THE OHIO STATE UNIVERSITY

Galilean-Invariant Effective Field Theory for the X(3872) (XEFT)

Liping He (何丽萍) [he.1011@osu.edu]

The Ohio State University

Eric Braaten (Ohio State U.),
Galilean-Invariant Effective Field Theory for the X(3872), **PRD 91, 114007 (2015)**

Eric Braaten, LH (Ohio State U.), Jun Jiang (Shandong U.),
Galilean-Invariant XEFT at Next-to-Leading Order , **arXiv: 2010.05801**

@ Institute of Theoretical Physics, Chinese Academy of Sciences
January 29, 2021

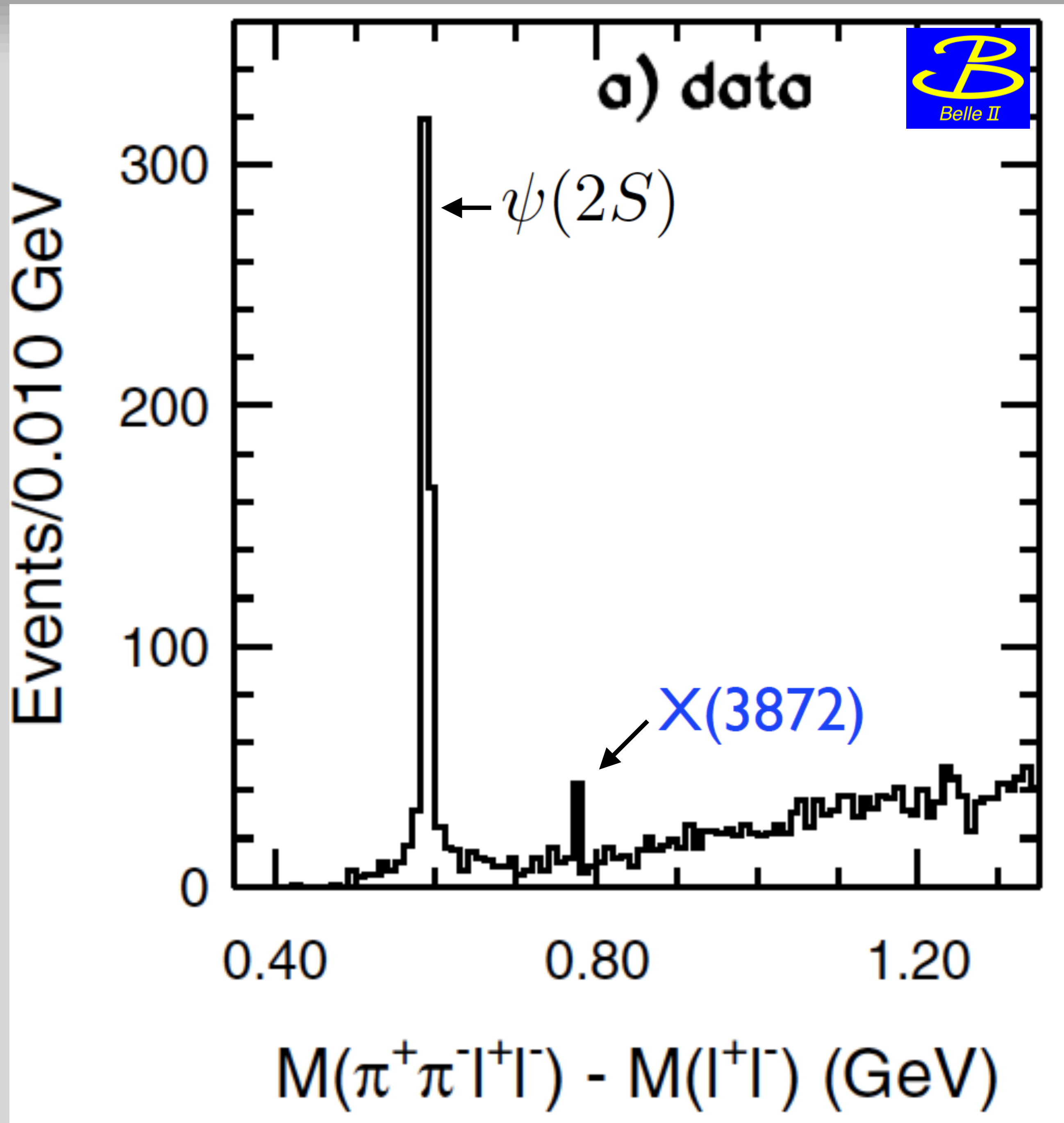
Outline

- Brief review of X(3872)
- Introduction to Original XEFT
- Galiean-invariant XEFT
  - ✦ Galilean invariance
  - ✦ Galilean-invariant Lagrangian and Feynman rules
  - ✦ Lagrangian and Feynman rules with a pair field
  - ✦ NLO pair propagator
  - ✦ $D^{*0}\bar{D}^0$ scattering
- Summary

Brief review of X(3872)

- **discovery:** Belle (2003)

$B^+ \to K^+ + X$ $\quad X \to J/\psi\pi^+\pi^-$



- **confirmation:** CDF (2003)

$p\bar{p} \to X + \text{anything}$

- **quantum numbers:** LHCb (2013)

$J^{PC} = 1^{++}$ → S-wave

- **mass:** LHCb (2020)

extremely close to $D^{*0}\bar{D}^0$ threshold

$$E_X \equiv M_X - (M_{D^{*0}} + M_{D^0}) = (-0.07 \pm 0.12)\ \text{MeV}$$
$$> -0.22\ \text{MeV} \quad \text{at } 90\%\ \text{CL}$$

- **width (Breit-Wigner):** LHCb (2020)

$$\Gamma_X = (1.19 \pm 0.19)\ \text{MeV}$$

Brief review of X(3872)

resonant coupling (fine-tuning of Mx to $D^{*0}D^0$ threshold)

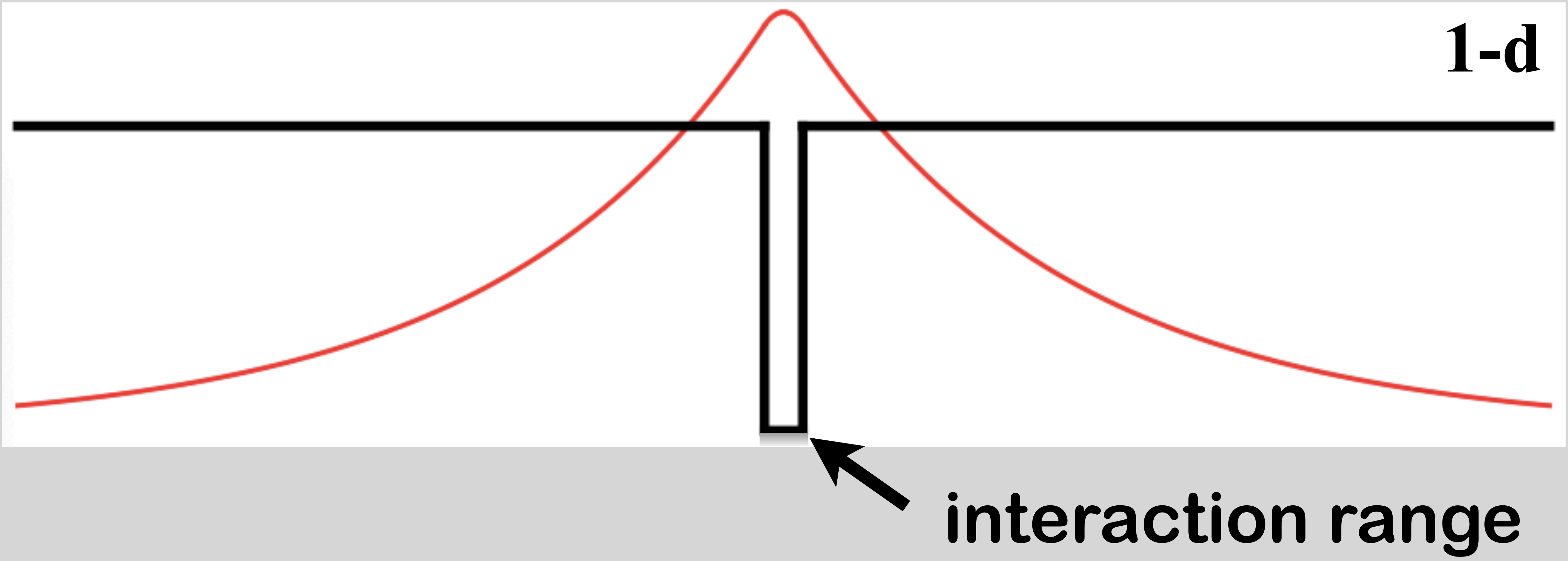
Nonrelativistic Quantum Mechanics:

- short-range interactions
- S-wave resonance close enough to threshold

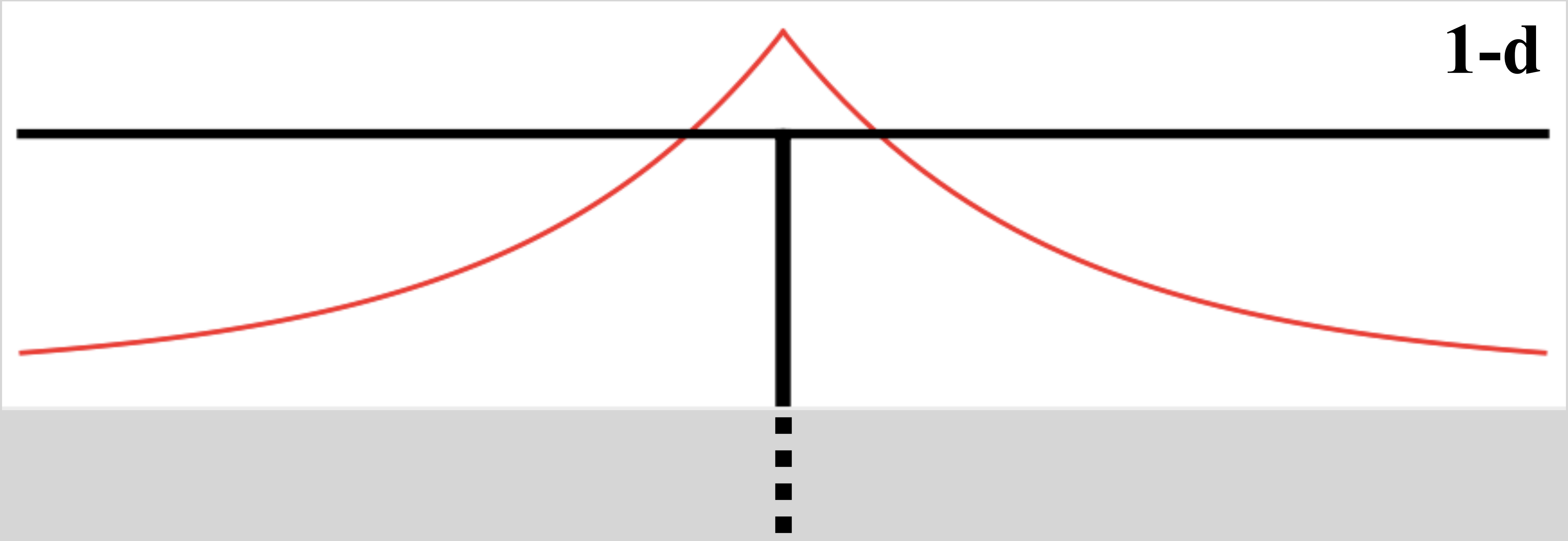
→
* large scattering length: |a| >> range
* Universal properties are determined by the binding energy E_X (or scattering length a = $1/\sqrt{2\mu\,|E_X|}$)

- universal wave function at r >> range: $\psi(r) \sim e^{-r/a}/r$
- scattering amplitude at k << 1/range: $f(k) = 1/(-1/a - i\,k)$

square well potential with short interaction range



zero-range limit: δ potential with fixed area



Brief review of X(3872)

What is the X(3872)?

$J^{PC} = 1^{++}$

$E_X \equiv M_X - (M_{D^{*0}} + M_{D^0}) = (-0.07 \pm 0.12)$ MeV
> -0.22 MeV at 90% CL

S-wave loosely bound charm-meson molecule!!

$$X = \frac{1}{\sqrt{2}}\left(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}\right)$$

What would the X(3872) be if not for fine-tuning of its mass to $D^{*0}D^0$ threshold?

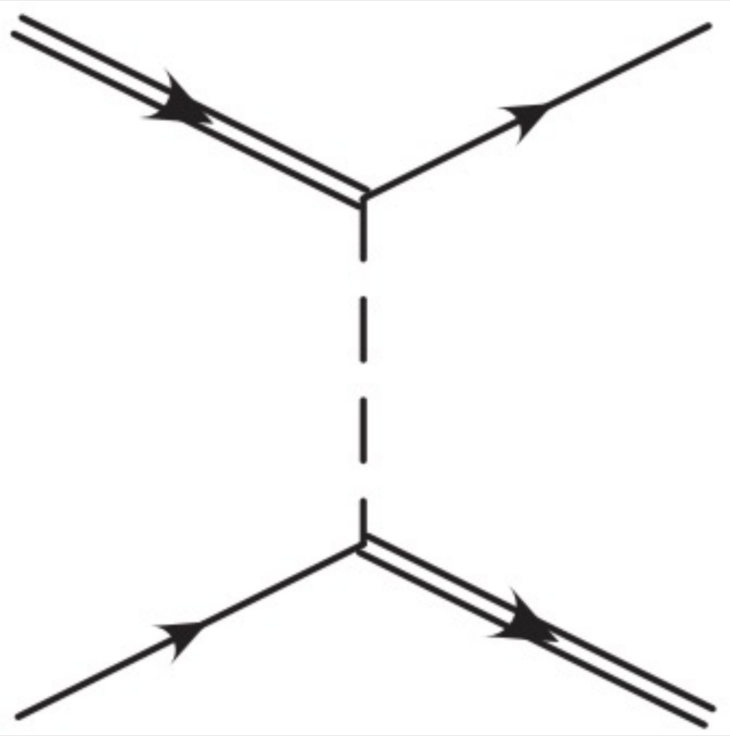
- P-wave charmonium state: $\chi_{c1}(2P)$? $\chi_{c1}(2P) = c\,\bar{c}$
- isospin-0 charm-meson molecule? $[\,(D^{*0}\bar{D}^0 + D^0\bar{D}^{*0}) + (D^{*+}D^- + D^+D^{*-})\,]/2$
- isospin-1 compact tetraquark? $[\,(cu)(\bar{c}\bar{u}) - (cd)(\bar{c}\bar{d})\,]/\sqrt{2}$
- other?

In all cases, X(3872) is transformed into charm-meson molecule by resonant coupling to $D^{*0}\,\bar{D}^0$ and $\bar{D}^{*0}\,D^0$

Original XEFT [Fleming, Kusunoki, Mehen, van Kolck, PRD 76, 034006(2007)]

effect of π exchange on the properties of X(3872)

low energy $D^{*0}\bar{D}^0 \longrightarrow D^0\bar{D}^{*0}$ scattering in HHχPT with one-pion exchange



$$\frac{g^2}{2f_\pi^2}\frac{\vec{\epsilon}^{\,*}\cdot\vec{q}\;\vec{\epsilon}\cdot\vec{q}}{\vec{q}^{\,2}-\mu^2}$$

$\boxed{\mu^2 = (M_{*0}-M_0)^2-m_0^2}$, $\mu \approx 45$ MeV

→ **pions** generate anomalously long-range effects and should be included as **explicit nonrelativistic degrees of freedom** in the description of the molecule.

- pion-exchange is characterized by small momentum ~ μ
- two-pion-and one-pion-exchange ratio is small

$$\frac{g^2 M_{DD^*}\mu}{4\pi f_\pi^2} \approx \frac{1}{20} - \frac{1}{10}$$

pion-exchange can be treated perturbatively

Original XEFT [Fleming, Kusunoki, Mehen, van Kolck, PRD 76, 034006(2007)]

Lagrangian relevant to low-energy S-wave DD* scattering

- Matching onto HHχPT yields D^0, D^{*0}, π^0 kinetic terms and the axial D^{*0}-D^0-π^0 coupling
- integrating all momentum scales much larger than μ
- including contact interactions to incorporate effects from shorter distance scales

$$\mathcal{L} = \boxed{\boldsymbol{D}^\dagger\Big(i\partial_0 + \frac{\vec{\nabla}^2}{2m_{D^*}}\Big)\boldsymbol{D} + D^\dagger\Big(i\partial_0 + \frac{\vec{\nabla}^2}{2m_D}\Big)D + \bar{\boldsymbol{D}}^\dagger\Big(i\partial_0 + \frac{\vec{\nabla}^2}{2m_{D^*}}\Big)\bar{\boldsymbol{D}} + \bar{D}^\dagger\Big(i\partial_0 + \frac{\vec{\nabla}^2}{2m_D}\Big)\bar{D} + \boldsymbol{\pi}^\dagger\Big(i\partial_0 + \frac{\vec{\nabla}^2}{2m_\pi} + \delta\Big)\boldsymbol{\pi}}$$
$$\boxed{+ \frac{g}{\sqrt{2}f_\pi}\frac{1}{\sqrt{2m_\pi}}(D\boldsymbol{D}^\dagger\cdot\vec{\nabla}\boldsymbol{\pi} + \bar{D}^\dagger\bar{\boldsymbol{D}}\cdot\vec{\nabla}\boldsymbol{\pi}^\dagger) + \text{H.c.}} \boxed{- \frac{C_0}{2}(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D})^\dagger\cdot(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D})}$$
$$\boxed{+ \frac{C_2}{16}(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D})^\dagger\cdot(\bar{\boldsymbol{D}}(\overleftrightarrow{\nabla})^2D + \boldsymbol{D}(\overleftrightarrow{\nabla})^2\bar{D}) + \text{H.c.} + \frac{B_1}{\sqrt{2}}\frac{1}{\sqrt{2m_\pi}}(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D})^\dagger\cdot D\bar{D}\vec{\nabla}\boldsymbol{\pi} + \text{H.c.}} + \cdots,$$

$$\overleftrightarrow{\nabla} = \overleftarrow{\nabla} - \overrightarrow{\nabla}$$

$$\boxed{+\frac{C_\pi}{2m_\pi}\left(D^\dagger\pi^\dagger D\pi + \bar{D}^\dagger\pi^\dagger\bar{D}\pi\right) + C_{0D}D^\dagger\bar{D}^\dagger D\bar{D}}$$

Dai, Guo, Mehen, PRD101, 054024(2020)

Original XEFT [Fleming, Kusunoki, Mehen, van Kolck, PRD 76, 034006(2007)]

Power counting

expansion in powers of Q $\boxed{p_D \sim p_{D^*} \sim p_\pi \sim \mu \sim \gamma \sim Q}$ $E_D \sim E_{D^*} \sim E_\pi \sim Q^2$

loop integration: $\int d^4p \sim Q^5$

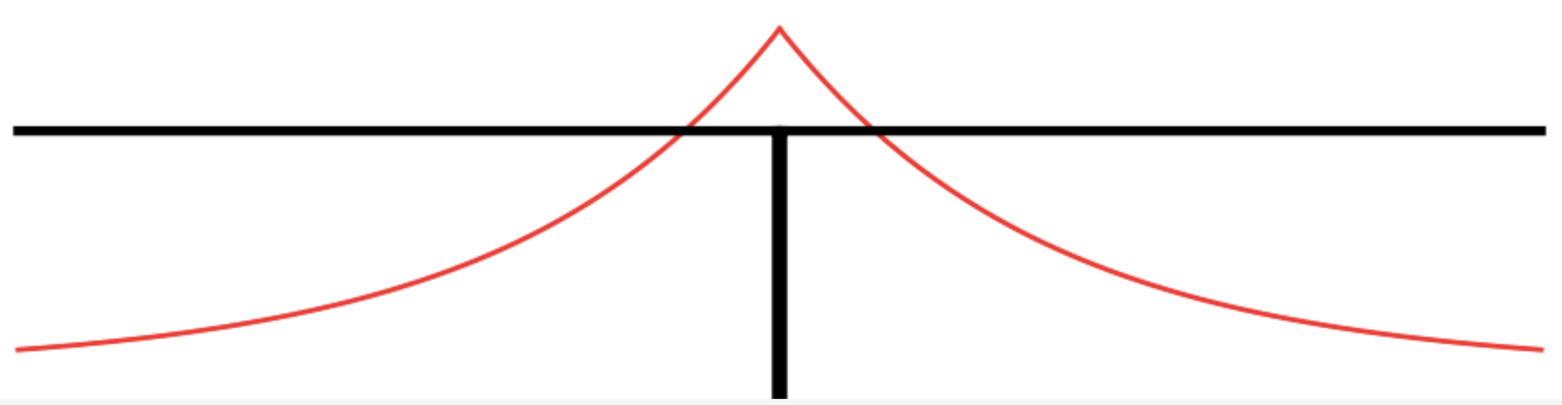
propagator: $\frac{1}{E - p^2/(2M)} \sim Q^{-2}$

✦ **Leading order (LO)**: contact interaction

$$-\frac{C_0}{2}(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D})^\dagger \cdot (\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D})$$

must be treated nonperturbatively to generate X(3872) bound state.

resonant coupling



✦ **Next-to-leading order (NLO)**:

- pion interaction

$$+\frac{g}{\sqrt{2}f_\pi}\frac{1}{\sqrt{2m_\pi}}(\boldsymbol{D}D^\dagger \cdot \vec{\nabla}\pi + \bar{D}^\dagger\bar{\boldsymbol{D}} \cdot \vec{\nabla}\pi^\dagger) + \text{H.c.}$$

- 2-derivative contact interaction

$$+\frac{C_2}{16}(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D})^\dagger \cdot (\bar{\boldsymbol{D}}(\overleftrightarrow{\nabla})^2 D + \boldsymbol{D}(\overleftrightarrow{\nabla})^2\bar{D}) + \text{H.c.}$$

- D*D (C=+) → DDπ transition

$$+\frac{B_1}{\sqrt{2}}\frac{1}{\sqrt{2m_\pi}}(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D})^\dagger \cdot D\bar{D}\vec{\nabla}\pi + \text{H.c.}$$

Galilean-invariant XEFT

- ✦ Galilean invariance
- ✦ Galilean-invariant Lagrangian and Feynman rules
- ✦ Lagrangian and Feynman rules with a pair field
- ✦ NLO pair propagator
- ✦ $D^{*0}\bar{D}^0$ scattering

Braaten (Ohio State U.),
Galilean-Invariant Effective Field Theory for the X(3872), **PRD 91, 114007 (2015)**

Braaten, LH (Ohio State U.), **Jiang** (Shandong U.),
Galilean-Invariant XEFT at Next-to-Leading Order , **arXiv: 2010.05801**

Galilean invariance

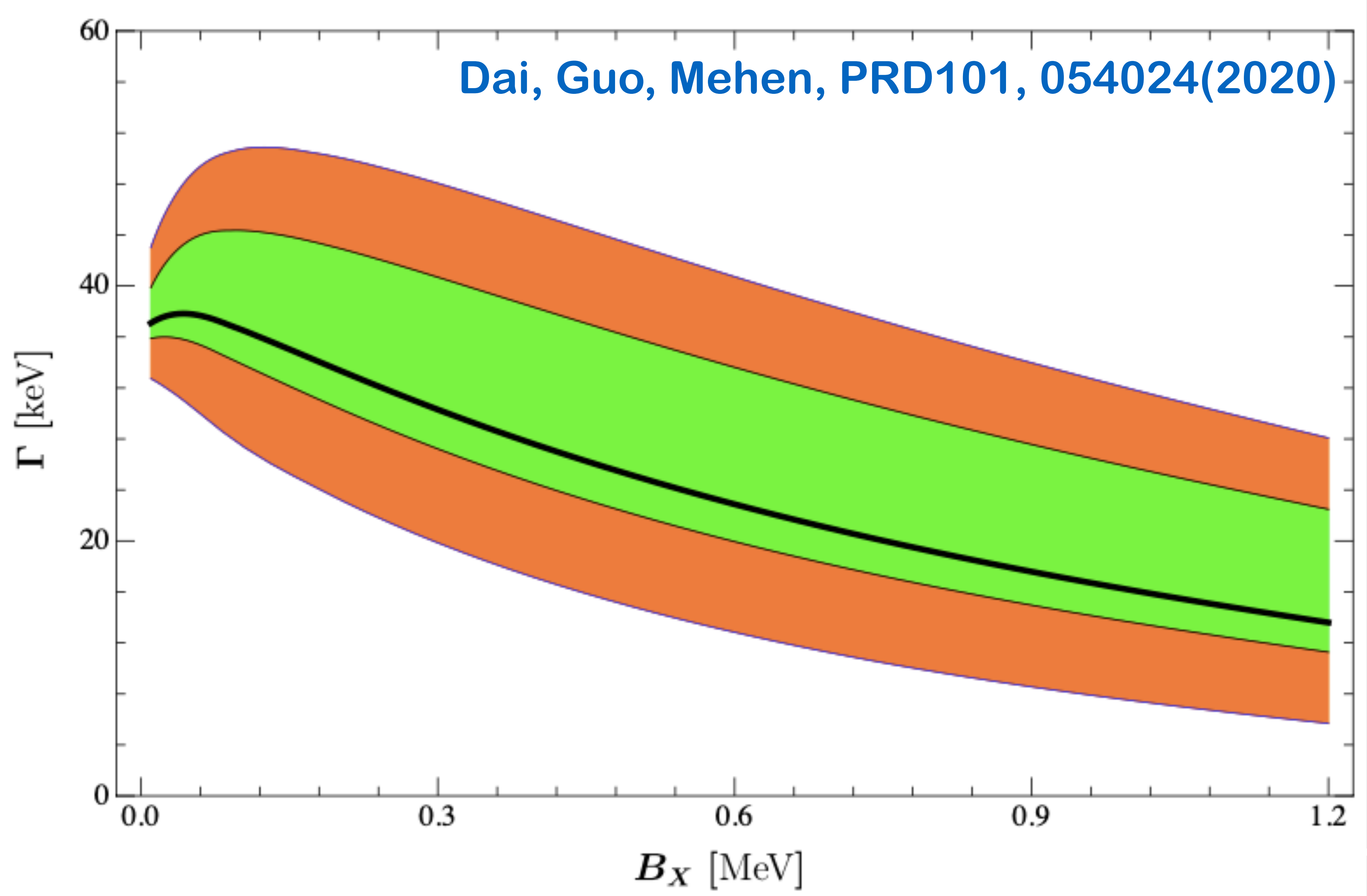
Problems in the original XEFT:

- not Galilean invariant
  - ✦ frame dependent
  - ✦ ultraviolet divergences are much less constrained
- renormalization scheme: difficult to take into account decays with momentum too large to be treated explicitly in XEFT, such as $D^* \rightarrow D\gamma$.

can be solved by Galilean invariance and a new renormalization scheme

- large error bands from NLO corrections in the decay $X(3872) \rightarrow D^0 D^0 \pi^0$

$X \rightarrow D^0 D^0 \pi^0$



Galilean invariance

Galilean invariance: a possible space-time symmetry of a NR theory
[Hagen, PRD5, 377(1972), Rosen, AJP40, 683 (1972)]

Galilean boost with velocity v:

$$E \longrightarrow E + \boldsymbol{v}\cdot\boldsymbol{p} + \frac{1}{2}mv^2$$
$$\boldsymbol{p} \longrightarrow \boldsymbol{p} + m\boldsymbol{v}.$$

- energy-momentum relation:

$$E(p) = \varepsilon + p^2/(2m)$$

- invariant energy

$$E_{\rm tot} - P_{\rm tot}^2/2m_{\rm tot}$$

Galilean invariance requires exact conservation of kinetic mass

✦ $D^{*0} \to D^0\,\pi^0$: mass is nearly conserved

$M_{*0} - M_0 - m_0 = \delta = 7.04$ MeV << m_0=135 MeV

→ makes a Galilean-invariant theory possible

kinetic mass: D^{*0} (M+m), D^0 (M), π^0 (m)

rest energy: D^{*0} ($E_* = \delta - i\Gamma_{*0}/2$), D^0 (0), π^0 (0)

✦ Conservations:

- number of charm quarks (antiquarks): $N_c = N_{D^{*0}} + N_{D^0}$, $N_{\bar c} = N_{\bar D^{*0}} + N_{\bar D^0}$
- pion number: $N_\pi = N_{\pi^0} + N_{D^{*0}} + N_{\bar D^{*0}}$

Galilean invariance

Galilean invariance requires interaction terms are invariant under Galilean boosts

term with ∇:

$\boxed{D^\dagger \cdot D\nabla\pi}$ $\sim p_\pi \rightarrow p_\pi + mv$

$\boxed{D[M\vec{\nabla} - m\overleftarrow{\nabla}]\pi}$ $\sim Mp_\pi - m\, p_0$

$\rightarrow M(p_\pi + mv) - m(p_0 + Mv)$

$= Mp_\pi - m\, p_0$

✦ **pion interaction term:**

$D^\dagger \cdot D\nabla\pi \rightarrow D^\dagger \cdot (D[M\vec{\nabla} - m\overleftarrow{\nabla}]\pi)\ /(M+m)$

✦ **In** $(\bar{D}D)^\dagger \cdot \bar{D}(\overleftrightarrow{\nabla})^2 D$

$(\overleftrightarrow{\nabla})^2 \rightarrow 4(M\vec{\nabla} - (M+m)\overleftarrow{\nabla})^2/(2M+m)^2$

✦ **In** $(\bar{D}D)^\dagger \cdot \bar{D}D\nabla\pi$

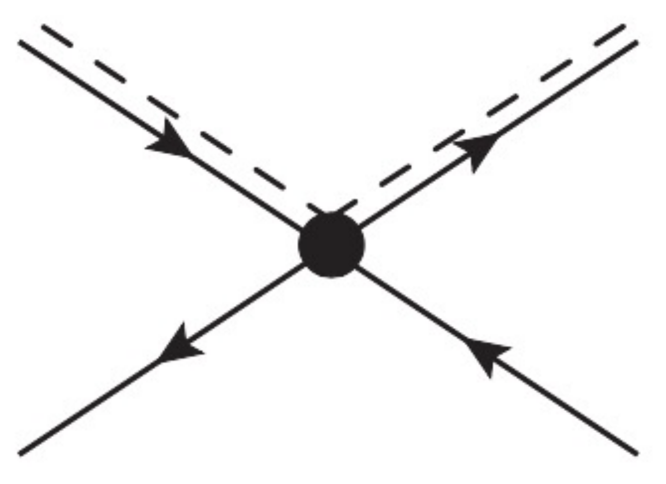
$\nabla \rightarrow (2M\vec{\nabla} - m\overleftarrow{\nabla})/(2M+m)$

Galilean-invariant Lagrangian and Feynman rules

- **Leading order (LO) term**

$$\mathcal{L}_{\mathrm{LO,int}} = -\frac{C_0}{2}\left(\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}}\right)^{\dagger} \cdot \left(\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}}\right)$$

$D^{*0}\bar{D}^{0} \to D^{*0}\bar{D}^{0}$



Feynman rules:

$$(-iC_0/2)\delta^{ij}$$

LO must be treated nonperturbatively to generate the bound state that can be identified with the X(3872)

Galilean-invariant Lagrangian and Feynman rules

Leading order (LO) term

LO must be treated nonperturbatively to generate the bound state that can be identified with the X(3872)

- ✦ $D^{*0}\bar{D}^0 \to D^{*0}\bar{D}^0$ **transition amplitude (in CM frame):**

  1/2 $(2\pi/\mu)\mathcal{A}(E)$

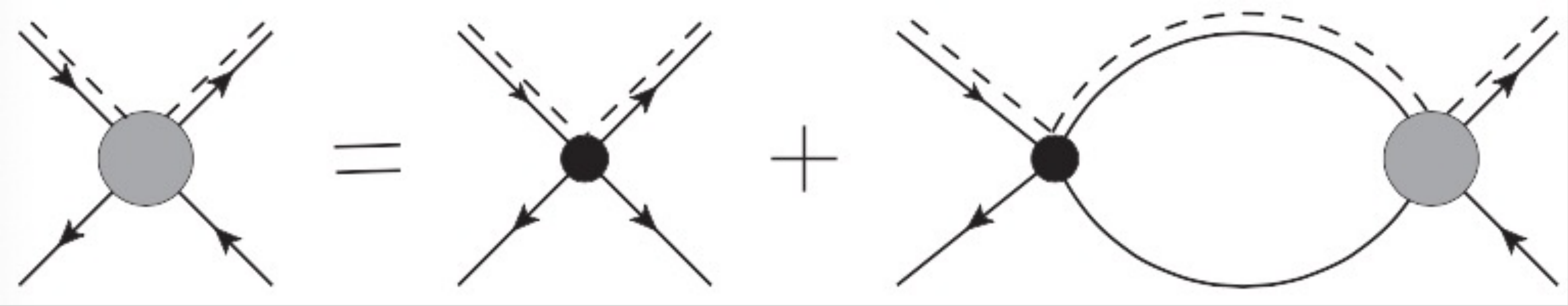
$$\mathcal{A}(E) = \frac{1}{-2\pi/(\mu C_0) - 4\pi J_1(E)}$$

- ✦ **dimensional regularization in d spatial dimensions:**
- ✦ **tuning C_0 to make A(E) finite as d→3:**

$$\mathcal{A}(E) = \frac{1}{-\gamma + \sqrt{-2\mu(E - E_*)}} \qquad (d = 3)$$

- ✦ **pole at the complex energy:**

Lippman-Schwinger integral equation:



$$J_1(E) = \frac{\Gamma(1 - d/2)}{(4\pi)^{d/2}}\Lambda^{3-d}\left[2\mu(E_* - E)\right]^{d/2-1}$$

$$\frac{2\pi}{\mu C_0} = \left(\frac{2}{d-2} - 2\right)\Lambda + \gamma$$

pole at d=2

$E_* = \delta - i\,\Gamma_{*0}/2$

complex binding momentum

$$E_{\text{pole,LO}} = E_* - \gamma^2/(2\mu)$$

Imaginary part of E_* gives contribution to the width of X (→D*D→DDπ,DDγ)

X also has other short-distance decays

Galilean-invariant Lagrangian and Feynman rules

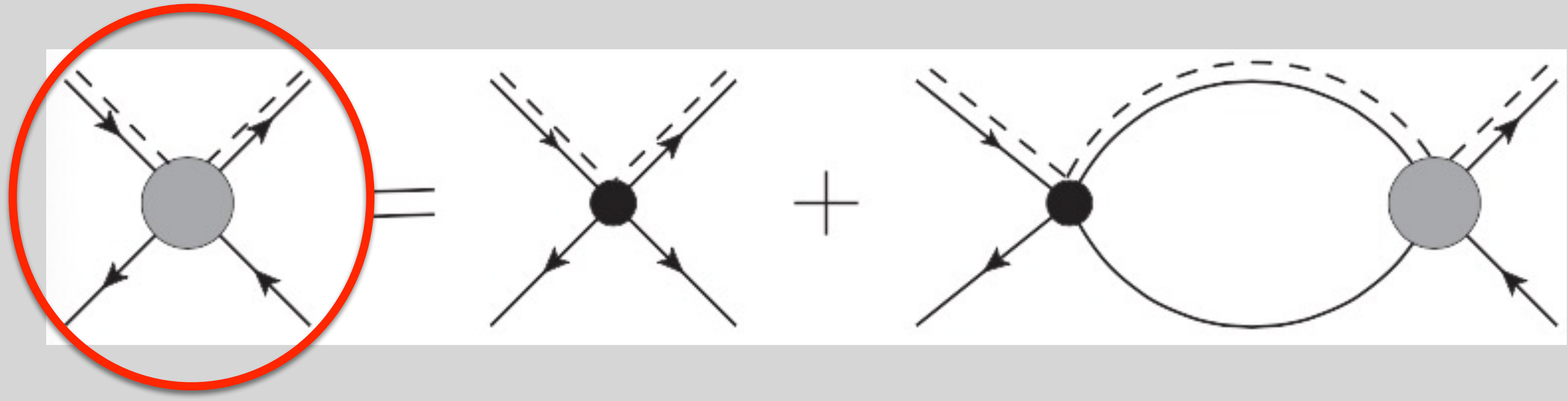
- **Leading order (LO) term**

LO must be treated nonperturbatively to generate the bound state that can be identified with the X(3872)

$$+i\left(\frac{1}{\sqrt{2}}\right)^2 \frac{2\pi}{\mu} \mathcal{A}(E_{\mathrm{cm}})\delta^{ij}$$

$$\mathcal{A}(E) = \frac{1}{-\gamma + \sqrt{-2\mu(E-E_*)}} \qquad (d=3)$$

$D^{*0}\bar{D}^0 \to D^{*0}\bar{D}^0$



invariant energy: $\boldsymbol{E}_{\mathrm{cm}} = \boldsymbol{E}_{\mathrm{tot}} - \frac{\boldsymbol{P}^2_{\mathrm{tot}}}{2(2M+m)}$

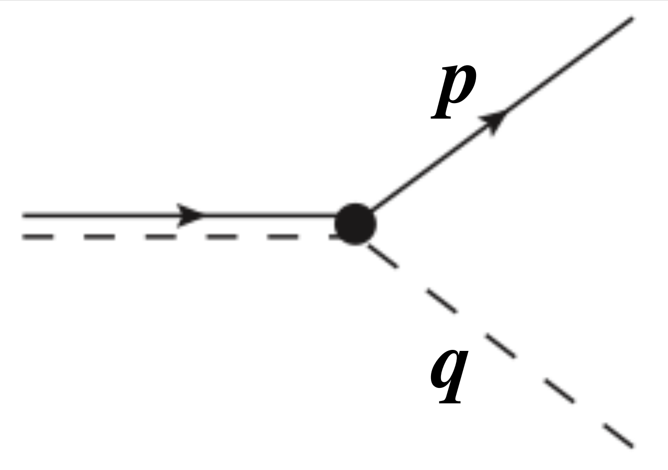
Galilean-invariant Lagrangian and Feynman rules

- **Next-to-Leading order (NLO) terms**

Feynman rules:

✦ **pion interaction (g):**

$$\mathcal{L}_{D^*\leftrightarrow D\pi} = \frac{g}{2\sqrt{m}f_\pi}\left[\boldsymbol{D}^\dagger\cdot(D\overleftrightarrow{\nabla}\pi)+(D\overleftrightarrow{\nabla}\pi)^\dagger\cdot\boldsymbol{D}+\bar{\boldsymbol{D}}^\dagger\cdot(\bar{D}\overleftrightarrow{\nabla}\pi)+(\bar{D}\overleftrightarrow{\nabla}\pi)^\dagger\cdot\bar{\boldsymbol{D}}\right]$$

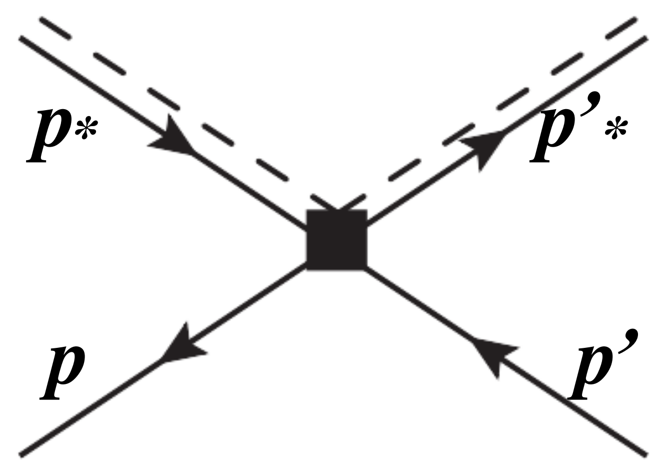


$D^{*0}\to D^0\pi^0$

$$\boxed{\frac{g}{2\sqrt{m}f_\pi}\frac{(M\boldsymbol{q}-m\boldsymbol{p})^i}{M+m}}$$

✦ **∇^2 interaction (g^2):**

$$\mathcal{L}_{\nabla^2} = \frac{C_2}{4(2M+m)^2}[(\bar{\boldsymbol{D}}\boldsymbol{D})^\dagger\cdot(\bar{D}[M\vec{\boldsymbol{\nabla}}-(M+m)\overleftarrow{\boldsymbol{\nabla}}]^2\boldsymbol{D})+(\bar{D}[M\vec{\boldsymbol{\nabla}}-(M+m)\overleftarrow{\boldsymbol{\nabla}}]^2\boldsymbol{D})^\dagger\cdot(\bar{\boldsymbol{D}}\boldsymbol{D})]$$

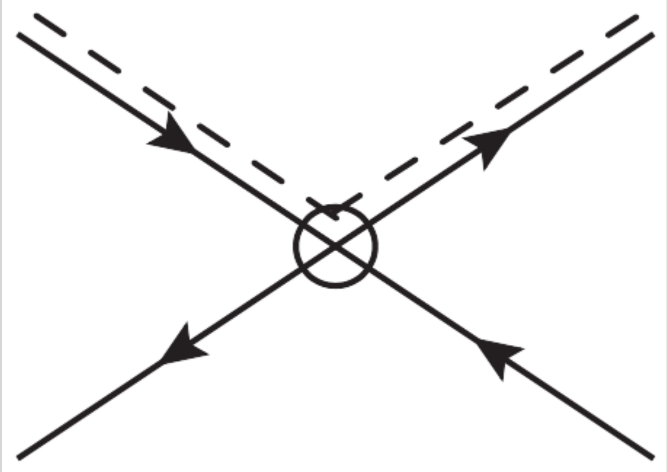


$D^{*0}\bar{D}^0\to D^{*0}\bar{D}^0$

$$\boxed{(-iC_2/4)\frac{((M+m)\boldsymbol{p}-M\boldsymbol{p}_*)^2+((M+m)\boldsymbol{p}'-M\boldsymbol{p}'_*)^2}{(2M+m)^2}\delta^{ij}}$$

✦ **counter terms (g^2):**

$$\mathcal{L}_{\text{counterterm}} = -\frac{\delta C_0}{2}(\bar{\boldsymbol{D}}\boldsymbol{D})^\dagger\cdot(\bar{\boldsymbol{D}}\boldsymbol{D})-\frac{\delta D_0}{2}(\bar{\boldsymbol{D}}\boldsymbol{D})^\dagger\cdot[i\partial_0+\nabla^2/(2(2M+m))](\bar{\boldsymbol{D}}\boldsymbol{D})$$



$$\boxed{(-i/2)\left[\delta C_0+\delta D_0\left(E-\frac{P^2}{2(2M+m)}\right)\right]\delta^{ij}}$$

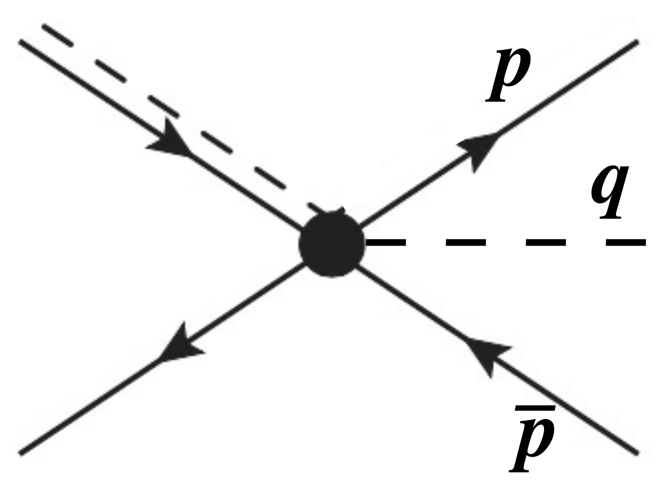
Galilean-invariant Lagrangian and Feynman rules

- **Next-to-Leading order (NLO) terms**

Feynman rules:

✦ $D^{*0}\bar{D}^0 \to D^0\bar{D}^0\pi^0$ term (g^3):

$$\mathcal{L}_{D\bar{D}\pi} = \frac{B_1}{\sqrt{2m}}\frac{1}{\sqrt{2}}\left[(\bar{D}\boldsymbol{D})^\dagger \cdot (D\bar{D}\overleftrightarrow{\boldsymbol{\nabla}}\pi) + (D\bar{D}\overleftrightarrow{\boldsymbol{\nabla}}\pi)^\dagger \cdot (\bar{D}\boldsymbol{D})\right]$$



$$\boxed{\frac{B_1}{2\sqrt{m}}\frac{(2M\boldsymbol{q} - m(\boldsymbol{p} + \bar{\boldsymbol{p}}))^i}{2M + m}}$$

✦ S-wave $D^0\bar{D}^0$ and $D^0\pi^0$ interaction terms

$$\cancel{+\frac{C_\pi}{2m_\pi}\left(D^\dagger\pi^\dagger D\pi + \bar{D}^\dagger\pi^\dagger\bar{D}\pi\right) + C_{0D}D^\dagger\bar{D}^\dagger D\bar{D}}$$

not required to cancel ultraviolet divergences

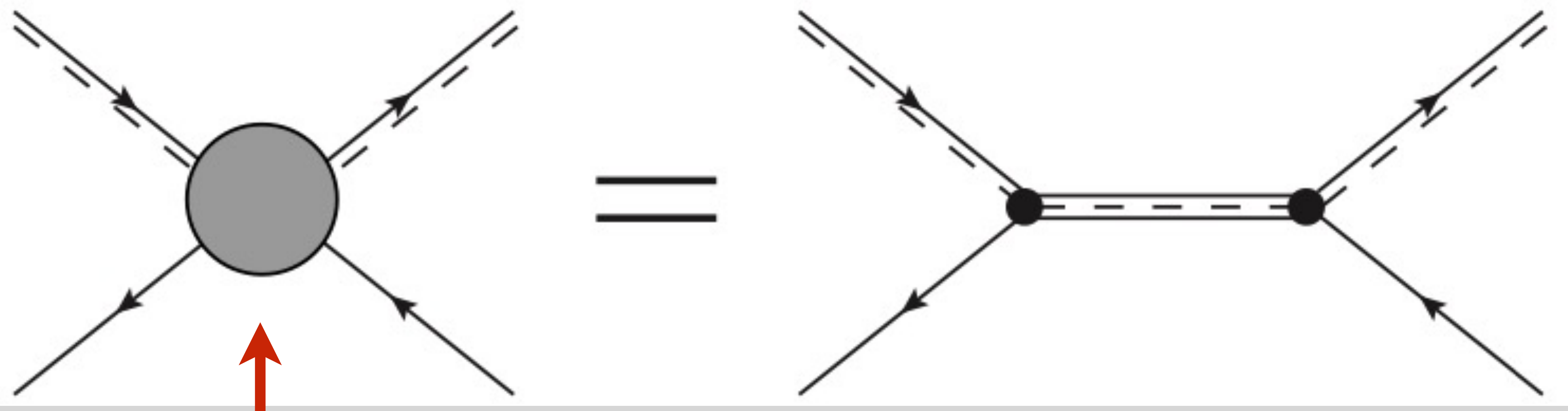
Lagrangian and Feynman rules with a pair field

- **LO with a pair field** (annihilates a pair of charm mesons in the resonant channel)

$$\boxed{\phi = \frac{C_0}{\sqrt{2}}\left(\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}}\right)}$$

$$\mathcal{L}_{\mathrm{LO,int}} = -\frac{C_0}{2}\left(\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}}\right)^{\dagger}\cdot\left(\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}}\right)$$

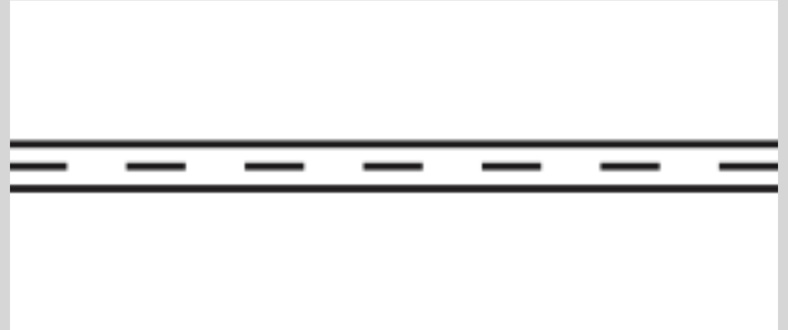
$$\boxed{= \frac{1}{C_0}\boldsymbol{\phi}^{\dagger}\cdot\boldsymbol{\phi} - \frac{1}{\sqrt{2}}\left[\left(\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}}\right)^{\dagger}\cdot\boldsymbol{\phi} + \boldsymbol{\phi}^{\dagger}\cdot\left(\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}}\right)\right]}$$



$D^{*0}\bar{D}^0 \to D^{*0}\bar{D}^0$

$$\boxed{+i\left(\frac{1}{\sqrt{2}}\right)^2\frac{2\pi}{\mu}\mathcal{A}(E_{\mathrm{cm}})\delta^{ij}}$$

pair propagator:



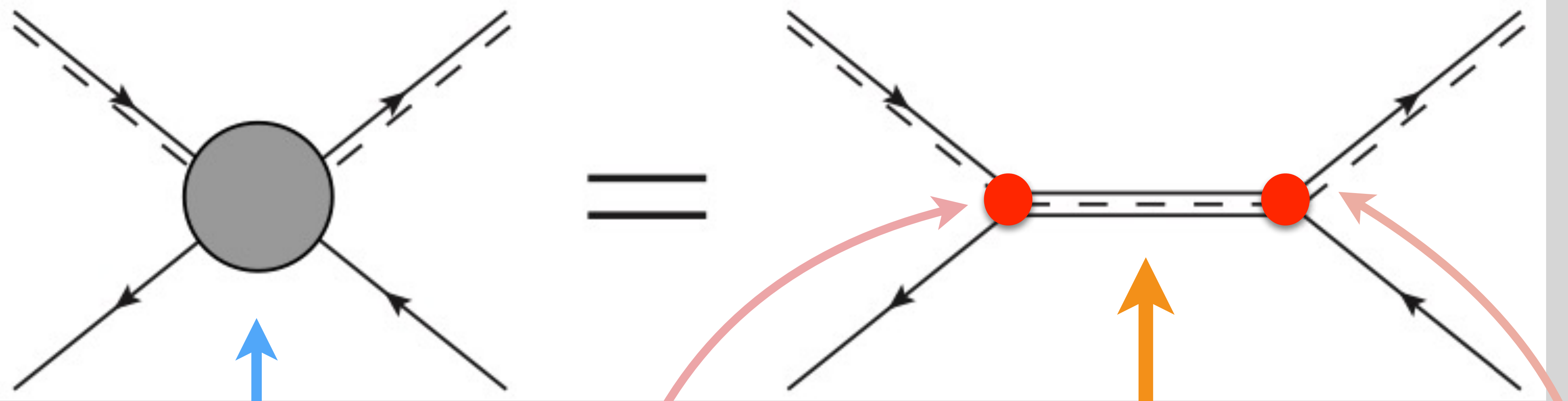
Lagrangian and Feynman rules with a pair field

- **LO with a pair field** (annihilates a pair of charm mesons in the resonant channel)

$$\boxed{\phi = \frac{C_0}{\sqrt{2}}\,(\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}})}$$

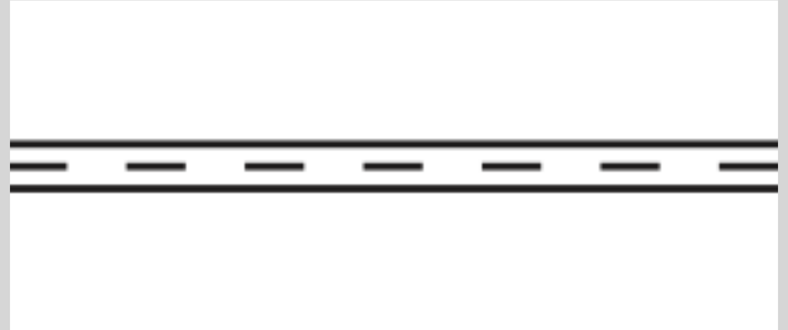
$$\mathcal{L}_{\mathrm{LO,int}} = -\frac{C_0}{2}\,(\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}})^\dagger \cdot (\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}})$$

$$\boxed{= \frac{1}{C_0}\boldsymbol{\phi}^\dagger \cdot \boldsymbol{\phi} - \frac{1}{\sqrt{2}}\left[(\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}})^\dagger \cdot \boldsymbol{\phi} + \boldsymbol{\phi}^\dagger \cdot (\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}})\right]}$$

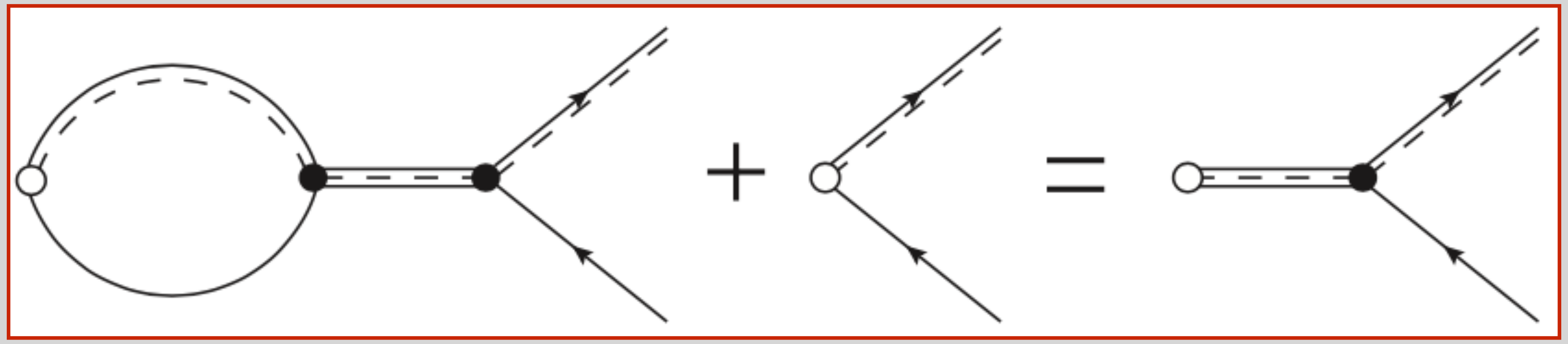


$$\boxed{+i\left(\frac{1}{\sqrt{2}}\right)^2 \frac{2\pi}{\mu}\mathcal{A}(E_{\mathrm{cm}})\delta^{ij}} = \boxed{-i\frac{1}{\sqrt{2}}\delta^{ij}} \times \boxed{-i\frac{2\pi}{\mu}\mathcal{A}(E_{\mathrm{cm}})\,\delta^{ij}} \times \boxed{-i\frac{1}{\sqrt{2}}\delta^{ij}}$$

pair propagator:



- **diagrammatic identity**



$$J_1(E)\,\mathcal{A}(E) + \frac{1}{4\pi} = -\frac{1}{2\mu C_0}\mathcal{A}(E) \longleftarrow \mathcal{A}(E) = \frac{1}{-2\pi/(\mu C_0) - 4\pi J_1(E)}$$

Lagrangian and Feynman rules with a pair field

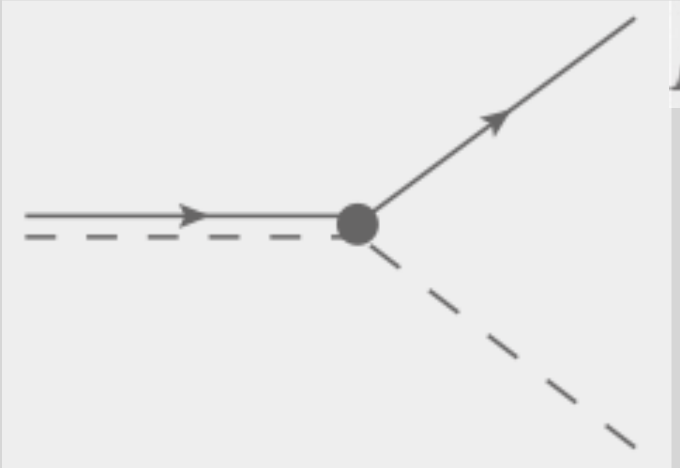
- **NLO with a pair field**

$$\phi = \frac{C_0}{\sqrt{2}}\left(\bar{D}\boldsymbol{D} + D\bar{\boldsymbol{D}}\right)$$

Feynman rules:

✦ **pion interaction:**

$$\mathcal{L}_{D^*\leftrightarrow D\pi} = \frac{g}{2\sqrt{m}f_\pi}\left[\boldsymbol{D}^\dagger\cdot(D\overleftrightarrow{\nabla}\pi) + (D\overleftrightarrow{\nabla}\pi)^\dagger\cdot\boldsymbol{D} + \bar{\boldsymbol{D}}^\dagger\cdot(\bar{D}\overleftrightarrow{\nabla}\pi) + (\bar{D}\overleftrightarrow{\nabla}\pi)^\dagger\cdot\bar{\boldsymbol{D}}\right]$$



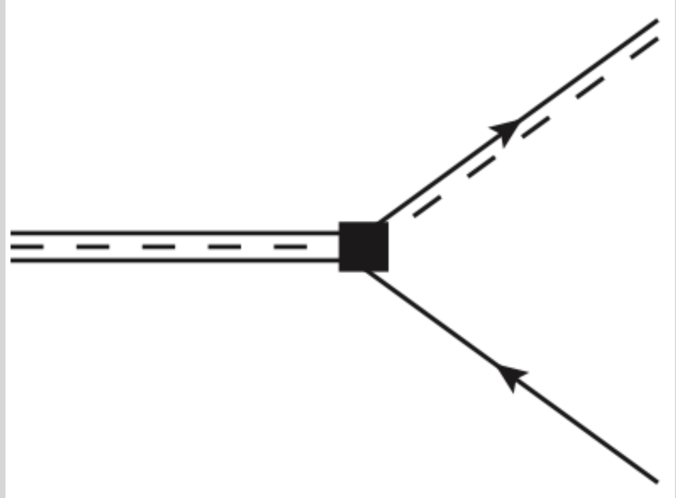
$D^{*0} \to D^0\pi^0$

$$\frac{g}{2\sqrt{m}f_\pi}\,\frac{(M\boldsymbol{q} - m\boldsymbol{p})^i}{M+m}$$

✦ **∇^2 interaction:**

$$\mathcal{L}_{\nabla^2} = \frac{C_2}{4(2M+m)^2}[(\bar{D}\boldsymbol{D})^\dagger\cdot(\bar{D}[M\overrightarrow{\nabla} - (M+m)\overleftarrow{\nabla}]^2\boldsymbol{D}) + (\bar{D}[M\overrightarrow{\nabla} - (M+m)\overleftarrow{\nabla}]^2\boldsymbol{D})^\dagger\cdot(\bar{D}\boldsymbol{D})]$$

$$= \frac{C_2}{2\sqrt{2}\,C_0}\left[\phi^\dagger\cdot\left(\bar{D}\overleftrightarrow{\nabla}^2\boldsymbol{D} + D\overleftrightarrow{\nabla}^2\bar{\boldsymbol{D}}\right) + \left(\bar{D}\overleftrightarrow{\nabla}^2\boldsymbol{D} + D\overleftrightarrow{\nabla}^2\bar{\boldsymbol{D}}\right)^\dagger\cdot\phi\right]$$

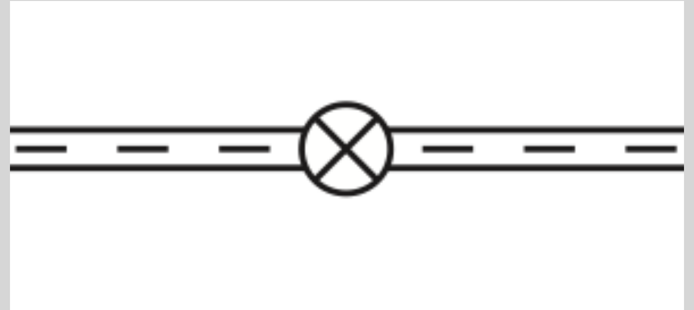


$$-i\frac{C_2}{2\sqrt{2}\,C_0}\,\frac{((M+m)\boldsymbol{p}_0 - M\boldsymbol{p}_1)^2}{(2M+m)^2}\,\delta^{ij}$$

✦ **counterterm:**

$$\mathcal{L}_{\text{counterterm}} = -\frac{\delta C_0}{2}(\bar{D}\boldsymbol{D})^\dagger\cdot(\bar{D}\boldsymbol{D}) - \frac{\delta D_0}{2}(\bar{D}\boldsymbol{D})^\dagger\cdot[i\partial_0 + \nabla^2/(2(2M+m))](\bar{D}\boldsymbol{D})$$

$$= -(1/C_0^2)\left[\delta C_0\,\phi^\dagger\cdot\phi + D_0\,\phi^\dagger\cdot(iD_t - E_*)\phi\right]$$

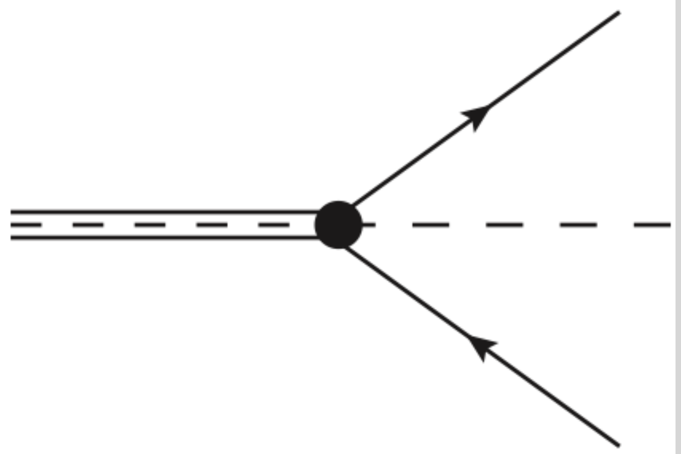


$$-i\frac{1}{C_0^2}\left[\delta C_0 + D_0\left(E_{\text{cm}} - E_*\right)\right]\delta^{ij}$$

✦ **$D^{*0}\bar{D}^0 \to D^0\bar{D}^0\pi^0$ term**

$$\mathcal{L}_{D\bar{D}\pi} = \frac{B_1}{\sqrt{2}}\frac{1}{\sqrt{2m_\pi}}(\bar{\boldsymbol{D}}D + \boldsymbol{D}\bar{D})^\dagger\cdot D\bar{D}\,\vec{\nabla}\boldsymbol{\pi} + \text{H.c.}$$

$$= \frac{B_1}{\sqrt{2m}\,C_0}\left[\phi^\dagger\cdot(D\bar{D}\overleftrightarrow{\nabla}\pi) + (D\bar{D}\overleftrightarrow{\nabla}\pi)^\dagger\cdot\phi\right]$$

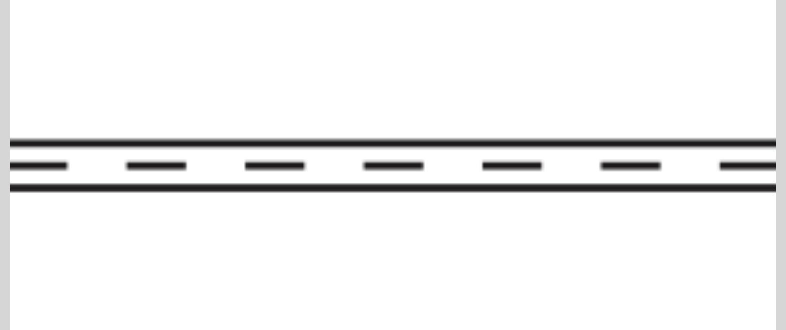


$$\frac{B_1}{\sqrt{2m}\,C_0}\,\frac{(2M\boldsymbol{q} - m(\boldsymbol{p}_0 + \boldsymbol{p}_0'))^i}{2M+m}$$

NLO pair propagator

A. Complete pair propagator

- **LO:**



$$-i\frac{2\pi}{\mu}\mathcal{A}(E_{\rm cm})\,\delta^{ij}$$

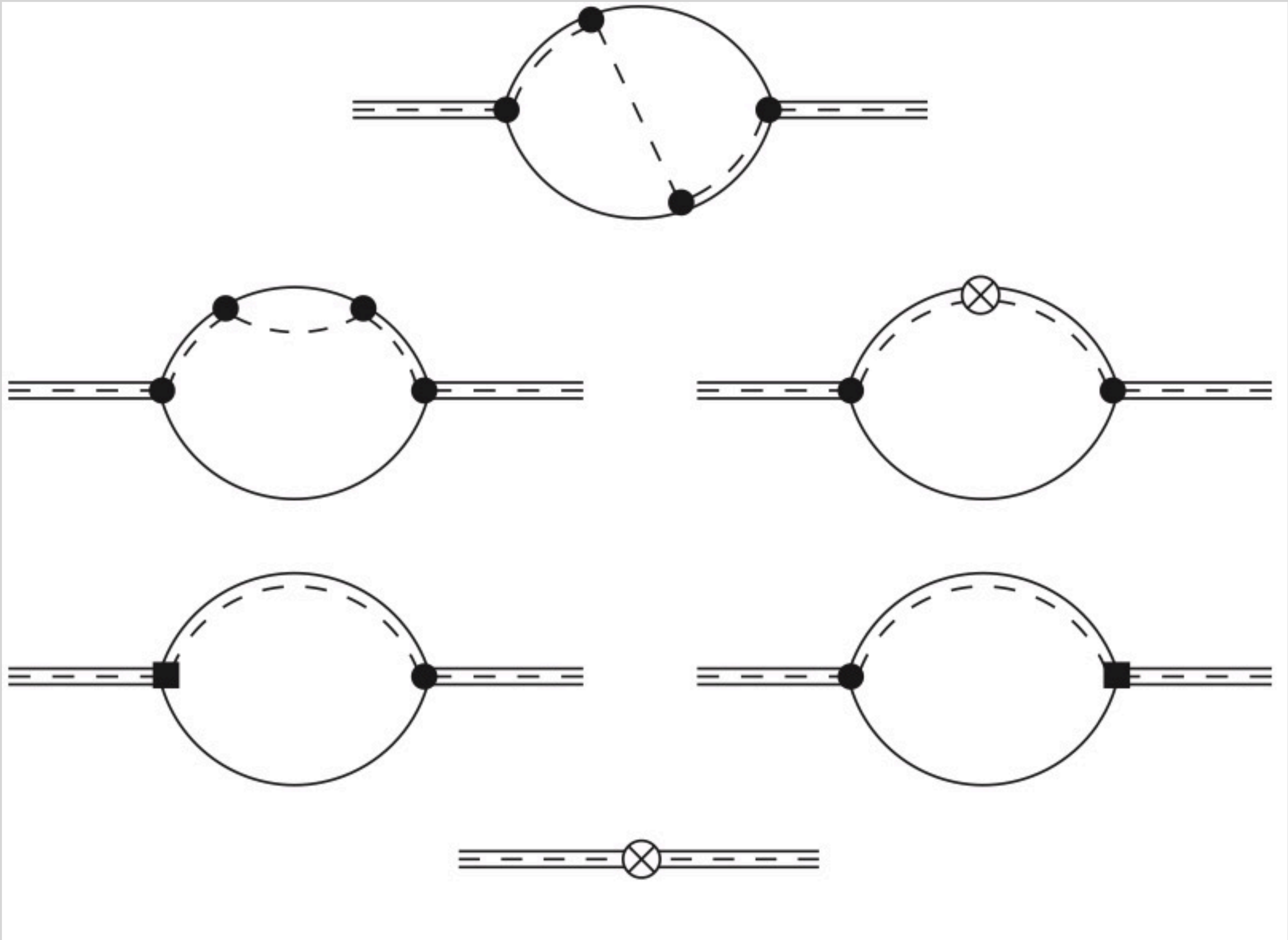
$$\mathcal{A}(E) = \frac{1}{-\gamma + \sqrt{-2\mu(E-E_*)}} \qquad (d=3)$$

summing the geometric series of pair self-energy diagrams

$$-i\frac{2\pi}{\mu}\frac{1}{\mathcal{A}(E_{\rm cm})^{-1} - \Pi_0(E_{\rm cm})}\,\delta^{ij}$$

$$\Pi_0(E) = g_\pi^2\,F(E) + (C_2/C_0)H(E) - (2\pi/(\mu C_0^2))\,[\delta C_0 + D_0(E-E_*)]$$

- **NLO (pair self-energy diagrams):**

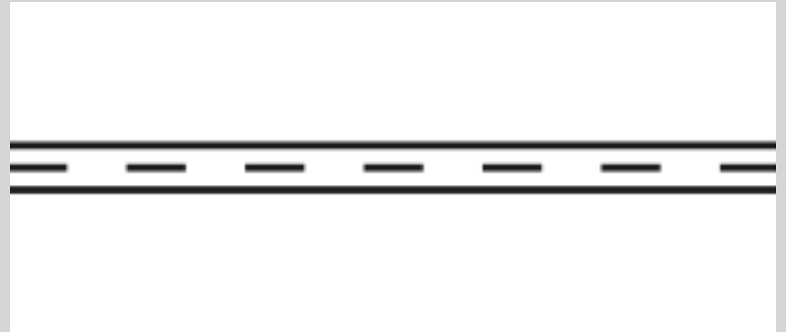


$$i[\mu/(2\pi)]\,\Pi_0(E)\,\delta^{ij}$$

NLO pair propagator

A. Complete pair propagator

- **LO:**



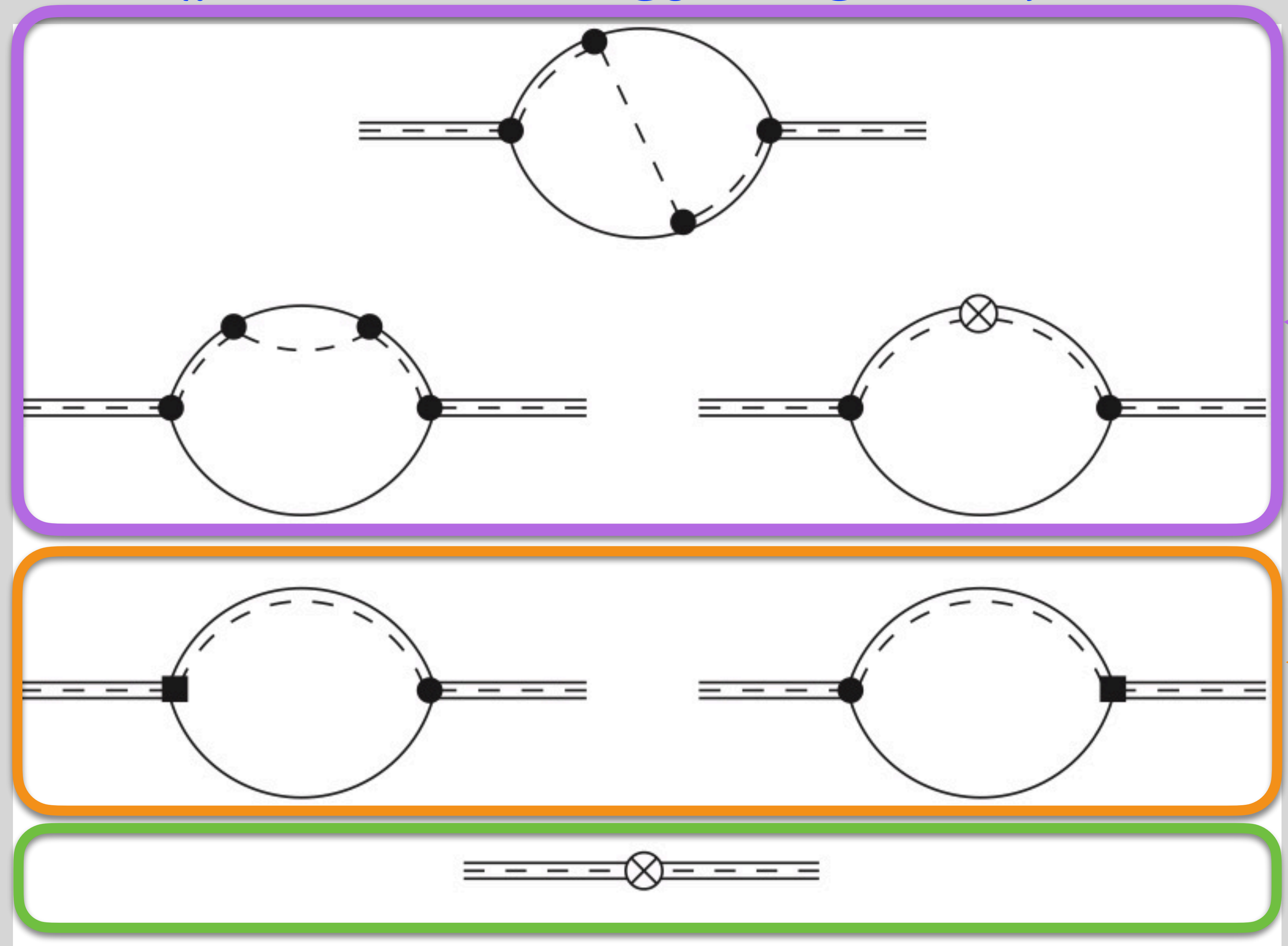
$$\boxed{-i\frac{2\pi}{\mu}\mathcal{A}(E_{\rm cm})\,\delta^{ij}}$$

$$\mathcal{A}(E) = \frac{1}{-\gamma+\sqrt{-2\mu(E-E_*)}} \qquad (d=3)$$

summing the geometric series of pair self-energy diagrams

$$\boxed{-i\frac{2\pi}{\mu}\frac{1}{\mathcal{A}(E_{\rm cm})^{-1}-\Pi_0(E_{\rm cm})}\,\delta^{ij}}$$

- **NLO (pair self-energy diagrams):**



$$i[\mu/(2\pi)]\,\Pi_0(E)\,\delta^{ij}$$

$$\Pi_0(E) = \boxed{g_\pi^2\,F(E)} + \boxed{(C_2/C_0)H(E)} \boxed{-(2\pi/(\mu C_0^2))\left[\delta C_0 + D_0(E-E_*)\right]}$$

$$H(E) = 8\pi\mu(E-E_*)\,J_1(E)$$

1-loop integral (D*D loop): $J_n(E) = \int_{\boldsymbol{p}} \frac{1}{[p^2-2\mu(E-E_*)]^n}$

in d dimensions: $= \frac{\Gamma(n-d/2)}{(4\pi)^{d/2}\Gamma(n)}\Lambda^{3-d}\left[2\mu(E_*-E)\right]^{d/2-n}$

NLO pair propagator

B. Renormalization

UV divergences can be cancelled order by order in the power counting by renormalization of the parameters of XEFT

$$-i\frac{2\pi}{\mu}\frac{1}{\mathcal{A}(E_{\rm cm})^{-1}-\Pi_0(E_{\rm cm})}\delta^{ij}$$

Renormalized self-energy Π(E): $Z_\phi\left[\mathcal{A}(E)^{-1}-\Pi_0(E)\right]=\mathcal{A}(E)^{-1}-\Pi(E)$

$$\mathcal{A}(E)=\frac{1}{-\gamma+\sqrt{-2\mu(E-E_*)}}\qquad (d=3)$$

$Z_\phi=1+\delta Z_\phi$ at NLO

$$\Pi(E)\;=\;g_\pi^2\,F(E)+(C_2/C_0)\,H(E)-(2\pi/(\mu C_0^2))\left[\delta C_0+D_0\,(E-E_*)\right]-\delta Z_\phi\,\mathcal{A}(E)^{-1}$$

choose δZ_ϕ to cancel all UV divergences

- ✦ pole in d-2 (linear UV divergence): all divergences can be cancelled if $Z_\phi=1+\left(\frac{2r\sqrt{1-r}\,g_\pi^2\,\Lambda}{(d-2)\pi}+\text{finite}\right)$
- ✦ pole in d-3 (logarithmic UV divergence)
  - Minimal subtraction (MS) renormalization scheme
  - Complex on-shell (COM) renormalization scheme
  - Complex threshold (CT) renormalization scheme

NLO pair propagator

C. Complex threshold (CT) renormalization scheme

(specifying the behavior of the renormalized pair propagator near the complex threshold E = E*)

expand in powers of $\kappa(E)$ [$\kappa_*=\kappa(0)$]:

$$\boxed{\mathcal{A}(E)^{-1} - \Pi(E) = -\gamma + \kappa(E) + \mathcal{O}\big(\kappa^2(E)\big)}$$

$$\kappa(E) = \sqrt{2\mu(E_* - E)}$$

$$H(E) = \kappa^3(E)$$

$$F(E) = f_0\,\kappa_*^2 + f_1\,\kappa_*\kappa(E) + f_2\,\kappa^2(E) + f_4\,\kappa^4(E)/\kappa_*^2 + \ldots.$$

$$\boxed{f_1 = i\frac{r^{5/2}}{3\pi\sqrt{1-r}}}$$

$$\boxed{\begin{aligned}\Pi(E) \;=\; & \left[f_0 g_\pi^2 \kappa_*^2 - 2\pi\delta C_0/(\mu C_0^2) + \delta Z_\phi \gamma\right] + \left[f_1 g_\pi^2 \kappa_* - \delta Z_\phi\right]\kappa(E) \\ & + \left[f_2 g_\pi^2 + \pi D_0/(\mu^2 C_0^2)\right]\kappa^2(E) + (C_2/C_0)\,\kappa^3(E) + g_\pi^2\,F_4(E)\end{aligned}}$$

$F_4(E)$: $O(\kappa^4)$

$$\mathcal{A}(E) = \frac{1}{-\gamma + \sqrt{-2\mu(E - E_*)}} \qquad (d=3) \longrightarrow \mathcal{A}(E)^{-1} = -\gamma + \kappa(E)$$

NLO pair propagator

C. Complex threshold (CT) renormalization scheme

(specifying the behavior of the renormalized pair propagator near the complex threshold E = E*)

expand in powers of $\kappa(E)$ [$\kappa_*=\kappa(0)$]:

$$\boxed{\mathcal{A}(E)^{-1} - \Pi(E) = -\gamma + \kappa(E) + \mathcal{O}\big(\kappa^2(E)\big)} \longrightarrow$$

CT scheme requires the total subtraction of the leading and κ(E) terms

$$H(E) = \kappa^3(E) \qquad F(E) = f_0\,\kappa_*^2 + f_1\,\kappa_*\kappa(E) + f_2\,\kappa^2(E) + f_4\,\kappa^4(E)/\kappa_*^2 + \ldots.$$

$$\begin{aligned}\Pi(E) \;=\; & \cancel{\left[f_0 g_\pi^2 \kappa_*^2 - 2\pi\delta C_0/(\mu C_0^2) + \delta Z_\phi \gamma\right]} + \cancel{\left[f_1 g_\pi^2 \kappa_* - \delta Z_\phi\right]\kappa(E)} \\ & + \left[f_2 g_\pi^2 + \pi D_0/(\mu^2 C_0^2)\right]\kappa^2(E) + (C_2/C_0)\,\kappa^3(E) + g_\pi^2\, F_4(E)\end{aligned}$$

d=3: poles in Re[f_0] and Re[f_2]

$$\boxed{\Pi(E) = \big(F_2 + i\,\mathrm{Im}[f_2]\big)\,g_\pi^2\,\kappa^2(E) + (C_2/C_0)\,\kappa^3(E) + g_\pi^2\,F_4(E)}$$

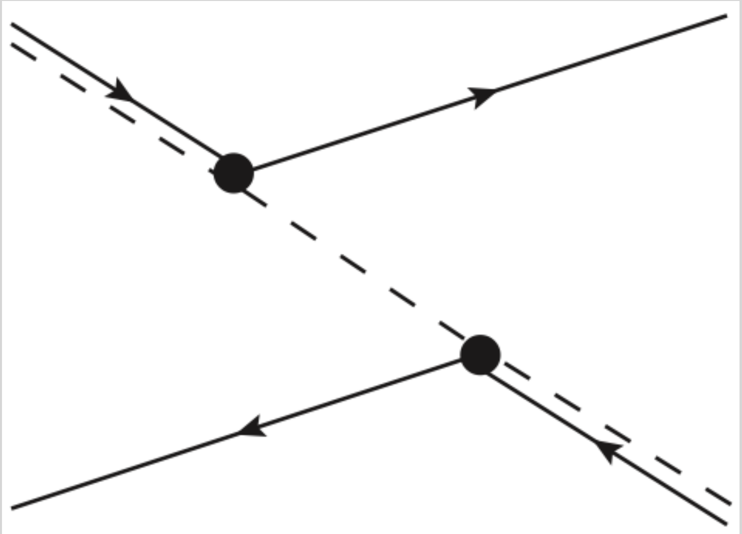
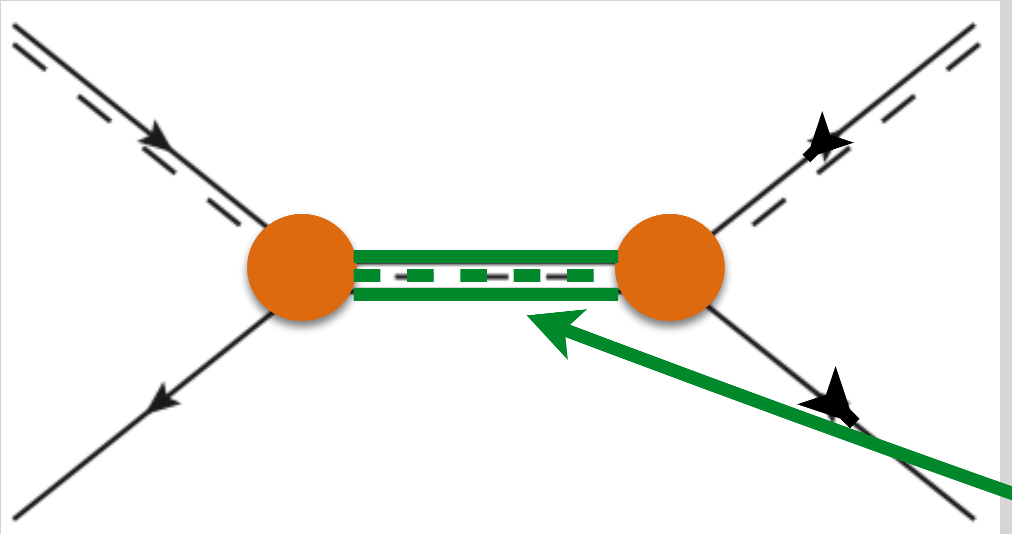
$D^{*0}\bar{D}^0$ scattering

A. NLO transition amplitude (D*0 off shell, D0 on shell, depends on 7 independent variables)

C = + S-wave transition tensor:

$$\boxed{\left\langle \mathcal{T}_+^{ij}(E,\boldsymbol{p},\boldsymbol{p}')\right\rangle_{\hat{\boldsymbol{p}},\hat{\boldsymbol{p}}'} = \frac{2\pi}{\mu}\mathcal{A}_{s+}(E,p,p')\,\delta^{ij}}$$

1/2 $D^{*0}\bar{D}^0 \to D^{*0}\bar{D}^0$, $D^{*0}\bar{D}^0 \to D^0\bar{D}^{*0}$, $D^0\bar{D}^{*0} \to D^{*0}\bar{D}^0$, and $D^0\bar{D}^{*0} \to D^0\bar{D}^{*0}$

 +  =

$$\mathcal{A}_{s+}(E,p,p') = \boxed{\mathcal{A}_\pi(E,p,p')} + W_{\pi 0}(E,p)\boxed{\frac{1}{\mathcal{A}(E)^{-1} - \Pi_0(E)}}W_{\pi 0}(E,p')$$

$$\boxed{\begin{aligned}\mathcal{A}_\pi(E,p,p') \;=\; -\frac{r g_\pi^2}{6\pi\sqrt{1-r}}\Bigg(&\frac{2(2-r)r\mu E - r(p^2+p'^2)}{4\sqrt{1-r}\,p\,p'} \\ &\times \log\frac{2r\mu E - p^2 - p'^2 + 2\sqrt{1-r}\,p\,p'}{2r\mu E - p^2 - p'^2 - 2\sqrt{1-r}\,p\,p'} - (2-r)\Bigg)\end{aligned}}$$

from

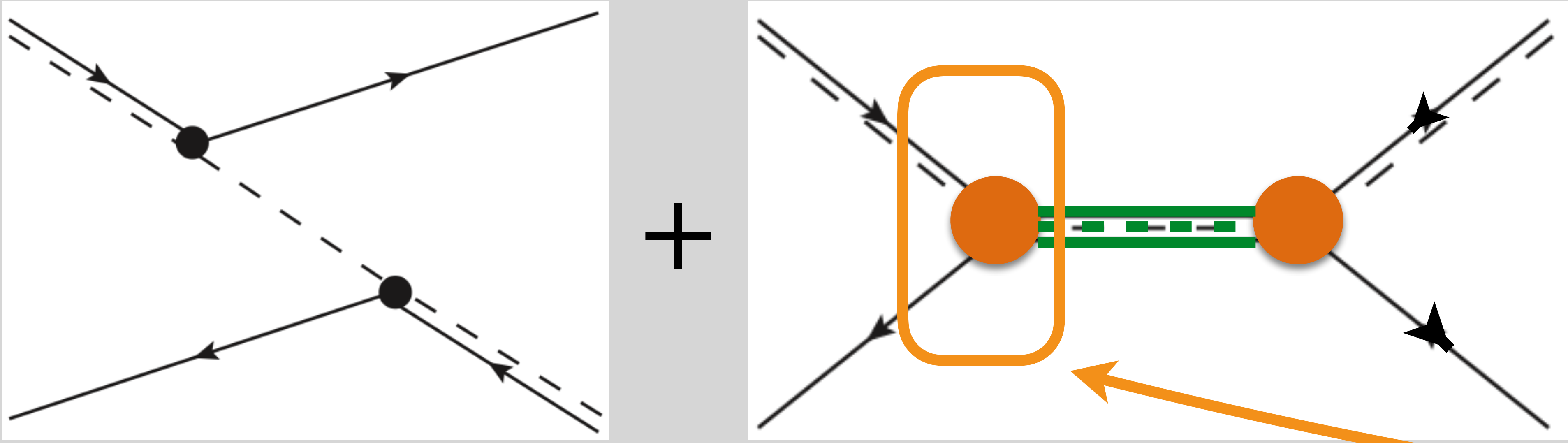
$$\boxed{-i\frac{2\pi}{\mu}\frac{1}{\mathcal{A}(E_{\rm cm})^{-1} - \Pi_0(E_{\rm cm})}\delta^{ij}}$$

$D^{*0}\bar{D}^0$ scattering

A. NLO transition amplitude (D*0 off shell, D0 on shell, depends on 7 independent variables)

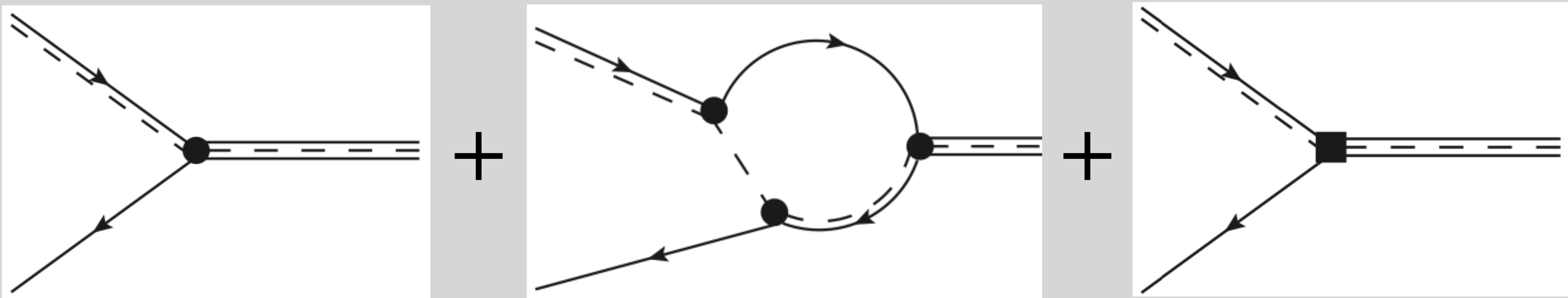
C = + S-wave transition tensor:

$$\boxed{\left\langle \mathcal{T}_+^{ij}(E,\boldsymbol{p},\boldsymbol{p}')\right\rangle_{\hat{\boldsymbol{p}},\hat{\boldsymbol{p}}'} = \frac{2\pi}{\mu}\mathcal{A}_{s+}(E,p,p')\,\delta^{ij}}$$



$$= \mathcal{A}_{s+}(E,p,p') = \mathcal{A}_\pi(E,p,p') + W_{\pi 0}(E,p)\,\frac{1}{\mathcal{A}(E)^{-1}-\Pi_0(E)}\,W_{\pi 0}(E,p')$$

$$\boxed{W_{\pi 0}(E,p) = 1 + g_\pi^2\,G(E,p) + (C_2/C_0)\,p^2/2}$$



$$G(E,p) = \frac{2r}{d\sqrt{1-r}}\Big(-r\,L_0(E,p) + r\left[2\mu E_* + 2(1-r)\mu E - p^2\right]L_1(E,p) + (2-r)J_1(E)\Big)$$

$D^{*0}\bar{D}^0$ scattering

B. Renormalization

transition amplitude:

$$\mathcal{A}_{s+}(E,p,p') = \mathcal{A}_\pi(E,p,p') + \boxed{W_{\pi 0}(E,p)\,\frac{1}{\mathcal{A}(E)^{-1} - \Pi_0(E)}\,W_{\pi 0}(E,p')}$$

$$W_{\pi 0}(E,p) = 1 + g_\pi^2\, G(E,p) + (C_2/C_0)\, p^2/2$$

$$\boxed{\mathcal{A}_{s+}(E,p,p') \;=\; \mathcal{A}_\pi(E,p,p') + \boxed{W_\pi(E,p)\,\frac{1}{\mathcal{A}(E)^{-1} - \Pi(E)}\,W_\pi(E,p')}}$$

$$\boxed{W_\pi(E,p) = 1 + g_\pi^2\, G(E,p) + (R_0/\gamma)\, p^2/4 + \delta Z_\phi/2}$$

✦ **Complex on-shell renormalization scheme (Braaten, 2015)** [same pole value and same residue at p=p'=0]

$$\frac{W_\pi^2(E,0)}{-\gamma_X + \kappa(E) - \Pi_{\rm COS}(E)} \longrightarrow \frac{-\gamma_X/\mu}{E - E_{\rm pole}} \qquad \text{as } E \to E_{\rm pole}$$

$$\begin{aligned} \Pi_{\rm COS}(E_{\rm pole}) &= 0, \\ \Pi'_{\rm COS}(E_{\rm pole}) &= \frac{\mu}{\gamma_X}\left[W_\pi^2(E_{\rm pole},0) - 1\right] \end{aligned}$$

$$W_\pi^2(E,0) = 1 + 2g_\pi^2\, G(E,0) + \delta Z_\phi$$

$$E_{\rm pole} = E_* - \gamma_X^2/(2\mu)$$

$$\boxed{\begin{aligned} \Pi_{\rm COS}(E) \;=\; & f_1\, g_\pi^2 \kappa_* \left[\kappa(E) - \frac{\gamma_X}{2} - \frac{1}{2\gamma_X}\kappa^2(E)\right] - \delta Z_\phi\,[\kappa(E) - \gamma_X] \\ & - \frac{g_\pi^2\, G(E_{\rm pole},0)}{\gamma_X}\left[\kappa^2(E) - \gamma_X^2\right] + \frac{R_0}{2\gamma_X}\left[\kappa^3(E) + \frac{\gamma_X^3}{2} - \frac{3\gamma_X}{2}\kappa^2(E)\right] + g_\pi^2\, F_{4,\rm sub}(E) \end{aligned}}$$

more complicated than that from CT scheme

$D^{*0}\bar{D}^0$ scattering

C. NLO S-wave scattering amplitude in C=+ channel (initial and final states are on-shell)

conservation of energy → p=p'

total energy: $E_p = E_* + p^2/(2\mu)$

T-matrix element: $\mathcal{T}_{s+}(p) = (2\pi/\mu)\left(\mathcal{A}_\pi(E_p,p,p) + \dfrac{W_\pi^2(E_p,p)}{(-\gamma - i\,p) - \Pi(E_p)}\right)$

in Complex threshold (CT) scheme

$$\mathcal{A}_\pi(E_p,p,p) = \frac{r g_\pi^2}{6\pi\sqrt{1-r}}\left(\frac{r\left[(2-r)\kappa_*^2 - r\,p^2\right]}{4\sqrt{1-r}\,p^2}\log\frac{r\kappa_*^2 - \left(1+\sqrt{1-r}\right)^2 p^2}{r\kappa_*^2 - \left(1-\sqrt{1-r}\right)^2 p^2} + (2-r)\right)$$

$$\delta Z_\phi = f_1 g_\pi^2 \kappa_*$$

$$\boxed{\Pi(E_p) = \cancel{(\delta Z_\phi - f_1 g_\pi^2 \kappa_*)\, i\, p} - \left(F_2 + i\,\mathrm{Im}[f_2]\right) g_\pi^2\, p^2 + i\frac{R_0}{2\gamma}p^3 + g_\pi^2\, F_4(E_p)}$$

compare with the effective range expansion: $\dfrac{2\pi/\mu}{\mathcal{T}(p)} = -\dfrac{1}{a} - i\,p + \dfrac{1}{2}r_e\,p^2 + \mathcal{O}(p^4)$

$D^{*0}\bar{D}^0$ scattering

D. Breakdown of effective range expansion

LO: $\boxed{\mathcal{T}_{s+,\mathrm{LO}}(p) = \frac{2\pi/\mu}{-\gamma - i\,p}}$

$$\frac{2\pi/\mu}{\mathcal{T}(p)} = -\frac{1}{a} - i\,p + \frac{1}{2} r_e\, p^2 + \mathcal{O}(p^4)$$

NLO: $\boxed{\mathcal{T}_{s+}(p) = (2\pi/\mu)\left(\mathcal{A}_\pi(E_p, p, p) + \frac{W_\pi^2(E_p, p)}{(-\gamma - i\,p) - \Pi(E_p)}\right)}$

inverse scattering length: $1/a_{s+} = \left(1 - i\,\frac{(2-r)r^{3/2}}{3\pi\sqrt{1-r}} g_\pi^2 \kappa_*\right)\gamma$

$$\frac{2\pi/\mu}{\mathcal{T}_{s+}(p)} \approx \exp\left(-i\,\frac{(2-r)r^{3/2}}{3\pi\sqrt{1-r}} g_\pi^2 \kappa_*\right)\left[-\gamma_{s+} - i\,p + \frac{1}{2} r_{s+}\, p^2 + \mathcal{O}(p^3)\right]$$

phase shift from the successive exchange of pions

breaks effective range expansion

Jansen, Hammer, Jia, PRD (2014)

long range effects (successive exchange of pions that are almost on their energy shell)**?**

$g_\pi^2 \to 0$

$$\frac{2\pi/\mu}{\mathcal{T}_{s+}(p)} = -\gamma - i\,p + \frac{R_0}{2} p^2 - \frac{3R_0^2}{16\gamma} p^4 + i\,\frac{R_0^2}{16\gamma^2} p^5 + \mathcal{O}(p^6)$$

Summary

XEFT: low-energy effective field theory for charm mesons and pions that provides a systematically improvable description of the X(3872) resonance

Galilean-invariant XEFT

- **Galilean invariance:**

  constraints the ultraviolet divergences and it significantly simplifies analytic results

- **Systematic treatment of the width of D*, which requires the LO binding momentum γ to be complex**
- **Introduce a pair field that annihilates a pair of charm mesons in the resonant channel:**

  simplifies calculations at NLO by making some cancellations of UV divergences between diagrams automatic

- **Introduce new renormalization schemes**
- **may provide solution to convergence problem of XEFT? (ongoing)**

Backup

$$\mathcal{L}_D = D^\dagger \left[i\partial_t + \nabla^2/(2M) \right] D,$$

$$\mathcal{L}_\pi = \pi^\dagger \left[i\partial_t + \nabla^2/(2m) \right] \pi.$$

$$\mathcal{L}_{D^*} = \boldsymbol{D}^\dagger \cdot \left[i\partial_t + \nabla^2/(2(M+m)) - E_* \right] \boldsymbol{D}$$

$$L_0(E,p) = i\,\frac{r^{1/2}}{4\pi}\sqrt{2\mu E - p^2},$$

$$L_1(E,p) = i\,\frac{1}{8\pi\sqrt{1-r}\,p}\log\frac{\sqrt{r}\,\sqrt{2\mu E - p^2} + i\,\kappa(E) + \sqrt{1-r}\,p}{\sqrt{r}\,\sqrt{2\mu E - p^2} + i\,\kappa(E) - \sqrt{1-r}\,p}$$

effective range expansion: a scattering amplitude can be expanded in powers of the relative momentum with only short-range interactions

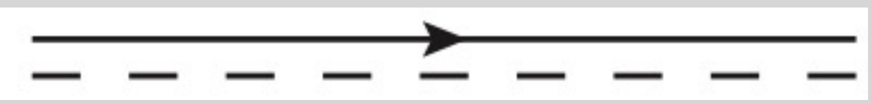
$$\frac{2\pi/\mu}{\mathcal{T}(p)} = -\frac{1}{a} - i\,p + \frac{1}{2} r_e\,p^2 + \mathcal{O}(p^4)$$

renormalization → $\boxed{C_0 \sim Q^{-1}}$

Backup

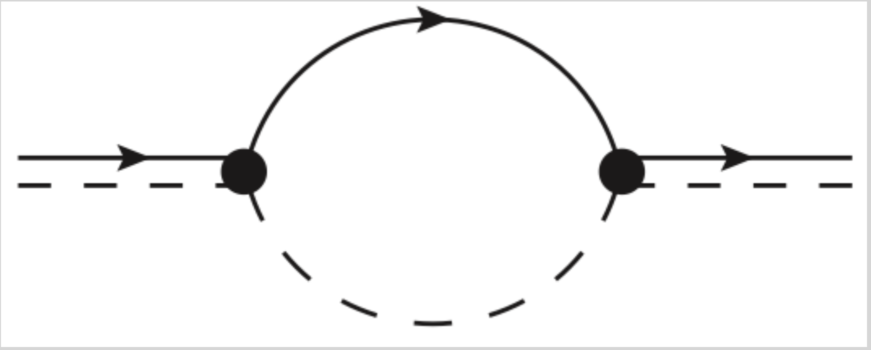
D^{*0} propagator

- **LO:**

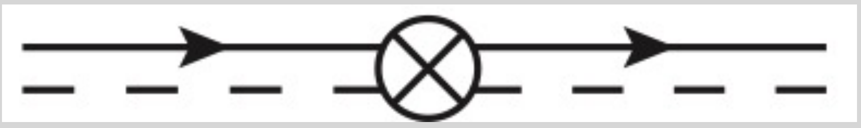


$$\boxed{\frac{i\,\delta^{ij}}{E - p^2/(2(M+m)) - E_*}}$$

- **NLO:** [summing a geometric series of 1-loop digram]



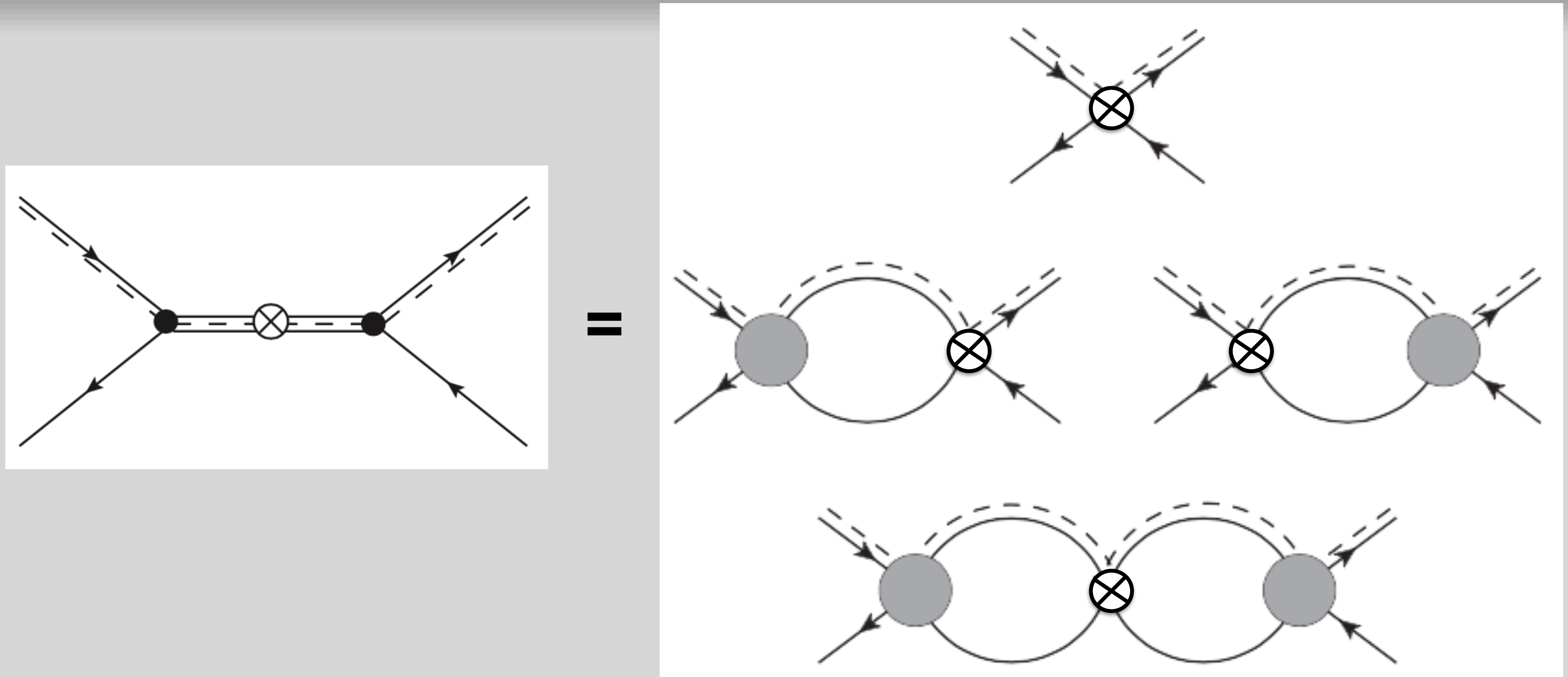
- **counter term**

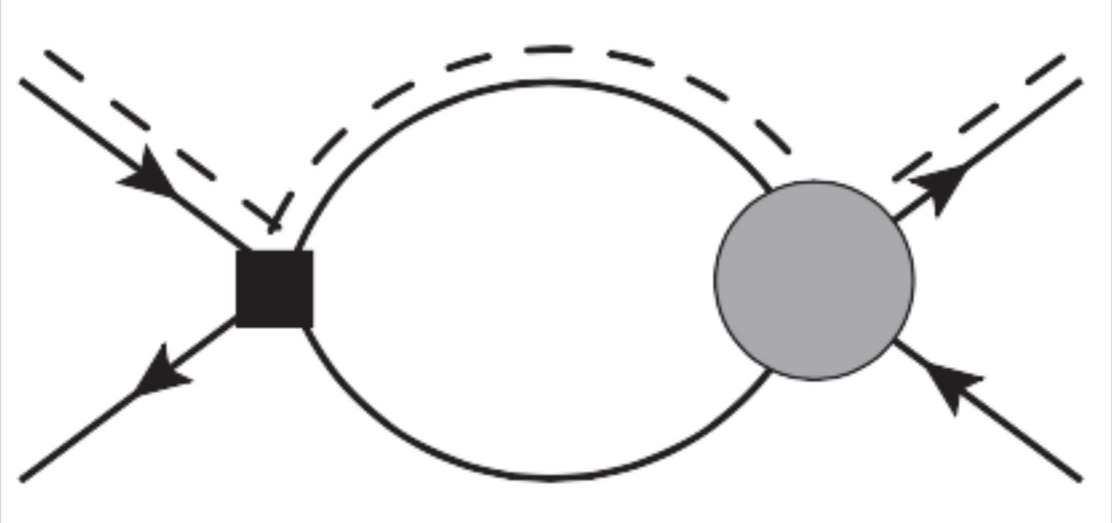


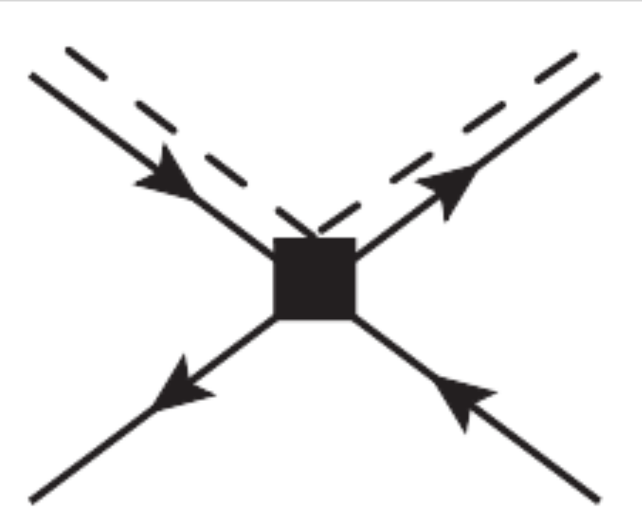
- **Complex on-shell (COS) renormalization scheme, the UV divergences are removed by subtractions at the complex pole energy $E_X - i\Gamma_X/2$.**

- **Include the width of D^{*0} → LO binding momentum γ is complex.**

Backup



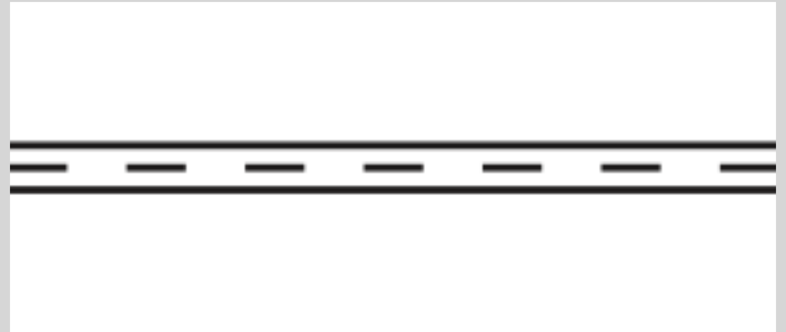




Backup —— NLO pair propagator

A. Complete pair propagator

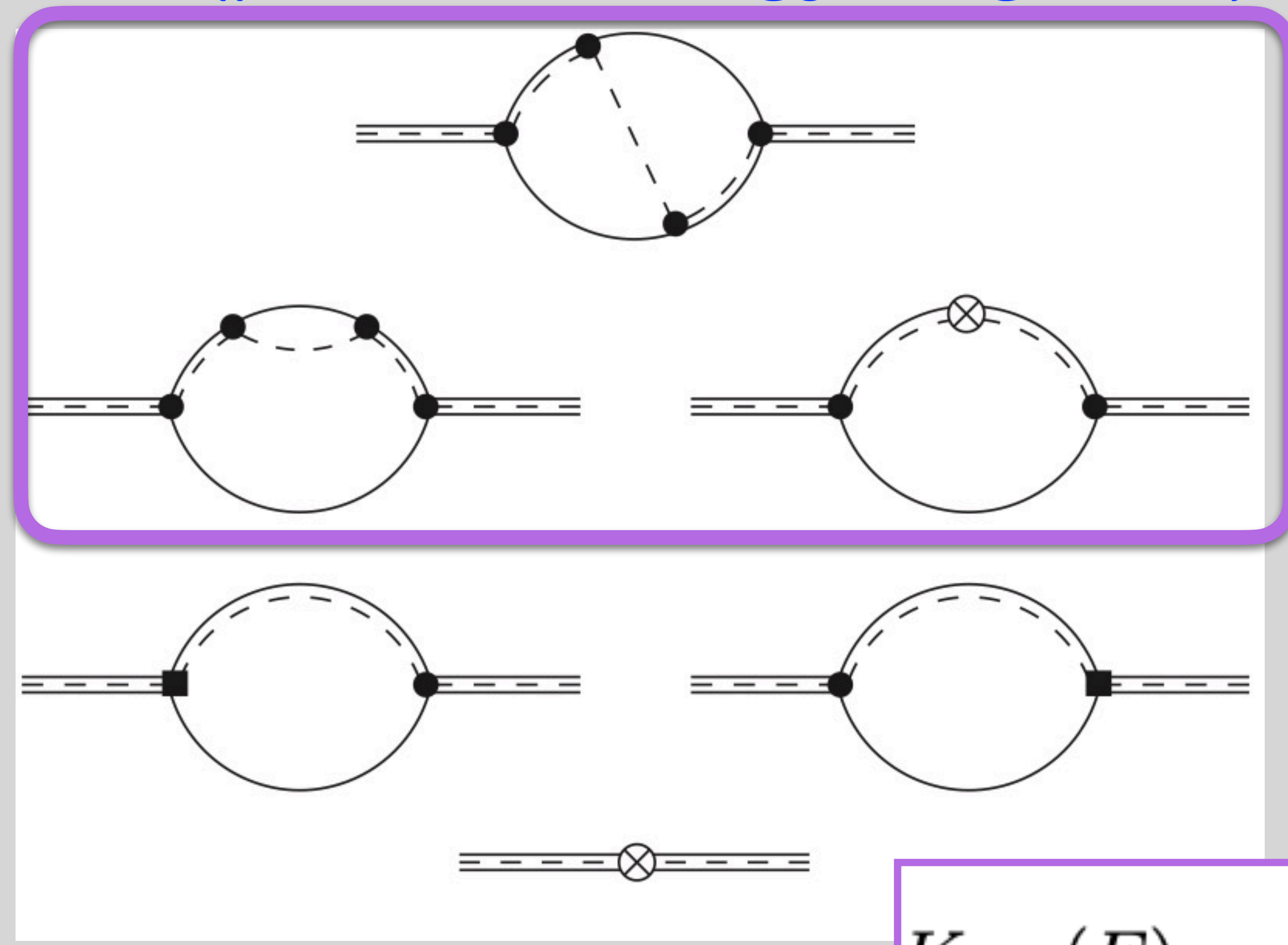
- **LO:**



$$\boxed{-i\frac{2\pi}{\mu}\mathcal{A}(E_{\rm cm})\,\delta^{ij}}$$

$$\mathcal{A}(E) = \frac{1}{-\gamma+\sqrt{-2\mu(E-E_*)}} \qquad (d=3)$$

- **NLO (pair self-energy diagrams):**



summing the geometric series of pair self-energy diagrams

$$-i\frac{2\pi}{\mu}\frac{1}{\mathcal{A}(E_{\rm cm})^{-1}-\Pi_0(E_{\rm cm})}\delta^{ij}$$

$$\Pi_0(E) = g_\pi^2\,F(E) + (C_2/C_0)H(E) - (2\pi/(\mu C_0^2))\left[\delta C_0 + D_0(E-E_*)\right]$$

$$F(E) = -\frac{8\pi r}{d}\left(\frac{1}{\sqrt{1-r}}\left[2K_{110}(E) - 2(2\mu E_* - r\mu E)K_{111}(E) - (2-r)J_1(E)^2\right]\right.$$
$$\left. + 2K_{110}(E) - 4\mu E_*\left[K_{120}(E) - rI_1(E_*)J_2(E)\right] - drI_1(E_*)J_1(E)\right),$$

2-loop integral:

$$K_{lmn}(E) = \int_{\boldsymbol{p}}\int_{\boldsymbol{q}} \frac{1}{[p^2-2\mu(E-E_*)]^m[q^2-2\mu(E-E_*)]^n} \times\frac{(2\mu)^{-l}}{[(\boldsymbol{p}+\boldsymbol{q})^2/(2m)+(p^2+q^2)/(2M)-E-i\epsilon]^l}$$

1-loop integral (Dπ loop):

$$I_n(E) = \int_{\boldsymbol{p}}\frac{1}{[p^2-2\mu_\pi E-i\epsilon]^n}$$