

Recent advances in ab initio modeling nuclear matrix elements for weak processes: progress and perspectives

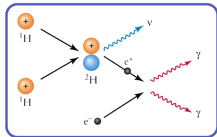
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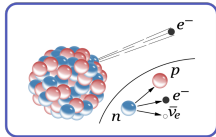
第二届中微子散射：理论、实验与唯象研讨会 (vSTEP 2025)
Oct 25, 2025, ITP, Beijing

- ① Introduction to nuclear weak processes
- ② Ab initio studies of single- β decay
 - g_A quenching problem and 2BC effect
 - Superallowed Fermi transitions
- ③ Ab initio studies of $0\nu\beta\beta$ decay
 - Lepton-number violation and $0\nu\beta\beta$ decay
 - The 2BC effect on NMEs
 - Modeling NMEs with operators from chiral EFT
- ④ Summary and outlook

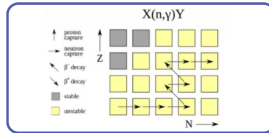
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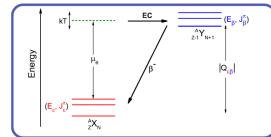
Proton-proton fusion



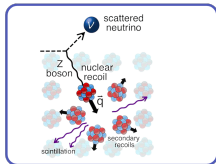
Nuclear beta decay



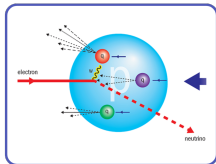
r-process



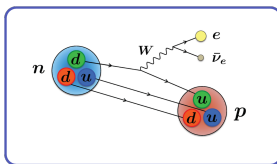
Urca process



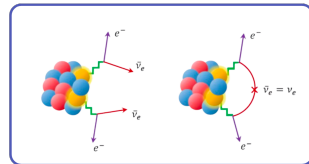
CEνNS



PV eN scattering



Superaligned Fermi transition



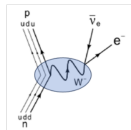
Double beta decay

- Nuclear weak processes are essential for understanding nuclear stability, nucleosynthesis, and new physics beyond the Standard Model.
- Nuclear matrix elements are crucial for the interpretation of experimental signals.

□ The studies on nuclear matrix elements (NME)

- Single-beta decay

$$T_{1/2}^{-1} = g_A^2 G_\beta |M_{fi}|^2 \frac{1}{m_e^2}$$

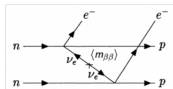


- Superalowed Fermi transition

$$M_F^2 = 2(1 - \delta_C)$$

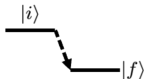
- Neutrinoless double-beta decay

$$T_{1/2}^{-1} = g_A^4 G_{0\nu} |M_{fi}|^2 \left| \frac{\langle m_{\beta\beta} \rangle}{m_e} \right|^2$$



where the NME is given by

$$M_{fi} = \langle f | \hat{O} | i \rangle$$



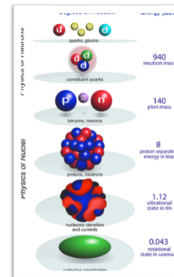
□ Towards an accurate calculation of NME

- “bridge” linking signals and their interpretation
- **Transition operators:** determined by weak interactions
- **Wave functions:** obtained from nuclear structure models

□ Challenges in Nuclear Many-Body Theory

- **Strong interaction:** *non-perturbative* at nuclear energy scales
- **Many-body problem:** complex

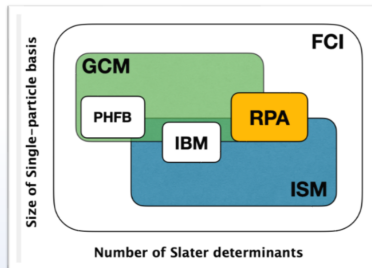
Accurately solving **nuclear wave functions** remains a **major challenge**, which **directly affects** the precision of NME calculations.



Status of theoretical studies

□ Frequently employed nuclear models

- Interacting shell model (ISM)
- Proton-neutron random-phase approx. (pn-QRPA)
- Projected shell model (PSM)、GCM
- Other methods: IBM, Mac-Mic model, etc.



Problems to be resolved

□ Uncertainty from nuclear forces

- EDFs: parameters of pairing force and tensor force, etc.
- ISM: parameters of the interaction matrix elements

□ Uncertainty from truncations in model space

- EDF: large s.p. basis, missing many-body correlation
- ISM: full many-body correlation, small model space

□ Uncertainty from transition operators:

- axial-vector coupling g_A : quenching factor
- Higher-order operators: two-body currents, etc.

Large model dependence in predicted NMEs, difficult to constrain or reduce.

□ Basic idea and advantages

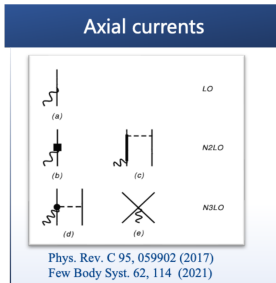
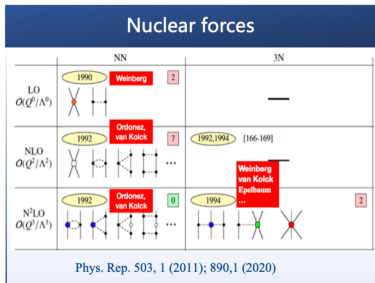
- Nuclear forces & transition operators: consistently constructed within **chiral EFT** (N, π)
- The theoretical uncertainty: controlled by the power-counting $(Q/\Lambda)^n$ with $Q \approx 140$ MeV, $\Lambda \approx 700$ MeV)
- Nuclear wave functions: solved using **exact or systematically improvable many-body methods**



● Nuclear ab initio methods

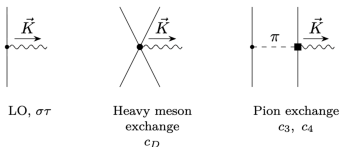
- ✓ 无芯壳模型 (NCSM)
- ✓ [介质相似重整化群方法 \(IMSRG\)](#)
- ✓ 格林函数蒙特卡罗方法 (GFMC)
- ✓ 耦合团簇理论 (CC)
- ✓ 多体微扰论方法 (MBPT)
- ✓ Brueckner Hartree-Fock (BHF)
- ✓ ...

Frontiers in Physics 8, 379 (2020)



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One and two-body current (2BC)



$$\mathbf{J}_{i,1b}(p^2) = g_A(p^2) \boldsymbol{\sigma}_i - g_P(p^2) \frac{\mathbf{p}(\mathbf{p} \cdot \boldsymbol{\sigma}_i)}{2m} + i(g_M + g_V) \frac{\boldsymbol{\sigma}_i \times \mathbf{p}}{2m} - g_V \frac{\mathbf{p}}{2m}$$

$$\mathbf{J}_{12} = -\frac{g_A}{F_\pi^2} \left[2d_1(\boldsymbol{\sigma}_1\tau_1^- + \boldsymbol{\sigma}_2\tau_2^-) + d_2 \boldsymbol{\sigma}_\times \tau_\times^- \right] - \frac{g_A}{2F_\pi^2} \frac{1}{m_\pi^2 + k^2} \left[\left(c_4 + \frac{1}{4m} \right) \mathbf{k} \times (\boldsymbol{\sigma}_\times \times \mathbf{k}) \tau_\times^- + 4c_3 \mathbf{k} \cdot (\boldsymbol{\sigma}_1\tau_1^- + \boldsymbol{\sigma}_2\tau_2^-) \mathbf{k} - \frac{i}{2m} \mathbf{k} \cdot (\boldsymbol{\sigma}_1 - \boldsymbol{\sigma}_2) \mathbf{q} \tau_\times^- \right]$$

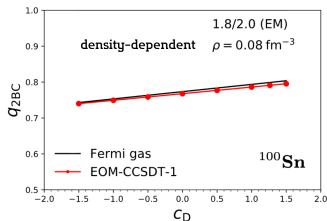
Note: $\boldsymbol{\sigma}_\times = \boldsymbol{\sigma}_1 \times \boldsymbol{\sigma}_2$

Normal-order one-body (NO1B)

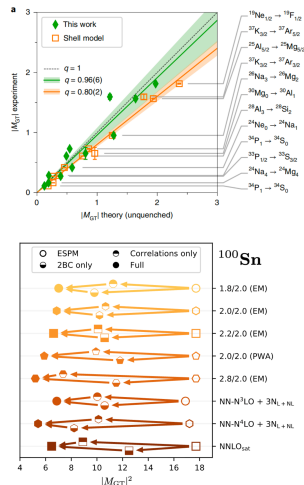
Effective one-body current: (Fermi gas)

J. Menendez et al., PRL107, 062501 (2011)

$$\mathbf{J}_{i,2b}^{\text{eff}} = -g_A \boldsymbol{\sigma}_i \tau_i^- \left[\frac{\rho}{F_\pi^2} \left[g_A \Lambda_\chi + \frac{2}{3} c_3 \frac{\mathbf{p}^2}{4m_\pi^2 + \mathbf{p}^2} + I(\rho, P) \left(\frac{1}{3} (2c_4 - c_3) + \frac{1}{6m} \right) \right] \right]$$

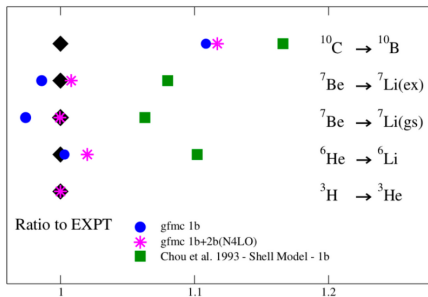


P. Gysbers et al., Nature Physics 15, 428 (2019)



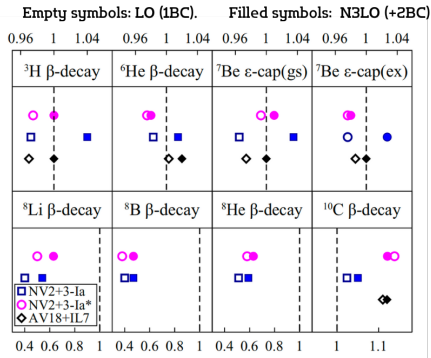
The Quantum Monte Carlo calculation

AV18+IL7 Hamiltonian



Pastore, et al., PRC 97, 022501 (2018).

Norfolk 2N+3N chiral interactions



G. B. King et al., PRC 102, 025501 (2020)

The 2BC increases the NME by 20–30%

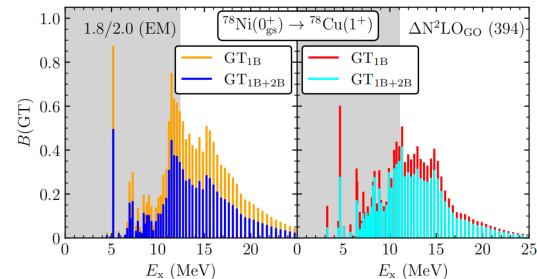
Open question: The 2BC effect is scale/scheme-dependent?

The 2BC effect on GT transition and beta-decay half-life

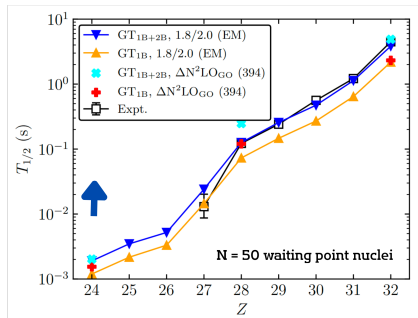
$$B(\text{GT}) = \frac{1}{2J_i + 1} |\langle J_f || \hat{O}_{\text{GT}} || J_i \rangle|^2$$

For the case dominated by the single-state transition

$$fT_{1/2} = \frac{\kappa}{B(\text{F}) + B(\text{GT})}$$



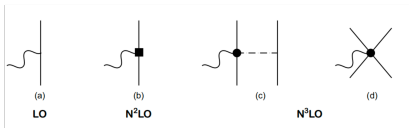
The 2B currents systematically reduces the B(GT) across the whole energy region



The 2B currents leads to longer total half-lives

Z. Li, T. Miyagi, A. Schwenk, arXiv:2509.06812 [nucl-th]

Reduced Gamow-Teller matrix element (RGTM)

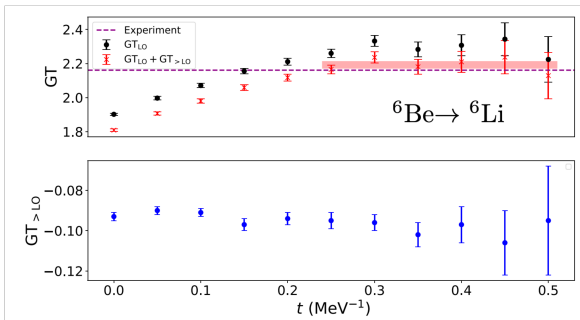


$$\mathcal{J} = \mathcal{J}_{\text{LO}} + \mathcal{J}_{\text{N}^2\text{LO}} + \mathcal{J}_{\text{N}^3\text{LO}}$$

$$\text{GT} = \frac{\sqrt{2J_f + 1}}{g_A} \frac{\langle J_f M_f | \mathcal{J} | J_i M_i \rangle}{\langle J_i M_i, 11 | J_f M_f \rangle}.$$

$$= \text{GT}_{\text{LO}} + \text{GT}_{\text{N}^2\text{LO}} + \text{GT}_{\text{N}^3\text{LO}}$$

Results from the lattice EFT calculation



	LO	N ² LO	N ³ LO(OPE)	N ³ LO(CT)	Total-LO	Total	Expt. [96]
RGTME	2.289(19)	-0.024(2)	-0.104(1)	0.032(1)	-0.096(2)	2.192(20)	2.161(4)

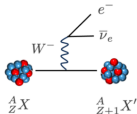
The 2B currents reduces the RGTME less than 5% .

T. Wang, X. Feng, B. N. Lu, PRC112, 025502 (2025)

Superaligned Fermi transition between isobaric analog states

- A pure Fermi transition between two nuclear states that belong to the same isospin multiplet

$$\Delta T = 0, \quad \Delta T_z = \pm 1, \quad \Delta J = 0, \quad \Delta \pi = 0$$



- The CKM matrix

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Unitarity of the first row of the CKM matrix

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 \pm 10^{-3} ?$$

- The beta-decay half-life

$$\frac{1}{ft} = \frac{G_F^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2} (1 + \Delta_R^V) (1 + \delta'_R) (1 - \delta_C + \delta_{NS})$$

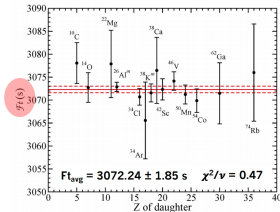
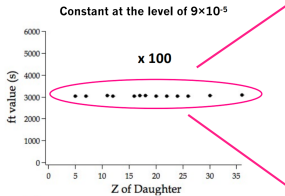
Radiative correction on single-nucleon

Nucleus-dependent radiative correction

QED correction (electron energy, nuclear charge)

isospin symmetry breaking

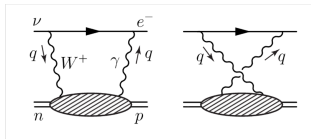
$$M_F = \langle f | \tau^+ | i \rangle = M_F^{(0)} (1 - \delta_C/2)$$



J. C. Hardy and I. S. Towner, PRC102, 045501(2020)

$$\mathcal{F}t = ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C)$$

Hadronic γW -box contribution to the radiative correction



$$\Delta_R^V = 0.02361(38)$$



$$\Delta_R^V = 0.02467(22)$$

C. Y. Seng et al., PRL121, 241804 (2018)

$$|V_{ud}|^2 = \frac{2984.432(3) \text{ s}}{\mathcal{F}t(1 + \Delta_R^V)}$$

$$\rightarrow V_{ud} = 0.97361(5)_{\text{exp}}(6)_{\delta'_R}(4)_{\delta_C}(28)_{\delta_{NS}}(10)_{\Delta_R^V}$$

$$|V_{us}| = 0.2243(5); |V_{ub}| = 0.00394(36)$$

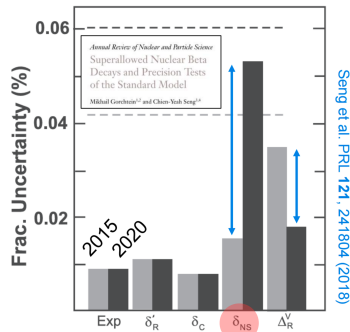
$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9983(4)$$

Underestimated SM uncertainties or BSM physics?

2.5 σ deviation from unity!!!

Uncertainties in the corrections

Hardy et al. PRC 102, 045501 (2020)

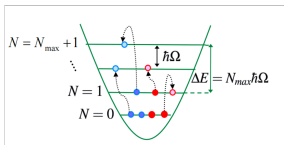


The largest contributor to the uncertainty is shifted to the nucleus-dependent correction.

Precision determination of this correction becomes crucial.

ab initio calculation of nuclear-structure-dependent radiative correction

NCSM+chiral NN+3N interaction



$$\Delta_R^V + \delta_{\text{NS}} = 2 \square_{\gamma W}^{b, \text{nuc}} + \dots,$$

$$\delta_{\text{NS}} = 2 \left\{ (\square_{\gamma W}^{b, \text{nuc}})_{\text{ab}} - (\square_{\gamma W}^{b, n})_{\text{el}} + \delta(\square_{\gamma W}^{b, n})_{\text{sh}} \right\}$$

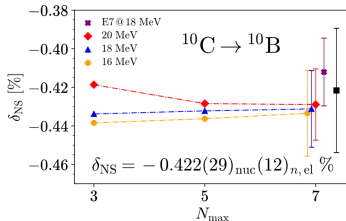
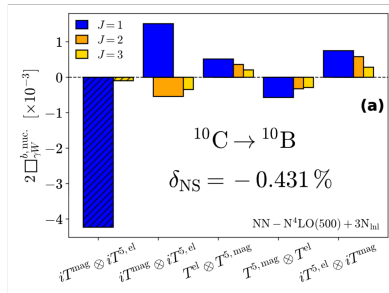
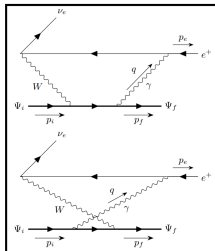
$$\delta_{\text{NS}} = -0.422(14)_{\text{PME}}(4)_{\Omega}(9)_{\chi}(24)_{\text{sh}}(12)_{n, \text{el}} \%$$

compared to the prediction of phenomenological shell models

$$\delta_{\text{NS}} = -0.400(50) \%$$

1.6x reduction in the total uncertainty.

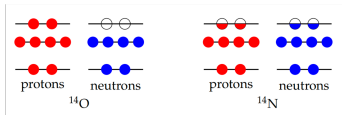
M. Gennari et al., PRL134, 012501 (2025)



The isospin-symmetry breaking correction from the VS-IMSRG calculation

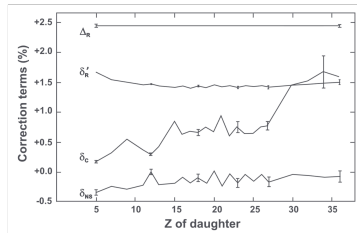
$$\delta_C = 1 - |M_F|^2/2$$

$$M_F = \langle \Psi_f || \tau || \Psi_i \rangle$$

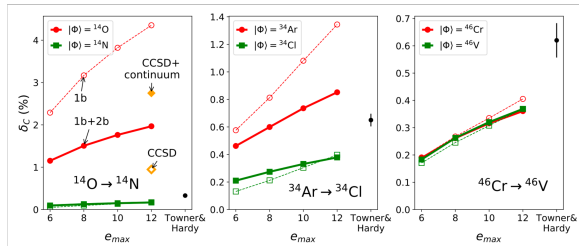


The results shows dramatic reference-dependence and 2BC effect.

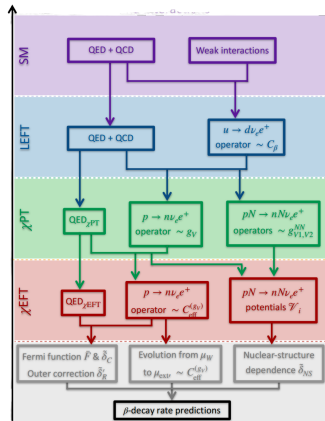
S. R. Stroberg, arXiv:2109.13462v1 [nucl-th]



J. C. Hardy and I. S. Towner (2020)

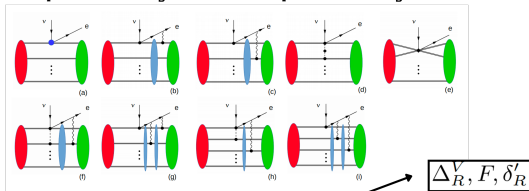


EFT-based master formula for superallowed Fermi transition



V. Cirigliano et al., PRC110, 055502 (2024)

Representative diagrams for RC to superallowed decays



- The half-life

$$\frac{1}{t} = \frac{G_F^2 |V_{ud}|^2 m_e^5}{\pi^3 \log 2} [C_{\text{eff}}^{(g_V)}(\mu)]^2 \times [1 + \bar{\delta}'_R(\mu)] (1 + \bar{\delta}_{\text{NS}}) (1 - \bar{\delta}_C) \bar{f}(\mu),$$

- The phase-space factor $\bar{f}(\mu) = \frac{1}{m_e^5} \int_{m_e}^{E_0} dE_e w_0(E_e) \tilde{C}(E_e) \bar{F}(\beta, \mu)$,

- The QMC calculation $^{14}\text{O} \rightarrow ^{14}\text{N}$

LEC of the CT term

$$V_{ud} = 0.97364(10)_{\text{exp}}(12)_{g_V}(22)_{\mu}(12)_{\delta_C}(43)_{g_V^{NN}}(20)_{\delta_{\text{NS}}^E}$$

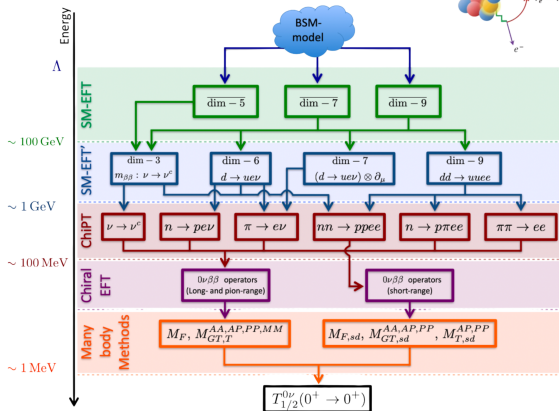
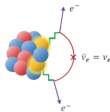
in comparison Ref. [J. C. Hardy and I. S. Towner, PRC102, 045501(2020)]

$$V_{ud}^{[3]}[^{14}\text{O}] = 0.97405(13)_{\text{exp}}(9)_{\Delta_R^V}(12)_{\delta_C}(31)_{\delta_{\text{NS}}} [37]_{\text{total}},$$

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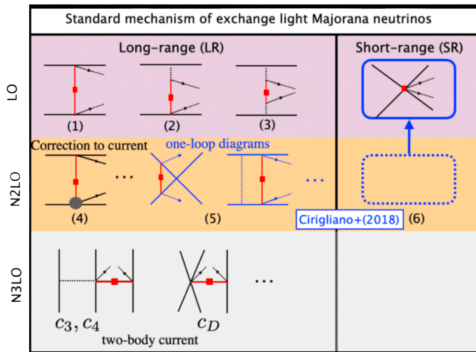
Lepton-number violating operators for neutrinoless double-beta decay

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + (2\bar{\nu}_e)$$



The $0\nu\beta\beta$ decay operators in the chiral EFT

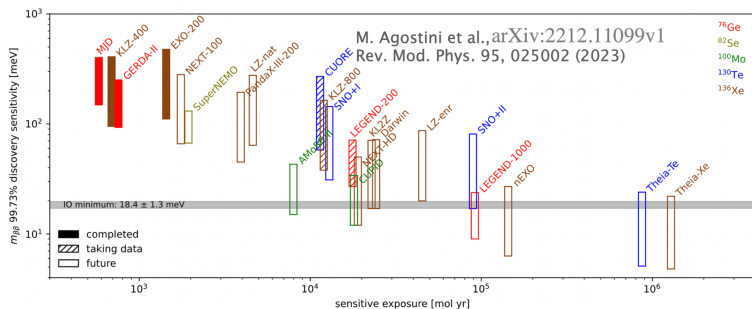
Power Counting: $\nu = 2A + 2L - 2 + \sum_i \left(\frac{n_f}{2} + d - 2 + n_e \right)_i$



- The $0\nu\beta\beta$ decay rate in the standard mechanism:

$$\langle m_{\beta\beta} \rangle \equiv \left| \sum_{j=1}^3 U_{ej}^2 m_j \right| = \left[\frac{m_e^2}{g_A^4 G_{0\nu} T_{1/2}^{0\nu} |M^{0\nu}|^2} \right]^{1/2}$$

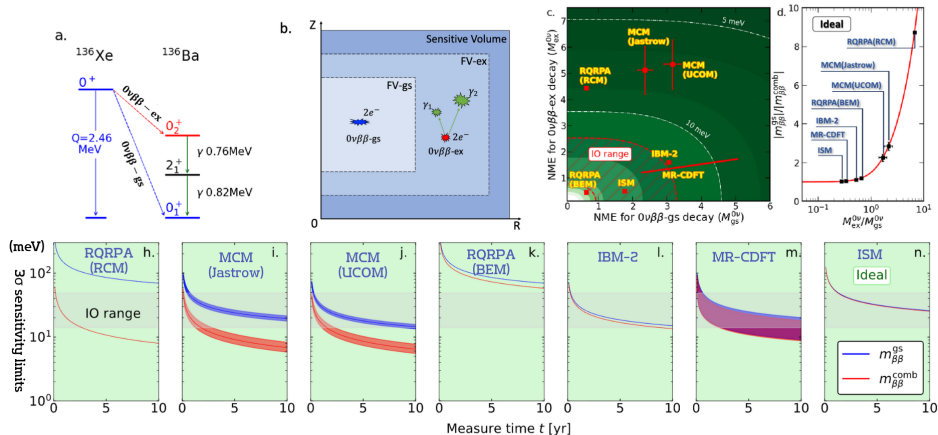
- Whether or not the ton-scale experiments are able to cover the entire parameter space for the IO case depends strongly on the accuracy of the NME.



A strategy to enhance the sensitivity of $0\nu\beta\beta$ decay

Combined multi-transition analysis

10-year data acquisition on XLZD and the PandaX-xT



The effectiveness strongly depends on the NMEs, which currently exhibit significant discrepancies across different nuclear models.

Ding, Han, Wang, JMY, arXiv2508.17413 [nucl-th]

A proposal to reduce the uncertainty of the NMEs

- angular distribution of heavy-ion collision

$$\frac{dN_{\text{ch}}}{d^2\mathbf{p}} \propto \frac{dN_{\text{ch}}}{p_T dp_T} \left(1 + \sum_{n>1} 2v_n \cos n(\phi - \phi_n) \right)$$

where v_2 is strongly correlated with the initial-state ellipticity parameter, ε_2

- observables of interest

$$\Delta(v_2\{2\}) \equiv \frac{1}{2} \Delta(\langle v_2^2 \rangle),$$

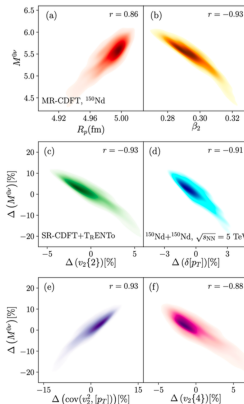
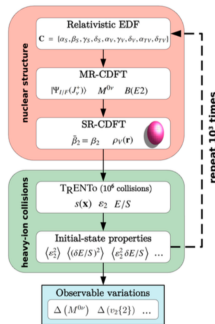
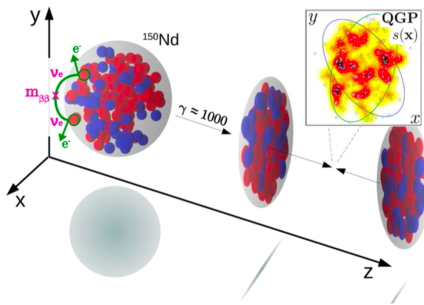
$$\Delta(\delta[p_T]) \equiv \frac{1}{2} \Delta(\langle (\delta[p_T])^2 \rangle),$$

$$\Delta(\text{cov}(v_2^2, [p_T])) \equiv \frac{1}{3} \Delta(\langle v_2^2 \delta[p_T] \rangle),$$

$$\Delta(v_2\{4\}) \equiv \frac{1}{4} \Delta(2\langle v_2^2 \rangle^2 - \langle v_2^4 \rangle).$$

- Relative variation of an observable

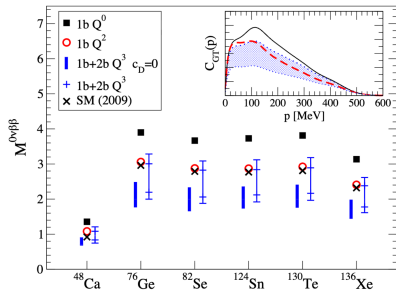
$$\Delta(O) = \frac{O - \langle O \rangle_c}{|\langle O \rangle_c|}$$



Valence-space shell model calculation

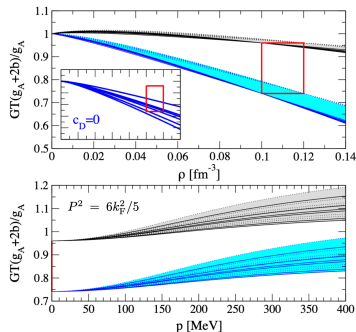
Fermi gas approximation for the core

$$\begin{aligned}
 J_{i,2b}^{\text{eff}} = & -g_A \sigma_i \tau_i^- \frac{\rho}{F_\pi^2} \left[\frac{c_D}{g_A \Lambda_\chi} + \frac{2}{3} c_3 \frac{\mathbf{p}^2}{4m_\pi^2 + \mathbf{p}^2} \right. \\
 & \left. + I(\rho, P) \left(\frac{1}{3} (2c_4 - c_3) + \frac{1}{6m} \right) \right],
 \end{aligned}$$



The 2BC reduces the NME from 10% to 35%

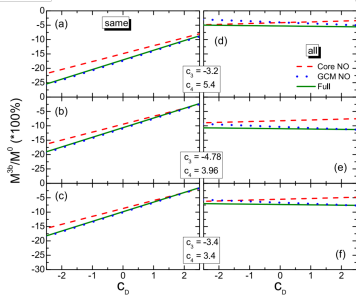
$$\begin{aligned}
 I(\rho, P) = & 1 - \frac{3m_\pi^2}{2k_F^2} + \frac{3m_\pi^3}{2k_F^3} \arccot \left[\frac{m_\pi^2 + \frac{P^2}{4} - k_F^2}{2m_\pi k_F} \right] \\
 & + \frac{3m_\pi^2}{4k_F^3 P} \left(k_F^2 + m_\pi^2 - \frac{P^2}{4} \right) \log \left[\frac{m_\pi^2 + (k_F - \frac{P}{2})^2}{m_\pi^2 + (k_F + \frac{P}{2})^2} \right].
 \end{aligned}$$



J. Menendez et al., PRL107, 062501 (2011)

Projected GCM calculation with a shell-model interaction

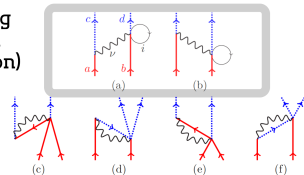
$$M^{3b} = - \sum_{abcdef} \langle abc | \hat{O}_{3b} | def \rangle \rho_{abc,def}^{3b}$$



- Only (a, b) are considered, quenching 5%–25%
- All the (a)-(f) are considered, quenching about 10%

L. J. Wang, J. Engel, JMY, PRC98, 031301 (2018)

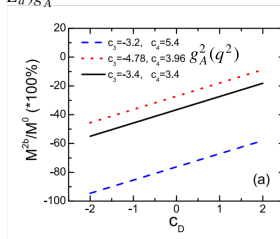
normal-ordering
approximation
(HF wave function)



product of one-body and two-body currents

$$\hat{O}_{c_D}^{2b} = \frac{2c_D R}{\pi m_N F_\pi^2} \sum_{k \neq l}^A \int dq \frac{[q g_A^2(q^2) - \frac{q^3 g_A(q^2) g_F(q^2)}{6m_N}]}{(q + E_d) g_A^2} \times \sigma_k \cdot \sigma_l \tau_k^- \tau_l^- \delta(\mathbf{r}).$$

If no dipole form factor,
the above term diverges
and needs a counterterm
to cancel the divergence



- Multi-reference in-medium generator coordinate method (IM-GCM)

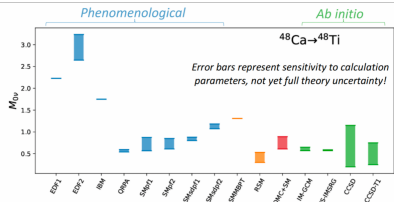
JMY et al., PRL124, 232501 (2020)

- Valence-space shell model+IMSRG (VS-IMSRG)

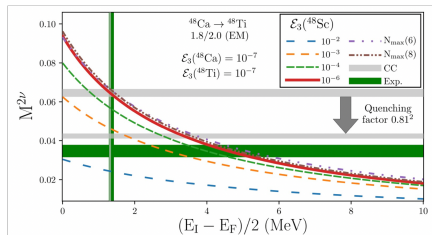
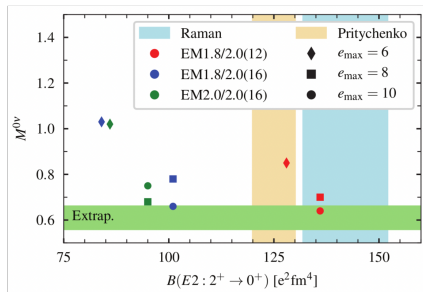
A. Belley et al., PRL126, 042502 (2021)

- Coupled-cluster with singlets, doublets, and partial triplets (CCSDT1) .

S. Novario et al., PRL126, 182502 (2021)



arXiv:2212.11099, adapted from J. Phys. G: Nucl. Part. Phys. 49, 120502 (2022)



IMSRG + Configuration interaction (IMCI-Q) method

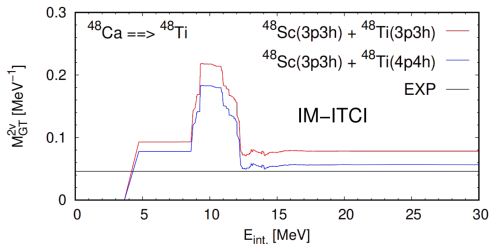
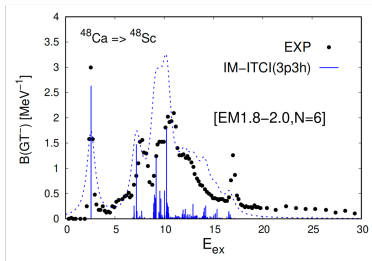


Table: $0\nu\beta\beta$ matrix elements for ^{48}Ca . ($\hbar\omega = 12$)

Force	Reference state	Cut off	$M_F^{0\nu}$	$M_{GT}^{0\nu}$	$M_T^{0\nu}$	$M^{0\nu}$
EM1.8/2.0	^{48}Ca (HF)	N=6, IM-ITCI(4p4h)	-0.376	1.035	-0.088	1.180
EM1.8/2.0	^{48}Ti (HFB)	N=6, IM-ITCI(4p4h)	-0.291	0.740	-0.088	0.833

Xin Lian, Chengrong Ding, Chunlin Bai, JMY, in preparation (2025)

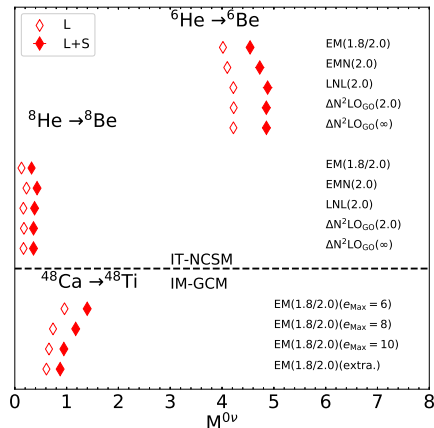
In comparison with the IM-GCM calculation

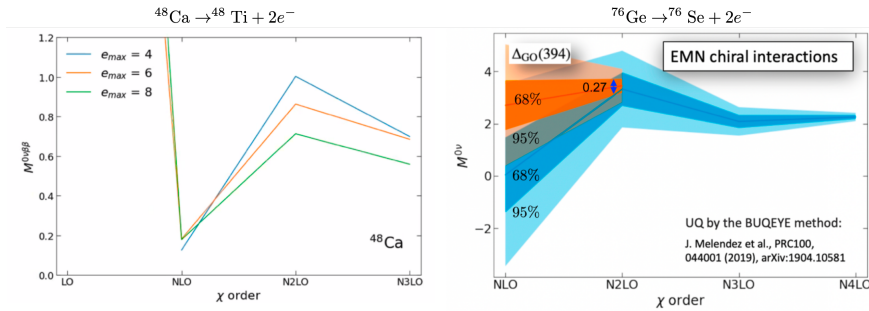
Interaction	$\hbar\omega$	NME		
		$e_{\max} = 6$	$e_{\max} = 8$	$e_{\max} = 10$
EM1.8/2.0	12	0.85	0.70	0.64
EM1.8/2.0	16	1.03	0.78	0.66

JMY et al., PRL124, 232501 (2020)

- A contact transition operator which could either enhance or quench the $0\nu\beta\beta$ decay, is needed to be promoted to LO to ensure renormalizability. V. Cirigliano et al., PRL120, 202001 (2018)
- We determine the unknown LEC g_{ν}^{NN} of the contact operator, consistent with the employed chiral interaction (EM1.8/2.0), based on the synthetic data for the process $2n \rightarrow 2p + 2e^{-}$. V. Cirigliano et al., PRL126, 172002 (2021)
- The contact term turns out to enhance the NME for ^{48}Ca by 43(7)%, thus reducing the half-life $T_{1/2}^{0\nu}$ significantly.

R. Wirth, JMY, H. Hergert, PRL127, 242502 (2021)





- The NME converges with respect to the chiral expansion order χ of nuclear forces for candidate nuclei ^{48}Ca and ^{76}Ge .
- The EFT truncation error is shrinking with the increase of χ expansion order ν .

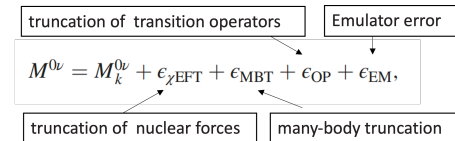
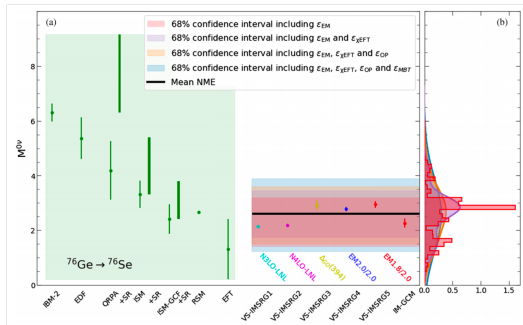


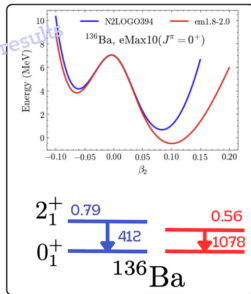
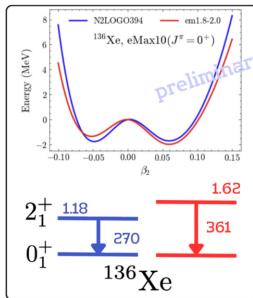
TABLE I. The recommended value for the total NME of $0\nu\beta\beta$ decay in ^{76}Ge , together with the uncertainties from different sources.

$M^{0\nu}$	ϵ_{LEC}	$\epsilon_{\chi\text{EFT}}$	ϵ_{MBT}	ϵ_{OP}	ϵ_{EM}
$2.60^{+1.28}_{-1.36}$	0.75	0.3	0.88	0.47	< 0.06

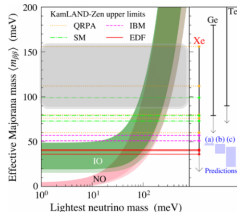


- Our recommended value $M^{0\nu} = 2.60^{+1.28}_{-1.36}$.
- Together with the best half-life limit: $> 1.8 \times 10^{26}$ yr, it sets the upper limit $\langle m_{\beta\beta} \rangle = 187^{+205}_{-62}$ meV, and the sensitivity of the next-generation experiment $\langle m_{\beta\beta} \rangle = 22^{+24}_{-7}$ meV, covering almost the entire range of IO hierarchy.

Extension to the $0\nu\beta\beta$ decay of ^{136}Xe

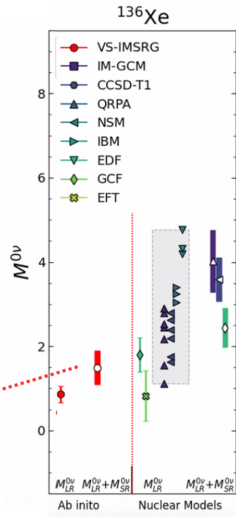


KamLAND-Zen: $T_{1/2}^{0\nu} > 2.3 \times 10^{26}$ yr at 90% C.L.



	IM-GCM	VS-IMSRG
NME	1.15(N2LOGO394) 1.34 (EM1.8-2.0)	[1.08, 1.90]
$\langle m_{\beta\beta} \rangle$	0.13-0.15 eV	0.09-0.16 MeV

large uncertainty



Ding, JMY, et al. in preparation (2025)

A. Belley et al., arXiv2307.15156 [nucl-th]

- ① Introduction to nuclear weak processes
- ② Ab initio studies of single- β decay
 - g_A quenching problem and 2BC effect
 - Superallowed Fermi transitions
- ③ Ab initio studies of $0\nu\beta\beta$ decay
 - Lepton-number violation and $0\nu\beta\beta$ decay
 - The 2BC effect on NMEs
 - Modeling NMEs with operators from chiral EFT
- ④ Summary and outlook

Precision study of nuclear weak processes (single and double-beta decay) is important for understanding physics both within and beyond the standard model.

- The ab initio framework based on the operators derived from chiral EFT allows to determine the NMEs of nuclear weak processes and their uncertainties.
- This ab initio framework has already been applied to compute the NMEs of both single and double-beta decay, where the contributions of higher-order operators have been examined. These studies are of importance for understanding nuclear weak processes with quantified uncertainties in the predicted NMEs.

Next

- Ab initio study of two-body current effect on $0\nu\beta\beta$ decay.
- Consideration of contributions from higher-order operators.
- More benchmark studies among different ab initio nuclear models.

Collaborators

- **SYSU**

C.R. Ding, Y. Li, X. Zhang, Y. X. Zhang

- **SCU**: Xin Lian, Chunlin Bai

- **SJU**: Ke Han, Shaobo Wang

- **MSU**: Heiko Hergert, R. Wirth

- **UNC**: Jonathan Engel

- **TRIUMF**: Antonie Belly, Jason Holt

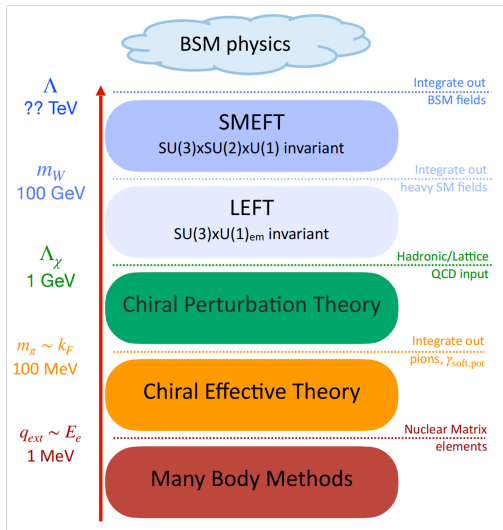
- **Tsukuba University**: Takayuki Miyagi

- **Notre-Dame U**: Ragnar Stroberg

- **UAM**: Benjamin Bally, Tomas Rodriguez

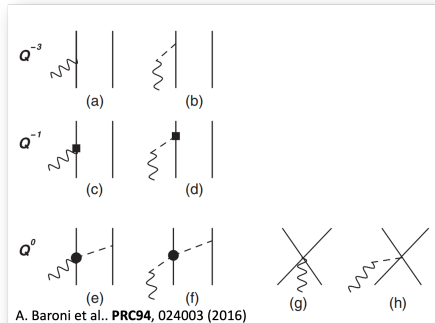
This work is supported in part by the National Natural Science Foundation of China (Grant Nos. 12375119 and 12141501), the Guangdong Basic and Applied Basic Research Foundation (2023A1515010936).

Thank you for your attention!



credit to Wouter Dekens

Axial currents in chiral EFT



$$\nu = -3 + \sum_i V_i \kappa_i$$

$$\kappa = d + \frac{3}{2}a + b + c - 4$$

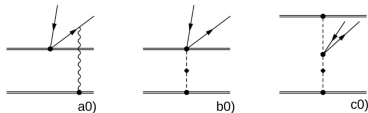
d: number of derivatives and/or insertions of M_π

a: number of nucleon fields

b: number of pion fields

c: number of external source

Two-body operators for δ_{NS}



$$\bar{\delta}_{\text{NS}} = \frac{2\langle f^{(0)} | \mathcal{V} | i^{(0)} \rangle}{g_V(\mu) M_F^{(0)}} = \alpha \left[\sum_{n=1} a_n \left(\frac{m_\pi}{\Lambda_\chi} \right)^n + E_0 R \sum_{n=0} b_n(E_\theta, m_e) \left(\frac{m_\pi}{\Lambda_\chi} \right)^n + \alpha \sum_{n=0} c_n \left(\frac{m_\pi}{\Lambda_\chi} \right)^n \dots \right]$$

- LO vertices lead to an operator that depends on external energies

$$\mathcal{V}_E^0 = \frac{1}{3} \left(\frac{1}{2} + \frac{4E_\theta}{E_0} \right) \mathcal{V}_E \quad \mathcal{V}_E = g_V \sum_{j < k} \theta^2 \frac{1}{\mathbf{q}^4} \left(\tau^{+(j)} P_\rho^{(k)} + P_\rho^{(j)} \tau^{+(k)} \right).$$

- at NLO, magnetic and recoil corrections generate energy-independent corrections

$$\mathcal{V}_0^{\text{mag}}(\mathbf{q}) = \sum_{j < k} \frac{e^2}{3} \frac{g_A}{m_N} \frac{1}{\mathbf{q}^2} \left(\sigma^{(j)} \cdot \sigma^{(k)} + \frac{1}{2} S^{(jk)} \right) \left[(1 + \kappa_p) \tau^{+(j)} P_\rho^{(k)} + \kappa_n \tau^{+(j)} P_n^{(k)} + (j \leftrightarrow k) \right],$$

$$\mathcal{V}_0^{\text{rec}}(\mathbf{q}, \mathbf{P}) = \sum_{j < k} \left[-i \frac{e^2 g_A}{4m_N} \frac{\tau^{+(j)} P_\rho^{(k)}}{\mathbf{q}^4} ((\mathbf{P}_j - \mathbf{P}_k) \times \mathbf{q}) \cdot \sigma^{(j)} - \frac{Z_\pi e^2 g_A^2}{m_N} \frac{\tau^{+(j)} \tau_3^{(k)}}{(\mathbf{q}^2 + m_\pi^2)^2} \sigma^{(j)} \cdot \mathbf{q} \sigma^{(k)} \cdot \mathbf{P}_j + (j \leftrightarrow k) \right],$$

- the magnetic moments induce a Coulomb-like potential in $^1S_0 \dots$ need isospin 1 and 2 contact interactions

$$\mathcal{V}_0^{\text{CT}} = e^2 (g_{V1}^{\text{W}} \mathcal{O}_1 + g_{V2}^{\text{W}} \mathcal{O}_2), \quad \mathcal{O}_1 = \sum_{j \neq k} \tau^{+(j)} \mathbb{1}_k, \quad \mathcal{O}_2 = \sum_{j < k} [\tau^{+(j)} \tau_3^{(k)} + (j \leftrightarrow k)].$$

V. Cirigliano et al., PRC110, 055502 (2024)

Counterterms for β decays

- introduce the dimensionless couplings $\tilde{g}_{V1, V2}^{\text{W}}$ as

$$g_{V1, V2}^{\text{W}} = \frac{1}{m_N} \left(\frac{m_N C_1 S_0}{4\pi} \right)^2 \tilde{g}_{V1, V2}^{\text{W}},$$

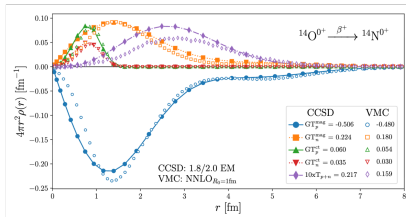
- the LO RGE

$$\frac{d\tilde{g}_{V1}^{\text{W}}}{d \log \mu} = -g_A(1 + \kappa_p + \kappa_n) = -1.12, \quad \frac{d\tilde{g}_{V2}^{\text{W}}}{d \log \mu} = -g_A(1 + \kappa_p - \kappa_n) = -5.99,$$

- to limit the number of fit parameters, if needed we could impose

$$\tilde{g}_{V1} = \tilde{g}_{V2} \frac{1 + \kappa_p + \kappa_n}{1 + \kappa_p - \kappa_n} \ll \tilde{g}_{V2}$$

Fitting g_{-V1} and g_{-V2} to superallowed data



Check the dependence on the interaction and on the regulator

A slide from E. Mereghetti's talk

Quantum Monte Carlo (QMC) methods

Variational Monte Carlo (VMC): minimize expectation value of Hamiltonian

$$E_V = \frac{\langle \Psi_V | H | \Psi_V \rangle}{\langle \Psi_V | \Psi_V \rangle} \geq E_0$$

$$|\Psi_V\rangle = \left[\prod_{i < j} \left(1 + U_{ij} + \sum_{k \neq i, j} \tilde{U}_{ijk} \right) \right] \left[\prod_{i < j} f_c(r_{ij}) \right] |\Phi_A(JMTT_3)\rangle$$

two- and three-body
correlation operators

cluster structure

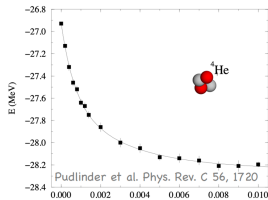


${}^8\text{Be} \sim 2\alpha$

Green's Function Monte Carlo (GFMC): improve VMC solution by propagating in imaginary time to remove contaminations from excited states

$$\Psi(\tau) = e^{-(H-E_0)\tau} \Psi_V = \sum_n e^{-(E_n-E_0)\tau} a_n \psi_n$$

$$\lim_{\tau \rightarrow \infty} \Psi(\tau) = a_0 \psi_0$$



Chambers-Wall, Pastore, Piarulli

Two-body current in normal-ordering approximation

$$J_{iA}^{\text{eff}}(q, Q, \rho) \equiv -g_A^{\text{eff}}(q, Q, \rho) \sigma_i \tau_i^+ \\ = -g_A \sigma_i \tau_i^+ \left\{ 1 - \frac{\rho}{f_\pi^2} \left[-\frac{c_D}{4} \frac{1}{g_A A_\chi} + \frac{2c_3}{3} \frac{q^2}{4m_\pi^2 + q^2} + \frac{l(\rho, Q)}{3} \left(2c_4 - c_3 + \frac{1}{2m_p} \right) \right] \right\}.$$

33

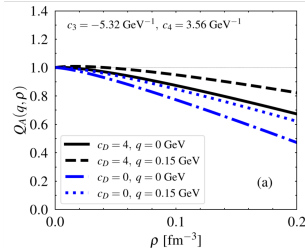
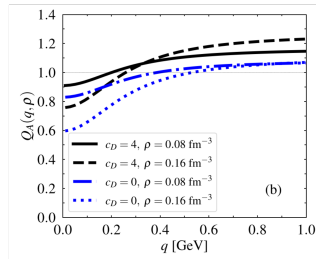
The quenching factor

$$\mathcal{Q}_A(q, \rho) \equiv g_A^{\text{eff}}(q, 0, \rho) / g_A = 1 + A[q]\rho + B\rho^{1/3} + C,$$

where the coefficients A , B and C are defined as

$$A[q] = \frac{c_D}{4f_\pi^2} \frac{1}{g_A A_\chi} - \frac{1}{3f_\pi^2} \left[\left(2c_4 - c_3 + \frac{1}{2m_p} \right) + 2c_3 \frac{q^2}{4m_\pi^2 + q^2} \right], \\ B = \frac{m_\pi^2}{f_\pi^2} \left(\frac{2}{3\pi^2} \right)^{2/3} \left(2c_4 - c_3 + \frac{1}{2m_p} \right), \\ C = -\frac{2m_\pi^3}{3\pi^2 f_\pi^2} \left(2c_4 - c_3 + \frac{1}{2m_p} \right) \tan^{-1} \left(\frac{k_F}{m_\pi} \right).$$

JMY, J. Meng, Y.F. Niu, P. Ring, PPNP 126, 103965 (2022)



The Lagrangian density for weak interaction at nuclear-energy scale,

$$\mathcal{L} = \sum_{i=V,A,S,T,P} (\bar{p} O_i n) \bar{e} O_i (C_i - C'_i \gamma_5) \nu$$

where the operator O_i specifies different types (standard and non-standard) of weak interactions, C_i, C'_i are couplings of dimension $[\text{mass}]^{-2}$, indicating the mass scale of particles mediating the interaction.

Significance

- Impact of nuclear β -decay rates on the abundances of r -process elements.
Kelsey A. Lund et al., Astrophys.J. 944, 144 (2023); Y. W. Hao et al., Phys. Rev. C 108, L062802 (2023)
- Matrix element of superallowed Fermi transition: test of the unitarity of CKM matrix. *V. Cirigliano et al., PRL133, 211801 (2024); PRC110, 055502(2024)*
- Constraints on the non-standard weak interaction with β decay:
 $C_i, C'_i \propto (M_W/\Lambda_{\text{BSM}})^{n \geq 2}$ with $i = S, P, T$. *L. Hayen, Annu. Rev. Nucl. Part. Sci. 74, 1 (2024)*

Remaining problems

- **Improvements in EDF-based QRPA?**: systematic studies of nuclear β decay, interplay between many-body correlations (PVC, 2p-2h, deformation, etc) and phenomenological parameters (g_A , $V_{pp}^{T=0,1}$, etc) E.M. Ney et al., Phys.Rev.C 102, 034326 (2020)
- **Significant discrepancies (by an order of magnitude or more) between theory and experiment**: M_{GT} and Q value.
- **g_A quenching problem**: ab initio studies restricted to light and near closed-shell nuclei. P. Gysbers et al., Nature Physics 15, 428 (2019); G.B. King et al. PRC107, 015503 (2023)

Recommendation

Development of state-of-the-art models (VS-IMSRG, IM-GCM, pn-QRPA guided by or based on chiral nuclear forces) for nuclear single- β decay and superallowed Fermi transition, considering two-body currents and radiative corrections, and examining convergence of matrix elements with respect to the chiral expansion orders.