SMEFT meets SM:

$U(2)^5$ flavour symmetry in semileptonic and leptonic B decays

Speaker: Min-Di Zheng (郑旻笛)

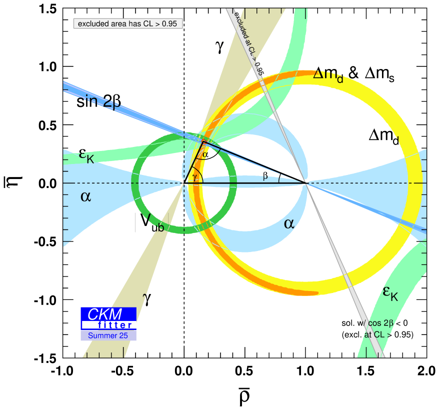
M.-C. Gao, X.-Q. Li, Y.-F. Wang, Y.-D. Yang, X.-B. Yuan, and M.-D. Zheng

Cross-Strait 2026, Guangzhou

Outlines

- Motivation
- $U(2)^5$ flavour symmetry
- Matching with CKM in SM
- Flavor structures in SMEFT
- Numerical discussions and conclusions

$U(2)^5$ flavour symmetry



CKMfitter 2025

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

"the success of the CKM picture be due to the existence of a suitable flavour symmetry appropriately broken in some definite direction, thus allowing a scale of new flavour physics phenomena sufficiently near to the Fermi scale to leave room for relatively small but nevertheless observable deviations from the SM in the flavour sector."

$U(2)_q \otimes U(2)_u \otimes U(2)_d$: act on the first two generations of quarks

$U(2)_\ell \otimes U(2)_e$: act the first two generations of leptons

A. Pomarol and D. Tommasini, hep-ph/9507462

R. Barbieri, G. R. Dvali and L. J. Hall, hep-ph/9512388

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & s_u s e^{-i\delta} \\ -\lambda & 1-\lambda^2/2 & c_u s \\ -s_d s e^{i(\delta+\alpha_u-\alpha_d)} & -s c_d & 1 \end{pmatrix}$$

R. Barbieri, G. Isidori, J. Jones-Perez, P. Lodone and D. M. Straub, 1105.2296

M. Bordone, C. Cornella, J. Fuentes-Martín and G. Isidori, 1805.09328

Plenty of literature on $U(2)^5$

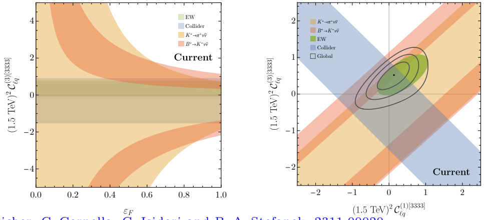
New physics in the third generation. A comprehensive SMEFT analysis and future prospects

Lukas Allwicher,[a] **Claudia Cornella,**[b] **Gino Isidori**[a] **and Ben A. Stefanek**[c]

[a] *Physik-Institut, Universität Zürich, CH-8057 Zürich, Switzerland*
[b] *PRISMA+ Cluster of Excellence & MITP, Johannes Gutenberg University, Mainz, Germany*
[c] *Physics Department, King's College London, Strand, London, WC2R 2LS, U.K.*

E-mail: lukas.allwicher@physik.uzh.ch, claudia.cornella@uni-mainz.de, isidori@physik.uzh.ch, benjamin.stefanek@kcl.ac.uk

Abstract: We present a comprehensive analysis of electroweak, flavor, and collider bounds on the complete set of dimension-six SMEFT operators in the $\mathrm{U}(2)^5$-symmetric limit. This



L. Allwicher, C. Cornella, G. Isidori and B. A. Stefanek, 2311.00020

Probing third-generation New Physics with $K \to \pi\nu\bar\nu$ and $B \to K^{(*)}\nu\bar\nu$

L. Allwicher,[1,*] M. Bordone,[2,†] G. Isidori,[1,‡] G. Piazza,[1,§] and A. Stanzione[3,4,¶]

[1] *Physik-Institut, Universität Zürich, CH-8057 Zürich, Switzerland*
[2] *Theoretical Physics Department, CERN, Geneva, Switzerland*
[3] *INFN, Sezione di Trieste, SISSA, Via Bonomea 265, 34136, Trieste, Italy*
[4] *SISSA International School for Advanced Studies, Via Bonomea 265, 34136, Trieste, Italy*

The recent observation of the $K^+ \to \pi^+\nu\bar\nu$ decay by NA62 is an important milestone in precision flavor physics. Together with evidence of $B^+ \to K^+\nu\bar\nu$ reported by Belle-II, they are the only FCNC decays involving third-family leptons where a precision close to the SM expectation has been reached. We study the implications of these recent results in the context of a new physics scenario aligned to the third generation, with an approximate $U(2)^5$ flavor symmetry acting on the light families. We find that the slight excess observed in both channels supports the hypothesis of non-standard TeV dynamics of this type, as also hinted at by other B-meson decays, consistently with bounds from colliders and electroweak observables. We further discuss how future improvements in precision could affect this picture, highlighting the discovery potential in these di-neutrino modes.

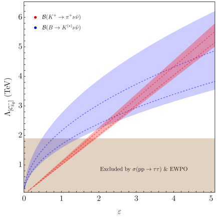


FIG. 5. Λ vs. ε plane, where Λ is defined by $C_{\ell q}^- = 1/\Lambda^2$. The blue and red areas indicate present 1σ constraints from $\mathcal{B}(K^+ \to \pi^+\nu\bar\nu)$ and $\mathcal{B}(B \to K^{(*)}\nu\bar\nu)$, setting $\kappa = 1$. The 95% CL exclusion limit from direct searches and EWPO are

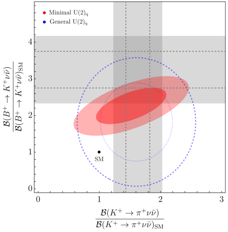


FIG. 6. Correlation between $\mathcal{B}(B^+ \to K^+\nu\bar\nu)$ and $\mathcal{B}(K^+ \to \pi^+\nu\bar\nu)$, normalized to their SM predictions. The red areas denote the parameter regions favored at 1σ and 2σ from a

L. Allwicher, M. Bordone, G. Isidori, G. Piazza and A. Stanzione, 2410.21444

Some issues need further research!

$$L_d \approx \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & s_b \\ -s_d\, s_b\, e^{-i(\alpha_d+\phi_q)} & -c_d\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix},$$

$$R_d \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_s}{m_b}\, s_b \\ 0 & -\frac{m_s}{m_b}\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix},$$

$$R_u \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_c}{m_t}\, s_t \\ 0 & -\frac{m_c}{m_t}\, s_t\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix},$$

$$L_e \approx \begin{pmatrix} c_e & -s_e & 0 \\ s_e & c_e & s_\tau \\ -s_e s_\tau & -c_e s_\tau & 1 \end{pmatrix},$$

$$R_e \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_\mu}{m_\tau}\, s_\tau \\ 0 & -\frac{m_\mu}{m_\tau}\, s_\tau & 1 \end{pmatrix},$$

with $V_{\rm CKM} = L_u^\dagger L_d$.

$$\mathcal{O}_{ledq} = (\bar{l}_\alpha e_\beta)(\bar{d}_i q_j)$$

$$\Lambda_S^{[ij\alpha\beta]} = (\Gamma_L^\dagger)^{\alpha j} \times \Gamma_R^{i\beta},$$

where, in the interaction basis,

$$\Gamma_L^{i\alpha} = \begin{pmatrix} x_{q\ell} V_q^i (V_\ell^\alpha)^* & x_q V_q^i \\ x_\ell (V_\ell^\alpha)^* & 1 \end{pmatrix}, \quad \Gamma_R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$\hat{\Gamma}_L = e^{i\phi_q} \begin{pmatrix} \Delta_{q\ell}^{de} & \Delta_{q\ell}^{d\mu} & \lambda_q^d \\ \Delta_{q\ell}^{se} & \Delta_{q\ell}^{s\mu} & \lambda_q^s \\ \lambda_\ell^e & \lambda_\ell^\mu & x_{q\ell}^{b\tau} \end{pmatrix} \approx e^{i\phi_q} \begin{pmatrix} 0 & 0 & \lambda_q^d \\ 0 & \Delta_{q\ell}^{s\mu} & \lambda_q^s \\ \lambda_\ell^e & \lambda_\ell^\mu & 1 \end{pmatrix},$$

$$\hat{\Gamma}_R \approx e^{i\phi_q} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{m_s}{m_b}\, s_b \\ 0 & -\frac{m_\mu}{m_\tau}\, s_\tau & 1 \end{pmatrix}.$$

$$\lambda_q^s = O(|V_q|), \qquad \lambda_\ell^\mu = O(|V_\ell|),$$

$$x_{q\ell}^{b\tau} = O(1), \qquad \Delta_{q\ell}^{s\mu} = O(\lambda_q^s \lambda_\ell^\mu),$$

$$\frac{\lambda_q^d}{\lambda_q^s} = \frac{\Delta_{q\ell}^{d\alpha}}{\Delta_{q\ell}^{s\alpha}} = \frac{V_{td}^*}{V_{ts}^*}, \qquad \frac{\lambda_\ell^e}{\lambda_\ell^\mu} = \frac{\Delta_{q\ell}^{ie}}{\Delta_{q\ell}^{i\mu}} = s_e.$$

J. Fuentes-Martín, G. Isidori, J. Pagès and K. Yamamoto, 1909.02519

Theoretical framework

- $U(2)^5$ symmetry:

$$U(2)^5 = U(2)_q \otimes U(2)_u \otimes U(2)_d \otimes U(2)_\ell \otimes U(2)_e .$$

- Quantum number (q, u, d, ℓ, e):

$$q = \begin{pmatrix} q_{12} \sim (2,1,1,1,1) \\ q_3 \sim (1,1,1,1,1) \end{pmatrix}, \quad u = \begin{pmatrix} u_{12} \sim (1,2,1,1,1) \\ u_3 \sim (1,1,1,1,1) \end{pmatrix}, \quad d = \begin{pmatrix} d_{12} \sim (1,1,2,1,1) \\ d_3 \sim (1,1,1,1,1) \end{pmatrix}$$

$$\ell = \begin{pmatrix} \ell_{12} \sim (1,1,1,2,1) \\ \ell_3 \sim (1,1,1,1,1) \end{pmatrix}, \quad e = \begin{pmatrix} e_{12} \sim (1,1,1,1,2) \\ e_3 \sim (1,1,1,1,1) \end{pmatrix},$$

- Minimal *spurions*:

$$\Delta_e \sim (1,1,1,2,\bar{2}) \ , \qquad \Delta_u \sim (2,\bar{2},1,1,1) \ , \qquad \Delta_d \sim (2,1,\bar{2},1,1) \ ,$$

$$V_\ell \sim (1,1,1,2,1) \ , \qquad V_q \sim (2,1,1,1,1)$$

Yukawa interaction

- Minimal *spurions*:

$$\Delta_e \sim (1,1,1,2,\bar{2})\ , \qquad \Delta_u \sim (2,\bar{2},1,1,1)\ , \qquad \Delta_d \sim (2,1,\bar{2},1,1)\ ,$$
$$V_\ell \sim (1,1,1,2,1)\ , \qquad V_q \sim (2,1,1,1,1)$$

$$(\bar{Q}_q, \bar{Q}_3) \begin{pmatrix} Y_d^{2\times 2} & Y_d^{2\times 1} \\ Y_d^{1\times 2} & Y_d^{33} \end{pmatrix} \begin{pmatrix} d_q \\ d_3 \end{pmatrix} H$$

$$\begin{pmatrix} Y_d^{2\times 2} & Y_d^{2\times 1} \\ Y_d^{1\times 2} & Y_d^{33} \end{pmatrix} \sim \begin{pmatrix} 2\otimes\bar{2} & 2\otimes\bar{1} \\ 1\otimes\bar{2} & 1\otimes\bar{1} \end{pmatrix} \qquad \text{(quantum numbers: } \ q\otimes d)$$

- Yukawa matrices:

$$Y_u = \begin{pmatrix} \alpha_u\Delta_u & \beta_t V_q \\ \beta_t' V_q^\dagger\Delta_u & \alpha_t \end{pmatrix}, \qquad Y_d = \begin{pmatrix} \alpha_d\Delta_d & \beta_b V_q \\ \beta_b' V_q^\dagger\Delta_d & \alpha_b \end{pmatrix}, \qquad Y_e = \begin{pmatrix} \alpha_e\Delta_e & \beta_\tau V_\ell \\ \beta_\tau' V_\ell^\dagger\Delta_e & \alpha_\tau \end{pmatrix}$$

Yukawa structure

- Yukawa matrices:

$$Y_u = y_t \begin{pmatrix} \Delta_u & x_t V_q \\ 0 & 1 \end{pmatrix}, \qquad Y_d = y_b \begin{pmatrix} \Delta_d & x_b V_q \\ 0 & 1 \end{pmatrix}, \qquad Y_e = y_\tau \begin{pmatrix} \Delta_e & x_\tau V_\ell \\ 0 & 1 \end{pmatrix}$$

- Yukawa matrices in interaction basis:

$$Y_u = |y_t| \begin{pmatrix} U_q^\dagger O_u^\intercal \hat{\Delta}_u & |V_q|\,|x_t|\, e^{i\phi_q}\, \vec{n} \\ 0 & 1 \end{pmatrix}, \qquad Y_d = |y_b| \begin{pmatrix} U_q^\dagger \hat{\Delta}_d & |V_q|\,|x_b|\, e^{i\phi_q}\, \vec{n} \\ 0 & 1 \end{pmatrix},$$

$$Y_e = |y_\tau| \begin{pmatrix} O_e^\intercal \hat{\Delta}_e & |V_\ell|\,|x_\tau|\, \vec{n} \\ 0 & 1 \end{pmatrix}$$

  - $\hat{\Delta}_u^{1,2} = \epsilon_{u,c}, \quad \hat{\Delta}_d^{1,2} = \epsilon_{d,s}, \quad \hat{\Delta}_e^{1,2} = \epsilon_{e,\mu}, \quad \vec{n} = (0,1)^T, \quad |V_q||x_{t,b,\tau}| = \epsilon_{t,b,\tau}$
  - Unitary U_q and Orthogonal $O_{u,e}$ matrices:

$$U_q = \begin{pmatrix} c_d & s_d\, e^{i\alpha_d} \\ -s_d\, e^{-i\alpha_d} & c_d \end{pmatrix}, \qquad O_u = \begin{pmatrix} c_u & s_u \\ -s_u & c_u \end{pmatrix}, \qquad O_e = \begin{pmatrix} c_e & s_e \\ -s_e & c_e \end{pmatrix}$$

Diagonalization

- Unitary transformations:

$$\mathrm{diag}(Y_d) = \hat{Y}_d = L_d'^{\dagger} \begin{pmatrix} U_q^\dagger & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} U_q & 0 \\ 0 & 1 \end{pmatrix} |y_b| \begin{pmatrix} U_q^\dagger \hat{\Delta}_d & |V_q|\,|x_b|\,e^{i\phi_q}\,\vec{n} \\ 0 & 1 \end{pmatrix} R_d'$$

$$= L_d'^{\dagger} \begin{pmatrix} U_q^\dagger & 0 \\ 0 & 1 \end{pmatrix} |y_b| \begin{pmatrix} \epsilon_d & 0 & \epsilon_b s_d e^{i(\alpha_d+\phi_q)} \\ 0 & \epsilon_s & \epsilon_b c_d e^{i\phi_q} \\ 0 & 0 & 1 \end{pmatrix} R_d'$$

- Y_d related to the interaction basis while $Y_d' = \begin{pmatrix} U_q & 0 \\ 0 & 1 \end{pmatrix} Y_d$ related to a new basis
- Arbitrary phase:

$$P_u = \begin{pmatrix} e^{i\phi_x} & 0 & 0 \\ 0 & e^{i\phi_y} & 0 \\ 0 & 0 & e^{i\phi_z} \end{pmatrix}, \qquad P_d = \begin{pmatrix} e^{i\phi_a} & 0 & 0 \\ 0 & e^{i\phi_b} & 0 \\ 0 & 0 & e^{i\phi_c} \end{pmatrix},$$

- diagonalize Y_d:

$$L_d^\dagger Y_d R_d = P_d^\dagger L_d'^{\dagger} Y_d R_d' P_d = P_d^\dagger L_d''^{\dagger} Y_d' R_d' P_d = P_d^\dagger \hat{Y}_d P_d = \hat{Y}_d = |y_b|\mathrm{diag}(\epsilon_d,\, \epsilon_s,\, 1 + \frac{1}{2}\epsilon_b^2)$$

  - $L_d = \begin{pmatrix} U_q^\dagger & 0 \\ 0 & 1 \end{pmatrix} L_d'' P_d,\qquad R_d = R_d' P_d,\qquad \epsilon_s \sim \epsilon_b \approx \mathcal{O}(\epsilon = m_d/m_s),\ \epsilon_d \approx \mathcal{O}(\epsilon^2)$

Get $V_{\rm CKM}^{\rm U(2)}$

- General CKM:

$$V_{\rm CKM}^{\rm U(2)} = L_u^\dagger L_d = \begin{pmatrix} c_u e^{i\alpha_d - i\theta + i\phi_a - i\phi_x} & s_u e^{-i\theta + i\phi_b - i\phi_x} & x_{bt}\lambda e^{i\phi_c - i\phi_x} \\ -s_u e^{i\alpha_d - i\delta + i\phi_a - i\phi_y} & c_u e^{-i\delta + i\phi_b - i\phi_y} & x_{bt}\chi e^{i\phi_c - i\phi_y} \\ -x_{bt} s_d e^{i\phi_a - i\phi_z} & -x_{bt} c_d e^{i\phi_b - i\phi_z} & e^{i\phi_c - i\phi_z} \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

  - $x_{bt} = \epsilon_b - \epsilon_t, \quad \lambda e^{i\theta} = (c_d s_u + s_d c_u e^{i\alpha_d}), \quad \chi e^{i\delta} = (c_d c_u - s_d s_u e^{i\alpha_d})$

- Matching with Wolfstein parameterization directly:
  - $\phi_z = \phi_b = \phi_c,\ \phi_x = \phi_b - \theta,\ \phi_y = \phi_b - \delta,\ \phi_a = \phi_b - \alpha_d$

$$V_{\rm CKM}^{\rm U(2)} = \begin{pmatrix} c_u & s_u & x_{bt}(c_u s_d e^{i\alpha_d} + s_u c_d) \\ -s_u & c_u & x_{bt}(c_u c_d - s_u s_d e^{i\alpha_d}) \\ -x_{bt} s_d e^{-i\alpha_d} & -x_{bt} c_d & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

Matching with Wolfstein parameterization directly

$$V_{\text{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4),$$

$$V_{\text{CKM}}^{\text{U(2)}} = \begin{pmatrix} c_u & s_u & x_{bt}(c_u s_d e^{i\alpha_d} + s_u c_d) \\ -s_u & c_u & x_{bt}(c_u c_d - s_u s_d e^{i\alpha_d}) \\ -x_{bt} s_d e^{-i\alpha_d} & -x_{bt} c_d & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

We set

- $s_d \approx \mathcal{O}(\lambda)$, and $\epsilon \approx \mathcal{O}(\lambda^2)$ expected
- $\lambda = s_u, \quad A = \frac{x_{bt} c_d}{s_u^2}, \quad \rho = 1 + \frac{s_d \cos\alpha_d}{c_d s_u}, \quad \eta = -\frac{s_d \sin\alpha_d}{c_d s_u}$

We can get

- $-x_{bt} s_d e^{-i\alpha_d} = A\lambda^3(1-\rho-i\eta) = V_{\text{CKM}}^{td}$
- $s_d = |V_{\text{CKM}}^{td} c_d / A\lambda^2|, \quad e^{-i\alpha_d} = (V_{\text{CKM}}^{td}/V_{\text{CKM}}^{ts})(c_d/s_d)$

Matching with Wolfstein parameterization directly

$$V_{\text{CKM}} = \begin{pmatrix} c_u & s_u & x_{bt}(c_u s_d e^{i\alpha_d} + s_u c_d) \\ -s_u & c_u & x_{bt}(c_u c_d - s_u s_d e^{i\alpha_d}) \\ -x_{bt} s_d e^{-i\alpha_d} & -x_{bt} c_d & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

$$= \begin{pmatrix} 1 - \lambda^2/2 & \lambda & V_{\text{CKM}}^{ub} \\ -\lambda & 1 - \lambda^2/2 & V_{\text{CKM}}^{cb} \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4),$$

Use power counting for λ:

$$\begin{aligned} V_{\text{CKM}}^{ub} =& (A\lambda^3 + x_{bt} s_d \cos\alpha_d) + i x_{bt} s_d \sin\alpha_d + \mathcal{O}(\lambda^4) \\ =& A\lambda^3(\rho - i\eta) + \mathcal{O}(\lambda^4) \\ V_{\text{CKM}}^{cb} =& A\lambda^2 + \mathcal{O}(\lambda^4) \end{aligned}$$

Comparison with the literature ($\mathcal{O}(\epsilon)$)

Our results

$$L_d = \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & \epsilon_b \\ -s_d \epsilon_b e^{-i(\phi_q+\alpha_d)} & -c_d \epsilon_b e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & {\color{red}0} \\ 0 & {\color{red}0} & e^{-i\phi_q} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & {\color{red}0} \\ 0 & {\color{red}0} & e^{-i\phi_q} \end{pmatrix}$$

$$L_d = \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & s_b \\ -s_d\, s_b\, e^{-i(\alpha_d+\phi_q)} & -c_d\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & {\color{red}\frac{m_s}{m_b}\, s_b} \\ 0 & {\color{red}-\frac{m_s}{m_b}\, s_b\, e^{-i\phi_q}} & e^{-i\phi_q} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & {\color{red}\frac{m_c}{m_t}\, s_t} \\ 0 & {\color{red}-\frac{m_c}{m_t}\, s_t\, e^{-i\phi_q}} & e^{-i\phi_q} \end{pmatrix}$$

D. A. Faroughy, G. Isidori, F. Wilsch and K. Yamamoto, 2005.05366

Comparison with the literature ($\mathcal{O}(\epsilon)$, while $\mathcal{O}(\epsilon, \epsilon^2)$ for $23, 32$ parts)

Our results

$$L_d = \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & \epsilon_b \\ -s_d \epsilon_b e^{-i(\phi_q+\alpha_d)} & -c_d \epsilon_b e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon_s \epsilon_b c_d \\ 0 & -\epsilon_s \epsilon_b c_d e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \chi \epsilon_c \epsilon_t e^{i\delta} \\ 0 & -\chi \epsilon_c \epsilon_t e^{-i(\phi_q+\delta)} & e^{-i\phi_q} \end{pmatrix}$$

$$L_d = \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & s_b \\ -s_d\, s_b\, e^{-i(\alpha_d+\phi_q)} & -c_d\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_s}{m_b}\, s_b \\ 0 & -\frac{m_s}{m_b}\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_c}{m_t}\, s_t \\ 0 & -\frac{m_c}{m_t}\, s_t\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

D. A. Faroughy, G. Isidori, F. Wilsch and K. Yamamoto, 2005.05366

Two-step matching

- Matching with Standard parameterization
  - $\phi_z = \phi_c = \phi_y, \phi_x = \phi_b - \theta, \phi_a = \phi_b - \alpha_d, x = y = -z = a = b = -c = 1$

$$V_{\rm CKM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix},$$

$$V_{\rm CKM}^{\rm U(2)} = \begin{pmatrix} c_u & s_u & \lambda x_{bt}e^{-i(-\theta-\phi_b+\phi_z)} \\ -s_u e^{i(-\delta-\phi_b+\phi_z)} & c_u e^{-i(\delta+\phi_b-\phi_z)} & \chi x_{bt} \\ -s_d x_{bt} e^{-i(\alpha_d+\phi_b-\phi_z)} & -c_d x_{bt} e^{-i(\phi_b-\phi_z)} & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

- Then matching with Wolfstein parameterization
  - further choose $\phi_z = \phi_b - \delta$

$$V_{\rm CKM}^{\rm U(2)} = \begin{pmatrix} c_u & s_u & \lambda x_{bt}e^{-i(-\theta+\delta)} \\ -s_u & c_u & \chi x_{bt} \\ -\frac{(\lambda e^{-i\theta} - c_d s_u) x_{bt} e^{i\delta}}{c_u} & -c_d x_{bt} e^{i\delta} & 1 \end{pmatrix}$$

Two-step matching

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4),$$

$$V_{\rm CKM}^{\rm U(2)} = \begin{pmatrix} c_u & s_u & \lambda x_{bt} e^{-i(-\theta+\delta)} \\ -s_u & c_u & \chi x_{bt} \\ -\frac{(\lambda e^{-i\theta} - c_d s_u) x_{bt} e^{i\delta}}{c_u} & -c_d x_{bt} e^{i\delta} & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

We set:

- $s_u = \lambda, \quad c_d x_{bt} = A\lambda^2, \quad \lambda x_{bt} e^{i\theta} = A\lambda^3(\rho - i\eta)$, and $\epsilon \approx \mathcal{O}(\lambda^2)$ expected
- $\delta \approx \mathcal{O}(\lambda^2)$, then $\tan\delta = \frac{s_d s_u s_{\alpha_d}}{c_d c_u - s_d s_u c_{\alpha_d}} \approx \mathcal{O}(\lambda^2) \to s_d \sim \mathcal{O}(\lambda)$

Then all the other matrix elements can be matched with Wolfstein parameterization before the order $\mathcal{O}(\lambda^4)$

Comparison with the literature ($\mathcal{O}(\epsilon)$)

Our results

$$L_d = \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & \epsilon_b e^{-i\delta} \\ -s_d \epsilon_b e^{-i(\phi_q+\alpha_d)} & -c_d \epsilon_b e^{-i\phi_q} & e^{-i(\phi_q+\delta)} \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i(\phi_q+\delta)} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{-i(\phi_q+\delta)} \end{pmatrix}$$

$$L_d = \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & s_b \\ -s_d\, s_b\, e^{-i(\alpha_d+\phi_q)} & -c_d\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_s}{m_b}\, s_b \\ 0 & -\frac{m_s}{m_b}\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_c}{m_t}\, s_t \\ 0 & -\frac{m_c}{m_t}\, s_t\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

D. A. Faroughy, G. Isidori, F. Wilsch and K. Yamamoto, 2005.05366

Comparison with the literature ($\mathcal{O}(\epsilon)$, while $\mathcal{O}(\epsilon, \epsilon^2)$ for $23, 32$ parts)

Our results

$$L_d = \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & \epsilon_b e^{-i\delta} \\ -s_d \epsilon_b e^{-i(\phi_q+\alpha_d)} & -c_d \epsilon_b e^{-i\phi_q} & e^{-i(\phi_q+\delta)} \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon_s \epsilon_b c_d e^{-i\delta} \\ 0 & -\epsilon_s \epsilon_b c_d e^{-i\phi_q} & e^{-i(\phi_q+\delta)} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \chi \epsilon_c \epsilon_t \\ 0 & -\chi \epsilon_c \epsilon_t e^{-i(\phi_q+\delta)} & e^{-i(\phi_q+\delta)} \end{pmatrix}$$

$$L_d = \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & s_b \\ -s_d\, s_b\, e^{-i(\alpha_d+\phi_q)} & -c_d\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_s}{m_b}\, s_b \\ 0 & -\frac{m_s}{m_b}\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_c}{m_t}\, s_t \\ 0 & -\frac{m_c}{m_t}\, s_t\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

D. A. Faroughy, G. Isidori, F. Wilsch and K. Yamamoto, 2005.05366

Operators in SMEFT

- Semileptonic and leptonic decays at tree-level:

$$
\begin{aligned}
\mathcal{Q}_{lq}^{(1)} &= (\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t), & \mathcal{Q}_{lq}^{(3)} &= (\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t), \\
\mathcal{Q}_{ld} &= (\bar{l}_p \gamma_\mu l_r)(\bar{d}_s \gamma^\mu d_t), & \mathcal{Q}_{qe} &= (\bar{q}_p \gamma^\mu q_r)(\bar{e}_s \gamma^\mu e_t), \\
\mathcal{Q}_{ed} &= (\bar{e}_p \gamma_\mu e_r)(\bar{d}_s \gamma^\mu d_t), & \mathcal{Q}_{ledq} &= (\bar{l}_p^j e_r)(\bar{d}_s q_t^j), \\
\mathcal{Q}_{lequ}^{(1)} &= (\bar{l}_p^j e_r)\varepsilon_{ik}(\bar{q}_s^k u_t), & \mathcal{Q}_{lequ}^{(3)} &= (\bar{l}_p^j \sigma_{\mu\nu} e_r)\varepsilon_{ik}(\bar{q}_s^k \sigma^{\mu\nu} u_t),
\end{aligned}
$$

- Quark and lepton bilinear:

$$
\begin{aligned}
&\Gamma_{qq}\bar{q}\gamma^\mu q, \quad \Gamma_{dd}\bar{d}\gamma^\mu d, \quad \Gamma_{dq}\bar{d}q, \quad \Gamma_{qu}\bar{q}u, \quad \Gamma_{qu}\bar{q}\sigma^\mu u, \\
&\Gamma_{ll}\bar{l}\gamma_\mu l, \quad \Gamma_{ee}\bar{e}\gamma_\mu e, \quad \Gamma_{le}\bar{l}e, \quad \Gamma_{le}\bar{l}\sigma_{\mu\nu} e,
\end{aligned}
$$

Wilson coefficients in SMEFT

Flavour basis:

$$\mathcal{Q}_{lq}^{(1)} = (\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t), \qquad \mathcal{Q}_{lq}^{(3)} = (\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t),$$

$$\mathcal{Q}_{ledq} = (\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$$

$$[\Lambda_{lq}^{(1)}]_{prst} \sim [\Lambda_{\ell q}^{(3)}]_{prst} = \begin{pmatrix} a_{ll} + a_{ll}^V V_l V_l^\dagger & x_l V_l \\ x_l^* V_l^\dagger & a_l' \end{pmatrix}_{pr} \times \begin{pmatrix} a_{qq} + a_{qq}^V V_q V_q^\dagger & x_q V_q \\ x_q^* V_q^\dagger & a_q' \end{pmatrix}_{st},$$

$$[\Lambda_{ledq}]_{prst} = \begin{pmatrix} x_e \Delta_e & x_l V_l \\ V_l^\dagger x_{el}^\dagger \Delta_e & x_e' \end{pmatrix}_{pr} \times \begin{pmatrix} x_d \Delta_d^\dagger & \Delta_d^\dagger x_{dq} V_q \\ x_q V_q^\dagger & x_d' \end{pmatrix}_{st}$$

Wilson coefficients in SMEFT

Flavour basis:

$$\mathcal{Q}_{lq}^{(1)} = (\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t), \qquad \mathcal{Q}_{lq}^{(3)} = (\bar{l}_p \gamma_\mu \tau^I l_r)(\bar{q}_s \gamma^\mu \tau^I q_t),$$
$$\mathcal{Q}_{ledq} = (\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$$

$$[\Lambda_{lq}^{(1)}]_{prst} \sim [\Lambda_{\ell q}^{(3)}]_{prst} = \begin{pmatrix} a_{ll} + a_{ll}^V V_l V_l^\dagger & x_l V_l \\ x_l^* V_l^\dagger & a_l' \end{pmatrix}_{pr} \times \begin{pmatrix} a_{qq} + a_{qq}^V V_q V_q^\dagger & x_q V_q \\ x_q^* V_q^\dagger & a_q' \end{pmatrix}_{st},$$
$$[\Lambda_{ledq}]_{prst} = \begin{pmatrix} x_e \Delta_e & x_l V_l \\ V_l^\dagger x_{el}^\dagger \Delta_e & x_e' \end{pmatrix}_{pr} \times \begin{pmatrix} x_d \Delta_d^\dagger & \Delta_d^\dagger x_{dq} V_q \\ x_q V_q^\dagger & x_d' \end{pmatrix}_{st}$$

Mass basis:

$$(\hat{\Gamma}^{d_R d_L})^\dagger = \begin{pmatrix} x_d \epsilon_d & 0 & -k_{sb} K_{ds}^* c_d \epsilon_b e^{-i\delta} \\ 0 & x_d \epsilon_s & -k_{sb} c_d \epsilon_b e^{-i\delta} \\ 0 & (x_d - k_{sb}') c_d \epsilon_s \epsilon_b e^{i\delta} & x_d' + (-k_{sb} + \frac{x_d'}{2}) \epsilon_b^2 \end{pmatrix}$$

where $K_{ds} = V_{td}/V_{ts}$, $k_{sb} = x_d' - \frac{x_q}{x_b}$, $k_{sb}' = x_d' - \frac{x_{dq}}{x_b}$

Numerical discussions

The $B \to K^{(*)}\ell\bar{\ell}$, $B \to K\nu\bar{\nu}$ and $K \to \pi\nu\bar{\nu}$ decays described by

$$\mathcal{L}_{\rm eff} = \frac{4G_F}{\sqrt{2}}\frac{\alpha}{2\pi}\left[\lambda^t_{sd}L^{\nu}_{\ell,sd}\mathcal{O}^{\nu}_{\ell,sd} + \lambda^t_{bs}L^{\nu}_{\ell,bs}\mathcal{O}^{\nu}_{\ell,bs} + \lambda^t_{bs}L_9\mathcal{O}_9\right] + \text{h.c.},$$

where $\lambda^q_{ij} = V^*_{qi}V_{qj}$, $\mathcal{O}^{\nu}_{\ell,ij} = (\bar{d}^i_L\gamma_\mu d^j_L)(\bar{\nu}^\ell_L\gamma^\mu\nu^\ell_L)$, $\mathcal{O}_9 = \frac{1}{2}(\bar{s}_L\gamma_\mu b_L)(\bar{\ell}\gamma^\mu\ell)$

General structure of quark sector in flavour basis:

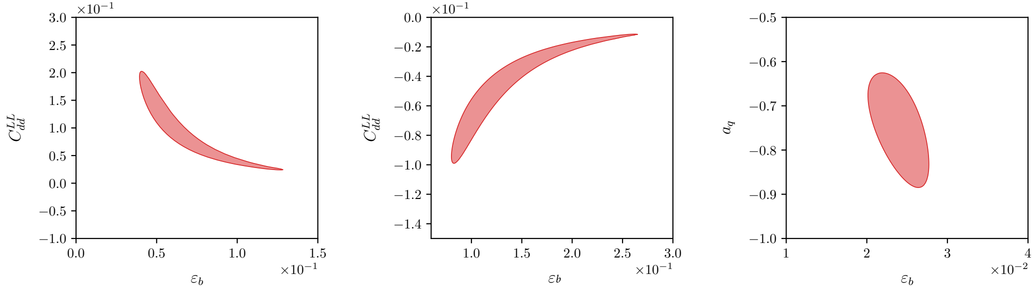
$$[\Gamma^{(1,3)}_{lq}]_{st} = \begin{pmatrix} a_{qq} + a^V_{qq}V_qV_q^\dagger & x_qV_q \\ x^*_qV^\dagger_q & a'_q \end{pmatrix}_{st}$$

Numerical discussions

General form:

$$[\Gamma^{(1,3)}_{lq}]_{st} = \begin{pmatrix} a_{qq} + a^V_{qq} V_q V_q^\dagger & x_q V_q \\ x_q^* V_q^\dagger & a'_q \end{pmatrix}_{st}$$

- **(c)**: $a_{qq} = a'_q = C^{LL}_{dd}, \quad a^V_{qq} = 0, \quad x_q = x_q^* = |x_b| C^{LL}_{dd}$
- **(d)**: $a^V_{qq} = x_q = x_q^* = a'_q = C^{LL}_{dd}, \quad a_{qq} = 0$
- **(4)**: $a_{qq} - a'_q = a_q, \quad a^V_{qq} = 0, \quad x_q|V_q| = x_q^*|V_q| = \epsilon_b$



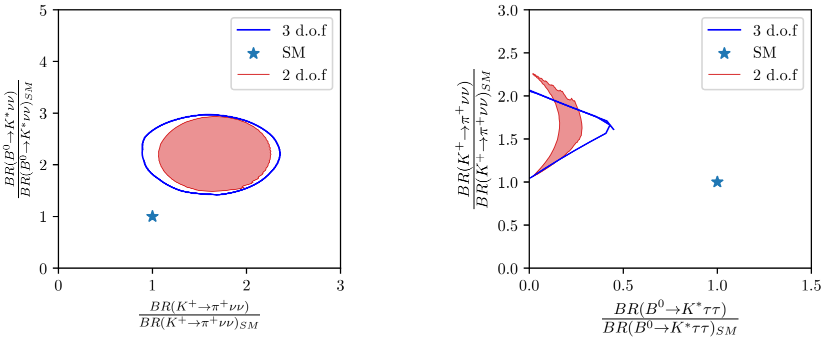
Numerical discussions

General form:

$$[\Gamma_{lq}^{(1,3)}]_{st} = \begin{pmatrix} a_{qq} + a_{qq}^{V} V_q V_q^{\dagger} & x_q V_q \\ x_q^* V_q^{\dagger} & a_q' \end{pmatrix}_{st}$$

(4): $a_{qq} - a_q' = a_q, \quad a_{qq}^V = 0, \quad x_q|V_q| = x_q^*|V_q| = \epsilon_b$

(1): $a_{qq} - a_q' = a_q, \quad a_{qq}^V = 0, \quad x_q|V_q| = x_q^*|V_q| = x_q$



Conclusions

In the $U(2)^5$ flavor symmetry framework, we obtain the following analytical results with strict power counting:

- the CKM of $U(2)^5$ matching with Standard model.
- the flavour structures in Wilson coefficients of semileptonic and leptonic B decays.

Then some numerical calculations are also performed.

Thank you !!

Power-counting of *spurions*

$$\Delta_e \sim (1,1,1,2,\bar{2})\ , \qquad \Delta_u \sim (2,\bar{2},1,1,1)\ , \qquad \Delta_d \sim (2,1,\bar{2},1,1)\ ,$$
$$V_\ell \sim (1,1,1,2,1)\ , \qquad V_q \sim (2,1,1,1,1)$$

$$\mathcal{L}_{\rm eff} = C_S(\bar{l}_p^j \bar{d}_s)\Lambda_{pstr}(q_t^j e_r) + \cdots$$

$$\begin{aligned}\Lambda_{pstr} =& A\{V_q^{\dagger n_q'}, V_q^{n_q}, V_l^{\dagger n_l'}, V_l^{n_l}, \Delta_d^{\dagger n_d'}, \Delta_d^{n_d}, \Delta_u^{\dagger n_u'}, \Delta_u^{n_u}, \Delta_e^{\dagger n_e'}, \Delta_e^{n_e}\} \\ & \sim (f(n_q + n_d + n_u - n_q' - n_d' - n_u'), f(n_u' - n_u), f(n_d' - n_d), \\ & f(n_l + n_e - n_l' - n_e'), f(n_e' - n_e))\end{aligned}$$

- quantum-number function: $f(-1) = \bar{2}$, $f(0) = 1$, and $f(1) = 2$.
- fermion-generation variables: Q, D, L, $E = 1$ (first two generations) or 0 (the 3rd one).

$$n_q + n_d - n_q' - n_d' = -Q, \qquad n_l + n_e - n_l' - n_e' = L,$$
$$n_d' - n_d = D, \qquad n_u' - n_u = 0 \qquad n_e' - n_e = -E.$$

Wilson coefficients

$$[\Lambda^{(1)}_{lq}]_{prst} \sim [\Lambda^{(3)}_{\ell q}]_{prst} = \begin{pmatrix} a_l & x_l V_l \\ x_l^* V_l^\dagger & a_l' \end{pmatrix}_{pr} \times \begin{pmatrix} a_q & x_q V_q \\ x_q^* V_q^\dagger & a_q' \end{pmatrix}_{st},$$

$$[\Lambda_{ld}]_{prst} = \begin{pmatrix} a_l & x_l V_l \\ x_l^* V_l^\dagger & a_l' \end{pmatrix}_{pr} \times \begin{pmatrix} a_d & \Delta_d^\dagger x_{dq} V_q \\ V_q^\dagger x_{dq}^\dagger \Delta_d & a_d' \end{pmatrix}_{st},$$

$$[\Lambda_{qe}]_{prst} = \begin{pmatrix} a_q & x_q V_q \\ x_q^* V_q^\dagger & a_q' \end{pmatrix}_{pr} \times \begin{pmatrix} a_e & \Delta_e^\dagger x_{el} V_l \\ V_l^\dagger x_{el}^\dagger \Delta_e & a_e' \end{pmatrix}_{st},$$

$$[\Lambda_{ed}]_{prst} = \begin{pmatrix} a_e & \Delta_e^\dagger x_{el} V_l \\ V_l^\dagger x_{el}^\dagger \Delta_e & a_e' \end{pmatrix}_{pr} \times \begin{pmatrix} a_d & \Delta_d^\dagger x_{dq} V_q \\ V_q^\dagger x_{dq}^\dagger \Delta_d & a_d' \end{pmatrix}_{st},$$

$$[\Lambda_{ledq}]_{prst} = \begin{pmatrix} x_e \Delta_e & x_l V_l \\ V_l^\dagger x_{el}^\dagger \Delta_e & x_e' \end{pmatrix}_{pr} \times \begin{pmatrix} x_d \Delta_d^\dagger & \Delta_d^\dagger x_{dq} V_q \\ x_q V_q^\dagger & x_d' \end{pmatrix}_{st},$$

$$[\Lambda^{(1)}_{lequ}]_{prst} \sim [\Lambda^{(3)}_{lequ}]_{prst} = \begin{pmatrix} x_e \Delta_e & x_l V_l \\ V_l^\dagger x_{el}^\dagger \Delta_e & x_e' \end{pmatrix}_{pr} \times \begin{pmatrix} x_u \Delta_u & x_q V_q \\ V_q^\dagger x_{uq}^\dagger \Delta_u & x_u' \end{pmatrix}_{st}.$$

Rotating to the mass basis

$\mathcal{Q}_{lq}^{(1)} = (\bar{l}_p \gamma_\mu l_r)(\bar{q}_s \gamma^\mu q_t)$:

$$\hat{\Gamma}^{ee} = \begin{pmatrix} a_l - s_e^2 k'_{l\tau}\epsilon_\tau^2 & -c_e s_e k'_{l\tau}\epsilon_\tau^2 & s_e k_{l\tau}\epsilon_\tau \\ -c_e s_e k'_{l\tau}\epsilon_\tau^2 & a_l - c_e^2 k'_{l\tau}\epsilon_\tau^2 & c_e k_{l\tau}\epsilon_\tau \\ s_e k_{l\tau}\epsilon_\tau & c_e k_{l\tau}\epsilon_\tau & a'_l + k'_{l\tau}\epsilon_\tau^2 \end{pmatrix},$$

$$\hat{\Gamma}^{dd} = \begin{pmatrix} a_q - s_d^2 k'_{qb}\epsilon_b^2 & -c_d s_d k'_{qb} K^*_{ds}\epsilon_b^2 & s_d k_{qb} K^*_{ds}\epsilon_b \\ -c_d s_d k'_{qb} K_{ds}\epsilon_b^2 & a_q - c_d^2 k'_{qb}\epsilon_b^2 & c_d k_{qb}\epsilon_b \\ s_d k_{qb} K_{ds}\epsilon_b & c_d k_{qb}\epsilon_b & a'_q + k'_{qb}\epsilon_b^2 \end{pmatrix},$$

$$\hat{\Gamma}^{\nu\nu} = \begin{pmatrix} a_l & 0 & 0 \\ 0 & a_l & \frac{x_l}{x_\tau}\epsilon_\tau \\ 0 & \frac{x_l}{x_\tau}\epsilon_\tau & a'_l \end{pmatrix},$$

$$\hat{\Gamma}^{uu} = \begin{pmatrix} a_q - \lambda^2\epsilon_t(\frac{2x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) & -\lambda\chi e^{-i(\delta-\theta)}\epsilon_t(\frac{2x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) & \lambda e^{i\theta}(\frac{x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) \\ -\lambda\chi e^{i(\delta-\theta)}\epsilon_t(\frac{2x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) & a_q - \chi^2\epsilon_t(\frac{2x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) & \chi e^{i\delta}(\frac{x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) \\ \lambda e^{-i\theta}(\frac{x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) & \chi e^{-i\delta}(\frac{x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) & a'_q + \epsilon_t(\frac{2x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) \end{pmatrix},$$

where $k_{l\tau} = a_l - a'_l + \frac{x_l}{x_\tau}$, $k'_{l\tau} = a_l - a'_l + \frac{2x_l}{x_\tau}$, $k_{qb} = a_q - a'_q + \frac{x_q}{x_b}$, $k'_{qb} = a_q - a'_q + \frac{2x_q}{x_b}$, $x_{qq} = a_q - a'_q$.

Rotating to the mass basis

$\mathcal{Q}_{ledq} = (\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$:

$$\hat{\Gamma}^{e_L e} = \begin{pmatrix} x_e\epsilon_e & 0 & -k_{\mu\tau}s_e\epsilon_\tau \\ 0 & x_e\epsilon_\mu & -k_{\mu\tau}c_e\epsilon_\tau \\ 0 & (x_e - k'_{\mu\tau})c_e\epsilon_\mu\epsilon_\tau & x'_e + (-k_{\mu\tau} + \frac{x'_e}{2})\epsilon_\tau^2 \end{pmatrix},$$

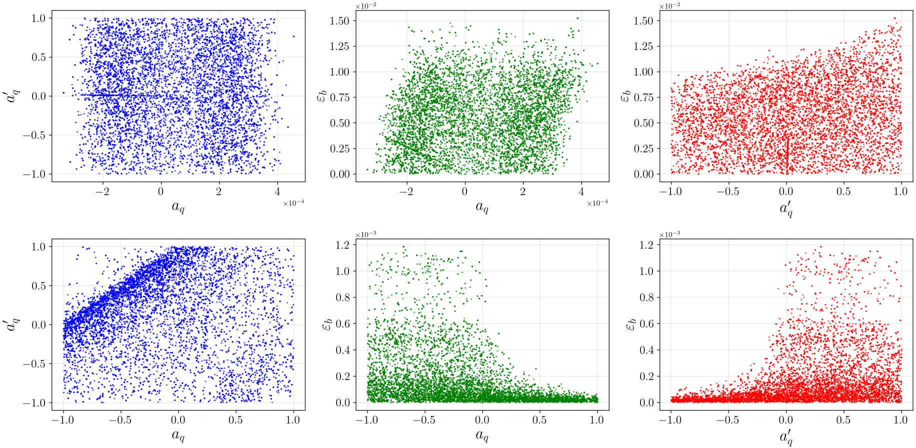
$$(\hat{\Gamma}^{dd_L})^\dagger = \begin{pmatrix} x_d\epsilon_d & 0 & -k_{sb}K^*_{ds}c_d\epsilon_b \\ 0 & x_d\epsilon_s & -k_{sb}c_d\epsilon_b \\ 0 & (x_d - k'_{sb})c_d\epsilon_s\epsilon_b & x'_d + (-k_{sb} + \frac{x'_d}{2})\epsilon_b^2 \end{pmatrix},$$

$$\hat{\Gamma}^{\nu e} = \begin{pmatrix} x_e c_e\epsilon_e & -x_e s_e\epsilon_\mu & 0 \\ x_e s_e\epsilon_e & x_e c_e\epsilon_\mu & \frac{x_l}{x_\tau}\epsilon_\tau \\ 0 & -k'_{\mu\tau}c_e\epsilon_\mu\epsilon_\tau & x'_e \end{pmatrix},$$

$$(\hat{\Gamma}^{du_L})^\dagger = \begin{pmatrix} x_d c_u\epsilon_d & x_d s_u\epsilon_s & c_d(s_u + c_u K^*_{ds})(\frac{x_q}{x_b}\epsilon_b - x'_d\epsilon_t) \\ -x_d s_u\epsilon_d & x_d c_u\epsilon_s & c_d(c_u - s_u K^*_{ds})(\frac{x_q}{x_b}\epsilon_b - x'_d\epsilon_t) \\ 0 & -c_d k'_{sb}\epsilon_b\epsilon_s + x_d c_d\epsilon_t\epsilon_s & x'_d + \frac{x_q}{x_b}\epsilon_b\epsilon_s - \frac{x'_d}{2}\epsilon_t^2 \end{pmatrix},$$

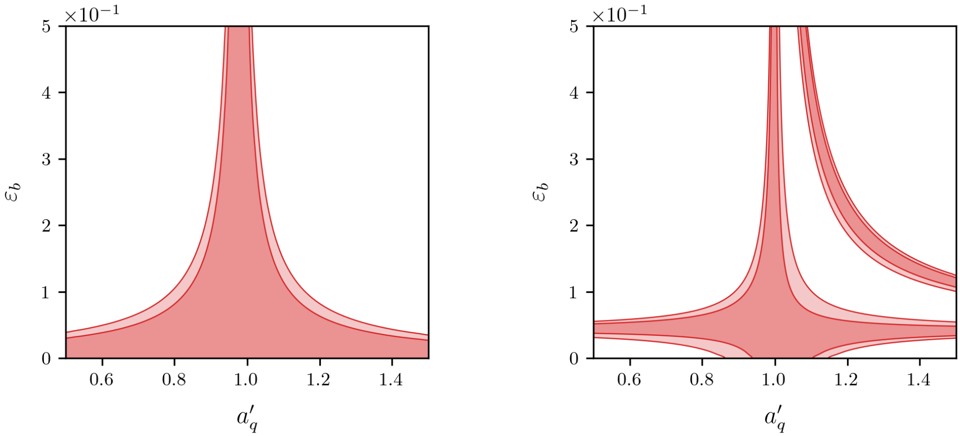
where $K_{ds} = V_{td}/V_{ts}$, $k_{\mu\tau} = x'_e - \frac{x_l}{x_\tau}$, $k_{sb} = x'_d - \frac{x_q}{x_b}$, $k'_{\mu\tau} = x'_e - \frac{x_{el}}{x_\tau}$, $k'_{sb} = x'_d - \frac{x_{dq}}{x_b}$.

Numerical discussions



Parameters of $C_{lq}^{(1)}$ constrained by the leptonic processes of $b \to d\ell^+\ell^-$ (upper) and the $b \to s\ell^+\ell^-$ (lower).

Numerical discussions



Parameters of C_{ledq} constrained by the leptonic processes of $b \to s\tau^+\tau^-$ process (left) and $b \to u\tau\nu$ (right).