基础物理与数学科学学院
School of Fundamental Physics and Mathematical Sciences

Effective field theory approach to neutrino interactions from high to low energies

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νSTEP (Bei Jing)

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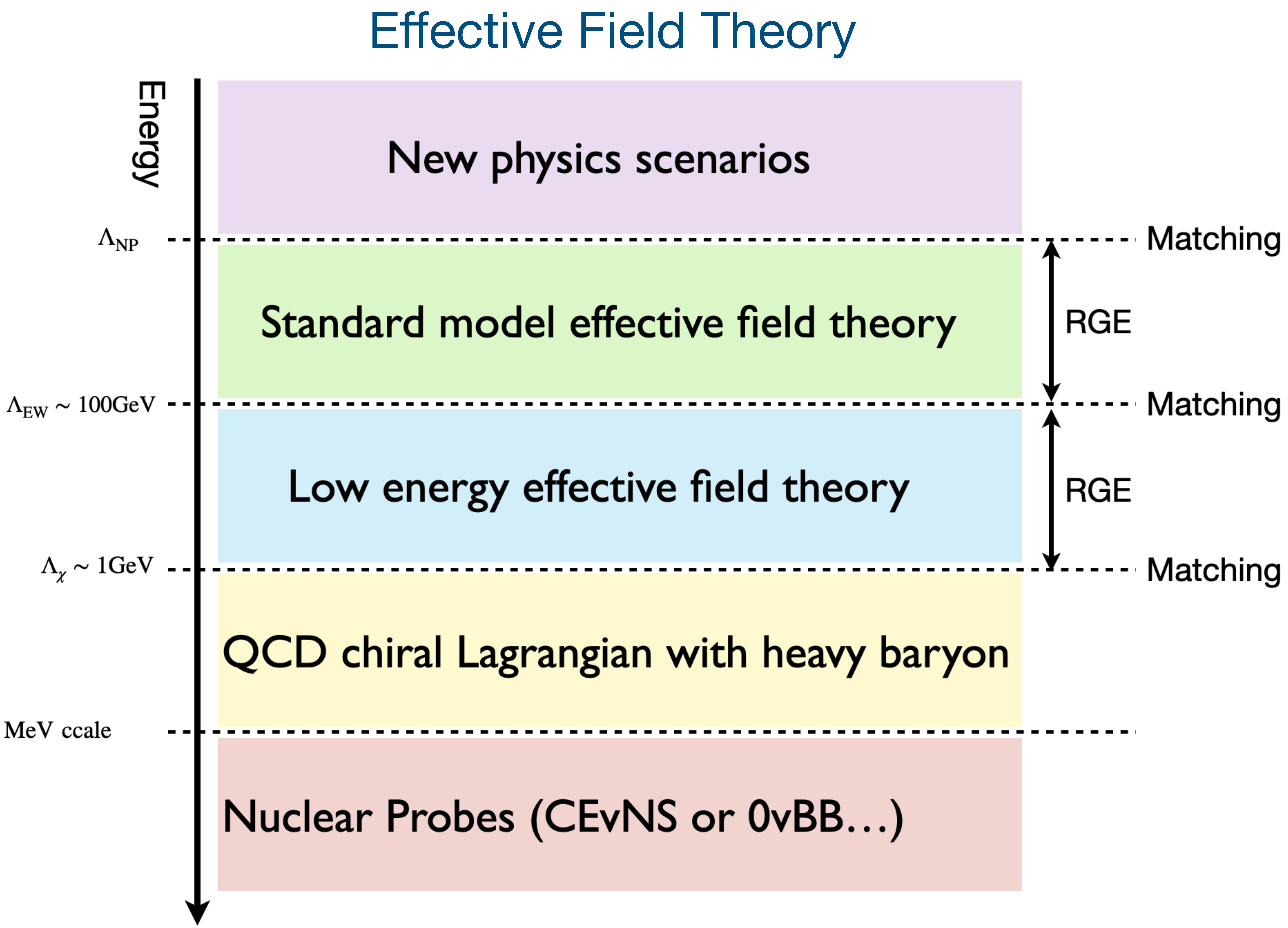
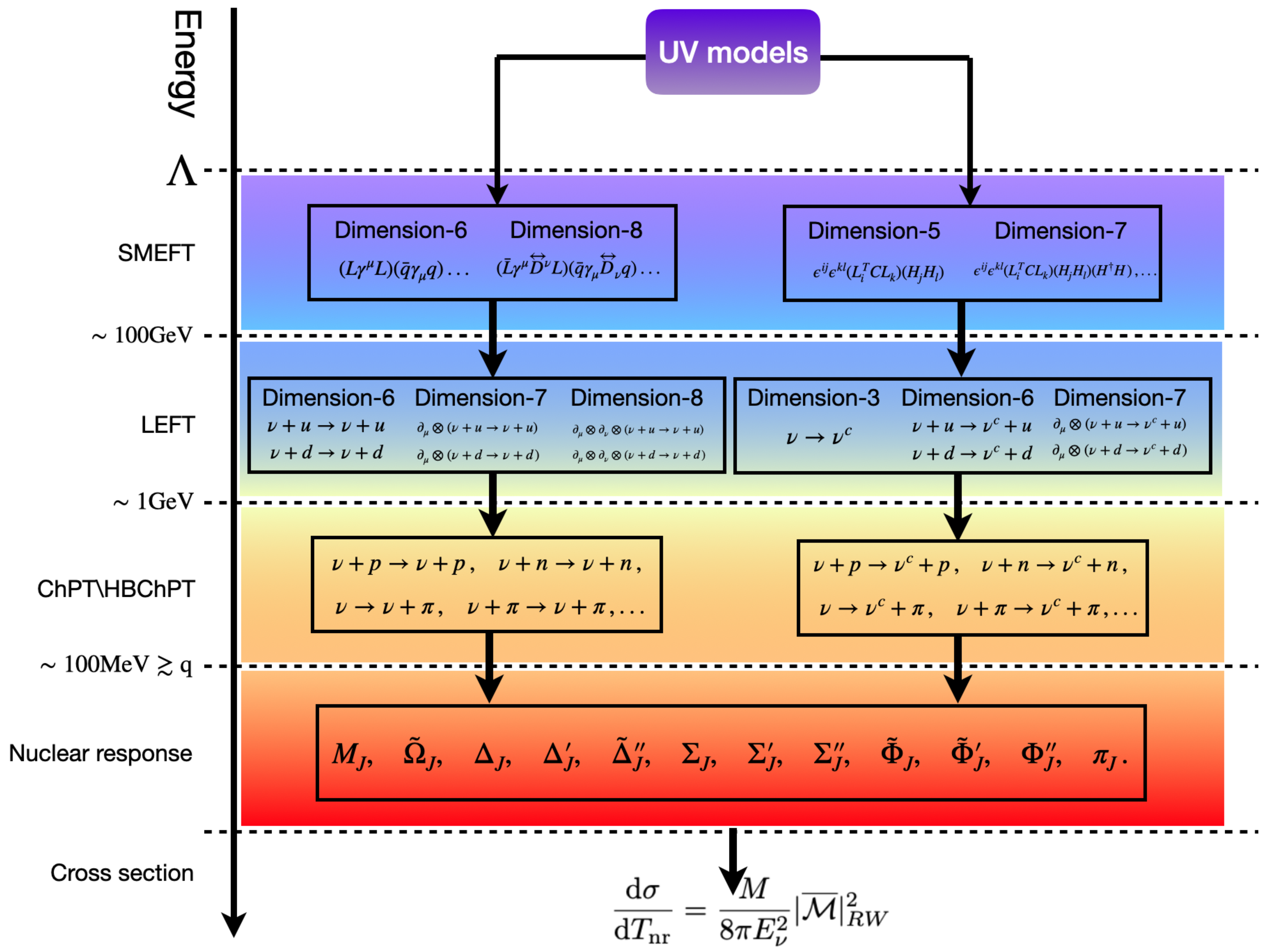
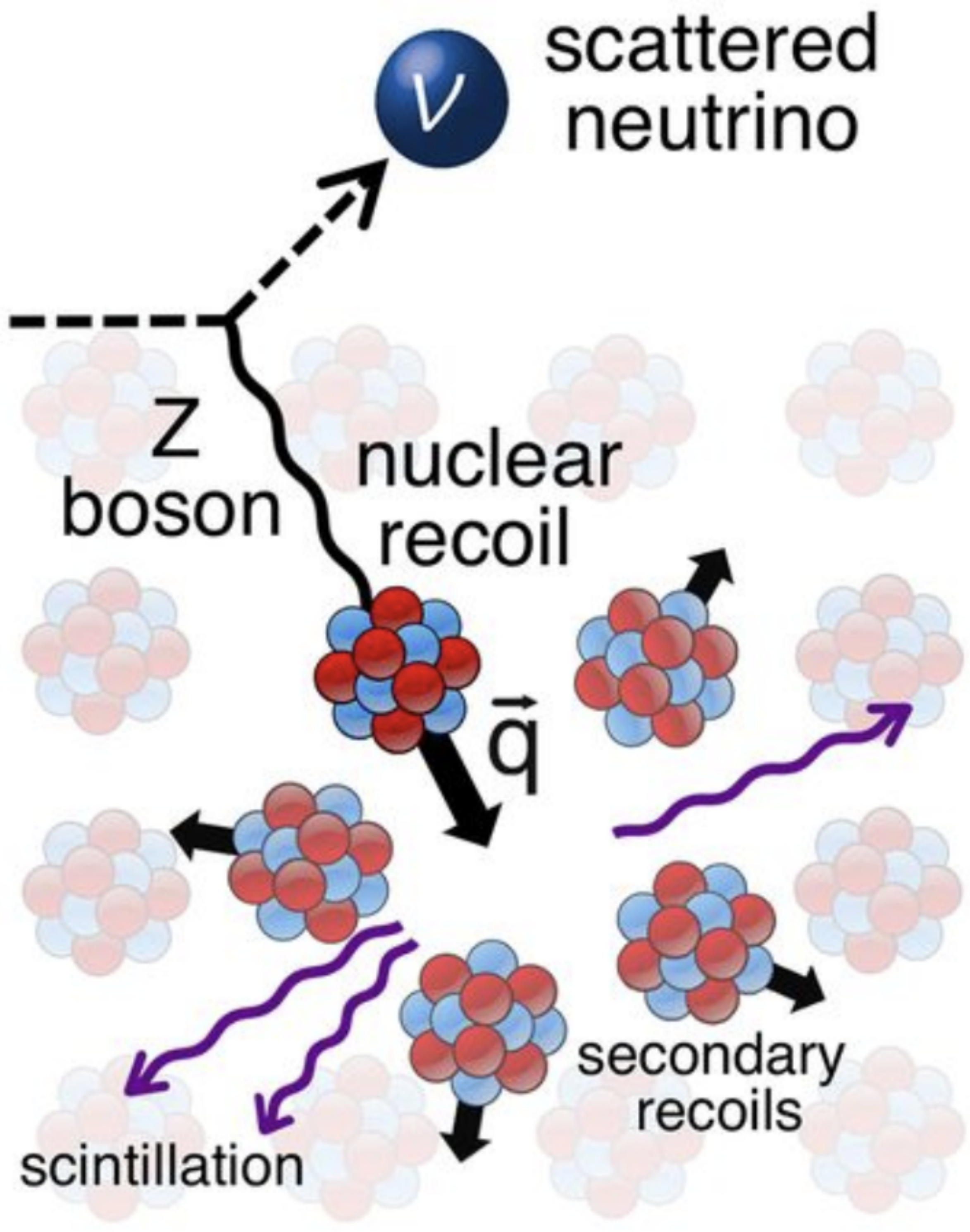
Introduction



Example: Coherent Elastic Neutrino-Nucleus Scattering

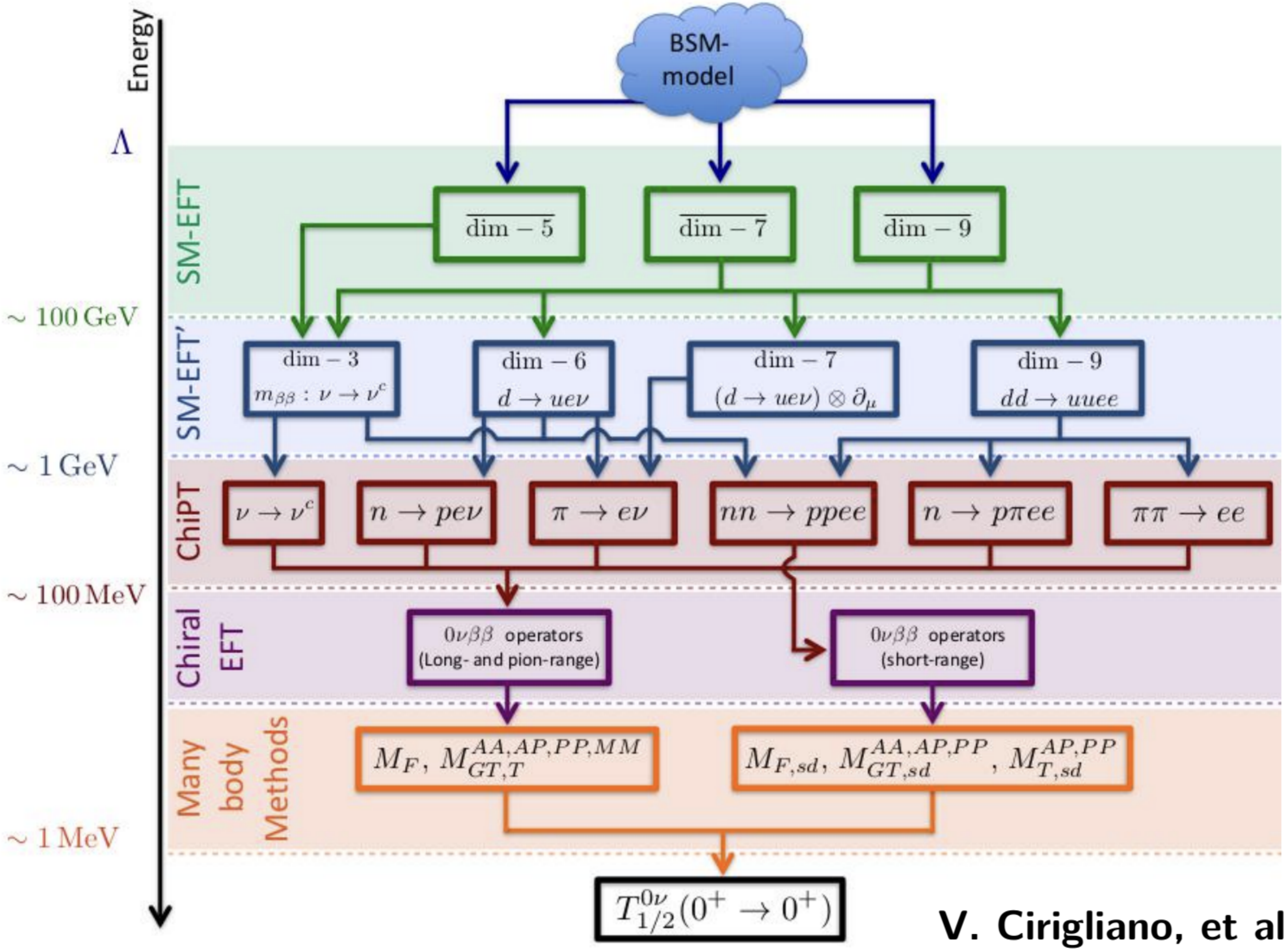


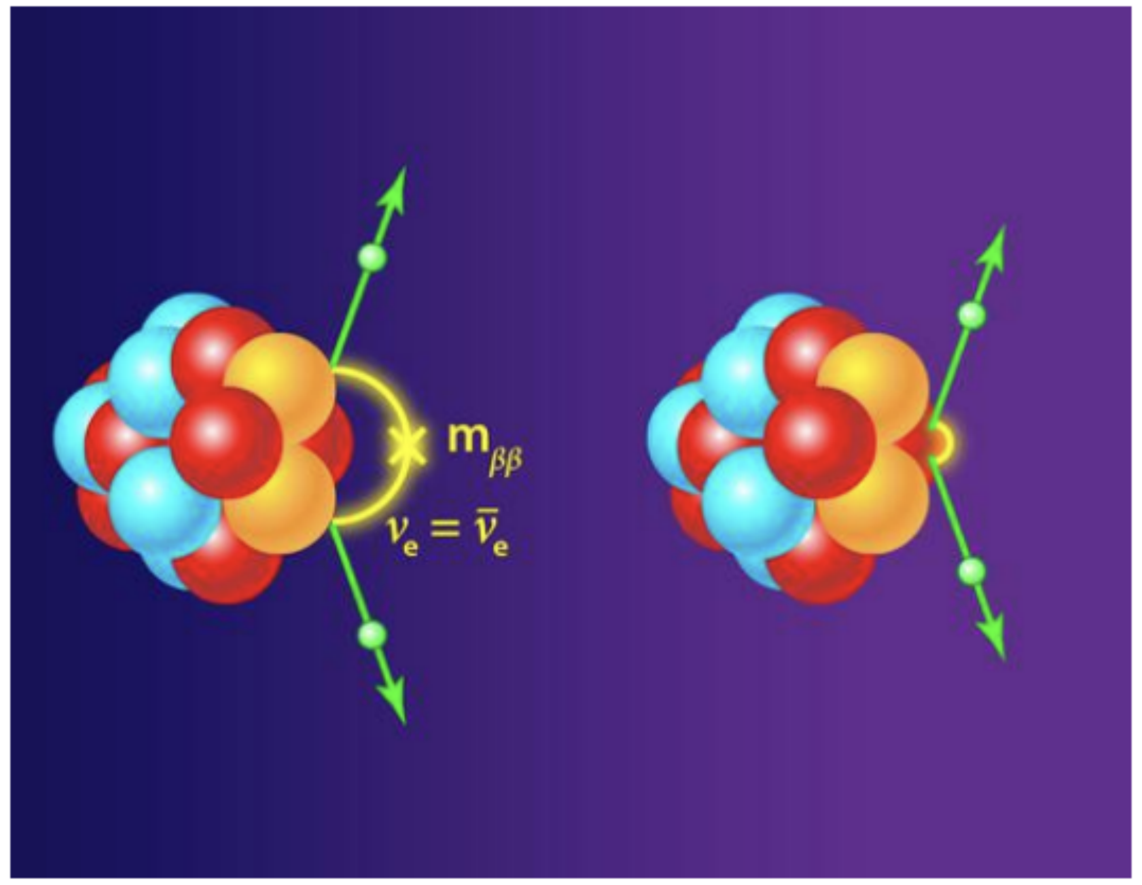


D.Z Freedman, "Coherent Neutrino Nucleus Scattering as a Probe of the Weak Neutral Current", Phys.Rev.D 9(1974) 1389-1392

COHERENT Collaboration, "Observation of Coherent Elastic Neutrino-Nuclues Scattering", Science 357 (2017) 6356, 1123-1126

Example: Neutrinoless Double Beta Decay





V. Cirigliano, et al, 1806.02780 (JHEP)

Effective Field Theory

<table>
<tr><th></th><th colspan="2">SMEFT</th><th colspan="2">LEFT</th><th colspan="2">ChPT</th></tr>
<tr><td>scale</td><td></td><td colspan="2">100 GeV</td><td colspan="2">1 GeV</td><td></td></tr>
<tr><td>operator</td><td>✓</td><td></td><td>✓</td><td></td><td>Y/N</td><td></td></tr>
<tr><td>RGE</td><td colspan="2">✓</td><td colspan="2">✓</td><td colspan="2"></td></tr>
<tr><td>matching</td><td></td><td colspan="2">✓</td><td colspan="2">Y/N</td><td></td></tr>
</table>

Y for SM interactions

N for BSM interactions

How to systematically match LEFT to ChPT?

LEFT operators and chiral operators

CEvNs LEFT operators

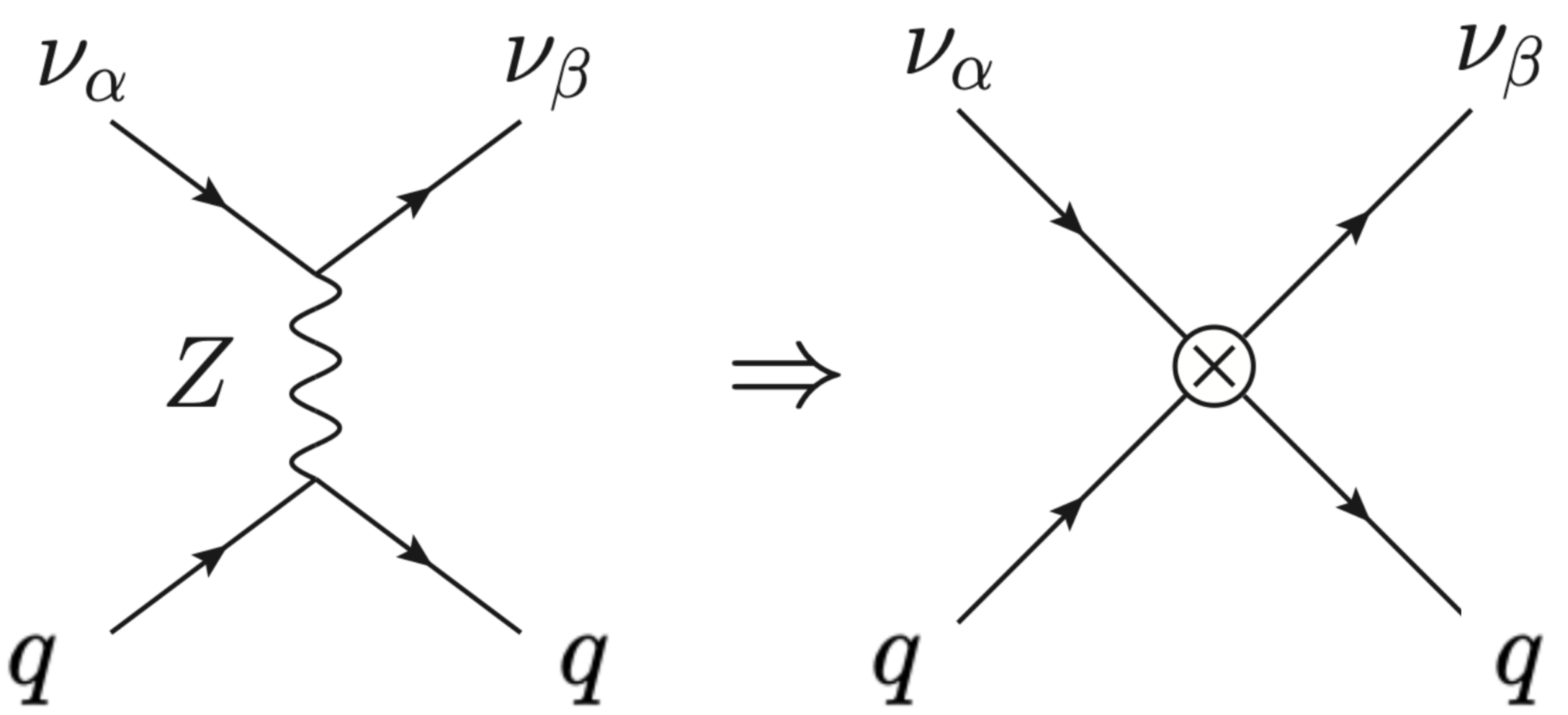
$$\mathcal{L}_{\rm LEFT} = \sum_{d,a} \hat{\mathcal{C}}_a^d \mathcal{O}_a^d\,, \quad \hat{\mathcal{C}}_a^d = \frac{\mathcal{C}_a^d}{\Lambda_{\rm EW}{}^{d-4}}\,, \qquad q = (u, d)$$

$\Delta L = 0$

$$\mathcal{O}_1^{6p/n} = (\bar\nu_{L\alpha}\gamma^\mu\nu_{L\beta})(\bar q\gamma_\mu\tau^{p/n}q)\,, \qquad \mathcal{O}_2^{6p/n} = (\bar\nu_{L\alpha}\gamma^\mu\nu_{L\beta})(\bar q\gamma^5\gamma_\mu\tau^{p/n}q)\,,$$
$$\mathcal{O}_1^{7p/n} = (\bar\nu_{L\alpha}\gamma^\mu\nu_{L\beta})(\bar q\overleftrightarrow{\partial}_\mu\tau^{p/n}q)\,, \qquad \mathcal{O}_2^{7p/n} = (\bar\nu_{L\alpha}\gamma^\mu\nu_{L\beta})(\bar q\gamma^5\overleftrightarrow{\partial}_\mu\tau^{p/n}q)\,,$$
$$\mathcal{O}_1^{8p/n} = (\bar\nu_{L\alpha}\gamma^\mu\nu_{L\beta})\partial^2(\bar q\gamma_\mu\tau^{p/n}q)\,, \qquad \mathcal{O}_2^{8p/n} = (\bar\nu_{L\alpha}\gamma^\mu\nu_{L\beta})\partial^2(\bar q\gamma^5\gamma_\mu\tau^{p/n}q)\,.$$
$$\mathcal{O}_3^{8p/n} = (\bar\nu_{L\alpha}\gamma^\mu\overleftrightarrow{\partial}^\nu\nu_{L\beta})(\bar q\gamma_\mu\overleftrightarrow{\partial}_\nu\tau^{p/n}q)\,, \qquad \mathcal{O}_4^{8p/n} = (\bar\nu_{L\alpha}\gamma^\mu\overleftrightarrow{\partial}^\nu\nu_{L\beta})(\bar q\gamma^5\gamma_\mu\overleftrightarrow{\partial}_\nu\tau^{p/n}q)\,,$$
$$\mathcal{O}_5^{8p/n} = (\bar\nu_{L\alpha}\gamma^\mu\nu_{L\beta})(\bar q\gamma^\nu T^A\tau^{p/n}q)G^A_{\mu\nu}\,, \qquad \mathcal{O}_6^{8p/n} = (\bar\nu_{L\alpha}\gamma^\mu\nu_{L\beta})(\bar q\gamma^5\gamma^\nu T^A\tau^{p/n}q)G^A_{\mu\nu}\,,$$
$$\mathcal{O}_7^{8p/n} = (\bar\nu_{L\alpha}\gamma^\mu\nu_{L\beta})(\bar q\gamma^\nu T^A\tau^{p/n}q)\tilde G^A_{\mu\nu}\,, \qquad \mathcal{O}_8^{8p/n} = (\bar\nu_{L\alpha}\gamma^\mu\nu_{L\beta})(\bar q\gamma^5\gamma^\nu T^A\tau^{p/n}q)\tilde G^A_{\mu\nu}\,,$$
$$\mathcal{O}_9^{8} = (\bar\nu_{L\alpha}\gamma^\mu\overleftrightarrow{\partial}^\nu\nu_{L\beta})G^A_{\mu\rho}G^{A\rho}_\nu\,,$$

$\Delta L = 2$

$$\mathcal{O}_1^{5} = (\nu_{L\alpha}^T C\sigma^{\mu\nu}\nu_{L\beta})F_{\mu\nu}\,, \qquad \mathcal{O}_3^{6p/n} = (\nu_{L\alpha}^T C\sigma^{\mu\nu}\nu_{L\beta})(\bar q\sigma_{\mu\nu}\tau^{p/n}q)\,,$$
$$\mathcal{O}_4^{6p/n} = (\nu_{L\alpha}^T C\nu_{L\beta})(\bar q\tau^{p/n}q)\,, \qquad \mathcal{O}_5^{6p/n} = (\nu_{L\alpha}^T C\nu_{L\beta})(\bar q\gamma^5\tau^{p/n}q)\,,$$



$$\hat{\mathcal{C}}_1^{6p/n}\Big|_{\rm SM} = \mp\frac{G_F}{\sqrt2}\left(1 - \frac{8(4)}{3}\sin^2\theta_W\right)\delta_{\alpha\beta}\,,$$
$$\hat{\mathcal{C}}_2^{6p/n}\Big|_{\rm SM} = \pm\frac{G_F}{\sqrt2}\delta_{\alpha\beta}\,,$$
$$\hat{\mathcal{C}}_1^{8p/n}\Big|_{\rm SM} = \mp\frac{G_F^2}{2}\left(1 - \frac{8(4)}{3}\sin^2\theta_W\right)\delta_{\alpha\beta}\,,$$
$$\hat{\mathcal{C}}_2^{8p/n}\Big|_{\rm SM} = \pm\frac{G_F^2}{2}\delta_{\alpha\beta}\,.$$

LEFT operators and chiral operators

0vbb LEFT operators

$$\mathcal{L}_{\Delta L=2} = -\frac{1}{2}(m_\nu)_{ij}\,\nu^T_{L,\,i}C\nu_{L,\,j} + \dots$$

$$\mathcal{L}^{(6)}_{\Delta L=2} = \frac{2G_F}{\sqrt{2}}\Bigg(C^{(6)}_{\mathrm{VL},ij}\,\bar{u}_L\gamma^\mu d_L\,\bar{e}_{R,i}\,\gamma_\mu\,C\bar{\nu}^T_{L,j} + C^{(6)}_{\mathrm{VR},ij}\,\bar{u}_R\gamma^\mu d_R\,\bar{e}_{R,i}\,\gamma_\mu\,C\bar{\nu}^T_{L,j}$$

$$+ C^{(6)}_{\mathrm{SR},ij}\,\bar{u}_L d_R\,\bar{e}_{L,i}\,C\bar{\nu}^T_{L,j} + C^{(6)}_{\mathrm{SL},ij}\,\bar{u}_R d_L\,\bar{e}_{L,i}\,C\bar{\nu}^T_{L,j} + C^{(6)}_{\mathrm{T},ij}\,\bar{u}_L\sigma^{\mu\nu}d_R\,\bar{e}_{L,i}\sigma_{\mu\nu}\,C\bar{\nu}^T_{L,j}\Bigg) + \mathrm{h.c.}$$

$$\mathcal{L}^{(7)}_{\Delta L=2} = \frac{2G_F}{\sqrt{2}v}\left(C^{(7)}_{\mathrm{VL},ij}\,\bar{u}_L\gamma^\mu d_L\,\bar{e}_{L,i}\,C\,i\overleftrightarrow{\partial}_\mu\bar{\nu}^T_{L,j} + C^{(7)}_{\mathrm{VR},ij}\,\bar{u}_R\gamma^\mu d_R\,\bar{e}_{L,i}\,Ci\overleftrightarrow{\partial}_\mu\bar{\nu}^T_{L,j}\right) + \mathrm{h.c.}$$

$$\mathcal{L}^{(9)}_{\Delta L=2} = \frac{1}{v^5}\sum_i\left[\left(C^{(9)}_{i\mathrm{R}}\,\bar{e}_R C\bar{e}^T_R + C^{(9)}_{i\mathrm{L}}\,\bar{e}_L C\bar{e}^T_L\right)O_i + C^{(9)}_i\bar{e}\gamma_\mu\gamma_5 C\bar{e}^T\,O^\mu_i\right],$$

V. Cirigliano, et al, 1806.02780 (JHEP)

LEFT operators and chiral operators

Chiral Lagrangian

The QCD Lagrangian with external source

$$\mathcal{L} = \mathcal{L}^0_{\text{QCD}} + \mathcal{L}_{\text{ext}} \ ,$$

$$\mathcal{L}_{\text{ext}} = \bar{q}\gamma^\mu \left(v_\mu + \gamma^5 a_\mu\right) q - \bar{q}\left(s - i\gamma^5 p\right) q + \bar{q}\sigma_{\mu\nu}\bar{t}^{\mu\nu} q \ ,$$

$SU(2)_L \times SU(2)_R$ chiral symmetry

symmetry breaking $\longrightarrow$

The chiral Lagrangian

$$\mathcal{L}_2 = \frac{F_0^2}{4}\mathrm{Tr}\left[D_\mu U (D^\mu U)^\dagger\right] + \frac{F_0^2}{4}\mathrm{Tr}\left(\chi U^\dagger + U\chi^\dagger\right)$$

$$\mathcal{L}^{(1)}_{\pi N} = \bar{N}_v (i\nabla^\mu v_\mu + g_A u^\mu S_\mu) N_v$$

$SU(2)_V$ symmetry

$$U(x) = \exp\left(i\frac{\phi(x)}{F_0}\right), \qquad N = \begin{pmatrix} p \\ n \end{pmatrix}$$

u basis

$$u(x) = \sqrt{U} = \exp\left(i\frac{\vec{\phi}\cdot\vec{\tau}}{2F_0}\right) \qquad u \to R u K^\dagger = K u L^\dagger, \quad u^\dagger \to L u^\dagger K^\dagger = K u^\dagger R^\dagger$$

$$\chi_\pm = u^\dagger \chi u^\dagger \pm u \chi^\dagger u,$$

$$u_\mu = i\left[u^\dagger(\partial_\mu - i r_\mu) u - u(\partial_\mu - i\ell_\mu) u^\dagger\right] \qquad \boxed{X \to K X K^\dagger, \quad K \in SU(2)_V} \qquad X = \chi_\pm, u^\mu, f_\pm^{\mu\nu}$$

$$f_\pm^{\mu\nu} = u F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu} u,$$

External source and systematic spurion method

External source

$$\mathcal{L}_{\text{ext}} = \bar{q}\gamma^\mu \left(v_\mu + \gamma^5 a_\mu\right) q - \bar{q}\left(s - i\gamma^5 p\right) q + \bar{q}\sigma_{\mu\nu}\bar{t}^{\mu\nu} q \ ,$$

where

$$\begin{aligned} u_\mu &= i\{u^\dagger(\partial_\mu - ir_\mu)u - u(\partial_\mu - i\ell_\mu)u^\dagger\} \ , \\ \chi_\pm &= u^\dagger \chi u^\dagger \pm u\chi^\dagger u \ , \\ F^\pm_{\mu\nu} &= u^\dagger F^R_{\mu\nu} u \pm u F^L_{\mu\nu} u^\dagger \ , \end{aligned}$$

$$\begin{aligned} \chi &= 2B(s+ip) \ , \\ F^{\mu\nu}_R &= \partial^\mu r^\nu - \partial^\nu r^\mu - i[r^\mu, r^\nu] \ , \quad r_\mu = v_\mu + a_\mu \ , \\ F^{\mu\nu}_L &= \partial^\mu \ell^\nu - \partial^\nu \ell^\mu - i[\ell^\mu, \ell^\nu] \ , \quad \ell_\mu = v_\mu - a_\mu \ , \end{aligned}$$

$$\mathcal{L}_2' = \frac{F_0^2}{4}\langle u^\mu u_\mu + \chi_+ \rangle$$

$$\begin{aligned} \mathcal{L}_4' = {} & L_1 \langle u_\mu u^\mu \rangle \langle u_\nu u^\nu \rangle + L_2 \langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle + L_3 \langle \chi_+ \rangle \langle u^\mu u_\mu \rangle \\ & + L_4 \left\langle f_+^{\mu\nu} u_\mu u_\nu \right\rangle + L_5 \left\langle f_+^{\mu\nu} f_{+\mu\nu} \right\rangle + L_6 \left\langle f_+^{\mu\nu} f_{+\mu\nu} \right\rangle \\ & + L_7 \langle \chi_+ \chi_+ \rangle + L_8 \langle \chi_- \chi_- \rangle + L_9 \langle \chi_+ \rangle \langle \chi_+ \rangle + L_{10} \langle \chi_- \rangle \langle \chi_- \rangle \end{aligned}$$

The external source method is limited by the interactions (V/A, S/P, T).

External source and systematic spurion method

Systematic spurion method

$$(\bar{q}_L \Gamma \Sigma_L q_L), \quad (\bar{q}_R \Gamma \Sigma_R q_R), \quad (\bar{q}_R \Gamma \Sigma q_L), \quad (\bar{q}_L \Gamma \Sigma^\dagger q_R) ,$$

$$\begin{pmatrix} q_L \\ q_R \\ \bar{q}_L \\ \bar{q}_R \\ \hat{\chi} \\ \hat{\chi}^\dagger \\ \Sigma \\ \Sigma^\dagger \\ \Sigma_L \\ \Sigma_R \\ e_L \\ e_R \\ \nu_L \\ \bar{e}_L \\ \bar{e}_R \\ \bar{\nu}_L \end{pmatrix} \to \begin{pmatrix} L q_L \\ R q_R \\ \bar{q}_L L^\dagger \\ \bar{q}_R R^\dagger \\ R \hat{\chi} L^\dagger \\ L \hat{\chi}^\dagger R^\dagger \\ R \Sigma L^\dagger \\ L \Sigma^\dagger R^\dagger \\ L \Sigma_L L^\dagger \\ R \Sigma_R R^\dagger \\ e_L \\ e_R \\ \nu_L \\ \bar{e}_L \\ \bar{e}_R \\ \bar{\nu}_L \end{pmatrix} , \quad L \in SU(2)_L \, , \; R \in SU(2)_R$$

$$\begin{aligned} \hat{\chi}_\pm &= u\, \hat{\chi}^\dagger\, u \pm u^\dagger\, \hat{\chi}\, u^\dagger \, , \\ \Sigma_\pm &= u\, \Sigma^\dagger\, u \pm u^\dagger\, \Sigma\, u^\dagger \, , \\ Q_\pm &= u^\dagger\, \Sigma_R\, u \pm u\, \Sigma_L\, u^\dagger \, . \\ \hat{u}_\mu &\equiv u_\mu|_{r_\mu=0\, ,\ell_\mu=0} \end{aligned}$$

$$\begin{pmatrix} \hat{u}_\mu \\ \Sigma_\pm \\ \chi_\pm \\ Q_\pm \\ N \\ e_L \\ e_R \\ \nu_L \\ \bar{e}_L \\ \bar{e}_R \\ \bar{\nu}_L \end{pmatrix} \to \begin{pmatrix} V \hat{u}_\mu V^\dagger \\ V \Sigma_\pm V^\dagger \\ V \chi_\pm V^\dagger \\ V Q_\pm V^\dagger \\ V N \\ e_L \\ e_R \\ \nu_L \\ \bar{e}_L \\ \bar{e}_R \\ \bar{\nu}_L \end{pmatrix} , \quad V \in SU(2)_V \, .$$

External sources can be regarded as spurions

$$\begin{aligned} (v_\mu - a_\mu)' &= L(v_\mu - a_\mu + i\partial_\mu)L^\dagger \\ (v_\mu + a_\mu)' &= R(v_\mu + a_\mu + i\partial_\mu)R^\dagger \\ (s - ip)' &= L(s - ip)R^\dagger \end{aligned}$$

J. Gasser, H. Leutwyler, Annals Phys. 158 (1984) 142

External source and systematic spurion method

Young tensor technique

Lorentz structure

$$\text{Scalar field: } \phi \in (0,0) \sim 1$$

$$\text{Left-handed spinor field: } \psi \in (\frac{1}{2},0) \sim \lambda_\alpha$$

$$\text{Right-handed spinor field: } \psi^\dagger \in (0,\frac{1}{2}) \sim \tilde{\lambda}_{\dot\alpha}$$

$$\text{Left-handed field strength tensor: } F_L = \frac{F - i\tilde{F}}{2} \in (1,0) \sim \lambda_\alpha\lambda_\beta$$

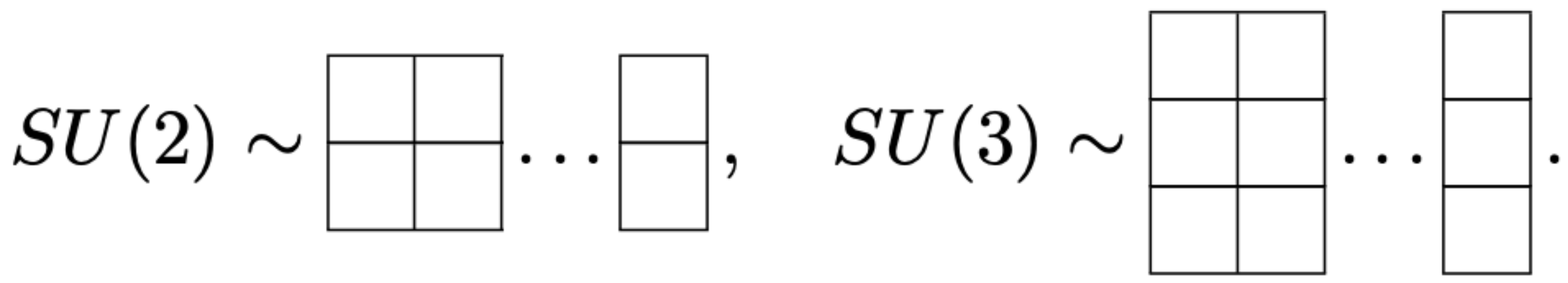
$$\text{Right-handed field strength tensor: } F_R = \frac{F + i\tilde{F}}{2} \in (0,1) \sim \tilde{\lambda}_{\dot\alpha}\tilde{\lambda}_{\dot\beta}$$

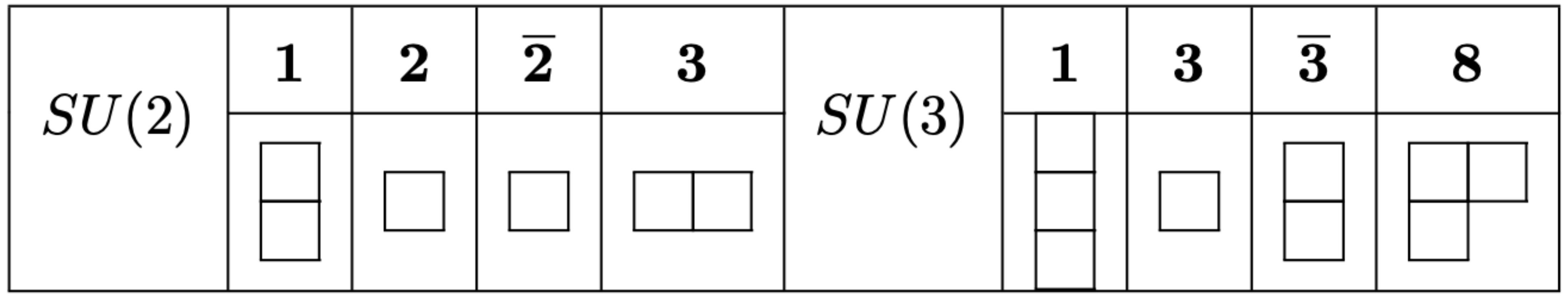
$$\text{Derivative: } D \in (1,1) \sim \lambda_\alpha\tilde{\lambda}_{\dot\alpha}\,,$$

$$\epsilon^{\alpha\beta}\lambda^i_\beta\lambda^j_\alpha = \langle ij\rangle\,, \quad \tilde{\lambda}^i_{\dot\alpha}\tilde{\lambda}^j_{\dot\beta}\epsilon^{\dot\beta\dot\alpha} = [ij]\,,$$

$$\mathcal{B} = \prod^{n}\langle ij\rangle\prod^{\tilde{n}}[kl]\,,$$

Gauge structure





Li, Ren, Xiao, Yu, Zheng, 2007.07899
Li, Ren, Xiao, Yu, Zheng, 2201.04639
Li, Ren, Shu, Xiao, Yu, Zheng, 2005.00008

SMEFT, LEFT, …..

External source and systematic spurion method

Young tensor technique for chiral Lagrangian

Lorentz structure

Fields	$SO(1,3)$
u_μ	(1/2,1/2)
$f_{\pm\mu\nu}$	(1,0)⊕(0,1)
$\tilde{f}_{\pm\mu\nu}$	(1,0)⊕(0,1)
$\Sigma_\pm$	(0,0)
$\langle\Sigma_\pm\rangle$	(0,0)
N_L	(1/2,0)
N_R	(0,1/2)

$$\text{Lorentz}: \quad \sigma^{\mu\nu}_{\alpha\beta}, \bar{\sigma}^{\mu\nu}_{\dot{\alpha}\dot{\beta}}, \sigma^{\mu}_{\alpha\dot{\alpha}}, \bar{\sigma}^{\mu\dot{\alpha}\alpha}, \epsilon^{\alpha\beta}, \tilde{\epsilon}^{\dot{\alpha}\dot{\beta}}$$

Internal structure

$SU(2)$	**1**	**2**	$\bar{\mathbf{2}}$	**3**
	$\ydiagram{1,1}$	$\ydiagram{1}$	$\ydiagram{1}$	$\ydiagram{2}$

$$SU(2): \quad \epsilon^{IJK}, \delta^{IJ}, (\tau^I)_i^j, \epsilon_{ij}, \epsilon^{ij},$$

Chiral Lagrangian

$$N = \begin{pmatrix} N_L \\ N_R \end{pmatrix} \qquad \gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}$$

$$\mathcal{B}_1 = \delta^{IJ}(N_L^\dagger \bar{\sigma}^\mu \tau^I N_L) u_\mu^J \langle\Sigma_+\rangle\,, \quad \mathcal{B}_2 = \delta^{IJ}(N_R^\dagger \sigma^\mu \tau^I N_R) u_\mu^J \langle\Sigma_+\rangle\,.$$

$$\mathcal{B}_1 + \mathcal{B}_2 = (\overline{N}\gamma^\mu u_\mu N)\langle\Sigma_+\rangle\,, \quad -\mathcal{B}_1 + \mathcal{B}_2 = (\overline{N}\gamma^5\gamma^\mu u_\mu N)\langle\Sigma_+\rangle\,,$$

$$\begin{pmatrix} \hat{u}_\mu \\ \Sigma_\pm \\ \chi_\pm \\ Q_\pm \\ N \\ e_L \\ e_R \\ \nu_L \\ \bar{e}_L \\ \bar{e}_R \\ \bar{\nu}_L \end{pmatrix} \to \begin{pmatrix} V\hat{u}_\mu V^\dagger \\ V\Sigma_\pm V^\dagger \\ V\chi_\pm V^\dagger \\ VQ_\pm V^\dagger \\ VN \\ e_L \\ e_R \\ \nu_L \\ \bar{e}_L \\ \bar{e}_R \\ \bar{\nu}_L \end{pmatrix}, \quad V \in SU(2)_V\,.$$

Song, Sun, Yu, 2404.15047 (JHEP)

The difference is the fermion part !

External source and systematic spurion method

Matching rules

The information used in the matching is the spurions, the CP properties, and the non-quark fields.

- The spurions in the LEFT operators remains unchanged in the matching.
- The CP transformation properties of the LEFT and chiral operators are the same.
- The leptons parts are identical in the LEFT operators and chiral operators.

	Σ_+	Σ_-	Q_+	Q_-	$\hat{\chi}_+$	$\hat{\chi}_-$	$\hat{u}_\mu$
P	$+$	$-$	$+$	$-$	$+$	$-$	$-$
C	$+$	$+$	$+$	$-$	$+$	$+$	$+$

External source and systematic spurion method

Naive dimension analysis

The NDA master formula of the LEFT:

$$\frac{\Lambda_{\text{EW}}^4}{16\pi^2}\left[\frac{\partial}{\Lambda_{\text{EW}}}\right]^{N_p}\left[\frac{4\pi F}{\Lambda_{\text{EW}}^2}\right]^{N_F}\left[\frac{4\pi\psi}{\Lambda_{\text{EW}}^{3/2}}\right]^{N_\psi}$$

The NDA master formula of the ChPT:

$$f^2\Lambda_\chi^2\left[\frac{\partial}{\Lambda_\chi}\right]^{N_p}\left[\frac{\psi}{f\sqrt{\Lambda_\chi}}\right]^{N_\psi}\left[\frac{F}{\Lambda_\chi f}\right]^{N_A}$$

To relate the two scales,

$$\frac{\Lambda_{\text{EW}}^4}{16\pi^2}\left[\frac{\Lambda_\chi}{\Lambda_{\text{EW}}}\right]^{N_p+2N_F+\frac{3}{2}N_\psi}\left[\frac{\partial}{\Lambda_\chi}\right]^{N_p}\left[\frac{F}{\Lambda_\chi^2}\right]^{N_F}\left[\frac{\psi}{\sqrt{\Lambda_\chi}f}\right]^{N_\psi}$$

$$=\left[\frac{\Lambda_\chi}{\Lambda_{\text{EW}}}\right]^{\mathcal{D}}\left(f^2\Lambda_\chi^2\left[\frac{\partial}{\Lambda_\chi}\right]^{N_p}\left[\frac{F}{\Lambda_\chi^2}\right]^{N_F}\left[\frac{\psi}{\sqrt{\Lambda_\chi}f}\right]^{N_\psi}\right).$$

The matching for neutrino interaction

Dimension-7 operators

$$\mathcal{O}_1^{(7)} = (\bar{\nu}_L \gamma^\mu \nu_L)[\bar{q} i \overleftrightarrow{D}_\mu (\Sigma^\dagger P_R + \Sigma P_L) q]$$

Type $\Sigma u \nu^2$: $(\bar{\nu}_L \gamma^\mu \nu_L)\langle \hat{u}_\mu \Sigma_\pm \rangle$ Wrong CP properties

Type $\Sigma u \chi \nu^2$: $(\bar{\nu}_L \gamma^\mu \nu_L)\langle \hat{u}_\mu [\chi_\pm, \Sigma_\pm] \rangle$ $\xrightarrow{\text{CP properties}}$

$$(\bar{\nu}_L \gamma^\mu \nu_L)\langle \hat{u}_\mu [\chi_+, \Sigma_-] \rangle$$
$$(\bar{\nu}_L \gamma^\mu \nu_L)\langle \hat{u}_\mu [\chi_-, \Sigma_+] \rangle$$

Type $D \Sigma u^2 \nu^2$:

$$(\bar{\nu}_L \gamma^\mu \nu_L)\langle \nabla^\nu \Sigma_\pm [u^\mu, u_\nu] \rangle$$
$$(\bar{\nu}_L \gamma^\mu \nu_L)\langle \Sigma_\pm [\nabla^\nu u_\mu, u_\nu] \rangle$$
$$(\bar{\nu}_L \gamma^\mu \nu_L)\langle \nabla^\nu \Sigma_\pm [u^\rho, u^\lambda] \rangle \epsilon_{\mu\nu\rho\lambda}$$

$\xrightarrow{\text{CP properties}}$

$$(\bar{\nu}_L \gamma^\mu \nu_L)\langle \nabla^\nu \Sigma_+ [u^\mu, u_\nu] \rangle$$
$$(\bar{\nu}_L \gamma^\mu \nu_L)\langle \Sigma_+ [\nabla^\nu u_\mu, u_\nu] \rangle$$
$$(\bar{\nu}_L \gamma^\mu \nu_L)\langle \nabla^\nu \Sigma_- [u^\rho, u^\lambda] \rangle \epsilon_{\mu\nu\rho\lambda}$$

The matching for neutrino interaction

Dimension-6 tensor operators

$$\mathcal{O}_1^{(6)} = (\bar{\nu}_L \sigma^{\mu\nu} e_R)[\bar{q}\sigma_{\mu\nu}(\Sigma^\dagger P_R + \Sigma P_L)q]$$

$$\mathcal{L}_{\rm chiral} = (\bar{\nu}_L \sigma^{\mu\nu} e_R)\langle \Sigma_+ [\hat{u}_\mu, \hat{u}_\nu]\rangle + \varepsilon_{\mu\nu\rho\sigma}(\bar{\nu}_L \sigma^{\mu\nu} e_R)\langle \Sigma_- [\hat{u}_\rho, \hat{u}_\sigma]\rangle + \ldots$$

External source method

$$\mathcal{L}_{{\rm ext},\,T} = \bar{q}\sigma_{\mu\nu}\bar{t}^{\mu\nu} q$$

$$\mathcal{L}_{4,\pi} = \Lambda_2 \langle (u^\dagger t^{\mu\nu} u^\dagger + u t^{\mu\nu\dagger} u)[\hat{u}_\mu, \hat{u}_\nu]\rangle$$

$$\begin{aligned}
\bar{t}^{\mu\nu} &= P_L^{\mu\nu\lambda\rho}\, t_{\lambda\rho} + P_R^{\mu\nu\lambda\rho}\, t^\dagger_{\lambda\rho}\,, \\
t^{\mu\nu} &= P_L^{\mu\nu\lambda\rho}\, \bar{t}_{\lambda\rho}\,, \\
P_R^{\mu\nu\lambda\rho} &= \frac{1}{4}(g^{\mu\lambda} g^{\nu\rho} - g^{\nu\lambda} g^{\mu\rho} + i\,\varepsilon^{\mu\nu\lambda\rho})\,, \\
P_L^{\mu\nu\lambda\rho} &= \left(P_R^{\mu\nu\lambda\rho}\right)^\dagger .
\end{aligned}$$

The matching for neutrino interaction

Four quark part

$$(\bar{q}_L\Gamma\,\Sigma_L q_L)(\bar{q}_L\Gamma\,\Sigma_L q_L)\,,\qquad (\bar{q}_R\Gamma\,\Sigma_R q_R)(\bar{q}_R\Gamma\,\Sigma_R q_R)\,,$$
$$(\bar{q}_L\Gamma\,\Sigma_L q_L)(\bar{q}_R\Gamma\,\Sigma_R q_R)\,,\qquad (\bar{q}_L\Gamma\,\Sigma^\dagger q_R)(\bar{q}_L\Gamma\,\Sigma q_R)\,,$$
$$(\bar{q}_L\Gamma\,\Sigma^\dagger q_R)(\bar{q}_R\Gamma\,\Sigma^\dagger q_L)\,,\qquad (\bar{q}_R\Gamma\,\Sigma q_L)(\bar{q}_R\Gamma\,\Sigma q_L)\,,$$
$$(\bar{q}_L\Gamma\,\Sigma_L q_L)(\bar{q}_R\Gamma\,\Sigma q_L)\,,\qquad (\bar{q}_L\Gamma\,\Sigma_L q_L)(\bar{q}_L\Gamma\,\Sigma^\dagger q_R)\,,$$
$$(\bar{q}_R\Gamma\Sigma_R q_R)(\bar{q}_R\Gamma\,\Sigma q_L)\,,\qquad (\bar{q}_R\Gamma\Sigma_R q_R)(\bar{q}_L\Gamma\,\Sigma^\dagger q_R)\,.$$

Chiral representation

$$(\bar{\mathbf{2}}_L\otimes\mathbf{2}_L,\bar{\mathbf{2}}_R\otimes\mathbf{2}_R)\,,$$
$$(\bar{\mathbf{2}}_L\otimes\bar{\mathbf{2}}_L,\mathbf{2}_R\otimes\mathbf{2}_R)\,,$$
$$(\mathbf{2}_L\otimes\mathbf{2}_L,\bar{\mathbf{2}}_R\otimes\bar{\mathbf{2}}_R)\,,$$
$$(\mathbf{2}_L\otimes\mathbf{2}_L\otimes\bar{\mathbf{2}}_L,\bar{\mathbf{2}}_R)\,,$$
$$(\mathbf{2}_L\otimes\bar{\mathbf{2}}_L\otimes\bar{\mathbf{2}}_L,\mathbf{2}_R)\,,$$
$$(\bar{\mathbf{2}}_L,\mathbf{2}_R\otimes\mathbf{2}_R\otimes\bar{\mathbf{2}}_R)\,,$$
$$(\mathbf{2}_L,\mathbf{2}_R\otimes\bar{\mathbf{2}}_R\otimes\bar{\mathbf{2}}_R)\,,$$
$$(\bar{\mathbf{2}}_L\otimes\mathbf{2}_L\otimes\bar{\mathbf{2}}_L\otimes\mathbf{2}_L,\mathbf{1})\,,$$
$$(\mathbf{1},\bar{\mathbf{2}}_R\otimes\mathbf{2}_R\otimes\bar{\mathbf{2}}_R\otimes\mathbf{2}_R)\,.$$

The combination of the spurions

$$\Sigma_L\sim(\mathbf{3}_L,\mathbf{1}_R)\,,\quad \Sigma_R\sim(\mathbf{1}_L,\mathbf{3}_R)\,,\quad \Sigma\sim(\bar{\mathbf{2}}_L,\mathbf{2}_R)\,,\quad \Sigma^\dagger\sim(\mathbf{2}_L,\bar{\mathbf{2}}_R)\,.$$

$$\Sigma_L\otimes\Sigma_L\,,\quad \Sigma_R\otimes\Sigma_R\,,\quad \Sigma_L\otimes\Sigma_R\,,$$
$$\Sigma\otimes\Sigma\,,\quad \Sigma^\dagger\otimes\Sigma^\dagger\,,\quad \Sigma\otimes\Sigma^\dagger\,,$$
$$\Sigma_L\otimes\Sigma\,,\quad \Sigma_L\otimes\Sigma^\dagger\,,\quad \Sigma_R\otimes\Sigma\,,\quad \Sigma_R\otimes\Sigma^\dagger\,.$$

Equivalence

The matching for neutrino interaction

Dimension-9 operators

$$O_1^{(9)} = \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\alpha \, \bar{q}_L^\beta \gamma^\mu \tau^+ q_L^\beta \,, \qquad O_1^{(9)\prime} = \bar{q}_R^\alpha \gamma_\mu \tau^+ q_R^\alpha \, \bar{q}_R^\beta \gamma^\mu \tau^+ q_R^\beta \,,$$

$$O_2^{(9)} = \bar{q}_R^\alpha \tau^+ q_L^\alpha \, \bar{q}_R^\beta \tau^+ q_L^\beta \,, \qquad O_2^{(9)\prime} = \bar{q}_L^\alpha \tau^+ q_R^\alpha \, \bar{q}_L^\beta \tau^+ q_R^\beta \,,$$

$$O_3^{(9)} = \bar{q}_R^\alpha \tau^+ q_L^\beta \, \bar{q}_R^\beta \tau^+ q_L^\alpha \,, \qquad O_3^{(9)\prime} = \bar{q}_L^\alpha \tau^+ q_R^\beta \, \bar{q}_L^\beta \tau^+ q_R^\alpha \,,$$

$$O_4^{(9)} = \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\alpha \, \bar{q}_R^\beta \gamma^\mu \tau^+ q_R^\beta \,,$$

$$O_5^{(9)} = \bar{q}_L^\alpha \gamma_\mu \tau^+ q_L^\beta \, \bar{q}_R^\beta \gamma^\mu \tau^+ q_R^\alpha \,,$$

$$O_6^{\mu(9)} = \left(\bar{q}_L \tau^+ \gamma^\mu q_L\right)\left(\bar{q}_L \tau^+ q_R\right) \,, \qquad O_6^{\mu(9)\prime} = \left(\bar{q}_R \tau^+ \gamma^\mu q_R\right)\left(\bar{q}_R \tau^+ q_L\right) \,,$$

$$O_7^{\mu(9)} = \left(\bar{q}_L T^A \tau^+ \gamma^\mu q_L\right)\left(\bar{q}_L T^A \tau^+ q_R\right) \,, \qquad O_7^{\mu(9)\prime} = \left(\bar{q}_R T^A \tau^+ \gamma^\mu q_R\right)\left(\bar{q}_R T^A \tau^+ q_L\right) \,,$$

$$O_8^{\mu(9)} = \left(\bar{q}_L \tau^+ \gamma^\mu q_L\right)\left(\bar{q}_R \tau^+ q_L\right) \,, \qquad O_8^{\mu(9)\prime} = \left(\bar{q}_R \tau^+ \gamma^\mu q_R\right)\left(\bar{q}_L \tau^+ q_R\right) \,,$$

$$O_9^{\mu(9)} = \left(\bar{q}_L T^A \tau^+ \gamma^\mu q_L\right)\left(\bar{q}_R T^A \tau^+ q_L\right) \,, \qquad O_9^{\mu(9)\prime} = \left(\bar{q}_R T^A \tau^+ \gamma^\mu q_R\right)\left(\bar{q}_L T^A \tau^+ q_R\right)$$

basis trans. $\Longrightarrow$

$$O_{XY} = \left(\bar{q} X \tau^+ q\right)\left(\bar{q} Y \tau^+ q\right), \quad X, Y = V, A, S, P$$

$$V = \gamma^\mu, A = \gamma^\mu \gamma^5, S = 1, P = \gamma^5$$

CP property	four-quark operator	p^0	p^2
$C+P+$	O_{VV}	$\langle Q_- Q_- \rangle$	$\langle Q_- \hat{u}^\mu Q_- \hat{u}_\mu \rangle$
$C+P+$	O_{AA}	$\langle Q_+ Q_+ \rangle$	$\langle Q_+ \hat{u}^\mu Q_+ \hat{u}_\mu \rangle$
$C-P-$	O_{VA}	$\langle Q_- Q_+ \rangle$	$\langle Q_- \hat{u}^\mu Q_+ \hat{u}_\mu \rangle$

$$\langle Q_+ Q_+ \rangle = 2 \left\langle U^\dagger \tau^+ U \tau^+ \right\rangle$$

$$\langle Q_- Q_- \rangle = -2 \left\langle U^\dagger \tau^+ U \tau^+ \right\rangle$$

$$\langle Q_- Q_+ \rangle = \left\langle U^\dagger \tau^+ U \tau^+ \right\rangle - \left\langle U \tau^+ U^\dagger \tau^+ \right\rangle = 0$$

four-quark operators: $0\nu\beta\beta$ decay **M. L. Graesser, 1606.04549 (JHEP)**

Summary

- We propose a systematic spurion method for the matching of LEFT to ChPT, which is particularly useful for LEFT at higher dimensions and for ChPT at higher orders of p.

- This method avoids the redundancy for ChPT at higher orders of p by Young tensor technique.

- The systematic spurion matching is illustrated for the LEFT operators relevant to neutrino physics.

Thanks !