.80	-41.51	0.14
E_2	66.94	-145.26	-55.01	-38.14	72.58	119.94	-36.23	62.89
B	10.40	112.64	-4.00	9.60	23.65	-122.56	-12.73	-19.93
Tree	619.26	-6.26	615.57	-67.49	904.75	-14.21	877.08	-222.06
$P_f^{C_1}$	58.44	0.00	58.44	0.00	2.90	0.00	2.90	0.00
$P_{nf}^{C_1}$	1.24	-115.38	-0.53	-1.12	11.16	-95.25	-1.02	-11.11
$P_f^{C_1} + P_{nf}^{C_1}$	57.91	-1.11	57.90	-1.12	11.27	-80.38	1.88	-11.11
P^{C_2}	13.36	-116.10	-5.88	-12.00	14.93	71.96	4.62	14.20
$P^{E_1^u}$	9.48	-87.62	0.39	-9.47	8.83	114.44	-3.65	8.04
P^B	1.36	-51.30	0.85	-1.06	1.55	-159.86	-1.46	-0.53
$P^{E_1^d} + P^{E_2}$	3.87	-98.18	-0.55	-3.83	1.41	-12.55	1.37	-0.31
Penguin	59.45	-27.54	52.71	-27.49	10.65	74.93	2.77	10.28

$$S(P_f^{C_1}) = -\frac{G_F}{\sqrt{2}} f_h V_{tb} V_{td}^* \left(\frac{C_3}{3} + C_4 + \frac{C_9}{3} + C_{10} + R_1^{\pi} \left(\frac{C_5}{3} + C_6 + \frac{C_7}{3} + C_8 \right) \right)$$

$$\left[F_1(m_h^2)(M_{\Lambda_b} - M_p) + F_3(m_h^2)m_h^2 \right]$$

$$R_1^{\pi} \equiv \frac{2m_{\pi}^2}{(m_b - m_u)(m_u + m_d)}$$

$$P(P_f^{C_1}) = -\frac{G_F}{\sqrt{2}} f_h V_{tb} V_{td}^* \left(\frac{C_3}{3} + C_4 + \frac{C_9}{3} + C_{10} - R_2^{\pi} \left(\frac{C_5}{3} + C_6 + \frac{C_7}{3} + C_8 \right) \right)$$

$$\left[G_1(m_h^2)(M_{\Lambda_b} + M_p) - G_3(m_h^2)m_h^2 \right]$$

$$R_2^{\pi} \equiv \frac{2m_{\pi}^2}{(m_b + m_u)(m_u + m_d)}$$

➤ π/K

$$\Phi_{\pi(K)}(q, y) = \frac{i}{\sqrt{2N_c}} \left[\gamma_5 \not{q} \phi_{\pi(K)}^A(y) + m_0^{\pi(K)} \gamma_5 \phi_{\pi(K)}^P(y) + m_0^{\pi(K)} \gamma_5 (\not{q} \not{h} - 1) \phi_{\pi(K)}^T(y) \right]$$

$$\phi_{\pi(K)}^A(y) = \frac{f_{\pi(K)}}{2\sqrt{2N_c}} 6y(1-y) \left[1 + a_1^{\pi(K)} C_1^{3/2}(2y-1) + a_2^{\pi(K)} C_2^{3/2}(2y-1) + a_4^{\pi(K)} C_4^{3/2}(2y-1) \right],$$

$$\phi_{\pi(K)}^P(y) = \frac{f_{\pi(K)}}{2\sqrt{2N_c}} \left[1 + \left(0.45 - \frac{5}{2} \rho_{\pi(K)}^2 \right) C_2^{1/2}(2y-1) - 3 \left(-0.045 + \frac{9}{20} \rho_{\pi(K)}^2 (1 + 6a_2^{\pi(K)}) \right) C_4^{1/2}(2y-1) \right]$$

$$\phi_{\pi(K)}^T(y) = \frac{f_{\pi(K)}}{2\sqrt{2N_c}} (1-2y) \left[1 + 6 \left(0.0975 - \frac{7}{20} \rho_{\pi(K)}^2 - \frac{3}{5} \rho_{\pi(K)}^2 a_2^{\pi(K)} \right) (1 - 10y + 10y^2) \right], \quad (24)$$

➤ ρ/K^*

$$\Phi_V^L(q, \epsilon_L^*, y) = \frac{-1}{\sqrt{2N_c}} \left[m_V \not{\epsilon}_L^* \phi_V(y) + \not{\epsilon}_L^* \not{q} \phi_V^t(y) + m_V \phi_V^s(y) \right]_{\alpha\beta},$$

$$\langle P | (\bar{q}q')_{V \mp A} | 0 \rangle = \pm i f_P p_{P\mu},$$

$$\langle P | (\bar{q}q')_{S \mp P} | 0 \rangle = \pm i f_P m_{0P},$$

$$\Phi_V^T(q, \epsilon_T^*, y) = \frac{-1}{\sqrt{2N_c}} \left[m_V \not{\epsilon}_T^* \phi_V^v(y) + \not{\epsilon}_T^* \not{q} \phi_V^T(y) + m_V i \epsilon_{\mu\nu\rho\sigma} \gamma_5 \gamma^\mu \epsilon_T^{*\nu} v^\rho n^\sigma \phi_V^a(y) \right]_{\alpha\beta}, \quad (27)$$

$$\phi_V(y) = \frac{3f_V}{\sqrt{2N_c}} y(1-y) \left[1 + a_1^\parallel C_1^{3/2}(2y-1) + a_2^\parallel C_2^{3/2}(2y-1) \right], \quad (28)$$

$$\phi_V^T(y) = \frac{3f_V^T}{\sqrt{2N_c}} y(1-y) \left[1 + a_1^\perp C_1^{3/2}(2y-1) + a_2^\perp C_2^{3/2}(2y-1) \right], \quad (29)$$

$$\phi_V^t(y) = \frac{3f_V^T}{2\sqrt{2N_c}} (2y-1)^2, \quad (30)$$

$$\phi_V^s(y) = \frac{3f_V^T}{2\sqrt{2N_c}} (1-2y), \quad (31)$$

$$\phi_V^v(y) = \frac{3f_V}{8\sqrt{2N_c}} (1 + (2y-1)^2), \quad (32)$$

$$\phi_V^a(y) = \frac{3f_V}{4\sqrt{2N_c}} (1-2y), \quad (33)$$

$$\langle V | (\bar{q}q')_{V \mp A} | 0 \rangle = f_V m_V \epsilon_\mu^*,$$

$$\langle V | (\bar{q}q')_{S \mp P} | 0 \rangle = 0,$$

➤ a_1/K_1

$$\Phi_A^L(q, \epsilon_L^*, y) = \frac{-i}{\sqrt{2N_c}} \left[m_A \gamma_5 \not{\epsilon}_L^* \phi_A(y) + \not{\epsilon}_L^* \not{q} \gamma_5 \phi_A^t(y) + m_A \gamma_5 \phi_A^s(y) \right]_{\alpha\beta}, \quad (34)$$

$$\Phi_A^T(q, \epsilon_T^*, y) = \frac{-i}{\sqrt{2N_c}} \left[m_A \gamma_5 \not{\epsilon}_T^* \phi_A^v(y) + \not{\epsilon}_T^* \not{q} \gamma_5 \phi_A^T(y) + m_A i \epsilon_{\mu\nu\rho\sigma} \gamma^\mu \epsilon_T^{*\nu} v^\rho n^\sigma \phi_A^a(y) \right]_{\alpha\beta}, \quad (35)$$

$$\phi_A(y) = \frac{3f_A}{\sqrt{2N_c}} y(1-y) \left[a_{0A}^{\parallel} + 3a_{1A}^{\parallel}(2y-1) + \frac{3}{2}a_{2A}^{\parallel}(5(2y-1)^2-1) \right], \quad (36)$$

$$\phi_A^T(y) = \frac{3f_A}{\sqrt{2N_c}} y(1-y) \left[a_{0A}^{\perp} + 3a_{1A}^{\perp}(2y-1) + \frac{3}{2}a_{2A}^{\perp}(5(2y-1)^2-1) \right]. \quad (37)$$

$$\phi_A^t(y) = \frac{f_A}{2\sqrt{2N_c}} \left[3a_{0A}^{\perp}(2y-1)^2 + \frac{3}{2}a_{1A}^{\perp}(2y-1)(3(2y-1)^2-1) \right], \quad (38)$$

$$\phi_A^s(y) = \frac{3f_A}{2\sqrt{2N_c}} (a_{0A}^{\perp} - a_{1A}^{\perp} - 2a_{0A}^{\perp}y + 6a_{1A}^{\perp}y - 6a_{1A}^{\perp}y^2), \quad (39)$$

$$\phi_A^v(y) = \frac{f_A}{2\sqrt{2N_c}} \left[\frac{3}{4}a_{0A}^{\parallel}(1+(2y-1)^2) + \frac{3}{2}a_{1A}^{\parallel}(2y-1)^3 \right], \quad (40)$$

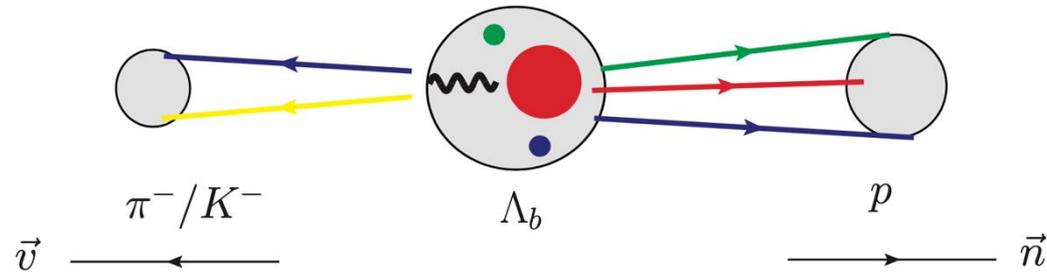
$$\phi_A^a(y) = \frac{3f_A}{4\sqrt{2N_c}} (a_{0A}^{\parallel} - a_{1A}^{\parallel} - 2a_{0A}^{\parallel}y + 6a_{1A}^{\parallel}y - 6a_{1A}^{\parallel}y^2).$$

$$\langle A | (\bar{q}q')_{V\mp A} | 0 \rangle = \mp i f_A m_A \epsilon_\mu^*,$$

$$\langle A | (\bar{q}q')_{S\mp P} | 0 \rangle = 0,$$

➤ mixing angle $\theta_{K_1} \sim 30^\circ/60^\circ$

$$\begin{pmatrix} |K_1(1270)\rangle \\ |K_1(1400)\rangle \end{pmatrix} = \begin{pmatrix} \sin\theta_{K_1} & \cos\theta_{K_1} \\ \cos\theta_{K_1} & -\sin\theta_{K_1} \end{pmatrix} \begin{pmatrix} |K_{1A}\rangle \\ |K_{1B}\rangle \end{pmatrix}$$



$$p_i = \frac{m_i}{\sqrt{2}}(1, 1, \mathbf{0}_T),$$

$$p_f = \frac{m_i}{\sqrt{2}}(\eta^+, \eta^-, \mathbf{0}_T),$$

$$q = p_i - p_f = \frac{m_i}{\sqrt{2}}(1 - \eta^+, 1 - \eta^-, \mathbf{0}_T),$$

$$k_1 = \left(\frac{m_i}{\sqrt{2}}, \frac{m_i}{\sqrt{2}}x_1, \mathbf{k}_{1T}\right),$$

$$k_2 = \left(0, \frac{m_i}{\sqrt{2}}x_2, \mathbf{k}_{2T}\right),$$

$$k_3 = \left(0, \frac{m_i}{\sqrt{2}}x_3, \mathbf{k}_{3T}\right),$$

$$k'_1 = \left(\frac{m_i}{\sqrt{2}}\eta^+x'_1, 0, \mathbf{k}'_{1T}\right),$$

$$k'_2 = \left(\frac{m_i}{\sqrt{2}}\eta^+x'_2, 0, \mathbf{k}'_{2T}\right),$$

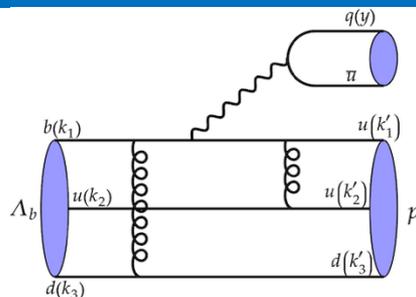
$$k'_3 = \left(\frac{m_i}{\sqrt{2}}\eta^+x'_3, 0, \mathbf{k}'_{3T}\right),$$

$$q_1 = \left(0, \frac{m_i}{\sqrt{2}}y(1 - \eta^-), \mathbf{q}_T\right), \quad q_2 = \left(0, \frac{m_i}{\sqrt{2}}(1 - y)(1 - \eta^-), -\mathbf{q}_T\right),$$

for T, E and P diagrams

$$k_1 = \left(\frac{m_i}{\sqrt{2}}(1 - x_3), \frac{m_i}{\sqrt{2}}(1 - x_2), \mathbf{k}_{1T}\right), \quad k_2 = \left(0, \frac{m_i}{\sqrt{2}}x_2, \mathbf{k}_{2T}\right), \quad k_3 = \left(\frac{m_i}{\sqrt{2}}x_3, 0, \mathbf{k}_{3T}\right)$$

for B and C' diagrams



- **SCET:** power counting of baryon is different from meson
 - heavy-to-light form factor is **factorizable at leading power** and **no end-point singularity**.

$$\xi_{\Lambda_b \rightarrow \Lambda} = f_{\Lambda_b} \Phi_{\Lambda_b}(x_i) \otimes J(x_i, y_i) \otimes f_{\Lambda} \Phi_{\Lambda}(y_i)$$
 - leading power: $\xi_{\Lambda_b \rightarrow \Lambda}(q^2 = 0) = -0.012$ [W.Wang, 2011]
 - Total form factors: $\xi_{\Lambda_b \rightarrow \Lambda}(q^2 = 0) = 0.18$ [Y.L.Shen, Y.M.Wang, 2016]
- **PQCD:** previous results of $\Lambda_b \rightarrow p$ form factors show **high-twist contributions dominant**

	Lattice/exp.	PQCD (Lü,Wang,et.al.,2009)	PQCD (Han,et.al.,2022)
$f_1^{\Lambda_b \rightarrow p}(q^2 = 0)$	~ 0.18	0.002 ± 0.001	0.27 ± 0.12

	twist-3	twist-4	twist-5	twist-6	total
exponential					
twist-2	0.0007	-0.00007	-0.0005	-0.000003	0.0001
twist-3 ⁺⁻	-0.0001	0.002	0.0004	-0.000004	0.002
twist-3 ⁻⁺	-0.0002	0.0060	0.000004	0.00007	0.006
twist-4	0.01	0.00009	0.25	0.0000007	0.26
total	0.01	0.008	0.25	0.00007	$0.27 \pm 0.09 \pm 0.07$

$$(Y_{\Lambda_b})_{\alpha\beta\gamma}(x_i, \mu) = \frac{1}{8\sqrt{2}N_c} \left\{ f_{\Lambda_b}^{(1)}(\mu) [M_1(x_2, x_3) \gamma_5 C^T]_{\gamma\beta} + f_{\Lambda_b}^{(2)}(\mu) [M_2(x_2, x_3) \gamma_5 C^T]_{\gamma\beta} \right\} [\Lambda_b(p)]_\alpha$$

$$M_1(x_2, x_3) = \frac{\not{x}_2 \not{x}_3}{4} \psi_3^{+-}(x_2, x_3) + \frac{\not{x}_3 \not{x}_2}{4} \psi_3^{-+}(x_2, x_3),$$

$$M_2(x_2, x_3) = \frac{\not{x}_2}{\sqrt{2}} \psi_2(x_2, x_3) + \frac{\not{x}_3}{\sqrt{2}} \psi_4(x_2, x_3),$$

$$\psi_2(x_2, x_3) = \frac{x_2 x_3}{\omega_0^4} m_{\Lambda_b}^4 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

$$\psi_3^{+-}(x_2, x_3) = \frac{2x_2}{\omega_0^3} m_{\Lambda_b}^3 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

$$\psi_3^{-+}(x_2, x_3) = \frac{2x_3}{\omega_0^3} m_{\Lambda_b}^3 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

$$\psi_4(x_2, x_3) = \frac{1}{\omega_0^2} m_{\Lambda_b}^2 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

[P.Ball, V.M.Braun, E.Gardi, 2008]

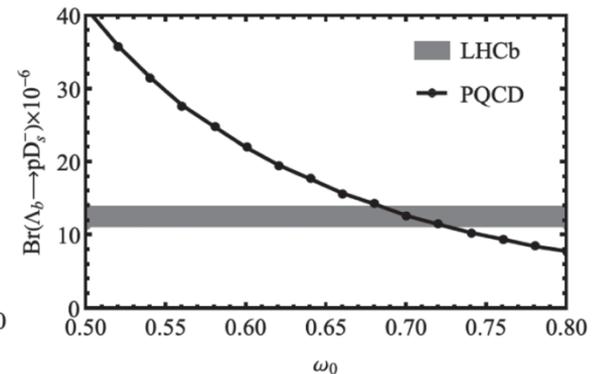
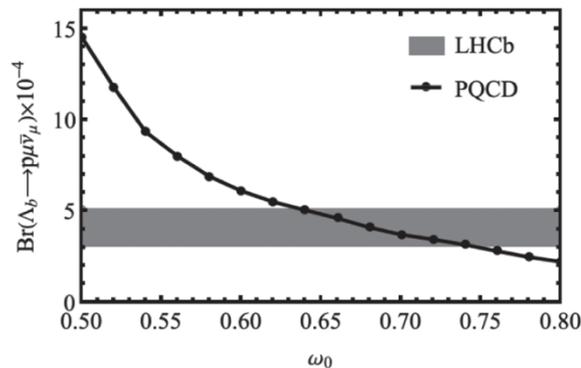
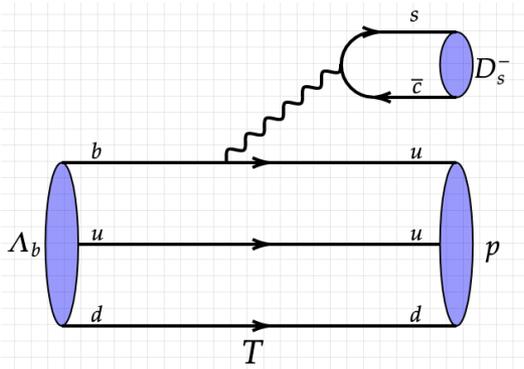
[G.Bell, T.Feldmann, Y.M.Wang, M.W.Y.Yip, 2013]

[Y.M.Wang, Y.L.Shen, 2016]

- (LHCb,2212.12574) recently measured the branching fraction:

$$Br(\Lambda_b \rightarrow p D_s^-) = (12.6 \pm 1.3)\%$$

- This mode has only W-external emission diagram, used to determine the parameter $\omega_0 = 0.7 \pm 0.1 GeV$



$$\begin{aligned}
 (\bar{Y}_P)_{\alpha\beta\gamma}(x'_i, \mu) &\equiv \frac{1}{2\sqrt{2}N_c} \int \prod_{l=2}^3 \frac{dz_l^- dz_l}{(2\pi)^3} e^{ik_l \cdot z_l} \epsilon^{i'j'k'} \langle p(p') | T[\bar{u}_\alpha^{i'}(0) \bar{u}_\beta^{j'}(z_2) \bar{d}_\gamma^{k'}(z_3)] | 0 \rangle \\
 &= \frac{-1}{8\sqrt{2}N_c} \left\{ S_1 m_p C_{\beta\alpha}(\bar{N}^+ \gamma_5)_\gamma + S_2 m_p C_{\beta\alpha}(\bar{N}^- \gamma_5)_\gamma + P_1 m_p (C \gamma_5)_{\beta\alpha} \bar{N}_\gamma^+ \right. \\
 &\quad + P_2 m_p (C \gamma_5)_{\beta\alpha} \bar{N}_\gamma^- + V_1 (C \not{P})_{\beta\alpha} (\bar{N}^+ \gamma_5)_\gamma + V_2 (C \not{P})_{\beta\alpha} (\bar{N}^- \gamma_5)_\gamma \\
 &\quad + V_3 \frac{m_p}{2} (C \gamma_\perp)_{\beta\alpha} (\bar{N}^+ \gamma_5 \gamma^\perp)_\gamma + V_4 \frac{m_p}{2} (C \gamma_\perp)_{\beta\alpha} (\bar{N}^- \gamma_5 \gamma^\perp)_\gamma + V_5 \frac{m_p^2}{2P_z} (C \not{z})_{\beta\alpha} (\bar{N}^+ \gamma_5)_\gamma \\
 &\quad + V_6 \frac{m_p^2}{2P_z} (C \not{z})_{\beta\alpha} (\bar{N}^- \gamma_5)_\gamma + A_1 (C \gamma_5 \not{P})_{\beta\alpha} (\bar{N}^+)_\gamma + A_2 (C \gamma_5 \not{P})_{\beta\alpha} (\bar{N}^-)_\gamma \\
 &\quad + A_3 \frac{m_p}{2} (C \gamma_5 \gamma_\perp)_{\beta\alpha} (\bar{N}^+ \gamma^\perp)_\gamma + A_4 \frac{m_p}{2} (C \gamma_5 \gamma_\perp)_{\beta\alpha} (\bar{N}^- \gamma^\perp)_\gamma + A_5 \frac{m_p^2}{2P_z} (C \gamma_5 \not{z})_{\beta\alpha} (\bar{N}^+)_\gamma \\
 &\quad + A_6 \frac{m_p^2}{2P_z} (C \gamma_5 \not{z})_{\beta\alpha} (\bar{N}^-)_\gamma - T_1 (iC \sigma_{\perp P})_{\beta\alpha} (\bar{N}^+ \gamma_5 \gamma^\perp)_\gamma - T_2 (iC \sigma_{\perp P})_{\beta\alpha} (\bar{N}^- \gamma_5 \gamma^\perp)_\gamma \\
 &\quad - T_3 \frac{m_p}{P_z} (iC \sigma_{Pz})_{\beta\alpha} (\bar{N}^+ \gamma_5)_\gamma - T_4 \frac{m_p}{P_z} (iC \sigma_{zP})_{\beta\alpha} (\bar{N}^- \gamma_5)_\gamma - T_5 \frac{m_p^2}{2P_z} (iC \sigma_{\perp z})_{\beta\alpha} (\bar{N}^+ \gamma_5 \gamma^\perp)_\gamma \\
 &\quad - T_6 \frac{m_p^2}{2P_z} (iC \sigma_{\perp z})_{\beta\alpha} (\bar{N}^- \gamma_5 \gamma^\perp)_\gamma + T_7 \frac{m_p}{2} (C \sigma_{\perp\perp'})_{\beta\alpha} (\bar{N}^+ \gamma_5 \sigma^{\perp\perp'})_\gamma \\
 &\quad \left. + T_8 \frac{m_p}{2} (C \sigma_{\perp\perp'})_{\beta\alpha} (\bar{N}^- \gamma_5 \sigma^{\perp\perp'})_\gamma \right\},
 \end{aligned}$$

[V.Braun, R.J.Fries, N.Mahnke, E.Stein (2000)]
[V.Braun, A.Lenz, M.Wittmann, Phys.Rev.D (2006)]
[RQCD, Eur.Phys.J:A (2019)]

Twist classification of the distribution amplitudes in Eq. (2.9)

	twist-3	twist-4	twist-5	twist-6
Vector	V_1	V_2, V_3	V_4, V_5	V_6
Pseudo-vector	A_1	A_2, A_3	A_4, A_5	A_6
Tensor	T_1	T_2, T_3, T_7	T_4, T_5, T_8	T_6
Scalar		S_1	S_2	
Pseudo-scalar		P_1	P_2	

	$f_N (10^{-3})$	$\lambda_1 (10^{-2})$	$\lambda_2 (10^{-2})$	V_1^d	A_1^u	f_1^d	f_2^d	f_1^u
QCDSR	5.0 ± 0.5	-2.7 ± 0.9	5.4 ± 1.9	0.23 ± 0.03	0.38 ± 0.15	0.4 ± 0.05	0.22 ± 0.05	0.07 ± 0.05
LQCD	3.67 ± 0.06	-4.02 ± 0.38	8.37 ± 0.43	0.288 ± 0.007	0.096 ± 0.010

➤ π/K

$$\Phi_{\pi(K)}(q, y) = \frac{i}{\sqrt{2N_c}} \left[\gamma_5 \not{q} \phi_{\pi(K)}^A(y) + m_0^{\pi(K)} \gamma_5 \phi_{\pi(K)}^P(y) + m_0^{\pi(K)} \gamma_5 (\not{y} - 1) \phi_{\pi(K)}^T(y) \right]$$

$$\phi_{\pi(K)}^A(y) = \frac{f_{\pi(K)}}{2\sqrt{2N_c}} 6y(1-y) \left[1 + a_1^{\pi(K)} C_1^{3/2}(2y-1) + a_2^{\pi(K)} C_2^{3/2}(2y-1) + a_4^{\pi(K)} C_4^{3/2}(2y-1) \right],$$

$$\phi_{\pi(K)}^P(y) = \frac{f_{\pi(K)}}{2\sqrt{2N_c}} \left[1 + \left(0.45 - \frac{5}{2} \rho_{\pi(K)}^2 \right) C_2^{1/2}(2y-1) - 3 \left(-0.045 + \frac{9}{20} \rho_{\pi(K)}^2 (1 + 6a_2^{\pi(K)}) \right) C_4^{1/2}(2y-1) \right]$$

$$\phi_{\pi(K)}^T(y) = \frac{f_{\pi(K)}}{2\sqrt{2N_c}} (1-2y) \left[1 + 6 \left(0.0975 - \frac{7}{20} \rho_{\pi(K)}^2 - \frac{3}{5} \rho_{\pi(K)}^2 a_2^{\pi(K)} \right) (1-10y+10y^2) \right], \quad (24)$$

$$\langle P | (\bar{q}q')_{V \mp A} | 0 \rangle = \pm i f_P p_\mu,$$

$$\langle P | (\bar{q}q')_{S \mp P} | 0 \rangle = \pm i f_P m_{0P},$$

[P.Ball, R.Zwicky, Phys.Rev.D (2005)]
 [P.Ball, V.Braun, A.Lenz, JHEP (2006)]

Results of $\Lambda_b \rightarrow p\pi^-$

TABLE V. Invariant amplitudes of the $\Lambda_b \rightarrow p\pi^-$ decay classified by topologies without the CKM matrix elements. in unit of 10^{-9}

$\Lambda_b \rightarrow p\pi^-$	$ S $	$\delta^S(^{\circ})$	Real(S)	Imag(S)	$ P $	$\delta^P(^{\circ})$	Real(P)	Imag(P)
T^f	707.17	0.00	707.17	0.00	1004.44	0.00	1004.44	0.00
T^{nf}	51.72	-96.64	-5.98	-51.38	267.72	-97.92	-36.90	-265.17
$T^f + T^{nf}$	703.07	-4.19	701.19	-51.38	1003.22	-15.33	967.54	-265.17
C_2	29.37	154.96	-26.61	12.43	41.51	179.80	-41.51	0.14
E_2	66.94	-145.26	-55.01	-38.14	72.58	119.94	-36.23	62.89
B	10.40	112.64	-4.00	9.60	23.65	-122.56	-12.73	-19.93
Tree	619.26	-6.26	615.57	-67.49	904.75	-14.21	877.08	-222.06
PC_1^f	58.44	0.00	58.44	0.00	2.90	0.00	2.90	0.00
PC_1^{nf}	1.24	-115.38	-0.53	-1.12	11.16	-95.25	-1.02	-11.11
$PC_1^f + PC_1^{nf}$	57.91	-1.11	57.90	-1.12	11.27	-80.38	1.88	-11.11
PC_2	13.36	-116.10	-5.88	-12.00	14.93	71.96	4.62	14.20
PE_1^u	9.48	-87.62	0.39	-9.47	8.83	114.44	-3.65	8.04
PB	1.36	-51.30	0.85	-1.06	1.55	-159.86	-1.46	-0.53
$PE_1^d + PE_2$	3.87	-98.18	-0.55	-3.83	1.41	-12.55	1.37	-0.31
Penguin	59.45	-27.54	52.71	-27.49	10.65	74.93	2.77	10.28

$$\begin{aligned}
 |T^f| &\gg |E_2| \gtrsim |PC_1^f| \gtrsim |T^{nf}| > |C_2| > |PC_2| > |B| \\
 &\gtrsim |PE_1^u| > |PE_1^d| \sim |PB| \sim |PE_2| \sim |PC_1^{nf}| \quad (S \text{ wave}), \\
 |T^f| &\gg |T^{nf}| \gg |E_2| > |C_2| > |B| > |PC_2| \gtrsim |PC_1^{nf}| \\
 &\gtrsim |PE_1^u| > |PC_1^f| > |PE_1^d| \sim |P^B| \sim |PE_2| \quad (P \text{ wave}),
 \end{aligned}$$

Results of $\Lambda_b \rightarrow p\pi^-$

TABLE V. Invariant amplitudes of the $\Lambda_b \rightarrow p\pi^-$ decay classified by topologies without the CKM matrix elements. in unit of 10^{-9}

$\Lambda_b \rightarrow p\pi^-$	$ S $	$\delta^S(^{\circ})$	Real(S)	Imag(S)	$ P $	$\delta^P(^{\circ})$	Real(P)	Imag(P)
T^f	707.17	0.00	707.17	0.00	1004.44	0.00	1004.44	0.00
T^{nf}	51.72	-96.64	-5.98	-51.38	267.72	-97.92	-36.90	-265.17
$T^f + T^{nf}$	703.07	-4.19	701.19	-51.38	1003.22	-15.33	967.54	-265.17
C_2	29.37	154.96	-26.61	12.43	41.51	179.80	-41.51	0.14
E_2	66.94	-145.26	-55.01	-38.14	72.58	119.94	-36.23	62.89
B	10.40	112.64	-4.00	9.60	23.65	-122.56	-12.73	-19.93
Tree	619.26	-6.26	615.57	-67.49	904.75	-14.21	877.08	-222.06
PC_1^f	58.44	0.00	58.44	0.00	2.90	0.00	2.90	0.00
PC_1^{nf}	1.24	-115.38	-0.53	-1.12	11.16	-95.25	-1.02	-11.11
$PC_1^f + PC_1^{nf}$	57.91	-1.11	57.90	-1.12	11.27	-80.38	1.88	-11.11
PC_2	13.36	-116.10	-5.88	-12.00	14.93	71.96	4.62	14.20
PE_1^u	9.48	-87.62	0.39	-9.47	8.83	114.44	-3.65	8.04
PB	1.36	-51.30	0.85	-1.06	1.55	-159.86	-1.46	-0.53
$PE_1^d + PE_2$	3.87	-98.18	-0.55	-3.83	1.41	-12.55	1.37	-0.31
Penguin	59.45	-27.54	52.71	-27.49	10.65	74.93	2.77	10.28

$$\frac{|C_2|}{|T|} \sim \frac{|E_2|}{|T|} \sim \frac{|B|}{|T|} \sim \mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)$$

$$\frac{|C_2|}{|T|} = 0.042, \quad \frac{|E_2|}{|T|} = 0.095, \quad \frac{|B|}{|T|} = 0.014 \quad (S \text{ wave}),$$

$$\frac{|C_2|}{|T|} = 0.041, \quad \frac{|E_2|}{|T|} = 0.072, \quad \frac{|B|}{|T|} = 0.024 \quad (P \text{ wave}),$$

Results of $\Lambda_b \rightarrow pK^-$

TABLE VII. Same as Table V but for the $\Lambda_b \rightarrow pK^-$ decay.

$\Lambda_b \rightarrow pK^-$	$ S $	$\delta^S(^{\circ})$	Real(S)	Imag(S)	$ P $	$\delta^P(^{\circ})$	Real(P)	Imag(P)
T^f	865.44	0.00	865.44	0.00	1230.64	0.00	1230.64	0.00
T^{nf}	53.41	-102.81	-11.84	-52.08	343.23	-96.76	-40.43	-340.84
$T^f + T^{nf}$	855.18	-3.49	853.60	-52.08	1238.05	-15.98	1190.21	-340.84
E_2	89.06	-138.10	-66.28	-59.48	94.13	122.31	-50.31	79.56
Tree	795.18	-8.06	787.31	-111.55	1169.46	-12.91	1139.90	-261.28
PC_1^f	76.43	0.00	76.43	0.00	3.30	180.00	-3.30	0.00
PC_1^{nf}	1.14	-134.10	-0.79	-0.82	13.85	-94.36	-1.05	-13.81
$PC_1^f + PC_1^{nf}$	75.64	-0.62	75.64	-0.82	14.48	-107.50	-4.35	-13.81
PE_1^u	11.80	-89.53	0.10	-11.80	11.02	115.62	-4.76	9.93
PE_1^d	7.53	-101.53	-1.50	-7.38	2.67	51.53	1.66	2.09
Penguin	76.88	-15.08	74.23	-20.00	7.66	-166.53	-7.45	-1.79

$$\begin{aligned} \frac{|T^f(pK)|}{|T^f(p\pi)|} &= 1.22, & \frac{|T^{nf}(pK)|}{|T^{nf}(p\pi)|} &= 1.03, & \frac{|E_2(pK)|}{|E_2(p\pi)|} &= 1.33 \text{ (S wave),} \\ \frac{|T^f(pK)|}{|T^f(p\pi)|} &= 1.23, & \frac{|T^{nf}(pK)|}{|T^{nf}(p\pi)|} &= 1.28, & \frac{|E_2(pK)|}{|E_2(p\pi)|} &= 1.29 \text{ (P wave).} \end{aligned}$$

[H.Y.Cheng, S.Oh, JHEP (2011)]