

Relativistic electromagnetic structure of a moving nucleon

Yi Chen

Based on:
[PRD 106, 116024 (2022)]
[PRD 107, 096003 (2023)]
[arXiv: 26xx.xxxx (to appear)]

thanks to:

Cédric Lorc é, Qun Wang, Yang Li, Guang-Peng Zhang, ...

第十届“海峡两岸粒子物理和宇宙学研讨会” (Cross-Strait 2026)

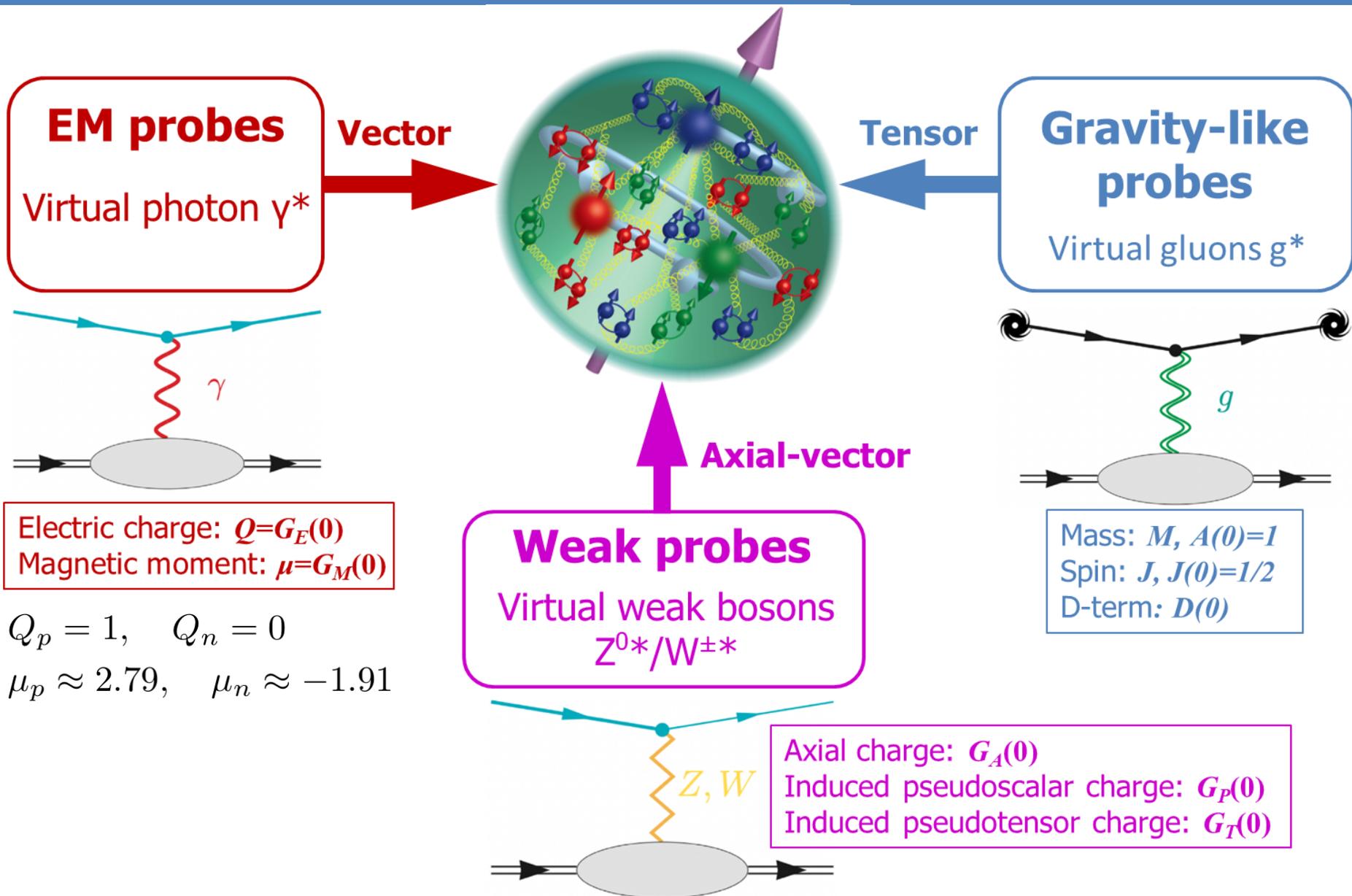
Outline

1. Introduction and motivations
2. Quantum phase-space approach and relativistic spatial distributions
3. Relativistic electromagnetic structure of a moving spin-1/2 hadron
4. Summary and outlook

致谢：

特别感谢“第十届海峡两岸粒子物理和宇宙学研讨会”组委会！
谢谢您给了我一个交流和展示近期研究工作的机会！

Probing the internal structures of a hadron



Structure dictates properties

■ Analogy:

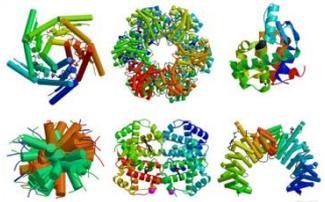
“Symmetry dictates interactions”

对称性决定动力学

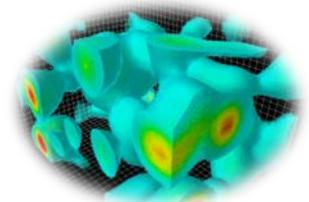
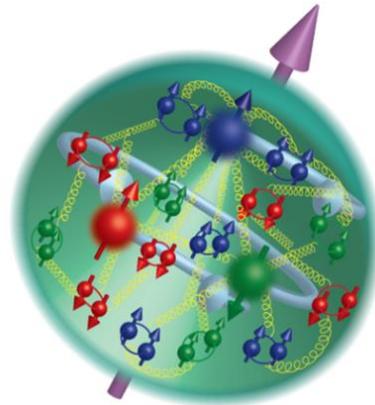


“Structure dictates properties”

结构决定性质



different proteins



QCD vacuum

■ Hadron structures are highly non-trivial and complicated!

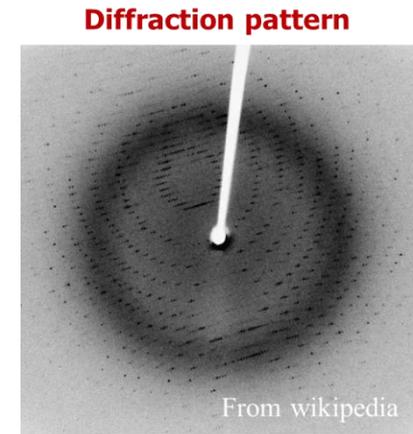
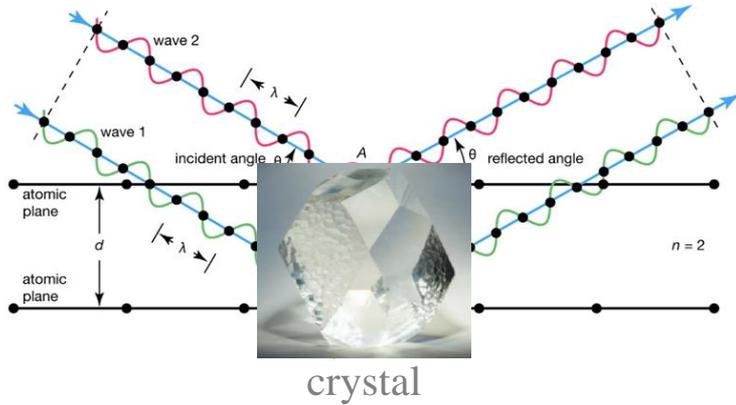
(1). Hadron structures are closely associated with the nonperturbative QCD dynamics between the internal quark and gluon degrees of freedom.

(2). The QCD vacuum itself is also highly non-trivial, due to quantum fluctuations (loop effects, pair creations and annihilations), non-trivial topologies, instanton/sphaleron transitions, and etc.

(3). On top of the QCD dynamics and non-trivial QCD vacuum structure, hadron structures are also affected by electromagnetic and weak interactions.

How to probe the internal structure of a system?

◆ Classically, e.g. using the x-ray diffraction for crystals

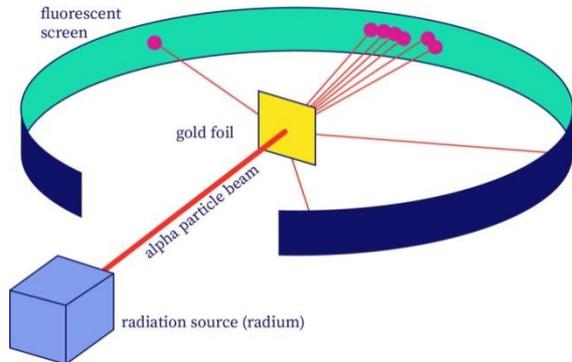


$$\propto |\mathcal{A}_{\text{scatt}}|^2$$

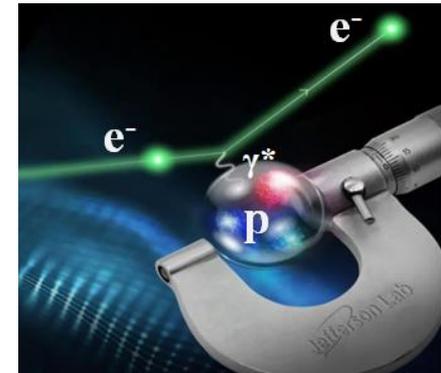
Scattering amplitude: $\mathcal{A}_{\text{scatt}} \propto F(\mathbf{q}) = \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} \rho(\mathbf{r})$ $\mathbf{q} = \mathbf{k} - \mathbf{k}'$

Form factor
Target distribution

◆ Experimentally, e.g. using modern Rutherford scattering



Rutherford alpha scattering experiment (1909)



Jefferson lab (JLab) e-p elastic scattering exps. 5

Comparison of elastic scattering at different scales

- Crystals & atoms:

$$d \approx 10^{-10} \text{ m} \quad \rightarrow \quad \hbar\omega \approx 10^4 \text{ eV} \quad \rightarrow$$

Method (probe)

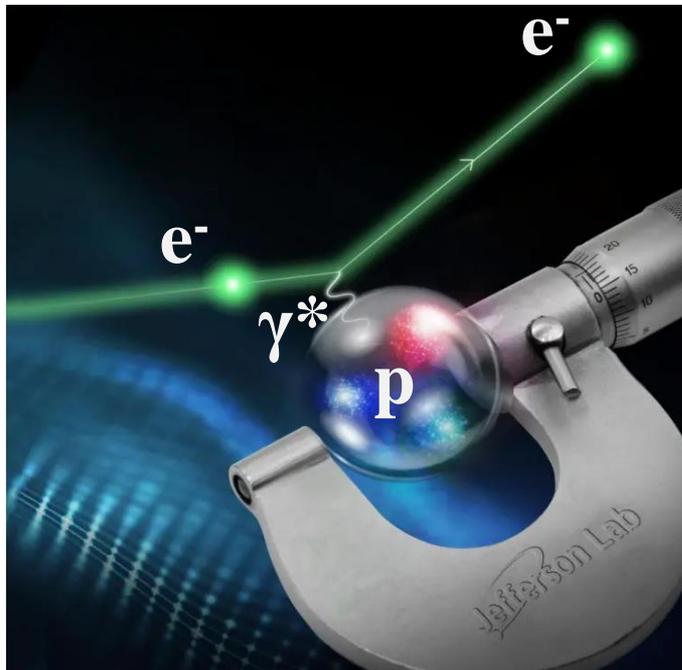
x-ray

- Nuclei & nucleon:

$$d \approx 10^{-15} \text{ m} \quad \rightarrow \quad \hbar\omega \approx 10^9 \text{ eV} \quad \rightarrow$$

**high-energy
electron scattering**

→ **Larger recoil for lighter targets** (e.g., nucleon and pion). 



- **Elastic electron-hadron scattering:**

$$e^-(k, r) + h(p, s) \rightarrow e^-(k', r') + h(p', s')$$

$h \sim$ **hadron**

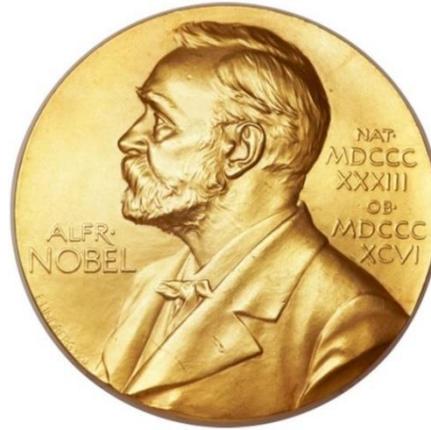
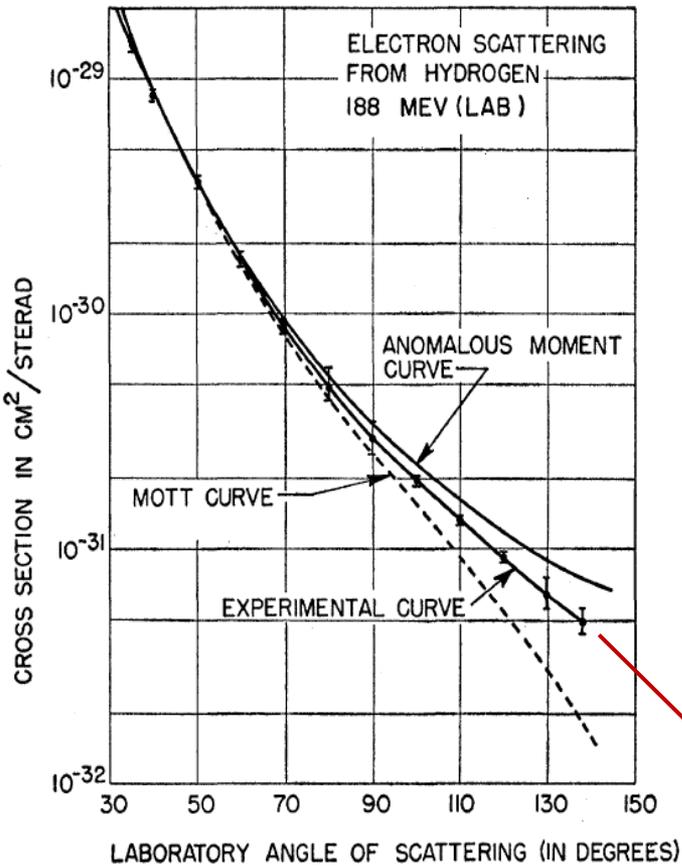
Spin-0: e.g., pion, kaon, ...

Spin-1/2: e.g., nucleon, ...

Spin-1: e.g., deuteron, ρ meson, ...

Spin-3/2: e.g., Δ baryon, ...

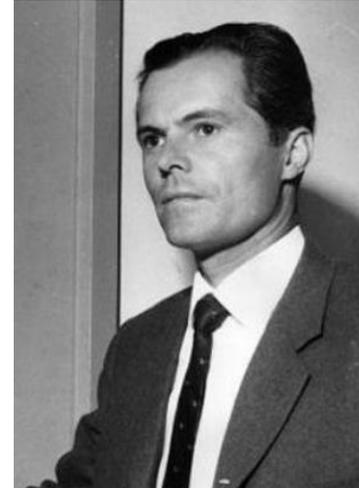
Electron-nucleon elastic scattering



Nobel prize in physics (1961)



Robert Hofstadter



Rudolf Mössbauer

➔ **Proton is not a point-like particle!**

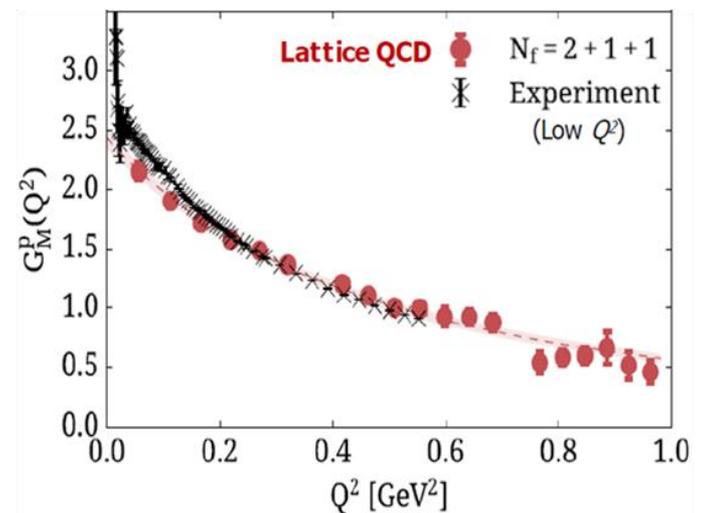
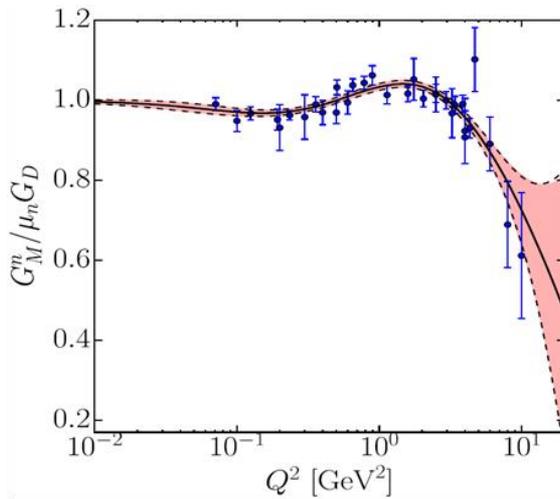
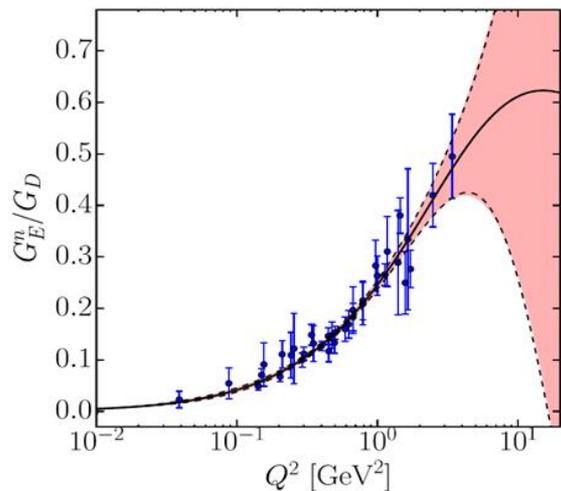
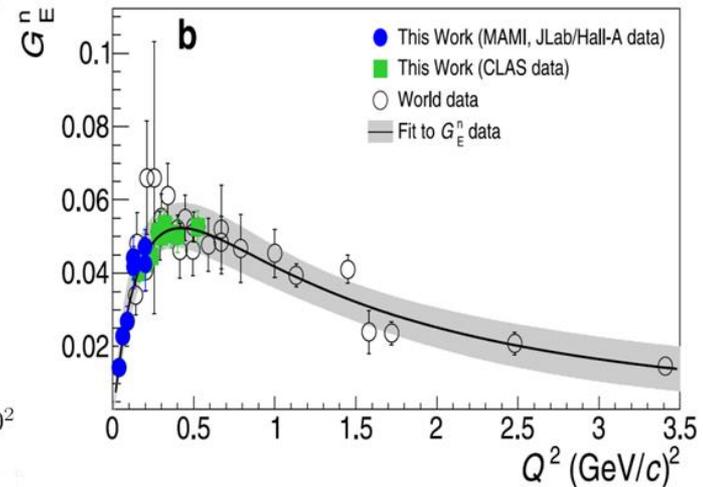
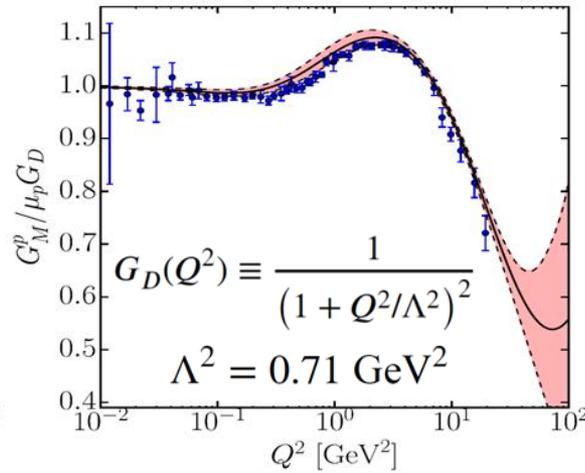
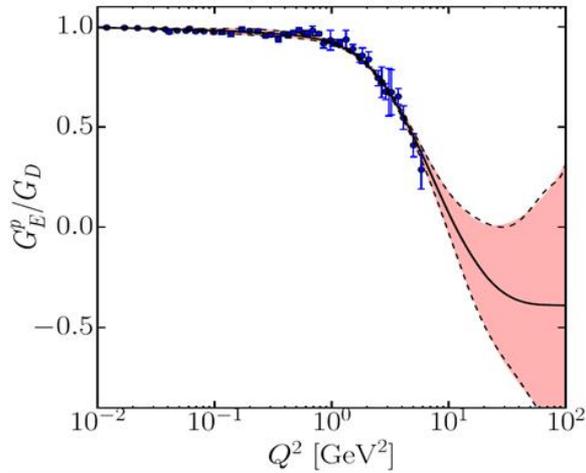
$r_E^p \approx 0.78$ fm (**Sachs electric charge radius**)

$$r_E = \sqrt{\langle r_E^2 \rangle} \quad \langle r_E^2 \rangle \equiv -\frac{6}{G_E(0)} \left. \frac{dG_E(Q^2)}{dQ^2} \right|_{Q^2=0}$$

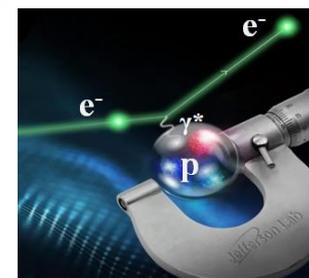
[R. Hofstadter and R. W. McCallister. "Electron Scattering from the Proton", PR. 98, 217 (1955)]
 [Ernst, Sachs, and Wali, PR 119, 1105 (1960); Sachs, PR 126, 2256 (1962)]

[W. Xiong et al. (PRad), Nature (London) 575, 147 (2019)] $r_E^p \approx (0.831 \pm 0.014)$ fm

Nucleon electromagnetic form factors (FFs)



- [Z. Ye et al., PLB777, 8 (2018)]
- [Bradford et al., Nucl. Phys. B, Proc. Suppl. 159, 127 (2006)]
- [Atac et al., Nat. Commun. (2021) 12, 1759]
- [A1 Collaboration, PRL 105, 242001 (2010)]
- [Alexandrou et al., PRD 100, 014509 (2019)]
- [W. Xiong et al. (PRad), Nature 575, 147 (2019)] ...



→ $J^\mu(\mathbf{r}) = ?$

World-wide experimental facilities

 **Jefferson Lab**


EicC

 **Brookhaven
National Laboratory**
EIC


@PSI


Mainz Microtron MAMI
(PRES)

 **DARK
MESA** ...
MAGIX @ MESA, Mainz


@BEPC


(e.g. NICA-SPD)


(Super-Kamiokande, T2K)


@ LHC near ATLAS


HIAF @ Hui Zhou


**超级陶粲装置
Super Tau-Charm Facility**

 **Fermilab**
(MiniBooNE, MINERvA, MicroBooNE, NOvA, ICARUS, ANNIE)



...

Different methods/formalisms for density interpretation

- **Traditional/tilted light-front formalism:**

[M. Burkardt, Int. J. Mod. Phys. A 18, 173 (2003)]...

[M. Burkardt, PRD 62 (2000) 071503(R) [erratum: PRD 66 (2002) 119903(E)]...

[G. Miller, PRL 99 (2007) 112001; PRC 79 (2009) 055204; Rev. Nucl. Part. Sci. 60, 1 (2010); PRC 99 (2019) 035202]...

[Carlson & Vanderhaeghen, PRL 100 (2008) 032004]

[Y. Guo, X. Ji and K. Shiells, NPB 969 (2021) 115440]

[A. Freese and G. Miller, PRD 105 (2022) 014003; PRD 108 (2023) 034008; PRD 107 (2023) 074036; PRD 108 (2023) 094026]...

- **Dimensional counting+ZAMF (zero average momentum frame):**

[G. N. Fleming, Phys. Reality Math. Descrip. 357 (1974)]

[Epelbaum, Gegelia, Lange, Meißner & Polyakov, PRL 129 (2022) 012001]

[Panteleeva, Epelbaum, Gegelia & Meißner, PRD 106 (2022) 056019; EPJC 83 (2023) 617; JHEP 07 (2023) 237; arXiv: 2412.05050 [hep-ph]]

[C. E. Carlson, arXiv:2208.00826 [hep-ph]]

[Alharazin, Sun, Epelbaum, Gegelia & Meißner, JHEP 02, 163 (2023)]...

- **Quantum phase-space approach/Wigner distribution:**

[Belitsky, Ji & Yuan, PRD 69 (2004) 074014]

[Cédric Lorcé, PRL 125 (2020) 232002; PRD105 (2022) 096032]

[J.-Y. Kim & Hy.-Ch. Kim, PRD104 (2021) 074003;]

[YC, Cédric Lorcé, PRD106 (2022) 116024; PRD107 (2023) 096003]

[YC, Yang Li, Cédric Lorcé, Qun Wang, PRD 110, L091503 (2024); JHEP 04(2025)232]...

- **Traditional/covariant moments expansion:**

[X. Ji and Y. Liu, PRD 106 (2022) 034028]

[Yang Li, Wen-bo Dong, Yi-liang Yin, Qun Wang & James Vary, PLB 838 (2023) 137676; arXiv: 2405.06892 [hep-ph]]...

- **Light-front Wigner distribution:** [Y. Han, T. Liu & B. Ma, PLB 830 (2022) 137127]...

Quantum phase-space approach

- **Phase-space representation:**

$$P = (p' + p)/2 \quad R = (r + r')/2$$

$$\langle \Psi | \hat{O}(x) | \Psi \rangle = \sum_{s',s} \int \frac{d^3 P}{(2\pi)^3} d^3 R \rho_{\Psi}^{s's}(\mathbf{R}, \mathbf{P}) \langle \hat{O} \rangle_{\mathbf{R}, \mathbf{P}}^{s's}(x)$$

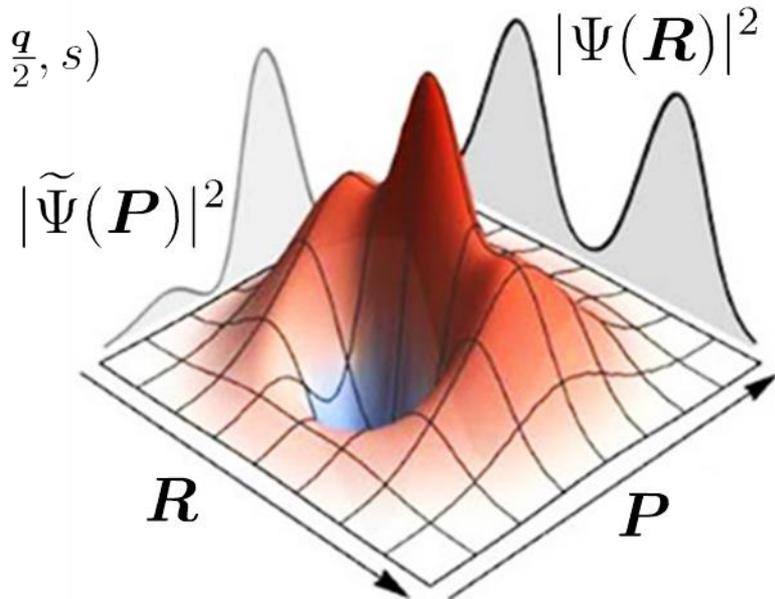
$$\tilde{\Psi}(p, s) \equiv \frac{\langle p, s | \Psi \rangle}{\sqrt{2p^0}}$$

- **Wigner distribution:**

$$\begin{aligned} \rho_{\Psi}^{s's}(\mathbf{R}, \mathbf{P}) &\equiv \int d^3 z e^{-i\mathbf{P}\cdot\mathbf{z}} \Psi^*(\mathbf{R} - \frac{\mathbf{z}}{2}, s') \Psi(\mathbf{R} + \frac{\mathbf{z}}{2}, s) \\ &= \int \frac{d^3 q}{(2\pi)^3} e^{-i\mathbf{q}\cdot\mathbf{R}} \tilde{\Psi}^*(\mathbf{P} + \frac{\mathbf{q}}{2}, s') \tilde{\Psi}(\mathbf{P} - \frac{\mathbf{q}}{2}, s) \end{aligned}$$

- **Quasi-probabilistic densities:**

$$\begin{aligned} \int d^3 R \rho_{\Psi}^{s's}(\mathbf{R}, \mathbf{P}) &= \tilde{\Psi}^*(\mathbf{P}, s') \tilde{\Psi}(\mathbf{P}, s), \\ \int \frac{d^3 P}{(2\pi)^3} \rho_{\Psi}^{s's}(\mathbf{R}, \mathbf{P}) &= \Psi^*(\mathbf{R}, s') \Psi(\mathbf{R}, s). \end{aligned}$$



[Wigner, PR40 (1932) 749]

[Hillery, O'Connell, Scully, Wigner, PR106 (1984) 121]

[Bialynicki-Birula, Gornicki, Rafelski, PRD 44 (1991) 1825]

[Belitsky, Ji, and Yuan, Phys. Rev. D 69, 074014 (2004)]

[Cédric Lorcé, PRL 125 (2020) 232002]

[YC & Cédric Lorcé, PRD 107, 096003 (2023)]

Relativistic spatial distributions

- **Internal distribution** (for a state « localized » in phase-space) $\mathbf{r} = \mathbf{x} - \mathbf{R} = (\mathbf{b}_\perp, r_z)$

$$\langle \hat{O} \rangle_{\mathbf{R}, \mathbf{P}}^{s' s}(t, \mathbf{r}) = \int \frac{d^3 \Delta}{(2\pi)^3} e^{i\Delta^0 t - i\Delta \cdot \mathbf{r}} \frac{\langle P + \frac{\Delta}{2}, s' | \hat{O}(0) | P - \frac{\Delta}{2}, s \rangle}{2\sqrt{p'^0 p^0}}$$

- **Elastic condition:**

$$\Delta^0 = p'^0 - p^0 = \frac{\Delta \cdot P}{P^0} = 0 \quad \rightarrow \quad \text{No energy transfer} \quad \rightarrow \quad \text{Time independence}$$

$$\Delta = p' - p \quad P = (p' + p)/2$$

[Lorcé, Mantovani, Pasquini, PLB776 (2018) 38]

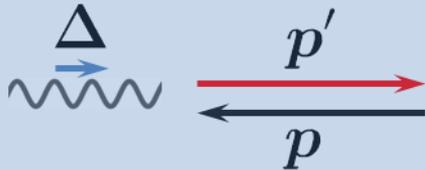
[Lorcé, EPJC78 (2018) 9, 785]

[Lorcé, Moutarde, Trawinski, EPJC79 (2019) 89]

[YC & Cédric Lorcé, PRD 107, 096003 (2023)]

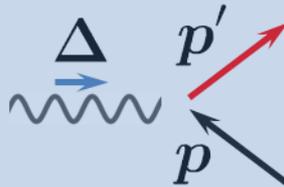
- **Three cases:**

$$P = 0$$



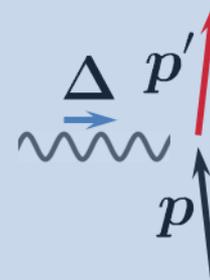
BF
(Breit frame)

$$P \text{ finite}$$



EF
(Elastic frame)

$$|P| \rightarrow \infty$$



IMF (Infinite momentum frame)

\approx

LF
(Light-front)

Relativistic BF, EF & LF distributions

- **Breit frame (BF) distributions:**

$$O_B(\mathbf{r}) \equiv \langle \hat{O} \rangle_{\mathbf{0},0}^{s'_B s_B}(\mathbf{r}) = \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\Delta \cdot \mathbf{r}} \frac{\langle p'_B, s'_B | \hat{O}(0) | p_B, s_B \rangle}{2P_B^0} \Big|_{P_B=0}$$

- **Elastic frame (EF) distributions:**

$$O_{\text{EF}}(\mathbf{b}_\perp; P_z) \equiv \int dr_z \langle \hat{O} \rangle_{\mathbf{0},P}^{s'_s}(\mathbf{r})$$

$$= \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{\langle p', s' | \hat{O}(0) | p, s \rangle}{2P^0} \Big|_{\Delta_z=|P_\perp|=0}$$

The standard definition frame for different 3D mean-square radii.

$$O_{\text{EF}}(\mathbf{b}_\perp; 0) = \int dr_z O_B(\mathbf{r}) \quad O_{\text{IMF}}(\mathbf{b}_\perp) = \lim_{P_z \rightarrow \infty} O_{\text{EF}}(\mathbf{b}_\perp; P_z)$$

- **Light-front (LF) distributions:**

$$O_{\text{LF}}(\mathbf{b}_\perp; P^+) \equiv \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{\text{LF} \langle p', \lambda' | \hat{O}(0) | p, \lambda \rangle_{\text{LF}}}{2P^+} \Big|_{\Delta^+=|P_\perp|=0}$$

$$a^\pm = (a^0 \pm a^3)/\sqrt{2}, \quad a^\mu = [a^+, a^-, \mathbf{a}_\perp]$$

Matrix elements of a general spin-1/2 hadron

◆ Hadronic matrix elements:

$$\langle p', s' | \hat{j}^\mu(0) | p, s \rangle = e \bar{u}(p', s') \Gamma^\mu(p, p') u(p, s)$$

Vertex function: $\Gamma^\mu(p', p) = \underbrace{\gamma^\mu F_1(Q^2)}_{\text{Dirac}} + \frac{i\sigma^{\mu\nu}\Delta_\nu}{2M} \underbrace{F_2(Q^2)}_{\text{Pauli}}$

$$\begin{aligned} \Delta &= p' - p \\ P &= (p' + p)/2 \\ &= \frac{MP^\mu}{P^2} \underbrace{G_E(Q^2)}_{\text{Electric}} + \frac{i\epsilon^{\mu\alpha\beta\lambda}\Delta_\alpha P_\beta \gamma_\lambda \gamma^5}{2P^2} \underbrace{G_M(Q^2)}_{\text{Magnetic}} \end{aligned}$$

(1). Electric charge:

$$q_e = F_1(0) = G_E(0)$$

(2). Magnetic moment:

$$\mu = G_M(0) \quad \kappa = F_2(0)$$

$$= q_e + \kappa$$

◆ Sachs electromagnetic FFs:

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2) \quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

◆ Classical electromagnetic four-current:

$$J^\mu = \underbrace{J_C^\mu}_{\text{Electric}} + \underbrace{J_P^\mu}_{\text{Magnetic}}, \quad J_P^\mu = \partial_\alpha P^{\alpha\mu}$$

$$\begin{aligned} J^0 &= \rho_c + \rho_P \\ \mathbf{J} &= \rho_c \mathbf{v} + \nabla \times \mathbf{M} + \partial_0 \mathcal{P} \\ \rho_P &= -\nabla \cdot \mathcal{P} \\ \rho_M &= -\nabla \cdot \mathbf{M} \end{aligned}$$

[Cédric Lorcé, PRL125 (2020) 232002]

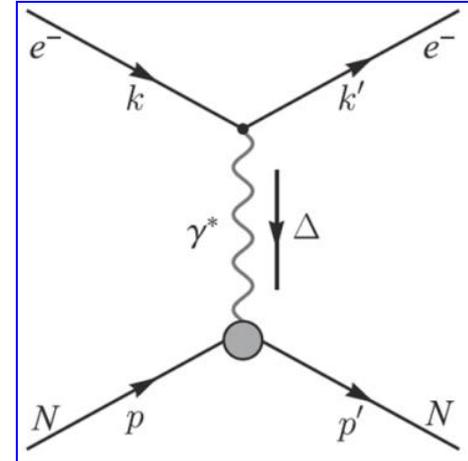
[Yang Li, Wen-bo Dong, Yi-liang Yin, Qun Wang & James Vary, PLB 838 (2023) 137676]

[YC & Cédric Lorcé, PRD 106, 116024 (2022); PRD 107, 096003 (2023)]

Elastic electron-hadron scattering

◆ **A typical reaction of elastic electron-hadron scattering:**

$$e^-(k, r) + h(p, s) \rightarrow e^-(k', r') + h(p', s')$$



Four-momentum transfer: $\Delta = p' - p = q = (\Delta^0, \mathbf{\Delta})$

$$Q^2 = -q^2 = -\Delta^2 \geq 0$$

◆ **Spin-0 hadron:**

$$\left(\frac{d\sigma}{d\Omega}\right) / \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = F^2(Q^2)$$

Breit frame



$$\Delta^0 = 0$$

$$Q^2 = \mathbf{q}^2$$

$$F(\mathbf{q}^2) = \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} \rho_e(\mathbf{r})$$

$$\tilde{\epsilon} \rightarrow \epsilon = [1 + 2(1 + \tau) \tan^2 \frac{\theta}{2}]^{-1}$$

◆ **Spin-1/2 hadron:**

$$\left(\frac{d\sigma}{d\Omega}\right) / \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{1}{1 + \tau} \left[G_E^2(Q^2) + \frac{\tau}{\tilde{\epsilon}} G_M^2(Q^2) \right]$$

$$\tau \equiv \frac{Q^2}{4M^2}$$

Breit frame



$$\Delta^0 = 0$$

$$Q^2 = \mathbf{q}^2$$

$$\frac{G_E(\mathbf{q}^2)}{\sqrt{1 + \tau}} = \int d^3r e^{i\mathbf{q}\cdot\mathbf{r}} \rho_e(\mathbf{r})$$

A natural expectation

[Rosenbluth, PR79 (1950) 615]

[Hofstadter, RMP28 (1956) 214]

[Yennie, Lévy, Ravenhall, RMP 29 (1957) 144]

[Gao, Vanderhaeghen, RMP 94 (2022) 015002]

Relativistic Breit frame (BF) interpretation

◆ BF charge and current density:

$$J_B^0(\mathbf{r}) = e\delta_{s'_B s_B} \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta\cdot\mathbf{r}} \frac{G_E(\Delta^2)}{\sqrt{1+\tau}}$$

$$\mathbf{J}_B(\mathbf{r}) = \nabla \times \frac{e(\boldsymbol{\sigma})_{s'_B s_B}}{2M} \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta\cdot\mathbf{r}} \frac{G_M(\Delta^2)}{\sqrt{1+\tau}}$$

recoil factor $\frac{1}{\sqrt{1+\tau}} \approx 1 - \frac{\tau}{2} + \dots$

$$J^0 = \rho_c + \rho_P$$

$$\mathbf{J} = \rho_c \mathbf{v} + \nabla \times \mathbf{M} + \partial_0 \mathbf{P}$$

In the differential cross section, the **same factor** $1/\sqrt{1+\tau}$ is naturally there!

$$\left(\frac{d\sigma}{d\Omega}\right) / \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} = \frac{1}{1+\tau} \left[G_E^2(Q^2) + \frac{\tau}{\tilde{\epsilon}} G_M^2(Q^2) \right]$$

Electric **Magnetic**

$$= \bar{G}_E^2(Q^2) + \frac{\tau}{\tilde{\epsilon}} \bar{G}_M^2(Q^2)$$

$$G_{E,M} \rightarrow \bar{G}_{E,M} = G_{E,M}/\sqrt{1+\tau}$$

◆ 3D relativistic mean-square charge radius:

$$\langle r_{\text{ch}}^2 \rangle = \frac{\int d^3r r^2 J_B^0(\mathbf{r})}{\int d^3r J_B^0(\mathbf{r})} = \langle r_E^2 \rangle + \frac{3}{4M^2} \frac{3\bar{\lambda}^2}{4}$$

$$= -\frac{6}{\bar{G}_E(0)} \frac{d\bar{G}_E(Q^2)}{dQ^2} \Big|_{Q^2=0}$$

$$\langle r_E^2 \rangle \equiv \frac{6}{G_E(0)} \frac{dG_E(Q^2)}{dQ^2} \Big|_{Q=0}$$

traditional/standard
mean-square charge radius

Darwin-Foldy term (relativistic correction)

[Foldy and Wouthuysen, PR 78, 29 (1950)]

[Foldy, PR 87, 688 (1952)]

[Friar, et al., PRA 56, 4579 (1997)]

[YC & Cédric Lorcé, PRD 107, 096003 (2023)]

[YC, Y. Li, C. Lorcé, & Q. Wang, PRD 110, L091503 (2024)]

Relativistic Breit frame (BF) interpretation

- ◆ The relativistic recoil factor comes from relativistic normalization of Dirac spinors. It essentially ensures that the total charge is a Lorentz scalar!

$$\int d^3r \langle \hat{j}^0 \rangle_{R,P}(\mathbf{r}) = \frac{\langle P, s | \hat{j}^0(0) | P, s \rangle}{2P^0} = eG_E(0)$$

[Foldy and Wouthuysen, PR 78, 29 (1950)]

[Foldy, PR 87, 688 (1952)]

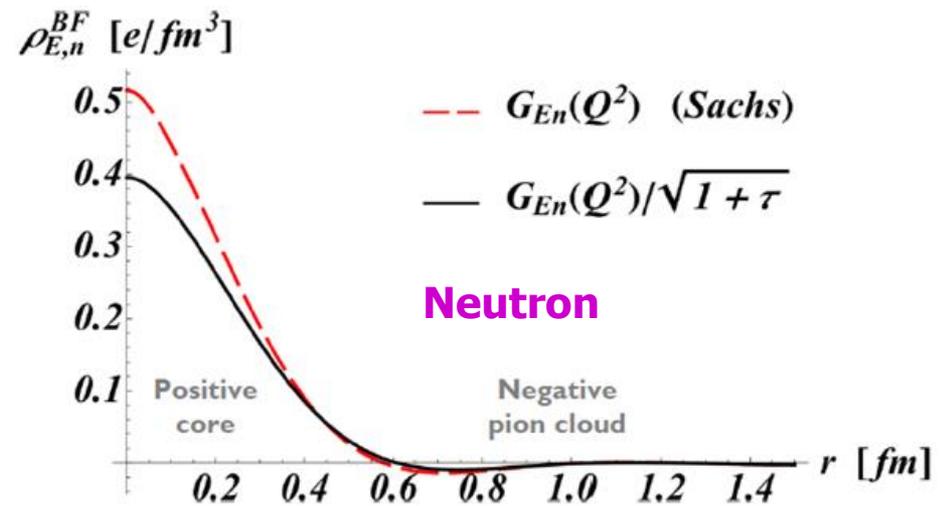
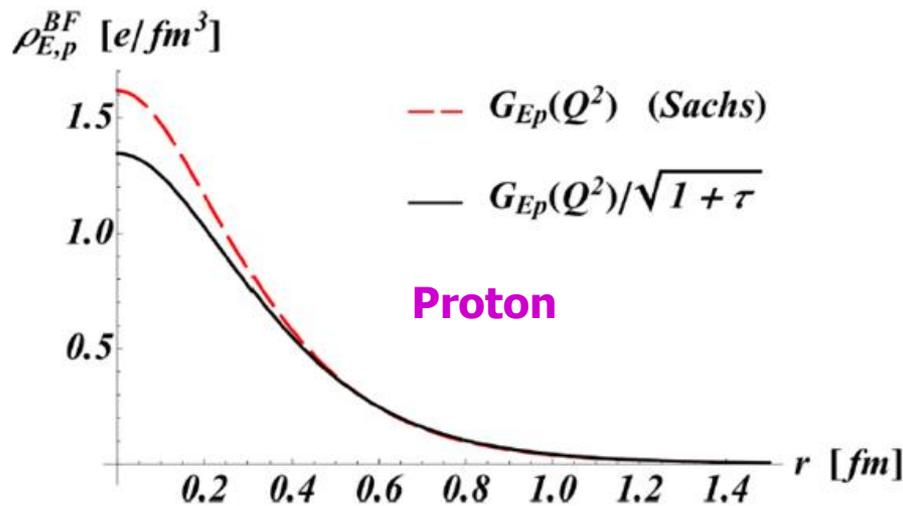
[Friar, et al., PRA 56, 4579 (1997)]

- ◆ Taylor expansion of $1/\sqrt{1+\tau}$:

$$\frac{1}{\sqrt{1+\tau}} \approx 1 - \frac{\tau}{2} + \frac{3\tau^2}{8} + \dots$$

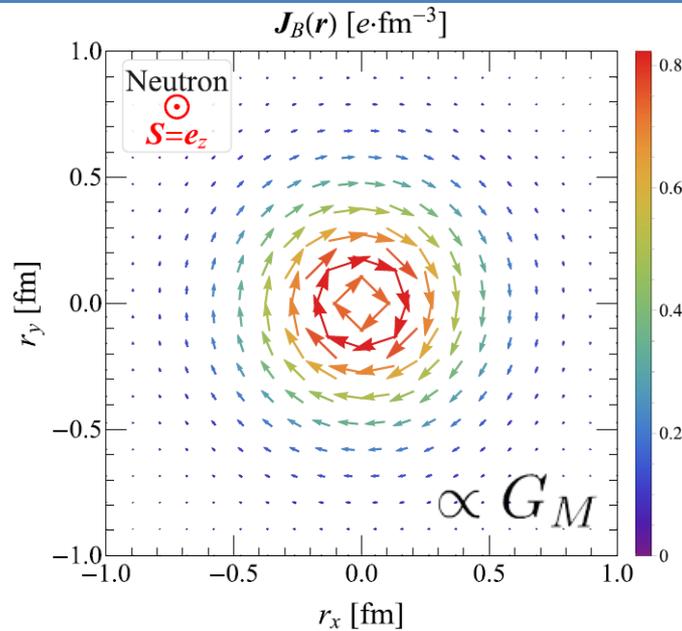
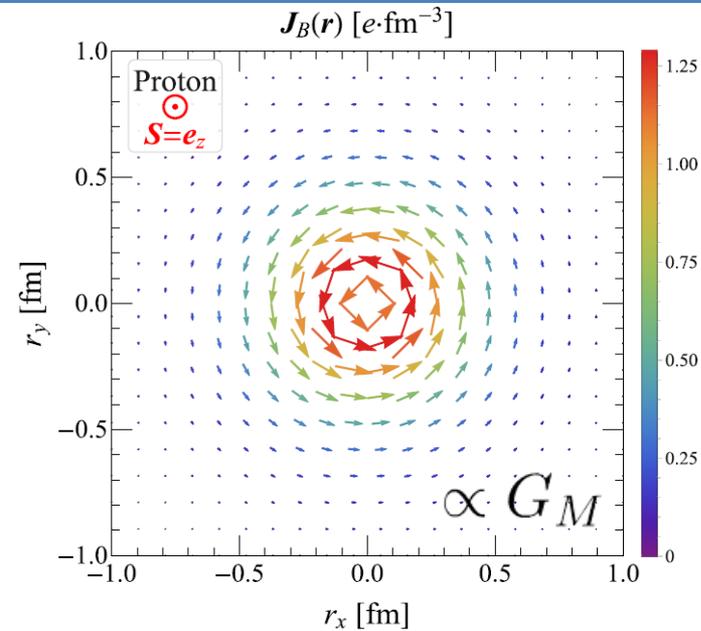
$$-\frac{\tau}{2} = -\frac{Q^2}{8M^2} \rightarrow \text{Darwin-Foldy term}$$

- ◆ Breit frame charge density of the nucleon:



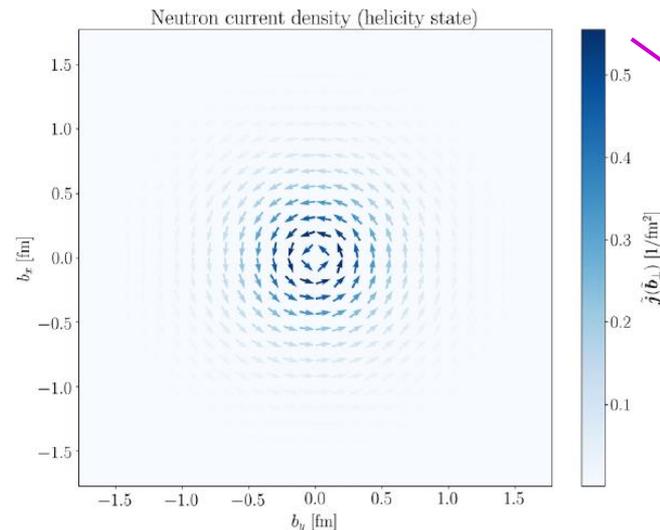
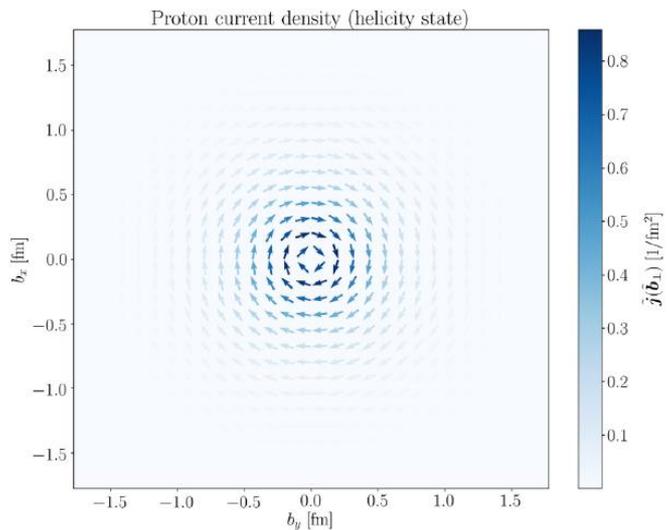
- [Cédric Lorcé, PRL 125, 232002 (2020)]
- [YC & Cédric Lorcé, PRD 106, 116024 (2022)]
- [YC & Cédric Lorcé, PRD 107, 096003 (2023)]

Relativistic Breit frame (BF) interpretation



$$\mu_p = G_M^p(0) \approx 2.79$$

$$\mu_n = G_M^n(0) \approx -1.91$$

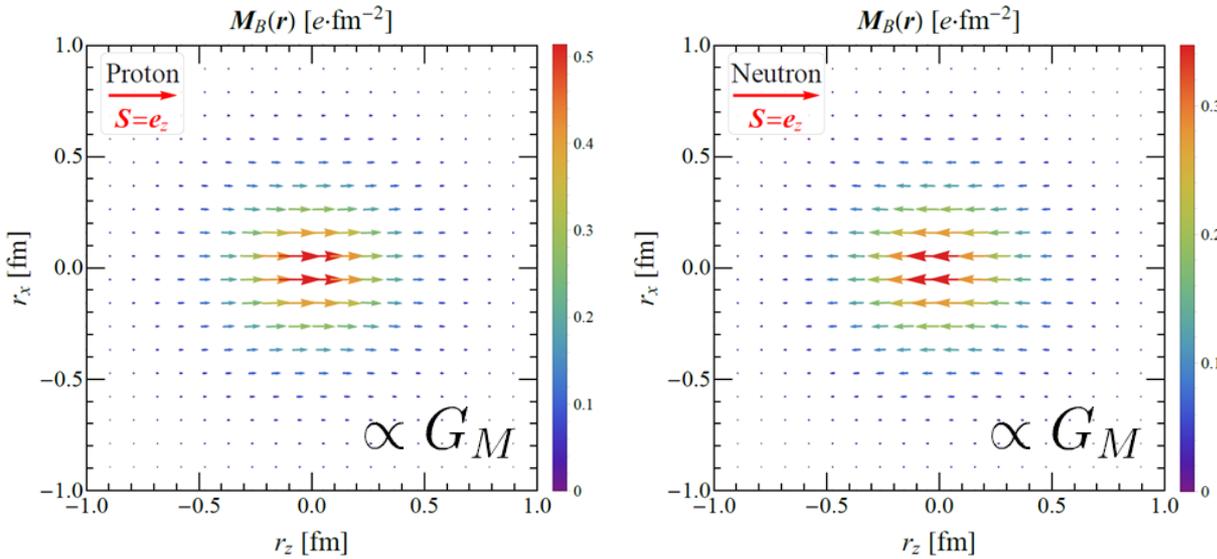


Our results were later on confirmed by Freese and Miller via the tilted light-front formalism.

Relativistic Breit frame (BF) interpretation

◆ Breit frame magnetization density:

$$\mathbf{M}_B(\mathbf{r}) = \frac{e}{2M} \int \frac{d^3\Delta}{(2\pi)^3} e^{-i\Delta \cdot \mathbf{r}} \left[\boldsymbol{\sigma} - \frac{\Delta(\Delta \cdot \boldsymbol{\sigma})}{4P_B^0(P_B^0 + M)} \right] \frac{M}{P_B^0} G_M(\Delta^2)$$



Polarization-magnetization tensor

$$P^{\mu\nu} = \begin{pmatrix} 0 & \mathcal{P}_x & \mathcal{P}_y & \mathcal{P}_z \\ -\mathcal{P}_x & 0 & -M_z & M_y \\ -\mathcal{P}_y & M_z & 0 & -M_x \\ -\mathcal{P}_z & -M_y & M_x & 0 \end{pmatrix}$$



\mathcal{P}, M

$$\langle r_M^2 \rangle \equiv - \frac{6}{G_M(0)} \left. \frac{dG_M(Q^2)}{dQ^2} \right|_{Q=0}$$

ρ MAGNETIC RADIUS

This is the rms magnetic radius, $\sqrt{\langle r_M^2 \rangle}$.

VALUE (fm)	DOCUMENT ID	TECN
0.851 ± 0.026	¹ LEE	15

◆ Relativistic mean-square magnetization radius:

$$\langle r_{\text{magn}}^2 \rangle = \frac{\int d^3r r^2 \hat{\mathbf{s}} \cdot \mathbf{M}_B(\mathbf{r})}{\int d^3r \hat{\mathbf{s}} \cdot \mathbf{M}_B(\mathbf{r})} = \langle r_M^2 \rangle + \frac{3}{2M^2}$$



by analogy with "mean-square spin radius"

[YC & Cédric Lorcé, PRD 107, 096003 (2023)]

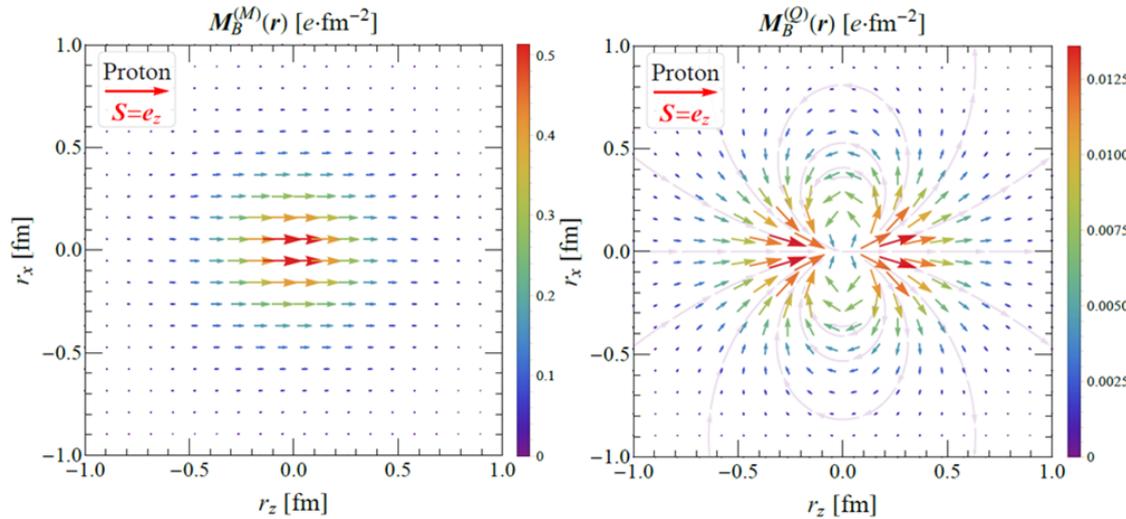
[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024); JHEP04, 132 (2025)]

Relativistic Breit frame interpretation

◆ **Effective magnetic charge distribution:** $\rho_M \equiv -\nabla \cdot \mathbf{M}$

$$\rho_{M,B}(\mathbf{r}) = \frac{e}{2M} \int \frac{d^3\Delta}{(2\pi)^2} e^{-i\Delta \cdot \mathbf{r}} \frac{\underbrace{(i\Delta \cdot \boldsymbol{\sigma})}_{\text{P-odd}} G_M(\Delta^2)}{1 + \tau}$$

→ No 3D mean-square effective magnetic charge radius



Multipole decomposition:

$$\mathbf{M}_B(\mathbf{r}) = \mathbf{M}_B^{(M)}(\mathbf{r}) + \mathbf{M}_B^{(Q)}(\mathbf{r})$$

NB: BF polarization density violates P and CP symmetries.

$$\mathcal{P}_B(\mathbf{r}) = \mathbf{0} \quad \longrightarrow \quad \rho_{P,B}(\mathbf{r}) = -\nabla \cdot \mathcal{P}_B = 0 \quad \longrightarrow \quad \text{well consistent with expectation}$$

◆ **Electric dipole moment (EDM):**

$$d_B = \int d^3r r J_B^0(\mathbf{r}) = \int d^3r r \rho_{P,B}(\mathbf{r}) = \int d^3r r \mathcal{P}_B(\mathbf{r}) = \mathbf{0}.$$

◆ **Magnetic dipole moment (MDM):**

$$\boldsymbol{\mu}_B = \int d^3r \mathbf{r} \times \mathbf{J}_B(\mathbf{r}) = \int d^3r r \rho_{M,B}(\mathbf{r}) = \int d^3r \frac{\mathbf{r} \times \mathbf{J}_B(\mathbf{r})}{2} = \sigma G_M(0) \frac{e}{2M}$$

Relativistic Light-front (LF) interpretation

PRL 99, 112001 (2007)

PHYSICAL REVIEW LETTERS

week ending
14 SEPTEMBER 2007

Charge Densities of the Neutron and Proton

Gerald A. Miller

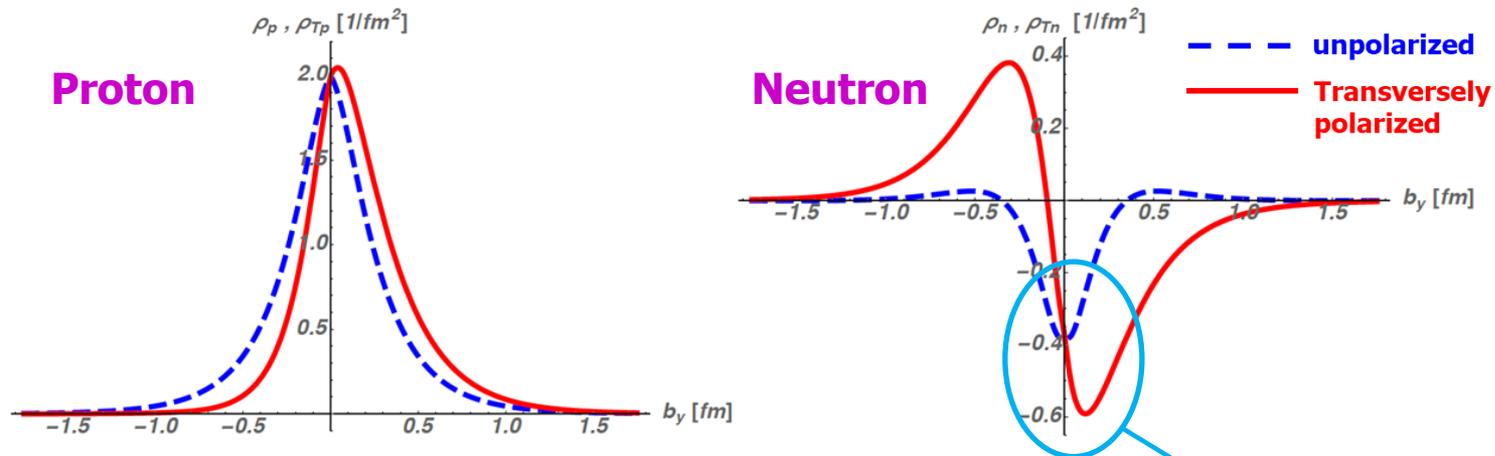
Department of Physics, University of Washington, Seattle, Washington 98195-1560, USA

(Received 18 May 2007; published 13 September 2007)

A model-independent analysis of the infinite-momentum-frame charge density of partons in the transverse plane is presented for the nucleon. We find that the neutron-parton charge density is negative at the center, so that the square of the transverse charge radius is positive, in contrast with many expectations. Additionally, the proton's central *d* quark charge density is larger than that of the *u* quark by about 30%. The proton (neutron) charge density has a long range positively (negatively) charged component.

◆ Generic LF distributions (strict probabilistic!!!):

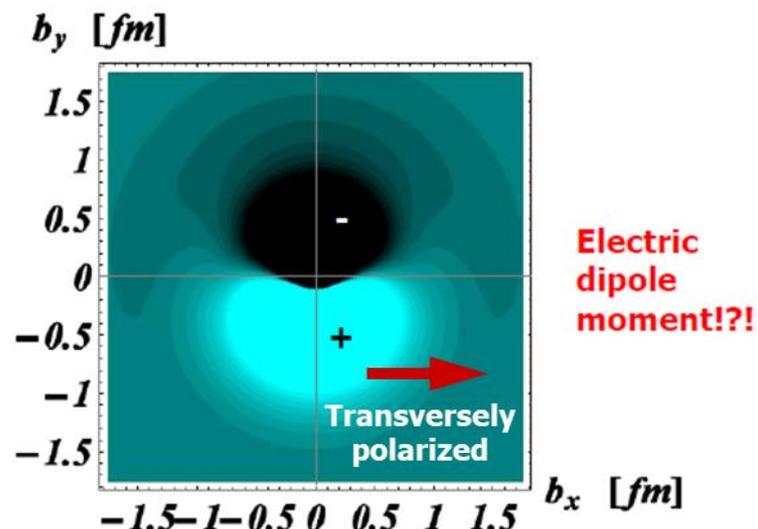
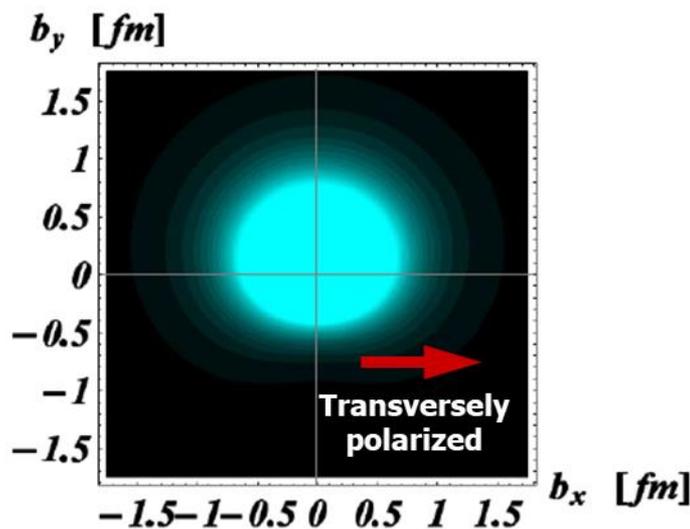
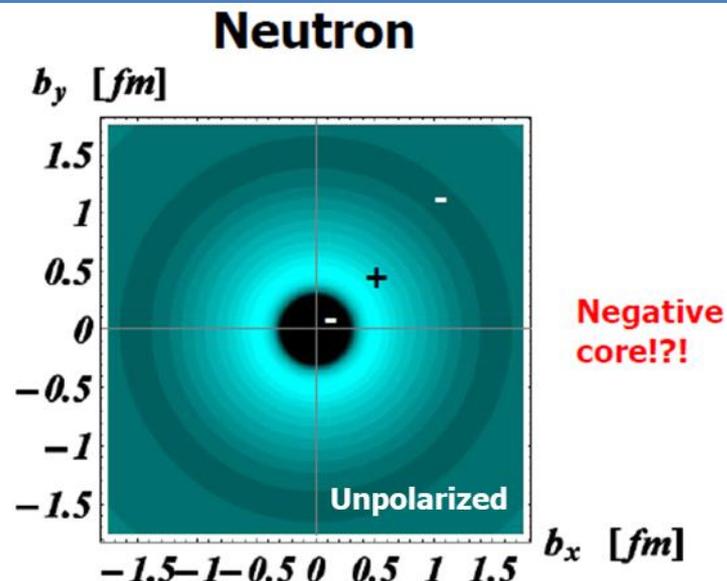
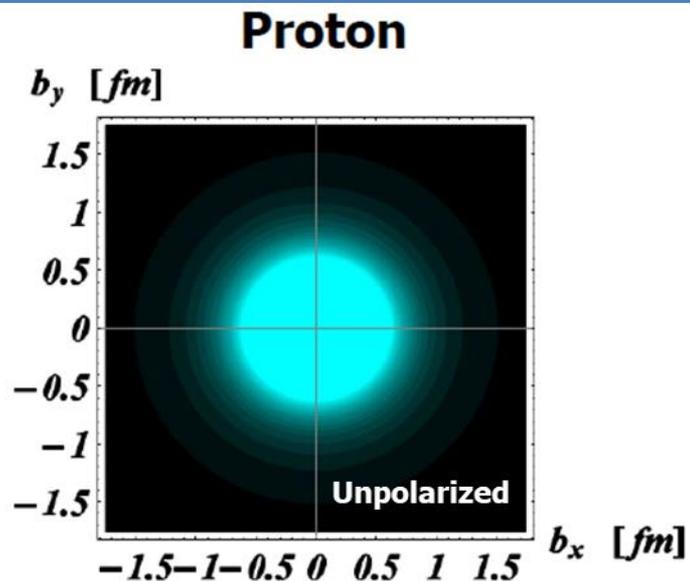
$$O_{\text{LF}}(\mathbf{b}_{\perp}; P^+) \equiv \int \frac{d^2 \Delta_{\perp}}{(2\pi)^2} e^{-i\Delta_{\perp} \cdot \mathbf{b}_{\perp}} \frac{\text{LF} \langle p', \lambda' | \hat{O}(0) | p, \lambda \rangle_{\text{LF}}}{2P^+} \Big|_{\Delta^+ = |\mathbf{P}_{\perp}| = 0}$$



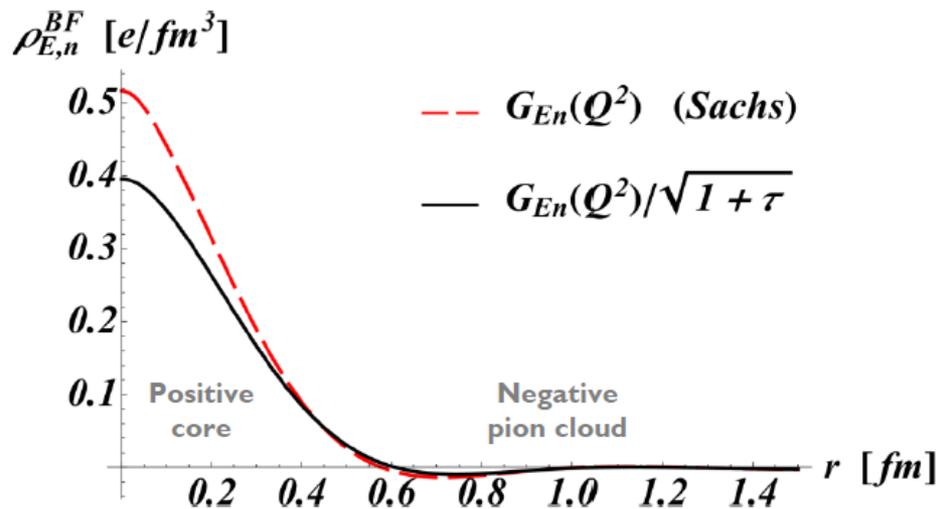
negative center!?

[G. A. Miller, PRL 99, 112001 (2007)]
[Carlson & Vanderhaeghen, PRL 100, 032004 (2008)]

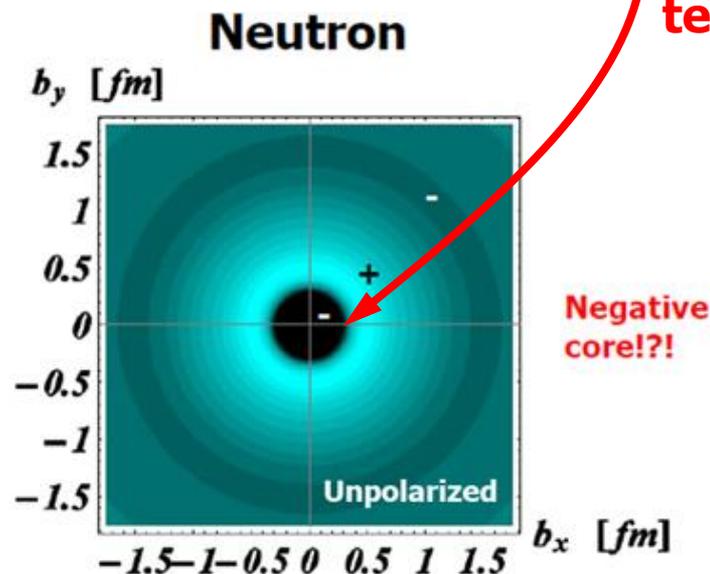
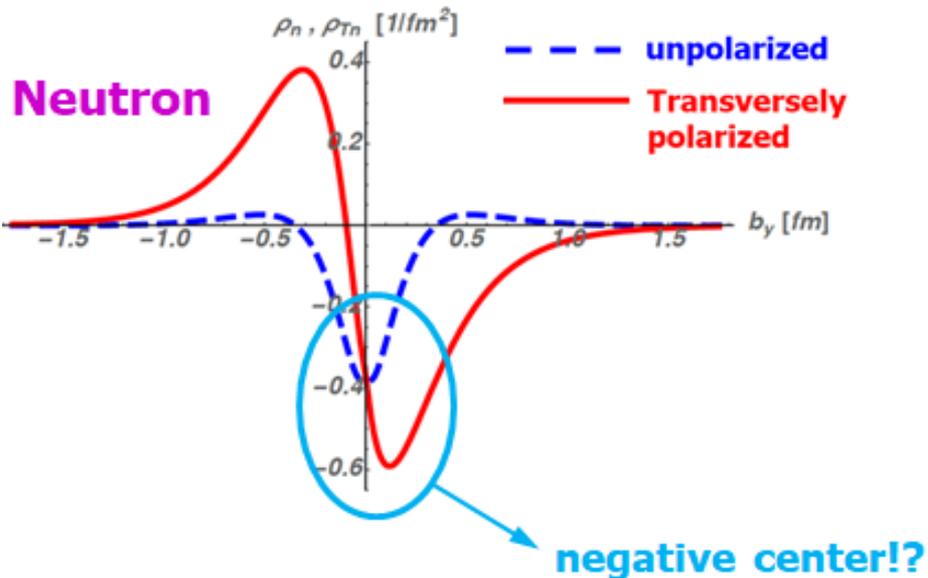
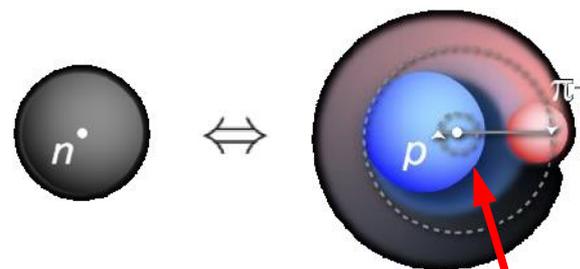
Some unusual observations on the LF



Neutron: 3D positive center → 2D negative center?!



Pion cloud picture (rest frame)



Relativistic elastic frame (EF) interpretation

◆ Poincaré symmetry:

$$\langle p', s' | \hat{j}^\mu(0) | p, s \rangle = \sum_{s'_B, s_B} \underbrace{D_{s'_B s_B}^{\dagger(j)}(p'_B, \Lambda) D_{s_B s}^{(j)}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^\mu{}_\nu}_{\text{Lorentz mixing}} \langle p'_B, s'_B | \hat{j}^\nu(0) | p_B, s_B \rangle$$

(1). Lorentz mixing

$$\begin{pmatrix} \tilde{J}_{\text{EF}}^0 \\ \tilde{J}_{\text{EF}}^z \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \tilde{J}_B^0 \\ \tilde{J}_B^z \end{pmatrix} \quad \sim \text{momentum-space amplitudes}$$

boost $\Lambda^\mu{}_\nu$

(2). Wigner spin rotation

Key insight: boost generators do not commute!

$$[\hat{K}^i, \hat{K}^j] = -i\epsilon^{ijk} \hat{J}^k$$

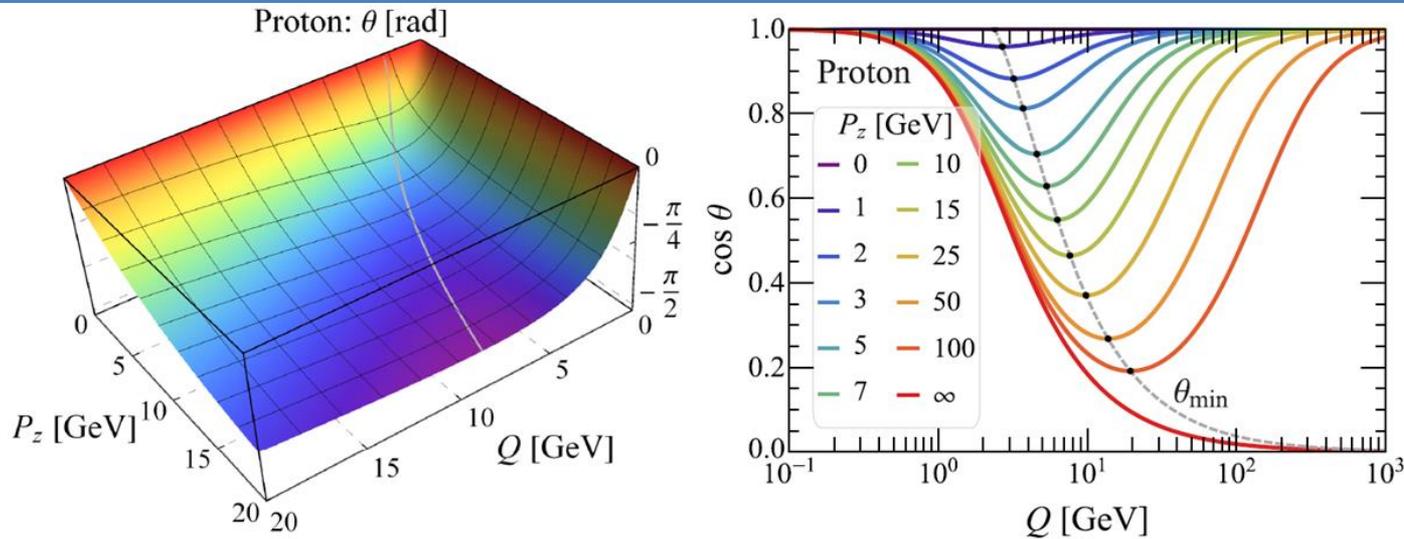
$$D^{(1/2)}(p_B, \Lambda) = \begin{pmatrix} \cos \frac{\theta}{2} & -e^{-i\phi\Delta} \sin \frac{\theta}{2} \\ e^{i\phi\Delta} \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{pmatrix}$$

θ : Wigner rotation angle

$$\cos \theta = \frac{P^0 + M(1 + \tau)}{(P^0 + M)\sqrt{1 + \tau}}, \quad \sin \theta = -\frac{\sqrt{\tau} P_z}{(P^0 + M)\sqrt{1 + \tau}}$$

- [Durand, De Celles, Marr, PR 126, 1882 (1962)]
- [Cédric Lorcé, PRL 125, 232002 (2020)]
- [YC & Cédric Lorcé, PRD 106, 116024 (2022)]
- [YC & Cédric Lorcé, PRD 107, 096003 (2023)]

Relativistic elastic frame (EF) interpretation



$$-\frac{\pi}{2} \leq \theta \leq 0$$

◆ Generic angular conditions for Wigner spin rotation:

$$\cos \theta = \frac{P^0 + M(1 + \tau)}{(P^0 + M)\sqrt{1 + \tau}}, \quad \sin \theta = -\frac{\sqrt{\tau}P_z}{(P^0 + M)\sqrt{1 + \tau}}$$

(1). At fixed P_z :

$$\cos \theta_{\min} = \frac{1 + 2\tau_{\min}}{(1 + \tau_{\min})^{3/2}} \longrightarrow \tau_{\min} = \frac{1}{2} + \sqrt{\frac{P_z^2}{M^2} + \frac{5}{4}}$$

(2). At $P_z \rightarrow \infty$:

$$\lim_{P_z \rightarrow \infty} \cos \theta = \frac{1}{\sqrt{1 + \tau}}, \quad \lim_{P_z \rightarrow \infty} \sin \theta = -\frac{\sqrt{\tau}}{\sqrt{1 + \tau}}$$

Relativistic elastic frame (EF) interpretation

◆ EF distributions (2D):

Wigner spin rotation

$$\begin{aligned}
 J_{\text{EF}}^0(\mathbf{b}_\perp; P_z) &= e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[\delta_{s's} \cos \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \sin \theta \right] \frac{G_E(\Delta_\perp^2)}{\sqrt{1+\tau}} \\
 &+ e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{P_z}{P^0} \left[-\delta_{s's} \sin \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \cos \theta \right] \frac{\sqrt{\tau} G_M(\Delta_\perp^2)}{\sqrt{1+\tau}} \\
 J_{z,\text{EF}}(\mathbf{b}_\perp; P_z) &= e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{P_z}{P^0} \left[\delta_{s's} \cos \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \sin \theta \right] \frac{G_E(\Delta_\perp^2)}{\sqrt{1+\tau}} \\
 &+ e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[-\delta_{s's} \sin \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \cos \theta \right] \frac{\sqrt{\tau} G_M(\Delta_\perp^2)}{\sqrt{1+\tau}}
 \end{aligned}$$

$$\underline{J_{\perp,\text{EF}}(\mathbf{b}_\perp; P_z)} = \delta_{s's} \frac{e\sigma_z}{2M} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} (\mathbf{e}_z \times i\boldsymbol{\Delta})_\perp \frac{P_B^0}{P^0} \frac{G_M(\Delta_\perp^2)}{\sqrt{1+\tau}}$$

free of Wigner spin rotation

suppressed by Lorentz contraction

In conclusion: Lorentz transformation and Wigner spin rotation indeed jointly play a central role for the structures of moving spinning hadrons.

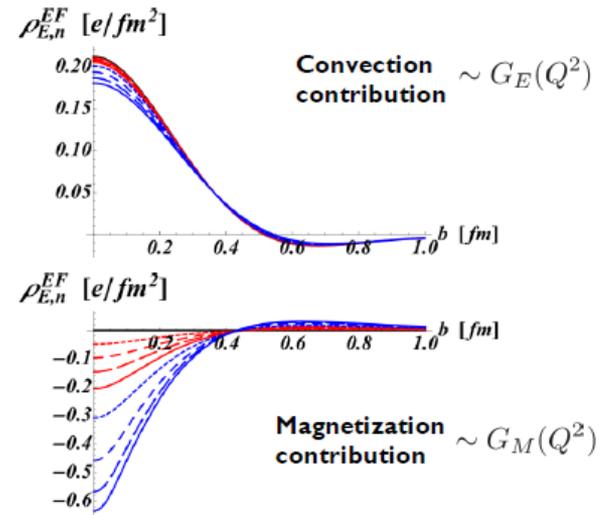
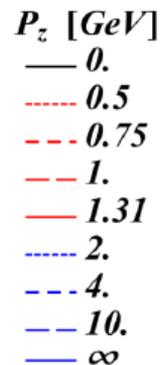
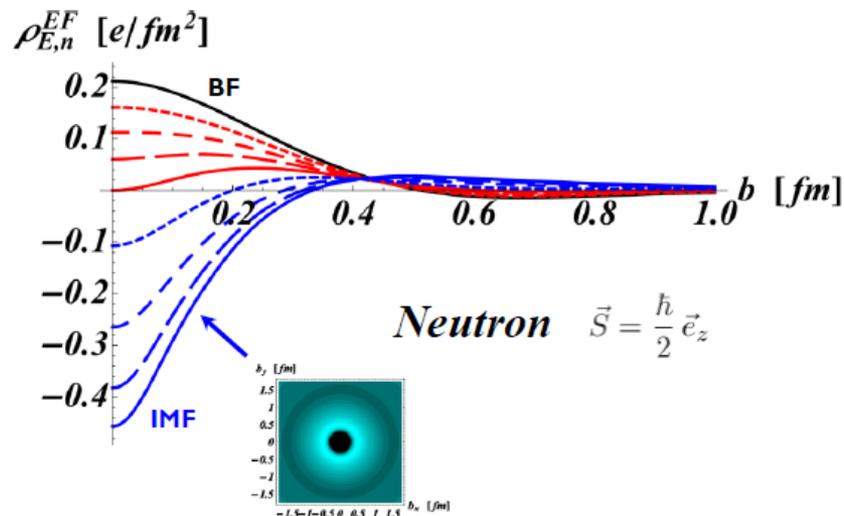
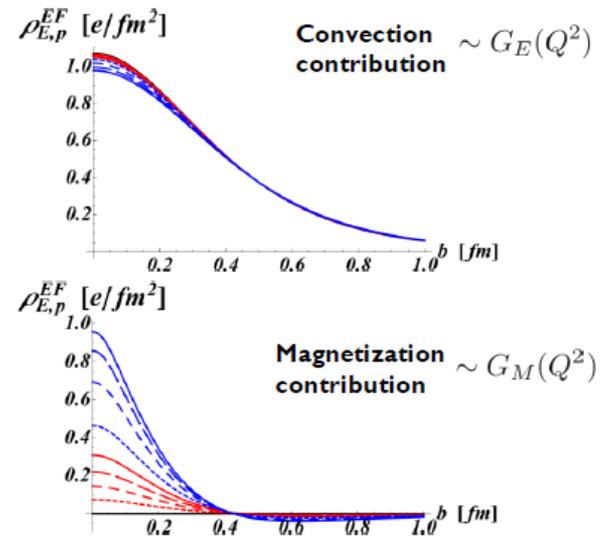
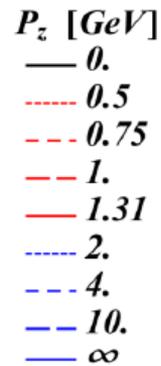
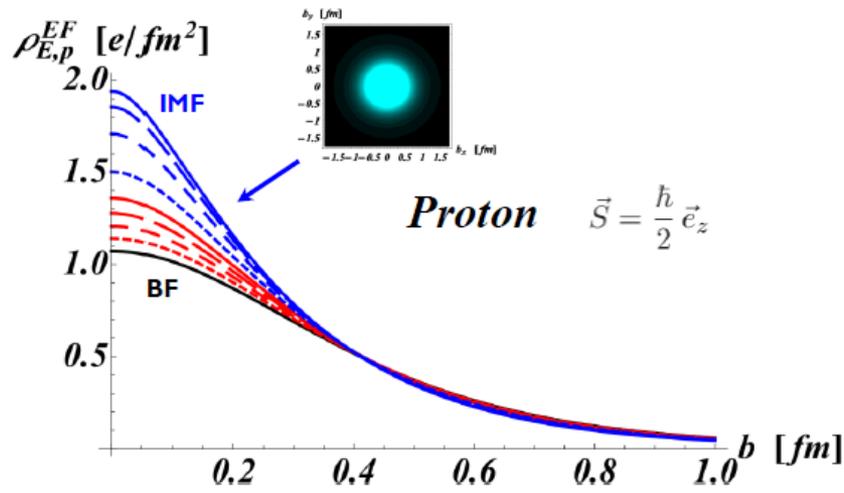
[Cédric Lorcé, Pierre Wang, PRD 105, 096032 (2022)]

[YC, Cédric Lorcé, PRD106 (2022) 116024]

[YC, Cédric Lorcé, PRD107 (2023) 096003]

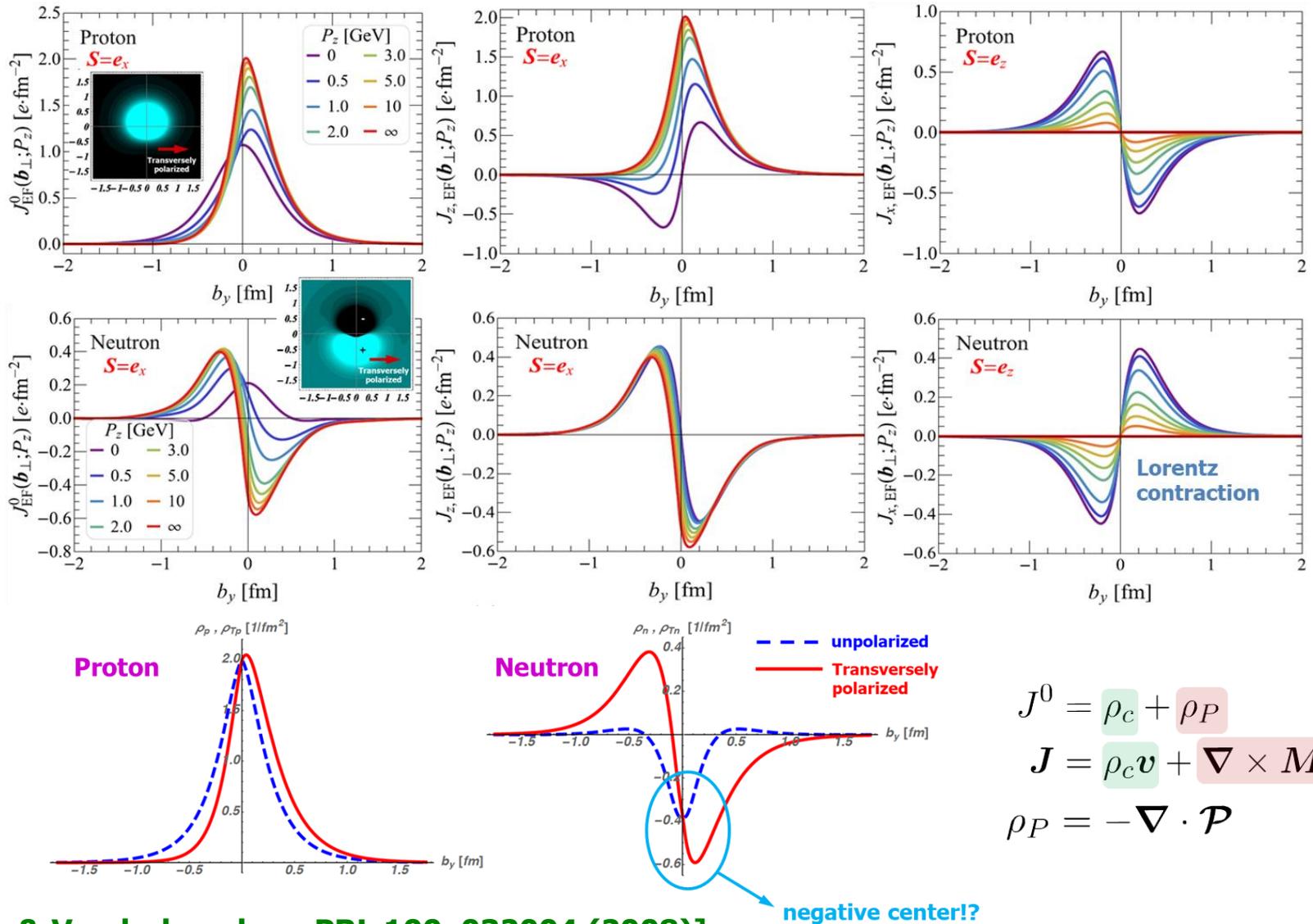
Relativistic elastic frame (EF) interpretation

◆ EF distributions (2D): unpolarized case



Relativistic elastic frame (EF) interpretation

◆ EF distributions (2D): polarized case



$$J^0 = \rho_c + \rho_P$$

$$\mathbf{J} = \rho_c \mathbf{v} + \nabla \times \mathbf{M} + \cancel{\partial_0 \mathcal{P}}$$

$$\rho_P = -\nabla \cdot \mathcal{P}$$

[Carlson & Vanderhaeghen, PRL 100, 032004 (2008)]
 [YC & Cédric Lorcé, PRD 106, 116024 (2022); PRD 107, 096003 (2023)]

Relativistic elastic frame (EF) interpretation

◆ In the IMF limit ($P_z \rightarrow \infty$): why it becomes $F_1(Q^2)$ and $F_2(Q^2)$ for LF J^+ ?

$$J_{\text{LF}}^+(\mathbf{b}_\perp; P^+) = e \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[\delta_{\lambda\lambda} F_1(\Delta_\perp^2) + \frac{(\boldsymbol{\sigma}_{\lambda\lambda} \times i\boldsymbol{\Delta})_z}{2M} F_2(\Delta_\perp^2) \right]$$

$$\lim_{P_z \rightarrow \infty} \frac{\cos \theta G_E(Q^2) - \sin \theta \sqrt{\tau} G_M(Q^2)}{\sqrt{1+\tau}} = \frac{G_E(Q^2) + \tau G_M(Q^2)}{1+\tau} = F_1(Q^2)$$

$$\lim_{P_z \rightarrow \infty} \frac{\sin \theta G_E(Q^2) + \cos \theta \sqrt{\tau} G_M(Q^2)}{\sqrt{\tau} \sqrt{1+\tau}} = \frac{G_M(Q^2) - G_E(Q^2)}{1+\tau} = F_2(Q^2)$$

unpolarized

Transversely polarized

◆ Besides, we found another analytic relations for LF J^- :

$$J_{\text{LF}}^-(\mathbf{b}_\perp; P^+) = e \int \frac{d^2\Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{P^-}{P^+} \left[\delta_{\lambda\lambda} G_1(\Delta_\perp^2) + \frac{(\boldsymbol{\sigma}_{\lambda\lambda} \times i\boldsymbol{\Delta})_z}{2M} G_2(\Delta_\perp^2) \right]$$

$$\lim_{P_z \rightarrow \infty} \frac{\cos \theta G_E(Q^2) + \sin \theta \sqrt{\tau} G_M(Q^2)}{\sqrt{1+\tau}} = \frac{(1-\tau)F_1(Q^2) - 2\tau F_2(Q^2)}{1+\tau} \equiv G_1(Q^2)$$

$$\lim_{P_z \rightarrow \infty} \frac{\sin \theta G_E(Q^2) - \cos \theta \sqrt{\tau} G_M(Q^2)}{\sqrt{\tau} \sqrt{1+\tau}} = -\frac{2F_1(Q^2) + (1-\tau)F_2(Q^2)}{1+\tau} \equiv G_2(Q^2)$$

unpolarized

Transversely polarized

new

→ $J_{\text{LF}}^+(\mathbf{b}_\perp; P^+) = J_{\text{EF}}^0(\mathbf{b}_\perp; \infty) = J_{\text{EF}}^z(\mathbf{b}_\perp; \infty) \sim$ generic relation

LF interpretation vs. EF interpretation

◆ LF interpretation:

$$J_{\text{LF}}^+(\mathbf{b}_\perp; P^+) = e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[\delta_{\lambda\lambda} F_1(\Delta_\perp^2) + \frac{(\boldsymbol{\sigma}_{\lambda\lambda} \times i\boldsymbol{\Delta})_z}{2M} F_2(\Delta_\perp^2) \right],$$

$$J_{\text{LF}}^-(\mathbf{b}_\perp; P^+) = e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left(\frac{P^-}{P^+} \right) \left[\delta_{\lambda\lambda} G_1(\Delta_\perp^2) + \frac{(\boldsymbol{\sigma}_{\lambda\lambda} \times i\boldsymbol{\Delta})_z}{2M} G_2(\Delta_\perp^2) \right],$$

$$J_{\perp, \text{LF}}(\mathbf{b}_\perp; P^+) = e(\sigma_z)_{\lambda\lambda} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{(\mathbf{e}_z \times i\boldsymbol{\Delta})_\perp}{2P^+} G_M(\Delta_\perp^2).$$

In total,
we need **5** EMFFs.

F_1, F_2

G_1, G_2

G_M

◆ EF interpretation:

$$J_{\text{EF}}^0(\mathbf{b}_\perp; P_z) = e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[\delta_{s's} \cos \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \sin \theta \right] \frac{G_E(\Delta_\perp^2)}{\sqrt{1+\tau}}$$

$$+ e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{P_z}{P^0} \left[-\delta_{s's} \sin \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \cos \theta \right] \frac{\sqrt{\tau} G_M(\Delta_\perp^2)}{\sqrt{1+\tau}}$$

$$J_{z, \text{EF}}(\mathbf{b}_\perp; P_z) = e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \frac{P_z}{P^0} \left[\delta_{s's} \cos \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \sin \theta \right] \frac{G_E(\Delta_\perp^2)}{\sqrt{1+\tau}}$$

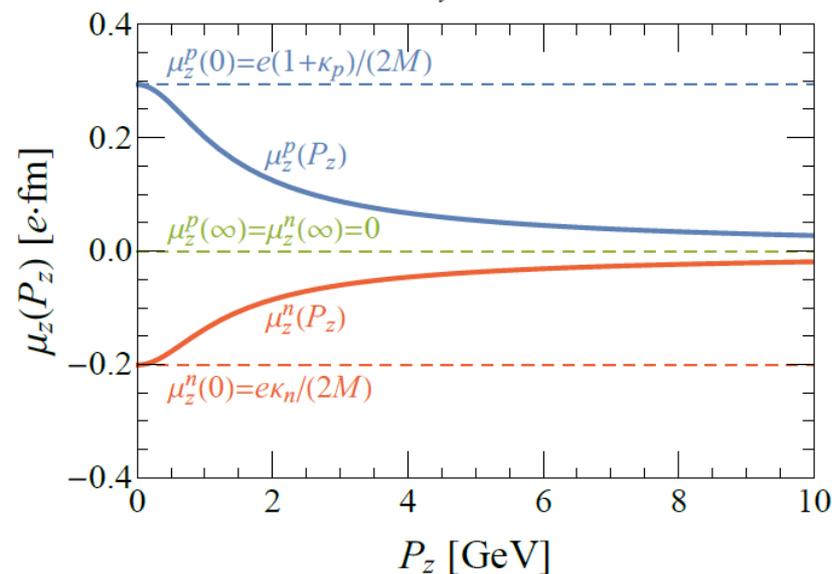
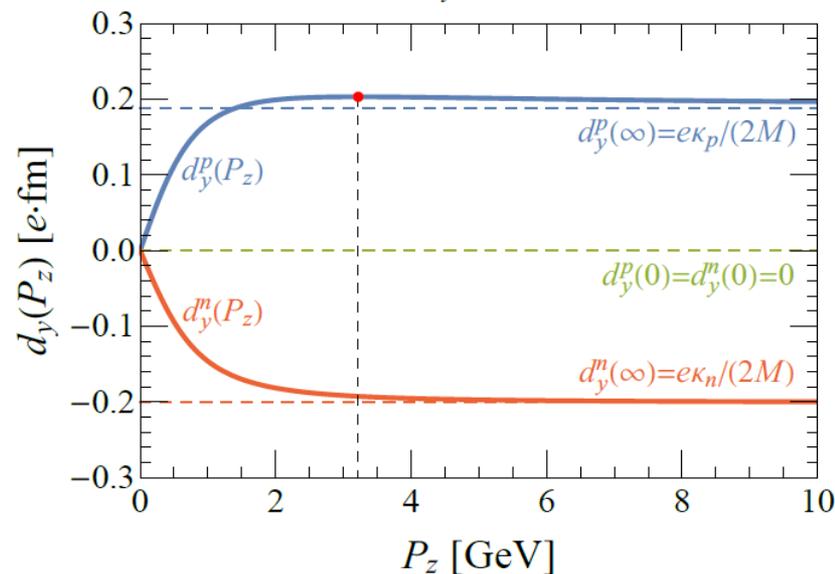
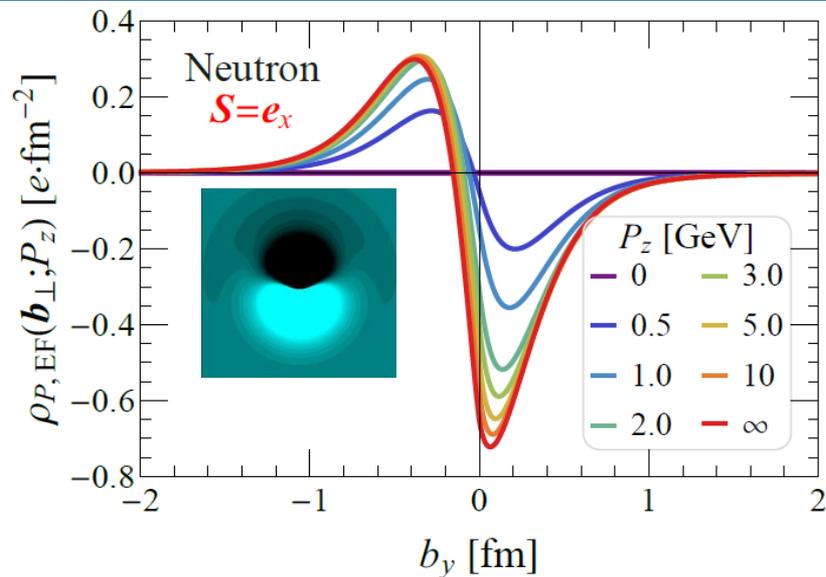
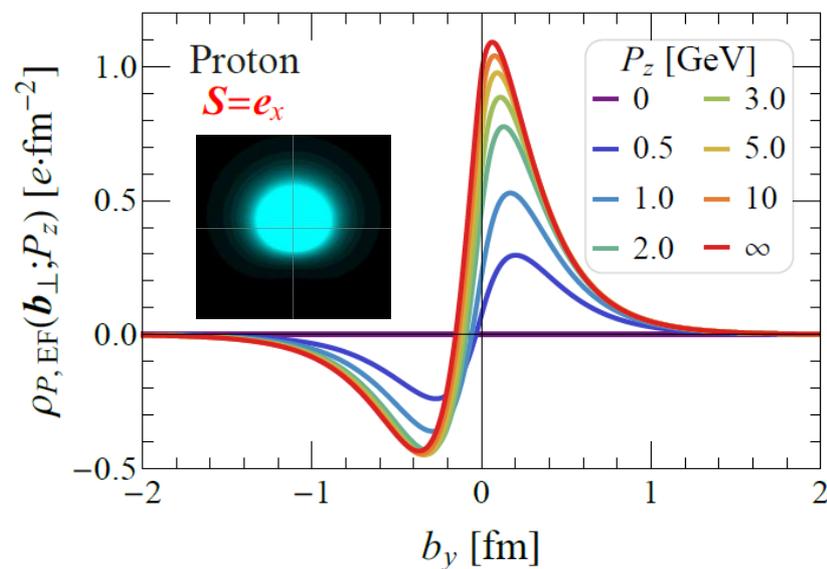
$$+ e \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} \left[-\delta_{s's} \sin \theta + \frac{(\boldsymbol{\sigma}_{s's} \times i\boldsymbol{\Delta})_z}{2M\sqrt{\tau}} \cos \theta \right] \frac{\sqrt{\tau} G_M(\Delta_\perp^2)}{\sqrt{1+\tau}}$$

$$J_{\perp, \text{EF}}(\mathbf{b}_\perp; P_z) = \delta_{s's} \frac{e\sigma_z}{2M} \int \frac{d^2 \Delta_\perp}{(2\pi)^2} e^{-i\Delta_\perp \cdot \mathbf{b}_\perp} (\mathbf{e}_z \times i\boldsymbol{\Delta})_\perp \frac{P_B^0}{P^0} \frac{G_M(\Delta_\perp^2)}{\sqrt{1+\tau}}$$

In total,
we need **2** EMFFs
and **1** Wigner
rotation angle.

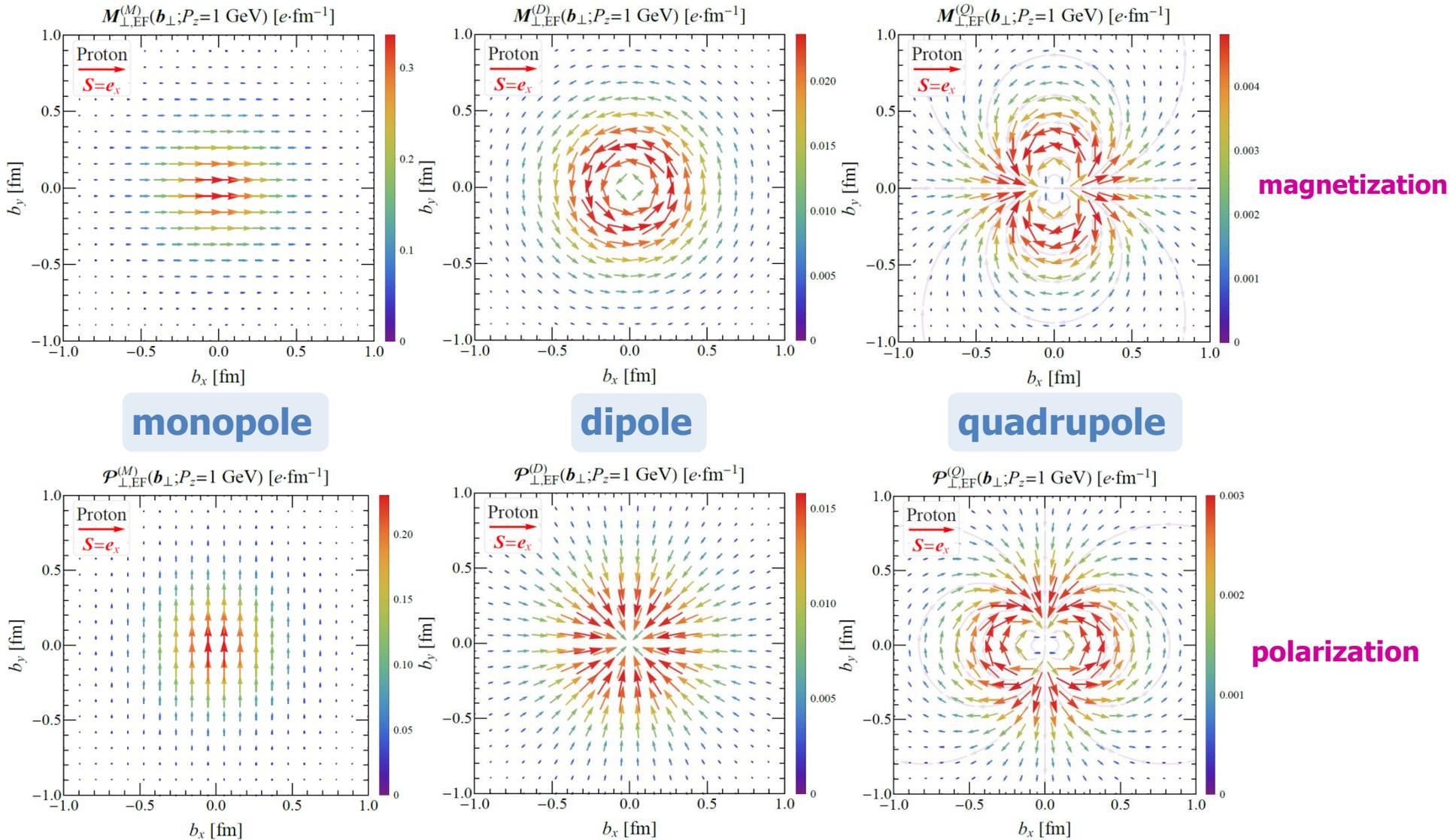
G_E, G_M, θ

Relativistic elastic frame (EF) interpretation



Induced EDM: $d = v \times \mu \sim$ manifestation of relativistic effect!

Relativistic elastic frame (EF) interpretation



$d = v \times \mu$ ← $\tilde{\mathcal{P}}^{(l)} = v \times \tilde{M}^{(l)} \sim$ **explicit manifestation of Ehrenfest's theorem!**
classical level valid at quantum level (in momentum space) to all orders!

Summary and outlook

- **Quantum phase-space approach allows one to define relativistic spatial distributions inside a hadron with arbitrary spin and arbitrary average momentum.**
- **For a spin-1/2 hadron, its electromagnetic vertex function has an one-to-one correspondence with the classical electromagnetic four-current, where convection and magnetization contributions are associated with $G_E(Q^2)$ and $G_M(Q^2)$, respectively.**
- **Of all possible elastic frames, the Breit frame distributions lead to the simplest multipole structures, which strongly suggests that Sachs electromagnetic FFs $G_E(Q^2)$ and $G_M(Q^2)$ should be interpreted as the more physical ones. In the moving case, polarization and magnetization contributions to the electromagnetic four-current distributions are crucial and significant.**
- **Any relativistic spatial distortions of a moving spinning hadron can be understood as a combination of Lorentz transformation and Wigner spin rotation. This is precisely protected by Poincaré symmetry.**

[YC & Cédric Lorcé, PRD 106, 116024 (2022)]

[YC & Cédric Lorcé, PRD 107, 096003 (2023)]

[YC et al., arXiv:26xx.xxxx (to appear)]

Thank you!

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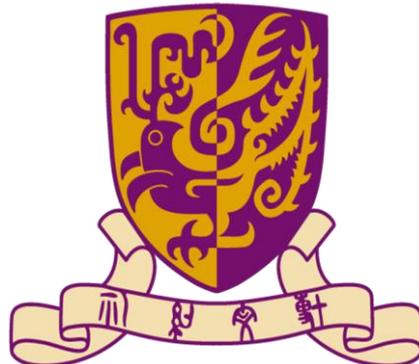


Guang-Peng Zhang

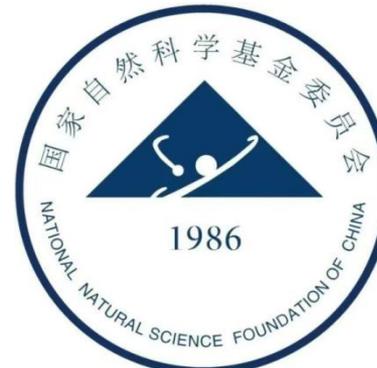
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Jian Zhou



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NSFC
“理论物理专款” No. 12547178

Backup: Covariant Lorentz transformation -- two key effects

- **Four-vector case (e.g., electromagnetic four-current):**

$$\langle p', s' | \hat{j}^\mu(0) | p, s \rangle = \sum_{s'_B, s_B} \underbrace{D_{s'_B s_B}^{\dagger(j)}(p'_B, \Lambda) D_{s_B s}^{(j)}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^\mu{}_\nu}_{\text{Lorentz mixing}} \langle p'_B, s'_B | \hat{j}^\nu(0) | p_B, s_B \rangle$$

- **Axial four-vector case (e.g., axial-vector four-current):**

$$\langle p', s' | \hat{j}_5^\mu(0) | p, s \rangle = \sum_{s'_B, s_B} \underbrace{D_{s'_B s_B}^{\dagger(j)}(p'_B, \Lambda) D_{s_B s}^{(j)}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^\mu{}_\nu}_{\text{Lorentz mixing}} \langle p'_B, s'_B | \hat{j}_5^\nu(0) | p_B, s_B \rangle$$

- **Tensor case (e.g., polarization-magnetization tensor):**

$$\langle p', s' | \hat{P}^{\mu\nu}(0) | p, s \rangle = \sum_{s'_B, s_B} \underbrace{D_{s'_B s_B}^{\dagger(j)}(p'_B, \Lambda) D_{s_B s}^{(j)}(p_B, \Lambda)}_{\text{Wigner rotation}} \underbrace{\Lambda^\mu{}_\alpha \Lambda^\nu{}_\beta}_{\text{Lorentz mixing}} \langle p'_B, s'_B | \hat{P}^{\alpha\beta}(0) | p_B, s_B \rangle$$

- (1). Lorentz mixing effect

$$\begin{pmatrix} \tilde{J}_{\text{EF}}^0 \\ \tilde{J}_{\text{EF}}^z \end{pmatrix} = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix} \begin{pmatrix} \tilde{J}_B^0 \\ \tilde{J}_B^z \end{pmatrix} \sim \text{momentum-space amplitudes}$$

boost $\Lambda^\mu{}_\nu$

- (2). Wigner rotation effect

Fundamental reason: boost generators of Lorentz group do not commute!

$$\left[\hat{K}^i, \hat{K}^j \right] = -i\epsilon^{ijk} \hat{J}^k \longrightarrow \text{Wigner rotation } D_{s_B s}^{(j)}(p_B, \Lambda)$$

[Durand, De Celles, Marr, PR 126, 1882 (1962)]

[Cédric Lorcé, PRL 125, 232002 (2020)]

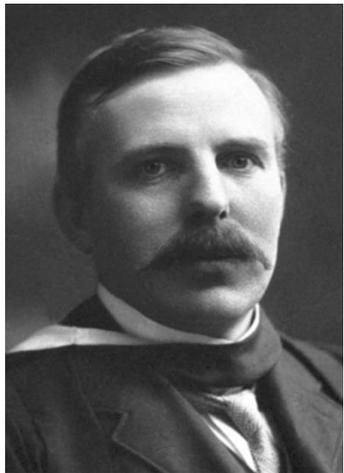
[YC & Cédric Lorcé, PRD 106, 116024 (2022)]

[YC & Cédric Lorcé, PRD 107, 096003 (2023)]

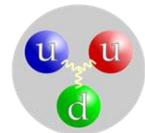
[YC, Y. Li, C. Lorcé, & Q. Wang. PRD 110, L091503 (2024); JHEP 04, 132 (2025)]

Why nucleons?

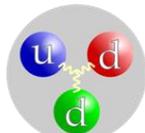
- **Nucleons (i.e. $N = p, n$):** known spin-1/2 composite systems, **data already exists**, **up-coming experiments** (MUSE, JLab 22 GeV, EIC, EicC, ...)
- **Lightest baryons:** ground-states, stable, abundant, building blocks of atomic nuclei responsible for $\geq 99\%$ visible matter mass in the universe, QCD laboratories, ...
- **Advanced experimental techniques:** well-developed, advanced, useful ways, ...
- **One of the top scientific goals of EIC:** 3D tomography of nucleon and nuclei.



E. Rutherford



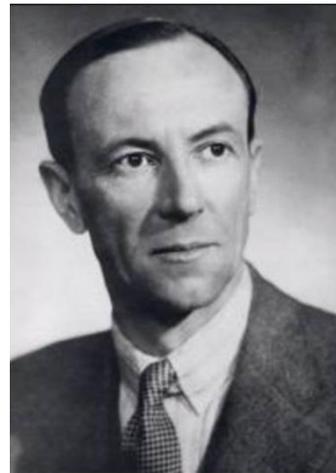
Proton (p)



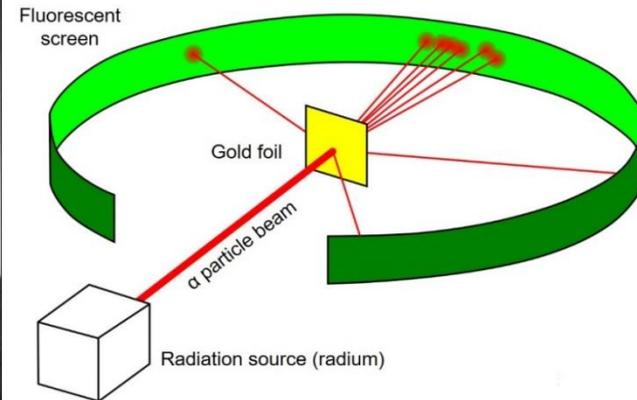
Neutron (n)



Nobel prize in physics
(←1908, 1935→)



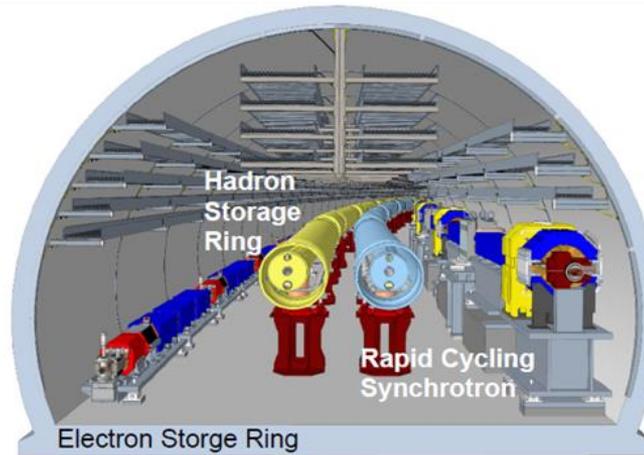
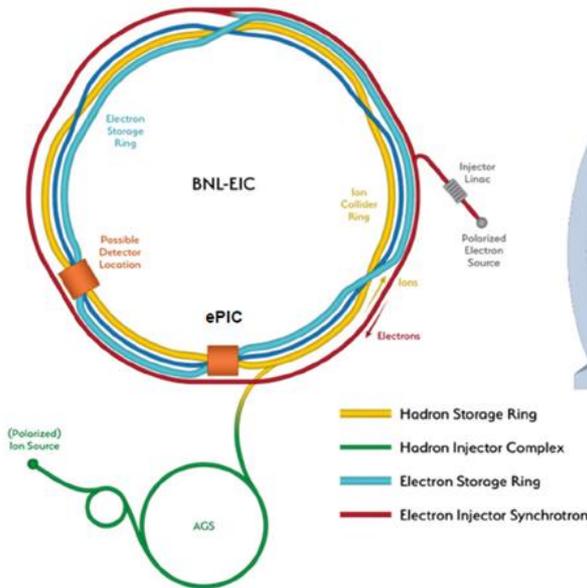
J. Chadwick



[E. Rutherford, J. Chadwick & C. Ellis, *Radiation from Radioactive Substances* (1930)]
[J. Chadwick. "Possible Existence of a Neutron". *Nature*, 129 (3252), 312 (1932)]

Why nucleons?

- ◆ **One of the key scientific goals of EIC is to perform 3D tomography or imaging of the internal structures of nucleon/nuclei.**



- ◆ **Nucleon are closely related to many big and key scientific questions:**

- (1). **Proton spin crisis, proton 3D spin structure;**
- (2). **Proton charge radius puzzle;**
- (3). **QCD confinement mechanism;**
- (4). **Mass origin of the proton;**
- (5). ...