Cosmological Signatures of Neutrino Seesaw mechanism

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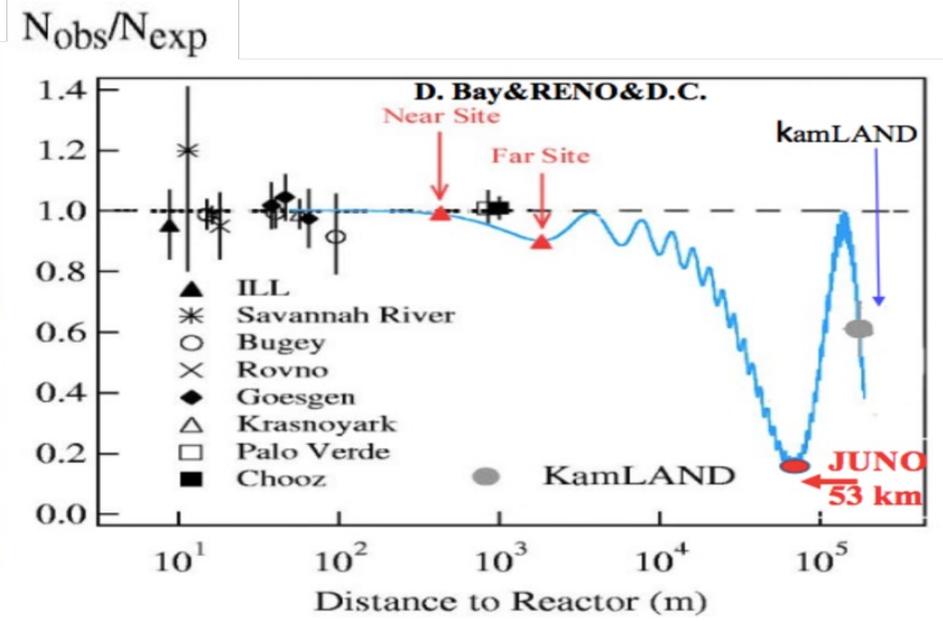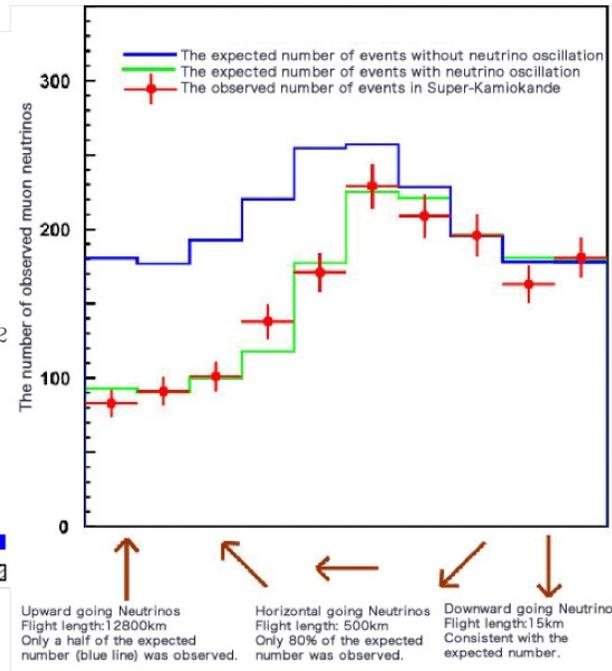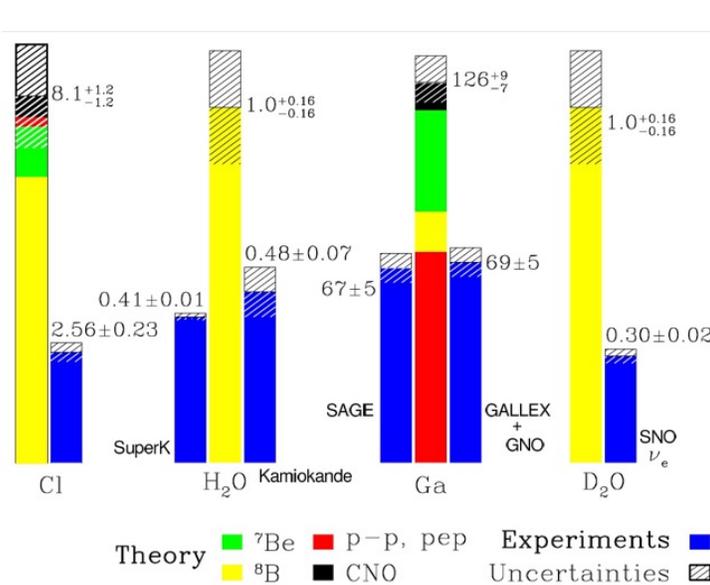
With Hongjian He, Linghao Song, Jingtao You, arXiv: 2412.21045(PRDL) , 2412.16033(PRD)

第十届海峡两岸粒子物理和宇宙学研讨会

2026.1.20

Neutrino oscillation indicates massive neutrinos

Solar Neutrino oscillations

θ_{12}

Atmospheric Neutrino Oscillations

θ_{23}

Reactor Neutrino Oscillations

θ_{13}

$$\Delta m_{21}^2 \simeq 7.42 \times 10^{-5} \text{ eV}^2$$

$$\left|\Delta m_{13}^2\right| \approx \left|\Delta m_{23}^2\right| \simeq 2.5 \times 10^{-3} \text{ eV}^2$$

Cosmological limit

Table 26.2: Summary of $\sum m_\nu$ constraints.   **PDG**

	Model	95% CL (eV)	Ref.
CMB alone			
Pl18[TT+lowE]	ΛCDM+$\sum m_\nu$	< 0.54	[24]
Pl18[TT,TE,EE+lowE]	ΛCDM+$\sum m_\nu$	< 0.26	[24]
CMB + probes of background evolution			
Pl18[TT,TE,EE+lowE] + BAO	ΛCDM+$\sum m_\nu$	< 0.13	[49]
Pl18[TT,TE,EE+lowE] + BAO	ΛCDM+$\sum m_\nu$+5 params.	< 0.515	[25]
CMB + LSS			
Pl18[TT+lowE+lensing]	ΛCDM+$\sum m_\nu$	< 0.44	[24]
Pl18[TT,TE,EE+lowE+lensing]	ΛCDM+$\sum m_\nu$	< 0.24	[24]
Pl18[TT,TE,EE+lowE]+ ACT[lensing]	ΛCDM+$\sum m_\nu$	< 0.12	[50]
CMB + probes of background evolution + LSS			
Pl18[TT,TE,EE+lowE] + BAO + RSD	ΛCDM+$\sum m_\nu$	< 0.10	[49]
Pl18[TT,TE,EE+lowE+lensing] + BAO + RSD + Shape	ΛCDM+$\sum m_\nu$	< 0.082	[51]
Pl18[TT+lowE+lensing] + BAO + Lyman-α	ΛCDM+$\sum m_\nu$	< 0.087	[52]
Pl18[TT,TE,EE+lowE] + BAO + RSD + SN + DES-Y1	ΛCDM+$\sum m_\nu$	< 0.12	[49]
Pl18[TT,TE,EE+lowE] + BAO + RSD + SN + DES-Y3	ΛCDM+$\sum m_\nu$	< 0.13	[53]

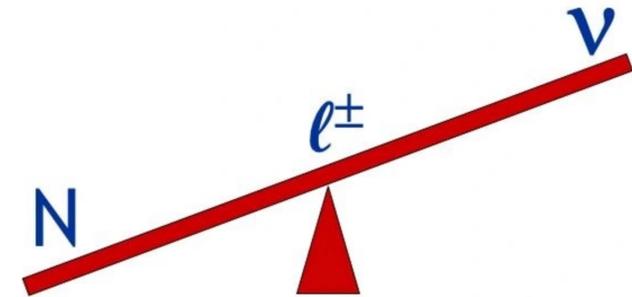**The heavy neutrino mass should be around 0.05 eV(IO)-0.06eV(NO)**

Origin of neutrino masses: seesaw mechanism

$$\mathcal{L} = \mathcal{L}_{\text{SM}} + y_\nu \tilde{H} \bar{L} N_R - \frac{1}{2} M_R \bar{N}_R^c N_R + h.c.$$

P. Minkowski；T. Yanagida； S. L. Glashow；
M. Gell-Mann, P. Ramond and R. Slansky

$$M = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

$$m_\nu \sim \frac{m_D^2}{M_R} = \frac{y_\nu^2 \langle h \rangle^2}{2 M_R}$$

- **Natural prediction of small neutrino masses**

- **Explain the baryon asymmetry of the universe: leptogenesis**

Baryogenesis Without Grand Unification, Fukugita and Yanagida, 1986′
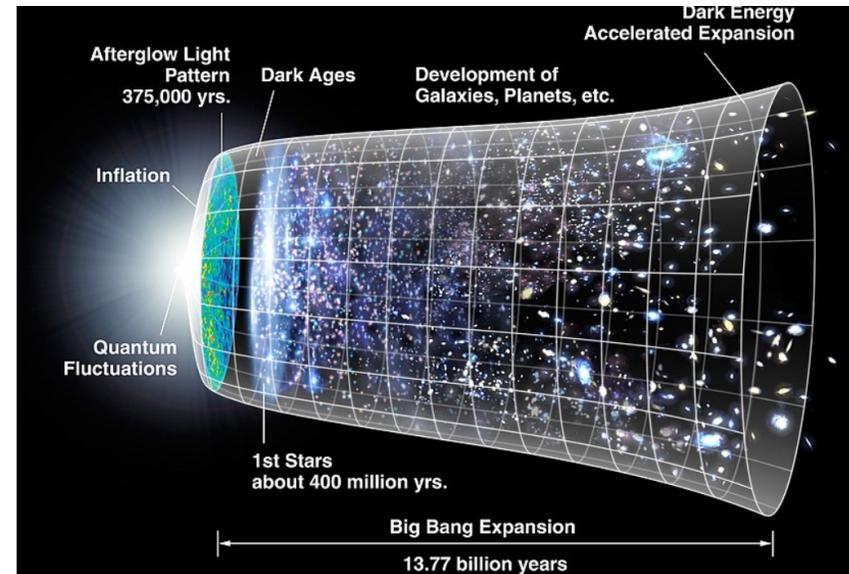
Seesaw mechanism

$$m_\nu \sim \frac{m_D^2}{M_R} = \frac{y_\nu^2 \langle h \rangle^2}{2 M_R}$$

If the Yukawa coupling is O(1)(as predicted by the GUT), the seesaw scale M_R should be around $10^{13\text{-}14}$ GeV, which is much beyond the reach of particle experiments.
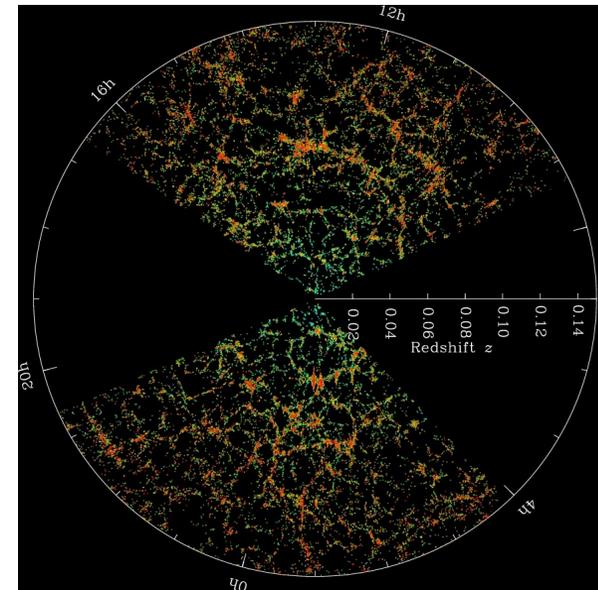
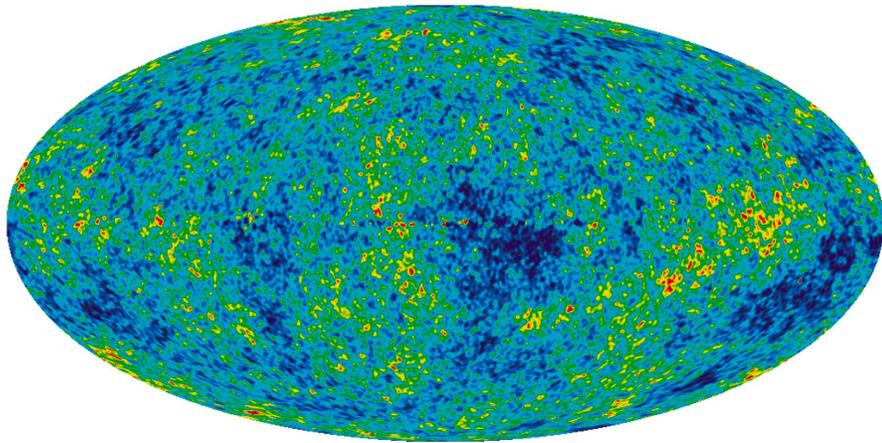**How to test such high scale seesaw?**

Inflation

Rapid expansion of the universe in the early time

- **Flatness problem**
- **Horizon problem**
- **Seeding the primordial anisotropies in CMB**

Inflation

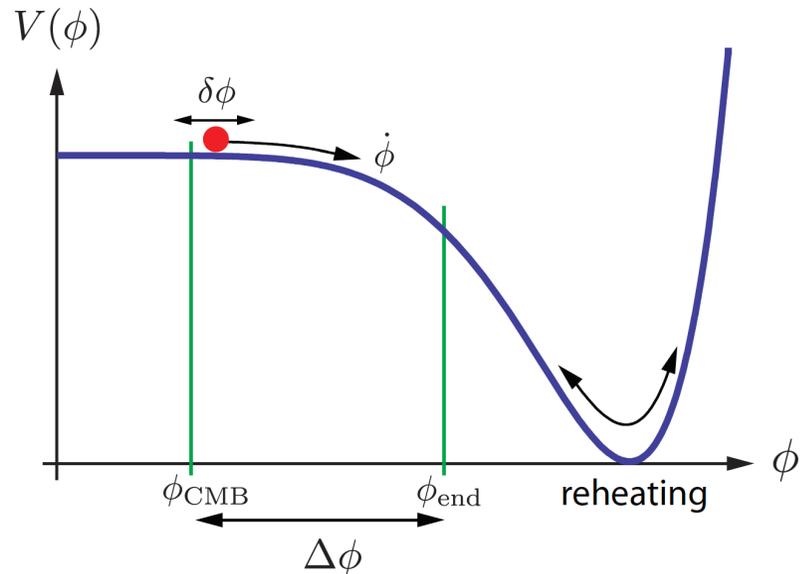**Stretching quantum fluctuations to large scale**

$$\frac{\delta T}{T} \sim 10^{-5}$$

Such small fluctuations finally develops the large structure of our universe

Slow-roll Inflation

Inflation is driven by a scalar field (inflaton)

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

Slow roll condition(epsilon, eta <<1)

$$\epsilon_{\mathrm{v}}(\phi) \equiv \frac{M_{\mathrm{pl}}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2 \qquad \eta_{\mathrm{v}}(\phi) \equiv M_{\mathrm{pl}}^2 \frac{V_{,\phi\phi}}{V}$$

- **Hubble parameter is nearly constant(de Sitter spacetime)**
- **After inflation, inflaton oscillates at the bottom of the potential and finally decays into SM particles, then reheats the universe(still no clear how it occurs)**

Slow-roll Inflation

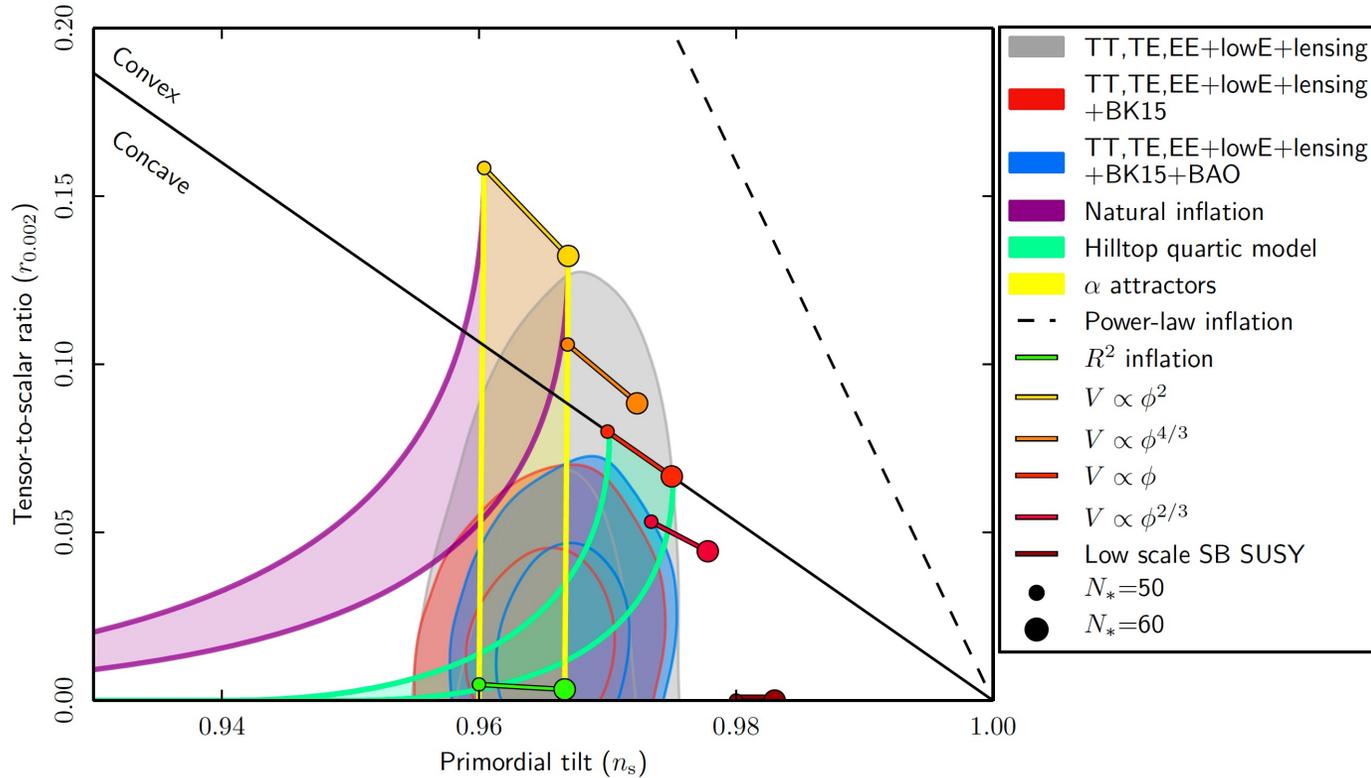**In a de Sitter universe, scalar fields get quantum fluctuation**
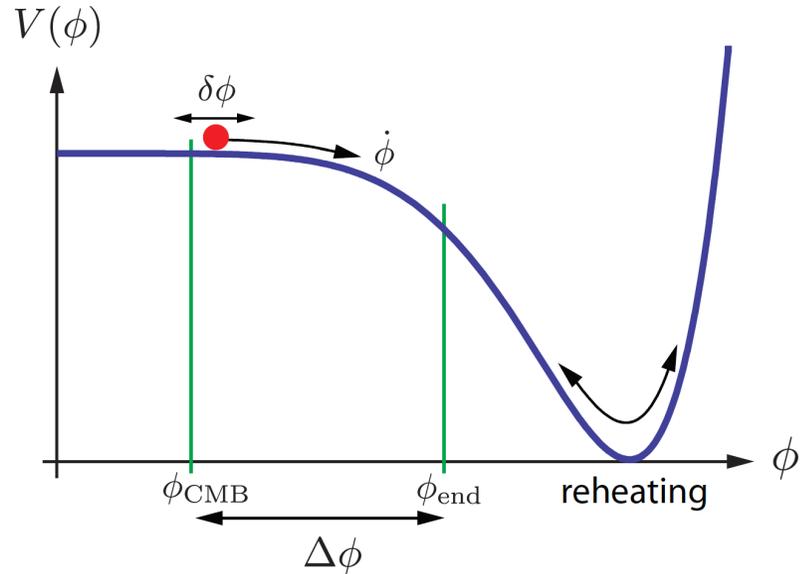
$$\langle \delta\phi_{\mathbf{k}} \, \delta\phi_{\mathbf{k'}} \rangle = (2\pi)^3 \, \delta(\mathbf{k} + \mathbf{k'}) \, \frac{2\pi^2}{k^3} \left(\frac{H}{2\pi} \right)^2$$

$$\Delta_{\mathrm{s}}^2(k) \approx \left. \frac{1}{24\pi^2} \frac{V}{M_{\mathrm{pl}}^4} \frac{1}{\epsilon_{\mathrm{v}}} \right|_{k=aH} \qquad \zeta = -\frac{H}{\dot\phi} \delta\phi$$

$$n_{\mathrm{s}} - 1 \equiv \frac{d\ln\Delta_{\mathrm{s}}^2}{d\ln k} = 2\eta_{\mathrm{v}} - 6\epsilon_{\mathrm{v}}$$

- **Quantum fluctuation of inflaton induces CMB anisotropies(or curvature perturbations)**
- **In the single field inflation, the fluctuations should be nearly gaussian and adiabatic, close to scale invariant**

Inflation

$$\Delta_{\mathrm{t}}^2(k) \approx \frac{2}{3\pi^2} \frac{V}{M_{\mathrm{pl}}^4}\bigg|_{k=aH}$$

$$r \equiv \frac{\Delta_{\mathrm{t}}^2}{\Delta_{\mathrm{s}}^2} = 16\epsilon_{\mathrm{v}}$$
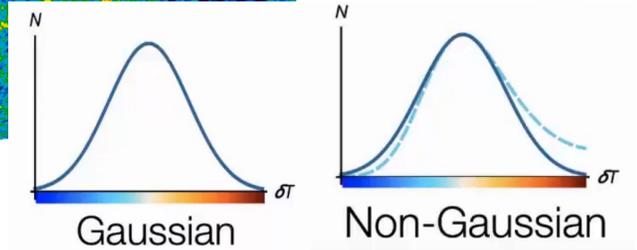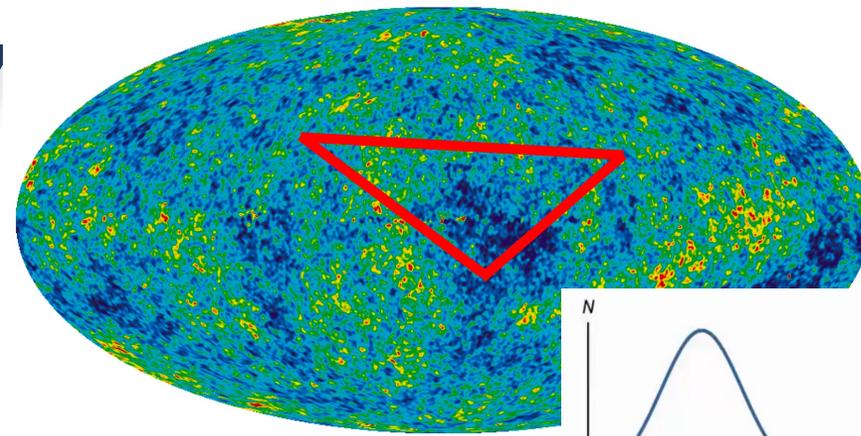
$$\epsilon_V < 0.0097$$

$$\eta_V = -0.010^{+0.007}_{-0.011}$$

$$\frac{H_*}{M_{\mathrm{Pl}}} < 2.5 \times 10^{-5}$$

- **Inflaton potential should be flat enough(shift-symmetry?)**
- **Hubble scale could be as high as 6*10^{13} GeV(close to seesaw scale), providing access to the high scale physics**

Non-Gaussianity

Gaussian | Non-Gaussian

Non-Gaussianity is sensitive to new physics

- **New physics could induce large non-Gaussianity : multi-field inflation models, modulated reheating, curvaton scenario...**

- **Current limit from Planck on local type $f_{NL} \sim$ O(10), future CMB observations, CMB S4, large scale structure observations DESI O(1), 21 cm tomography O(0.01-0.1)**

- **Non-Gaussianity could provide information to the new particle mass, spin, interactions: cosmological collider signals**

Nima Arkani-Hamed, Juan Maldacena, arXiv:1503.08043

Xingang Chen, Yi Wang, JCAP 04 (2010) 027

Cosmological collider signals

Bispectrum $\quad\quad \langle \zeta^3 \rangle \equiv (2\pi)^3 \delta_D(\mathbf{k}_{123}) \dfrac{A^2}{(k_1 k_2 k_3)^2} S(k_1, k_2, k_3) \quad P_\zeta(k) = A/k^3$

Massive particle coupling to the inflaton could induce

$$S_{\text{squeezed}} \propto \left(\frac{k_{\text{long}}}{k_{\text{short}}} \right)^{1/2 \pm i\mu} \quad\quad \mu = \sqrt{\frac{m^2}{H^2} - \frac{9}{4}}$$

P. Daniel Meerburg, Moritz Münchmeyer, Julian B. Muñoz, Xingang Chen, JCAP 03 (2017) 050
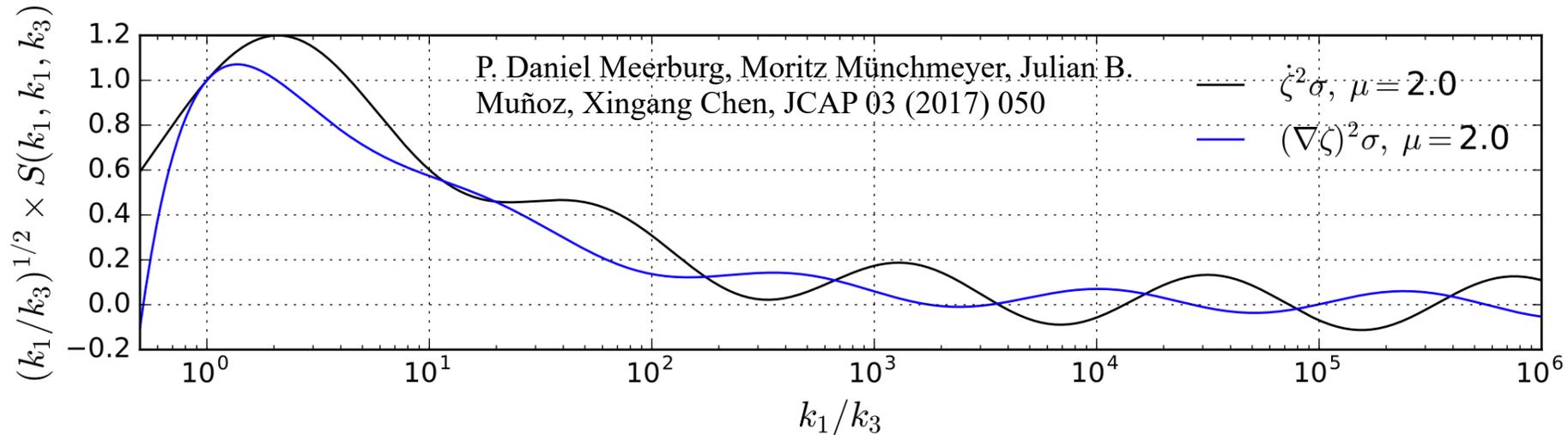
- **Probing new particles with mass around Hubble scale**
- **Signature would be highly suppressed when mass is large**

A minimal model

Minimal model incorporates inflation and seesaw

$$\Delta\mathcal{L} = \sqrt{-g}\left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \overline{N}_R i\slashed{\partial}N_R + \frac{1}{\Lambda}\partial_\mu\phi\,\overline{N}_R\gamma^\mu\gamma^5 N_R\right.$$
$$\left. + \left(-\frac{1}{2}M\overline{N_R^c}N_R - y_\nu\,\overline{L}_L\tilde{\mathbb{H}}N_R + \text{H.c.}\right)\right]$$

- V(phi) is the potential for inflation is unknown but denominated by the mass term after inflation
- Derivative coupling to keep the flatness of the inflaton potential(shift-symmetry, dim-4 coupling should be suppressed, otherwise the induced phi^4 potential would destroy the flatness of the potential)
- Lambda > 60 Hubble to keep perturbative unitarity
- After inflation, inflaton oscillates at the bottom of the potential until decays into heavy neutrinos (mphi > 2 mN). The heavy neutrinos quickly decay into SM particles and reheat the universe

Seesaw mechanism

Consequence of the seesaw mechanism

$$\mathcal{L} \supset \frac{1}{2} \bar{\psi}_L \mathbf{M}_\nu \psi_R + \text{h.c.}, \qquad \mathbf{M}_\nu = \begin{pmatrix} 0 & \frac{y_\nu h}{\sqrt{2}} \\ \frac{y_\nu h}{\sqrt{2}} & M \end{pmatrix}$$

$$m_\nu \simeq -\frac{y_\nu^2 h^2}{2M}, \qquad M_N \simeq M + \frac{y_\nu^2 h^2}{2M}$$

- **Light neutrino gets a mass**

- **Heavy neutrino mass are get lifted (h dependent)**

Decay rate of the inflaton is h dependent:

$$\Gamma \simeq \frac{m_\phi M^2}{4\pi\Lambda^2} \left[1 + \frac{1}{4} \left(\frac{y_\nu h}{M} \right)^2 \right]$$
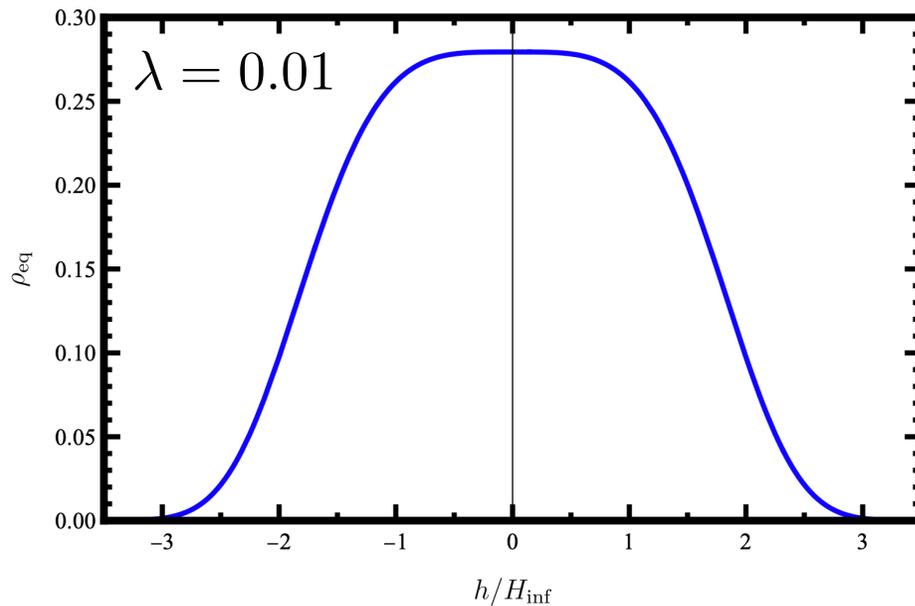
What happens to h in the early universe?

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Alexei A. Starobinsky, Jun'ichi Yokoyama,
Phys.Rev.D 50 (1994) 6357-6368

- **During inflation(de-Sitter universe), Higgs also gets quantum fluctuations**
- **The fluctuations reach a equilibrium state(F-P equation)**

$$\rho_{\text{eq}}(h) = \frac{2\lambda^{1/4}}{\Gamma(1/4)} \left(\frac{2\pi^2}{3}\right)^{1/4} \exp\left(\frac{-2\pi^2\lambda h^4}{3H_{\text{inf}}^4}\right)$$

$\lambda = 0.01$

D. Buttazzo, et al arXiv:1307.3536

$$\bar{h} = \sqrt{\langle h^2 \rangle} = \left[\int_{-\infty}^{+\infty} dh \, h^2 \rho_{\text{eq}}(h)\right]^{1/2} \simeq 0.363 \left(\frac{H_{\text{inf}}}{\lambda^{1/4}}\right)$$

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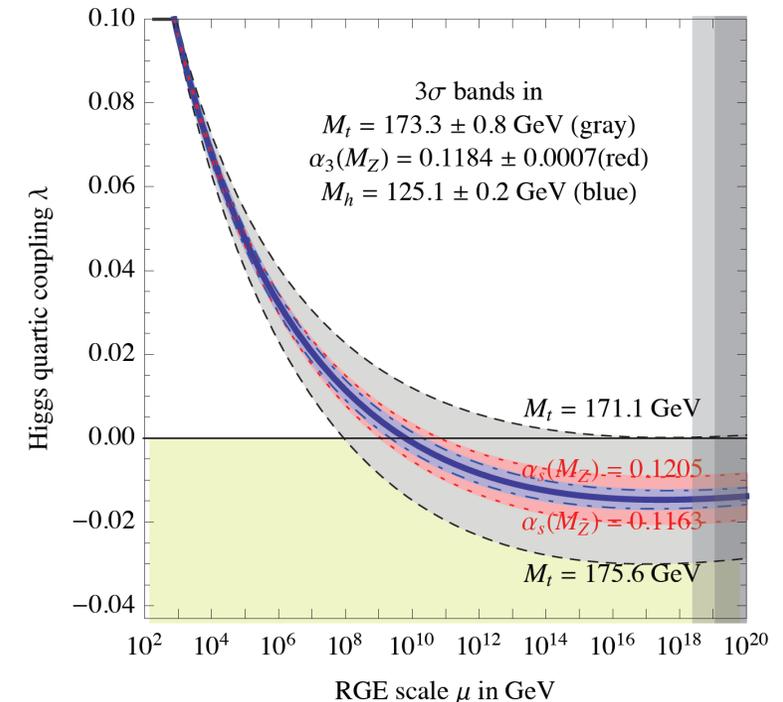
Inflaton oscillates at the bottom potential. If the inflaton potential is dominated by the mass term, the Universe is matter-dominated
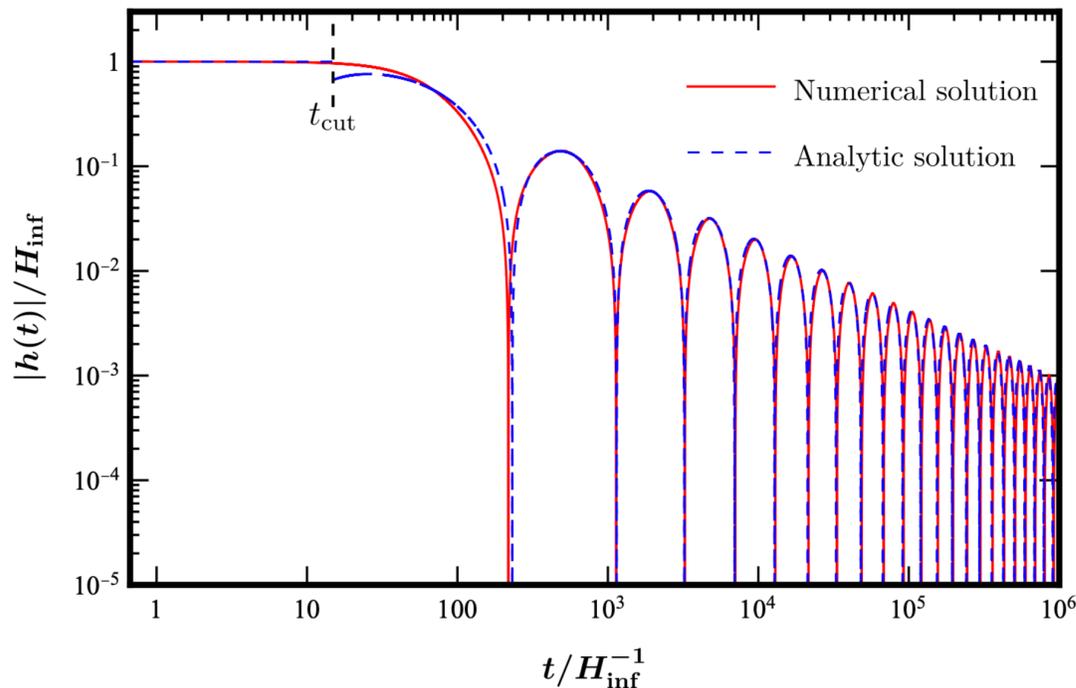
$$\ddot{h}(t) + \frac{2}{t}\dot{h}(t) + \lambda h^3(t) = 0$$

$$h(t) = \begin{cases} h_{\text{inf}}, & t \leq t_{\text{cut}} \\ A H_{\text{inf}} \left(\frac{h_{\text{inf}}}{H_{\text{inf}}\lambda}\right)^{\frac{1}{3}} (H_{\text{inf}} t)^{-\frac{2}{3}} \cos\left(\lambda^{\frac{1}{6}} |h_{\text{inf}}|^{\frac{1}{3}} \omega t^{\frac{1}{3}} + \theta\right), & t > t_{\text{cut}} \end{cases}$$

$$t_{\text{cut}} = \frac{\sqrt{2}}{3\sqrt{\lambda}\, h_{\text{inf}}}$$

$$A = 2^{1/3} 3^{-2/3} 5^{1/4} \simeq 0.9$$

$$\omega = \frac{\Gamma^2(3/4)}{\sqrt{\pi}} 2^{1/3} 3^{1/3} 5^{1/4} \simeq 2.3$$

$$\theta = -3^{-1/3} 2^{1/6}\omega - \arctan 2 \simeq -2.9$$

Higgs value would oscillate and decrease

Higgs modulated reheating

Considering decay rate of the inflaton is h dependent

$$\Gamma \simeq \frac{m_\phi M^2}{4\pi\Lambda^2}\left[1 + \frac{1}{4}\left(\frac{y_\nu h}{M}\right)^2\right]$$

Gia Dvali, Andrei Gruzinov, Matias Zaldarriaga, Phys.Rev. D69 (2004) 023505

- **Different patches of the universe reheat differently (modulated reheating)**

- **The curvature perturbation is generated by Higgs field**

- **Delta N formalism (zeta=delta N~ N- <N>)**

$$\zeta_h(t > t_{\text{reh}}, \mathbf{x}) = \delta N(\mathbf{x}) = N(\mathbf{x}) - \langle N(\mathbf{x})\rangle$$

$$= -\frac{1}{6}\left[\ln(\Gamma_{\text{reh}}) - \langle\ln(\Gamma_{\text{reh}})\rangle\right]$$

Higgs modulated reheating

Curvature perturbation contains two parts

$$\zeta = \zeta_\phi + \zeta_h \qquad\qquad \mathcal{P}_\zeta^{(\phi)} = \left(\frac{H}{\dot\phi}\right)^2 \mathcal{P}_\phi = \left(\frac{H}{\dot\phi}\right)^2 \frac{H^2}{4\pi^2}$$

Taylor expansion of the curvature perturbation

$$\zeta_h(\mathbf{x}) = -\frac{1}{6}\left[\frac{\Gamma_0'}{\Gamma_0}\delta h_{\mathrm{inf}}(\mathbf{x}) + \frac{\Gamma_0\Gamma_0'' - \Gamma_0'\Gamma_0'}{2\Gamma_0^2}\delta h_{\mathrm{inf}}^2(\mathbf{x})\right] \equiv z_1 \delta h_{\mathrm{inf}}(\mathbf{x}) + \frac{1}{2}z_2\delta h_{\mathrm{inf}}^2(\mathbf{x})$$

$$\mathcal{P}_\zeta^{(h)} = z_1^2 \mathcal{P}_{\delta h} = \frac{z_1^2 H^2}{4\pi^2} \qquad\qquad R = \left(\frac{\mathcal{P}_\zeta^{(h)}}{\mathcal{P}_\zeta^{(o)}}\right)^{1/2} = |z_1|\left(\frac{\mathcal{P}_{\delta h}}{\mathcal{P}_\zeta^{(o)}}\right)^{1/2}$$
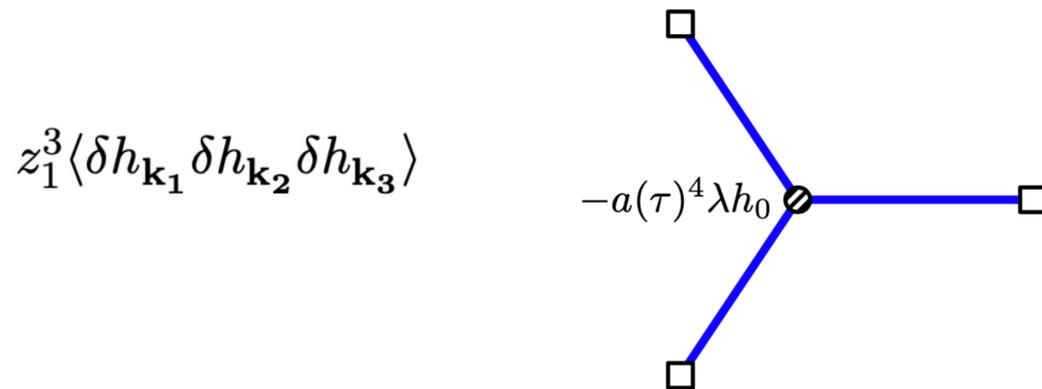
R should be less than 1

Considering the three point correlation function

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \zeta_{\mathbf{k}_3} \rangle_h = z_1^3 \langle \delta h_{\mathbf{k}_1} \delta h_{\mathbf{k}_2} \delta h_{\mathbf{k}_3} \rangle + z_1^2 z_2 \langle \delta h^4 \rangle (\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$$

First term is from Higgs self-coupling

$$z_1^3 \langle \delta h_{\mathbf{k}_1} \delta h_{\mathbf{k}_2} \delta h_{\mathbf{k}_3} \rangle$$

$-a(\tau)^4 \lambda h_0$

Calculated by in-in formalism/Schwinger-Keldysh formalism

Steven Weinberg, Phys.Rev.D 72 (2005) 043514, Phys.Rev.D 74 (2006) 023508

Xingang Chen, Yi Wang, Zhong-Zhi Xianyu, JCAP 1712 (2017) 006

Bispectrum

$$\langle \delta h_{\mathbf{k_1}} \delta h_{\mathbf{k_2}} \delta h_{\mathbf{k_3}} \rangle' = 12\lambda\bar{h}\,\mathrm{Im}\left(\int_{-\infty}^{\tau_f} a^4 \prod_{i=1}^{3} G_+ (\mathbf{k}_i, \tau)\, d\tau \right)$$

$$\mathrm{Im}\left(\int_{-\infty}^{\tau_f} a^4 \prod_{i=1}^{3} G_+ (\mathbf{k}_i, \tau)\, d\tau \right)$$

$$N_e = \log\left(\frac{a_{\mathrm{end}}}{a_k}\right) = \log\left(\frac{-\frac{1}{H\tau_f}}{\frac{k_t}{H}}\right) = -\log(k_t|\tau_f|) \sim 60$$

$$= \mathrm{Im}\int_{-\infty}^{\tau_f} \frac{d\tau}{(H\tau)^4} \cdot \frac{H^6}{8k_1^3 k_2^3 k_3^3} \left(\prod_{i=1}^{3}(1 - ik_i\tau) \right) e^{i(k_1+k_2+k_3)\tau}$$

$$= \frac{H^2}{24 k_1^3 k_2^3 k_3^3} \cdot \left\{ (k_1^3 + k_2^3 + k_3^3)[\log(k_t|\tau_f|) + \gamma - \frac{4}{3}] + k_1 k_2 k_3 - \sum_{a \neq b} k_a^2 k_b \right\}$$

Bispectrum

Second term is from non-linear evolution of the Higgs

$$z_1^2 z_2 \langle \delta h^4 \rangle (\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{z_1^2 z_2}{2} \int \frac{d^3 \mathbf{k}_0}{(2\pi)^3} \langle \delta h(\mathbf{k}_1) \delta h(\mathbf{k}_2) \delta h(\mathbf{k}_0) \delta h(\mathbf{k}_3 - \mathbf{k}_0) \rangle + (2\,\text{perm.})$$

$$= \frac{z_1^2 z_2}{2} \left[\int \frac{d^3 \mathbf{k}_0}{(2\pi)^3} \langle \delta h(\mathbf{k}_1) \delta h(\mathbf{k}_0) \rangle \langle \delta h(\mathbf{k}_2) \delta h(\mathbf{k}_3 - \mathbf{k}_0) \rangle + (\mathbf{k}_1 \leftrightarrow \mathbf{k}_2) \right] + (2\,\text{perm.})$$

$$= \frac{z_1^2 z_2}{2} \left[\int d^3 \mathbf{k}_0 \, (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_0) \delta^3(\mathbf{k}_2 + \mathbf{k}_3 - \mathbf{k}_0) \frac{H^4}{4 k_1^3 k_2^3} + (\mathbf{k}_1 \leftrightarrow \mathbf{k}_2) \right] + (2\,\text{perm.})$$

$$= (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) z_1^2 z_2 \left[\frac{H^4}{4 k_1^3 k_2^3} + (2\,\text{perm.}) \right].$$

Local type non-gaussianity

The local type non-gaussianity which is defined by Bardeen Potential $\quad \Phi \equiv \frac{3}{5}\zeta$
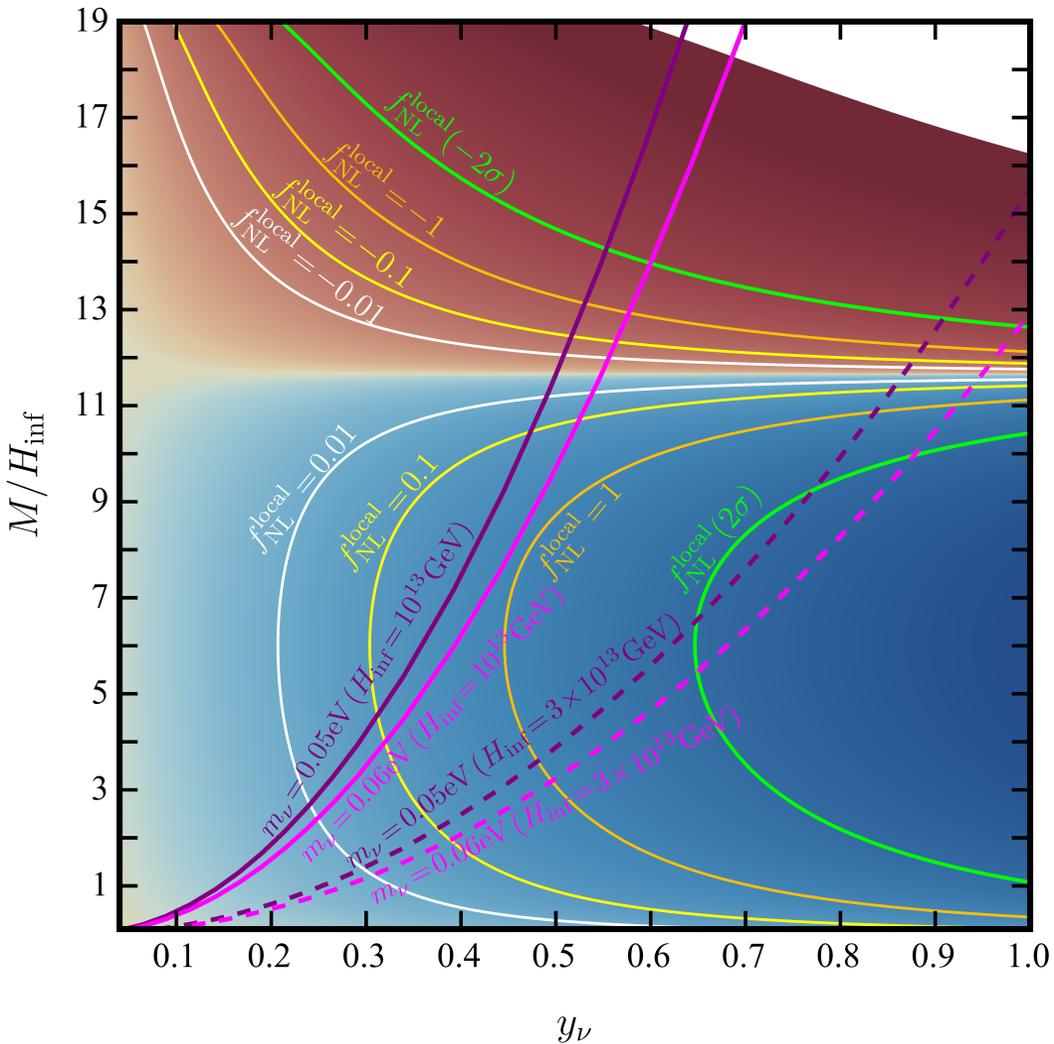
$$\langle \Phi_{\mathbf{k_1}} \Phi_{\mathbf{k_2}} \Phi_{\mathbf{k_3}} \rangle'_{\text{local}} = 2A^2 f_{\text{NL}}^{\text{local}} \left\{ \frac{1}{k_1^3 k_2^3} + \frac{1}{k_2^3 k_3^3} + \frac{1}{k_3^3 k_1^3} \right\}$$
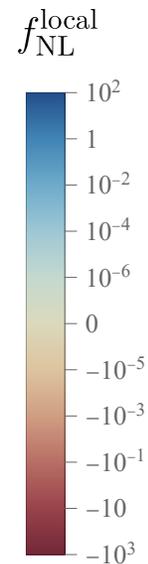
In the limit $k_1 \sim k_2 >> k_3$, we find

$$f_{\text{NL}}^{\text{local}} \simeq -\frac{10}{3} \frac{z_1^3 H^3}{(2\pi)^4 \mathcal{P}_\zeta^2} \left(\frac{\lambda \bar{h}}{2H} N_e - \frac{z_2 H}{4z_1} \right)$$

$f_{\text{NL}}^{\text{local}} = -0.9 \pm 5.1 \quad (68\% \text{ C.L., Planck 2018})$
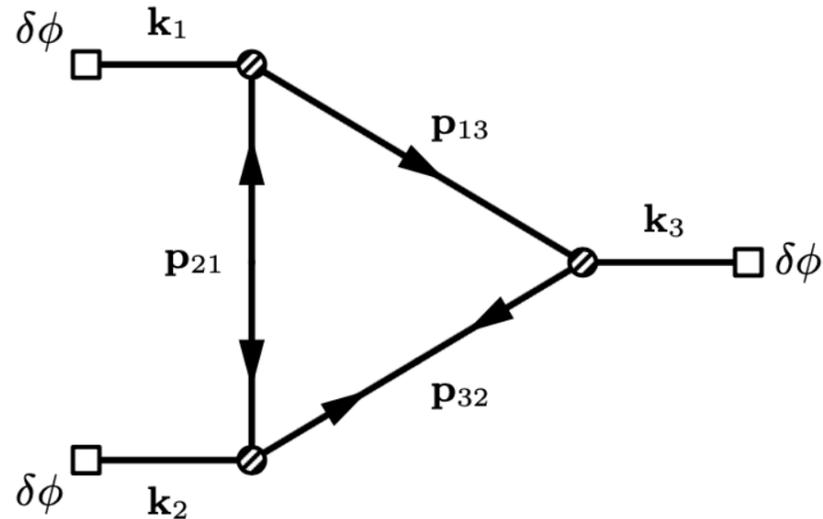
Local type non-gaussianity

Parameters	$\mathcal{P}_\zeta$	N_e	H_{inf}	m_ϕ	Λ	λ
Values	2.1×10^{-9}	60	$(1, 3) \times 10^{13}\,\text{GeV}$	$40 H_{\text{inf}}$	$60 H_{\text{inf}}$	0.01

- **Colored curves indicating future searches**
- **Parameter space with Yukawa O(1) could be probed by future observations**
- **The contribution from self-interaction and non-linear term are both important**
- **Interplaying with neutrino experiments(JUNO, DUNE for neutrino ordering)**

Cosmological collider signal

Prediction of cosmological collider signal: help to identify the mass of the right-handed neutrino

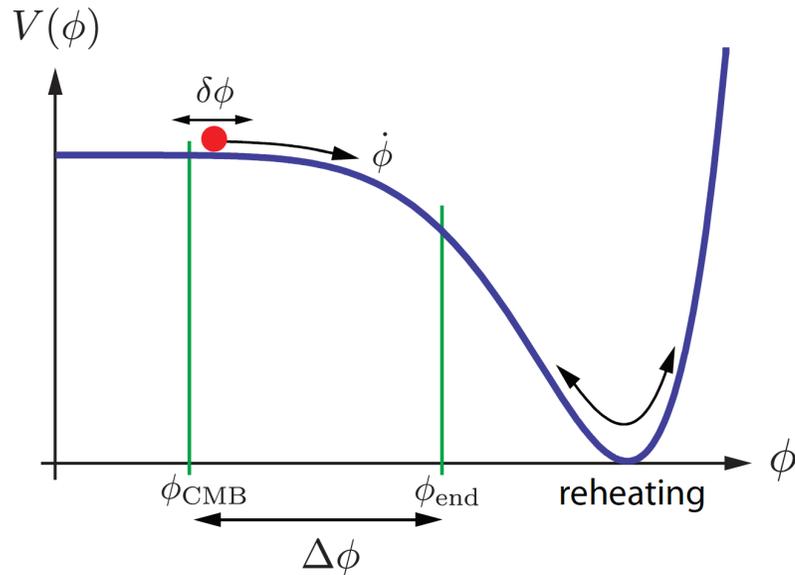**Challenge for calculation of fermion loop, results appear soon**

Summary

- **We propose a minimal model incorporating inflation and seesaw**

- **It provides new mechanism of reheating**

- **High scale seesaw can be probed by the non-Gaussianity which could be observed in near future CMB or LSS**

- **Cosmological collider signal to determine the mass of N_R**

Thanks!

Slow-roll Inflation

$$\epsilon_{\mathrm{v}}(\phi) \equiv \frac{M_{\mathrm{pl}}^2}{2} \left(\frac{V_{,\phi}}{V} \right)^2$$

$$\eta_{\mathrm{v}}(\phi) \equiv M_{\mathrm{pl}}^2 \frac{V_{,\phi\phi}}{V}$$

$$\Delta_{\mathrm{s}}^2(k) \approx \frac{1}{24\pi^2} \frac{V}{M_{\mathrm{pl}}^4} \frac{1}{\epsilon_{\mathrm{v}}} \bigg|_{k=aH}$$
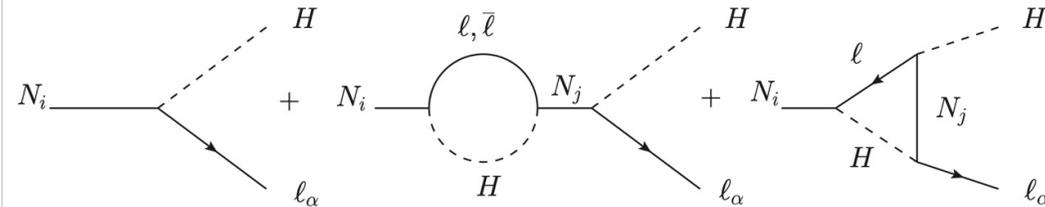
$$\Delta_{\mathrm{t}}^2(k) \approx \frac{2}{3\pi^2} \frac{V}{M_{\mathrm{pl}}^4} \bigg|_{k=aH}$$

$$r \equiv \frac{\Delta_{\mathrm{t}}^2}{\Delta_{\mathrm{s}}^2} = 16\epsilon_{\mathrm{v}}$$

Leptogenesis

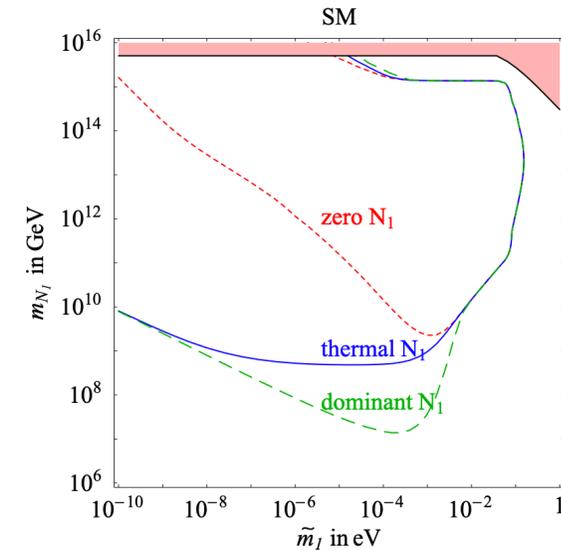Baryogenesis Without Grand Unification, Fukugita and Yanagida, 1986′

$$\mathcal{L}_I = \mathcal{L}_{SM} + i\overline{N_{R_i}}\slashed{\partial}N_{R_i} - \left(\frac{1}{2}M_i\overline{N_{R_i}^c}N_{R_i} + \epsilon_{ab}Y_{\alpha i}\overline{N_{R_i}}\ell_\alpha^a H^b + h.c.\right)$$

$$\epsilon_{i\alpha} = \frac{\gamma\left(N_i \to \ell_\alpha H\right) - \gamma\left(N_i \to \overline{\ell_\alpha}H^*\right)}{\sum_\alpha \gamma\left(N_i \to \ell_\alpha H\right) + \gamma\left(N_i \to \overline{\ell_\alpha}H^*\right)}$$

$$n_B = \frac{28}{79}(\mathcal{B} - \mathcal{L})_i$$

Mass of the right-handed neutrino should heavier than 10^7 GeV

S-K formalism

$$Q(\tau) \equiv \varphi^{A_1}(\tau, \mathbf{x}_1) \cdots \varphi^{A_N}(\tau, \mathbf{x}_N)$$

$$\langle Q(\tau) \rangle = \langle \Omega | \bar{F}(\tau, \tau_0) Q_I(\tau) F(\tau, \tau_0) | \Omega \rangle$$

$$F(\tau, \tau_0) = \mathrm{T} \exp \left(-\mathrm{i} \int_{\tau_0}^{\tau} \mathrm{d}\tau_1 \, H_I(\tau_1) \right),$$

$$\bar{F}(\tau, \tau_0) = \bar{\mathrm{T}} \exp \left(\mathrm{i} \int_{\tau_0}^{\tau} \mathrm{d}\tau_1 \, H_I(\tau_1) \right),$$

$$\begin{cases} G_{++}\left(\mathbf{k};\tau_1,\tau_2\right) & \equiv G_>\left(\mathbf{k};\tau_1,\tau_2\right)\theta(\tau_1-\tau_2) + G_<\left(\mathbf{k};\tau_1,\tau_2\right)\theta(\tau_2-\tau_1) \\ G_{+-}\left(\mathbf{k};\tau_1,\tau_2\right) & \equiv G_<\left(\mathbf{k};\tau_1,\tau_2\right) \\ G_{-+}\left(\mathbf{k};\tau_1,\tau_2\right) & \equiv G_>\left(\mathbf{k};\tau_1,\tau_2\right) \\ G_{--}\left(\mathbf{k};\tau_1,\tau_2\right) & \equiv G_<\left(\mathbf{k};\tau_1,\tau_2\right)\theta(\tau_1-\tau_2) + G_>\left(\mathbf{k};\tau_1,\tau_2\right)\theta(\tau_2-\tau_1) \end{cases}$$

$$G_>(k;\tau_1,\tau_2) \equiv u(\tau_1,k)u^*(\tau_2,k)$$
$$G_<(k;\tau_1,\tau_2) \equiv u^*(\tau_1,k)u(\tau_2,k)$$

$$\Box u_{\mathbf{k}} = \ddot{u}_{\mathbf{k}} + 3H\dot{u}_{\mathbf{k}} + \frac{\mathbf{k}^2}{a^2(t)}u_{\mathbf{k}} = 0$$

$$u_{\mathbf{k}}(\tau) = \frac{H}{\sqrt{2k^3}}\left[1 + ik\tau\right]e^{-ik\tau}$$

S-K formalism

Bulk-to-Boundary propagator

$$G_{\pm}\left(\mathbf{k},\tau\right) \equiv G_{\pm+}\left(\mathbf{k};\tau,\tau_f\right)$$

$$\tau \bullet\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!\square = G_{+}\left(\mathbf{k},\tau\right)$$

$$\tau \circ\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!\square = G_{-}\left(\mathbf{k},\tau\right)$$

$$\tau \oslash\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!-\!\!\!\square = G_{+}\left(\mathbf{k},\tau\right) + G_{-}\left(\mathbf{k},\tau\right)$$

$$G_{+}\left(\mathbf{k},\tau\right) = \frac{H^2}{2k^3}\left[1 - ik(\tau - \tau_f) + k^2\tau\tau_f\right]e^{ik(\tau-\tau_f)} \qquad G_{-}\left(\mathbf{k},\tau\right) \simeq \frac{H^2}{2k^3}\left[1 + ik\tau\right]e^{-ik\tau}$$

$$\simeq \frac{H^2}{2k^3}\left[1 - ik\tau\right]e^{ik\tau}$$

Higgs modulated reheating

Considering decay rate of the inflaton is h dependent

$$\Gamma \simeq \frac{m_\phi M^2}{4\pi\Lambda^2}\left[1+\frac{1}{4}\left(\frac{y_\nu h}{M}\right)^2\right]$$

Gia Dvali, Andrei Gruzinov, Matias Zaldarriaga, Phys.Rev. D69 (2004) 023505

- **Different patches of the universe reheat differently (modulated reheating)**
- **The curvature perturbation is generated by Higgs field**
- **Delta N formalism (from the end of inflation to the time after reheating completed)**

$$N(\mathbf{x}) = \int d\ln a(t) = \int_{t_{\text{inf}}}^{t_{\text{reh}}(\mathbf{x})} dt\, H(t) + \int_{t_{\text{reh}}(\mathbf{x})}^{t_{\text{f}}} dt\, H(t)$$

$$= \int_{\rho_{\text{inf}}}^{\rho_{\text{reh}}(h(\mathbf{x}))} d\rho\frac{H}{\dot\rho} + \int_{\rho_{\text{reh}}(h(\mathbf{x}))}^{\rho_{\text{f}}} d\rho\frac{H}{\dot\rho},$$

Higgs modulated reheating

Equation of state: $\qquad \dot{\rho} + 3H(1 + \omega)\rho = 0$

From matter-dominated universe to radiation dominated universe

$$N(\mathbf{x}) = -\frac{1}{3} \ln \frac{\rho_{\text{reh}}\big(h(\mathbf{x})\big)}{\rho_{\text{inf}}} - \frac{1}{4} \ln \frac{\rho_f}{\rho_{\text{reh}}\big(h(\mathbf{x})\big)}$$

Reheating occurs $\quad H_{\text{reh}} = \Gamma_{\text{reh}} \qquad 3H^2 M_p^2 = \rho$

Curvature perturbation in terms of the decay rate

$$\zeta_h(t > t_{\text{reh}}, \mathbf{x}) = \delta N(\mathbf{x}) = N(\mathbf{x}) - \langle N(\mathbf{x}) \rangle$$

$$= -\frac{1}{6} \Big[\ln(\Gamma_{\text{reh}}) - \langle \ln(\Gamma_{\text{reh}}) \rangle \Big]$$

Higgs modulated reheating

$$\zeta_h = -\frac{1}{6}\left[\ln(\Gamma_{\text{reh}}) - \langle\ln(\Gamma_{\text{reh}})\rangle\right]$$

$$\Gamma_{\text{reh}} = \Gamma\big(h(t_{\text{reh}})\big)$$

$$h(t_{\text{reh}}) = h(h_{\text{inf}}, t_{\text{reh}})$$

$$h(t) = A\left(\frac{h_{\text{inf}}}{\lambda}\right)^{\frac{1}{3}} t^{-\frac{2}{3}} \cos\left(\lambda^{\frac{1}{6}} h_{\text{inf}}^{\frac{1}{3}} \omega t^{\frac{1}{3}} + \theta\right)$$

n-point correlation function of zeta changes into n-point correlation function of hinf

Higgs during inflation

During inflation(de-Sitter universe), Higgs also gets quantum fluctuations

暴胀期间，希格斯场可以分为长波和短波两部分

Alexei A. Starobinsky, Jun'ichi Yokoyama, Phys.Rev.D 50 (1994) 6357-6368

$$h(\mathbf{x}, t) = h_L(\mathbf{x}, t) + \int \frac{d^3 k}{(2\pi)^3} \theta\big(k - \epsilon a(t) H_{\text{inf}}\big) \Big[a_{\mathbf{k}} h_{\mathbf{k}}(t) e^{-\mathrm{i}\mathbf{k}\cdot\mathbf{x}} + a_{\mathbf{k}}^{\dagger} h_{\mathbf{k}}^{*}(t) e^{\mathrm{i}\mathbf{k}\cdot\mathbf{x}} \Big]$$

$$h_{\mathbf{k}} = \frac{H_{\text{inf}}}{\sqrt{2k^3}} (1 + \mathrm{i}k\tau) e^{-\mathrm{i}k\tau}$$

长波部分可以用郎之万方程描述

$$\dot{h}_L(\mathbf{x}, t) = -\frac{1}{3H_{\text{inf}}} \frac{\partial V}{\partial h_L} + f(\mathbf{x}, t)$$

$$\langle f(\mathbf{x}_1, t_1) f(\mathbf{x}_2, t_2) \rangle = \frac{H_{\text{inf}}^3}{4\pi^2} \delta(t_1 - t_2) j_0\big(\epsilon a(t_1) H_{\text{inf}} |\mathbf{x}_1 - \mathbf{x}_2|\big)$$

带自相互作用的随机游走模型