Study on Rare B $\rightarrow$ $(K,\pi)\ell^+\ell^-$ Decays

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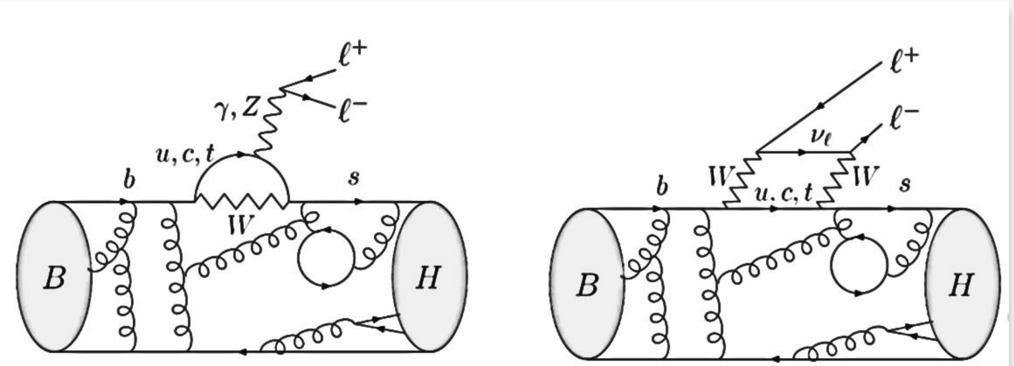
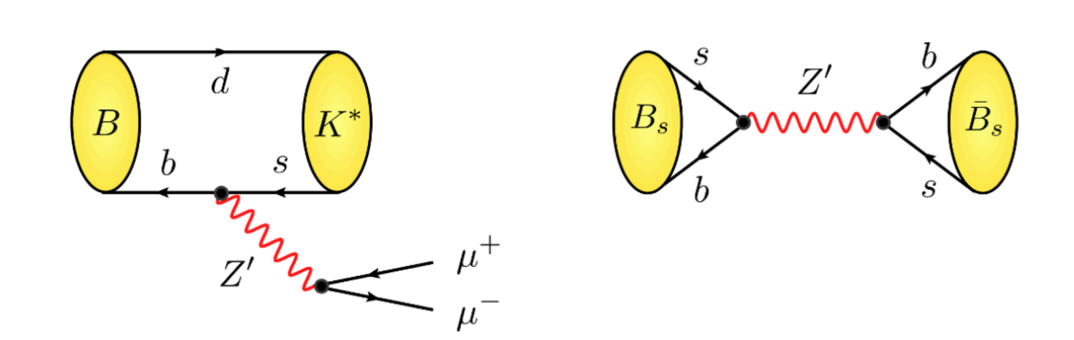
- QCD factorization of $B \to (K,\pi)\ell^+\ell^-$
- Weak annihilation amplitudes in $B \to (K,\pi)\ell^+\ell^-$
- Summary and outlook

Motivation

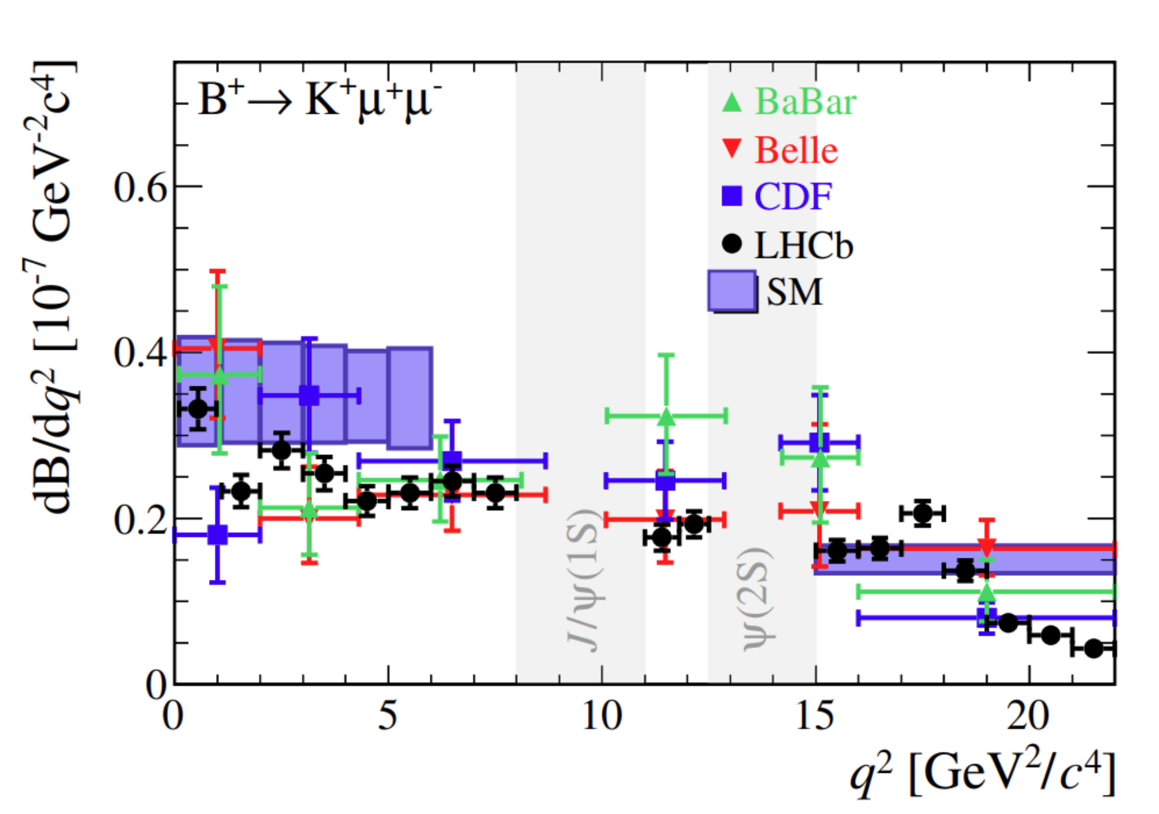
◆Flavor changing neutral current(FCNC) processes: suppressed in SM



◆Sensitive to new physics model



◆Branching ratios of $B \to K\ell^+\ell^-$ Sensitive to form factors



$$\mathcal{B}^{[1.1,2.0],SM}_{B^+\to K^+\mu^+\mu^-} = (0.33 \pm 0.03) \times 10^{-7}$$

$$\mathcal{B}^{[4.0,5.0],SM}_{B^+\to K^+\mu^+\mu^-} = (0.37 \pm 0.03) \times 10^{-7}$$

$$\mathcal{B}^{[5.0,6.0],SM}_{B^+\to K^+\mu^+\mu^-} = (0.37 \pm 0.03) \times 10^{-7}$$

$$\mathcal{B}^{[1.1,2.0],LHCb}_{B^+\to K^+\mu^+\mu^-} = (0.21 \pm 0.02) \times 10^{-7} (4.0\sigma)$$

$$\mathcal{B}^{[4.0,5.0],LHCb}_{B^+\to K^+\mu^+\mu^-} = (0.22 \pm 0.02) \times 10^{-7} (4.4\sigma)$$

$$\mathcal{B}^{[5.0,6.0],LHCb}_{B^+\to K^+\mu^+\mu^-} = (0.23 \pm 0.02) \times 10^{-7} (4.0\sigma)$$

[Albrecht etc. , 2107.04822]

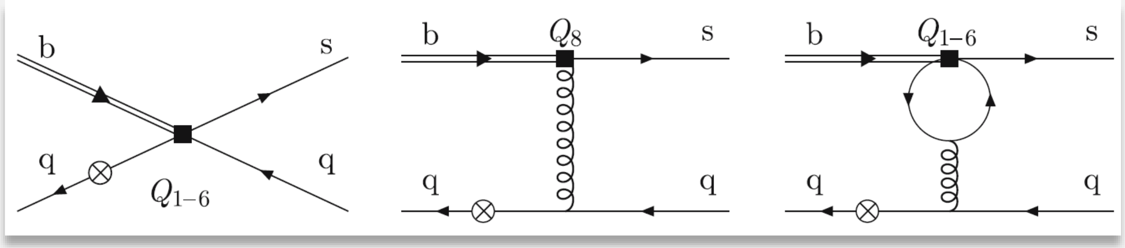
◆$R_{K^{(*)}}$: lepton flavor universality

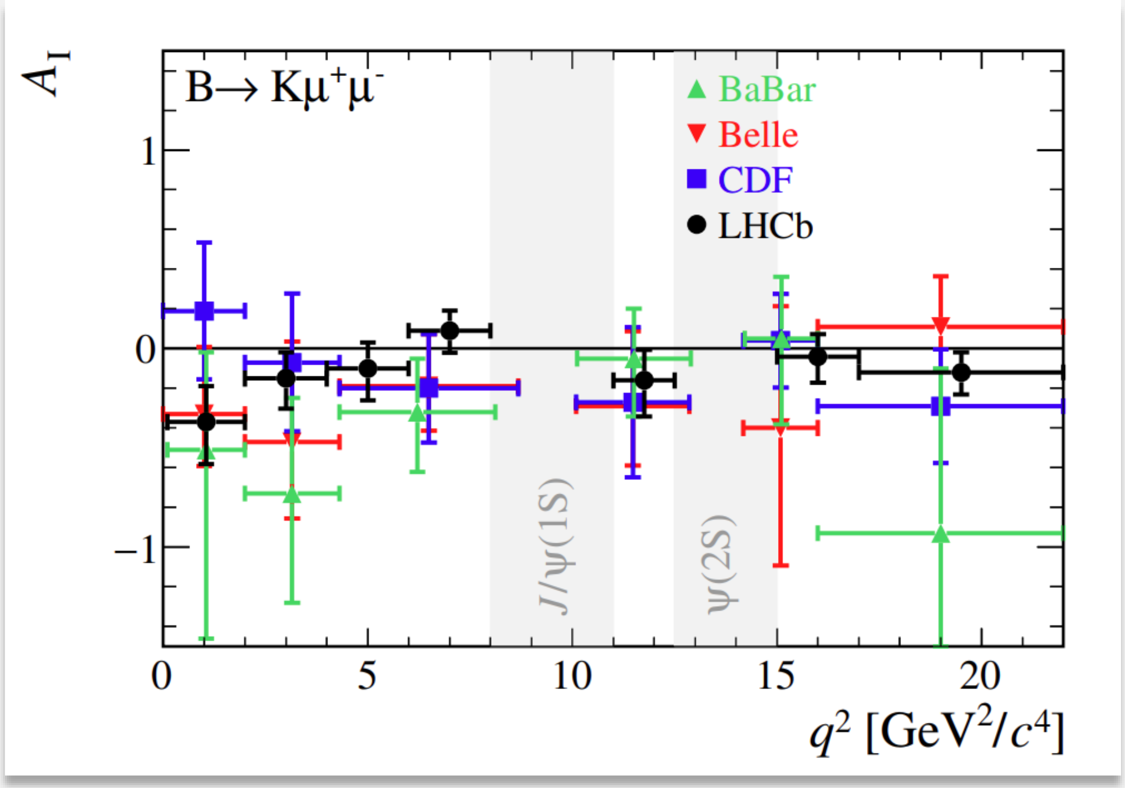
$$R_{K^{(*)}} = \mathcal{B}_{B^+\to K^+\mu^+\mu^-} / \mathcal{B}_{B^+\to K^+e^+e^-} = 1 + O(\alpha_{em}, m_\mu^2)$$

◆Isospin Asymmetries in $B \to K\ell^+\ell^-$

$$A_I = \frac{\Gamma\left[\overline{B^0} \to \overline{K^0}\ell^+\ell^-\right] - \Gamma[B^- \to K^-\ell^+\ell^-]}{\Gamma\left[\overline{B^0} \to \overline{K^0}\ell^+\ell^-\right] + \Gamma[B^- \to K^-\ell^+\ell^-]}$$

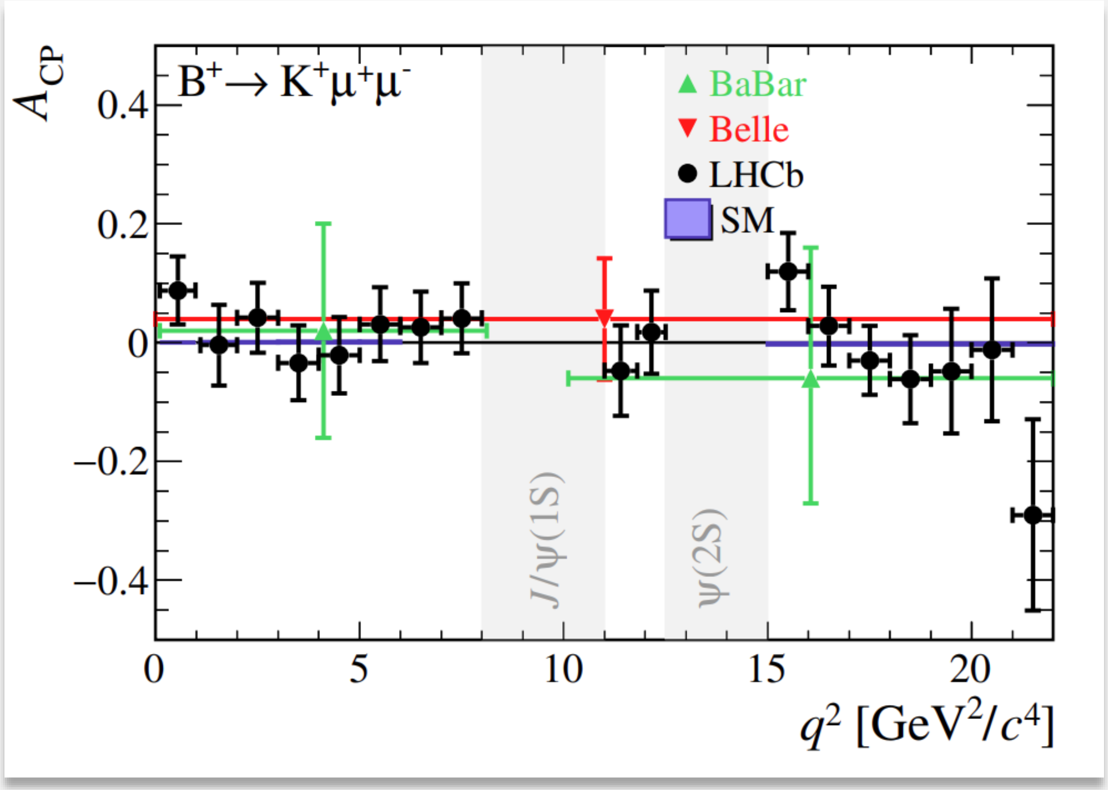
Sensitive to weak annihilation contribution





[Albrecht etc. , 2107.04822]

◆CP Asymmetries in $B \to K\ell^+\ell^-$



$$A_{CP} = \frac{d\Gamma[\bar{B} \to \bar{K}\ell^+\ell^-]/dq^2 - d\Gamma[B \to K\ell^+\ell^-]/dq^2}{d\Gamma[\bar{B} \to \bar{K}\ell^+\ell^-]/dq^2 + d\Gamma[B \to K\ell^+\ell^-]/dq^2}$$

$$A \sim V_{tb}V_{ts}^{*}P + V_{ub}V_{ud}^{*}T$$

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◆ $B \to \pi\ell^+\ell^-$: larger CP and isospin asymmetry are expected

◆ Measured branching ratio [LHCb, 1505.00414]

$$BR(B^+ \to \pi^+\ell^+\ell^-) = (1.83 \pm 0.24 \pm 0.05) \times 10^{-8}$$

◆ Direct CP asymmetry [LHCb, 1505.00414]

$$A_{CP}(B^+ \to \pi^+\ell^+\ell^-) = -0.11 \pm 0.12 \pm 0.01$$

◆Angular observables in $B \to K^* \ell^+ \ell^-$

$$\frac{d^2\Gamma}{dq^2 d\cos\theta_\ell d\cos\theta_K d\phi}$$

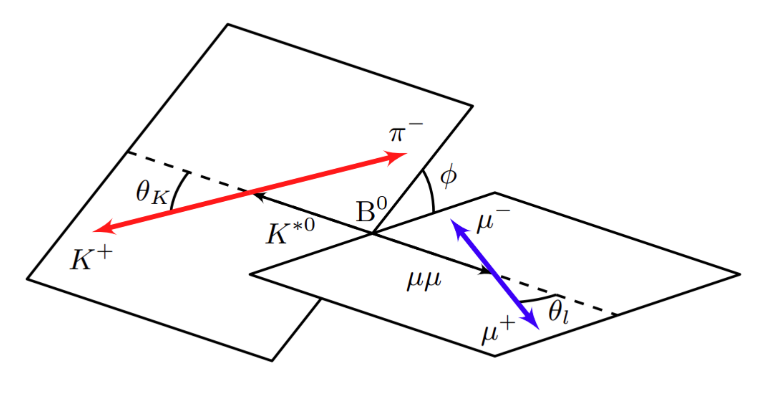
$$\propto \frac{9}{32\pi}[\frac{3}{4}(1-F_L)\sin^2\theta_K + F_L\cos^2\theta$$

$$+\frac{1}{4}(1-F_L)\sin^2\theta_K\cos 2\theta_\ell \text{-} F_L\cos^2\theta_K\cos 2\theta_\ell$$

$$+S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi + S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi$$

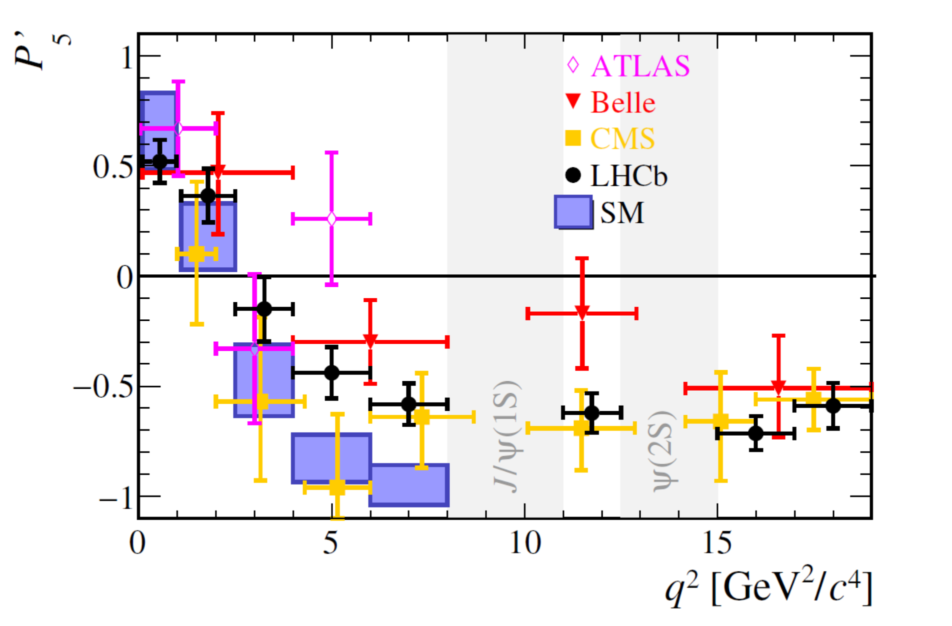
$$+S_5\sin 2\theta_K\sin\theta_\ell\cos\phi + S_6\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$

$$+S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi]$$



Typical Angular observable

$$P_i' = \frac{S_i}{\sqrt{F_L(1-F_L)}};\ i = 4,5,8$$



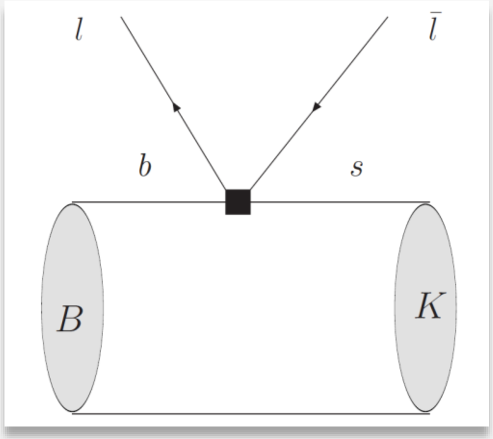
QCD factorization for $B \to P\ell^+\ell^-(P = K, \pi)$

◆The local contribution

$$A_{SM}^{local}(\bar{B} \to P\ell^+\ell^-) = \frac{G_F \alpha_{em} V_{tb} V_{tD}^*}{\sqrt{2}\pi}\left[C_9 L_V^\mu + C_{10} L_A^\mu\right] F_\mu^{B\to P}$$

$$O_{9V} = \frac{\alpha_e}{4\pi}(\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell)$$

$$O_{10A} = \frac{\alpha_e}{4\pi}(\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$



Local Form factors

$$F_\mu^{B\to K} = \langle K(p)|\bar{s}\gamma_\mu(1-\gamma_5)|\bar{B}(p+q)\rangle = \left[(2p+q)_\mu - \frac{m_B^2 - m_K^2}{q^2} q_\mu\right] f_+(q^2) + \frac{m_B^2 - m_K^2}{q^2} q_\mu f_0(q^2)$$

Local Form factors

- Lattice: calculation at large q^2+extrapolation

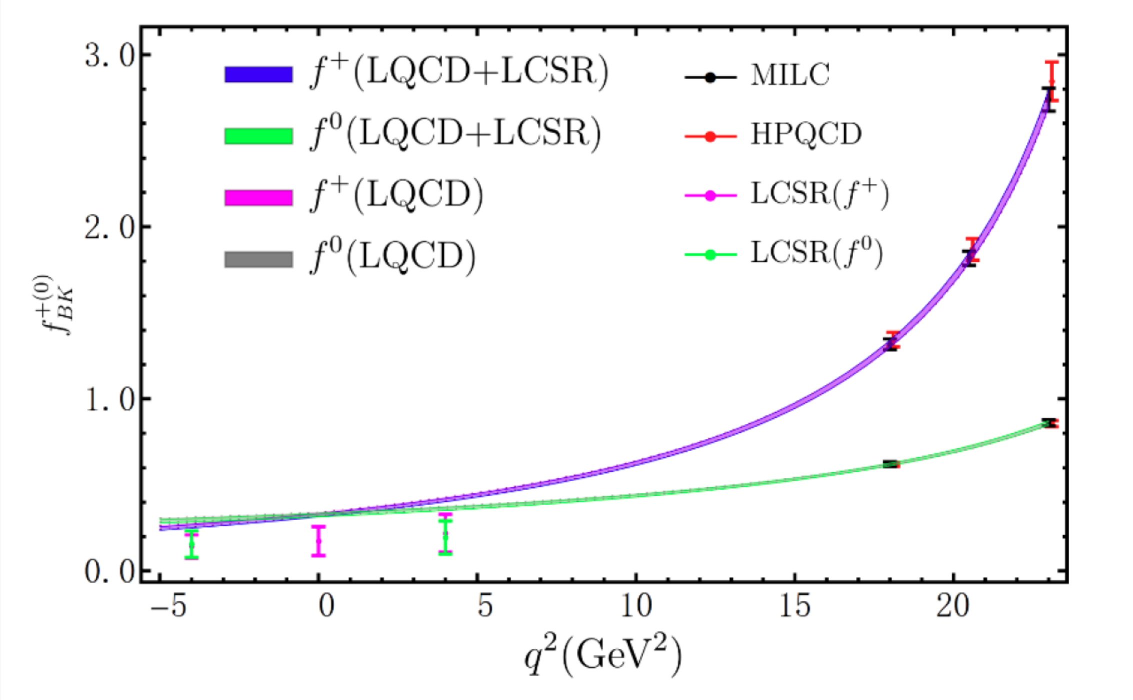
[Fermilab Lattice and MILC ,1507.01618; JLQCD, 2203.04938, B. Colquhoun et al., 1510.07446 1510.07446]
[Bailey etc., 1509.06325; HPQCD, 2207.12468]
[Fermilab Lattice and MILC, 1509.06325; HPQCD, 2207.12468]

- Light-meson LCSR: valid at small q^2

[Khodjamirian etc., 1103.2655; A. Bharucha, 1203.1359]

- B-meson LCSR: valid at small q^2

Including QCD corrections and power corrections to correlation functions
[Cui etc., 2212.11624; Lü etc. 1810.00819]



Combined BCL z-expansion fitting of the semileptonic B → K form factors

◆The nonlocal contribution

$$A_{SM}^{\text{nonlocal}}(\bar{B} \to P\ell^+\ell^-) = \frac{G_F\alpha_{em}}{\sqrt{2}\pi}\frac{L_V^\mu}{q^2}\{V_{tb}V_{tD}^*\left[2im_bC_7F_{T\mu}^{B\to P} + 16\pi^2 H_\mu^{(t)}\right] + V_{ub}V_{uD}^*\ 16\pi^2 H_\mu^{(u)}\}$$

$$H_\mu^{(u,t)} = iQ_p\int d^4x e^{iq\cdot x}\langle P(p)\left|T\left[\bar{p}\gamma_\mu p(x), H_{eff}^{u,t}\right]\right|\bar{B}(p+q)\rangle$$

- Large recoil region $4m_c^2 - q^2 \gg m_b\Lambda$: light-cone OPE

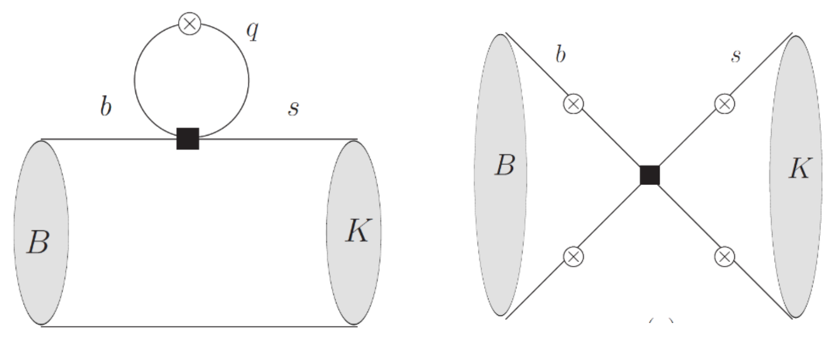
  [Beneke etc., hep-ph/0106067; Ali etc. hep-ph/0601034]

- Small recoil region $|q^2| \sim m_b^2$: (local OPE)

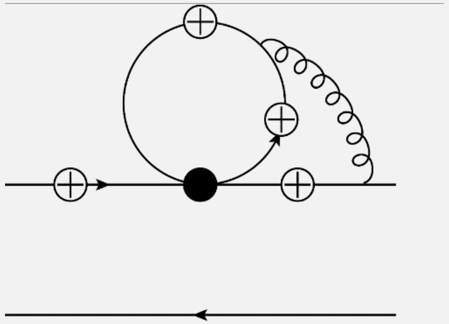
  B. Grinstein etc., hep-ph/0404250

  M. Beylich etc., 1101.5118

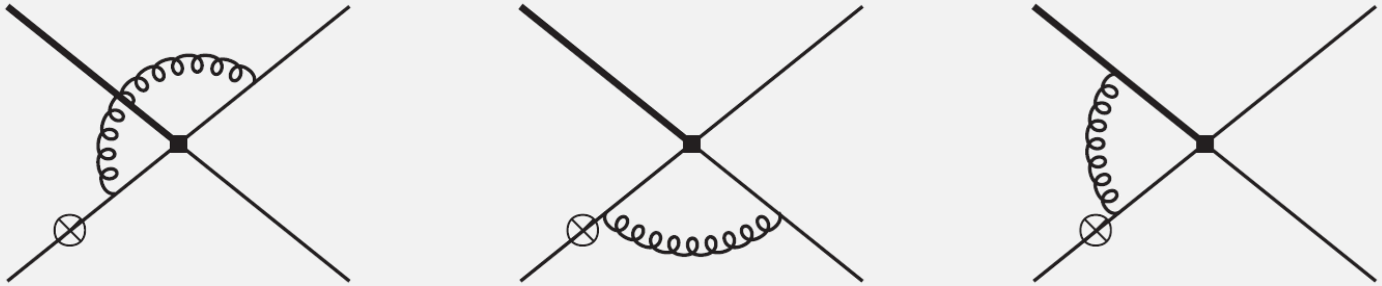
- Resonance region: violation of quark hadron duality



◆Arriving at a complete leading power study at $O(\alpha_s)$



● Penguin operator insertion



● NLO correction to annihilation diagram

NLO annihilation contribution

- Factorization formula [Huang etc., 2403.11258]

$$\mathcal{H}_\mu^{(t,u)} = \frac{m_b}{4\pi^2 m_B}\mathcal{T}_P^{(t,u)}(q^2)[q^2(2p+q)_\mu - (m_B^2 - m_K^2)q_\mu]$$

$$\mathcal{T}_{P,\mathrm{anni}}^{(u,t)}(q^2) = -\frac{\pi^2}{N_c}\frac{\tilde{F}_B f_P}{m_B}\sum_{m=\pm}\int_0^\infty \frac{d\omega}{\omega}\int_0^1 du\ T_{P,m}^{t,u}(\omega,u,\mu)\phi_{B,m}(\omega,\mu)\phi_P(u,\mu)$$

- Strategy of the calculation: two step matching

$$\mathrm{QCD} \longrightarrow \mathrm{SCET_I} \longrightarrow \mathrm{SCET_{II}}$$

$$\Downarrow \qquad\qquad \Downarrow$$

$$\sum C_i H_i\,(u,\mu) \qquad\qquad J_m\,(\omega,\mu)$$

$$T_{P,m}^{t,u}(\omega,u,\mu)=\sum C_i H_i(u,\mu) * J_m(\omega,\mu)$$

hard function[0911.3665,2002.03262]

$$Q_i = H_i^I * O^{(A0)} + H_i^{II} * O^{(B1)}$$

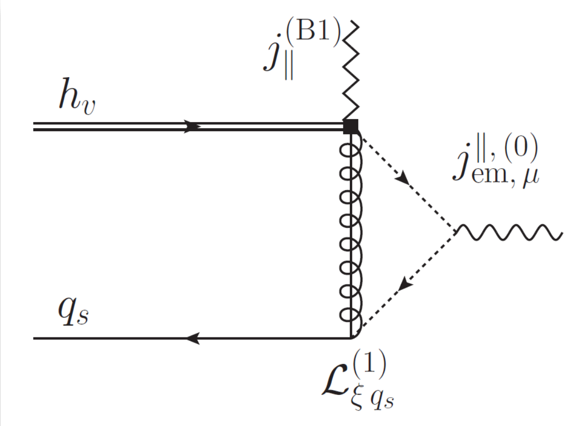
$$\mathcal{O}^{(A0)} = \left[(\bar\chi W_{\bar c})\,(t\bar n)\,\frac{\not{\bar n}}{2}\,(1-\gamma_5)\,\left(W_{\bar c}^\dagger \chi\right)(0)\right] \left[(\bar\xi W_c)\,(0)\,\not{n}\,(1-\gamma_5)\,h_v(0)\right],$$

$$\mathcal{O}^{(B1)} = \frac{1}{m_b}\left[(\bar\chi W_{\bar c})\,(t\bar n)\,\frac{\not{\bar n}}{2}\,(1-\gamma_5)\,\left(W_{\bar c}^\dagger \chi\right)(0)\right] \left[(\bar\xi W_c)\,(0)\,\frac{\not{n}}{2}\left[W_c^\dagger i\not{D}_{\perp c} W_c\right](sn)\,(1+\gamma_5)\,h_v(0)\right]$$

Jet function

$$\left\langle 0 \left| T\left\{ j_{em}^{(2)}, O^{(A0)} \right\} \right| \bar B \right\rangle_{FT} + \left\langle 0 \left| T\left\{ j_{em}^{(0)}, \int d^4y\, iL_{\xi q}^{(2)}(y), O^{(A0)} \right\} \right| \bar B \right\rangle_{FT} = \frac{1}{2}\tilde f_B m_B \frac{J_-^{(A0)}}{\bar n\cdot q-\omega} \otimes \phi_B^-$$

$$\left\langle 0 \left| T\left\{ j_{em}^{(0)}, \int d^4y\, iL_{\xi q}^{(1)}(y), O^{(B1)} \right\} \right| \bar B \right\rangle_{FT} = \frac{1}{2}\tilde f_B m_B \sum J_+^{(B1)} \otimes \phi_B^+$$



Resummation of large logarithms

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Hard function

$$\frac{d}{d\ln\mu} C_i H_i(u,\mu) = \left[-\Gamma_{cusp}(\alpha_s)\ln\frac{\mu}{m_b} + \gamma_h(\alpha_s)\right] C_i H_i(u,\mu) - \int_0^1 dv\,\Gamma_{ERBL}(v,u,\alpha_s)\, C_i H_i(v,\mu)$$

$$\Rightarrow C_i\ H_i(u,\mu) = e^{S_h(\mu_h,\mu)}\left(\frac{\mu_h}{m_b}\right)^{-a(\mu_h,\mu)}\int_0^1 U_{\text{ERBL}}(v,u,\mu_h,\mu)\, C_i H_i(u,\mu_h)$$

Jet function

$$\frac{d}{d\ln\mu}\frac{J_-(n\cdot q,\omega,\mu)}{\bar{n}\cdot q-1} = \left[\Gamma_{cusp}(\alpha_s)\ln\frac{\hat{\mu}^2}{\omega} + \gamma_{hc}(\alpha_s)\right] J_-(n\cdot q,\omega,\mu) - \int d\omega'\omega\Gamma_{LN,-}(\omega',\omega,\alpha_s)\frac{J_-(n\cdot q,\omega,\mu)}{\bar{n}\cdot q-1}$$

$$\Rightarrow \frac{J_-(n\cdot q,\omega,\mu)}{\bar{n}\cdot q-1} e^{S_h(\mu_{hc},\mu)}\int_0^\infty \frac{d\omega'}{\omega'}\left(\frac{\hat{\mu}_{hc}}{\omega'}\right)^{a(\mu_h c,\mu)} U_{LN,-}(\omega',\omega,\mu_{hc},\mu)\frac{J_-(n\cdot q,\omega,\mu c)}{\bar{n}\cdot q-1}$$

Convergence of the convolution

$$\int_0^\infty \frac{J_-(n\cdot q,\bar{n}\cdot q,\omega,\mu)}{\bar{n}\cdot q-\omega}\phi_B^-(\omega) \qquad \text{and} \qquad \int_0^\infty J_+(n\cdot q,\bar{n}\cdot q,\omega,\mu)\phi_B^+(\omega)$$

- $\omega \to 0$

$$\phi_B^-(\omega) \sim 1, \quad J_-(n\cdot q,\omega,\mu) \sim 1 \qquad\qquad \phi_B^+(\omega) \sim \omega, \quad J_+(n\cdot q,\omega,\mu) \sim 1$$

- $\omega \to \infty$

$$\phi_B^\pm(\omega) \sim \frac{1}{\omega}\ln\frac{\mu}{\omega} \qquad J_-(n\cdot q,\bar{n}\cdot q,\omega,\mu) \sim \ln^2\frac{\omega}{\bar{n}\cdot q} \qquad J_+(n\cdot q,\bar{n}\cdot q,\omega,\mu) \sim \frac{1}{\omega}\ln\frac{\omega}{\bar{n}\cdot q}$$

Phenomenology

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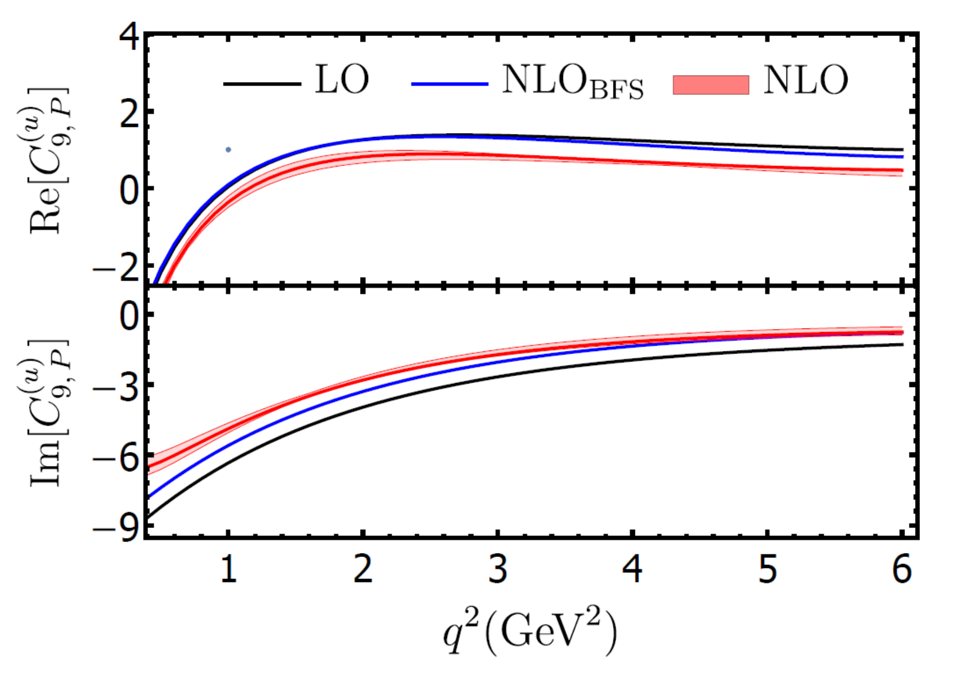
- Effective Wilson coefficient $C_9^i = C_9\delta^{it} + \frac{2m_b}{m_B}\frac{\mathcal{T}_P^{(i)}}{m_B}, i = u, t$

- C_9^u in $B \to \pi\ell^+\ell^-$:

NLO WA contribution： 35%(15%) reduction for real(imaginary) part of LO



- C_9^t in $B \to \pi\ell^+\ell^-$:

Only 3% corrections to $T_P^{(t)}$ of NLO(BFS)

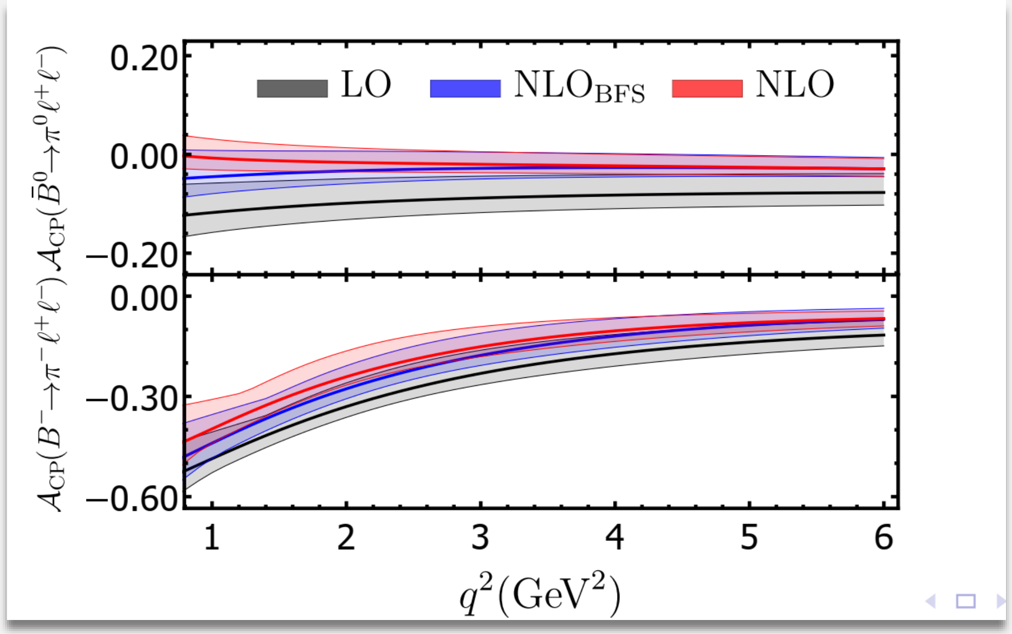
[Huang etc., 2403.11258]

Branching ratios：

WA brings negligible corrections

CP asymmetries in $B \to \pi \ell^+ \ell^-$

NLO WA contribution： 15% corrections



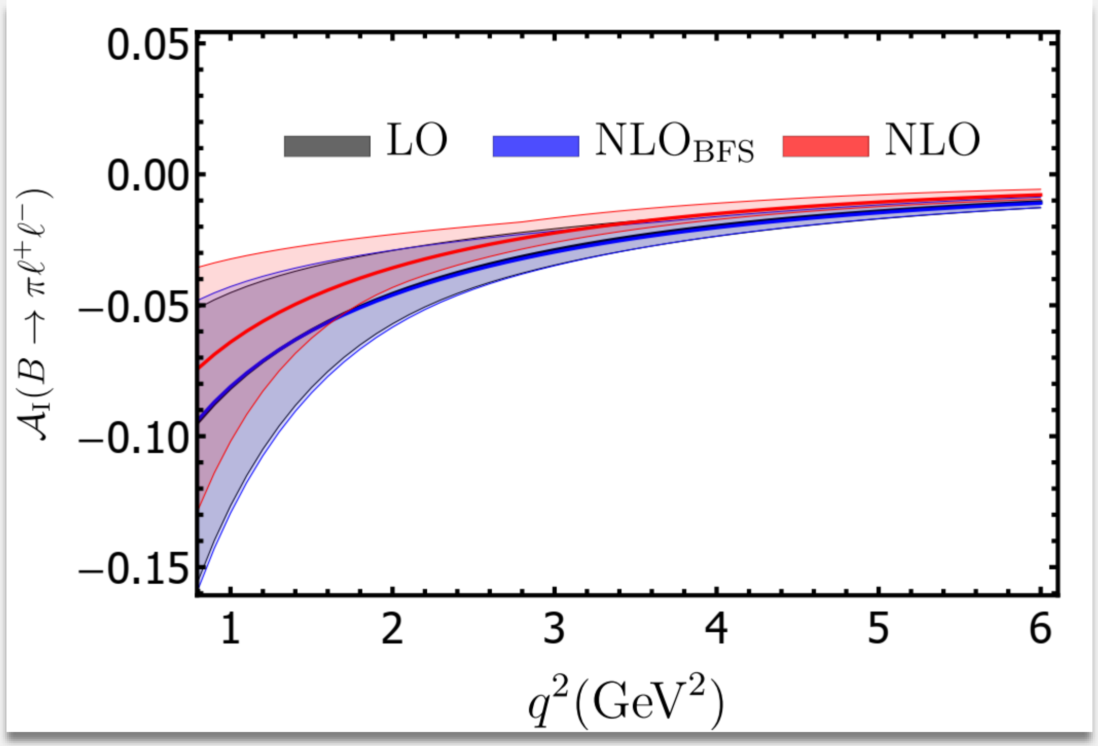
CP asymmetries in $B \to K \ell^+ \ell^-$

q^2	[1.1,2.0]	[2.0,3.0]	[2.0,3.0]	[4.0,5.0]	[5.0,6.0]
$B^- \to K^- \ell^+ \ell^-$	$1.08^{+0.09}_{-0.26}$	$0.67^{+0.09}_{-0.26}$	$0.44^{+0.12}_{-0.19}$	$0.33^{+0.12}_{-0.14}$	$0.27^{+0.10}_{-0.11}$
$\bar{B}^0 \to \bar{K}^0 \ell^+ \ell^-$	$0.01^{+0.08}_{-0.12}$	$0.04^{+0.07}_{-0.11}$	$0.06^{+0.07}_{-0.10}$	$0.08^{+0.07}_{-0.10}$	$0.09^{+0.07}_{-0.09}$

Isospin asymmetry

◆ NLO WA:

brings about 25% shift of A_I in $B \to \pi \ell^+ \ell^-$



◆ A_I in $B \to \pi \ell^+ \ell^-$ 3-6 times larger than $B \to K \ell^+ \ell^-$

Power corrections

- Long distance quark-loop
- Higher twist contribution
- ……

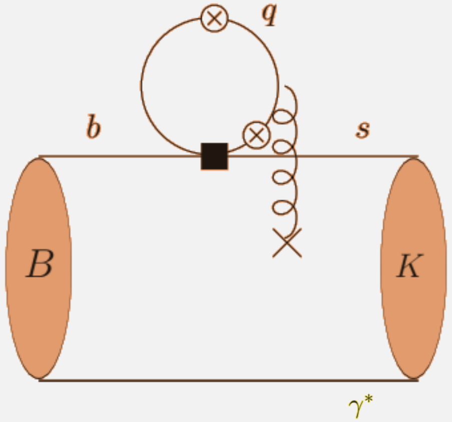
Long-distance quark loop

- Subleading power corrections: soft gluon emission

A. Khodjamirian etc., 1006.4945

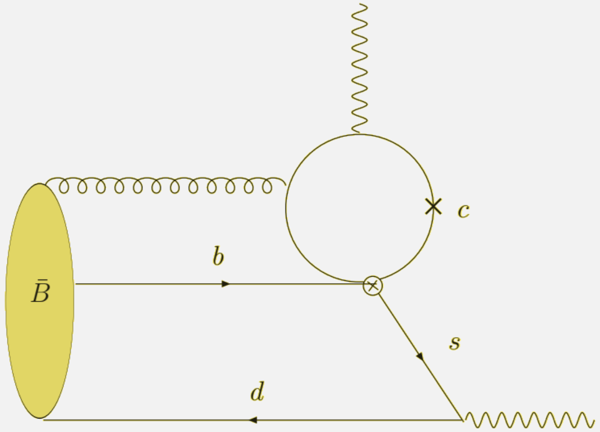
$$\left[\mathcal{H}_{\mu}^{(B\to K^{(*)})}(p,q)\right]_{nonfact} = 2C_1\langle K^{(*)}(p)|\widetilde{\mathcal{O}}_{\mu}(q)|B(p+q)\rangle\,,$$

$$\widetilde{\mathcal{O}}_{\mu}(q) = \int d\omega\, I_{\mu\rho\alpha\beta}(q,\omega)\bar{s}_L\gamma^{\rho}\delta[\omega - \frac{(in_+\mathcal{D})}{2}]\widetilde{G}_{\alpha\beta}b_L$$



- The matrix element: light-cone sum rules

$$\mathcal{F}_{\nu\mu}^{(B\to K)}(p,q) = i\int d^4y e^{ip\cdot y}\langle 0|T\{j_{\nu}^{K}(y)\widetilde{\mathcal{O}}_{\mu}(q)\}|B(p+q)\rangle$$



- A revisiting

N. Gubernari etc., 2011.09813

Including more B meson LCDAs

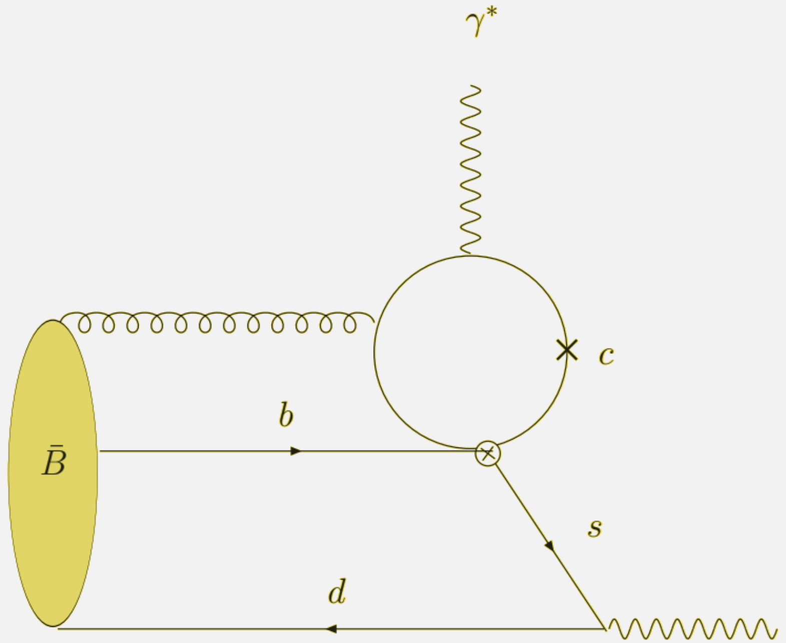
- Collinear gluon emission:

Mishira, 2505.16426

- Higher twist B-meson LCDAs

Such as: Four-particle twist-5 and twist-6 LCDAs

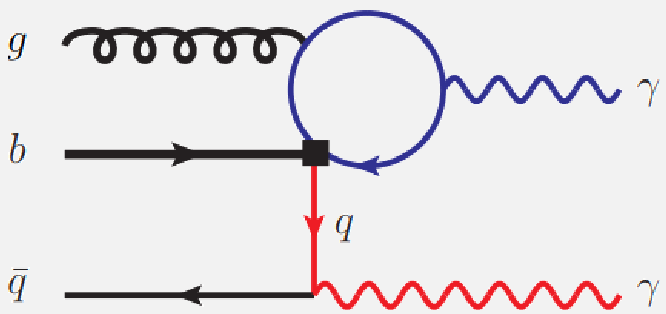
- Power counting of internal lines



Hard VS hard-collinear

$\Lambda_{QCD} \ll m_c \sim m_b$ $\Lambda_{QCD} \ll m_c \ll m_b$

- Long distance quark loop in $B \to \gamma\gamma$



- A novel soft function with different light-cone direction [Qin etc. *PRL.* 131 (2023) 9, 091902]

$$\langle 0|(\bar{q}_s S_n)(\tau_1 n) S_n^\dagger S_{\bar{n}}(0) S_{\bar{n}}^\dagger g_s G_{\mu\nu} S_{\bar{n}}(\tau_2 \bar{n}) \bar{n}^\nu n\cdot\gamma\gamma_\perp^\mu\gamma_5 S_{\bar{n}}^\dagger h_v(0)|\bar{B}_v\rangle$$

$$= 2F_{\rm B}(\mu) m_B \int_{-\infty}^{\infty} d\omega_1 d\omega_2 \ \mathrm{e}^{-i(\omega_1\tau_1+\omega_2\tau_2)} \Phi_G(\omega_1,\omega_2,\mu)$$

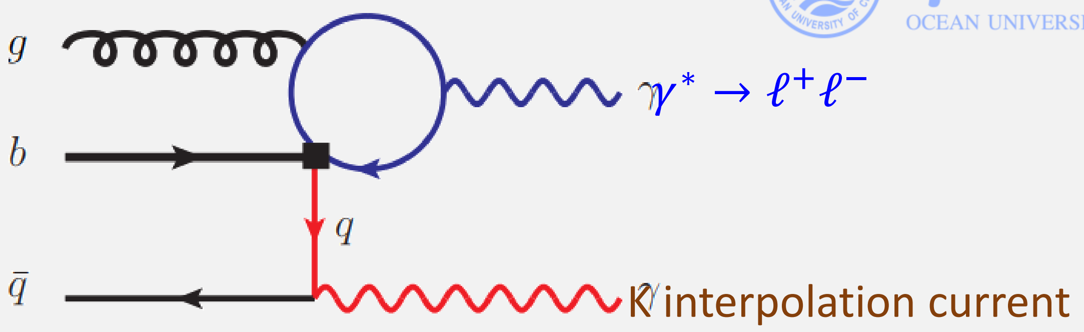
- Normalization

$$\int_{-\infty}^{\infty} d\omega_1 \, \Phi_G(\omega_1,\omega_2,\mu) = \int_0^{\infty} d\omega_1 \ \Phi_4(\omega_1,\omega_2,\mu)$$

$$\int_{-\infty}^{\infty} d\omega_2 \ \Phi_G(\omega_1,\omega_2,\mu) = \int_0^{\infty} d\omega_2 \ \Phi_5(\omega_1,\omega_2,\mu)$$

$$\int_{-\infty}^{\infty} d\omega_1 d\omega_2 \ \Phi_G(\omega_1,\omega_2,\mu) = \frac{\lambda_E^2+\lambda_H^2}{3}$$

- Generalized to $B \to K\ell\bar{\ell}$



- A novel soft function with different light-cone direction [Qin etc. *PRL.* 131 (2023) 9, 091902]

$$\langle 0|(\bar{q}_s S_n)(\tau_1 n) S_n^\dagger S_{\bar{n}}(0) S_{\bar{n}}^\dagger g_s G_{\mu\nu} S_{\bar{n}}(\tau_2 \bar{n}) \bar{n}^\nu n\cdot\gamma\gamma_\perp^\mu\gamma_5 S_{\bar{n}}^\dagger h_v(0)|\bar{B}_v\rangle$$

$$= 2F_{\rm B}(\mu) m_B \int_{-\infty}^{\infty} d\omega_1 d\omega_2 \ \mathrm{e}^{-i(\omega_1\tau_1+\omega_2\tau_2)} \Phi_G(\omega_1,\omega_2,\mu)$$

- Normalization

$$\int_{-\infty}^{\infty} d\omega_1 \, \Phi_G(\omega_1,\omega_2,\mu) = \int_0^{\infty} d\omega_1 \ \Phi_4(\omega_1,\omega_2,\mu)$$

$$\int_{-\infty}^{\infty} d\omega_2 \ \Phi_G(\omega_1,\omega_2,\mu) = \int_0^{\infty} d\omega_2 \ \Phi_5(\omega_1,\omega_2,\mu)$$

$$\int_{-\infty}^{\infty} d\omega_1 d\omega_2 \ \Phi_G(\omega_1,\omega_2,\mu) = \frac{\lambda_E^2+\lambda_H^2}{3}$$

Summary and outlook

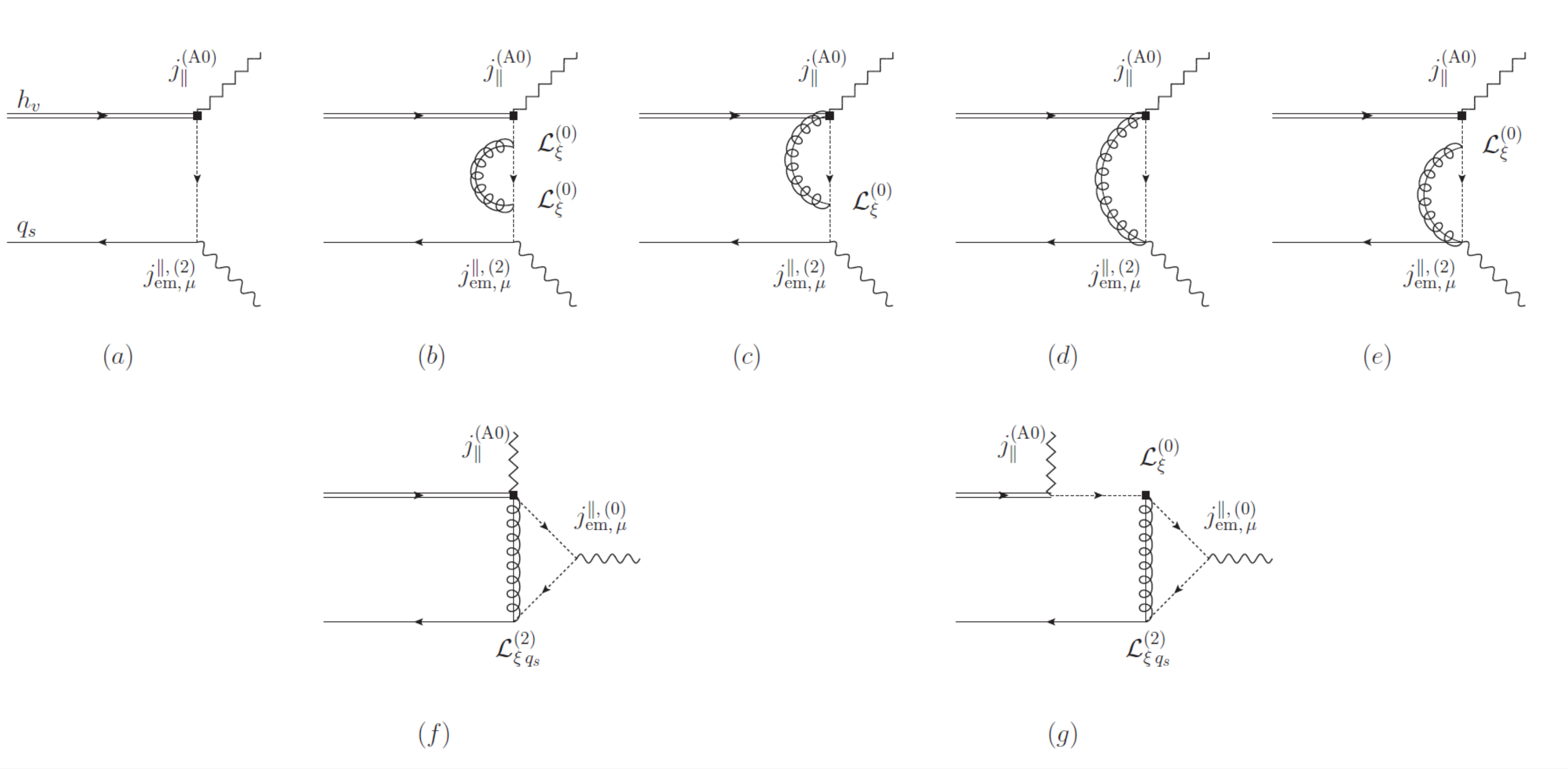
◆ We established the factorization formula for the weak annihilation contribution in $B \to P\ell^+\ell^-$ at leading power, and calculated NLO perturbation functions

◆ NLO weak annihilation contribution can bring sizable correction to the CP asymmetry and isospin asymmetry.

◆ The power suppressed contributions, such as long distance quark loop diagram, may have large impact on many observables

Jet function from A-type operators

$$\left\langle 0\left|T\left\{j_{em}^{(2)}, O^{(A0)}\right\}\right|\bar{B}\right\rangle_{FT} + \left\langle 0\left|T\left\{j_{em}^{(0)}, \int d^4y\, i\mathcal{L}_{\xi q}^{(2)}(y), O^{(A0)}\right\}\right|\bar{B}\right\rangle_{FT} = \frac{1}{2}\tilde{f}_B m_B \frac{J_-^{(A0)}}{\bar{n}\cdot q-\omega}\otimes\phi_B^-$$



NLO annihilation contribution

- Jet function from B-type operators

$$\left\langle 0 \left| T\left\{ j_{em}^{(0)}, \int d^4y\, i\mathcal{L}_{\xi q}^{(1)}(y), O^{(B1)} \right\} \right| \bar{B} \right\rangle_{FT} = \frac{1}{2} \tilde{f}_B m_B \sum J_{+}^{(B1)} \otimes \phi_B^{+}$$