NON-ABELIAN DOMAIN WALLS

BF, S. F. King, L. Marsili, S. Pascoli, J. Turner, Y-L. Zhou, 2409.16359

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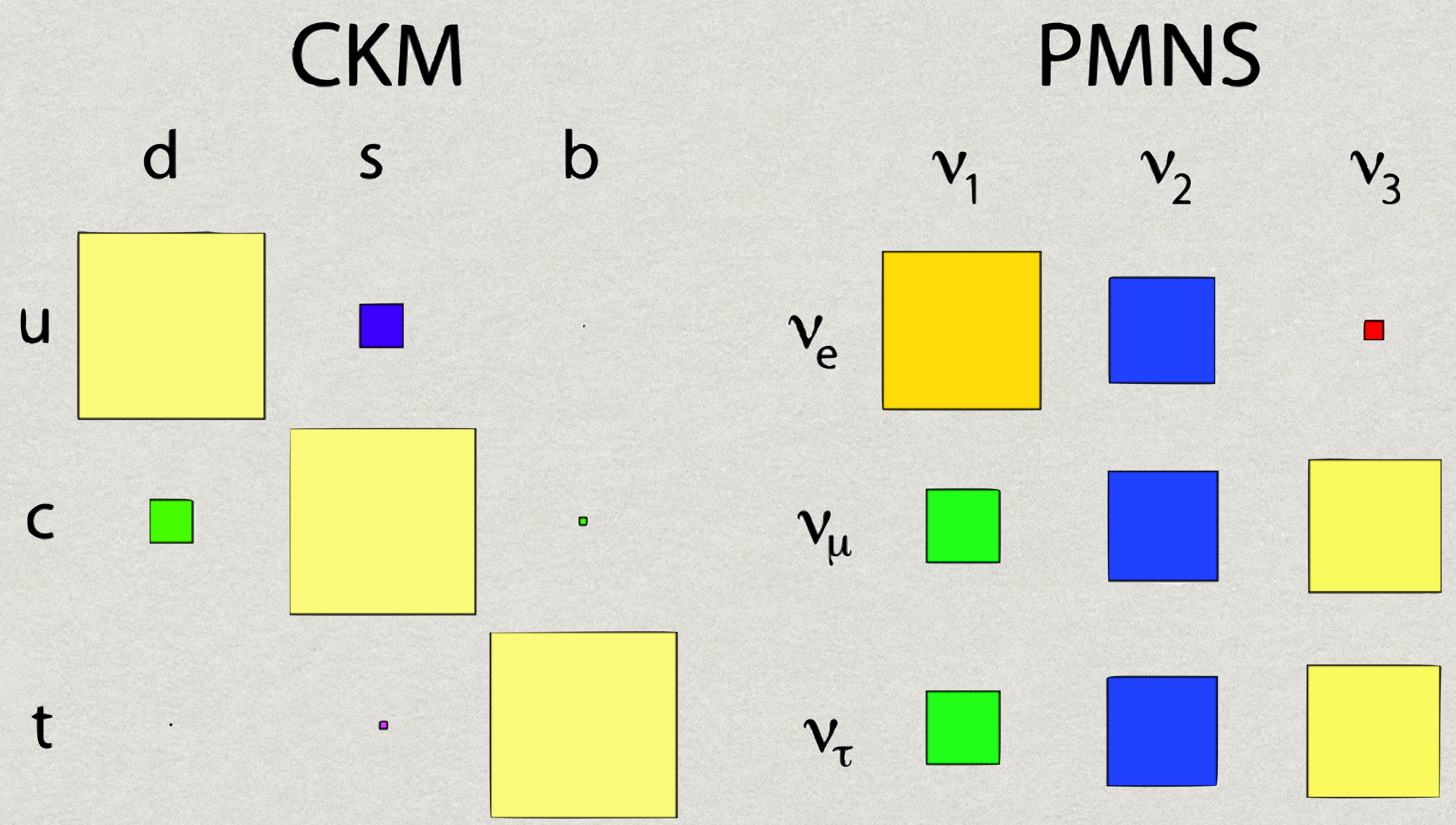
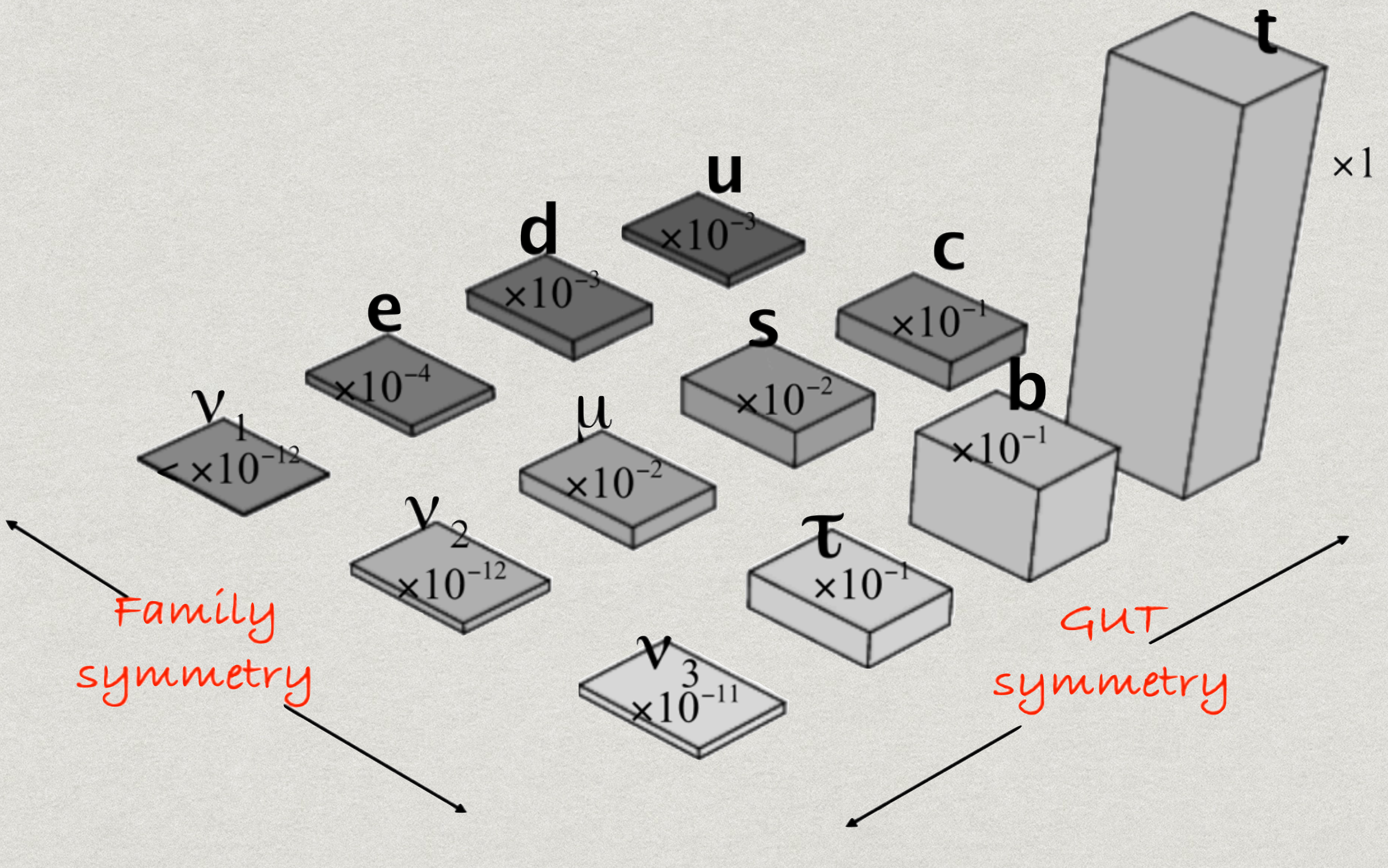
付博文 20 JAN 2026

第十届海峡两岸粒子物理和宇宙学研讨会

Flavour Problem

What is the origin of Quark and Lepton Mixing?





King's review 2017

Flavour Symmetries

- Continuous Abelian: Froggatt-Nielsen $\left(\frac{\langle\Phi\rangle}{\Lambda}\right)^n Hff$ Froggatt&Nielsen 1979
- Continuous non-Abelian: $SO(3)$ Wu 1998
- Discrete Abelian: $\mu - \tau$ reflection Xing&Zhao's review 2015
- Discrete non-Abelian: $A_4, S_4, A_5 \ldots$ Altarelli&Feruglio's review 2010
- Modular symmetry Feruglio 2017

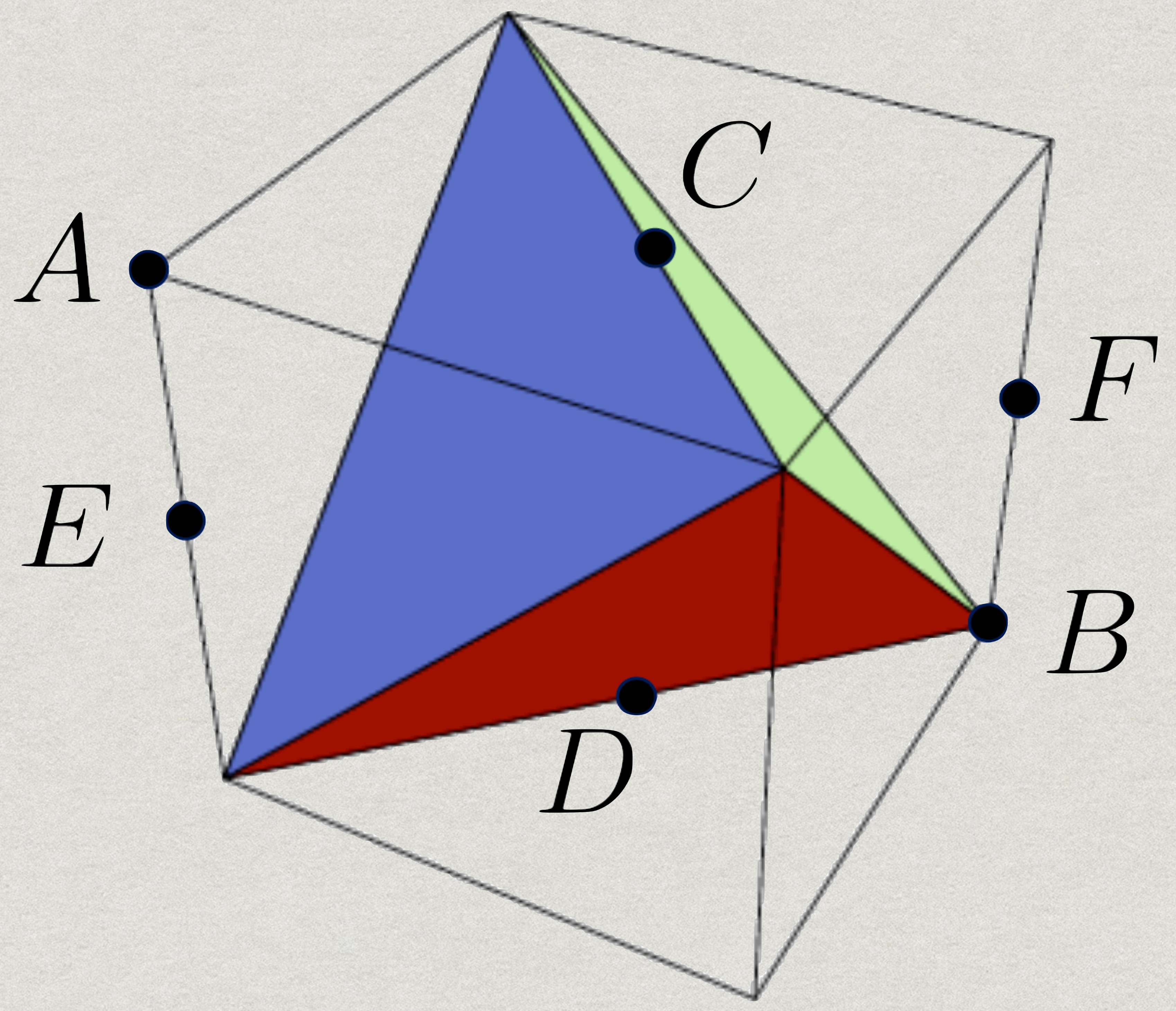
Discrete Symmetries

A_4: rigid rotation group of a tetrahedron

- 4 rotations by 120° clockwise (e.g. AB)
- 4 rotations by 120° anti-clockwise (e.g. AB)
- 3 rotations by 180° (e.g. CD)
- 1 unit operator

S_4: rigid rotation group of a cube

- 12 A_4 transformations
- 3 rotations by 90° clockwise (e.g. CD)
- 3 rotations by 90° anti-clockwise (e.g. CD)
- 6 rotations by 180° (e.g. EF)



King's review 2017

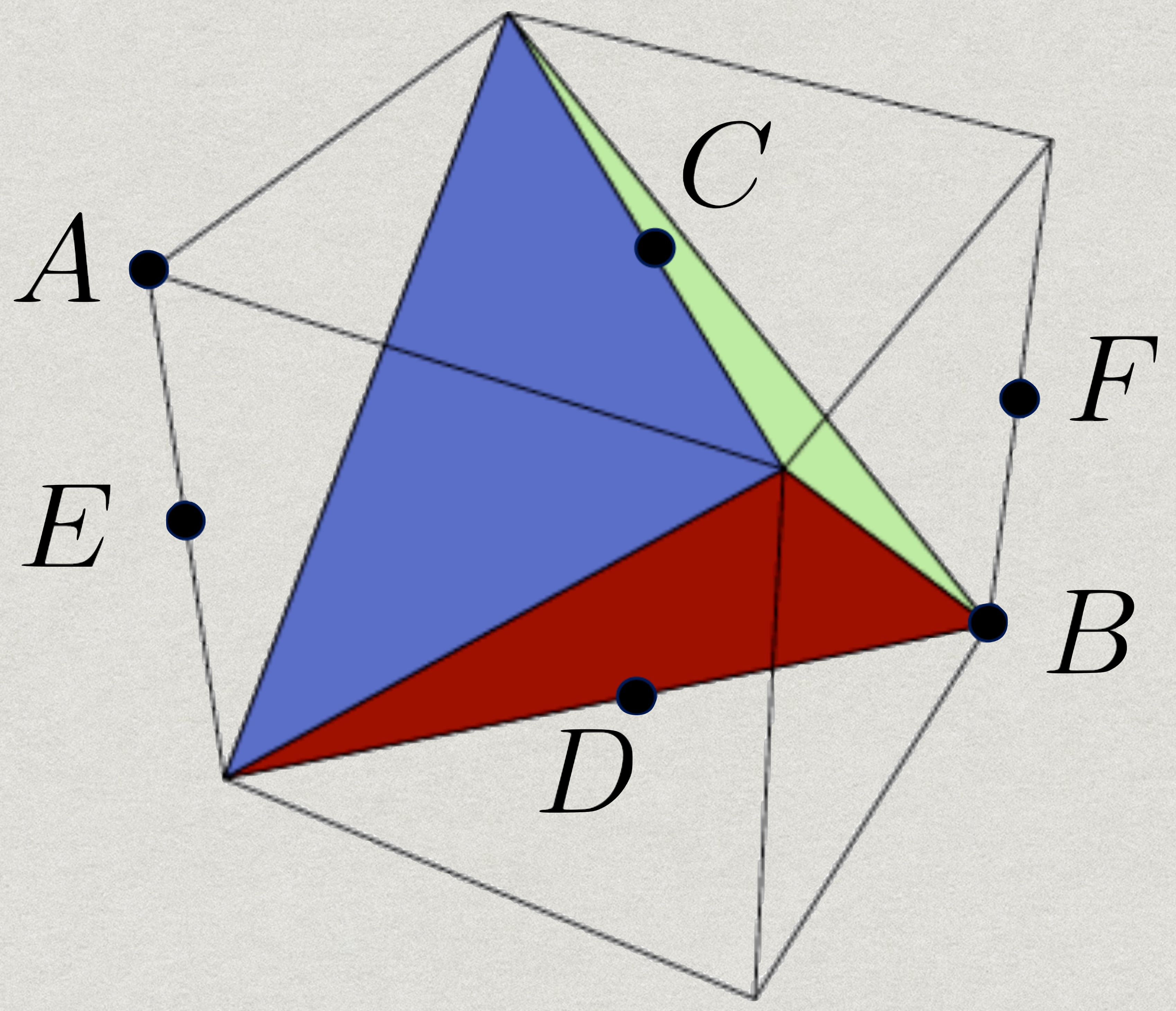
Discrete Symmetries

$$A_4\text{:}\quad T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$V(\phi) = -\frac{\mu^2}{2} I_1 + \frac{g_1}{4} I_1^2 + \frac{g_2}{2} I_2 + A\phi_1\phi_2\phi_3$$

$$S_4\text{:}\quad T = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \quad S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad U = \pm \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$V(\phi) = -\frac{\mu^2}{2} I_1 + \frac{g_1}{4} I_1^2 + \frac{g_2}{2} I_2 \qquad I_1 = \phi_1^2 + \phi_2^2 + \phi_3^2, \quad I_2 = \phi_1^2\phi_2^2 + \phi_2^2\phi_3^2 + \phi_3^2\phi_1^2$$



Vacuum Structure

A_4: $$V(\phi) = -\frac{\mu^2}{2} I_1 + \frac{g_1}{4} I_1^2 + \frac{g_2}{2} I_2 + A\phi_1\phi_2\phi_3$$

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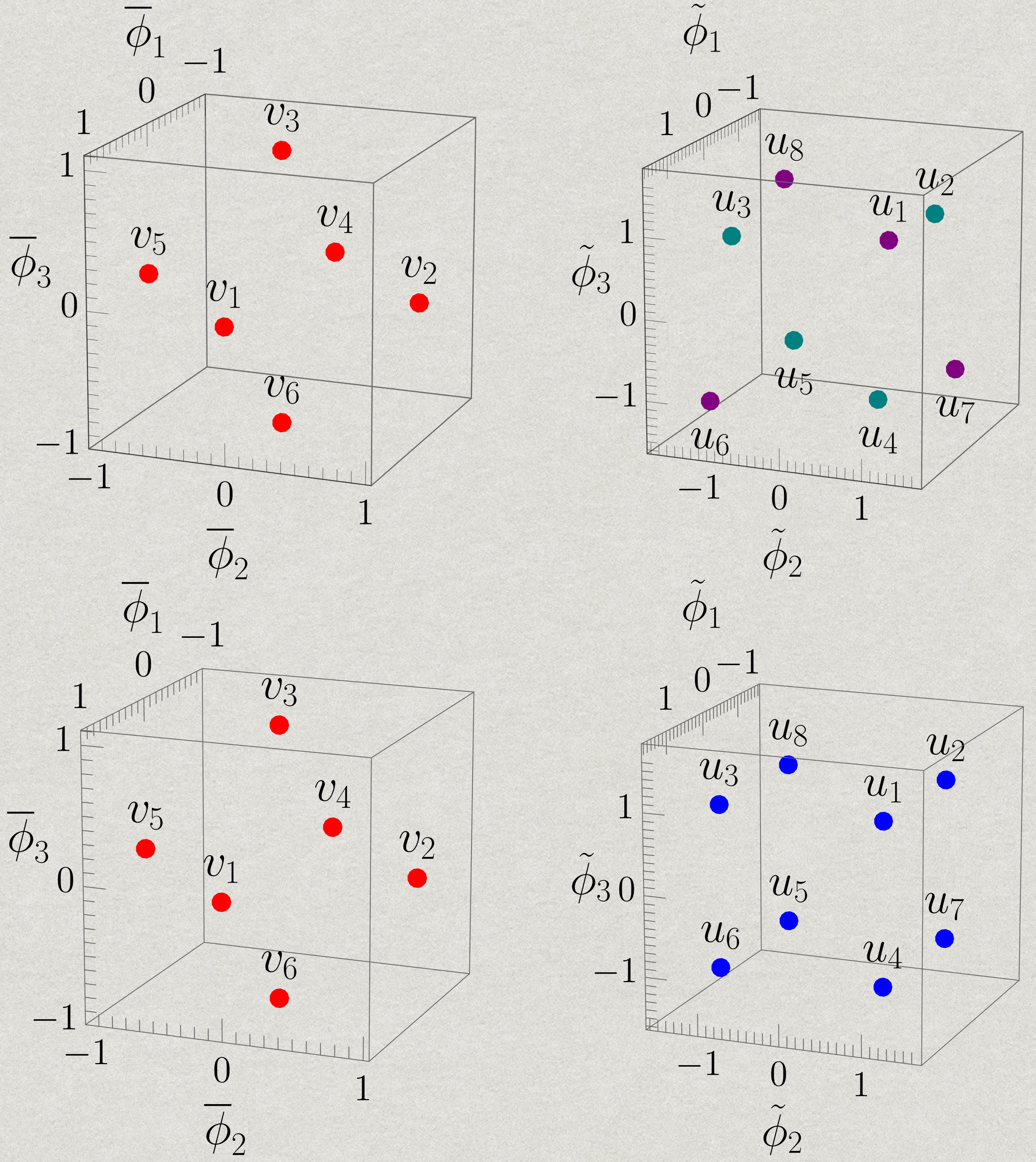

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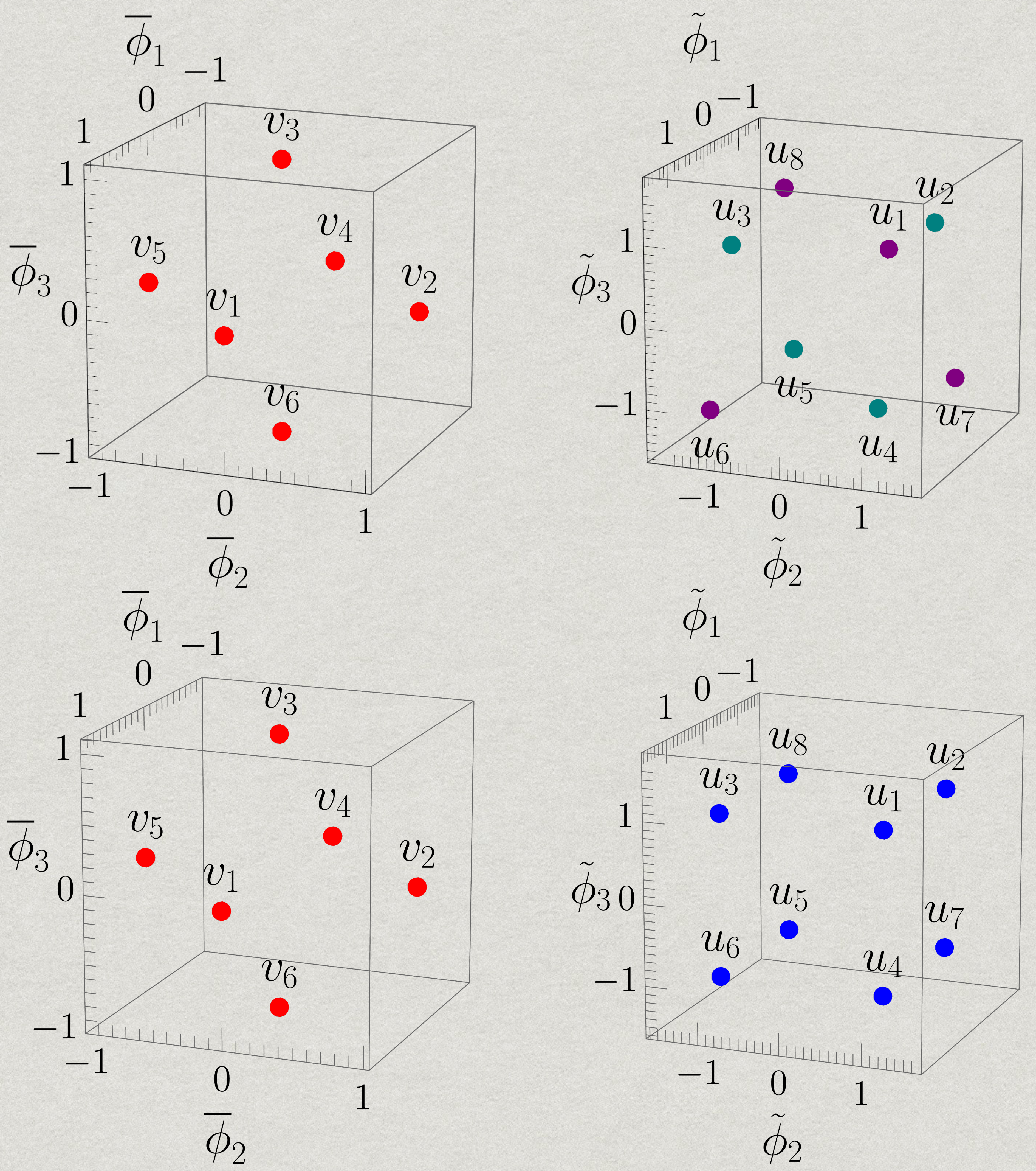

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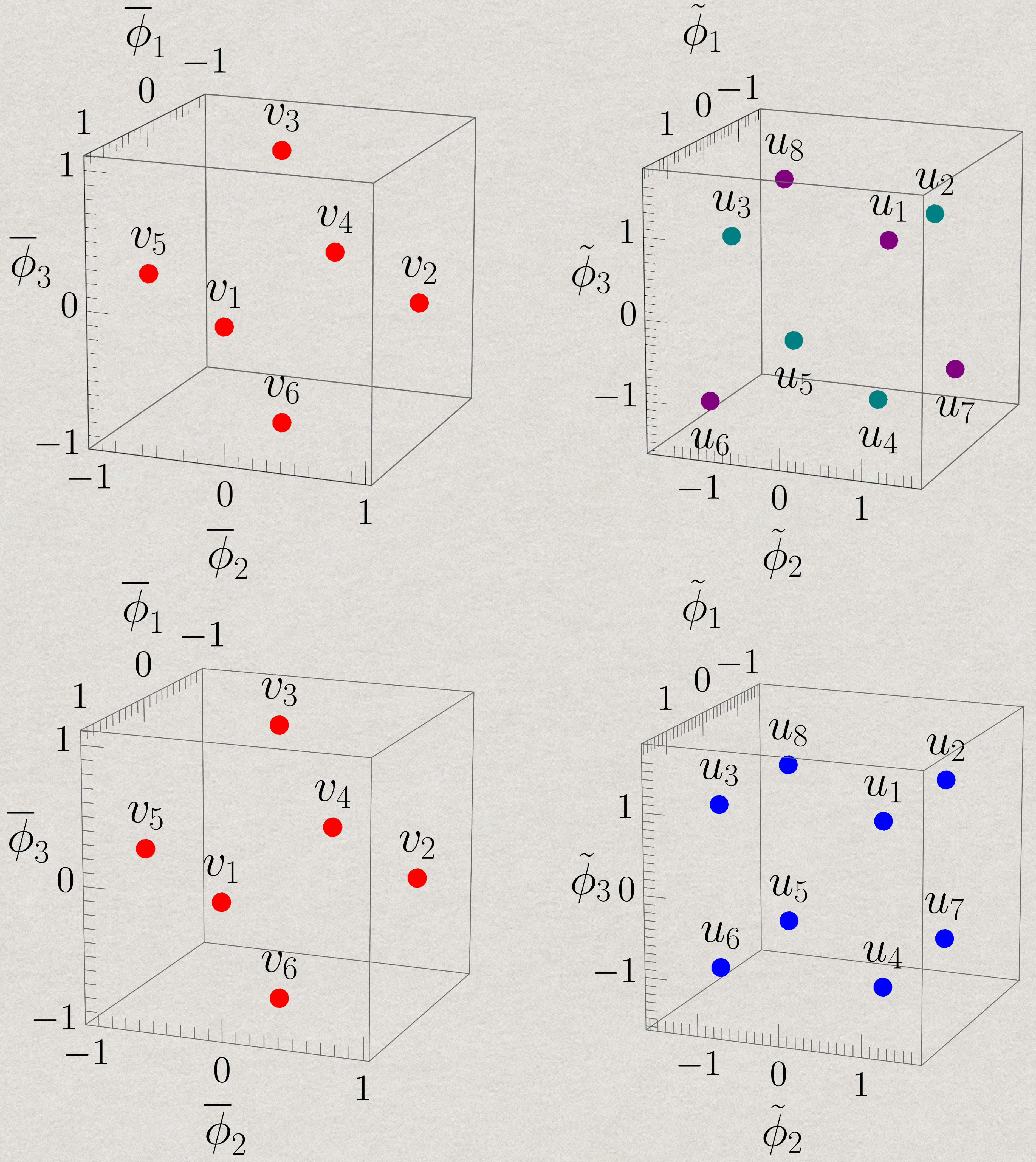
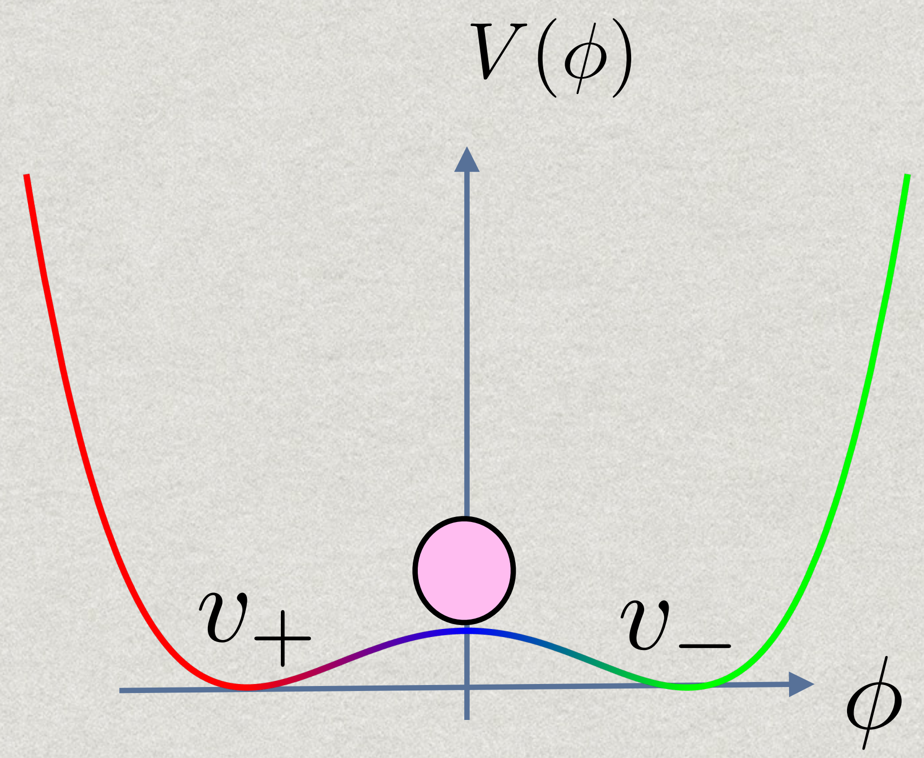


Meaning of different vacua?

Domain walls

Consider a real scalar field with $\mathbb{Z}_2$ -symmetric potential



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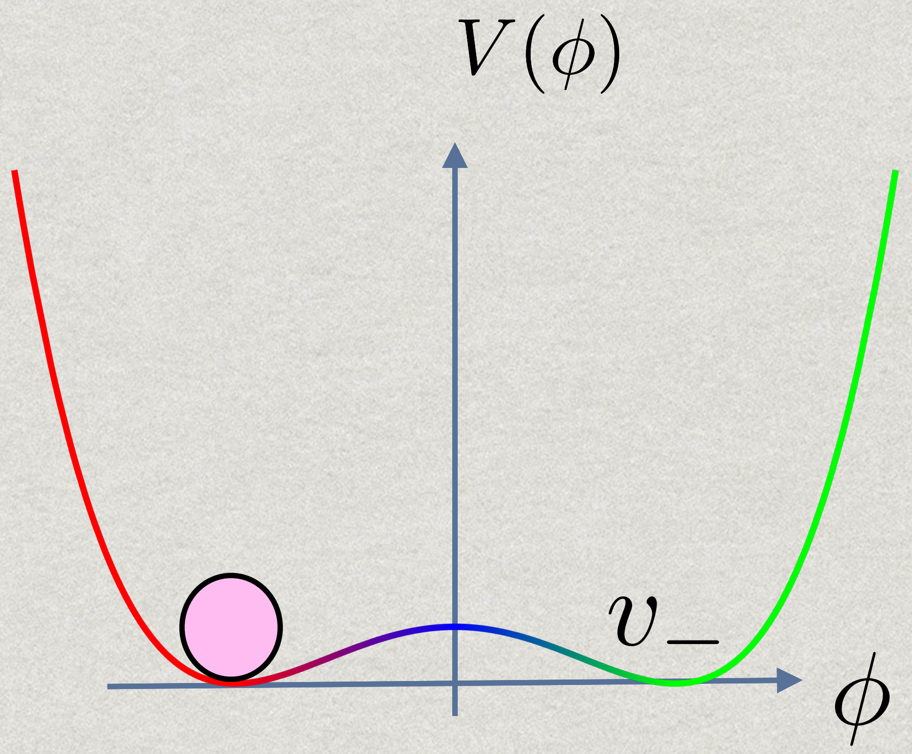

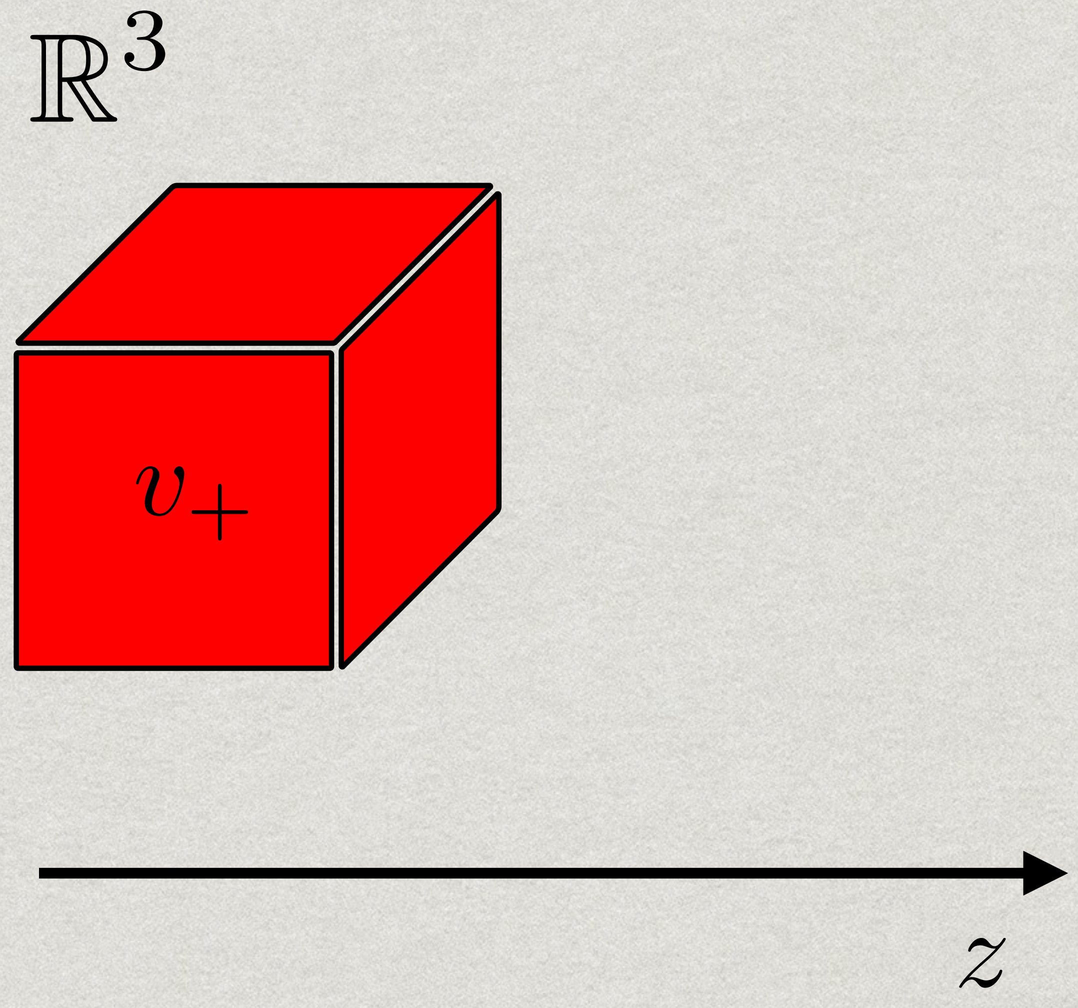

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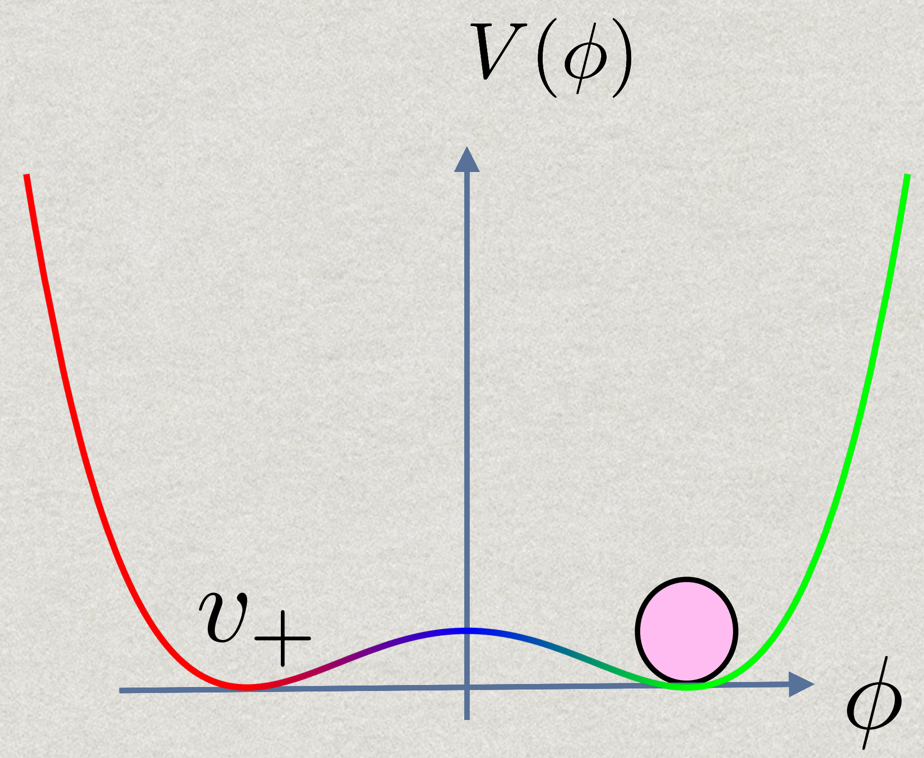

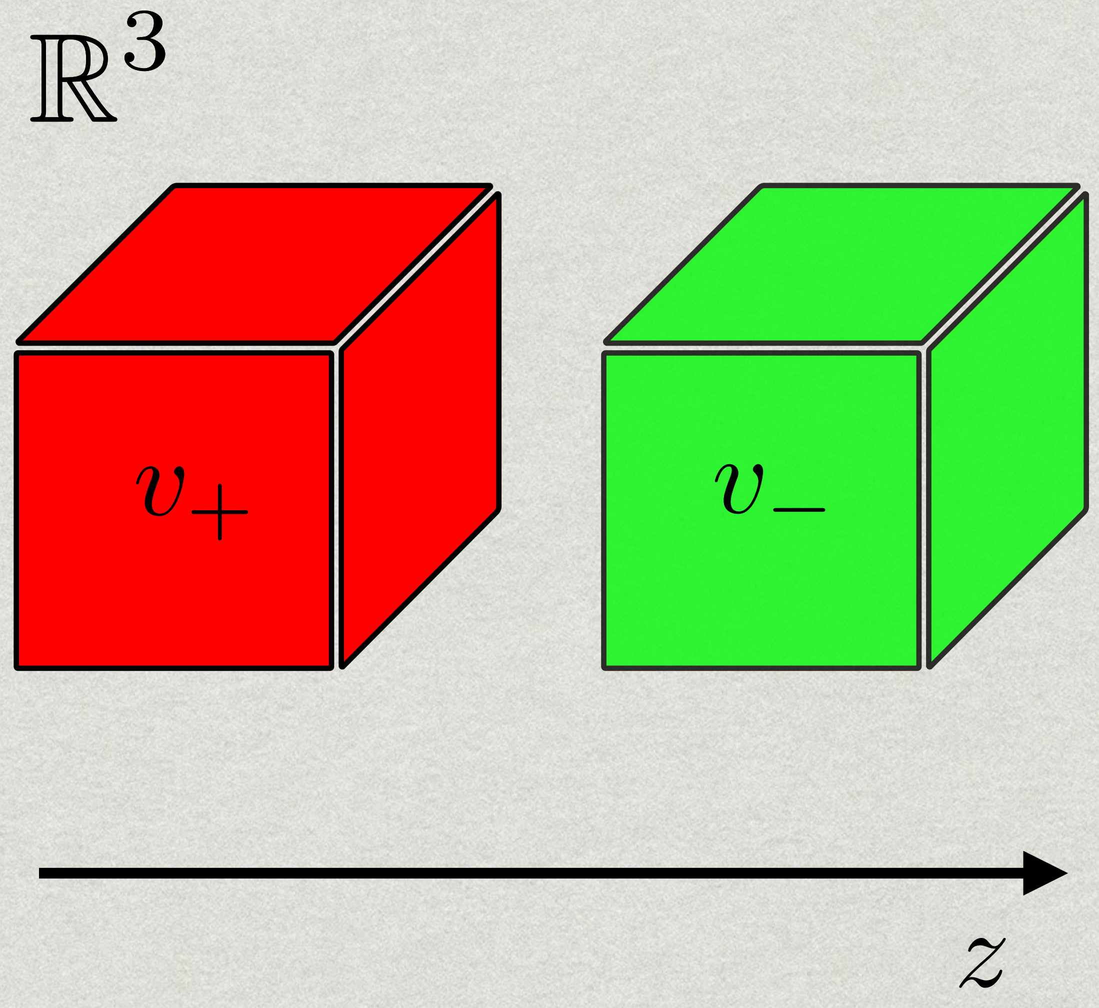

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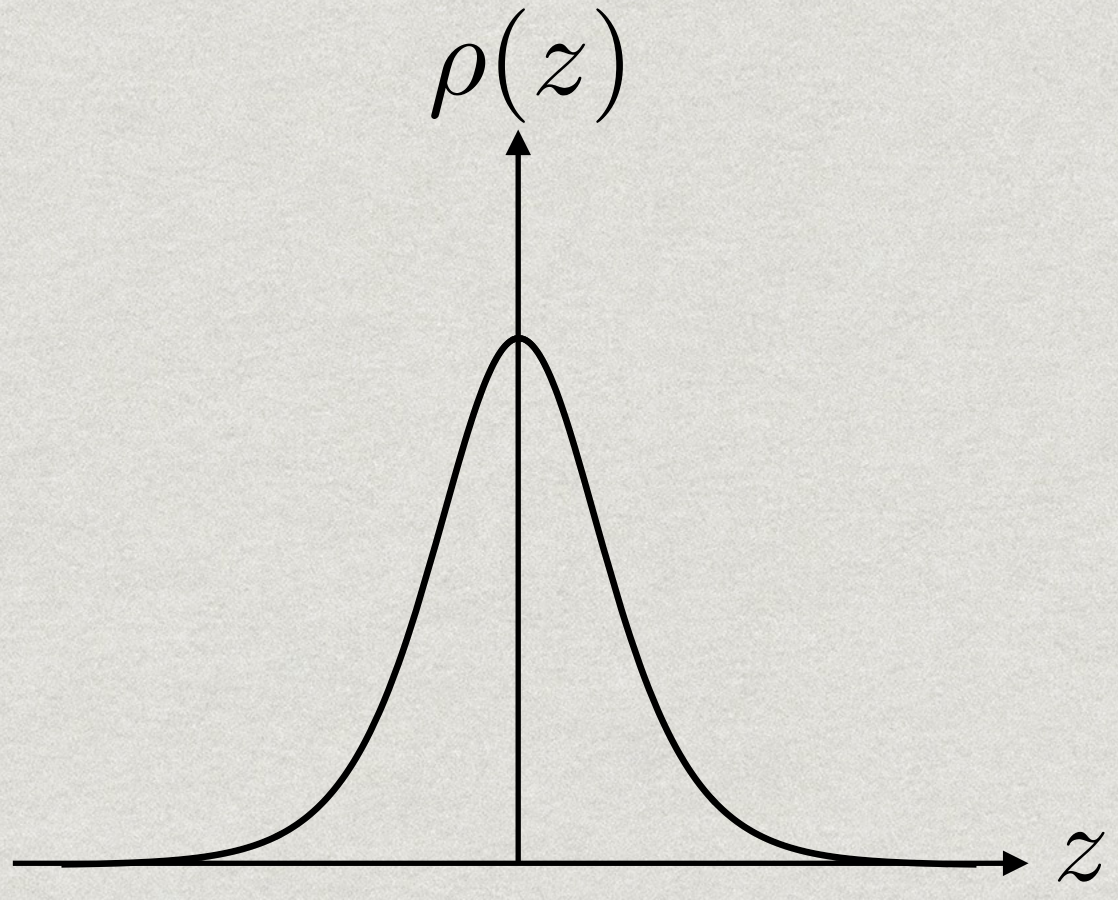
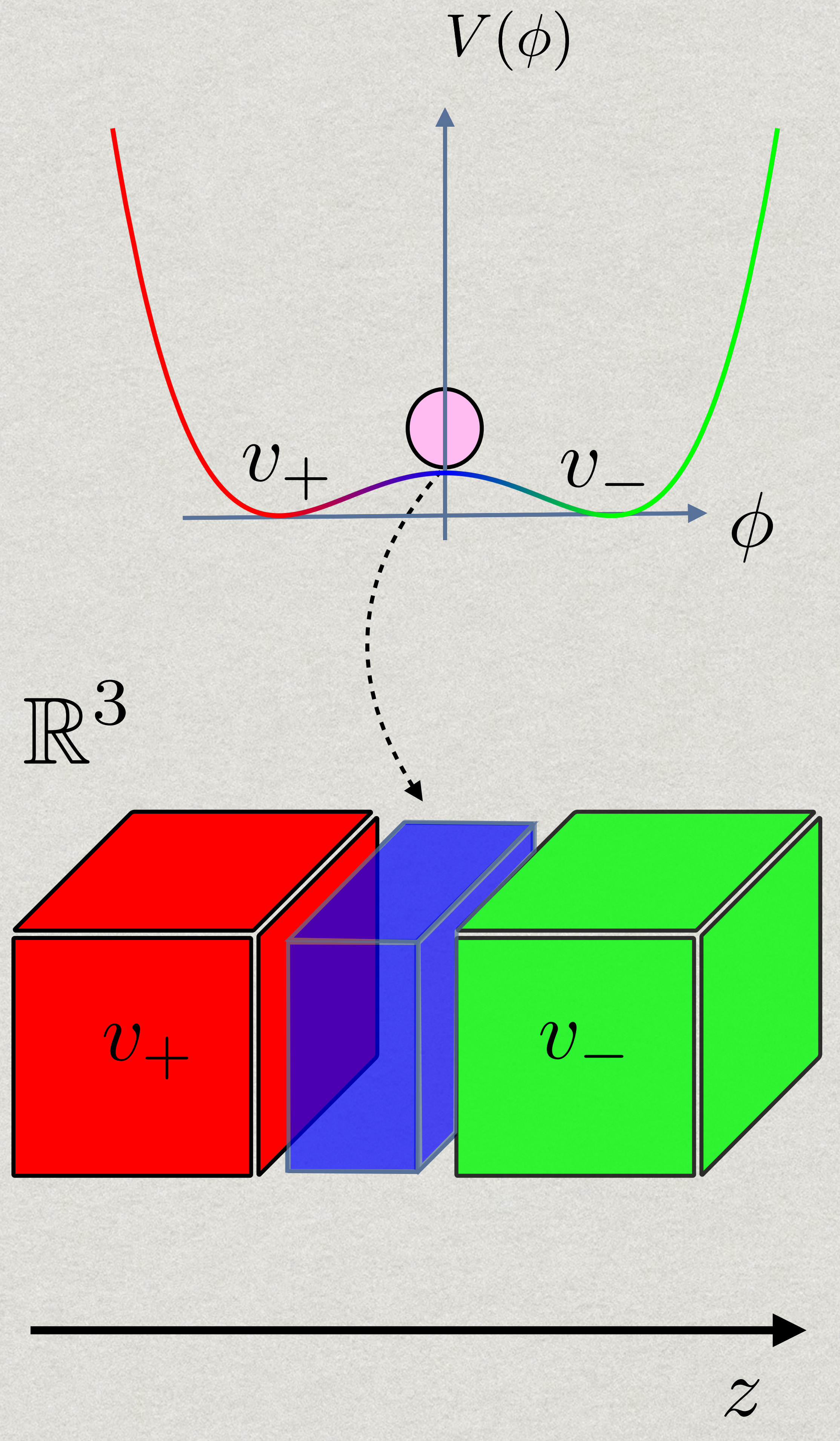
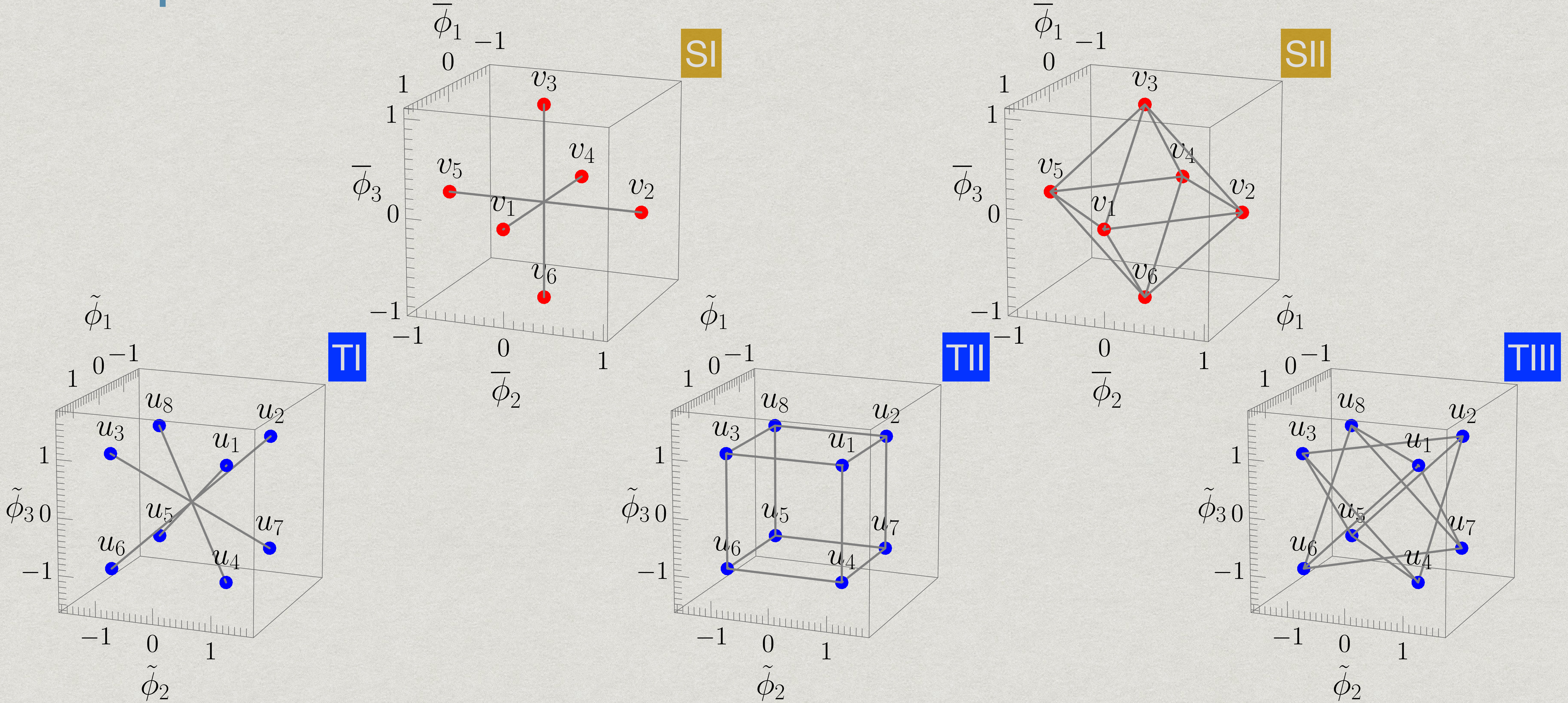




S_4 domain walls



A_4 domain walls

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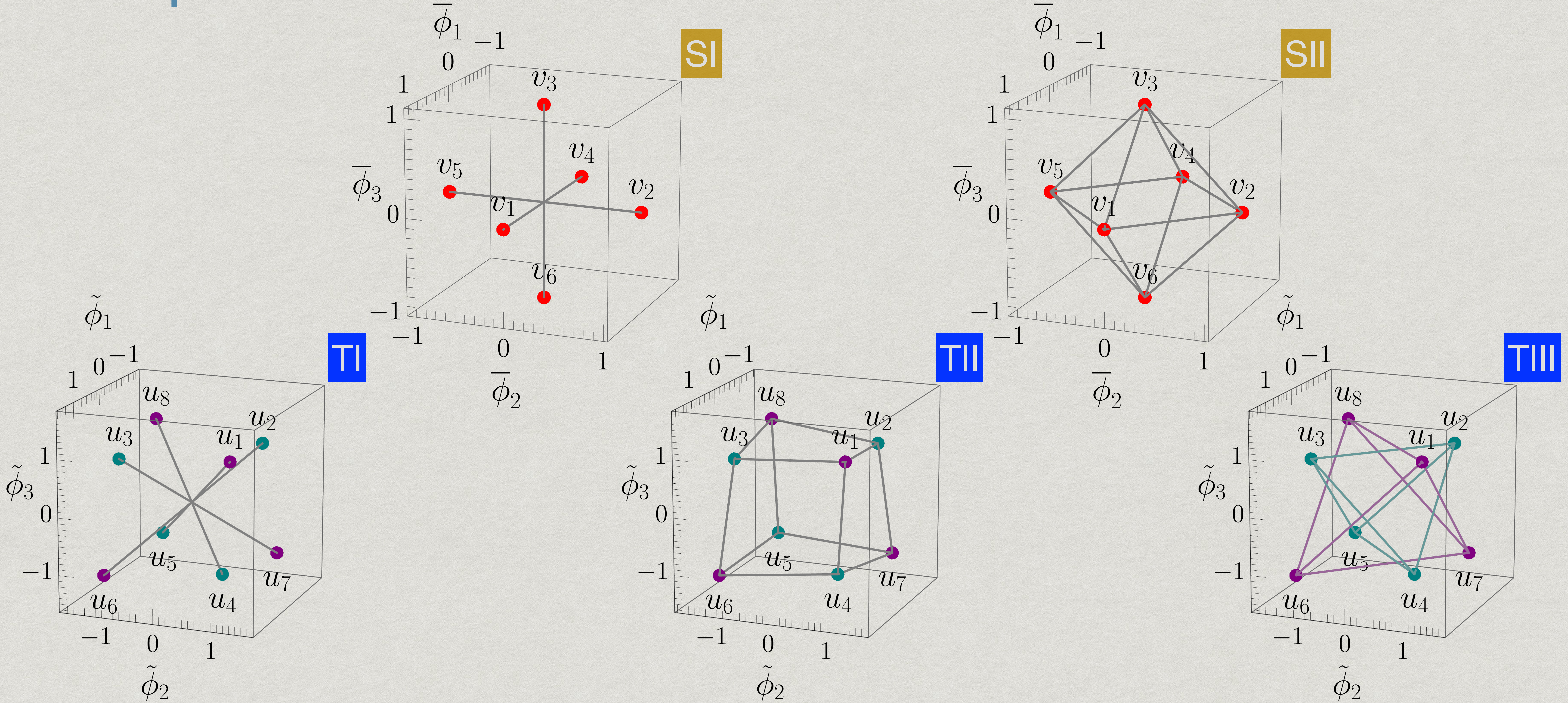
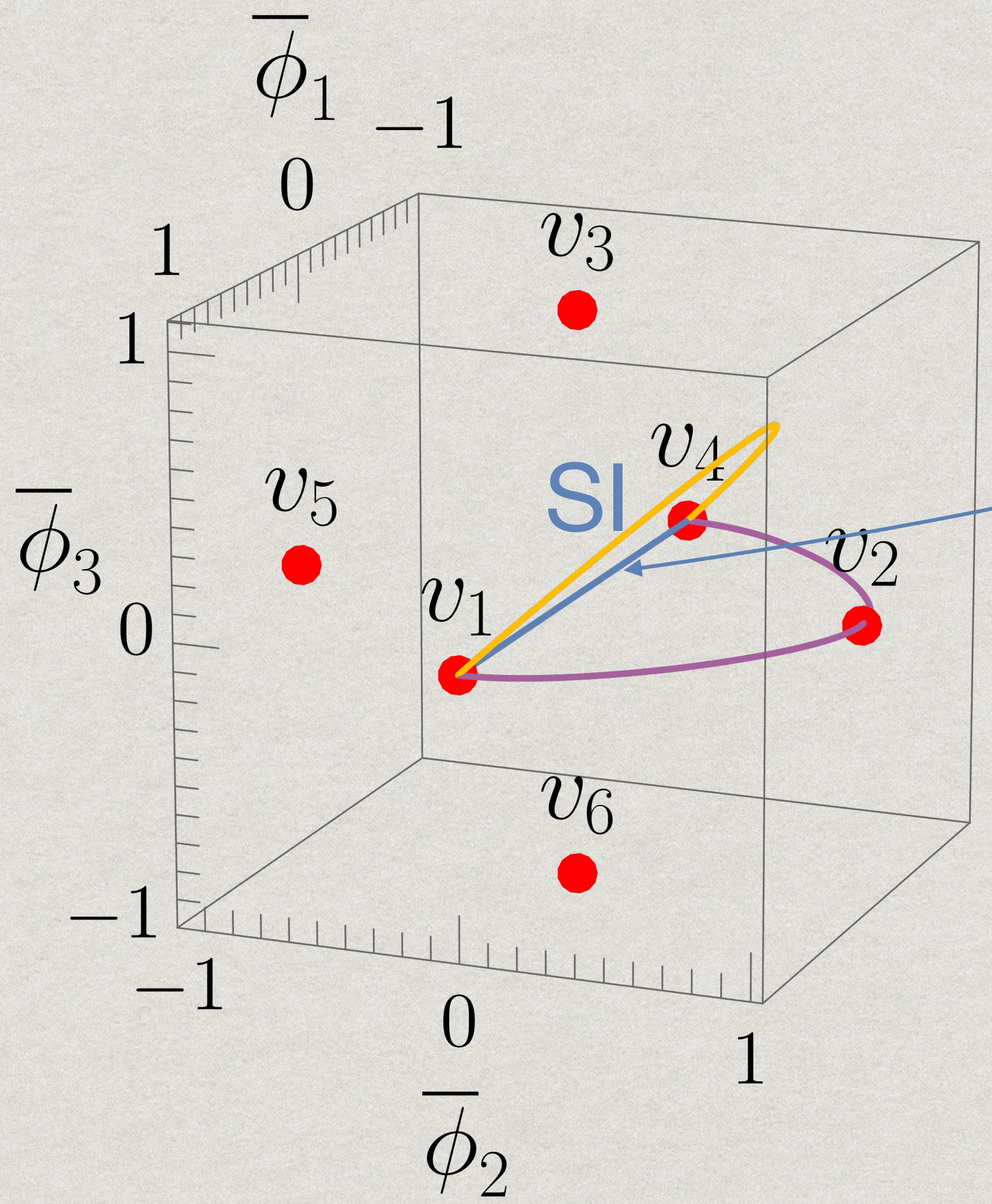


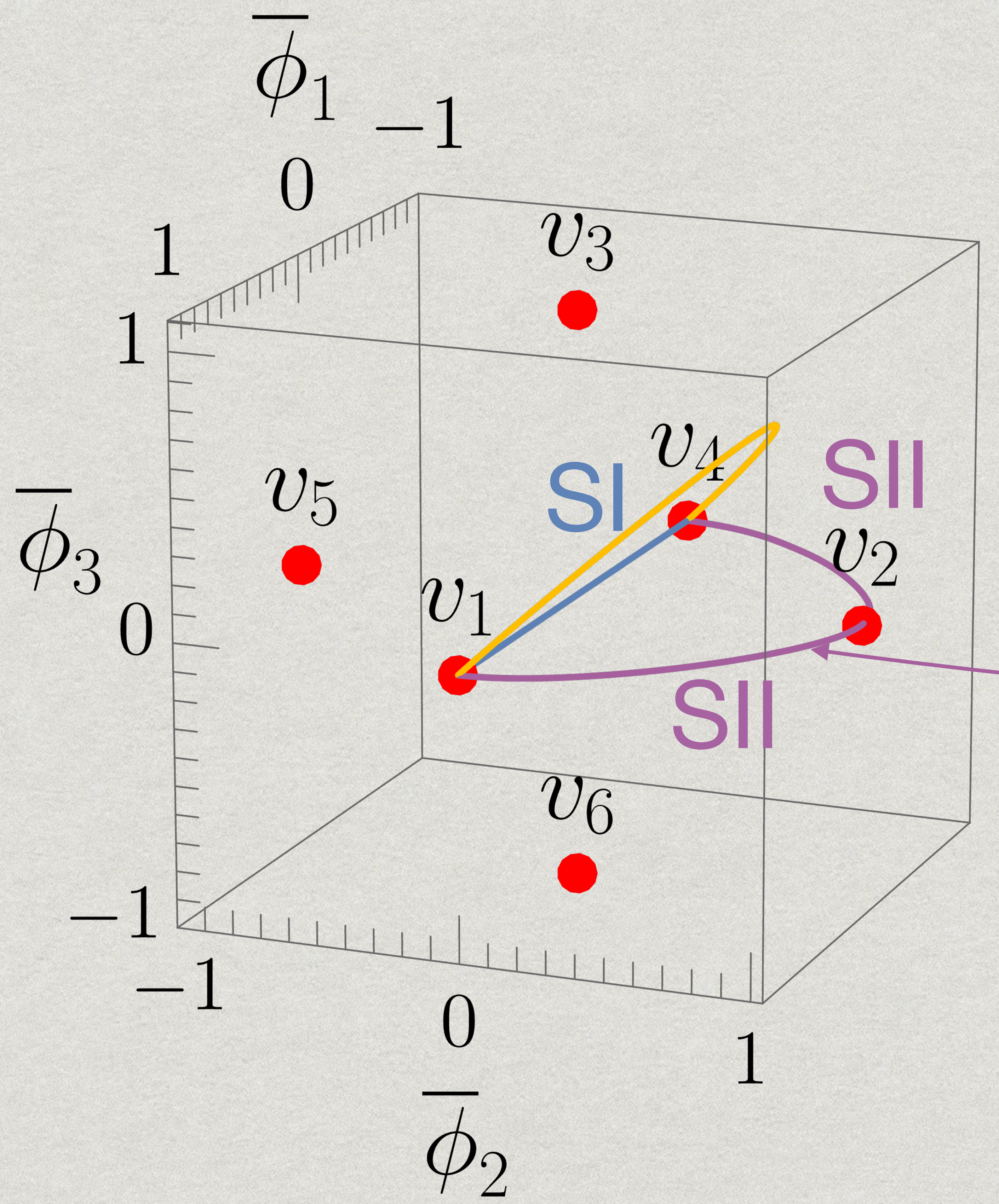
Stability of DWs



Straight line SI solution

Independent of $\beta = g_2/g_1$

Stability of DWs



Straight line SI solution

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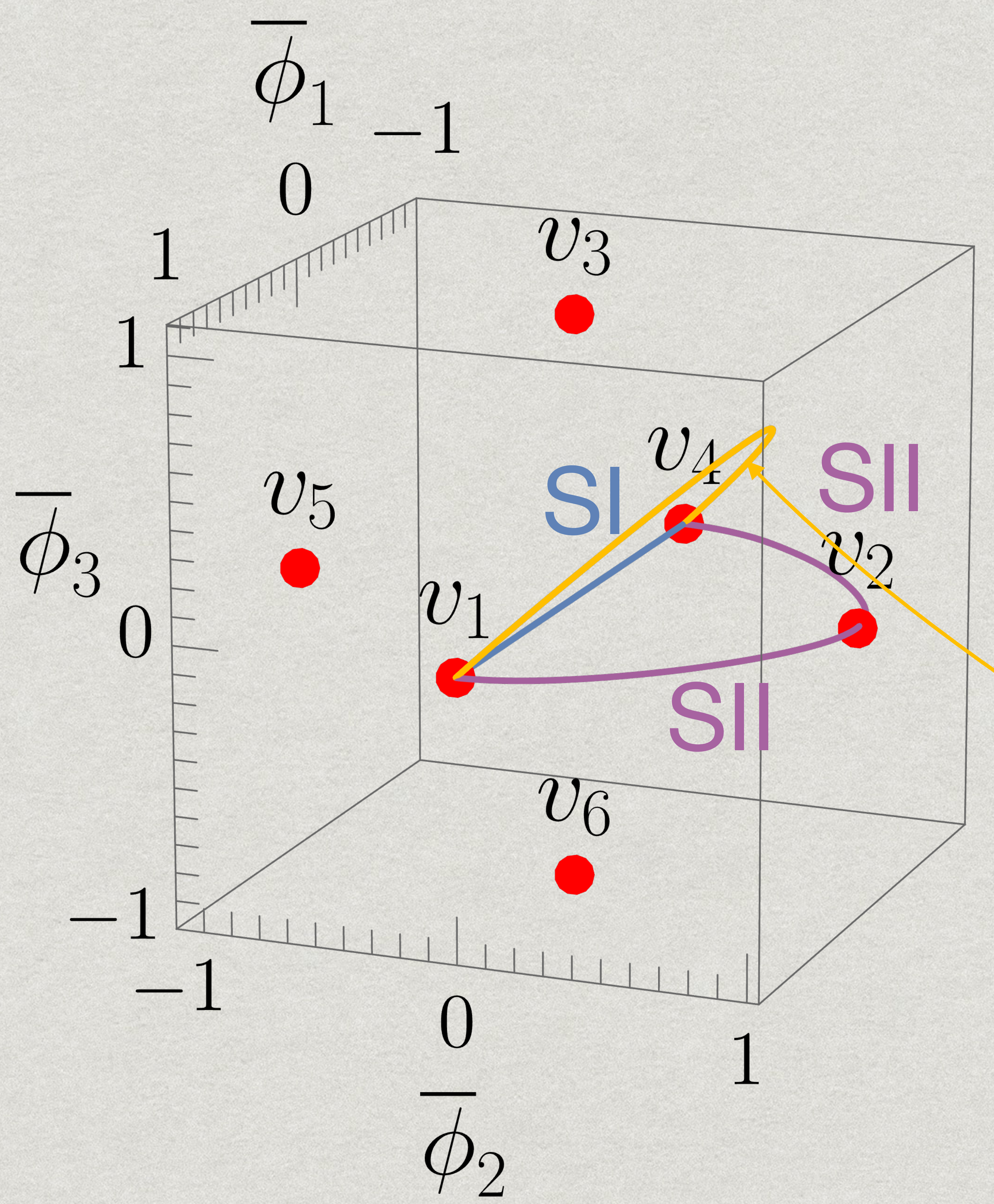
Two SIII solutions

with pitstop at v_2

Stability of DWs



Straight line SI solution

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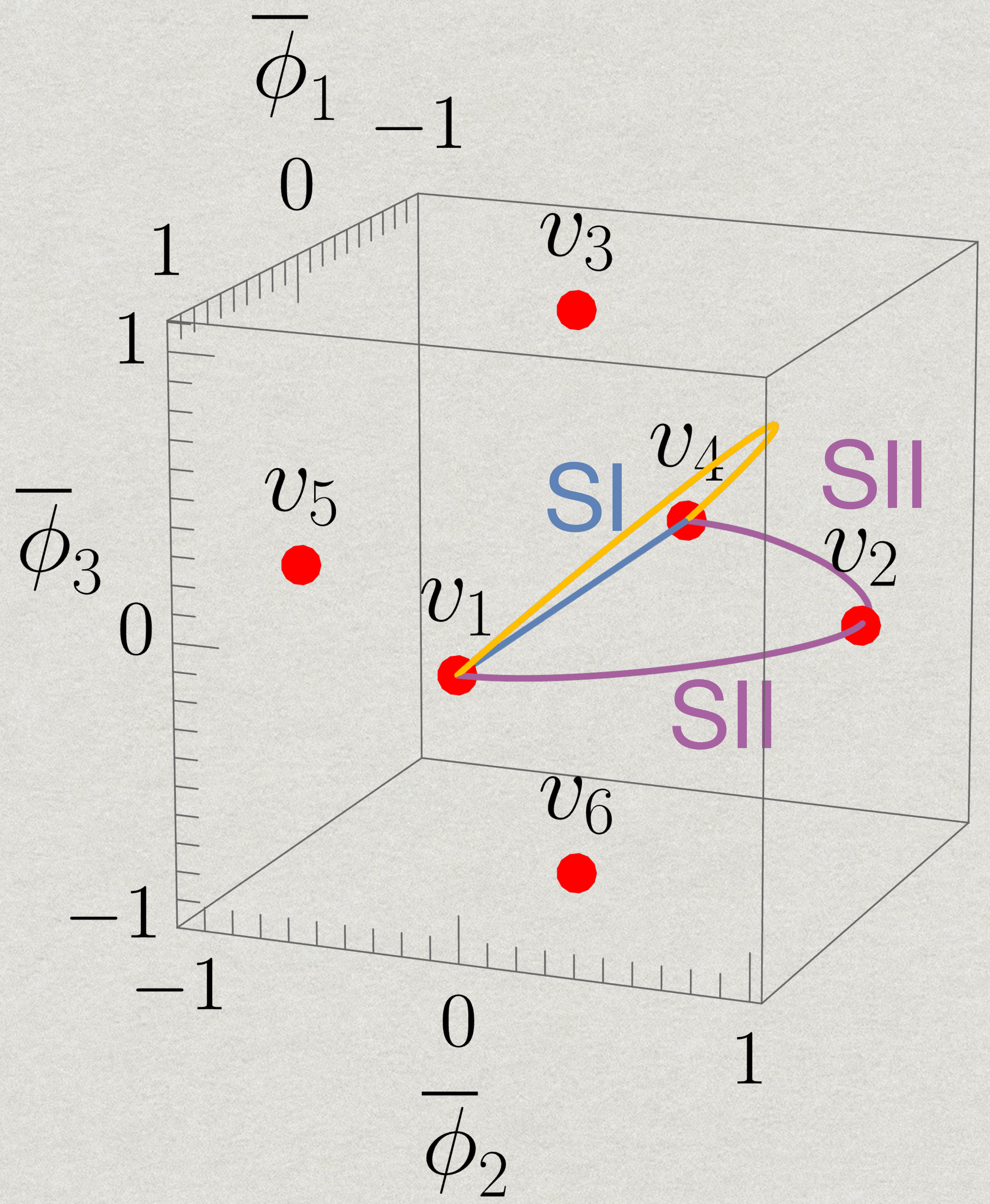
Two SIII solutions

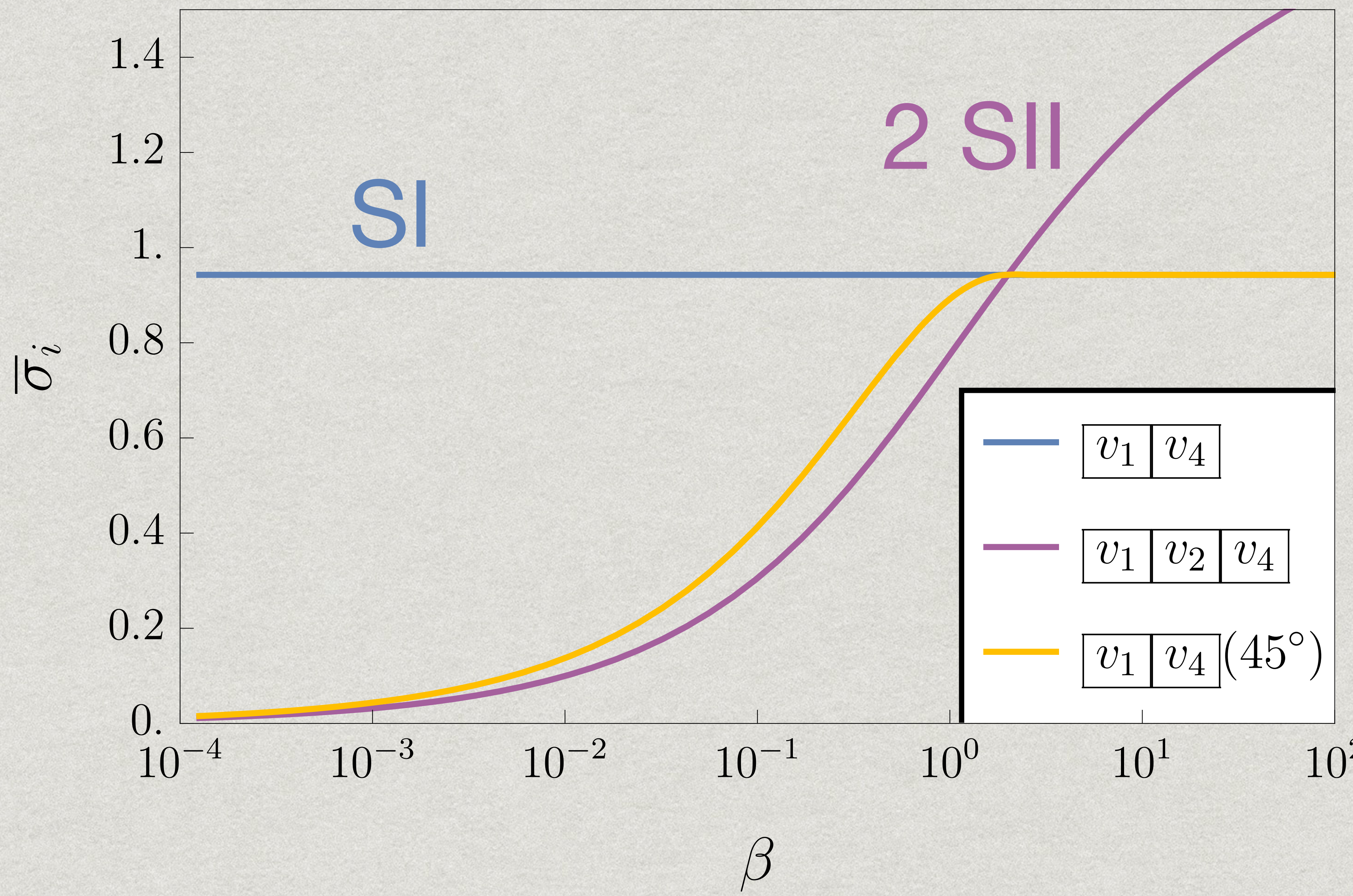
with pitstop at v_2

Intermediate solution

(still satisfies EoM)

Stability of DWs

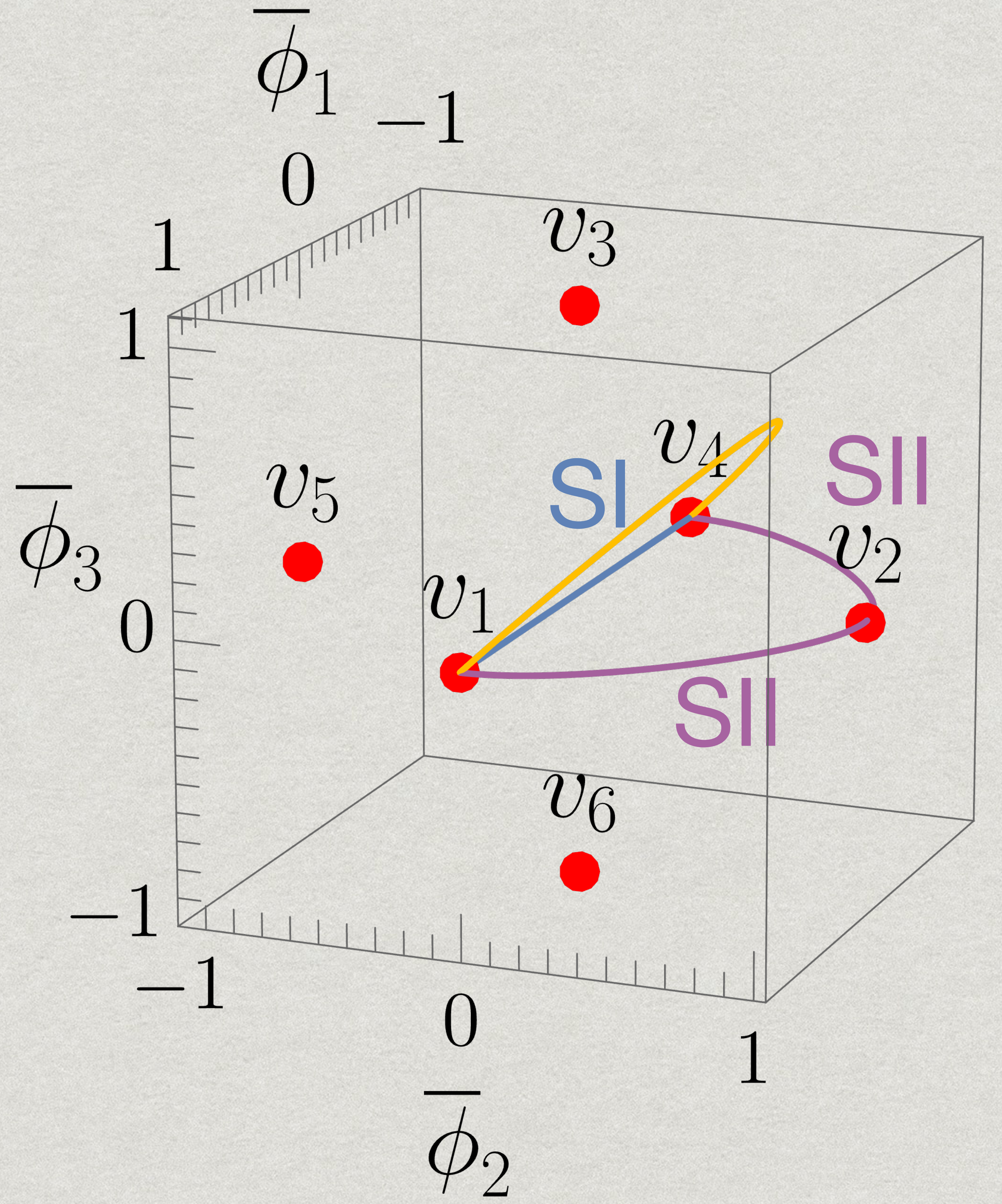


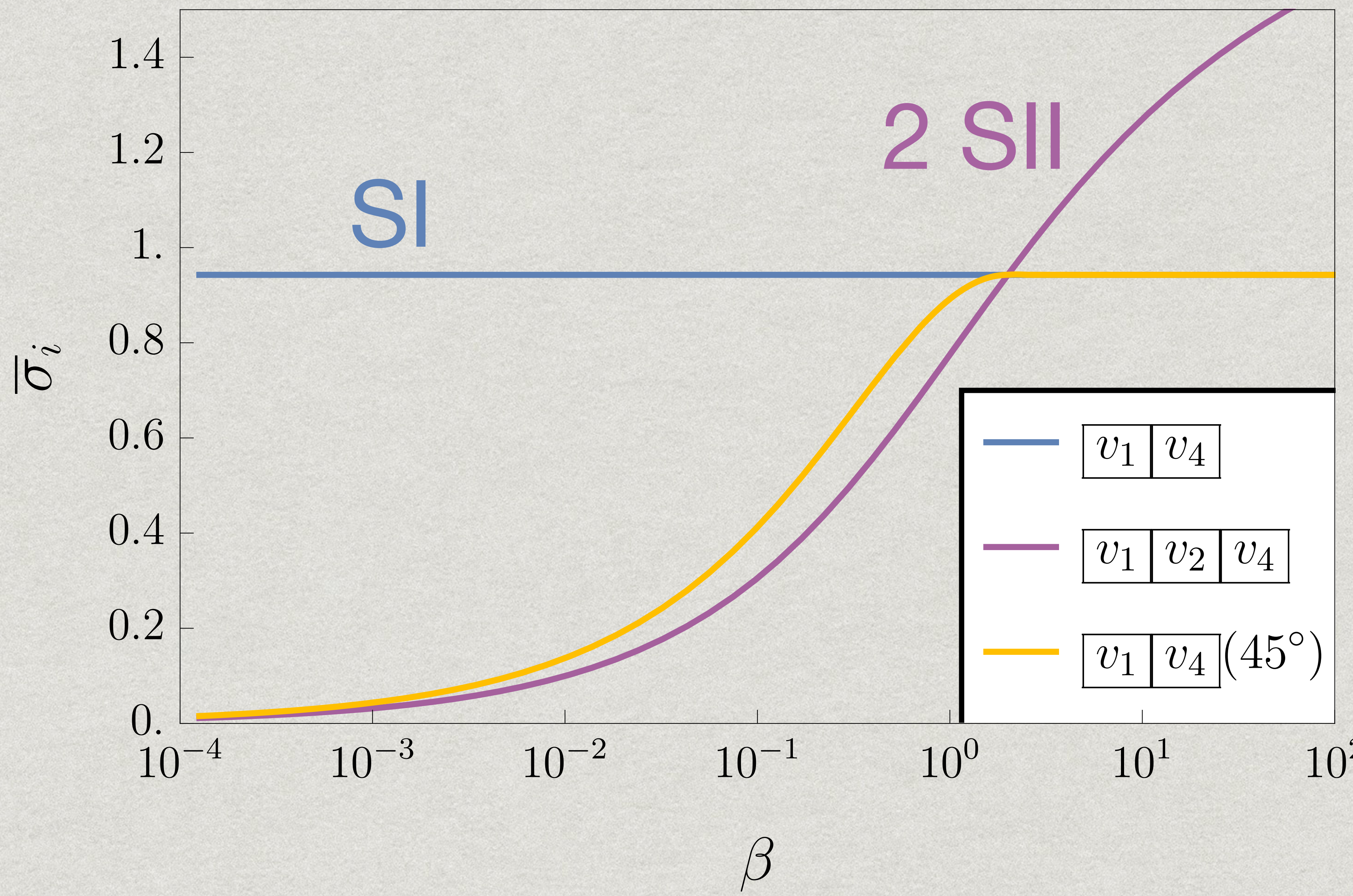


$$\beta \ll 2 : \sigma(\mathrm{SI}) > 2\sigma(\mathrm{SII})$$

SI DW **unstable** & would decay to SII

Stability of DWs

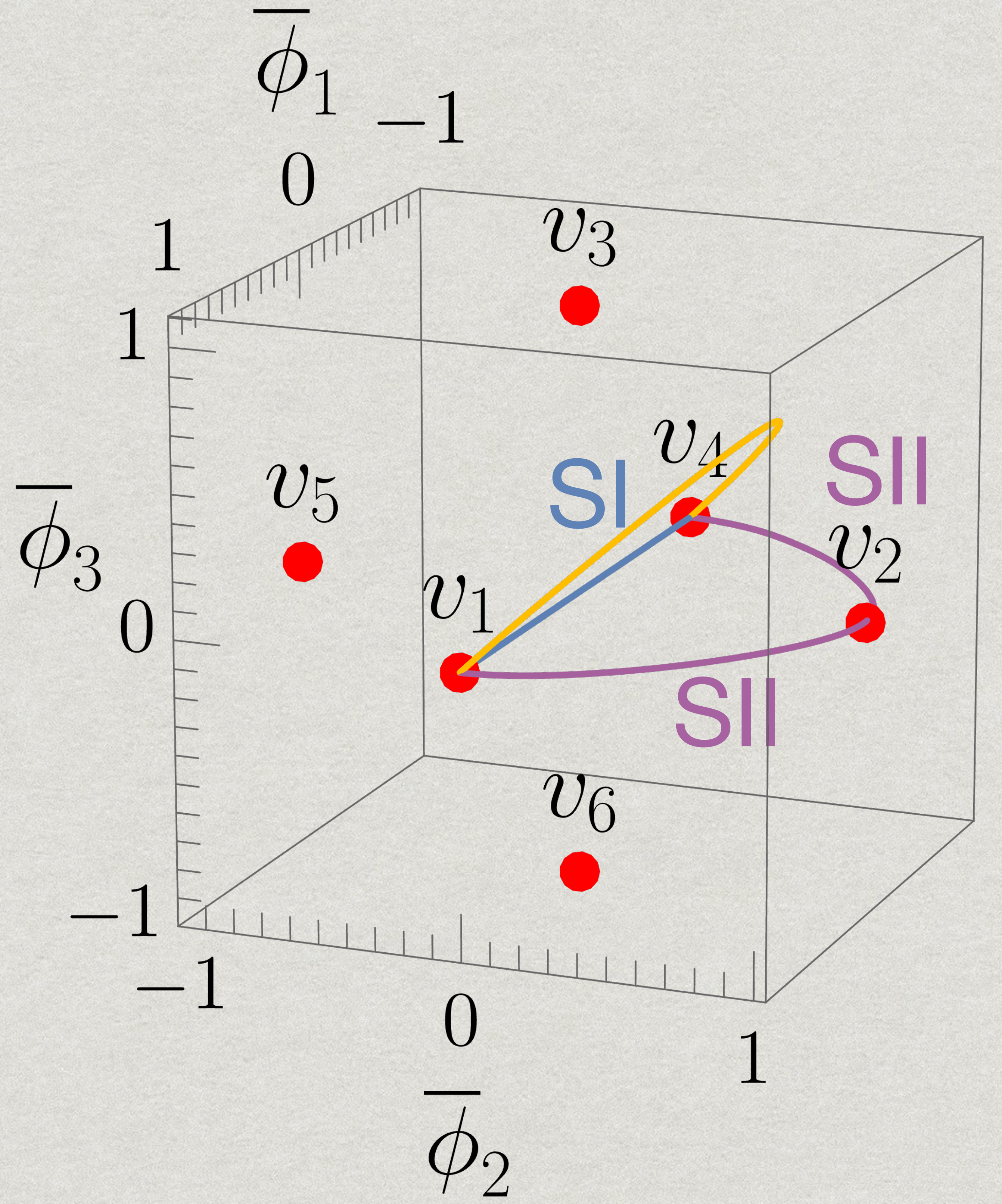


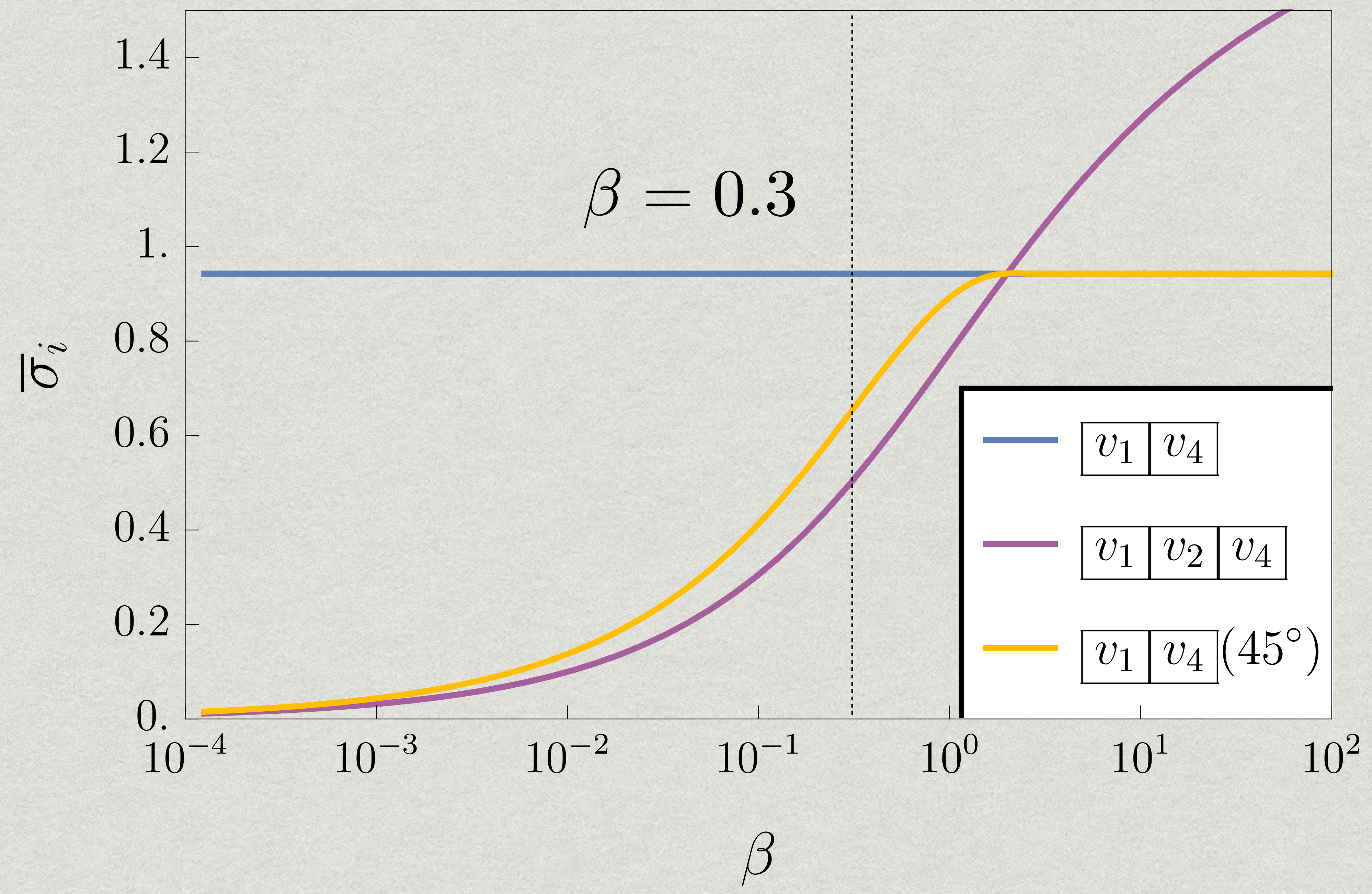


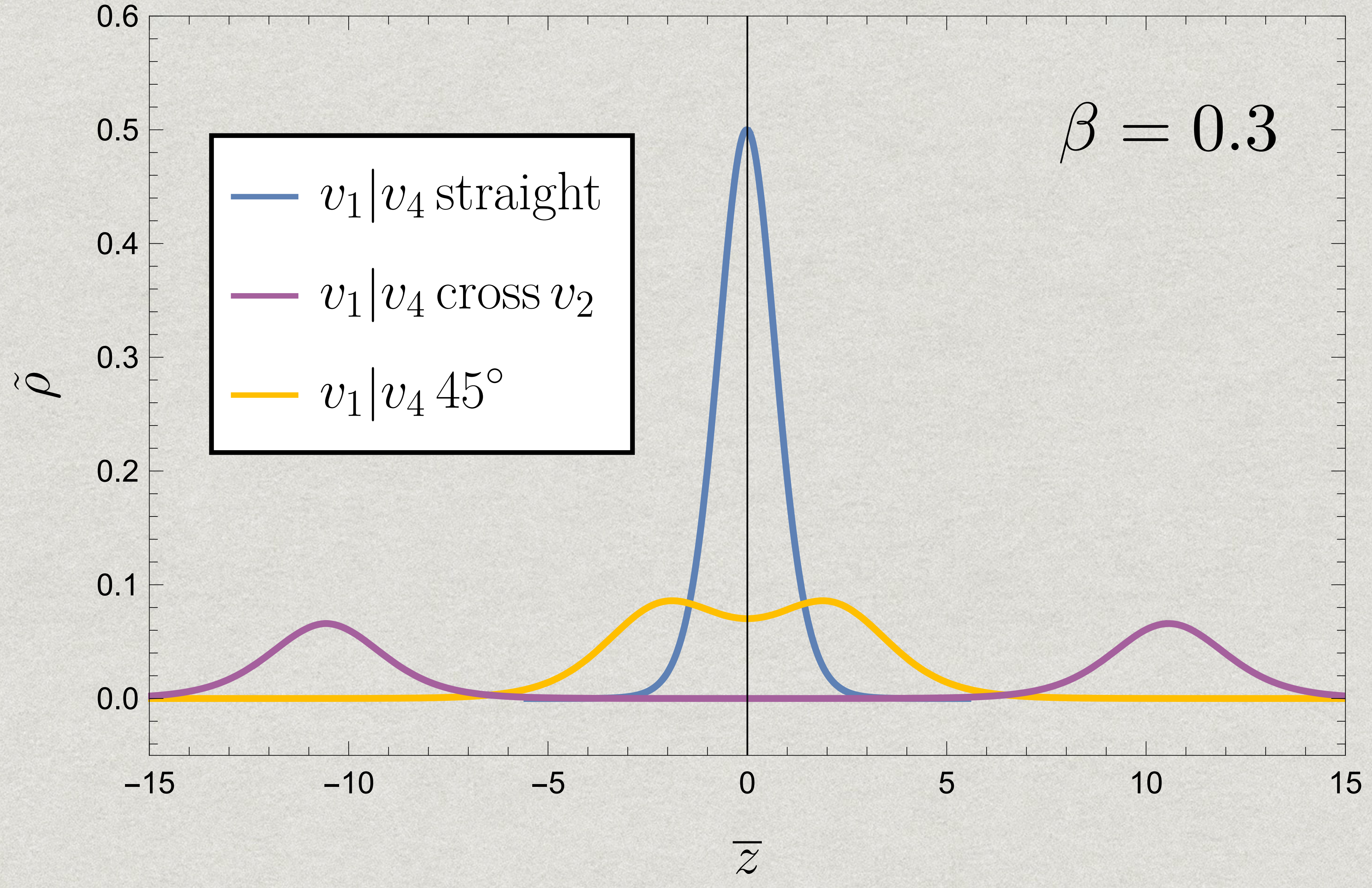
$$\beta \gg 2 : \sigma(\mathrm{SI}) < 2\sigma(\mathrm{SII})$$

SI DW **stable**

Stability of DWs







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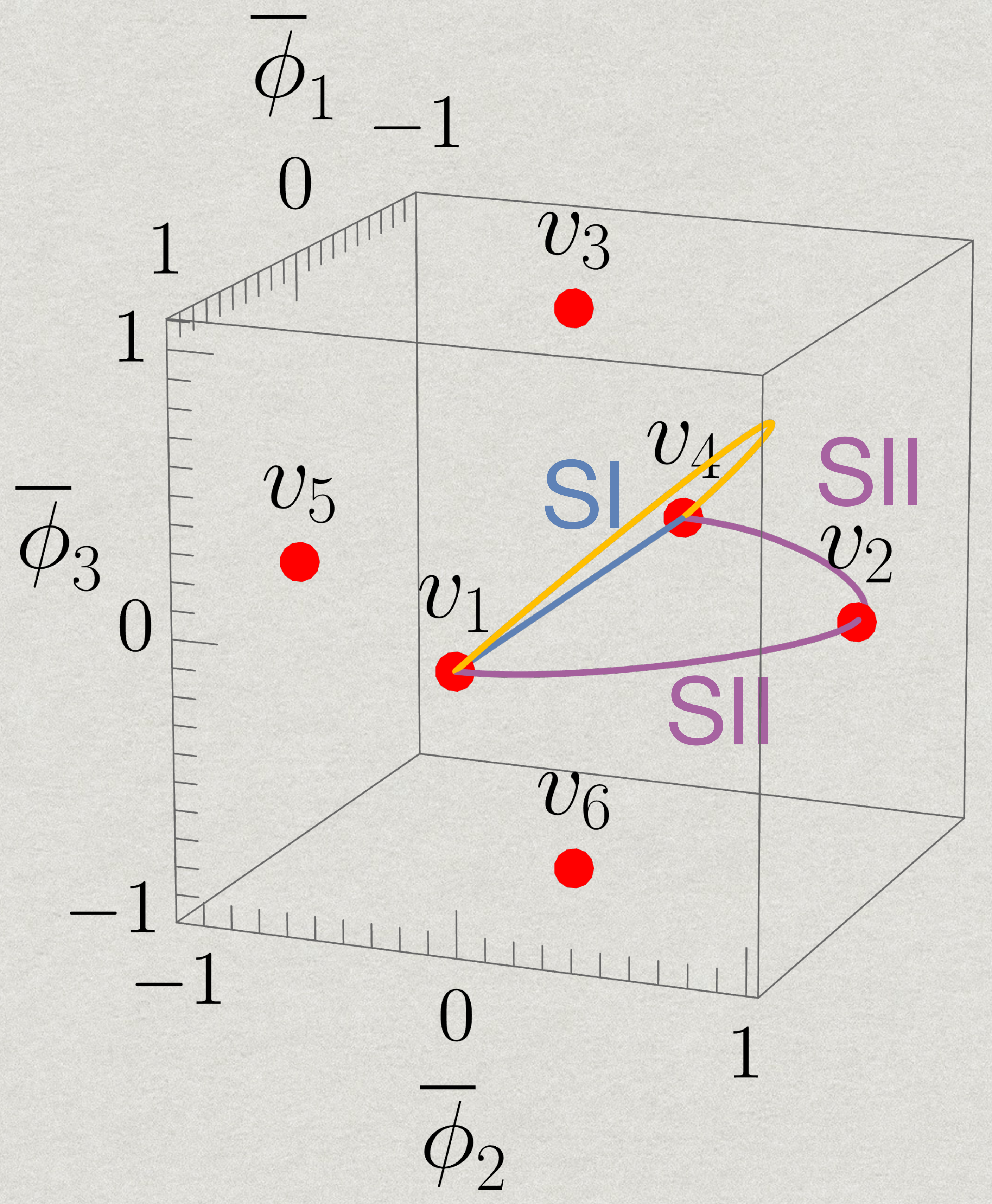
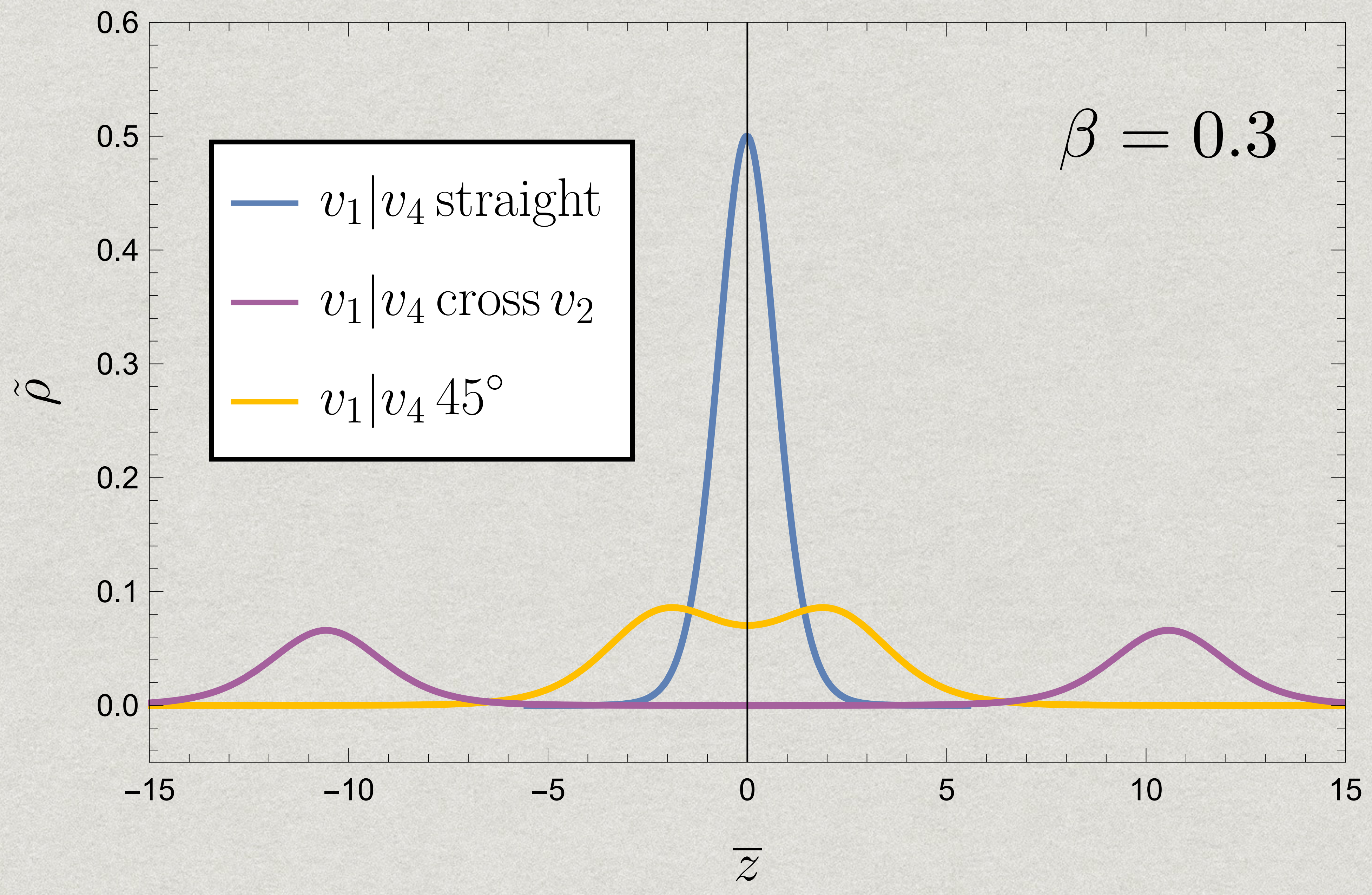


For $\beta = 0.3$, the SI-type DW will decay to two SII type DWs



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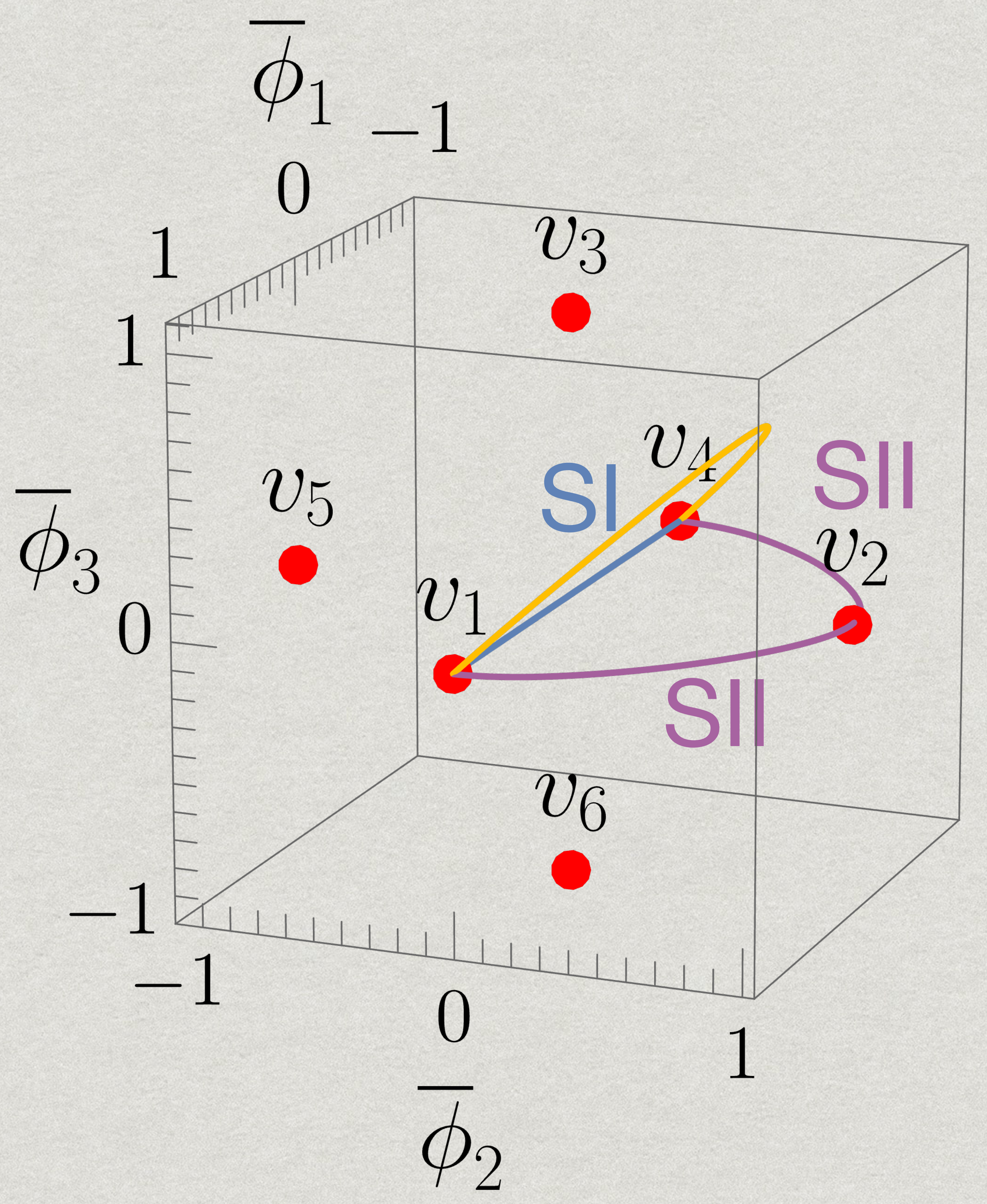

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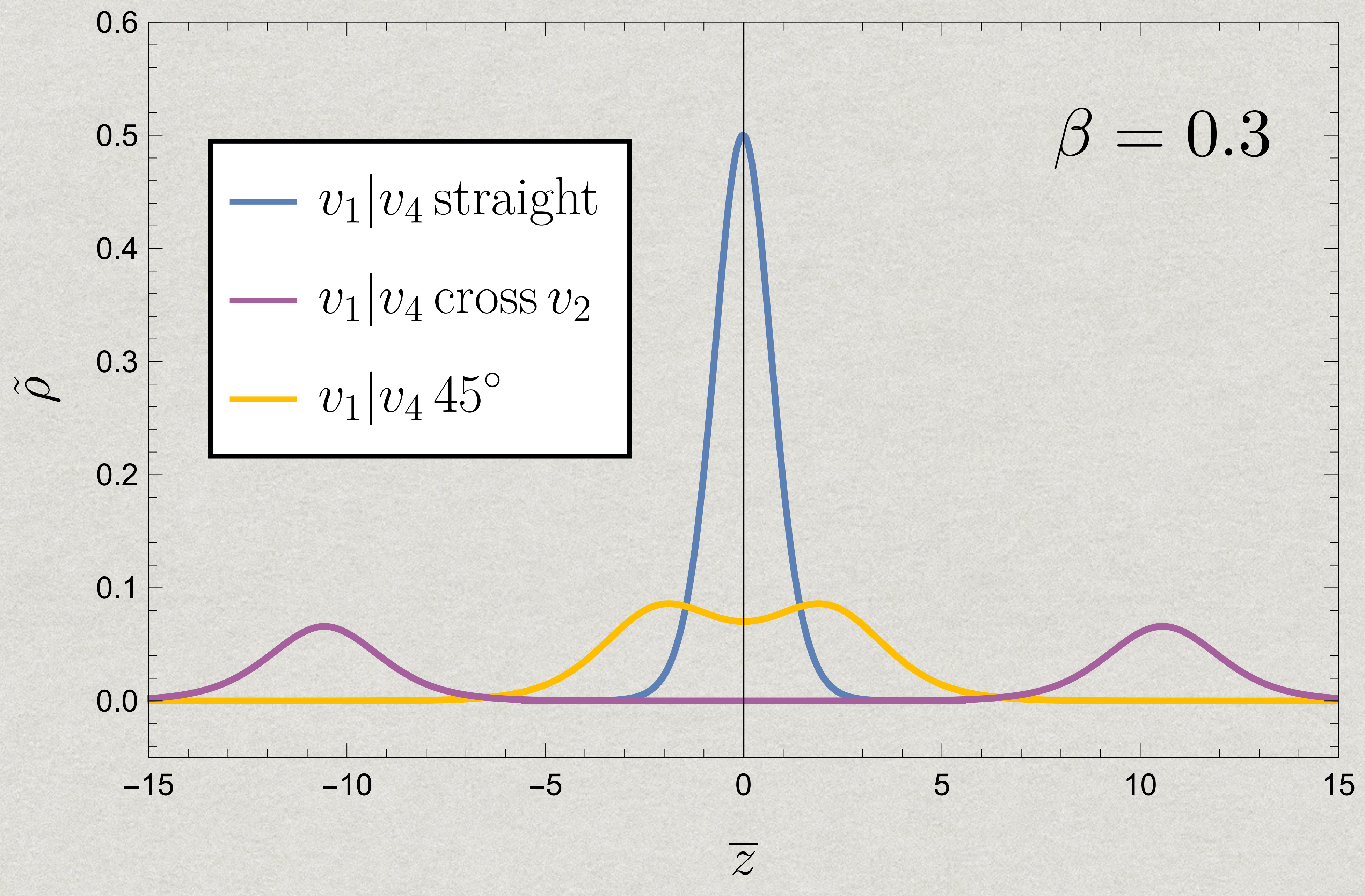
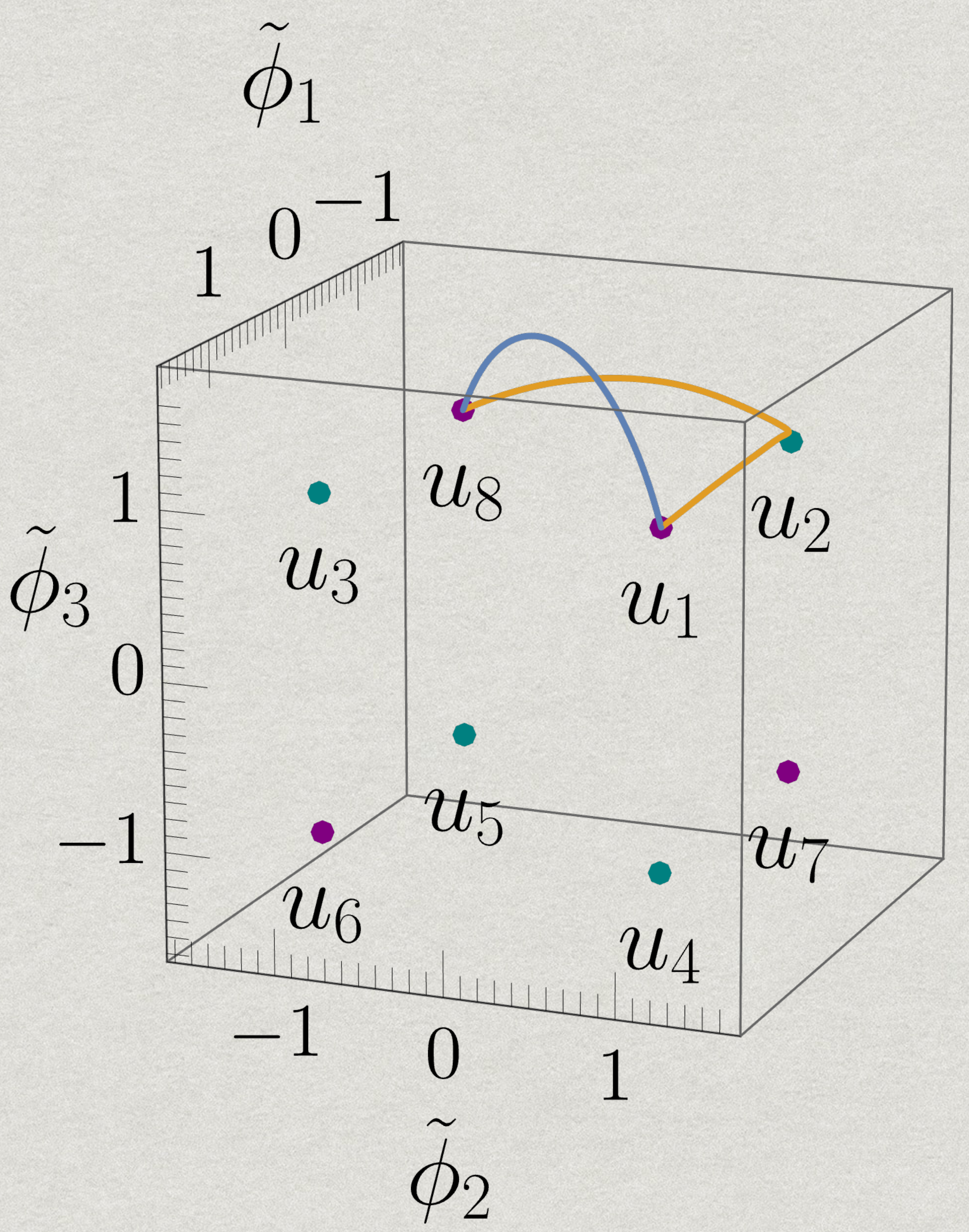
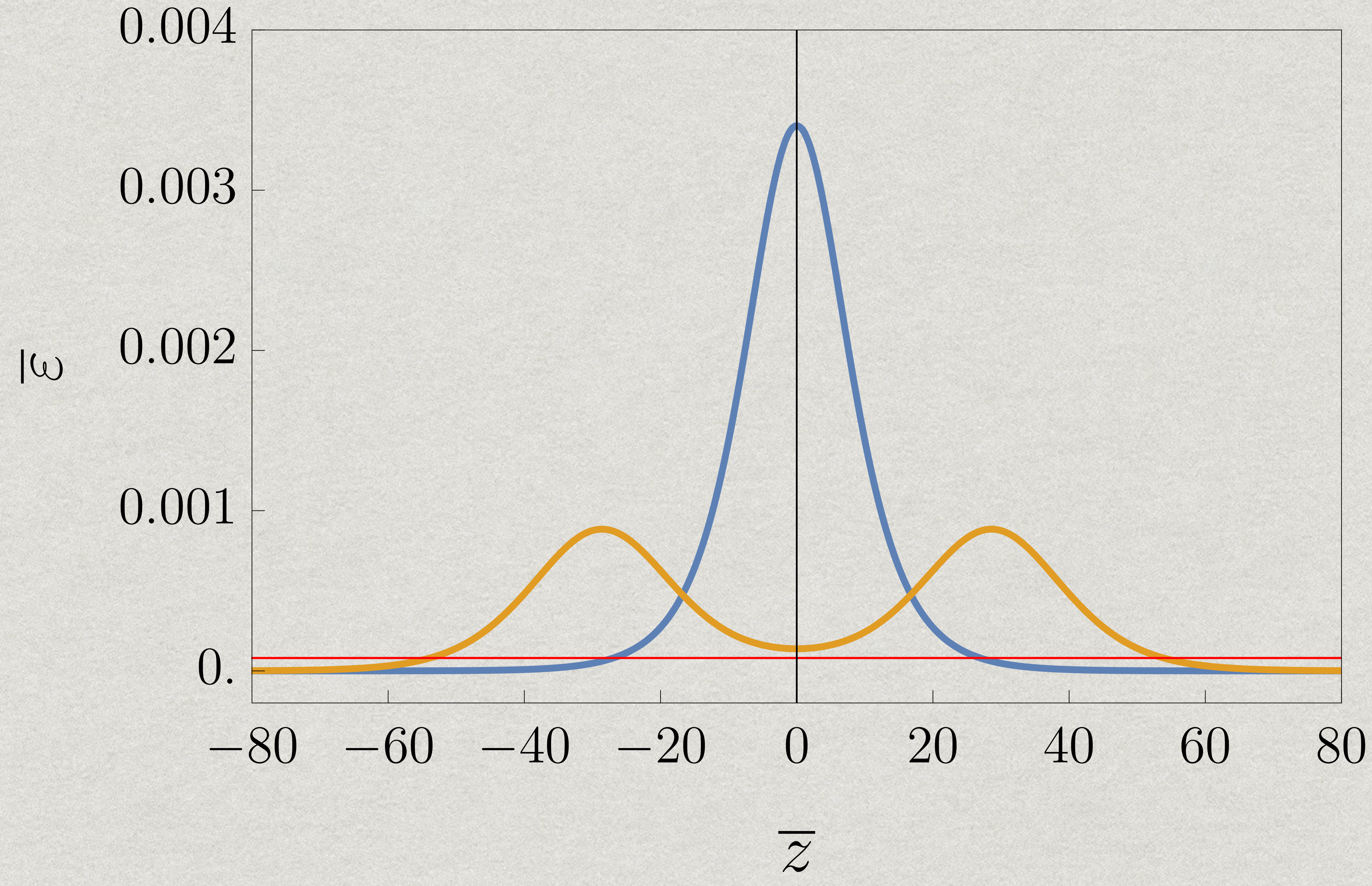


Stability of DWs

$$a = \frac{A}{2\mu\sqrt{3g_1 + 2g_2}} = -0.00003$$

$$\beta = g_2/g_1 = -0.01$$

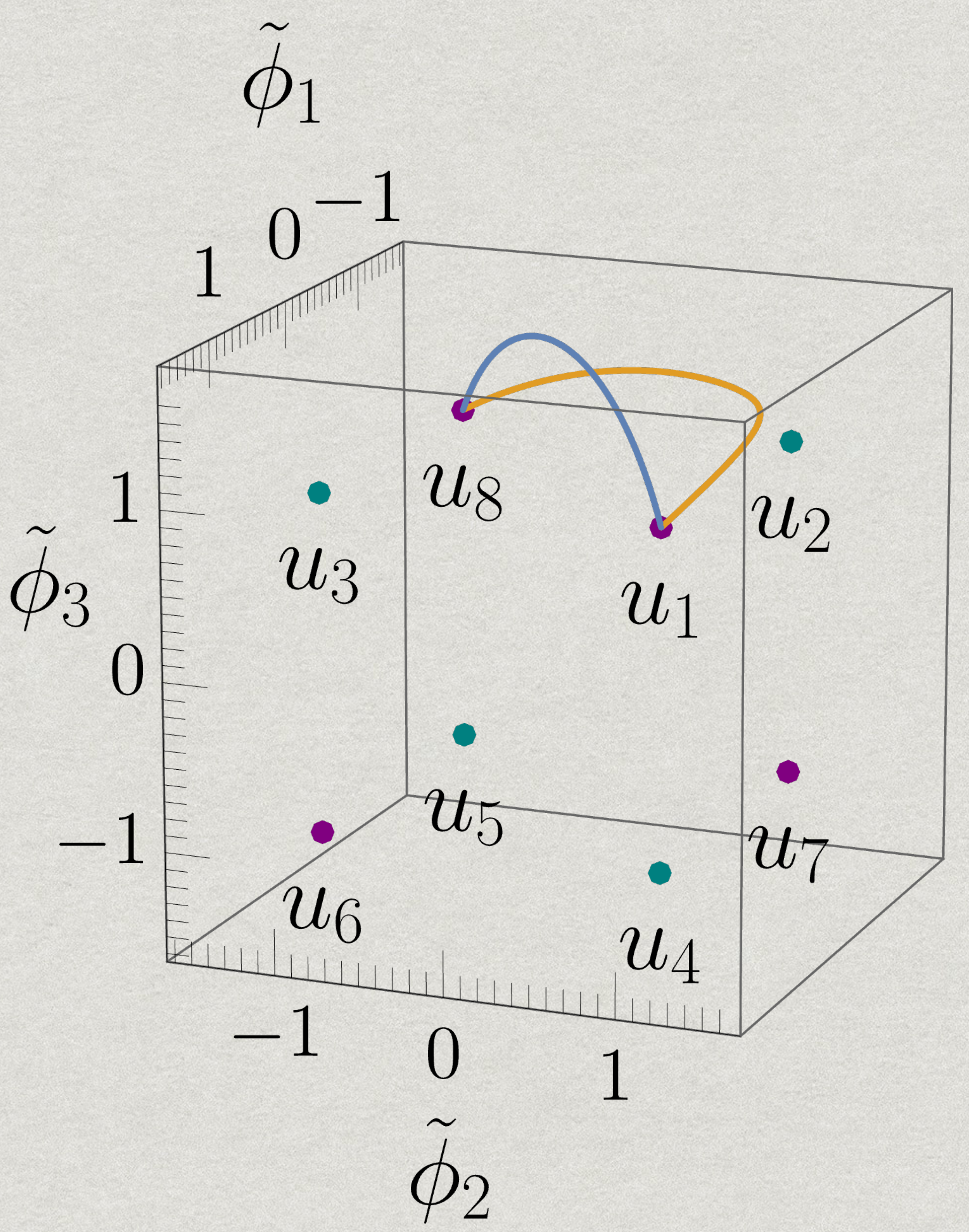


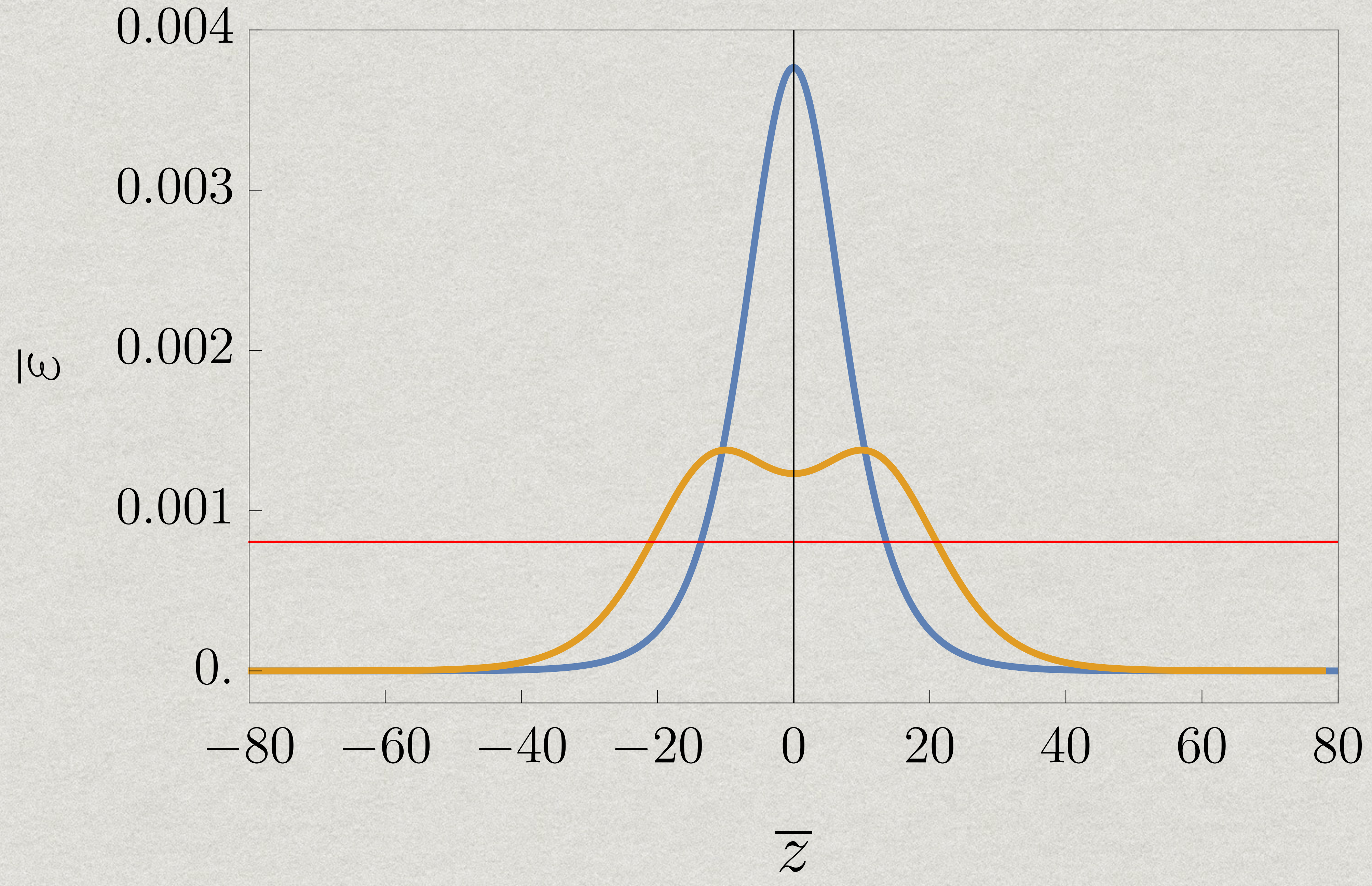


Stability of DWs

$$a = \frac{A}{2\mu\sqrt{3g_1 + 2g_2}} = -\,0.0003$$

$$\beta = g_2/g_1 = -\,0.01$$





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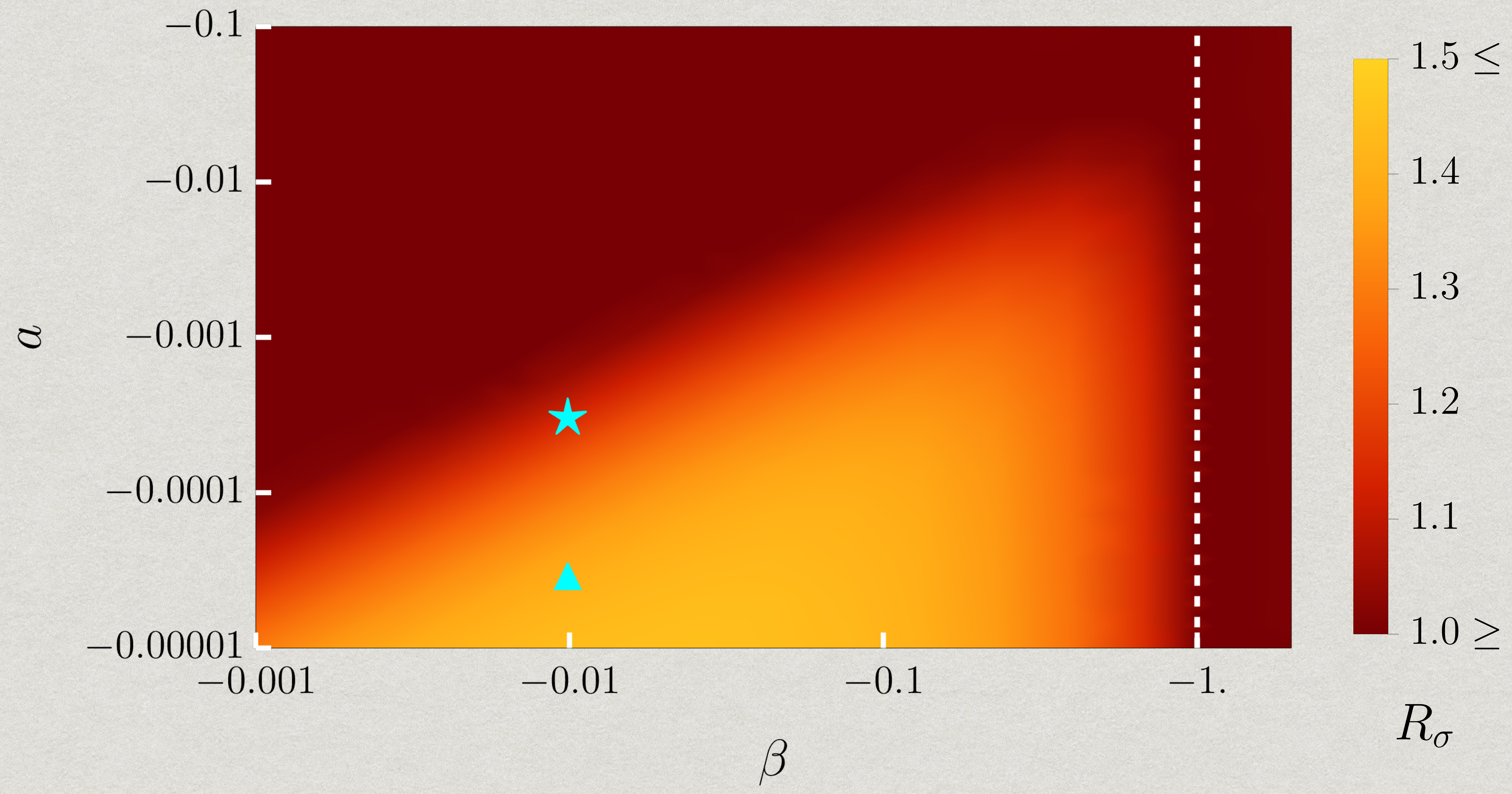

$$R_\sigma = \sigma_{1\mathrm{p}}/\sigma_{2\mathrm{p}}$$

Gravitational wave

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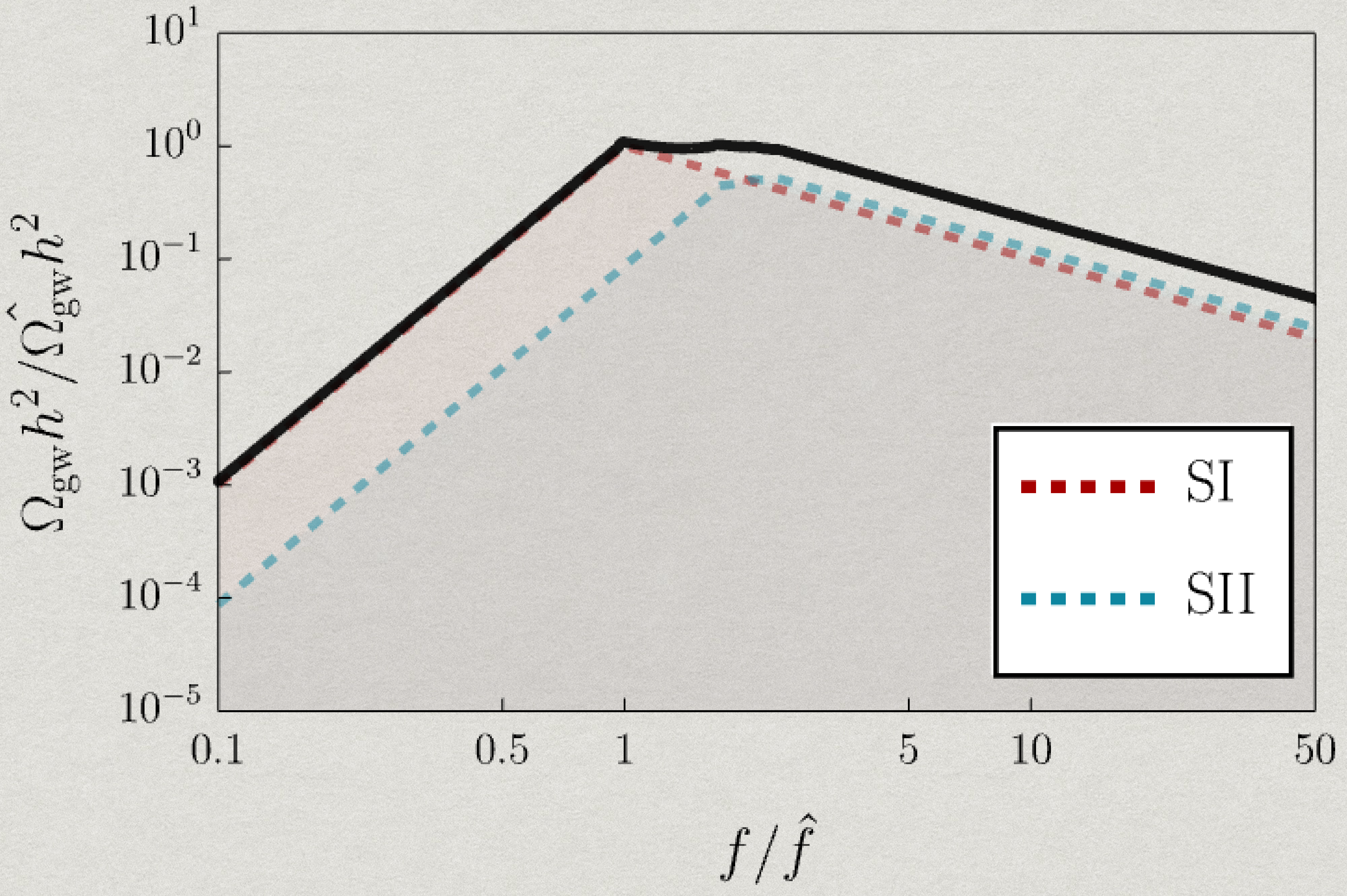

Summary and Outlook

* Non-abelian DWs have more interesting and non-trivial structure and phenomena
* If the DWs are stable, they can give rise to a unique multi-peak GW signal
* The signature of GW raised by unstable domain walls is still unexplored
* In realistic flavour models, the flavon fields are complex scalars and there may be more than one multiplets

TANKS !