强子物理在线论坛(Hadron Physics Online Forum),
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Hadron interaction matters in quest for axion/axion-like particle

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Introduction

Strong CP problem

$$\mathcal{L}_{\rm QCD} = \sum_q \bar{q}(i\not{D} - m_q e^{i\theta_q})q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \theta\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$

$$q \to e^{i\gamma_5\alpha}q \xrightarrow{\mathcal{D}q\mathcal{D}\bar{q} \to \left(e^{-i\alpha\frac{g_s^2}{16\pi^2}\int d^4x\, G\tilde{G}}\right)\mathcal{D}q\mathcal{D}\bar{q}} \theta_q \to \theta_q + 2\alpha\,, \qquad \theta \to \theta - 2\alpha$$

implying the invariant quantity: $\bar{\theta} = \theta + \theta_q$

$$\Rightarrow \quad \mathcal{L}_{\rm QCD} = \sum_q \bar{q}(i\not{D} - m_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \boxed{\bar{\theta}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

- $G_{\mu\nu}\tilde{G}^{\mu\nu} \sim \partial_\mu K^\mu$: **this total derivative is relevant, due to the nonperturbative QCD vacumm**
- **Naive guess from CKM:** $\theta_{\rm CPV} \sim O(1)$
- **Experimental constraints from neutron EDM:** $\bar{\theta} \leq 10^{-10}$

➢ **Strong CP problem: why $\bar{\theta}$ unnaturally tiny ?**

Peccei-Quinn mechanism to address strong CP problem

[Peccei,Quinn,PRL'77]

- **Promote constant $\bar{\theta}$ as a dynamical spin-0 field $a(x)$**

- **Impose a new global $U(1)_{PQ}$ symmetry**

$$\mathcal{L}_a = \frac{1}{2}(\partial_\mu a)^2 + \frac{a}{f_a}\frac{g_s^2}{32\pi^2}G\tilde{G}$$

$$a(x) \rightarrow a(x) + \kappa f_a \quad \Longrightarrow \quad S \rightarrow S + \frac{g_s^2\kappa}{32\pi^2}\int d^4x\, G\tilde{G}$$

(cancels $\bar{\theta}$ term)

- **Vafa-Witten theorem: VEV of $\langle a \rangle = 0$ in the vector-like theory, such as QCD**

Peccei-Quinn-Weinberg-Wilczek Axion

[Weinberg,PRL'78] [Wilzeck,PRL'78]

- **Weinberg and Wilczek: PQ mechanism indicates a pseudo-Nambu-Goldstone boson**
- **This pNGB strips off the unwanted strong CP phase !**

Wilczek names it as: Axion

中文：轴子 (轴矢流耦合)

$$\frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma_5 Q_a q$$

- **Original PQWW axion: $f_a \sim v_{EW} \approx 246$ GeV (visible axion)**

  quickly ruled out by experiments: $K \to \pi\, a$, $J/\psi \to \gamma\, a$, $\Upsilon \to \gamma\, a$, ...

  astrophysical constraints: Supernovae, Red giant, ... ($NN \to NNa$)

Generic effective axion Lagrangian for light-flavor quarks

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}$$
$$+ \frac{1}{4}g_{a\gamma}^0 aF\tilde{F} + \frac{\partial_\mu a}{2f_a}\bar{q}c_q^0\gamma^\mu\gamma_5 q$$

Diverse viable axion models

- **PQWW**: Y_{PQ} (SM fermion) $\neq 0$, $f_a \sim v_{EW}$ (ruled out)
- **KSVZ**: Y_{PQ} (SM fermion) $=0$, singlet Higgs and extra BSM fermions,

  $f_a >> v_{EW}$ (invisible axion)

  model-dependent terms vanish: $g_{a\gamma}^0=0$, $c_q^0=0$
- **DFSZ**: Y_{PQ} (SM fermion) $\neq 0$, extra singlet and doublet Higgs, $f_a >> v_{EW}$ (invisible)

  model-dependent terms retain: $g_{a\gamma}^0 \neq 0$, $c_q^0 \neq 0$
- **QCD axion / ALP (axion-like particle)**: bare axion mass term $m_{a,0}=0$ / $m_{a,0}\neq 0$

  QCD axion case: $m_a^2 \cong \frac{m_\pi^2 f_\pi^2}{4f_a^2}$ ($f_a >> f_\pi$, axion: a very light BSM particle)

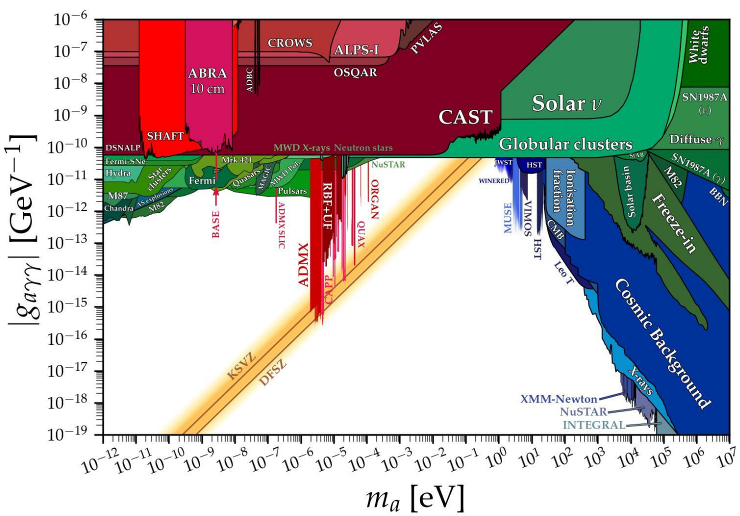
  ALP case: m_a and f_a are independent

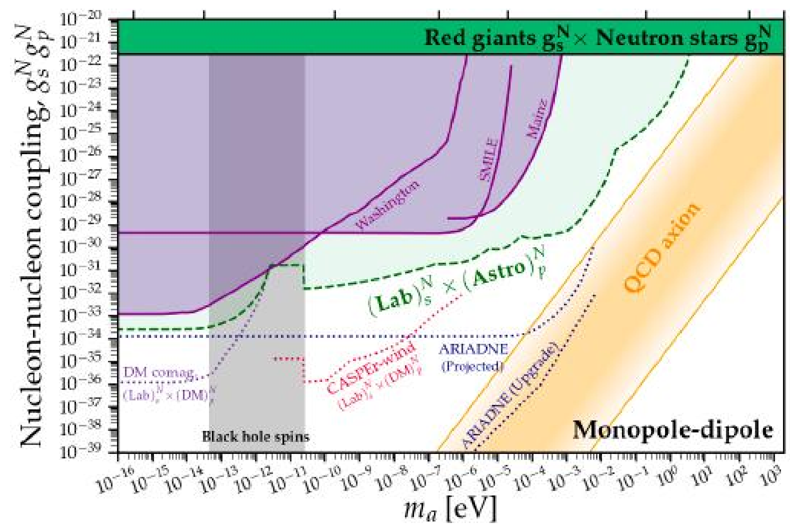
- **Various constraints from rather different experiments**

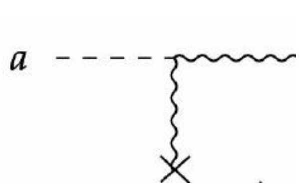
[Di Luzio, et al., Phy.Rep'20] [Sikivie, RMP'21] [Irastorza, Redondo,PPNP'18] ... ...

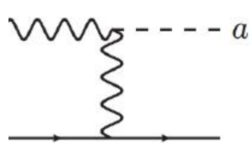
Cosmology, Astronomy, Colliders, Quantum precision measurements , Cavity Haloscope, ...

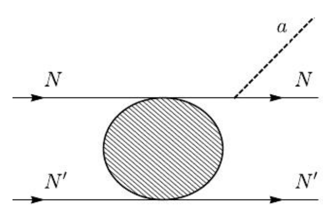
[O'Hare,Github, https://cajohare.github.io/AxionLimits/]

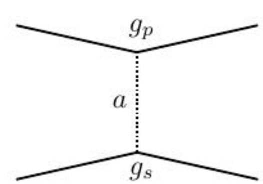












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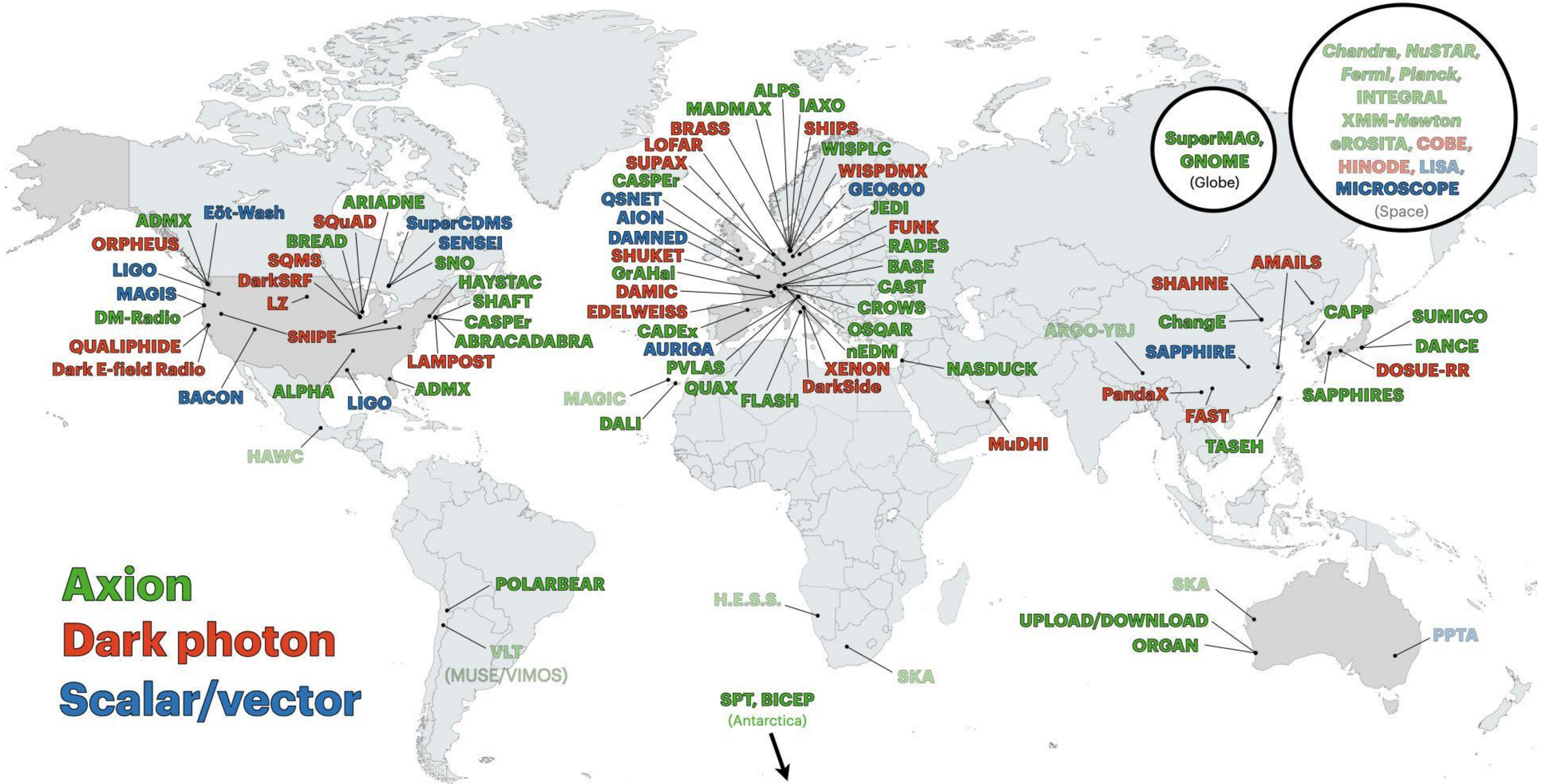
Experimental map in the quest of axion

[O'Hare,Github, https://cajohare.github.io/AxionLimits/]



- **Apart from dark-matter related search, other methods are also widely proposed:**

helioscopes (solar axion), thermal relics from cosmology, Supernovae, Stellar cooling, light shining through walls, fifth force, rare meson decays, missing energies@collider

… …

Axion chiral perturbation theory

Axion chiral perturbation theory (AχPT)

- We first focus on the QCD-like axion: $m_{a,0}\,(\neq 0) \ll f_a$ with model-independent $aG\tilde{G}$ interaction, i.e., the MODEL INDEPENDENT QCD axion interactions.
- Axion-hadron interactions are relevant at low energies.

$$\mathcal{L}_{\rm QCD}^{\rm axion} = \bar{q}(i\not{D} - M_q)q - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2}m_{a,0}^2 a^2 + \boxed{\frac{a}{f_a}\frac{\alpha_s}{8\pi}G_{\mu\nu}\tilde{G}^{\mu\nu}}$$

Two ways to proceed:

(1) Remove the $aG\tilde{G}$ term via the quark axial transformation

$$q \rightarrow e^{i\frac{a}{2f_a}\gamma_5 Q_a} q$$

$\mathrm{Tr}(Q_a) = 1$

$$-\frac{a\alpha_s}{8\pi f_a}G\tilde{G} - \frac{\partial_\mu a}{2f_a}\bar{q}\gamma^\mu\gamma_5 Q_a q \qquad M_q \rightarrow M_q(a) = e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a}$$

Mapping to χPT

$$\mathcal{L}_2 = \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + \chi_a U^\dagger + U\chi_a^\dagger \rangle + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\rm LO}$$

$$\chi_a = 2B_0 e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \qquad J_A^\mu|_{\rm LO} = -i\frac{F^2}{2}\langle Q_a(\partial^\mu U U^\dagger + U^\dagger \partial^\mu U)\rangle$$

- $Q_a = M_q^{-1}/\mathrm{Tr}(M_q^{-1})$ [Georgi,Kaplan,Randall, PLB'86]
- $J_A^\mu \partial_\mu a$ [Bauer, et al., PRL'21]

(2) Explicitly keep the $a G\widetilde{G}$ term and match it to χPT

Reminiscent:

QCD $U(1)_A$ anomaly that is caused by topological charge density $\omega(x) = \alpha_s G_{\mu\nu}\tilde{G}^{\mu\nu}/(8\pi)$ **is responsible for the massive singlet η_0.**

Axion could be similarly included as the η_0 via the U(3) χPT:

$$\mathcal{L}^{\text{LO}} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle\chi_+\rangle + \frac{F^2}{12}M_0^2 X^2$$

$$\mathcal{L}^{\text{NLO}} = L_5\langle u^\mu u_\mu \chi_+\rangle + \frac{L_8}{2}\langle\chi_+\chi_+ + \chi_-\chi_-\rangle - \frac{F^2\Lambda_1}{12}D^\mu X D_\mu X - \frac{F^2\Lambda_2}{12}X\langle\chi_-\rangle\,,$$

$$U = u^2 = e^{i\frac{\sqrt{2}\Phi}{F}}\,, \qquad \chi = 2B(s+ip)\,, \qquad \chi_\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u$$

$$u_\mu = iu^\dagger D_\mu U u^\dagger\,, \quad D_\mu U = \partial_\mu U - i(v_\mu + a_\mu)U + iU(v_\mu - a_\mu)$$

$$X = \log(\det U) - i\,\frac{a}{f_a} \qquad \Phi = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & \pi^+ & K^+ \\ \pi^- & \frac{-1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 & K^0 \\ K^- & \overline{K}^0 & \frac{-2}{\sqrt{6}}\eta_8 + \frac{1}{\sqrt{3}}\eta_0 \end{pmatrix}$$

- **Q_a is not needed in U(3) χPT.**
- **$M_0^2 = 6\tau/F^2$, with τ the topological susceptibility. Note that $M_0^2 \sim O(1/N_c)$.**
- **δ expansion scheme: $\delta \sim O(p^2) \sim O(m_q) \sim O(1/N_c)$.**
- **Axion interactions enter via the axion-meson mixing terms at LO.**

π-η-η'-a mixing in U(3) AχPT

[Gao,ZHG,Oller,Zhou,JHEP'23] [Gao,Hao,ZHG,et al.,EPJC'25]

LO

(mass mixing only)

$$\begin{pmatrix} \overline{\pi}^0 \\ \overline{\eta} \\ \overline{\eta}' \\ \overline{a} \end{pmatrix} = \begin{pmatrix} 1+v_{11} & -v_{12} & -v_{13} & -v_{14} \\ v_{12} & 1+v_{22} & -v_{23} & -v_{24} \\ v_{13} & v_{23} & 1+v_{33} & -v_{34} \\ v_{41} & v_{42} & v_{43} & 1+v_{44} \end{pmatrix} \begin{pmatrix} \pi^0 \\ \mathring{\eta} \\ \mathring{\eta}' \\ a \end{pmatrix}$$

$$v_{12} = -\frac{\epsilon}{\sqrt{3}} \frac{c_\theta - \sqrt{2} s_\theta}{m_\pi^2 - m_{\mathring{\eta}}^2}, \qquad v_{13} = -\frac{\epsilon}{\sqrt{3}} \frac{\sqrt{2} c_\theta + s_\theta}{m_\pi^2 - m_{\mathring{\eta}'}^2}, \qquad v_{23} = \frac{\sqrt{2} s_\theta^2 + c_\theta s_\theta - \sqrt{2} c_\theta^2}{3(m_{\mathring{\eta}'}^2 - m_{\mathring{\eta}}^2)} \epsilon, \qquad v_{41} = -\frac{M_0^2 \epsilon}{6(m_a^2 - m_\pi^2)} \frac{F}{f_a} \left[-\frac{(\sqrt{2} c_\theta - 2 s_\theta) s_\theta}{m_a^2 - m_{\mathring{\eta}}^2} + \frac{c_\theta (2 c_\theta + \sqrt{2} s_\theta)}{m_a^2 - m_{\mathring{\eta}'}^2} \right]$$

$$v_{42} = \frac{M_0^2 s_\theta}{\sqrt{6}(m_a^2 - m_{\mathring{\eta}}^2)} \frac{F}{f_a} - \frac{M_0^2 \epsilon}{3\sqrt{6}(m_a^2 - m_{\mathring{\eta}}^2)} \frac{F}{f_a} \left[\frac{c_\theta(-\sqrt{2} c_\theta^2 + c_\theta s_\theta + \sqrt{2} s_\theta^2)}{m_a^2 - m_{\mathring{\eta}'}^2} - \frac{s_\theta (2 c_\theta^2 + 2\sqrt{2} c_\theta s_\theta + s_\theta^2)}{m_a^2 - m_{\mathring{\eta}}^2} \right]$$

$$v_{43} = -\frac{M_0^2 c_\theta}{\sqrt{6}(m_a^2 - m_{\mathring{\eta}'}^2)} \frac{F}{f_a} - \frac{M_0^2 \epsilon}{3\sqrt{6}(m_a^2 - m_{\mathring{\eta}'}^2)} \frac{F}{f_a} \left[\frac{c_\theta (c_\theta^2 - 2\sqrt{2} c_\theta s_\theta + 2 s_\theta^2)}{m_a^2 - m_{\mathring{\eta}'}^2} - \frac{s_\theta(-\sqrt{2} c_\theta^2 + c_\theta s_\theta + \sqrt{2} s_\theta^2)}{m_a^2 - m_{\mathring{\eta}}^2} \right]$$

... ...

with

$$m_{\mathring{\eta}}^2 = \frac{M_0^2}{2} + m_K^2 - \frac{\sqrt{M_0^4 - \frac{4 M_0^2 \Delta^2}{3} + 4\Delta^4}}{2}, \qquad m_{\mathring{\eta}'}^2 = \frac{M_0^2}{2} + m_K^2 + \frac{\sqrt{M_0^4 - \frac{4 M_0^2 \Delta^2}{3} + 4\Delta^4}}{2}, \qquad \sin\theta = -\left(\sqrt{1 + \frac{\left(3M_0^2 - 2\Delta^2 + \sqrt{9 M_0^4 - 12 M_0^2 \Delta^2 + 36 \Delta^4}\right)^2}{32 \Delta^4}} \right)^{-1}$$

Physical masses after diagnolization

$$m_\eta^2 = m_{\mathring{\eta}}^2 + \frac{\epsilon}{3} (\sqrt{2} c_\theta + s_\theta)^2 + O(\epsilon^2) \qquad m_{\eta'}^2 = m_{\mathring{\eta}'}^2 + \frac{\epsilon}{3} (c_\theta - \sqrt{2} s_\theta)^2 + O(\epsilon^2)$$

$$\begin{aligned} m_a^2 = m_{a,0}^2 &+ \frac{M_0^2 F^2}{6 f_a^2} \left[1 + \frac{c_\theta^2 M_0^2}{m_{a,0}^2 - m_{\mathring{\eta}'}^2} + \frac{s_\theta^2 M_0^2}{m_{a,0}^2 - m_{\mathring{\eta}}^2} \right] \\ &+ \frac{M_0^4 F^2 \epsilon}{9 f_a^2} \left[\frac{s_\theta^2 (\sqrt{2} c_\theta + s_\theta)^2}{2(m_{a,0}^2 - m_{\mathring{\eta}}^2)^2} + \frac{c_\theta^2 (c_\theta - \sqrt{2} s_\theta)^2}{2(m_{a,0}^2 - m_{\mathring{\eta}'}^2)^2} \right. \\ &\left. + \frac{c_\theta s_\theta (\sqrt{2} c_\theta^2 - c_\theta s_\theta - \sqrt{2} s_\theta^2)}{(m_{a,0}^2 - m_{\mathring{\eta}}^2)(m_{a,0}^2 - m_{\mathring{\eta}'}^2)} \right] + O(\epsilon^2), \end{aligned}$$

$$\boldsymbol{m_a^2 = \frac{m_\pi^2 F^2}{4 f_a^2}}$$

[Weinberg,PRL'78]

(keep LO terms in m_π/m_K & m_π/M_0 & ε expansions)

NLO: (kinetic & mass mixing)

$$\mathcal{L} = \frac{1+\delta_k^{\eta}}{2}\partial_\mu\overline{\eta}\partial^\mu\overline{\eta} + \frac{1+\delta_k^{\eta'}}{2}\partial_\mu\overline{\eta}'\partial^\mu\overline{\eta}' + \delta_k^{\eta\eta'}\partial_\mu\overline{\eta}\partial^\mu\overline{\eta}' - \frac{m_{\overline{\eta}}^2+\delta_{m_{\eta}^2}}{2}\overline{\eta}\,\overline{\eta} - \frac{m_{\overline{\eta}'}^2+\delta_{m_{\eta'}^2}}{2}\overline{\eta}'\,\overline{\eta}' - \delta_{m^2}^{\eta\eta'}\overline{\eta}\,\overline{\eta}'$$
$$+ \frac{1+\delta_k^{\pi}}{2}\partial_\mu\overline{\pi}^0\partial^\mu\overline{\pi}^0 + \delta_k^{\pi\eta}\partial_\mu\overline{\pi}^0\partial^\mu\overline{\eta} + \delta_k^{\pi\eta'}\partial_\mu\overline{\pi}^0\partial^\mu\overline{\eta}' - \frac{m_{\overline{\pi}}^2+\delta_{m_{\pi}^2}}{2}\overline{\pi}^0\,\overline{\pi}^0 - \delta_{m^2}^{\pi\eta}\overline{\pi}^0\overline{\eta} - \delta_{m^2}^{\pi\eta'}\overline{\pi}^0\overline{\eta}'$$
$$+ \frac{1+\delta_k^{a}}{2}\partial_\mu\overline{a}\partial^\mu\overline{a} + \delta_k^{a\pi}\partial_\mu\overline{a}\partial^\mu\overline{\pi}^0 + \delta_k^{a\eta}\partial_\mu\overline{a}\partial^\mu\overline{\eta} + \delta_k^{a\eta'}\partial_\mu\overline{a}\partial^\mu\overline{\eta}' - \frac{m_{\overline{a}}^2+\delta_{m_{a}^2}}{2}\overline{a}\,\overline{a} - \delta_{m^2}^{a\pi}\overline{a}\,\overline{\pi}^0$$
$$- \delta_{m^2}^{a\eta}\overline{a}\,\overline{\eta} - \delta_{m^2}^{a\eta'}\overline{a}\,\overline{\eta}'$$

Separately handle the kinetic (x_{ij}) and mass (y_{ij}) mixing terms

$$\begin{pmatrix}\hat{\pi}^0\\ \hat{\eta}\\ \hat{\eta}'\\ \hat{a}\end{pmatrix} = \begin{pmatrix}1 & -y_{12} & -y_{13} & -y_{14}\\ y_{12} & 1 & -y_{23} & -y_{24}\\ y_{13} & y_{23} & 1 & -y_{34}\\ y_{14} & y_{24} & y_{34} & 1\end{pmatrix} \times \begin{pmatrix}1-x_{11} & -x_{12} & -x_{13} & -x_{14}\\ -x_{12} & 1-x_{22} & -x_{23} & -x_{24}\\ -x_{13} & -x_{23} & 1-x_{33} & -x_{34}\\ -x_{14} & -x_{24} & -x_{34} & 1-x_{44}\end{pmatrix}\begin{pmatrix}\overline{\pi}^0\\ \overline{\eta}\\ \overline{\eta}'\\ \overline{a}\end{pmatrix}$$

$$x_{11} = -\frac{\delta_k^{\pi}}{2},\quad x_{12} = -\frac{\delta_k^{\pi\eta}}{2},\quad x_{13} = -\frac{\delta_k^{\pi\eta'}}{2},\quad x_{14} = -\frac{\delta_k^{a\pi}}{2},\quad x_{22} = -\frac{\delta_k^{\eta}}{2},$$
$$x_{23} = -\frac{\delta_k^{\eta\eta'}}{2},\quad x_{24} = -\frac{\delta_k^{a\eta}}{2},\quad x_{33} = -\frac{\delta_k^{\eta'}}{2},\quad x_{34} = -\frac{\delta_k^{a\eta'}}{2},\quad x_{44} = -\frac{\delta_k^{a}}{2},$$

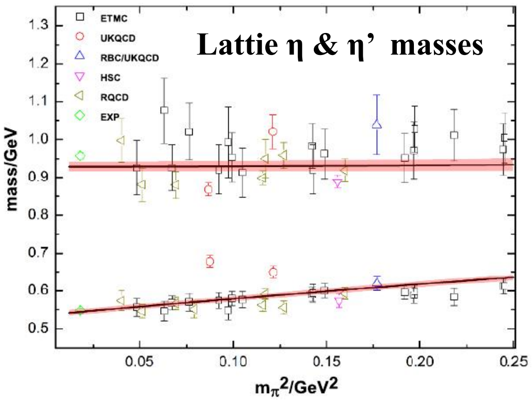
$$\delta_X \sim L_5, L_8, \Lambda_1, \Lambda_2$$

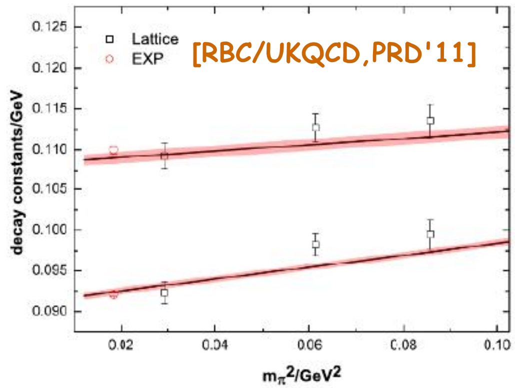
$$y_{12} = \frac{\delta_{m^2}^{\pi\eta} + x_{12}(m_{\overline{\eta}}^2 + m_{\overline{\pi}}^2)}{m_{\overline{\eta}}^2 - m_{\overline{\pi}}^2},\quad y_{13} = \frac{\delta_{m^2}^{\pi\eta'} + x_{13}(m_{\overline{\eta}'}^2 + m_{\overline{\pi}}^2)}{m_{\overline{\eta}'}^2 - m_{\overline{\pi}}^2},\quad y_{14} = \frac{\delta_{m^2}^{a\pi} + x_{14}(m_{a,0}^2 + m_{\overline{\pi}}^2)}{m_{a,0}^2 - m_{\overline{\pi}}^2},$$
$$y_{23} = \frac{\delta_{m^2}^{\eta\eta'} + x_{23}(m_{\overline{\eta}}^2 + m_{\overline{\eta}'}^2)}{m_{\overline{\eta}'}^2 - m_{\overline{\eta}}^2},\quad y_{24} = \frac{\delta_{m^2}^{a\eta} + x_{24}(m_{\overline{\eta}}^2 + m_{a,0}^2)}{m_{a,0}^2 - m_{\overline{\eta}}^2},\quad y_{34} = \frac{\delta_{m^2}^{a\eta'} + x_{34}(m_{\overline{\eta}'}^2 + m_{a,0}^2)}{m_{a,0}^2 - m_{\overline{\eta}'}^2}.$$

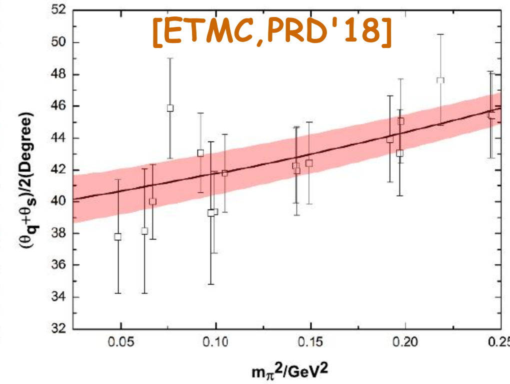
Fit to lattice data

[Gao,ZHG,Oller,Zhou,JHEP'23] [Gao,Hao,ZHG,et al.,EPJC'25]

Parameters	NLO Fit
F(MeV)	$91.05^{+0.42}_{-0.44}$
$10^3 \times L_5$	$1.68^{+0.05}_{-0.06}$
$10^3 \times L_8$	$0.88^{+0.04}_{-0.04}$
Λ_1	$-0.17^{+0.05}_{-0.05}$
Λ_2	$0.06^{+0.08}_{-0.09}$
χ^2/(d.o.f.)	219.9/(111–5)







Mixing pattern@NLO

$$\begin{pmatrix} \hat{\pi}^0 \\ \hat{\eta} \\ \hat{\eta}' \\ \hat{a} \end{pmatrix} = M^{\rm LO+NLO} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix}$$

$$M^{\rm LO+NLO} = \begin{pmatrix} 1+(0.015\pm0.001) & 0.017+(-0.007\pm0.001) & 0.009+(-0.011\pm0.001) & \frac{-12.8+(-0.13\pm0.02)}{f_a} \\ -0.019+(0.005\pm0.001) & 0.94+(0.21\pm0.01) & 0.33+(-0.21\pm0.03) & \frac{-34.3+(1.7^{+0.8}_{-0.7})}{f_a} \\ -0.003+(-0.001\pm0.000) & -0.33+(-0.18\pm0.02) & 0.94+(0.13^{+0.01}_{-0.02}) & \frac{-25.9+(0.2^{+0.4}_{-0.3})}{f_a} \\ \frac{12.1+(0.5\pm0.1)}{f_a} & \frac{23.8+(1.0^{+0.2}_{-0.1})}{f_a} & \frac{35.7+(1.7^{+0.2}_{-0.1})}{f_a} & 1+\frac{-921.5+(-56.6^{+7.9}_{-9.6})}{f_a^2} \end{pmatrix}$$

Mass decomposition@NLO

$$\begin{aligned} m_{\hat{\pi}} &= \left[134.9+(0.1\pm0.07)\right]\text{MeV}, \\ m_{\hat{K}} &= \left[492.1+(5.1^{+3.4}_{-3.3})\right]\text{MeV}, \\ m_{\hat{\eta}} &= \left[490.4+(61.1^{+10.0}_{-8.7})\right]\text{MeV}, \\ m_{\hat{\eta}'} &= \left[954.5+(-28.5^{+11.9}_{-10.9})\right]\text{MeV}, \\ m_{\hat{a}} &= \left[5.96+(0.12\pm0.02)\right]\mu\,\text{eV}\,\frac{10^{12}\,\text{GeV}}{f_a}, \end{aligned}$$

Two-photon couplings

$$\mathcal{L}_{WZW}^{\rm LO} = -\frac{3\sqrt{2}}{8\pi^2 F}\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2\Phi\rangle, \qquad Q = {\rm Diag}(\tfrac{2e}{3}, -\tfrac{e}{3}, -\tfrac{e}{3})$$

$$\mathcal{L}_{WZW}^{\rm NLO} = t_1\frac{32\sqrt{2}B}{F}\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle (M_q\Phi + \Phi M_q) Q^2\rangle + 16k_3\varepsilon_{\mu\nu\rho\sigma}\partial^\mu A^\nu \partial^\rho A^\sigma \langle Q^2\rangle \left(\frac{\sqrt{2}}{F}\langle\Phi\rangle - \frac{a}{f_a}\right)$$

* **Note: one needs the π-η-η'-*a* mixing as input to calculate $g_{a\gamma\gamma}$**

$$F^{\rm Exp}_{\pi^0\gamma\gamma} = 0.274 \pm 0.002{\rm GeV}^{-1},$$
$$F^{\rm Exp}_{\eta\gamma\gamma} = 0.274 \pm 0.006{\rm GeV}^{-1},$$
$$F^{\rm Exp}_{\eta'\gamma\gamma} = 0.344 \pm 0.008{\rm GeV}^{-1},$$

→

$$t_1 = -(3.8 \pm 2.4)\times 10^{-4}{\rm GeV}^{-2},$$
$$k_3 = (1.21 \pm 0.23)\times 10^{-4}$$

→

isospin limit(LO) isospin breaking(LO) NLO

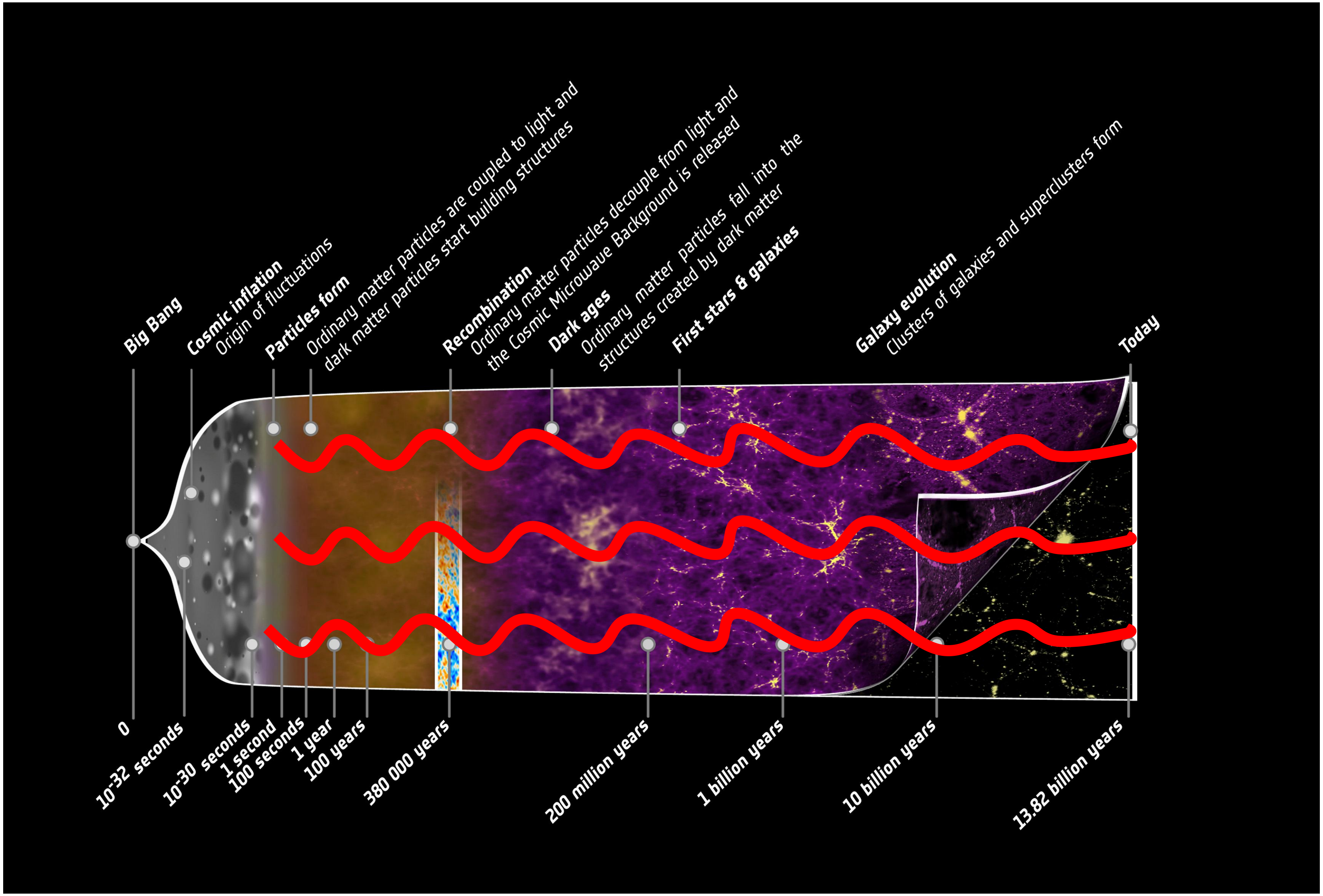
$$F_{a\gamma\gamma} = \frac{20.1 + 3.4 + (0.5 \pm 0.2)}{f_a}\times 10^{-3},$$

(IB corrections amount to be around 15%!)

$$g_{a\gamma\gamma} = 4\pi\alpha_{em}F_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a}(1.89 \pm 0.02).$$

which can be compared to: 1.92±0.04 [Grilli de Cortona, et al., JHEP'16] **and 2.05±0.03** [Lu, et al., JHEP'20]

Cosmology constraints on axion thermalization rate



- **Axions can be copiously produced from thermal bath in the early Universe.**
- **After decoupling, its thermal relics will leave imprints today.**

Cosmology constraints on axion thermalization rate

Axion thermal production in the early Universe : Extra radiation (ΔN_{eff})

Extra effective number of relativisitc d.o.f :

$$\Delta N_{\rm eff} \simeq \frac{4}{7}\left(\frac{43}{4g_{\star s}(T_D)}\right)^{\frac{4}{3}}$$

$g_{\star s}(T)$: effective number of entropy d.o.f at temperature T

T_D: axion decoupling temperature from the thermal medium

- **CMB constraint (Plank'18)** [Aghanim et al., 2020] **: $\Delta N_{eff} \leq 0.28$**
- **T_D : Instantaneous decoupling approximation**

$$\Gamma_a(T_D) = H(T_D)$$

Axion thermalization rate

$$\Gamma_a(T) = \frac{1}{n_a^{\rm eq}} \int d\widetilde{\Gamma} \sum |\mathcal{M}_{a-\rm SM}|^2 \, n_B(E_1) n_B(E_2) [1+n_B(E_3)][1+n_B(E_4)]$$

$$n_B(E) = 1/(e^{E/T}-1)$$

Axion-SM particle scattering amplitudes

Hubble expansion parameter

$$H(T) = T^2\sqrt{4\pi^3 g_*(T)/45}/m_{\rm Pl}$$

Key thermal channels of axion-SM scatterings at different temperatures

- ☞ $T_D \gtrsim 1$ GeV: $ag \leftrightarrow gg$.
  [Masso et al., 2002, Graf and Steffen, 2011]
- ☞ $T_D \lesssim 1$ GeV: Hadrons need to be included.
- ☞ $T_D \lesssim 200$ MeV: $a\pi \leftrightarrow \pi\pi$.
  [Chang and Choi, 1993, Hannestad et al., 2005, Giusarma et al., 2014, D'Eramo et al., 2022]

❑ **Reliable $a\pi$ interaction is crucial to determine Γ_a for $T_D < T_c \approx 155$ MeV**

- ➢ For a long time, only the LO $a\pi \leftrightarrow \pi\pi$ amplitude is employed to calculate Γ_a , e.g.,
  [Chang, Choi, PLB'93] [Hannestad, et al., JCAP'05] [Hannestad, et al., JCAP'05] [D'Eramo, et al., PRL'22] ... ...
- ➢ Recent NLO calculation of Γ_a : χPT invalid for $T_\chi > 70$ MeV **[Di Luzio, et al., PRL'21]**
- ➢ Chiral unitarization approach for $a\pi \leftrightarrow \pi\pi$: **[Di Luzio, et al., PRD'23]**
- ➢ **However, all the previous works have ignored thermal corrections to the $a\pi \leftrightarrow \pi\pi$ amplitudes. Fhe first estimation of such effect is given: [Wang, ZHG, Zhou, PRD'24]**

$$\Gamma_a(T) = \frac{1}{n_a^{\rm eq}} \int d\tilde{\Gamma} \sum |\mathcal{M}_{a\pi;\pi\pi}|^2 n_B(E_1) n_B(E_2) \, [1 + n_B(E_3)][1 + n_B(E_4)]$$

- ➢ **First realistic calculation of $aK \leftrightarrow \pi K$ shows significant contribution to axion thermalization rate: [Wang, ZHG, Zhou, PRD'25]**

Calculation of THERMAL $a\pi \leftrightarrow \pi\pi$ amplitudes at one-loop level

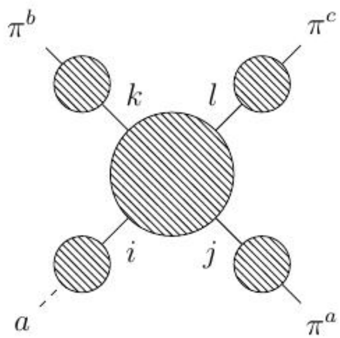
- Finite-temperature effects are included by imaginary time formalism (ITF), where [Kapusta and Gale, 2011, Bellac, 2011, Laine and Vuorinen, 2016]

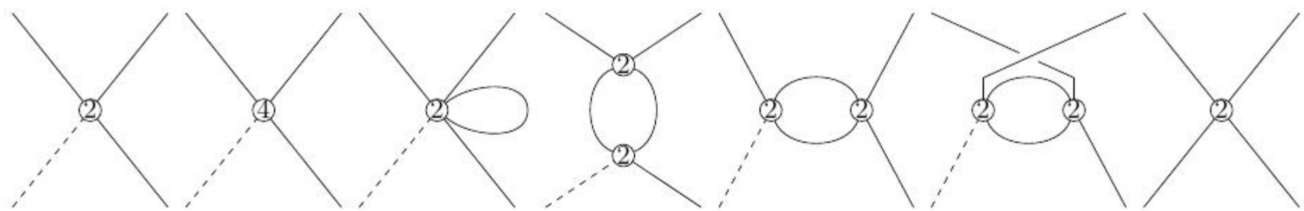
$$p^0 \to i\omega_n, \quad \text{with } \omega_n = 2\pi n T\,, n \in \mathbb{Z}\,,$$

$$-i\int \frac{\mathrm{d}^d q}{(2\pi)^d} \to -i\int_\beta \frac{\mathrm{d}^d q}{(2\pi)^d} \equiv T\sum_n \int \frac{\mathrm{d}^{d-1} q}{(2\pi)^{d-1}}\,.$$

- Compute the thermal Green functions in ITF

$$G^T_{a\pi^a;\pi^b\pi^c}(p_1,p_2;p_3,p_4) = \sum_{i,j,k,l} G_{ai}(p_1^2)G_{\pi^a j}(p_2^2)G_{k\pi^b}(p_3^2)G_{l\pi^c}(p_4^2)A_{ij;kl}(p_1,p_2;p_3,p_4)\,.$$





Feynman diagrams for amputated functions up to NLO.

- The effective Lagrangian at $\mathcal{O}(p^4)$

$$\begin{aligned}\mathcal{L}_4 \supset & \frac{l_3+l_4}{16}\left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle \left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle + \frac{l_4}{8}\left\langle \partial_\mu U \partial^\mu U^\dagger \right\rangle \left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle \\ & - \frac{l_7}{16}\left\langle \chi_a U^\dagger - U\chi_a^\dagger \right\rangle \left\langle \chi_a U^\dagger - U\chi_a^\dagger \right\rangle + \frac{h_1 - h_3 - l_4}{16}\left[\left(\left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle\right)^2 \right. \\ & \left. + \left(\left\langle \chi_a U^\dagger - U\chi_a^\dagger \right\rangle\right)^2 - 2\left\langle \chi_a U^\dagger \chi_a U^\dagger + U\chi_a^\dagger U \chi_a^\dagger \right\rangle\right] + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\mathrm{NLO}}\,,\end{aligned}$$

$$\begin{aligned}J_A^\mu|_{\mathrm{NLO}} \supset & - i l_1 \left\langle Q_a\left\{\partial^\mu U, U^\dagger\right\}\right\rangle \left\langle \partial_\nu U \partial^\nu U^\dagger \right\rangle \\ & - i\frac{l_2}{2}\left\langle Q_a\left\{\partial_\nu U, U^\dagger\right\}\right\rangle \left\langle \partial^\mu U \partial^\nu U^\dagger + \partial^\nu U \partial^\mu U^\dagger \right\rangle \\ & - i\frac{l_4}{4}\left\langle Q_a\left\{\partial^\mu U, U^\dagger\right\}\right\rangle \left\langle \chi_a U^\dagger + U\chi_a^\dagger \right\rangle\,.\end{aligned}$$

Unitarization of the partial-wave $a\pi \leftrightarrow \pi\pi$ amplitude

Inverse amplitude method (IAM)

$$\mathcal{M}^{\mathrm{IAM}}_{a\pi;IJ} = \frac{\left(\mathcal{M}^{(2)}_{a\pi;IJ}\right)^2}{\mathcal{M}^{(2)}_{a\pi;IJ} - \mathcal{M}^{(4)}_{a\pi;IJ}}$$

$$\mathcal{M}_{a\pi;IJ}(E_{cm}) = \frac{1}{2}\int_{-1}^{+1} \mathrm{d}\cos\theta\, \mathcal{M}_{a\pi;I}(E_{cm}, \cos\theta) P_J(\cos\theta)$$

$$\mathrm{Im}\mathcal{M}_{a\pi;IJ}(E_{cm}) \overset{s}{=} \frac{1}{2}\rho^T_{\pi\pi}(E_{cm})\mathcal{M}^{IJ*}_{\pi\pi;\pi\pi}\mathcal{M}_{a\pi;IJ}\,, \quad (E_{cm} > 2m_\pi)$$

$$\rho^T_{\pi\pi}(E_{cm}) = \frac{\sigma_\pi(E^2_{cm})}{16\pi}\left[1 + 2n_B(\frac{E_{cm}}{2})\right]\,, \quad \sigma_\pi(s) = \sqrt{1 - \frac{4m^2_\pi}{s}}\,, \quad n_B(E) = \frac{1}{e^{E/T} - 1}$$

- Resonances poles on the second Riemann sheet

	$f_0(500)/\sigma$		$\rho(770)$	
	$M_\sigma \pm i\frac{\Gamma_\sigma}{2}$	$\lvert f_a g_{\sigma a\pi}\rvert$	$M_\rho \pm i\frac{\Gamma_\rho}{2}$	$\lvert f_a g_{\rho a\pi}\rvert$
$T = 0$ MeV	$422 \pm i240$ MeV	0.032 GeV^2	$739 \pm i72$ MeV	0.035 GeV^2
$T = 100$ MeV*	$368 \pm i310$ MeV	0.037 GeV^2	$744 \pm i77$ MeV	0.036 GeV^2

*Only include s-channel unitary thermal correction.

Axion thermalization rate

- We calculate the axion rate by the temperature dependent $a\pi \to \pi\pi$ scattering amplitudes [Chang and Choi, 1993, Hannestad et al., 2005]

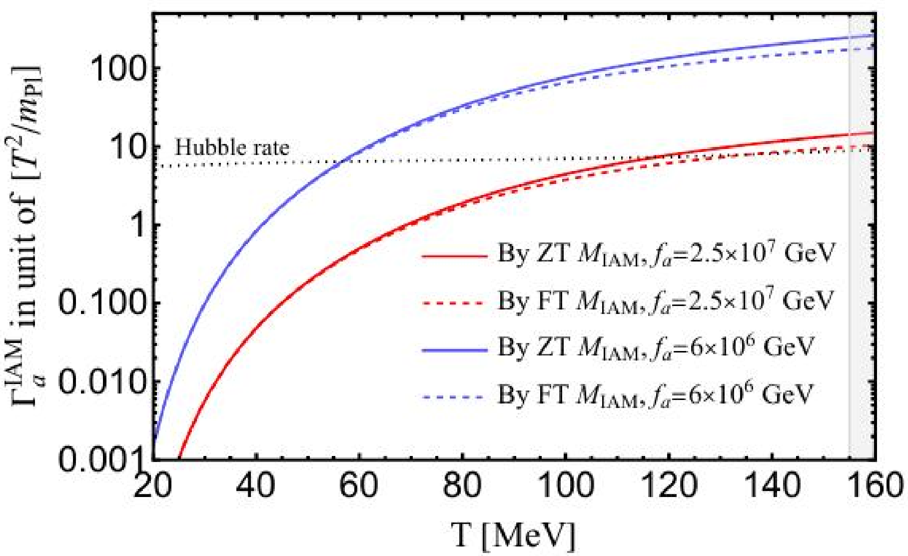
$$\Gamma_a(T) = \frac{1}{n_a^{\rm eq}} \int d\widetilde{\Gamma} \sum |\mathcal{M}_{a\pi;\pi\pi}|^2 n_B(E_1) n_B(E_2)[1+n_B(E_3)][1+n_B(E_4)] \,,$$

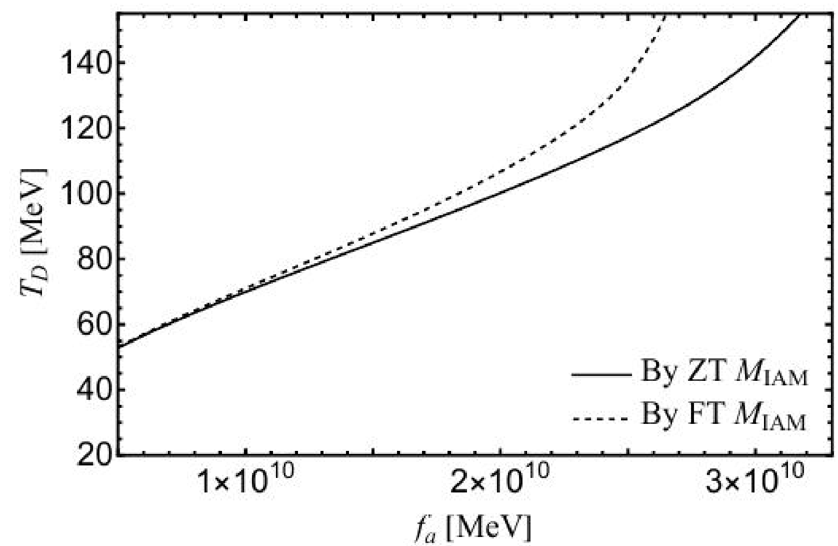
where the phase space integral

$$\int d\widetilde{\Gamma} = \int \left(\prod_{i=1}^{4} \frac{d^3 p_i}{(2\pi)^3} \frac{1}{2E_i} \right) (2\pi)^4 \delta^4(p_1 + p_2 - p_3 - p_4) \,.$$

Resulst from the thermal-IAM improved axion rates

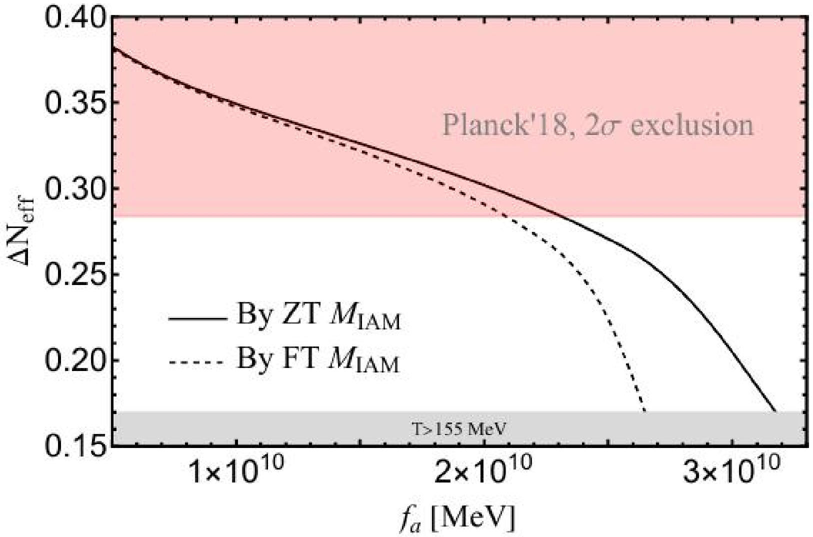
$$\Delta N_{\rm eff} \simeq \frac{4}{7} \left(\frac{43}{4 g_{\star s}(T_D)} \right)^{\frac{4}{3}}$$

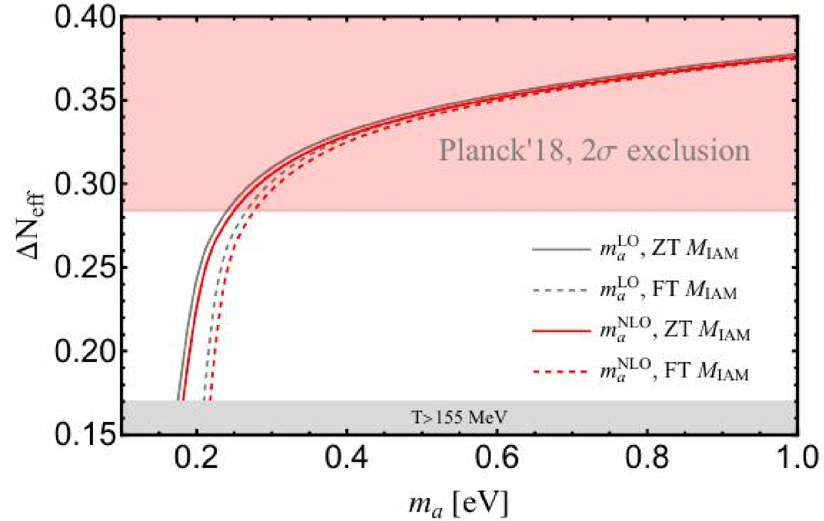




Updated bounds on the axion parameters

[Wang,ZHG,Zhou,PRD'24]





- The constrians

10% corrections are observed

	lower limit of f_a	upper limit of m_a by m_a^{LO}	upper limit of m_a by m_a^{NLO}
ZT	2.3×10^7 GeV	0.24 eV	0.25 eV
FT	2.1×10^7 GeV	0.27 eV	0.28 eV

- The QCD axion mass up to LO & NLO

$$m_a^2|_{\mathrm{LO}} = \gamma_{ud}\, m_\pi^2 \frac{F^2}{f_a^2}\,, \qquad \text{where} \quad \gamma_{ud} = \frac{m_u m_d}{(m_u + m_d)^2}\,,$$

$$m_a^2|_{\mathrm{NLO}} = \gamma_{ud}\, m_\pi^2 \frac{F^2}{f_a^2} \left\{ 1 - 2\frac{m_\pi^2}{(4\pi F)^2} \log\frac{m_\pi^2}{\mu^2} + 2\left[h_1^r(\mu^2) - h_3\right]\frac{m_\pi^2}{F^2} - 8 l_7 \gamma_{ud} \frac{m_\pi^2}{F^2} \right\}$$

Combined analyses with $a\pi \leftrightarrow \pi\pi$ & $aK \leftrightarrow \pi K$ channels

SU(3) Axion ChPT@LO

[Wang, ZHG, Zhou, PRD'25]

$$\mathcal{L}_2 = \frac{F_\pi^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + \chi(a) U^\dagger + U\chi^\dagger(a)\rangle - \frac{\partial_\mu a}{2f_a}\sum_{i=1}^{8} C_i J^\mu_{A,i}$$

$$\chi(a) = 2B_0 e^{-i\frac{a}{2f_a}Q_a} M_q e^{-i\frac{a}{2f_a}Q_a} \qquad Q_a = M_q^{-1}/\langle M_q^{-1}\rangle \qquad J^\mu_{A,i} = i\frac{F_\pi^2}{4}\langle \lambda_i\{\partial^\mu U, U^\dagger\}\rangle$$

(singlet component of axial currents neglected)

$$C_3 = \frac{z(1-r^2)}{2r + z(1+r)^2}\,, \qquad C_8 = \frac{z(1+r)^2 - 4r}{\sqrt{3}\,[2r + z(1+r)^2]} \qquad z = \frac{m_s}{\hat{m}}\,, r = \frac{m_u}{m_d}\,, \hat{m} = \frac{m_u + m_d}{2}$$

Unitarized partial-wave axion-meson/meson-meson amplitudes

Unitarized meson-meson Amp:

$$T_{IJ}^{\rm uni} = T_{IJ}^{(2)} . \left[T_{IJ}^{(2)} - T_{IJ}^{(4)\rm LECs} - T_{IJ}^{(2)} . \mathcal{G} . T_{IJ}^{(2)}\right]^{-1} . T_{IJ}^{(2)}$$

$$\mathrm{Im}T = T^\dagger . q/(8\pi\sqrt{s}) . T$$

Unitarized axion-meson Amp:

$$\vec{M}_{IJ}^{\rm uni} = T_{IJ}^{(2)} . \left[T_{IJ}^{(2)} - T_{IJ}^{(4)\rm LECs} - T_{IJ}^{(2)} . \mathcal{G} . T_{IJ}^{(2)}\right]^{-1} . \vec{M}_{IJ}^{(2)}$$

$$\mathrm{Im}\vec{M} = T^\dagger . q/(8\pi\sqrt{s}) . \vec{M}$$

$$\mathcal{G} = \mathrm{diag}(G_n, G_m, \cdots) \qquad G_n(s) = G(a^n_{\rm sc}, s, m_{n_1}, m_{n_2}) = -\frac{1}{(4\pi)^2}\left[a^n_{\rm sc} - 1 + \log\frac{m_{n_2}^2}{\mu^2} + \frac{m_{i}}{} - \frac{\sqrt{\lambda(s, m_{n_1}^2, m_{n_2}^2)}}{s}\log\frac{m_{n_1}^2 + m_{n_2}^2 - s + \sqrt{\lambda(s, m_{n_1}^2, m_{n_2}^2)}}{2m_{n_1}m_{n_2}}\right]$$

Example:

$$\begin{pmatrix} M^{uni}_{00,\pi\pi} \\ M^{uni}_{00,KK} \end{pmatrix} = \begin{pmatrix} \widehat{T}^{\pi\pi\to\pi\pi}_{00} & \widehat{T}^{\pi\pi\to KK}_{00} \\ \widehat{T}^{\pi\pi\to KK}_{00} & \widehat{T}^{KK\to KK}_{00} \end{pmatrix} \begin{pmatrix} M^{(2)}_{00,\pi\pi} \\ M^{(2)}_{00,KK} \end{pmatrix}$$

Relevant channels: $S + P$ waves

(1) $a\,\pi^0 \to \pi^+\pi^-\,,\ \pi^0\pi^0$

IJ=00: $f_0(500), f_0(980)$ [K-Kbar coupled-channel included]

IJ=20: nonresonant case [single ππ channel]

(2) $a\,\pi^+ \to \pi^+\pi^0$

IJ=11: $\rho(770)$ [K-Kbar coupled-channel included]

IJ=20: nonresonant case [single ππ channel, same as $a\pi^0$ case]

(3) $a\,K^+ \to \pi^+K^0\,,\ \pi^0K^+$

IJ=1/2 1: $K^*(892)$ [Kη coupled-channel included]

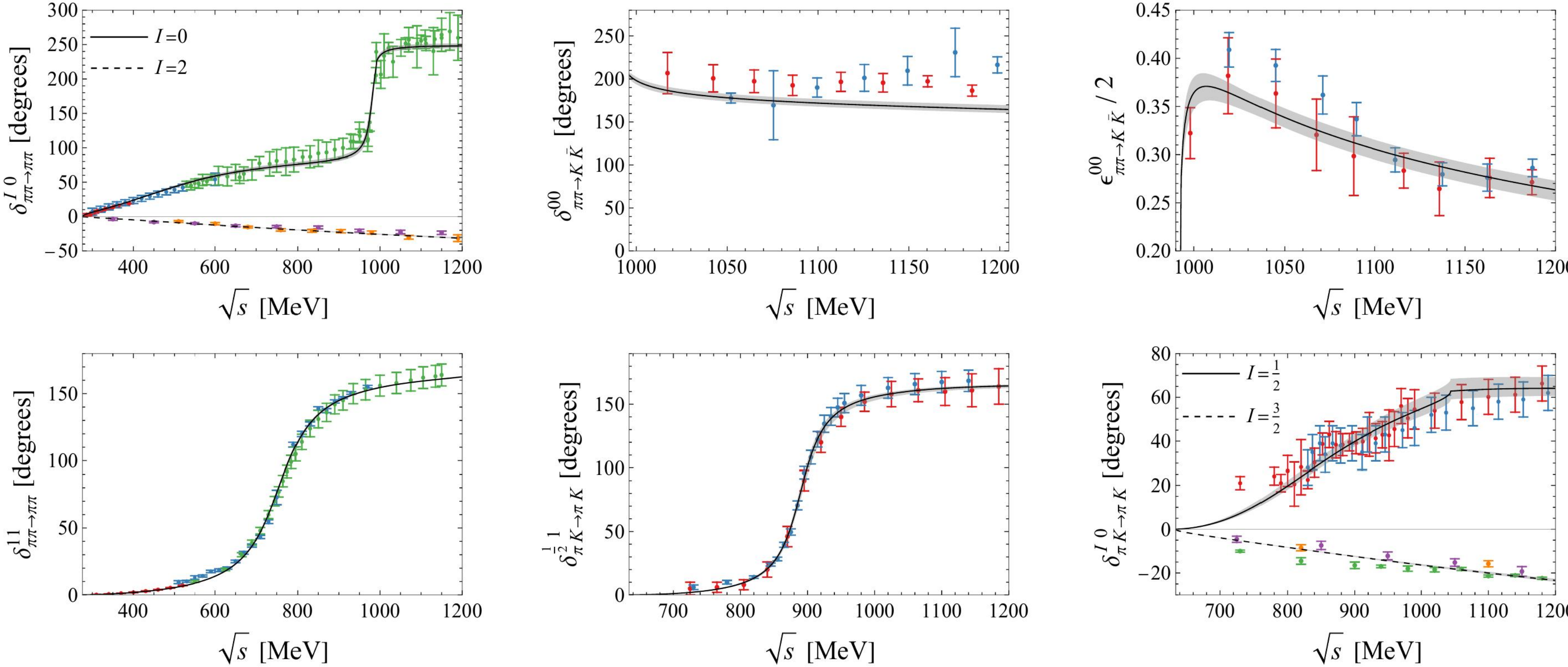
IJ=1/2 0: $K^*_0(700)$ [Kη coupled-channel included]

IJ=3/2 0: nonresonant case [single Kπ channel]

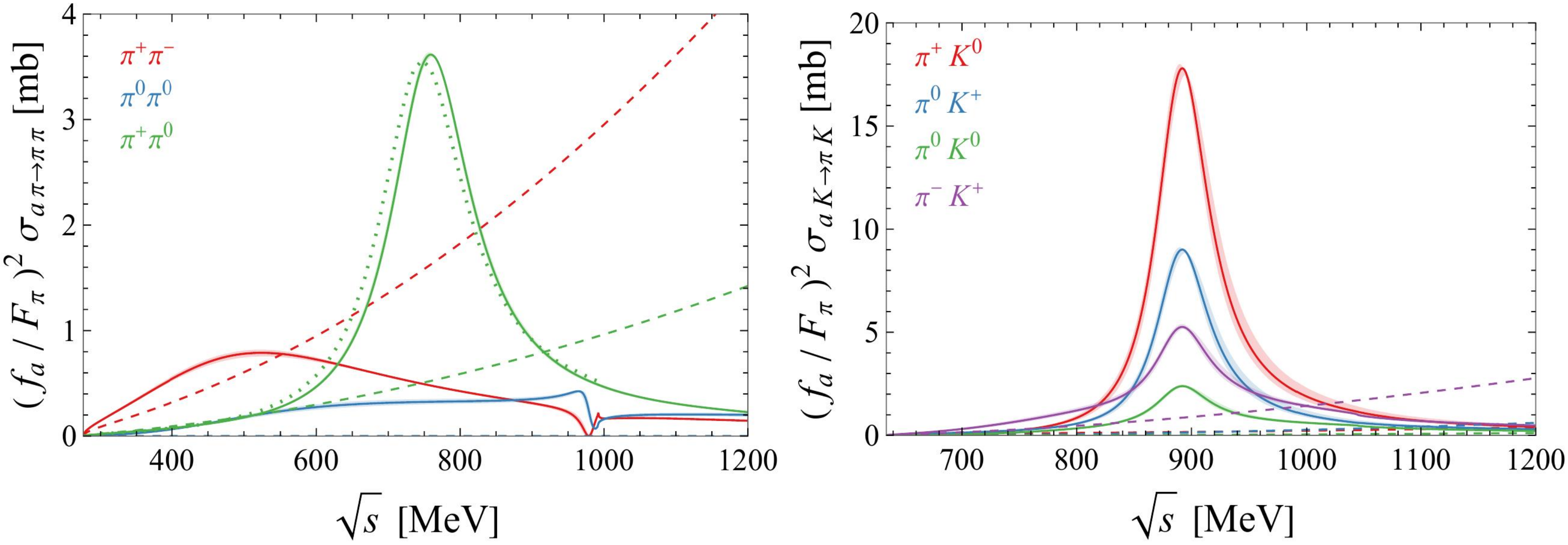
IJ=3/2 1: nonresonant case [neglected]

(4) $a\,K^0 \to \pi^-K^+\,,\ \pi^0K^0$ [similar as aK^+ case]

(5) Other channels can be obtained via the charge-conjugation symmetry.



Subst. const.		Low energy constants	
$a^{\pi\pi,00}_{\rm sc}$	$-0.49^{+0.24}_{-0.23}$	$\hat{L}_1\times10^3$	$0.33^{+0.02}_{-0.02}$
$a^{K\bar{K},00}_{\rm sc}$	$-1.51^{+0.20}_{-0.19}$	$\hat{L}_2\times10^3$	$0.97^{+0.05}_{-0.05}$
$a^{11}_{\rm sc}$	$-1.38^{+0.33}_{-0.26}$	$\hat{L}_3\times10^3$	$-2.71^{+0.10}_{-0.11}$
$a^{\frac{1}{2}0}_{\rm sc}$	$0.15^{+0.18}_{-0.21}$	$\hat{L}_4\times10^3$	$-0.77^{+0.09}_{-0.11}$
$a^{\frac{1}{2}1}_{\rm sc}$	$1.53^{+0.76}_{-0.80}$	$\hat{L}_5\times10^3$	$3.51^{+1.39}_{-1.62}$
		$\hat{L}_6\times10^3$	$-1.47^{+0.20}_{-0.24}$
		$\hat{L}_7\times10^3$	$-0.77^{+0.24}_{-0.18}$
		$\hat{L}_8\times10^3$	$4.05^{+0.37}_{-0.45}$

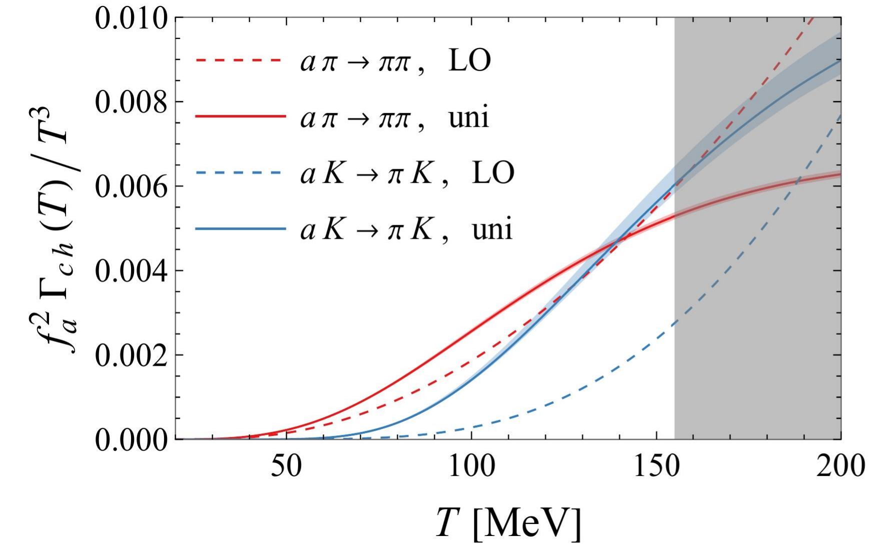


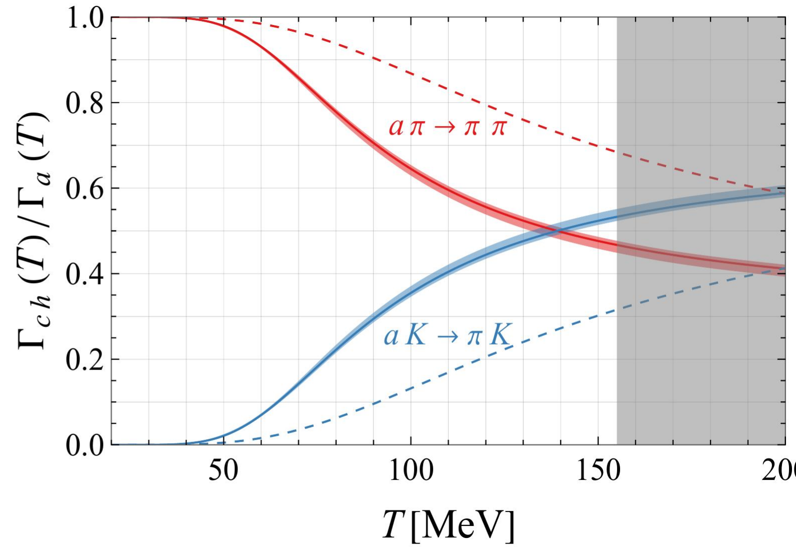
Resonance poles

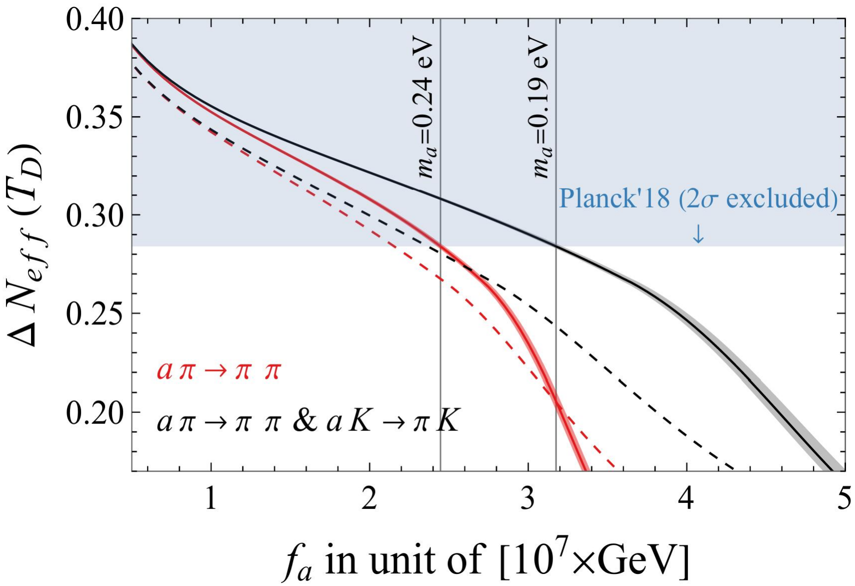
ρ: (754.3 - i67.9) MeV ; K^*: (889.5 - i28.0) MeV;

$f_0(500)$: 435.4- i238.0; $f_0(980)$: 981.1- i11.4; $K^*_0(700)$: 801.9- i195.2;

- **Clear enhancement from the unitarized amplitudes (solid lines)**
- **ρ(770) & K^*(892) lead to the most prominent effects**
- **Scalar resonances mostly give mild contributions**
- **aη related processes are much less important than the aK ones. (working in progress)**
- **axion-baryon is expected to be much suppressed, due to the heavy thresholds.**







[Wang, ZHG, Zhou, PRD'25]

$f_a \geq 2.45^{+0.03}_{-0.02} \times 10^7$ GeV **(aπ→ππ)**

$f_a \geq 3.18^{+0.04}_{-0.03} \times 10^7$ GeV **(aπ→ππ + aK→πK)**

- **Enhancement in $\tau \to \nu_\tau K a$ is also seen. However this belongs to a Cabibbo suppressed reaction. [Hao, Duan, ZHG, 2507.00383]**

Axion production in $\eta \rightarrow \pi\pi a$ decay

Axion production from $\eta \to \pi\pi a$ decay in SU(3) χPT

Why focus on axion in η decay:

- ✓ Valuable channel to search axion @colliders: many available experiments with large data samples of η/η' [BESIII, STCF, JLab, REDTOP, ...]
- ✓ $\eta \to \pi\pi\pi$ (IB suppressed), $\eta \to \pi\pi a$ (no IB suppression)
- ✓ $\eta \to \pi\pi a$: theoretically easier to handel than $\eta' \to \pi\pi a$ (next step)

Previous works:

- ❖ Most of them rely on leading-order χPT
- ❖ Possible issue: bulk contributions@LO χPT are constant terms, and potential large corrections from higher orders may result.
- ❖ Hadron resonance effects may lead to enhancements.

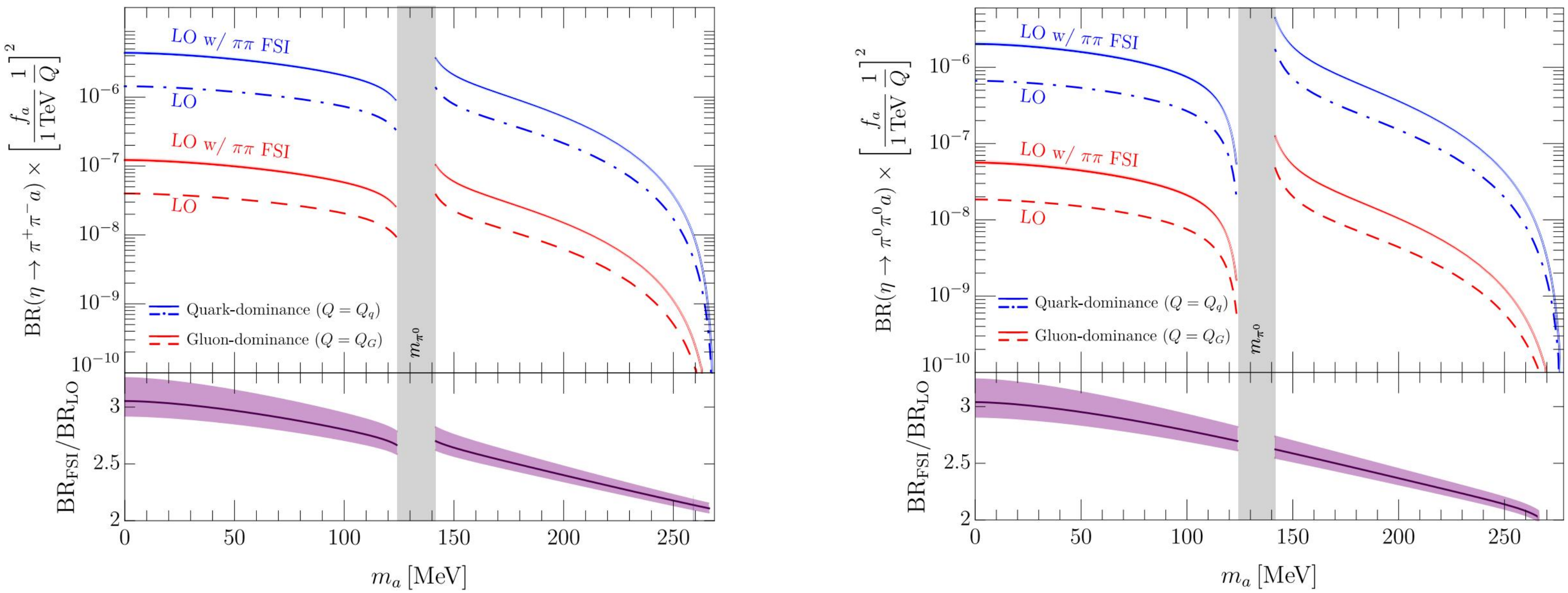
Advances in our work :

- ➢ Study of renormalization of $\eta \to \pi\pi a$ @1-loop level in SU(3) χPT
- ➢ To implement unitarization to the $\eta \to \pi\pi a$ χPT amplitude
- ➢ Uncertainty analyes in the phenomenological discussions

[Alves,Gonzalez-Solis,JHEP'24]

$$M_0(s) = P(s)\Omega_0^0(s)$$

η→ππa LO amplitude **Omnes function: ππ FSI**



Our improvements:

- ➢ **NLO perturbative decay amplitude include s- and $t(u)$-channel interactions perturbatively.**
- ➢ **The unitarized decay amplitude will be constructed to account for the s-channel $\pi\pi$ final state interaction (FSI) effect that respect the chiral symmetry.**
- ➢ **Dalitz plots will be explored to decode the dynamics in η→ππa.**

LO χPT Lagrangian $$\mathcal{L}_2 = \frac{F^2}{4}\langle \partial_\mu U \partial^\mu U^\dagger + \chi_a U^\dagger + U\chi_a^\dagger \rangle + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\rm LO} + \frac{1}{2}\partial_\mu a \partial^\mu a - \frac{1}{2} m_{a,0}^2 a^2$$

$$\chi_a = 2B_0 M(a) \qquad M(a) \equiv \exp\left(-i\frac{a}{2f_a}Q_a\right) M \exp\left(-i\frac{a}{2f_a}Q_a\right) \qquad J_A^\mu|_{\rm LO} = -i\frac{F^2}{2}\langle Q_a \left\{\partial^\mu U,\, U^\dagger\right\}\rangle$$

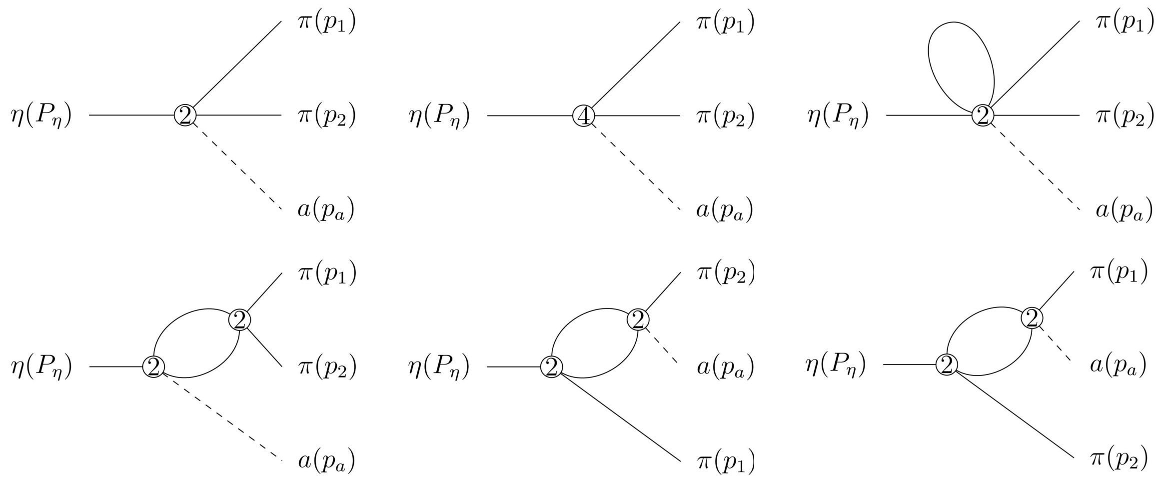
Note: we consider the octet part ($\bar{Q}_a$) of Q_a in SU(3) χPT

NLO χPT Lagrangian

$$\mathcal{L}_4 = L_1\langle \partial_\mu U \partial^\mu U^\dagger \rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \cdots + \frac{\partial_\mu a}{2f_a} J_A^\mu|_{\rm NLO},$$

$$J_A^\mu|_{\rm NLO} = -4iL_1 \langle \bar{Q}_a \{U^\dagger,\, \partial^\mu U\}\rangle \langle \partial_\nu U \partial^\nu U^\dagger \rangle + \cdots.$$

Feynman diagrams up to NLO



Parameters

Masses and F_π [MeV]				LECs $L_i^r(\mu)$ at $\mu = 770$ MeV (in unit of 10^{-3})							
m_π	m_K	m_η	F_π	L_1^r	L_2^r	L_3^r	L_4^r	L_5^r	L_6^r	L_7^r	L_8^r
137	496	548	92.1	1.0(1)	1.6(2)	−3.8(3)	0.0(3)	1.2(1)	0.0(4)	−0.3(2)	0.5(2)

[J. Bijnens and G. Ecker, Ann. Rev. Nucl. Part. Sci. 64, 149 (2014)]

✓ **Renomarlization condition is verified to be consistent with conventional ChPT.**

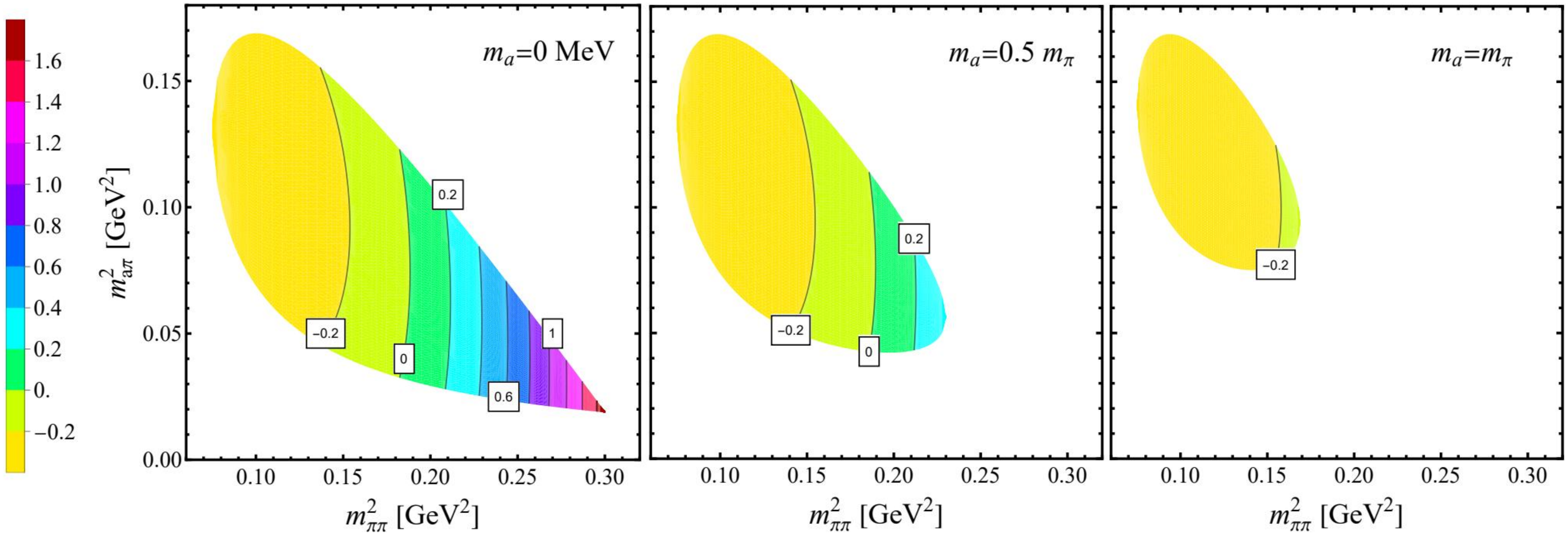
Observations:

- ➢ Strong isospin breaking effects enter the $\eta\to\pi\pi a$ amplitudes at the order of $(m_u-m_d)^2$
- ➢ In the isospin limit ($m_u=m_d$), the amplitudes with $\pi^+\pi^-$ and $\pi^0\pi^0$ in $\eta\to\pi\pi a$ processes are identical.

- **Dalitz plots to show the NLO/LO convergence**

$$\left(2\mathcal{M}^{(2)}_{\eta;\pi\pi a}\mathrm{Re}\left(\mathcal{M}^{(4)}_{\eta;\pi\pi a}\right)+\left|\mathcal{M}^{(4)}_{\eta;\pi\pi a}\right|^2\right)/\left|\mathcal{M}^{(2)}_{\eta;\pi\pi a}\right|^2$$

[Wang,ZHG,Lu,Zhou, JHEP'24]



Important lessons:

- ➢ Non-perturbative effect in the $\pi\pi$ subsystem can be important.
- ➢ Perturbative treatment of the $a\pi$ subsystem is justified.

Unitarization of the partial-wave η→ππa amplitude

$$\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s) = \frac{\mathcal{M}^{00,\mathrm{L}}_{\eta;\pi\pi a}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)} ,$$

$$G_{\pi\pi}(s) = -\frac{1}{(4\pi)^2}\left(\log\frac{m_\pi^2}{\mu^2} - \sigma_\pi(s)\log\frac{\sigma_\pi(s)-1}{\sigma_\pi(s)+1} - 1\right) ,$$

$$\mathcal{M}^{00,\mathrm{L}}_{\eta;\pi\pi a}(s) = \mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s) + \mathcal{M}^{00,(4)}_{\eta;\pi\pi a}(s) - G_{\pi\pi}(s)\,\mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s)\,T^{00,(2)}_{\pi\pi\to\pi\pi}(s) .$$

The unitarized amplitude satisfies the relation

$$\mathrm{Im}\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s) = \rho_{\pi\pi}(s)\mathcal{M}^{00,\mathrm{Uni}}_{\eta;\pi\pi a}(s)\left(T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s)\right)^* , \qquad (2m_\pi < \sqrt{s} < 2m_K)$$

with the unitarized PW ππ amplitude
$$T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s) = \frac{T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}$$

Unitarized PW amplitude based on LO η→ππa amplitude

$$\mathcal{M}^{00,\mathrm{Uni-LO}}_{\eta;\pi\pi a}(s) = \frac{\mathcal{M}^{00,(2)}_{\eta;\pi\pi a}(s)}{1 - G_{\pi\pi}(s) T^{00,(2)}_{\pi\pi\to\pi\pi}(s)} .$$

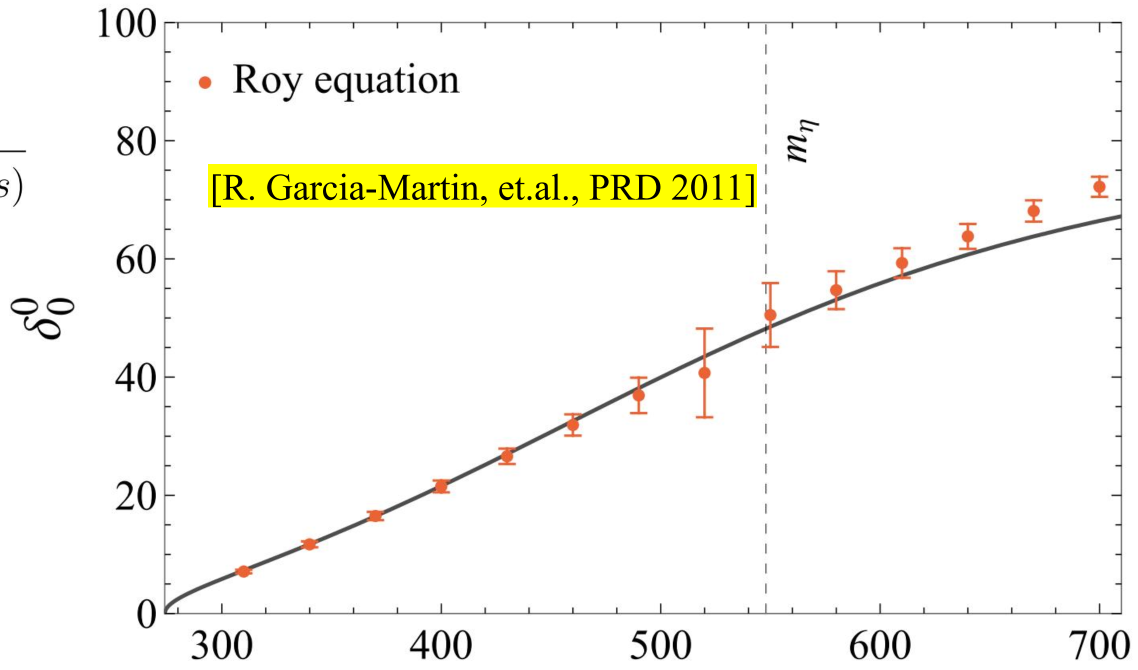
Resemble the method:

[Alves,Gonzalez-Solis,JHEP'24]

$$M_0(s) = P(s)\Omega_0^0(s)$$

Phase shifts from the unitarized PW $\pi\pi$ amplitude

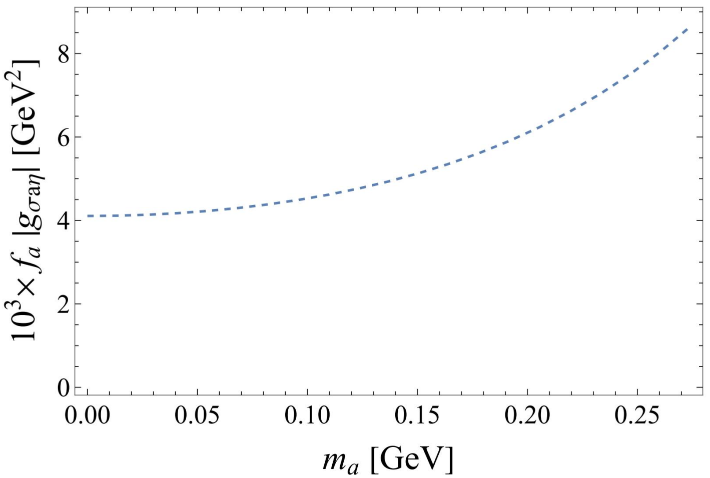
$$T^{00,\mathrm{Uni}}_{\pi\pi\to\pi\pi}(s) = \frac{T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}{1 - G_{\pi\pi}(s)T^{00,(2)}_{\pi\pi\to\pi\pi}(s)}$$



- **Pole position of $f_0(500)/\sigma$:**

$$\sqrt{s_\sigma} = 457 \pm i251 \text{ MeV}$$

$$\mathcal{M}^{00,\mathrm{Uni,II}}_{\eta;\pi\pi a}(s)\big|_{s\to s_\sigma} \sim -\frac{g_{\sigma\pi\pi}g_{\sigma a\eta}}{s - s_\sigma}$$



Predictions of the $\eta\to\pi\pi a$ branching ratios by varying m_a

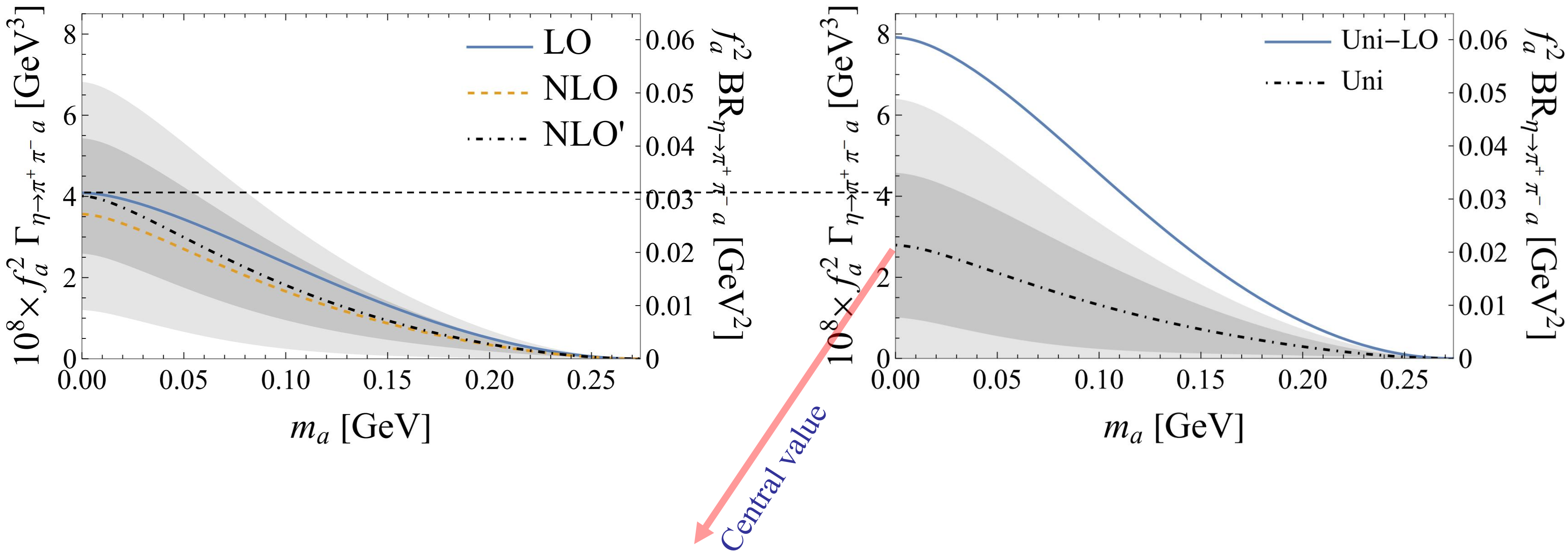
Uncertainty bands:

- **Lighter regions:**

L_1^r	L_2^r	L_3^r	L_4^r	L_5^r	L_6^r	L_7^r	L_8^r
1.0(1)	1.6(2)	−3.8(3)	0.0(3)	1.2(1)	0.0(4)	−0.3(2)	0.5(2)

- **Darker regions: freeze the 1/Nc suppressed ones (L_4,L_6,L_7)**

[Wang,ZHG,Lu,Zhou, JHEP'24]



$$\mathrm{BR}_{\eta\to\pi^+\pi^- a}\Big|_{m_a\to 0} = 2.1\times 10^{-2}\left(\frac{\mathrm{GeV}^2}{f_a^2}\right)$$

Possible detection channels: $a\to\gamma\gamma$, $a\to e^+e^-$, $a\to\mu^+\mu^-$

Summary & prospectives

- Chiral effecitive field theory provides a systematical and useful framework to study the axion-hadron reactions.
- Synergies of Lattice QCD, hadron phenomenologies and chiral EFT are demonstrated to be powerful to build axion amplitudes:

$a\to\gamma\gamma$; $a\pi\to\pi\pi$, $aK\to\pi K$; $\eta\to\pi\pi a$; (covered in this talk)

$\tau\to\nu_\tau Pa$; $\gamma N\to aN$; $eN\to eNa$; (skipped in this talk)

two-fermion potential with axion exchange; (skipped in this talk)

- Futher involvements with the experiments, cosmology, astronomy are needed to set up stronger constraints on axion parameters!

We are happy to make future collaborations with experts on those related fields!

谢谢！