pseudo-Nambu-Goldstone Boson dark matter

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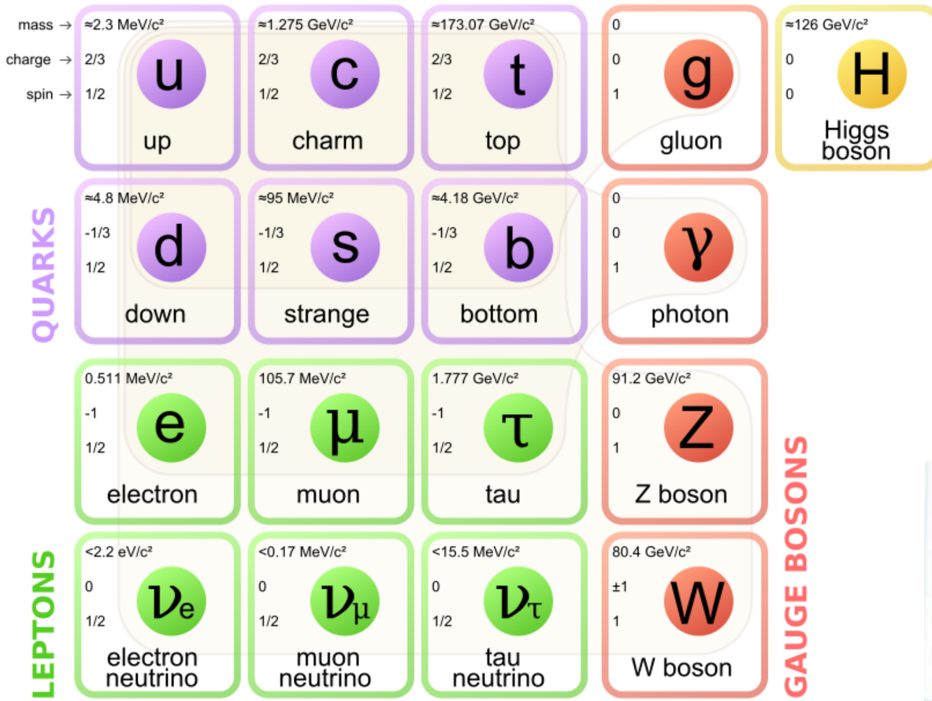
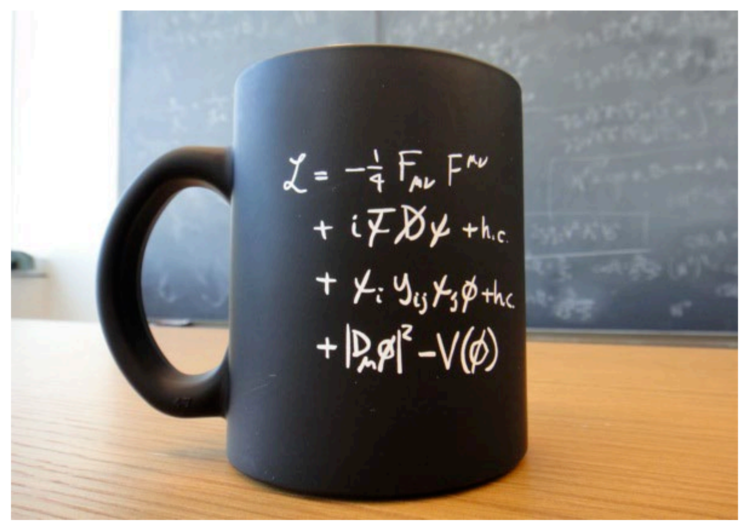
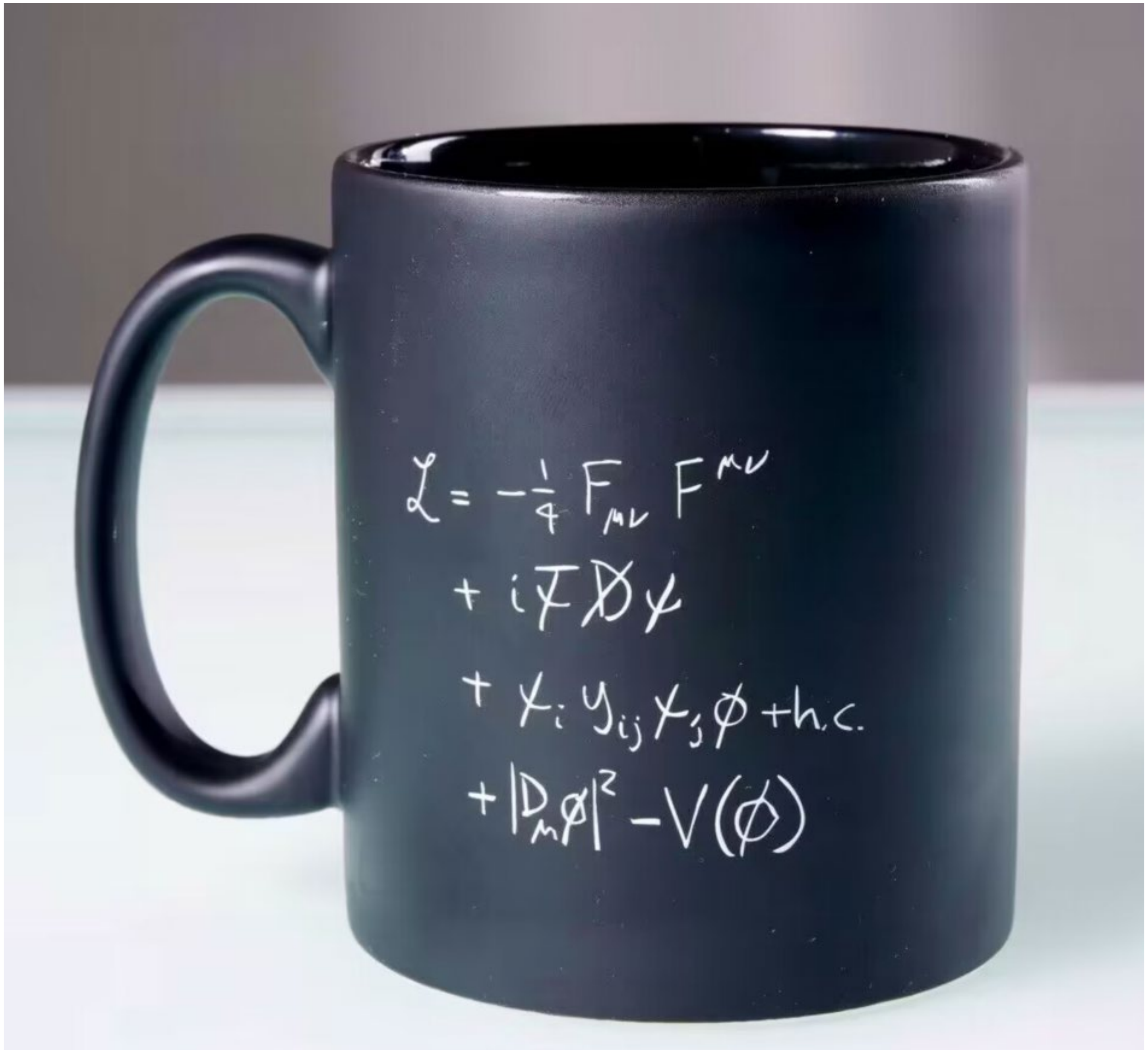
The standard model of particle physics



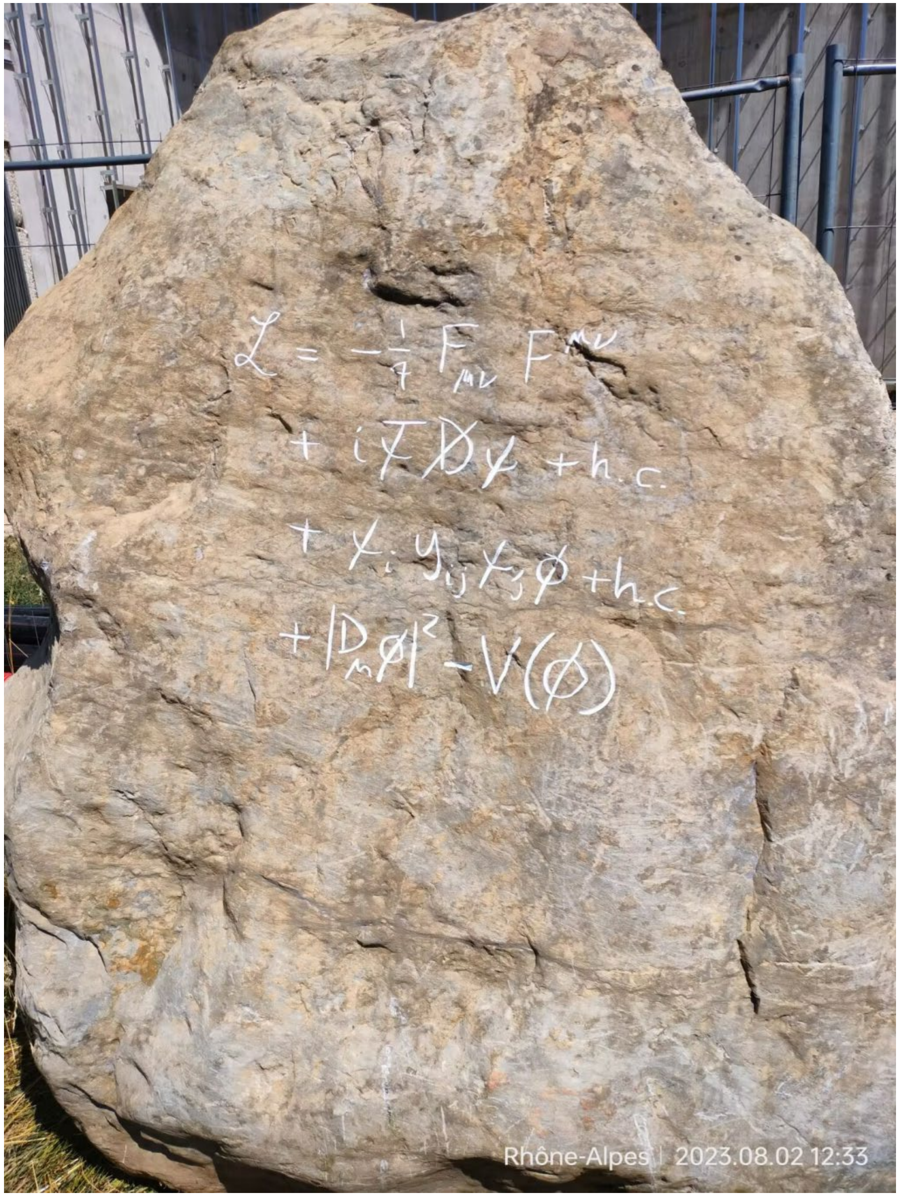
	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$	3	2	$\frac{1}{6}$
u_R	3	1	$\frac{2}{3}$
d_R	3	1	$-\frac{1}{3}$
$\ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1	2	$-\frac{1}{2}$
e_R	1	1	-1



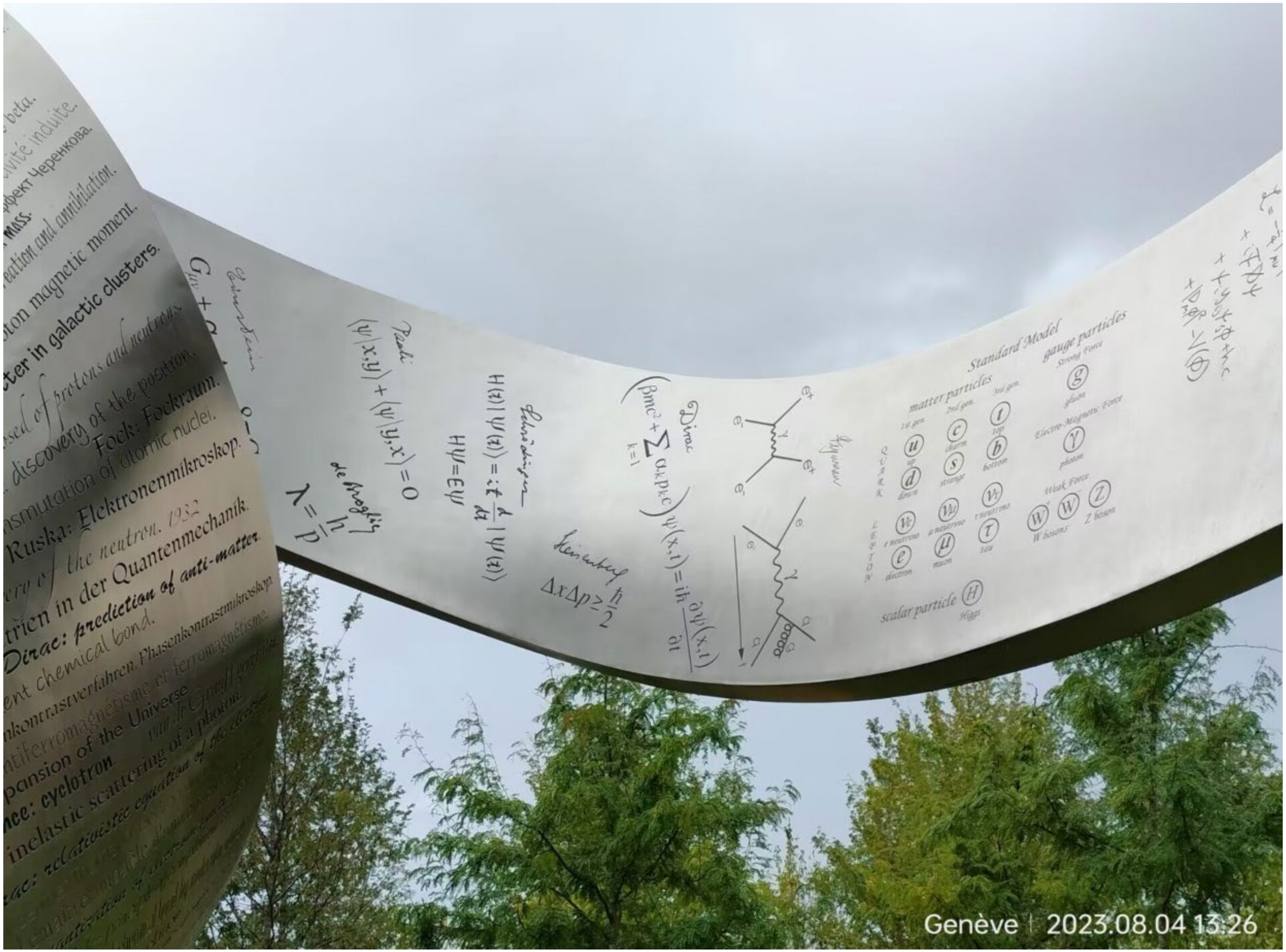
CERN的新版咖啡杯


$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$
$+ i \bar{\psi} \not{D} \psi$
$+ \psi_i y_{ij} \psi_j \phi + h.c.$
$+ |D_\mu \phi|^2 - V(\phi)$

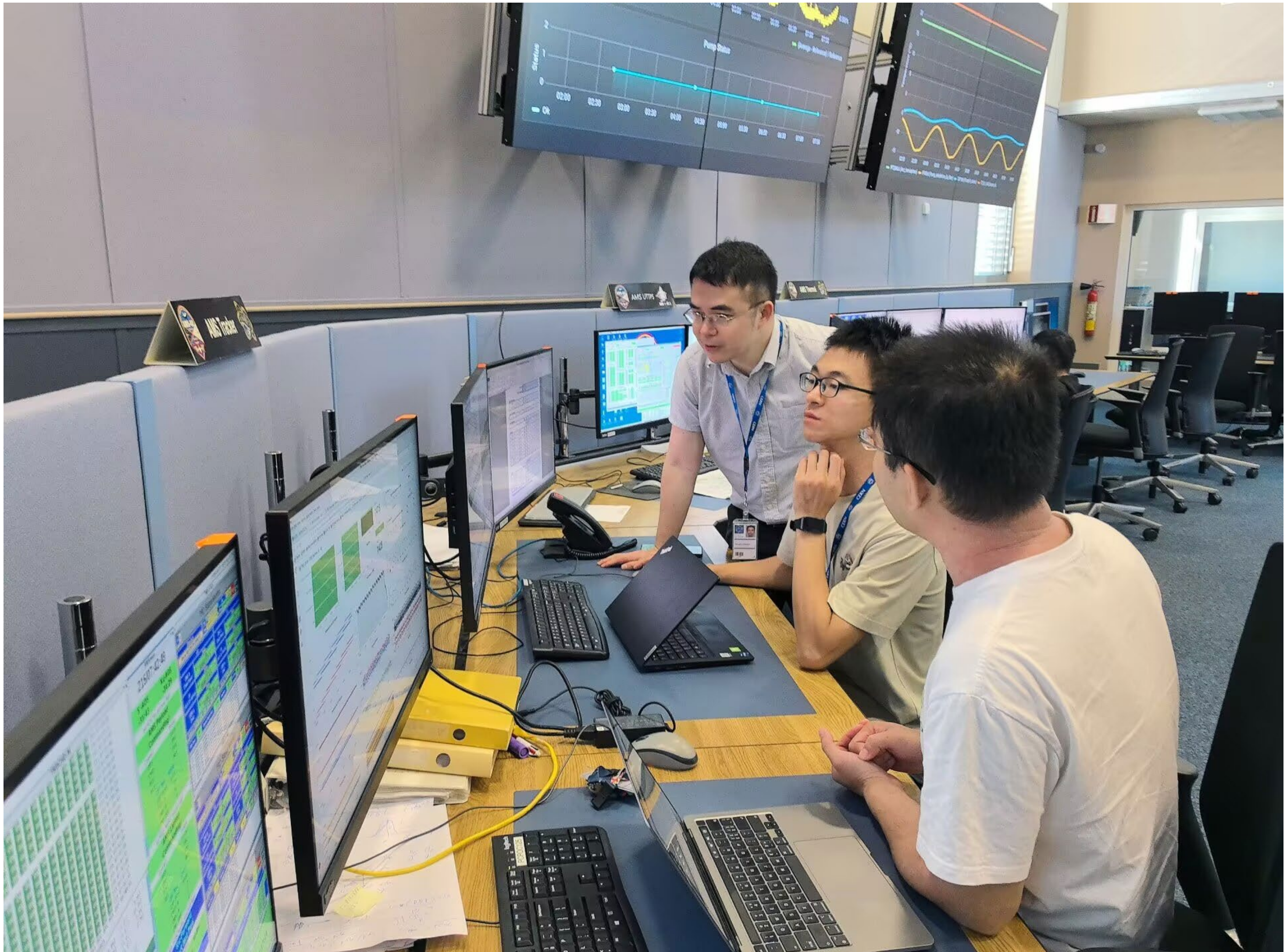
刻在CERN的石头上的标准模型的拉氏量



刻在CERN的铁皮上的标准模型的基本粒子、拉氏量和费曼图



中山大学师生参与AMS值班



Some **problems** of the standard model

- triviality

$$\lambda(Q) = \frac{\lambda(v)}{1 - \frac{3}{4\pi^3}\log\frac{Q^2}{v^2}\lambda(v)}$$

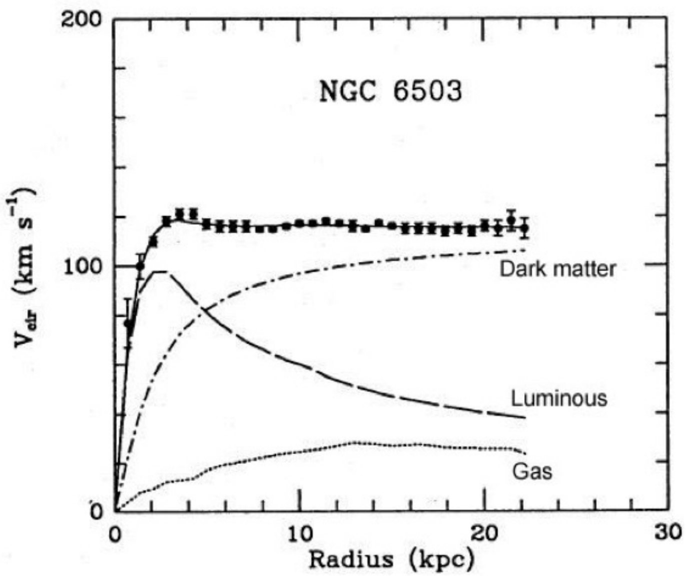
- hierarchy problem

$$m_h^2 \approx m_{tree}^2 - \frac{3}{8\pi^2}\lambda_t^2\Lambda^2 + \frac{1}{16\pi^2}g^2\Lambda^2 + \frac{1}{16\pi^2}\lambda^2\Lambda^2$$
$$\sim m_{tree}^2 - (200-20-10)(125GeV)^2\left(\frac{\Lambda}{10TeV}\right)^2$$

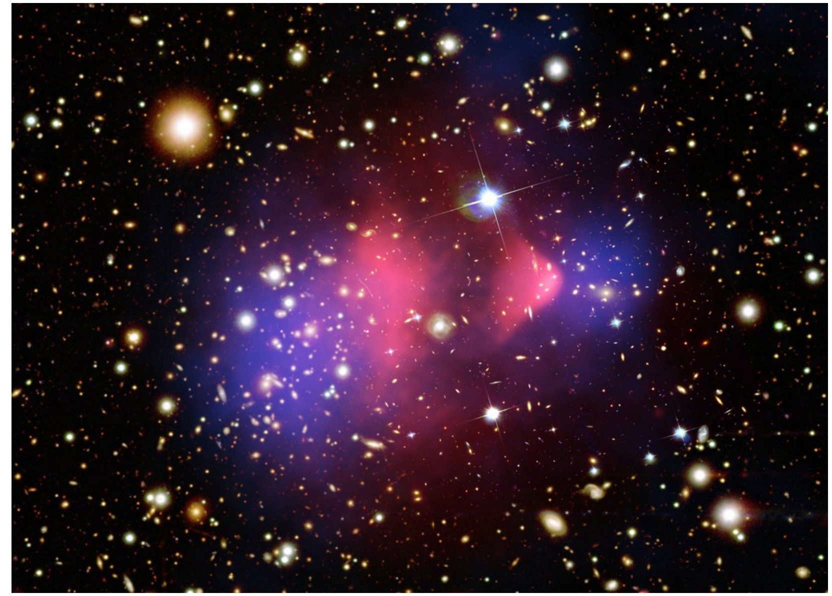
- too many parameters
- existence of **dark matter**
- smallness of neutrino mass
- matter/antimatter asymmetry
- vacuum stability

……

The SM may NOT be a final theory.



K.G. Begeman, A.H. Broels, R.H. Sanders. 1991. Mon.Not.RAS 249, 523.



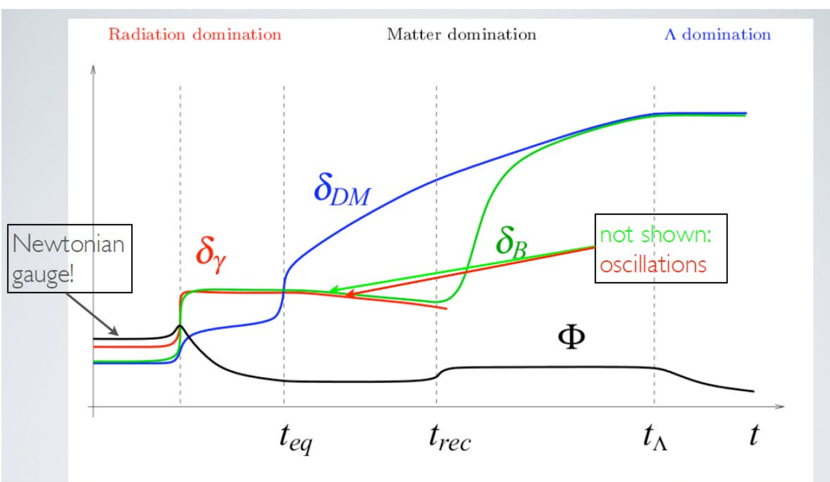
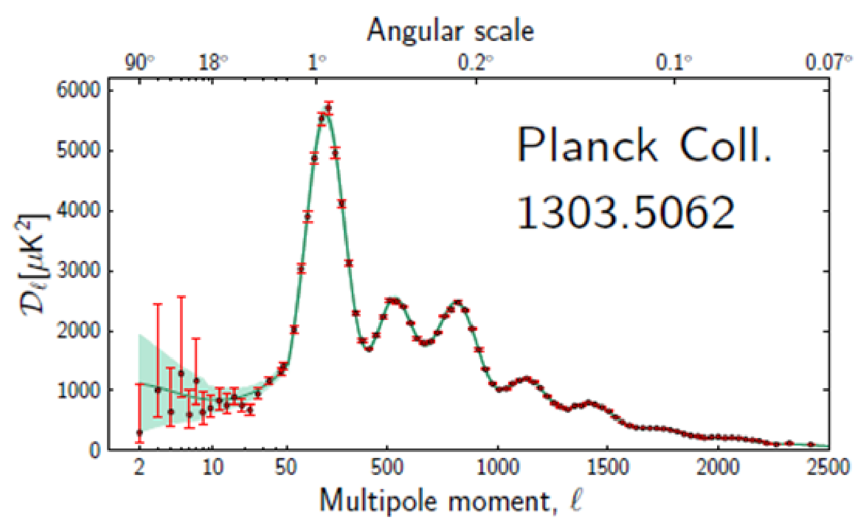
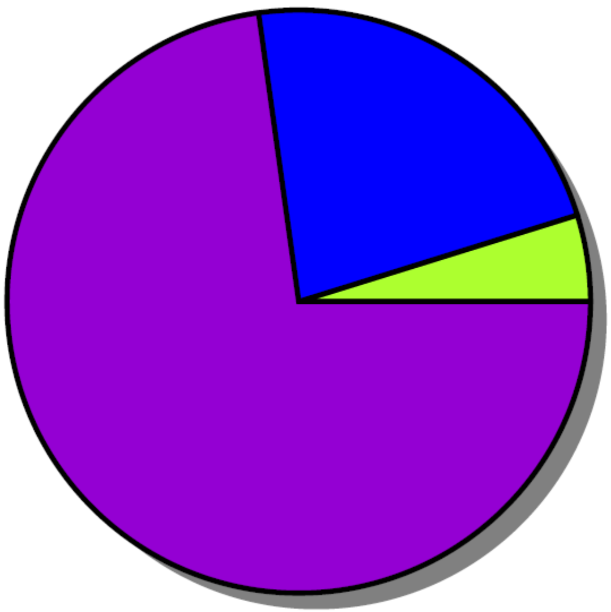


Figure 5: Schematic plot of the evolution of density pertirbations in different components. Here, $\delta = \delta\rho/\rho$, and Φ is the gravitational potential. The left dashed vertical line is the time of horizon crossing of a mode considered.

Rubakov & Vlasov, 1008.1704





Planck 2015

冷暗物质 (25.8%)

$$\Omega_c h^2 = 0.1186 \pm 0.0020$$

重子物质 (4.8%)

$$\Omega_b h^2 = 0.02226 \pm 0.00023$$

暗能量 (69.3%)

$$\Omega_\Lambda = 0.692 \pm 0.012$$

Inferred properties of dark matter

- **Dark (electrically neutral):** no light emitted from it
- **Nonbaryonic:** BBN & CMB observations
- **Long lived:** survived from early eras of the Universe to now
- **Colorless:** otherwise, it would bind with nuclei
- **Cold:** structure formation theory
- **Abundance:** more than 80% of all matter in the Universe

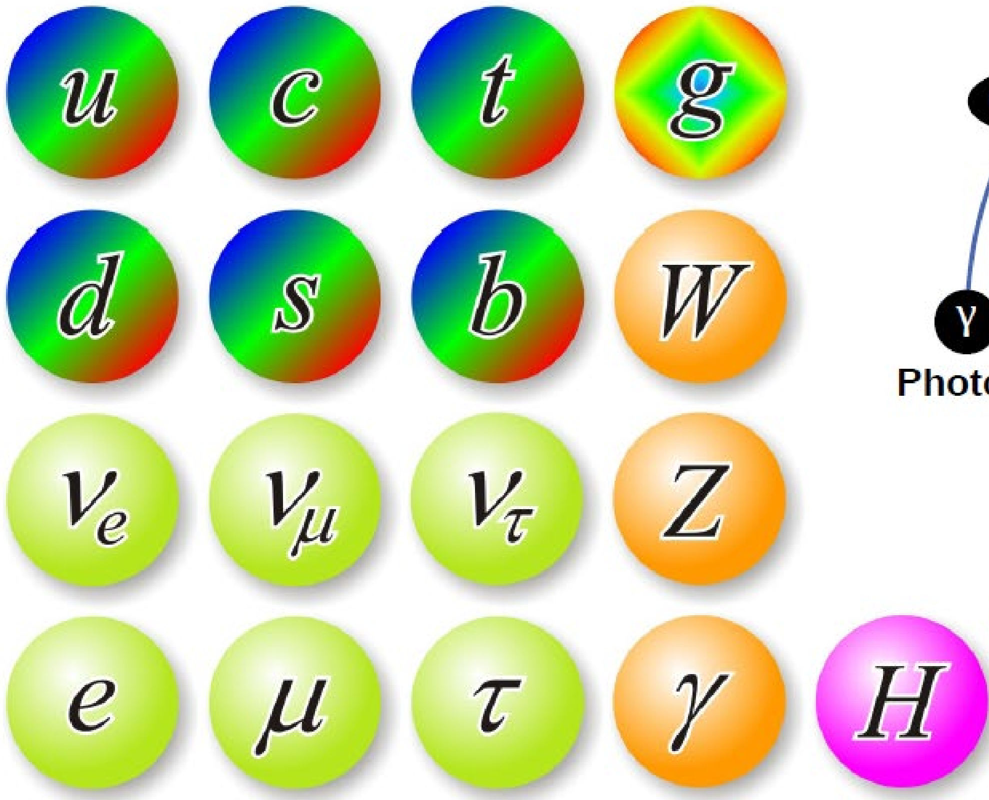
$$\rho_{\rm DM} \sim 0.4\ {\rm GeV/cm^3}\ \text{near the earth}$$

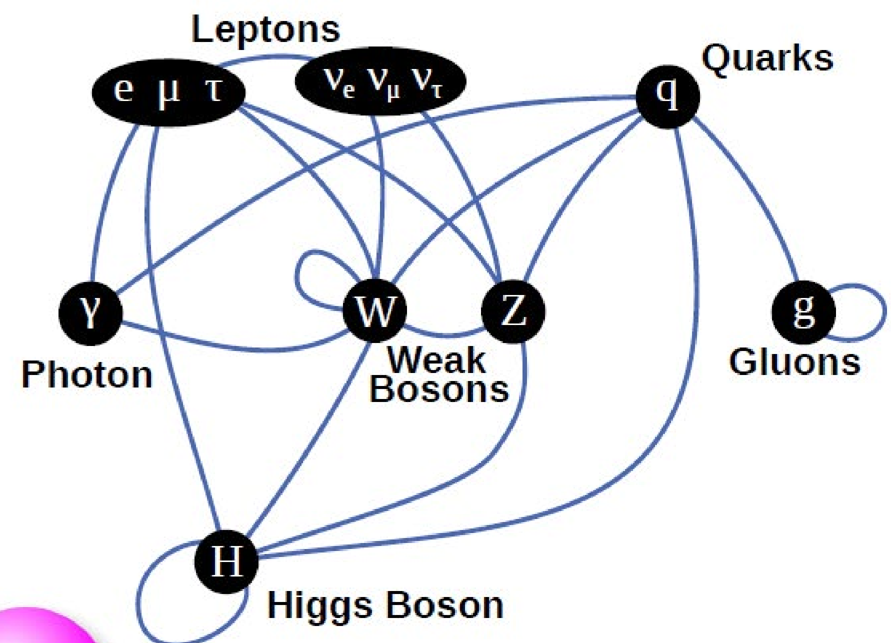
Standard model (SM) of particle physics

$SU(3)_C \times SU(2)_L \times U(1)_Y$ **gauge symmetry**

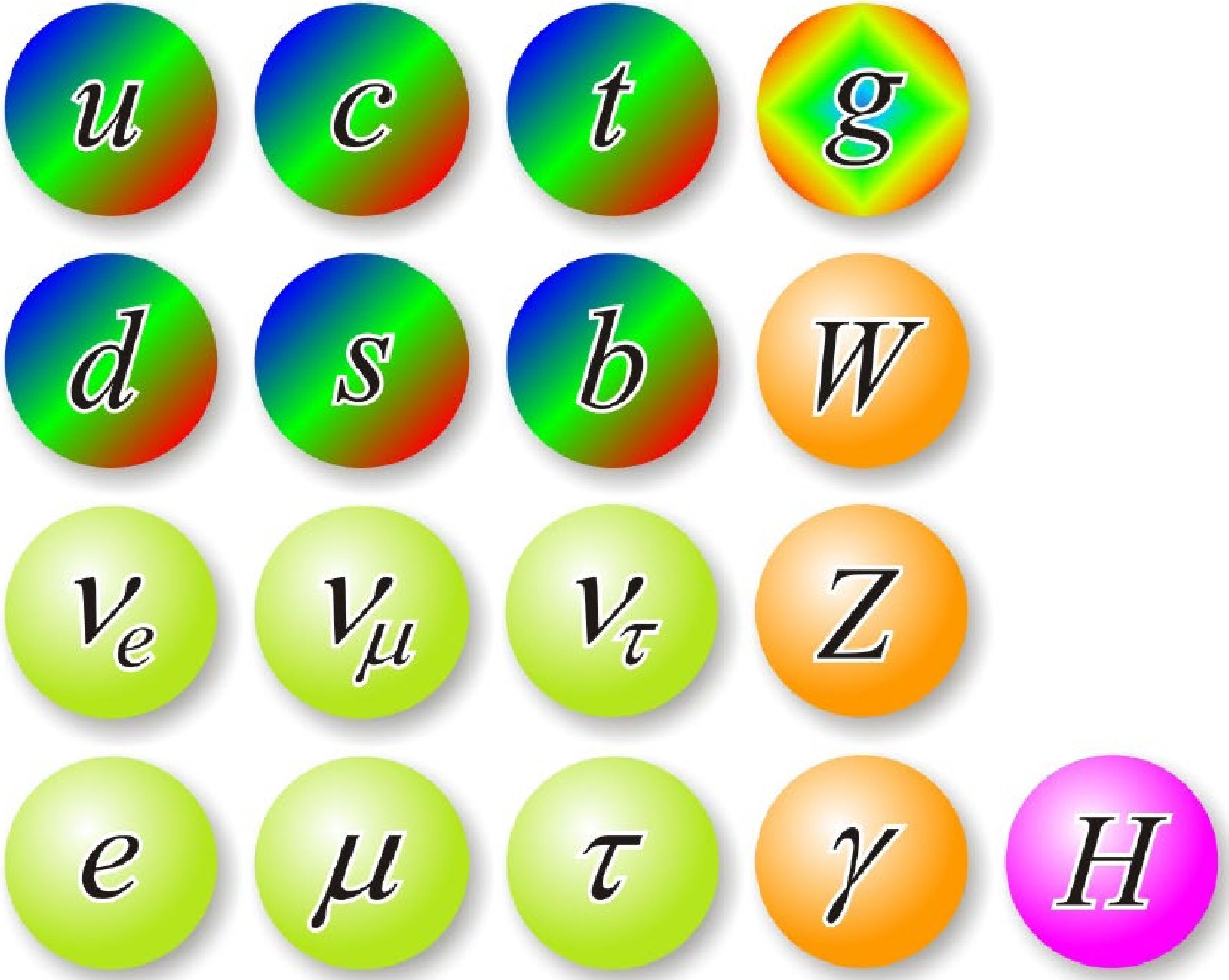
Spontaneous symmetry breaking of the Higgs field

$\Rightarrow$ Electroweak symmetry breaking & generating fermion masses





Are there dark matter candidates in the standard model?



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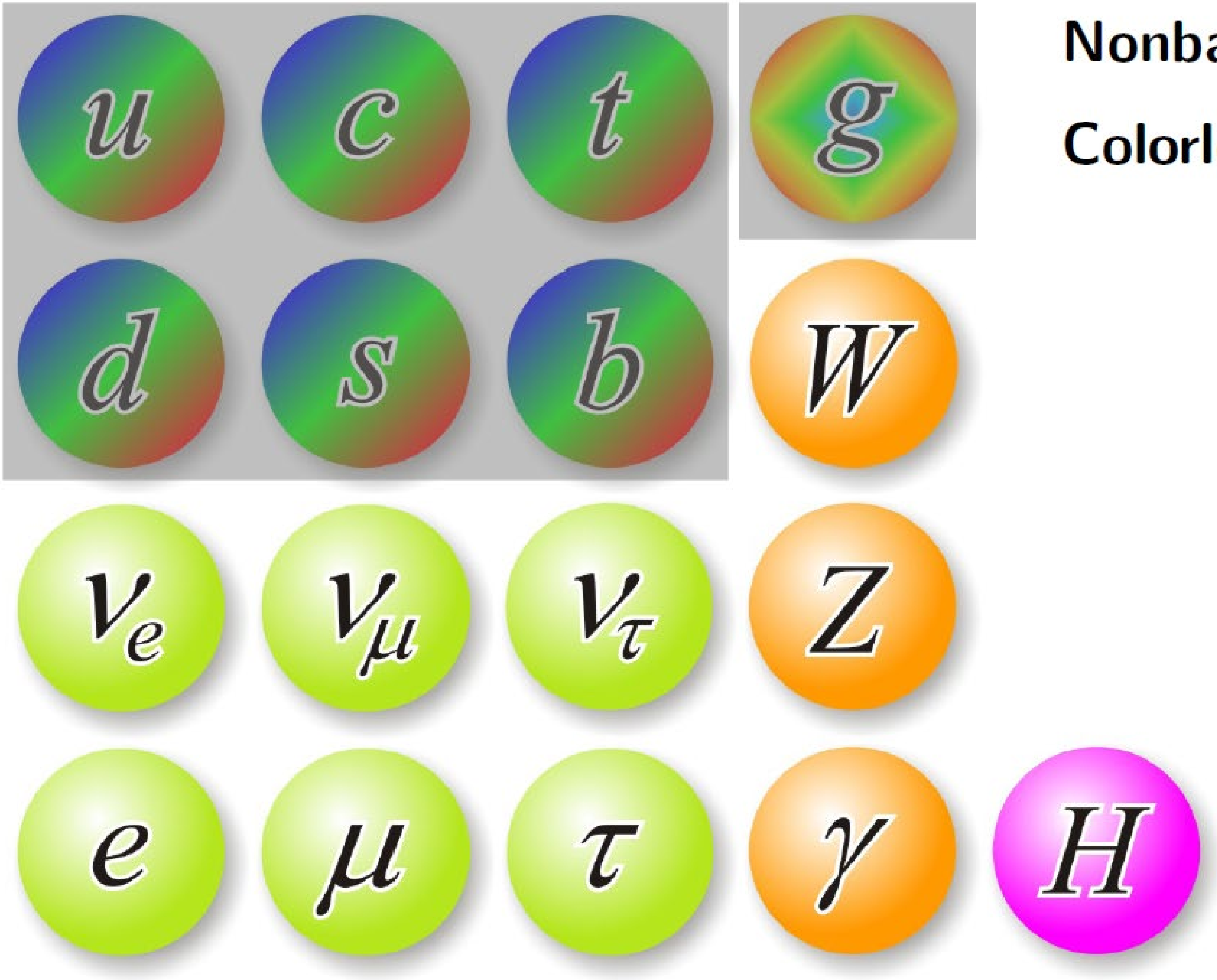
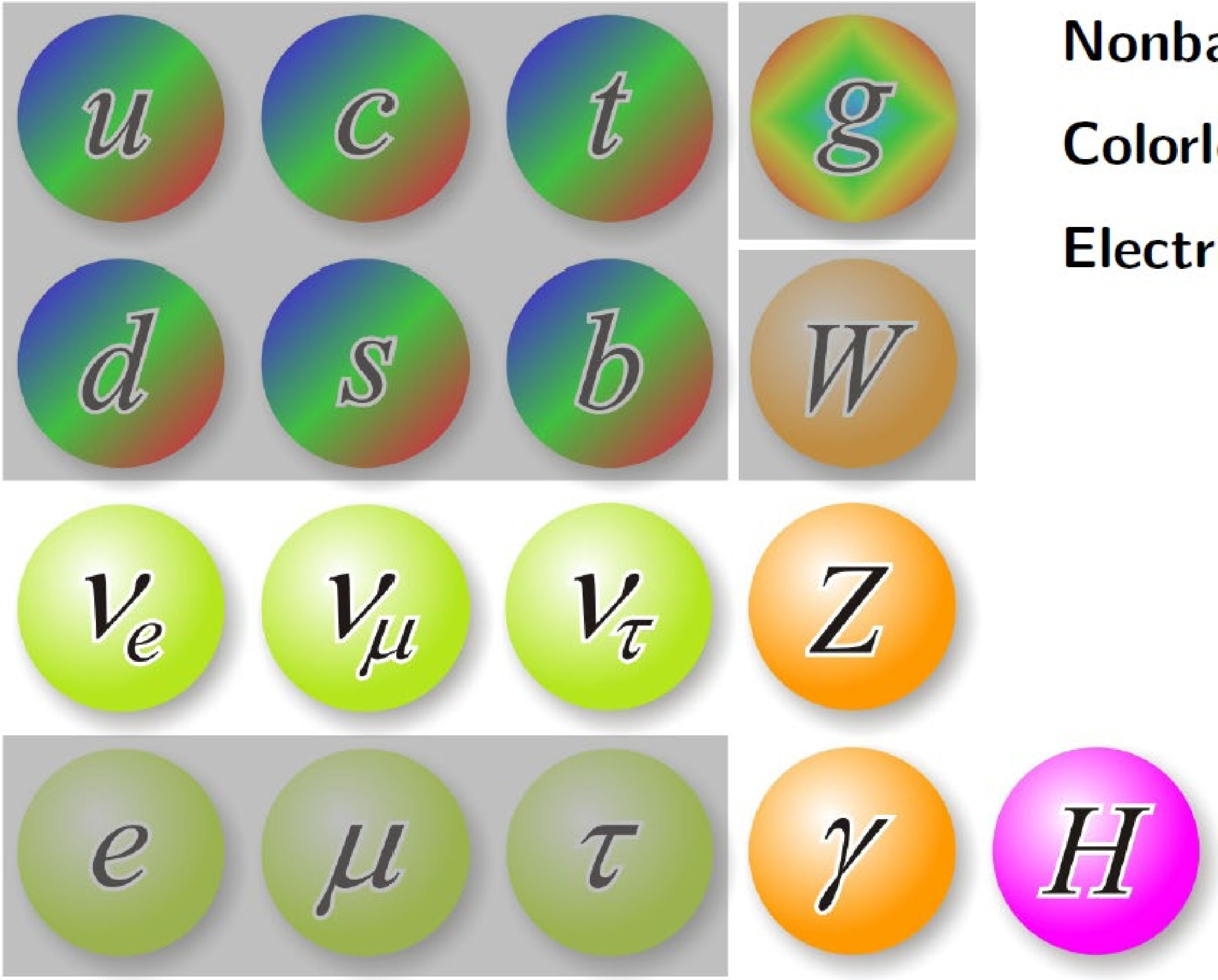


Nonbaryonic

Colorless

Are there dark matter candidates in the standard model?

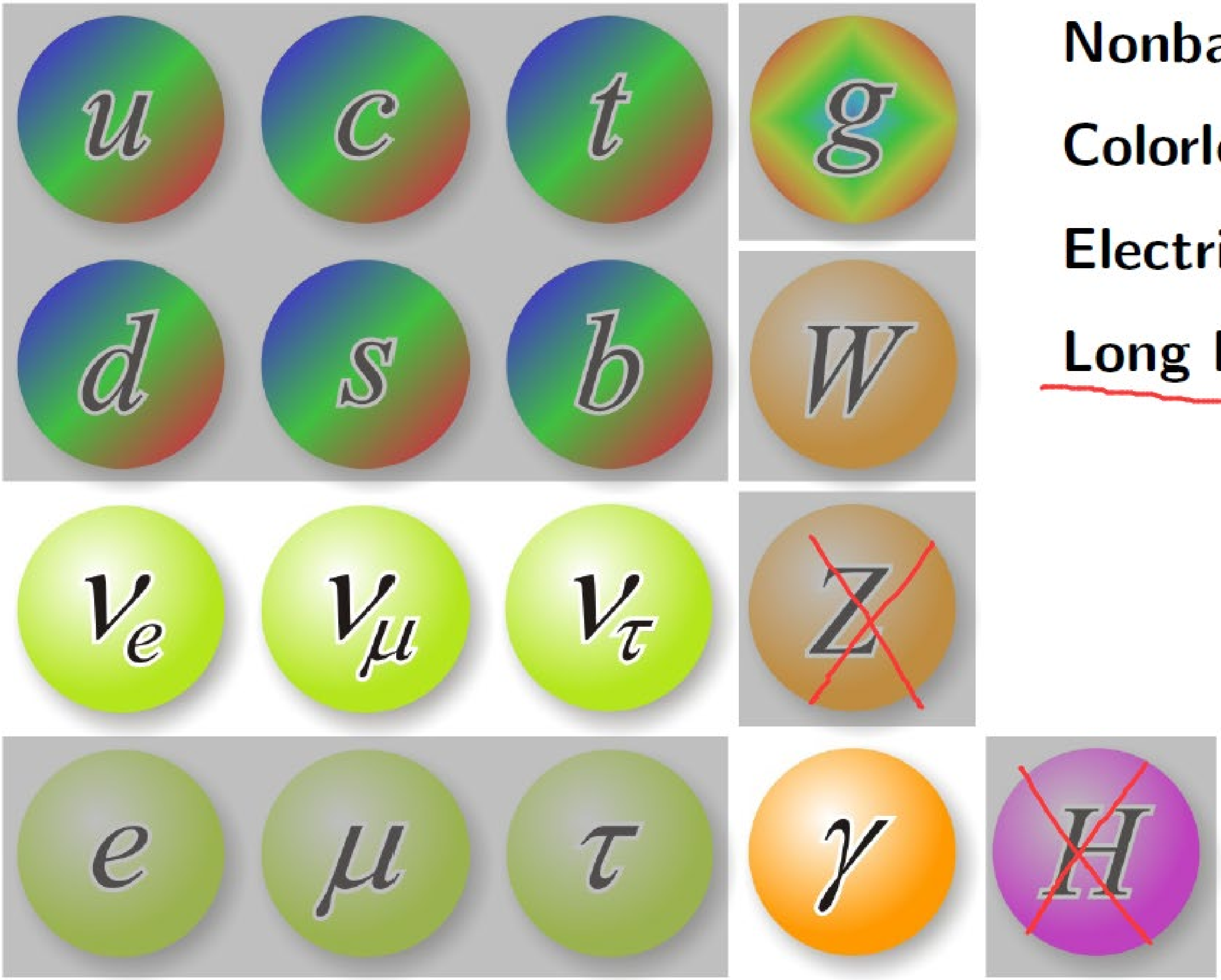


Nonbaryonic

Colorless

Electrically neutral

Are there dark matter candidates in the standard model?



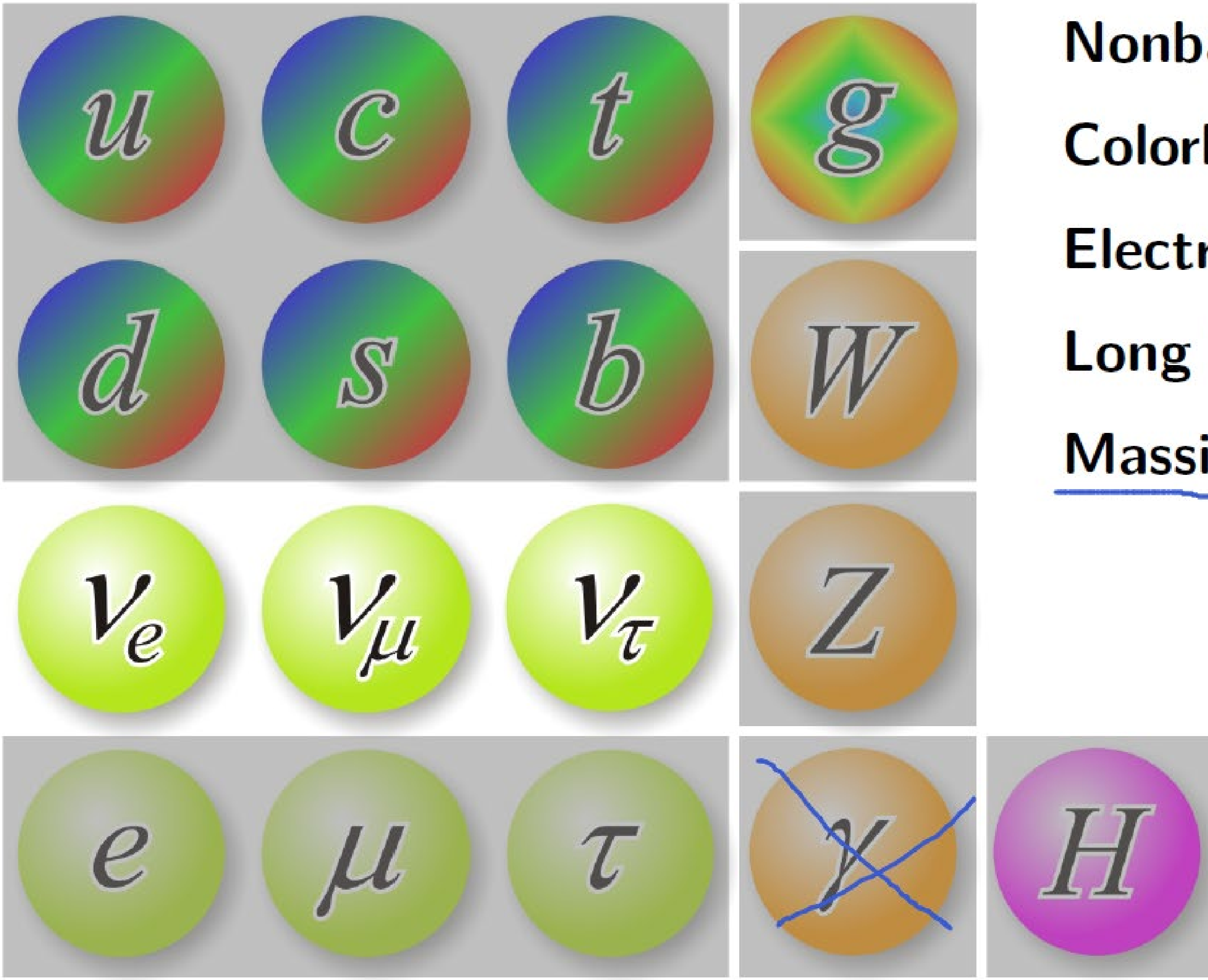
Nonbaryonic

Colorless

Electrically neutral

Long lived

Are there dark matter candidates in the standard model?



Nonbaryonic

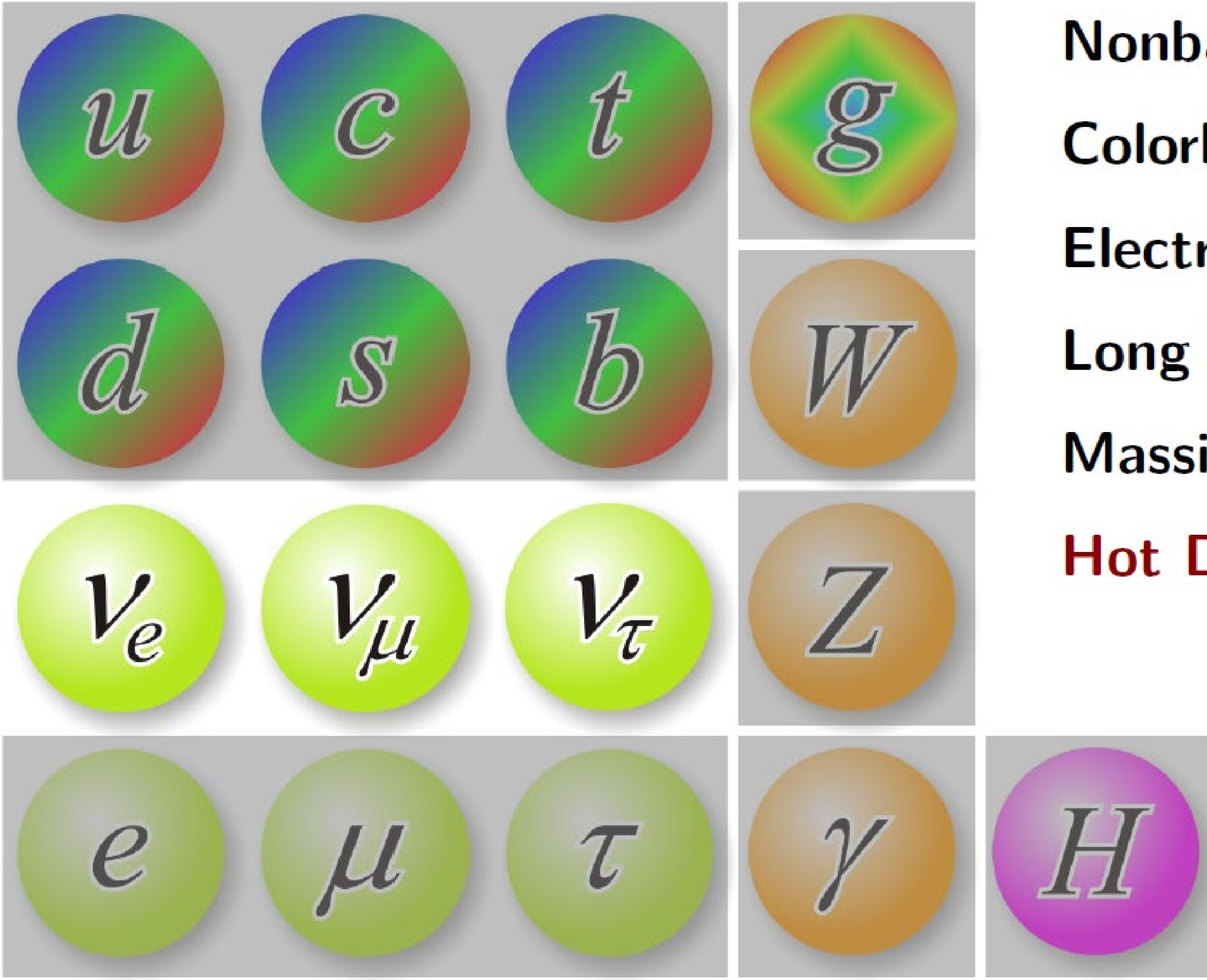
Colorless

Electrically neutral

Long lived

Massive

Are there dark matter candidates in the standard model?



Nonbaryonic

Colorless

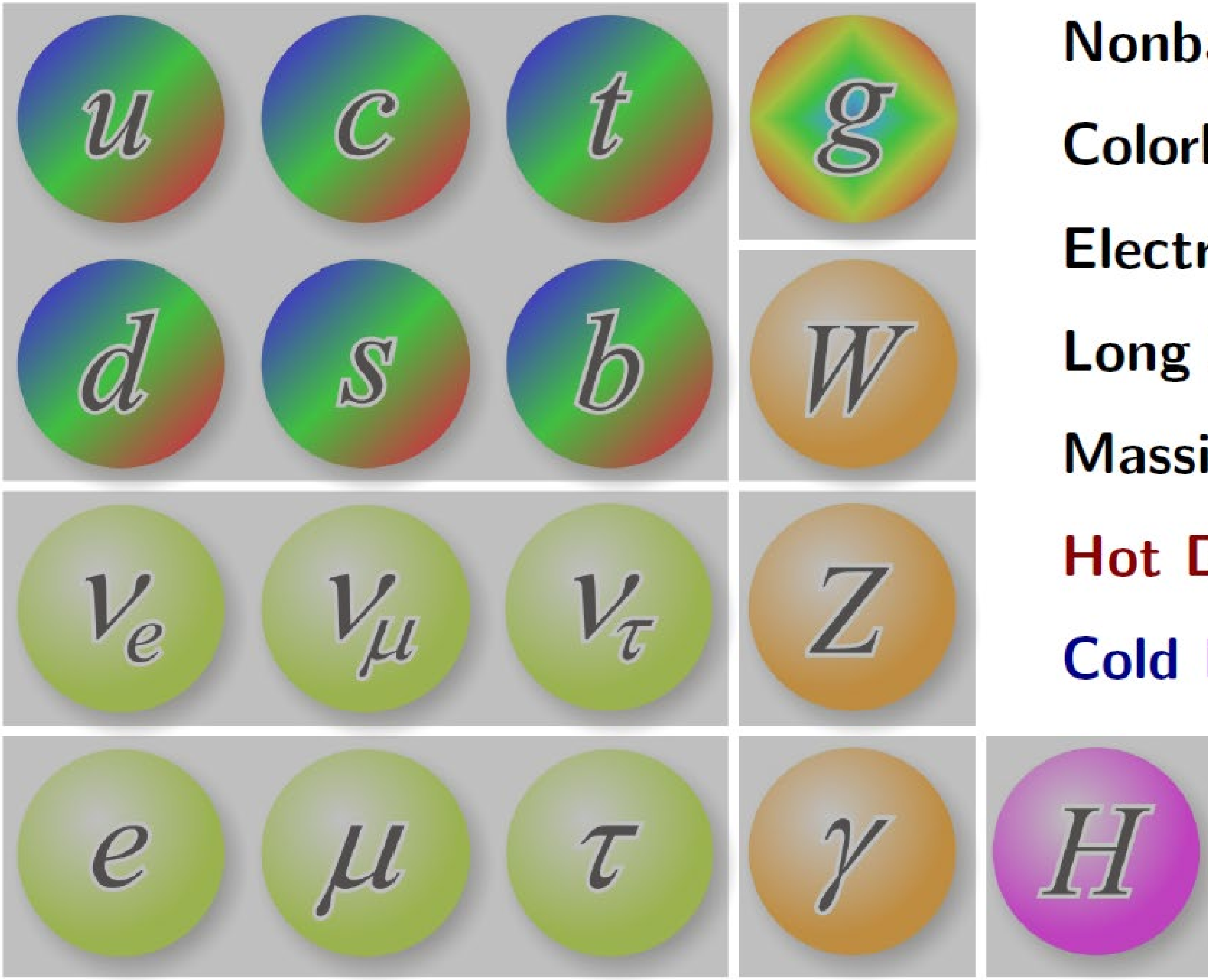
Electrically neutral

Long lived

Massive

Hot DM: neutrinos

Are there dark matter candidates in the standard model?



Nonbaryonic

Colorless

Electrically neutral

Long lived

Massive

Hot DM: neutrinos

Cold DM: none

WIMP miracle

The **relic density** of dark matter can be calculated by the Boltzmann equation:

$$\dot{n}_\chi + 3Hn_\chi = -\langle\sigma_{\rm ann} v\rangle\,[n_\chi^2 - (n_\chi^{\rm EQ})^2]$$

$$\Rightarrow \qquad \Omega_\chi h^2 \simeq \frac{3\times 10^{-27}\ {\rm cm}^3\ {\rm s}^{-1}}{\langle\sigma_{\rm ann} v\rangle}$$

Observed relic density

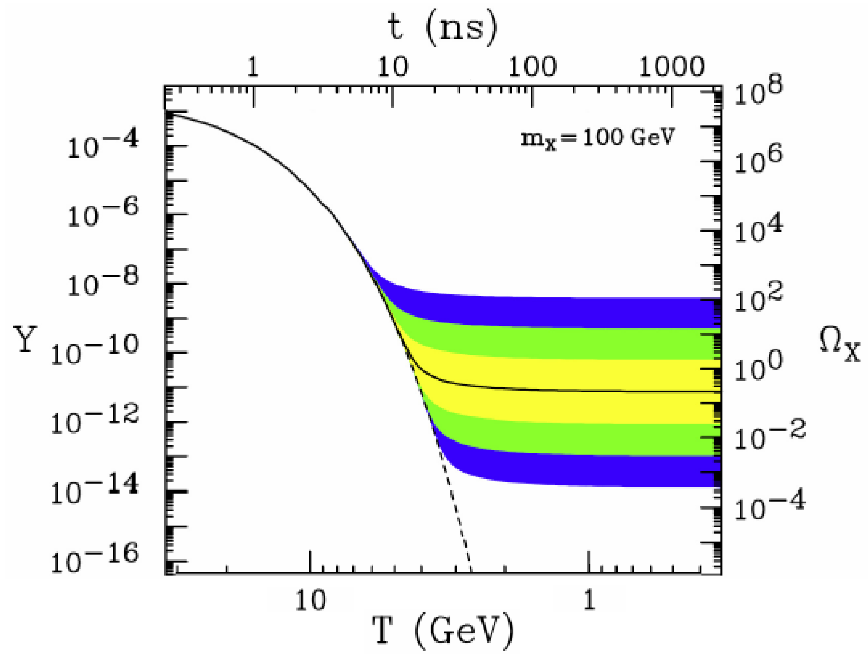
$\Downarrow$

$\langle\sigma_{\rm ann} v\rangle \sim \mathcal{O}(10^{-26})\ {\rm cm}^3\ {\rm s}^{-1}$

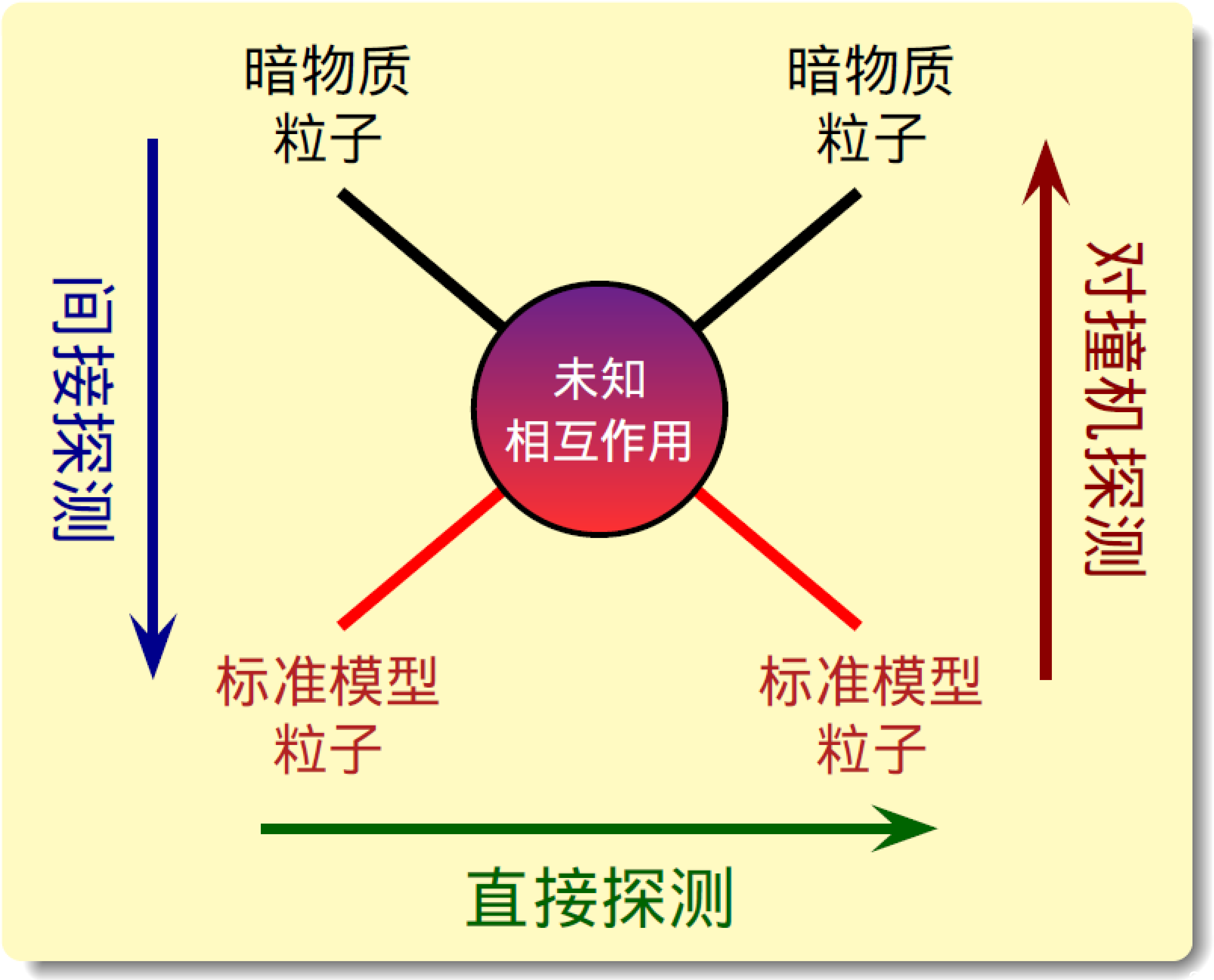
Typical value of weak interactions

$\Downarrow$

Weakly interacting massive particles (WIMPs) are wonderful candidates



[Feng, arXiv:1003.0904]



暗物质
粒子
暗物质
粒子
间接探测
未知
相互作用
对撞机探测
标准模型
粒子
标准模型
粒子
直接探测

Thermal Dark Matter

✨ Conventionally, **dark matter (DM)** is assumed to be a **thermal relic** remaining from the early Universe

🌙 DM relic abundance observation

👉 Particle mass $m_\chi \sim \mathcal{O}(\text{GeV}) - \mathcal{O}(\text{TeV})$

Interaction strength $\sim$ **weak strength**

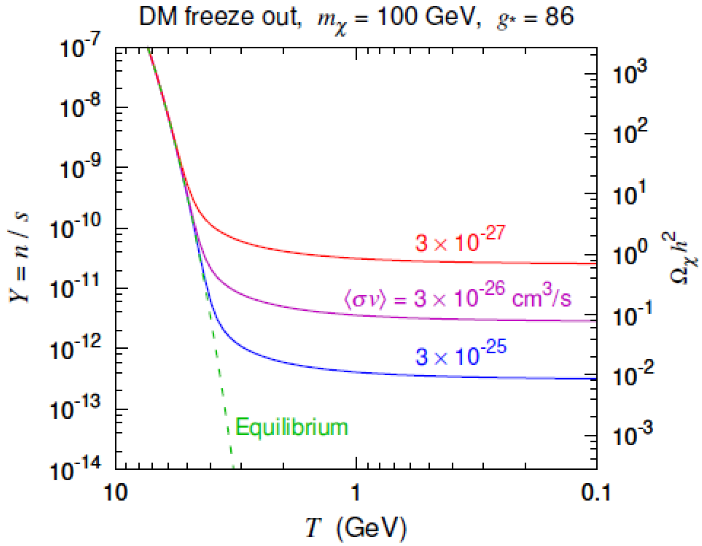
"Weakly interacting massive particles"

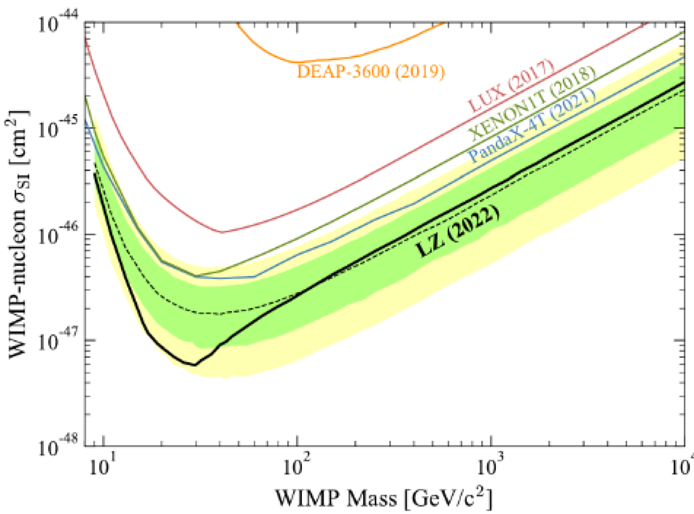
"WIMPs"

🔍 **Direct detection** for WIMPs

👉 **No robust signal found so far**

☁️ **Great challenge** to the thermal dark matter paradigm





[LZ Coll., 2207.03764]

pseudo-Nambu-Goldstone Dark Matter

[Gross, Lebedev, Toma, 1708.02253, PRL]

- **Standard model (SM) Higgs doublet** H, **complex scalar** S (SM singlet)
- Scalar potential respects a **softly broken global** $\mathrm{U}(1)$ **symmetry** $S \to e^{i\alpha} S$
- $\mathrm{U}(1)$ **symmetric:** $V_0 = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_H}{2}|H|^4 + \frac{\lambda_S}{2}|S|^4 + \lambda_{HS}|H|^2|S|^2$
- **Soft breaking:** $V_{\mathrm{soft}} = -\frac{\mu_S'^2}{4} S^2 + \mathrm{H.c.}$
- V_{soft} is **special**, and it can be justified by a **proper ultraviolet (UV) completion**
- H and S develop vacuum expectation values (VEVs) v and v_s

$$H \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h \end{pmatrix}, \quad S = \frac{1}{\sqrt{2}}(v_s + s + i\chi)$$

- The **soft breaking term** V_{soft} give a mass to χ: $m_\chi = \mu_S'$
- The **DM candidate** χ is a **stable pseudo-Nambu-Goldstone boson (pNGB)**
- Rotate **CP-even Higgs bosons** h and s to **mass eigenstates** h_1 and h_2

$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}, \quad m_{h_1,h_2}^2 = \frac{1}{2}\left(\lambda_H v^2 + \lambda_S v_s^2 \mp \frac{\lambda_S v_s^2 - \lambda_H v^2}{\cos 2\theta}\right)$$

DM-nucleon Scattering

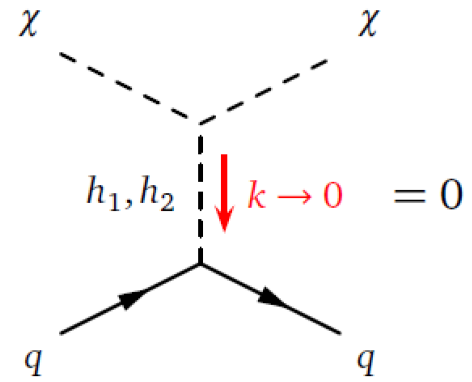
[Gross, Lebedev, Toma, 1708.02253, PRL]

✨ **DM-quark** interactions induce **DM-nucleon** scattering in direct detection

✏️ **DM-quark scattering amplitude** from Higgs portal interactions

$$\mathcal{M}(\chi q \to \chi q) \propto \frac{m_q s_\theta c_\theta}{v v_s} \left(\frac{m_{h_1}^2}{t - m_{h_1}^2} - \frac{m_{h_2}^2}{t - m_{h_2}^2} \right)$$

$$= \frac{m_q s_\theta c_\theta}{v v_s} \frac{t(m_{h_1}^2 - m_{h_2}^2)}{(t - m_{h_1}^2)(t - m_{h_2}^2)}$$



🔥 **Zero momentum transfer limit** $t = k^2 \to 0, \quad \mathcal{M}(\chi q \to \chi q) \to 0$

👉 DM-nucleon scattering cross section **vanishes** at tree level

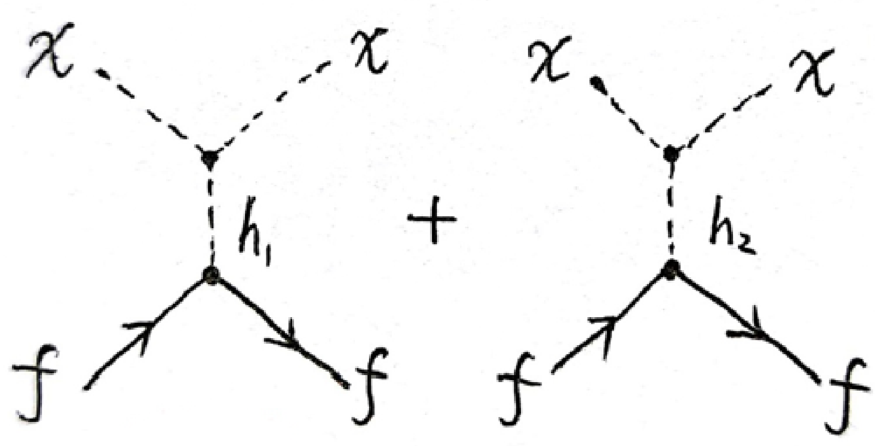
💡 Tree-level interactions of a **pNGB** are generally **momentum-suppressed**

🌥️ **One-loop corrections** typically lead to $\sigma_{\chi N}^{\rm SI} \lesssim \mathcal{O}(10^{-50})$ cm^2

[Azevedo *et al.*, 1810.06105, JHEP; Ishiwata & Toma, 1810.08139, JHEP]

👉 **Beyond capability** of current and near future direct detection experiments

The tree-level Feynman diagrams for scattering of the dark matter χ on the SM fermions f involve the t-channel exchange a single h_1 or h_2:



The interaction operators relevant to this scattering are:

$$h_1\chi^2, \quad h_2\chi^2, \quad h_1\bar{f}f, \quad h_2\bar{f}f$$

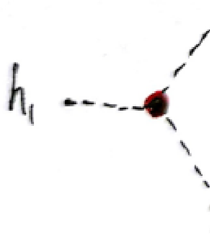
Let us figure out their coupling constants in this model.

There are only two terms in the potential V contributing to the effective operators $h_i\chi^2$ $(i = 1, 2)$:

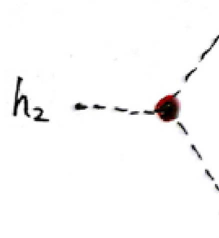
$$V \supset \frac{\lambda_S}{2}|S|^4 \supset \frac{\lambda_S v_s}{2} s\chi^2, \qquad V \supset \lambda_{HS}|H|^2|S|^2 \supset \frac{\lambda_{HS} v}{2} h\chi^2$$

$$\begin{aligned}
\Rightarrow \quad \mathcal{L} \supset -V \supset & -\frac{\lambda_S v_s}{2} s\chi^2 - \frac{\lambda_{HS} v}{2} h\chi^2 \\
&= -\frac{1}{2}\chi^2 \begin{pmatrix} \lambda_{HS} v & \lambda_S v_s \end{pmatrix} \begin{pmatrix} h \\ s \end{pmatrix} \\
&= -\frac{1}{2}\chi^2 \begin{pmatrix} \lambda_{HS} v & \lambda_S v_s \end{pmatrix} \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \\
&= -\frac{1}{2}(\lambda_{HS} v c_\theta - \lambda_S v_s s_\theta) h_1\chi^2 \\
&\quad -\frac{1}{2}(\lambda_{HS} v s_\theta + \lambda_S v_s c_\theta) h_2\chi^2 \\
&= \frac{1}{2}\frac{m_1^2 s_\theta}{v_s} h_1\chi^2 - \frac{1}{2}\frac{m_2^2 c_\theta}{v_s} h_2\chi^2
\end{aligned}$$

From the interactions we can read off the Feynman rules:



$$= i\frac{m_1^2 s_\theta}{v_s}$$



$$= -i\frac{m_2^2 c_\theta}{v_s}$$

We have used the relations:

$$\lambda_{HS} v c_\theta - \lambda_S v_S s_\theta = -\frac{m_1^2 s_\theta}{v_S}, \quad \lambda_{HS} v s_\theta + \lambda_S v_S c_\theta = \frac{m_2^2 c_\theta}{v_S}$$

$$\begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix}\begin{pmatrix} \lambda_H v^2 & \lambda_{HS} v v_s \\ \lambda_{HS} v v_s & \lambda_s v_s^2 \end{pmatrix}\begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} = \begin{pmatrix} m_1^2 & \\ & m_2^2 \end{pmatrix}$$

$$\rightarrow \underbrace{\begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix}\begin{pmatrix} \lambda_H v^2 & \lambda_{HS} v v_s \\ \lambda_{HS} v v_s & \lambda_s v_s^2 \end{pmatrix}}_{\begin{pmatrix} * & \lambda_{HS} v v_s c_\theta - \lambda_s v_s^2 s_\theta \\ * & \lambda_{HS} v v_s s_\theta + \lambda_s v_s^2 c_\theta \end{pmatrix}} = \underbrace{\begin{pmatrix} m_1^2 & \\ & m_2^2 \end{pmatrix}\begin{pmatrix} c_\theta & -s_\theta \\ s_\theta & c_\theta \end{pmatrix}}_{\begin{pmatrix} * & -m_1^2 s_\theta \\ * & m_2^2 c_\theta \end{pmatrix}}$$

$$\rightarrow \begin{cases} \lambda_{HS} v v_s c_\theta - \lambda_s v_s^2 s_\theta = -m_1^2 s_\theta \\ \lambda_{HS} v v_s s_\theta + \lambda_s v_s^2 c_\theta = m_2^2 c_\theta \end{cases}$$

Considering the Higgs-fermion interactions in the SM

$$\mathcal{L} \supset -\sum_f \frac{m_f}{v} h \bar{f} f$$

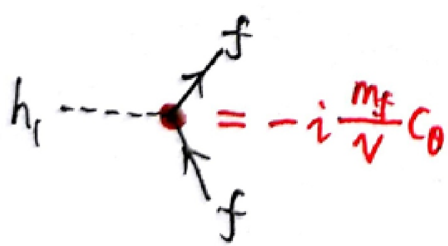
together with

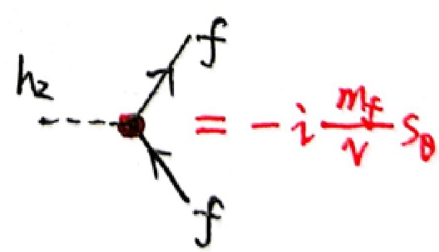
$$\begin{pmatrix} h \\ s \end{pmatrix} = \begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \quad \Rightarrow \quad h = h_1 c_\theta + h_2 s_\theta$$

we have

$$\mathcal{L} \supset -\sum_f \frac{m_f}{v} (h_1 c_\theta + h_2 s_\theta) \bar{f} f$$

from which we can read off the Feynman rules:

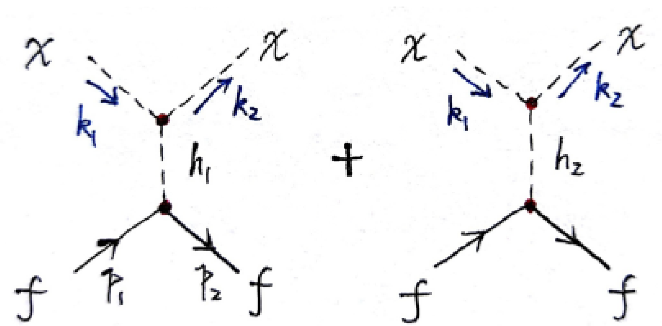




Thus, the tree-level $\chi + f \to \chi + f$ scattering amplitude is

$$
\begin{aligned}
& i\mathcal{M}\left[\chi(k_1) + f(p_1) \to \chi(k_2) + f(p_2)\right] \\
&= \bar{u}(p_2)\left(-i\frac{m_f}{v}c_\theta\right)u(p_1)\frac{i}{q^2 - m_1^2}\, i\frac{m_1^2 s_\theta}{v_s} \\
&\quad + \bar{u}(p_2)\left(-i\frac{m_f}{v}s_\theta\right)u(p_1)\frac{i}{q^2 - m_2^2}\left(-i\frac{m_2^2 c_\theta}{v_s}\right) \\
&= i\bar{u}(p_2)u(p_1)\frac{m_f c_\theta s_\theta}{v v_s}\left(\frac{m_1^2}{q^2 - m_1^2} - \frac{m_2^2}{q^2 - m_2^2}\right) \\
&= i\bar{u}(p_2)u(p_1)\frac{m_f c_\theta s_\theta}{v v_s}\,\frac{t(m_1^2 - m_2^2)}{(t - m_1^2)(t - m_2^2)} \xrightarrow{t \to 0} 0
\end{aligned}
$$

where $q \equiv k_1 - k_2 = p_2 - p_1$ and $t \equiv q^2$.



The previous calculation can be written in terms of matrices.
The Feynman rules in matrix language are

- **Propagator**

$$\left\langle \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \begin{pmatrix} h_1 & h_2 \end{pmatrix} \right\rangle = \begin{pmatrix} \frac{i}{q^2 - m_1^2} & \\ & \frac{i}{q^2 - m_2^2} \end{pmatrix} = \begin{pmatrix} \frac{i}{t - m_1^2} & \\ & \frac{i}{t - m_2^2} \end{pmatrix}$$

- **Vertices**

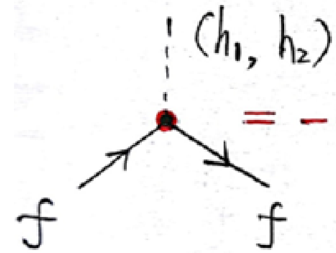
$$\mathcal{L} \supset \frac{1}{2} \frac{m_1^2 s_\theta}{v_s} h_1 \chi^2 - \frac{1}{2} \frac{m_2^2 c_\theta}{v_s} h_2 \chi^2$$

$$= \frac{1}{2} \chi^2 \left(\frac{m_1^2 s_\theta}{v_s} , - \frac{m_2^2 c_\theta}{v_s} \right) \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$$

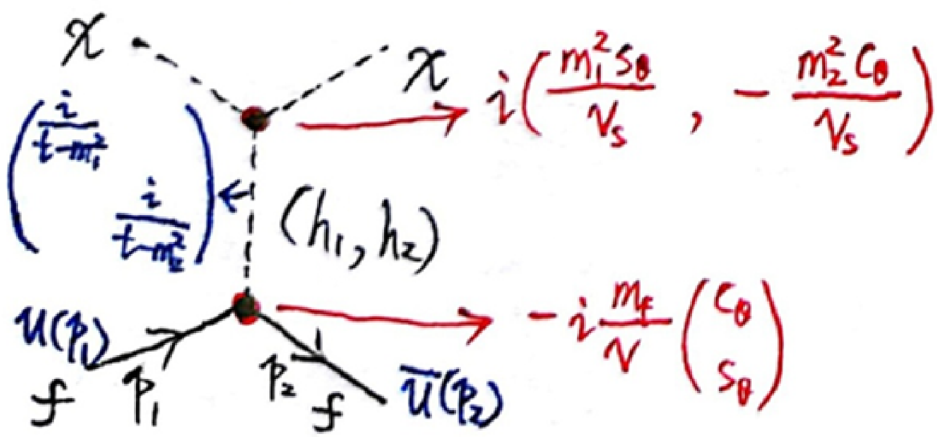
$$= i \left(\frac{m_1^2 s_\theta}{v_s} , - \frac{m_2^2 c_\theta}{v_s} \right)$$

$$\mathcal{L} \supset - \sum_f \frac{m_f}{v} (h_1 c_\theta + h_2 s_\theta) \bar{f} f$$

$$= - \sum_f \frac{m_f}{v} \bar{f} f \, (h_1 , h_2) \begin{pmatrix} c_\theta \\ s_\theta \end{pmatrix}$$



$$= - i \frac{m_f}{v} \begin{pmatrix} c_\theta \\ s_\theta \end{pmatrix}$$



Thus, the tree-level $\chi + f \to \chi + f$ scattering amplitude is

$$i\mathcal{M} = i\begin{pmatrix} \frac{m_1^2 s_\theta}{v_s} & -\frac{m_2^2 c_\theta}{v_s} \end{pmatrix} \begin{pmatrix} \frac{i}{t-m_1^2} & \\ & \frac{i}{t-m_2^2} \end{pmatrix} \left(-i\frac{m_f}{v}\right) \begin{pmatrix} c_\theta \\ s_\theta \end{pmatrix} \bar{u}(p_2)u(p_1)$$

$$= i\bar{u}(p_2)u(p_1)\frac{m_f c_\theta s_\theta}{v v_s}\left(\frac{m_1^2}{t-m_1^2} - \frac{m_2^2}{t-m_2^2}\right) \xrightarrow{t\to 0} 0$$

We can also calculate in the interaction basis (*i.e.*, in terms of the states *h* and *s*). The Feynman rules in this basis are

$$\begin{pmatrix} h \\ s \end{pmatrix} = \underbrace{\begin{pmatrix} c_\theta & s_\theta \\ -s_\theta & c_\theta \end{pmatrix}}_{\equiv\, O} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = O \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \quad \Longleftrightarrow \quad (h \;\; s) = (h_1 \;\; h_2)\, O^T$$

➤ **Propagator**

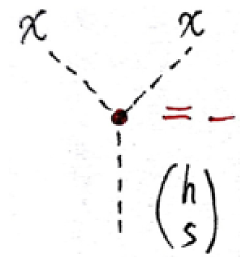
$$\left\langle \begin{pmatrix} h \\ s \end{pmatrix} (h \;\; s) \right\rangle = O \left\langle \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} (h_1 \;\; h_2) \right\rangle O^T$$

$$= O \begin{pmatrix} \frac{i}{q^2 - m_1^2} & \\ & \frac{i}{q^2 - m_2^2} \end{pmatrix} O^T$$

➤ **Vertices**

$$\mathcal{L} \supset -\frac{1}{2} \lambda_{HS} v\, h \chi^2 - \frac{1}{2} \lambda_S v_S\, s \chi^2$$

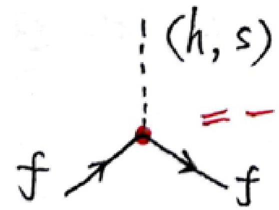
$$= -\frac{1}{2} \chi^2 \left(\lambda_{HS} v, \; \lambda_S v_S \right) \begin{pmatrix} h \\ s \end{pmatrix}$$



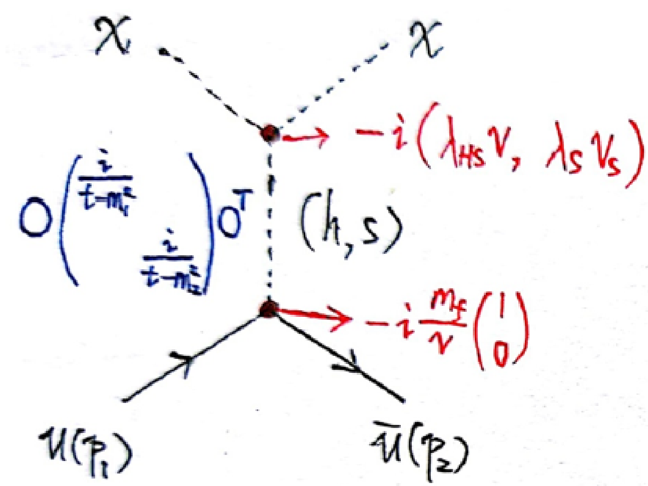
$$= -i \left(\lambda_{HS} v, \; \lambda_S v_S \right)$$

$$\mathcal{L} \supset -\sum_f \frac{m_f}{v} h \bar{f} f$$

$$= -\sum_f \frac{m_f}{v} \bar{f} f \,(h, \; s) \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



$$= -i \frac{m_f}{v} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$



$$i\mathcal{M} = -i\begin{pmatrix}\lambda_{HS}v & \lambda_S v_s\end{pmatrix} O \begin{pmatrix}\frac{i}{t-m_1^2} & \\ & \frac{i}{t-m_2^2}\end{pmatrix} O^T \left(-i\frac{m_f}{v}\right)\begin{pmatrix}1\\0\end{pmatrix}\bar{u}(p_2)u(p_1)$$

$$\xrightarrow{t\to 0} i\frac{m_f}{v}\bar{u}(p_2)u(p_1)\begin{pmatrix}\lambda_{HS}v & \lambda_S v_s\end{pmatrix} O \begin{pmatrix}\frac{1}{m_1^2} & \\ & \frac{1}{m_2^2}\end{pmatrix} O^T \begin{pmatrix}1\\0\end{pmatrix}$$

$$= i\frac{m_f}{v}\bar{u}(p_2)u(p_1)\begin{pmatrix}\lambda_{HS}v & \lambda_S v_s\end{pmatrix}\left(M^2_{\mathrm{even}}\right)^{-1}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= i\frac{m_f}{v}\bar{u}(p_2)u(p_1)\begin{pmatrix}\lambda_{HS}v & \lambda_S v_s\end{pmatrix}\frac{1}{\det\left(M^2_{\mathrm{even}}\right)}\begin{pmatrix}\lambda_S v_s^2 & -\lambda_{HS}v v_s\\ -\lambda_{HS}v v_s & \lambda_H v^2\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$$

$$i\mathcal{M} \xrightarrow{t\to 0} i\frac{m_f}{v}\frac{\bar{u}(p_2)u(p_1)}{\det\left(M^2_{\text{even}}\right)}\begin{pmatrix}\lambda_{HS}v & \lambda_S v_s\end{pmatrix}\begin{pmatrix}\lambda_S v_s^2 & -\lambda_{HS}vv_s \\ -\lambda_{HS}vv_s & \lambda_H v^2\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= i\frac{m_f}{v}\frac{\bar{u}(p_2)u(p_1)}{\det\left(M^2_{\text{even}}\right)}\begin{pmatrix}0 & *\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}$$

$$= 0$$

Once again, we see that the tree-level $\chi + f \to \chi + f$ scattering amplitude vanishes in the limit of zero momentum transfer.

pNBG DM and Two Higgs Doublets

[XM Jiang, CF Cai, ZH Yu, YP Zeng, **HH Zhang**, 1907.09684, PRD]

- **Two Higgs doublets** Φ_1 and Φ_2 with $Y = 1/2$, **complex scalar singlet** S
- Scalar potential respects a **softly broken global** $\mathrm{U}(1)$ **symmetry** $S \to e^{i\alpha} S$
- Scalar potential constructed with Φ_1 and Φ_2

$$V_1 = m_{11}^2|\Phi_1|^2 + m_{22}^2|\Phi_2|^2 - m_{12}^2(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) + \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \frac{\lambda_5}{2}[(\Phi_1^\dagger\Phi_2)^2 + (\Phi_2^\dagger\Phi_1)^2]$$

- $\mathrm{U}(1)$ **symmetric** potential terms involving S

$$V_2 = -m_S^2|S|^2 + \frac{\lambda_S}{2}|S|^4 + \kappa_1|\Phi_1|^2|S|^2 + \kappa_2|\Phi_2|^2|S|^2$$

- Quadratic term **softly breaking** the global $\mathrm{U}(1)$: $V_{\text{soft}} = -\frac{m_S'^2}{4}S^2 + \text{H.c.}$
- Φ_1, Φ_2, and S develop VEVs v_1, v_2 and v_s

$$\Phi_1 = \begin{pmatrix} \phi_1^+ \\ (v_1 + \rho_1 + i\eta_1)/\sqrt{2} \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ (v_2 + \rho_2 + i\eta_2)/\sqrt{2} \end{pmatrix}, \quad S = \frac{v_s + s + i\chi}{\sqrt{2}}$$

- χ is a **stable pNGB** with $m_\chi = m_S'$, acting as a **DM candidate**

Physical Scalars

- Rotations of **charged scalars** and **CP-odd scalars**:

$$\begin{pmatrix}\phi_1^+\\ \phi_2^+\end{pmatrix} = R(\beta)\begin{pmatrix}G^+\\ H^+\end{pmatrix},\quad \begin{pmatrix}\eta_1\\ \eta_2\end{pmatrix} = R(\beta)\begin{pmatrix}G^0\\ a\end{pmatrix},\quad R(\beta) = \begin{pmatrix}\cos\beta & -\sin\beta\\ \sin\beta & \cos\beta\end{pmatrix},\quad \tan\beta = \frac{v_2}{v_1}$$

- $G^\pm$ and G^0 are **massless Nambu-Goldstone bosons** eaten by $W^\pm$ and Z
- $H^\pm$ and a are **physical states**
- Mass terms for **CP-even scalars** $\mathcal{L}_{\text{mass}} \supset -\frac{1}{2}\left(\rho_1,\ \rho_2,\ s\right)\mathcal{M}^2_{\rho s}\begin{pmatrix}\rho_1\\ \rho_2\\ s\end{pmatrix}$

$$\mathcal{M}^2_{\rho s} = \begin{pmatrix}\lambda_1 v_1^2 + m_{12}^2\tan\beta & \lambda_{345}v_1v_2 - m_{12}^2 & \kappa_1 v_1 v_s\\ \lambda_{345}v_1v_2 - m_{12}^2 & \lambda_2 v_2^2 + m_{12}^2\cot\beta & \kappa_2 v_2 v_s\\ \kappa_1 v_1 v_s & \kappa_2 v_2 v_s & \lambda_S v_s^2\end{pmatrix},\quad \lambda_{345} \equiv \lambda_3 + \lambda_4 + \lambda_5$$

$$\begin{pmatrix}\rho_1\\ \rho_2\\ s\end{pmatrix} = O\begin{pmatrix}h_1\\ h_2\\ h_3\end{pmatrix},\quad O^{\mathrm{T}}\mathcal{M}^2_{\rho s}O = \mathrm{diag}(m_{h_1}^2, m_{h_2}^2, m_{h_3}^2),\quad m_{h_1} \le m_{h_2} \le m_{h_3}$$

- **One of h_i** should behave like the **125 GeV SM Higgs boson**

Four Types of Yukawa Couplings

- If all fermions with the same quantum numbers just couple to the one **same** Higgs doublet, **flavor-changing neutral currents (FCNCs)** will be **absent** at tree level

  [Glashow & Weinberg, PRD 15, 1958 (1977); Paschos, PRD 15, 1966 (1977)]

- Yukawa interactions for the **fermion mass eigenstates**

$$\mathcal{L}_{\rm Y} = \sum_{f=\ell_j,d_j,u_j} \left[-m_f \bar{f} f - \frac{m_f}{v}\left(\sum_{i=1}^{3} \xi^f_{h_i} h_i \bar{f} f + \xi^f_a a \bar{f} i\gamma_5 f\right)\right]$$

$$- \frac{\sqrt{2}}{v}[H^+(\xi^{\ell_i}_a m_{\ell_i} \bar{\nu}_i P_{\rm R} \ell_i + \xi^{d_j}_a m_{d_j} V_{ij} \bar{u}_i P_{\rm R} d_j + \xi^{u_i}_a m_{u_i} V_{ij} \bar{u}_i P_{\rm L} d_j) + {\rm H.c.}]$$

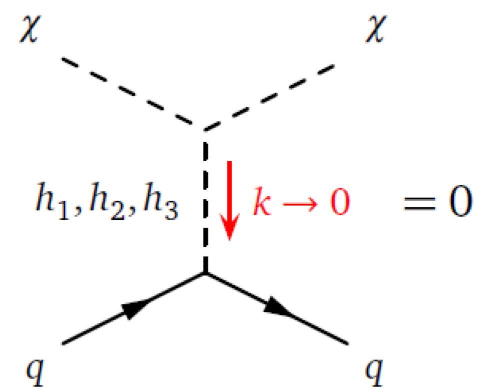
	Type I	Type II	Lepton specific	Flipped
$\xi^{\ell_j}_{h_i}$	$O_{2i}/\sin\beta$	$O_{1i}/\cos\beta$	$O_{1i}/\cos\beta$	$O_{2i}/\sin\beta$
$\xi^{d_j}_{h_i}$	$O_{2i}/\sin\beta$	$O_{1i}/\cos\beta$	$O_{2i}/\sin\beta$	$O_{1i}/\cos\beta$
$\xi^{u_j}_{h_i}$	$O_{2i}/\sin\beta$	$O_{2i}/\sin\beta$	$O_{2i}/\sin\beta$	$O_{2i}/\sin\beta$
$\xi^{\ell_j}_a$	$\cot\beta$	$-\tan\beta$	$-\tan\beta$	$\cot\beta$
$\xi^{d_j}_a$	$\cot\beta$	$-\tan\beta$	$\cot\beta$	$-\tan\beta$
$\xi^{u_j}_a$	$-\cot\beta$	$-\cot\beta$	$-\cot\beta$	$-\cot\beta$

Vanishing of DM-nucleon Scattering

💡 Take the **type-I Yukawa couplings** as an example

⭐ **Higgs portal** interactions $\mathcal{L}_{h_i\chi^2} = \frac{1}{2}\sum_{i=1}^{3} g_{h_i\chi^2}\, h_i \chi^2$

$$g_{h_i\chi^2} = -\kappa_1 v_1 O_{1i} - \kappa_2 v_2 O_{2i} - \lambda_S v_s O_{3i}$$



✨ **DM-quark scattering** amplitude

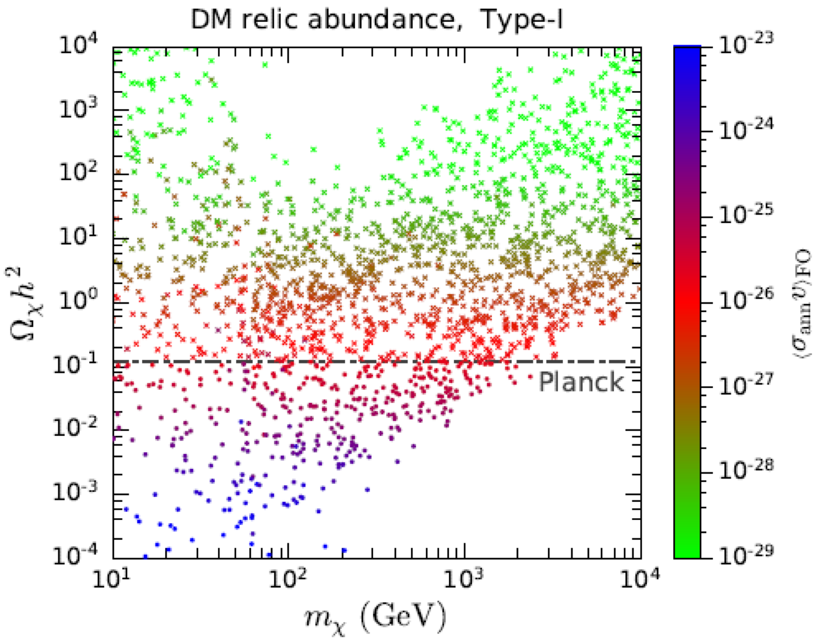
$$\mathcal{M}(\chi q \to \chi q) \propto \frac{m_q}{v \sin\beta}\left(\frac{g_{h_1\chi^2} O_{21}}{t - m_{h_1}^2} + \frac{g_{h_2\chi^2} O_{22}}{t - m_{h_2}^2} + \frac{g_{h_3\chi^2} O_{23}}{t - m_{h_3}^2}\right)$$

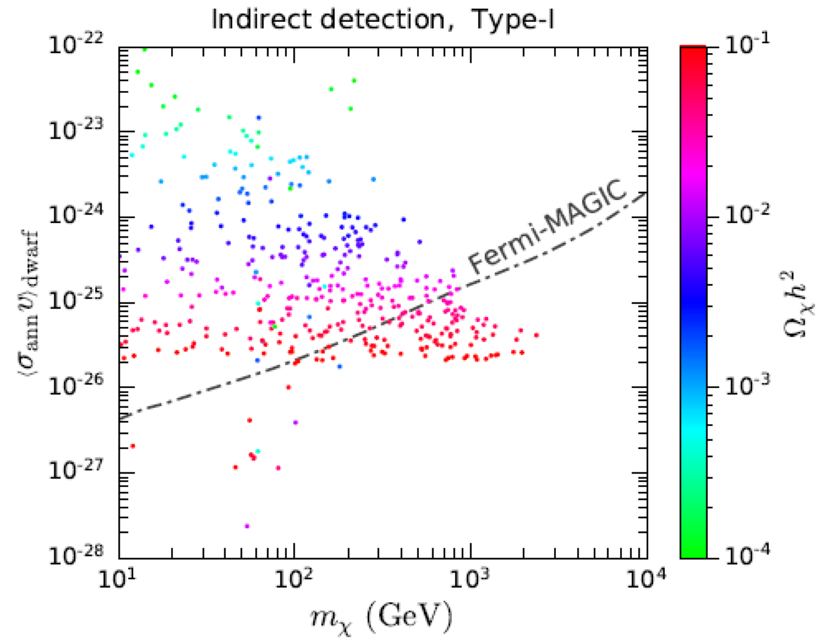
$$\xrightarrow{t\to 0} \frac{m_q}{v \sin\beta}\begin{pmatrix}\kappa_1 v_1 & \kappa_2 v_2 & \lambda_S v_s\end{pmatrix} O \begin{pmatrix} m_{h_1}^{-2} & & \\ & m_{h_2}^{-2} & \\ & & m_{h_3}^{-2}\end{pmatrix} O^{\mathrm{T}} \begin{pmatrix}0\\1\\0\end{pmatrix}$$

$$= \frac{m_q}{v \sin\beta}\begin{pmatrix}\kappa_1 v_1 & \kappa_2 v_2 & \lambda_S v_s\end{pmatrix} (\mathcal{M}_{\rho s}^2)^{-1} \begin{pmatrix}0\\1\\0\end{pmatrix} = 0$$

Interaction basis expression

DM Relic Abundance and Indirect Detection





- **Planck** observed DM relic abundance $\Omega_{\rm DM}h^2 = 0.1186 \pm 0.0020$
  [Planck coll., 1502.01589, Astron. Astrophys.]
- **Colored dots:** $\Omega_\chi h^2$ is **equal** or **lower** than observation
- **Colored crosses:** χ is **overproduced**, contradicting standard cosmology
- The parameter points with $m_\chi \gtrsim 100$ GeV and $\Omega_\chi h^2 \sim 0.1$ are **not excluded** by Fermi-LAT and MAGIC γ-ray observations of dwarf spheroidal galaxies
  [MAGIC & Fermi-LAT, 1601.06590, JCAP]

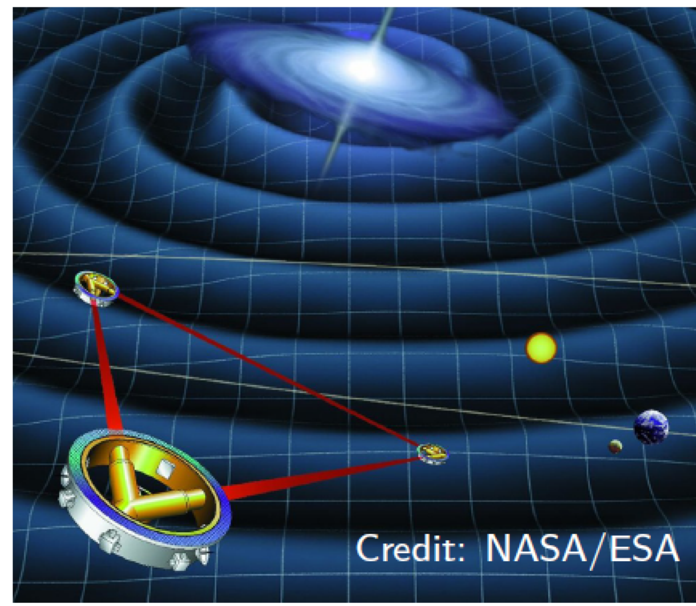
Gravitational Waves from First-order Phase Transition

[Z Zhang, CF Cai, XM Jiang, YL Tang, ZH Yu, **HH Zhang**, 2102.01588, JHEP]

🔭 The discovery of **gravitational waves (GWs)** by LIGO in 2015 opens a new window to new physics models

🔥 Introducing new scalar fields may change the **electroweak phase transition** to be a **first-order phase transition (FOPT)**

💥 Cosmological FOPT may induce a **stochastic GW background** with $f \sim \mathrm{mHz}$



👉 Potential signals in **future space-borne GW interferometers** like **TianQin**, **Taiji**, **LISA**, **BBO**, and **DECIGO**

😟 However, the **original pNGB DM model** can only result in **second-order** phase transitions

[Kannike & Raidal, 1901.03333, PRD]

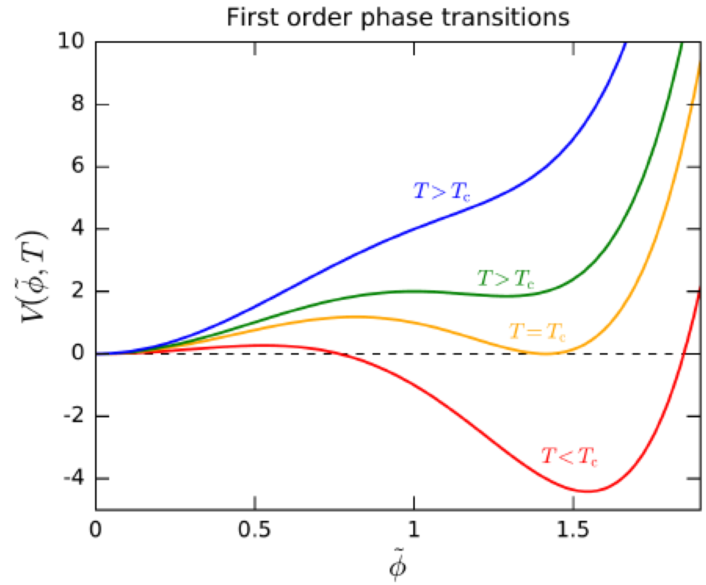
🤔 The situation may be different in the **2HDM extension of pNGB DM**

Effective Potential

Different **local minima** in the **effective potential** V_{eff} of the scalar fields

Different **phases** Phase transitions

We assume that only the CP-even neutral scalar fields (ρ_1, ρ_2, s) develop VEVs in the cosmological history



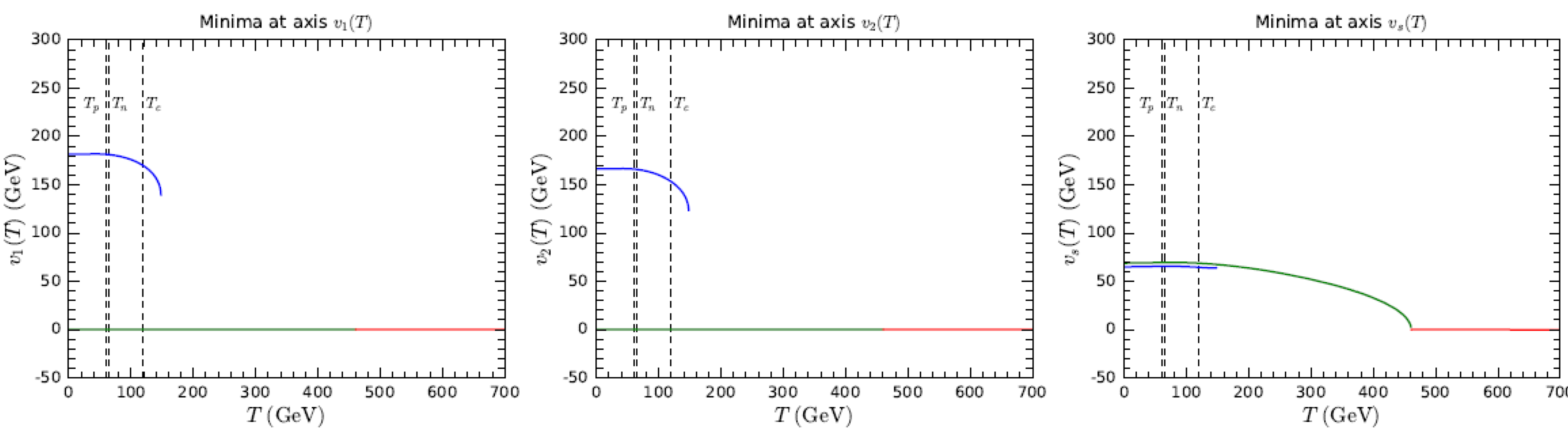
As a function of the **classical background fields** $(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{s})$ and the **temperature** T,

$$V_{\text{eff}}(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{s}, T) = V_0 + V_1 + V_{\text{CT}} + V_{\text{1T}} + V_{\text{D}}$$

- Tree-level potential V_0
- 1-loop zero-temperature corrections V_1
- Counter terms V_{CT} for keeping the VEV positions and the renormalized mass-squared matrix of the CP-even neutral scalars
- 1-loop finite-temperature corrections $V_{\text{1T}}(T)$
- Daisy diagram contributions $V_{\text{D}}(T)$ beyond 1-loop at finite temperatures

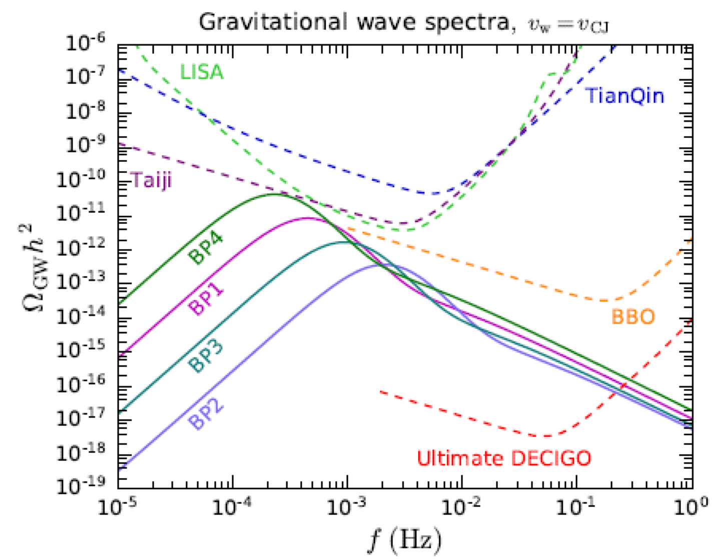
Temperature Evolution of Local Minima

- We utilize `CosmoTransitions` to analyze the phase transitions
- At sufficiently high temperatures, the only minimum in the effective potential is the **gauge symmetric minimum** $(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{s}) = (0, 0, 0)$
- As the Universe cools down, **extra minima** may appear
- **Multi-step** cosmological phase transitions typically occur in this model
- If there are two coexisted minima separated by a **high barrier**, a **strong FOPT** could take place, resulting in **stochastic gravitational waves**
- At last, the system is trapped at the **true vacuum** $(\tilde{\rho}_1, \tilde{\rho}_2, \tilde{s}) = (v_1, v_2, v_s)$



Benchmark Points (BPs)

	BP1	BP2	BP3	BP4
Type	I	I	II	II
v_s (GeV)	542.40	384.26	64.987	138.82
m_χ (GeV)	117.88	78.191	134.03	76.678
m_{12}^2 (10^4 GeV^2)	2.0210	0.015876	17.696	15.042
$\tan\beta$	2.8616	3.2654	0.91655	1.1732
λ_1	2.1496	2.1882	1.5297	0.87839
λ_2	0.80887	0.85479	1.2074	0.80222
λ_3	2.3925	2.2628	1.5741	2.8002
λ_4	3.0027	1.4715	5.3967	4.4643
λ_5	-6.2187	-4.0567	-7.8556	-7.5755
λ_S	3.4048	2.5502	6.0689	4.8644
κ_1	-1.4852	1.0295	0.80378	-0.38075
κ_2	1.1727	-1.2142	-0.83745	-0.14591
m_{h_1} (GeV)	125.11	91.459	125.38	124.87
m_{h_2} (GeV)	282.02	124.77	158.83	307.56
m_{h_3} (GeV)	1014.5	641.83	650.98	582.08
m_a (GeV)	664.75	496.49	911.87	874.04
$m_{H^\pm}$ (GeV)	402.96	280.94	655.60	631.66
$\langle\sigma_{\rm ann}v\rangle_{\rm dwarf}$ (10^{-26} cm^3/s)	1.30	0.368	1.72	0.682
α	**0.240**	**0.160**	**0.181**	**0.346**
$\tilde{\beta}^{-1}$ (10^{-2})	**1.33**	**0.402**	**0.771**	**2.15**
$T_{\rm p}$ (GeV)	55.3	74.9	60.2	47.2
$\rm SNR_{LISA}$	**96.6**	**37.7**	**60.1**	**120**
$\rm SNR_{Taiji}$	**83.3**	**23.9**	**42.3**	**155**
$\rm SNR_{TianQin}$	**5.50**	**2.39**	**3.07**	**9.20**



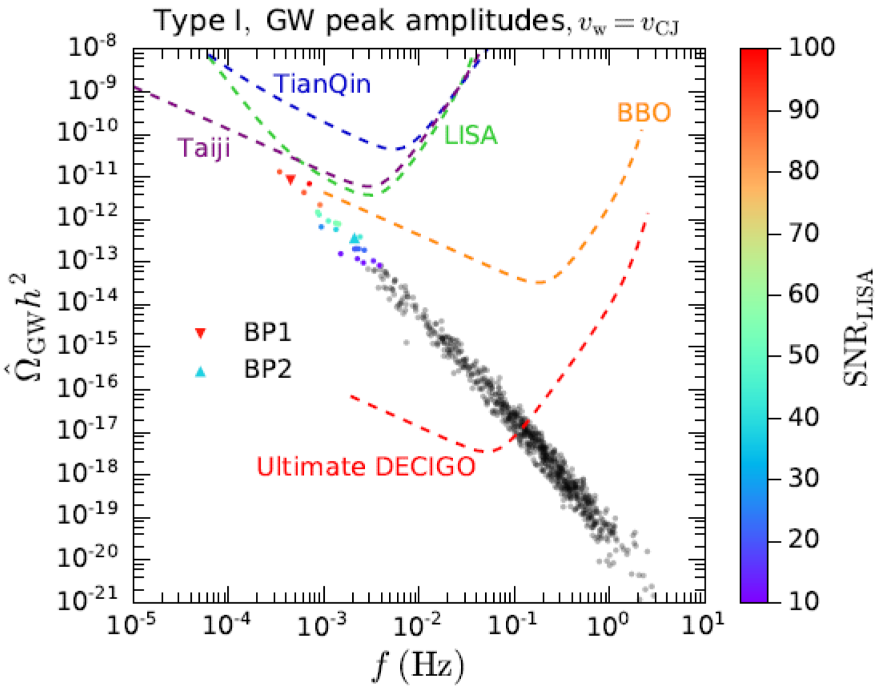
For a practical observation time $\mathcal{T}$, the **signal-to-noise ratio** is

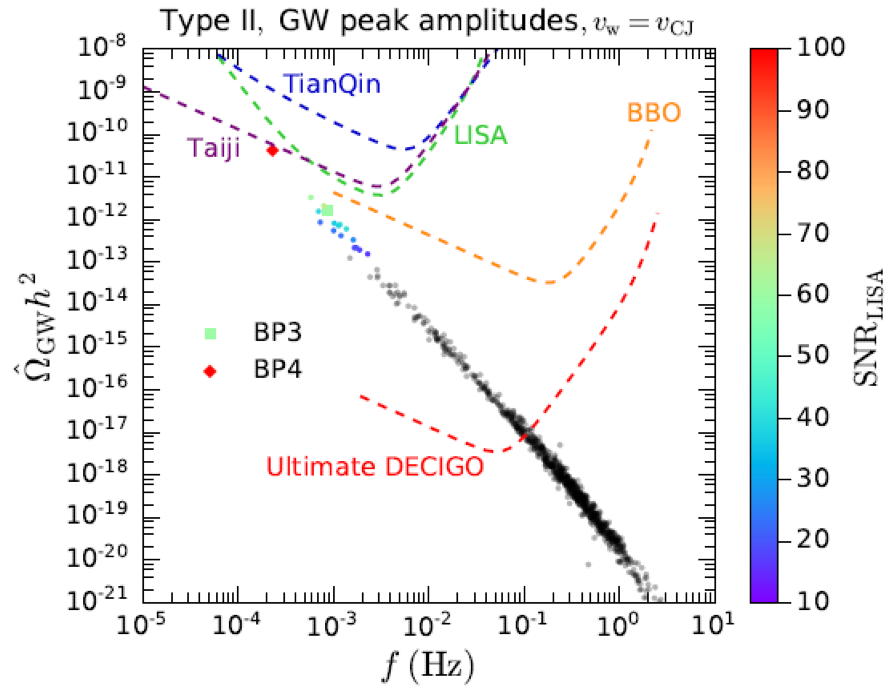
$$\text{SNR} \equiv \sqrt{\mathcal{T}\int_{f_{\min}}^{f_{\max}} \frac{\Omega_{\rm GW}^2(f)}{\Omega_{\rm sens}^2(f)}\,df}$$

Take $\mathcal{T} = 3$ yr for LISA, Taiji, TianQin

The detection threshold can be chosen as $\rm SNR_{thr} = 10$

Peak Amplitudes and Signal-to-noise Ratios





The **colored points** leads to $\mathrm{SNR}_{\mathrm{LISA}} > 10$, promising to be probed by **LISA**

Based on current information, the sensitivity of **Taiji** could be similar to **LISA**, while **TianQin** would be less sensitive due to its **shorter arm length**

Far future plans aiming at $f \sim \mathcal{O}(0.1)$ Hz, like **BBO** and **DECIGO**, may explore much more parameter points

小结

- pNGB暗物质可以自然地解释暗物质直接探测的零结果。

- 扩充这类模型的标量部分（例如增加二重态标量）仍然可以保留这种相消机制，而且有更丰富的唯象学，还可望产生一级相变引力波信号。