Strong backreaction of gauge quanta produced during inflation and the sourced GWs

Sep 29, 2025, Beijing

Based on:

2502.13158, Jian-Feng He, Kai-Ge Zhang, Chengjie Fu, Zong-Kuan Guo

2507.02611, Kai-Ge Zhang, Jian-Feng He, Chengjie Fu, Zong-Kuan Guo

Jian-Feng He

ITP CAS

1 An Overview

Given that the energy scale of inflation far exceeds that of the standard model of particle physics, it is natural to anticipate the presence of many beyond-standard model (BSM) fields during inflation.

On the other hand, there are many axion in BSMs:

- axion-like particles
- QCD axion
- axion from string theory

So it's natural to study axion during inflation

1 An Overview

some early literatures (axion as inflaton) see: [1011.1500, N. Barnaby, M. Peloso] [1101.1525, L. Sorbo] [1102.4333, N. Barnaby et al.] [1110.3327, N. Barnaby et al.]

Lagrangian with an inflaton and a spectator axion:

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2} R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - U(\chi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4f}\chi\tilde{F}^{\mu\nu}F_{\mu\nu}$$

Equation of Motion (background)

$$H^2 = \frac{1}{3M_{\rm pl}^2}\rho\,, \qquad \ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -\frac{1}{2M_{\rm pl}^2}P\,, \qquad \ddot{\chi} + 3H\dot{\chi} + U_{,\chi} = \frac{1}{f}\langle \boldsymbol{E}\cdot\boldsymbol{B}\rangle$$

strong backreaction condition

$$f^{-1}\langle \vec{E}\cdot\vec{B}\rangle \sim U_{,\chi}$$

E_i and B_i are defined by

$$E_i(t) = -\dot{A}_i/a,$$
$$B_i(t) = \epsilon_{ijk}\partial_j A_k/a^2.$$

1 An Overview

E^2 and B^2 also appear in energy density and presure

$$\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + V(\phi) + U(\chi) + \frac{1}{2}\langle \boldsymbol{E}^2 + \boldsymbol{B}^2 \rangle \, ,$$
$$P = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 - V(\phi) - U(\chi) + \frac{1}{6}\langle \boldsymbol{E}^2 + \boldsymbol{B}^2 \rangle \, .$$

The ensemble averages are computed by integral over the gauge field

$$\langle \boldsymbol{E}\cdot\boldsymbol{B} \rangle = -\frac{1}{4\pi^2 a^4}\sum_{\lambda=\pm}\lambda\int_0^\infty \mathrm{d}k k^3 \frac{\mathrm{d}}{\mathrm{d}\tau}|A^\lambda(k)|^2,$$

$$\langle E^2 \rangle = \frac{1}{2\pi^2 a^4}\sum_{\lambda=\pm}\int_0^\infty \mathrm{d}k k^2 \left|\frac{\mathrm{d}A^\lambda(k)}{\mathrm{d}\tau}\right|^2,$$

$$\langle B^2 \rangle = \frac{1}{2\pi^2 a^4}\sum_{\lambda=\pm}\int_0^\infty \mathrm{d}k k^4 |A^\lambda(k)|^2.$$

1 An Overview

Decomposition of the gauge field

$$\hat{A}_i(t, \boldsymbol{x}) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}} \sum_{\lambda=\pm} \mathrm{e}^{\,\mathrm{i}\,\boldsymbol{k}\cdot\boldsymbol{x}} \epsilon_i^{\lambda}(\hat{k}) A^{\lambda}(t, \boldsymbol{k}) \hat{a}^{\lambda}(\boldsymbol{k}) + \mathrm{h.c.}$$

where the basis of the left/right hand circular polarization is defined by

$$k_i \epsilon_i^{\pm}(\hat{k}) = 0, \;\; \varepsilon_{ijk} k_j \epsilon_k^{\pm}(\hat{k}) = \mp \mathrm{i}\, k \epsilon_i^{\pm}(\hat{k}),$$
$$\epsilon_i^{\pm}(\hat{k}) \epsilon_i^{\pm}(\hat{k}) = 0, \;\; \epsilon_i^{\pm}(\hat{k}) \epsilon_i^{\mp}(\hat{k}) = 1.$$

Note the non-vanishing cross product $\vec{k} \times \vec{\epsilon}$ implies the basis is "rotating".

1 An Overview

Tachyonic instability happen when the effective mass becomes negative;
Outside the horizon, oscillation frequency exponentially decrease and **finally freeze**

Equation of motion of the gauge field

$$\ddot{A}^{\pm}(\boldsymbol{k}) + H\dot{A}^{\pm}(\boldsymbol{k}) + \boxed{\left(\frac{k^2}{a^2} \mp \frac{k}{a}\frac{1}{f}\dot{\chi}\right)} A^{\pm}(\boldsymbol{k}) = 0$$

$$A^{\pm''}(\boldsymbol{k}) + (k^2 \pm 2\xi aHk)A^{\pm}(\boldsymbol{k}) = 0$$

Definition of the dimensionless parameter ξ

$$\xi \equiv -\frac{\dot{\chi}}{2fH}$$

In our study, $\dot{\chi} < 0$, thus the minus mode A^- is the enhanced mode. The initial condition is the Bunch-Davis vacuum

$$A^{\pm}_{\mathrm{BD}}(\boldsymbol{k}) \equiv A(\tau, \boldsymbol{k})|_{k \gg aH} = \frac{1}{\sqrt{2k}} \mathrm{e}^{-\mathrm{i}k\tau}$$

1 An Overview

Equations

$$\frac{\ddot{a}}{a} + \frac{1}{2}\left(\frac{\dot{a}}{a}\right)^2 = -\frac{1}{2M_{\rm pl}^2} P$$

$$P = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 - V(\phi) - U(\chi) + \frac{1}{6}\langle \boldsymbol{E}^2 + \boldsymbol{B}^2 \rangle$$

$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0$$

$$\ddot{\chi} + 3H\dot{\chi} + U_{,\chi} = \frac{1}{f}\langle \boldsymbol{E} \cdot \boldsymbol{B} \rangle$$

$$\ddot{A}^{\pm}(\boldsymbol{k}) + H\dot{A}^{\pm}(\boldsymbol{k}) + \left(\frac{k^2}{a^2} \mp \frac{k}{a}\frac{1}{f}\dot{\chi}\right) A^{\pm}(\boldsymbol{k}) = 0$$

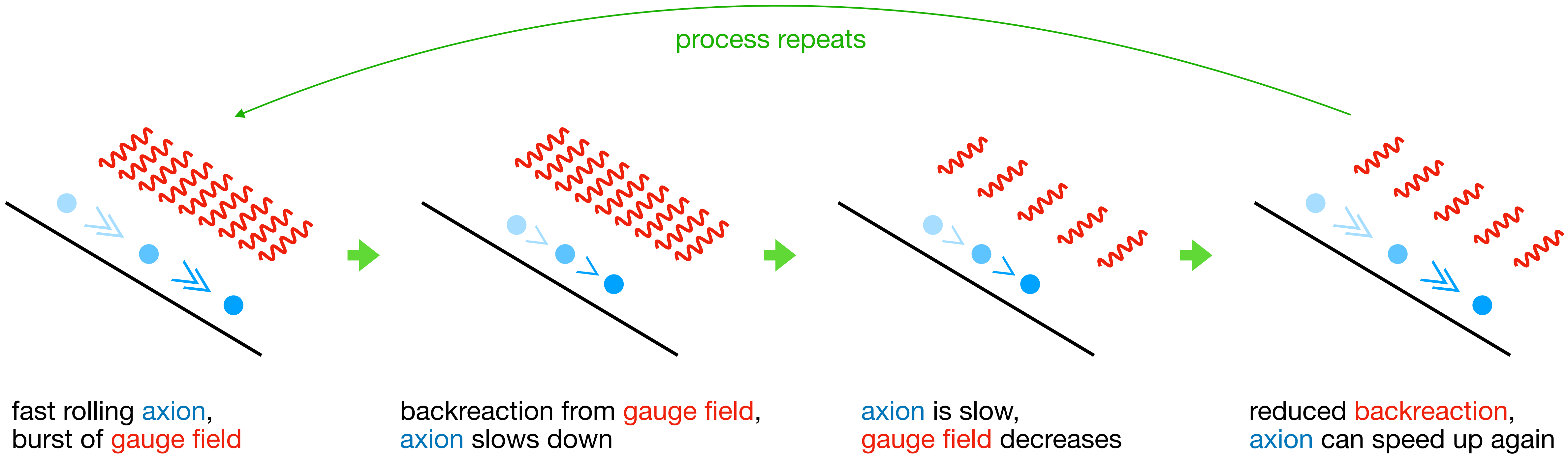
Pseudo Code

```
for it < itmax:
  // background evolution
  rk4_iterate(phi, dphi, chi, dchi,
              a, da, EEplusBB_ave, EB_ave)

  for iq < iqmax:
    // gauge field evolution
    rk2_iterate(Aq)

    // check if tachyonic instability
      happen, then sum to compute
      backreaction
    if mass_eff < 0:
      EB_ave += EB_integrand(Aq)
      EEplusBB_ave += EEplusBB_integrand(Aq)
```

1 An Overview



From Fig. 1 of [2407.13405, A. Caravano, M. Peloso, Unveiling the nonlinear dynamics of a rolling axion during inflation]

An Overview

Tensor perturbations of FRW background

$$\mathrm{d}s^2 = a^2(\tau)\left[-\mathrm{d}\tau^2 + (\delta_{ij} + h_{ij}(\tau,\boldsymbol{x}))\mathrm{d}x_i\mathrm{d}x_j\right]$$

Evolution of the sourced tensor perturbation

$$h''_{ij} + 2\,\frac{a'}{a}\,h'_{ij} - \Delta\,h_{ij} = \frac{2}{M_P^2}\,\Pi_{ij}{}^{lm}\,T^{EM}_{lm}$$

Stress-energy tensor and the traceless-transverse projection tensor are

$$T^{\mathrm{EM}}_{ij} = -a^2(E_iE_j + B_iB_j) + \delta_{ij}(\dots)$$

$$\Pi^{\pm}_{lm}(\boldsymbol{k}) \equiv \epsilon^{\mp}_{l}(\hat{k})\epsilon^{\mp}_{m}(\hat{k})$$

Decomposition of the tensor mode

$$\hat{h}_{ij}(\tau,\boldsymbol{x})=\int\frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}}\sum_{\lambda=+,-}(\epsilon_i^\lambda(\hat{k})\epsilon_j^\lambda(\hat{k})\hat{a}^\lambda(\boldsymbol{k})h^\lambda(\tau,\boldsymbol{k})\,\mathrm{e}^{\,\mathrm{i}\boldsymbol{k}\cdot\boldsymbol{x}}+\mathrm{h.c.})$$

Solution of the polarization ±

$$\hat{h}^\pm(\tau,\boldsymbol{k})=-\frac{2H^2}{M_{\mathrm{pl}}^2}\int_{-\infty}^{\tau}\mathrm{d}\tau' G_k(\tau,\tau')\tau'^2\int\frac{\mathrm{d}^3\boldsymbol{q}}{(2\pi)^{3/2}}\Pi_{lm}^{\pm}(\boldsymbol{k})\Big[\tilde{A}_l'(\boldsymbol{q},\tau')\hat{A}_m'(\boldsymbol{k}-\boldsymbol{q},\tau')$$
$$-\varepsilon_{lab}q_a\hat{A}_b(\boldsymbol{q},\tau')\varepsilon_{mcd}(k_c-q_c)\hat{A}_d(\boldsymbol{k}-\boldsymbol{q},\tau')\Big]$$

Green's function of the de-Sitter background is

$$G_k(\tau,\tau')=\frac{1}{k^3\tau'^2}\big[(1+k^2\tau\tau')\sin(k(\tau-\tau'))-k(\tau-\tau')\cos(k(\tau-\tau'))\big]\Theta(\tau-\tau')$$

1 An Overview

Power spectrum

$$\langle h^{\pm}(k,\tau)h^{\pm}(k',\tau)\rangle \equiv \frac{2\pi^2}{k^3}\mathcal{P}_h^{\pm}(k,\tau)\delta(\boldsymbol{k}+\boldsymbol{k}')$$

$$\mathcal{P}_h^{\lambda}(k,\tau_{\rm end}) = \frac{H^4k^3}{\pi^4 M_{\rm pl}^4}\int_0^{\infty} q^2 {\rm d}q \int_{-1}^{1} {\rm d}u \Bigg[\left|\epsilon_i^{\lambda}(\boldsymbol{k})\epsilon_i^{-}(-\boldsymbol{q})\right|^2 \left|\epsilon_j^{\lambda}(\boldsymbol{k})\epsilon_j^{-}(\boldsymbol{q}-\boldsymbol{k})\right|^2$$
$$\times \left|\int_{-\infty}^{0} {\rm d}\tau' \tau'^2 G_k(\tau_{\rm end},\tau')\Big(A'^{-}(\tau',\boldsymbol{q})A'^{-}(\tau',\boldsymbol{k}-\boldsymbol{q}) + q|\boldsymbol{k}-\boldsymbol{q}|A^{-}(\tau',\boldsymbol{q})A^{-}(\tau',\boldsymbol{k}-\boldsymbol{q})\Big)\right|^2\Bigg]$$

Energy spectrum, where $\Omega_{\rm r,0}$ is the the current fraction of the radiation energy density

$$\Omega_{\rm GW,0}(k)h^2 = \frac{\Omega_{r,0}h^2}{24}\left(\mathcal{P}_h^{+}(k,\tau_{\rm end}) + \mathcal{P}_h^{-}(k,\tau_{\rm end})\right)$$

1 Overview of Spectator Axion

2 Model 1: Linear Potential

3 Model 2: Cosine Potential

4 Conclusion

2 Model 1: Linear Potential

[2502.13158, Jian-Feng He, Kai-Ge Zhang, Chengjie Fu, Zong-Kuan Guo]

2.1 Model

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - U(\chi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4f}\chi\tilde{F}^{\mu\nu}F_{\mu\nu}$$

Since any potential can be locally treated as linear, we choose a linear potential to investigate the influence of the slope:

$$V(\phi) = V_0\left[1 - \exp\left(-\sqrt{\frac{2}{3}}\frac{\phi}{M_{\rm pl}}\right)\right]^2$$

Starobinsky potential

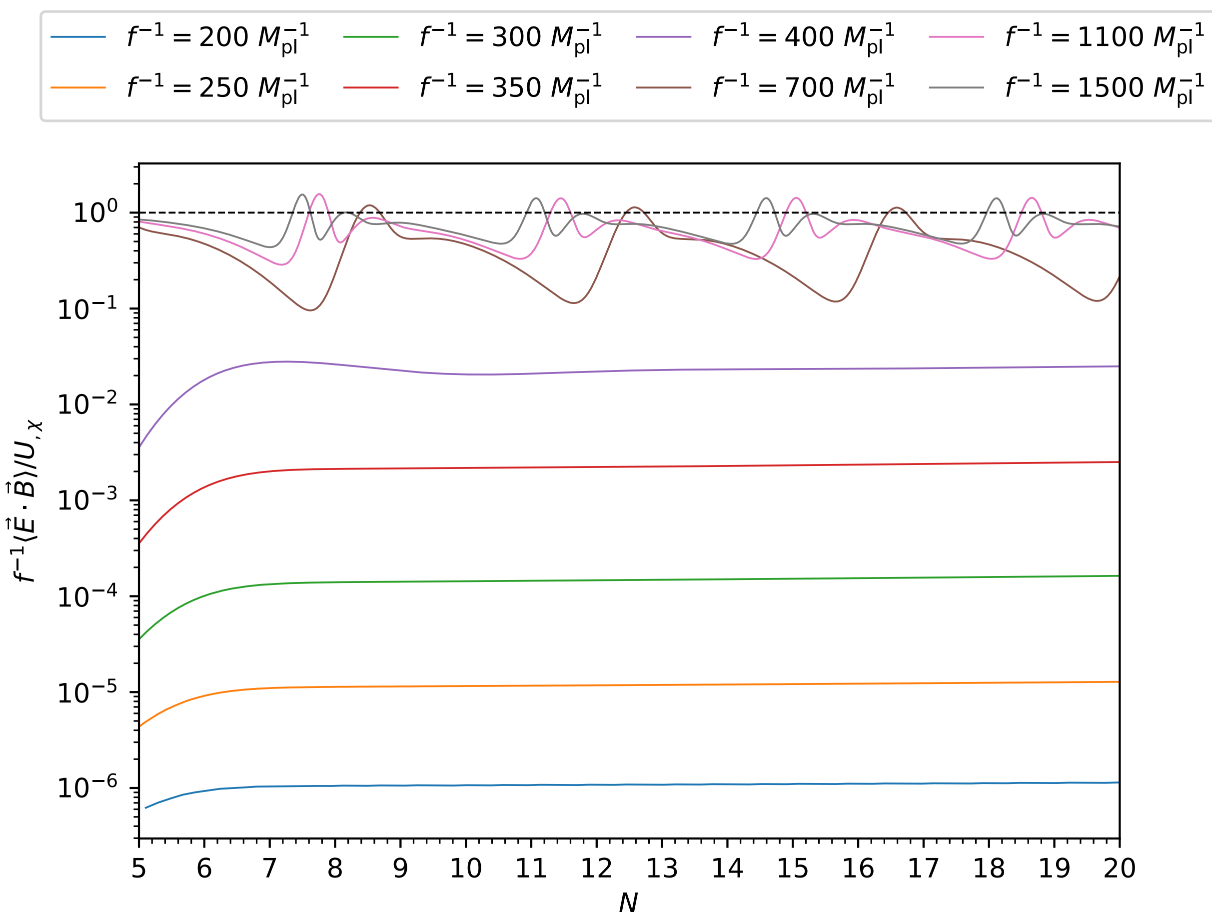
$$U(\chi) = c\chi$$

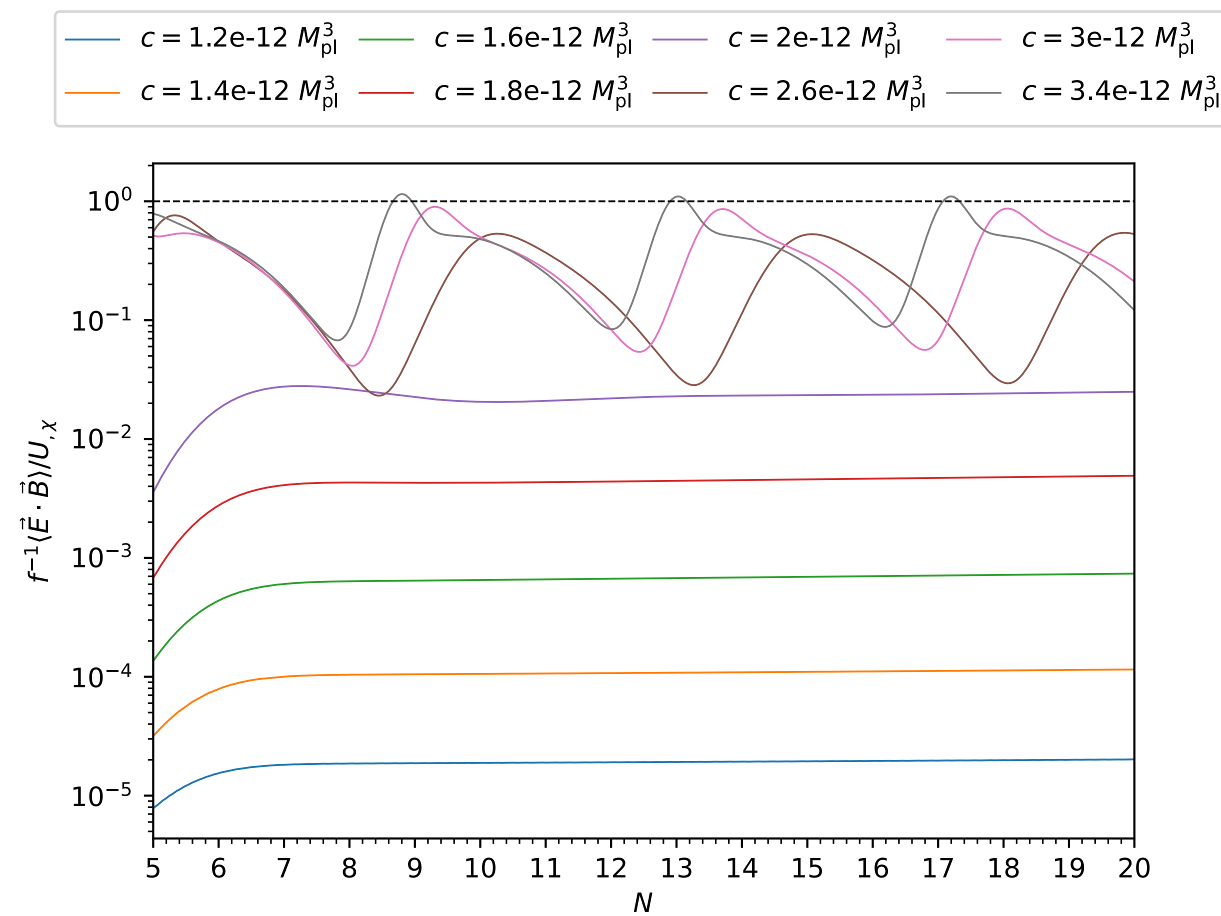
linear potential

2 Model 1: Linear Potential

[2502.13158, Jian-Feng He, Kai-Ge Zhang, Chengjie Fu, Zong-Kuan Guo]

2.2 Evolution of the Backreaction



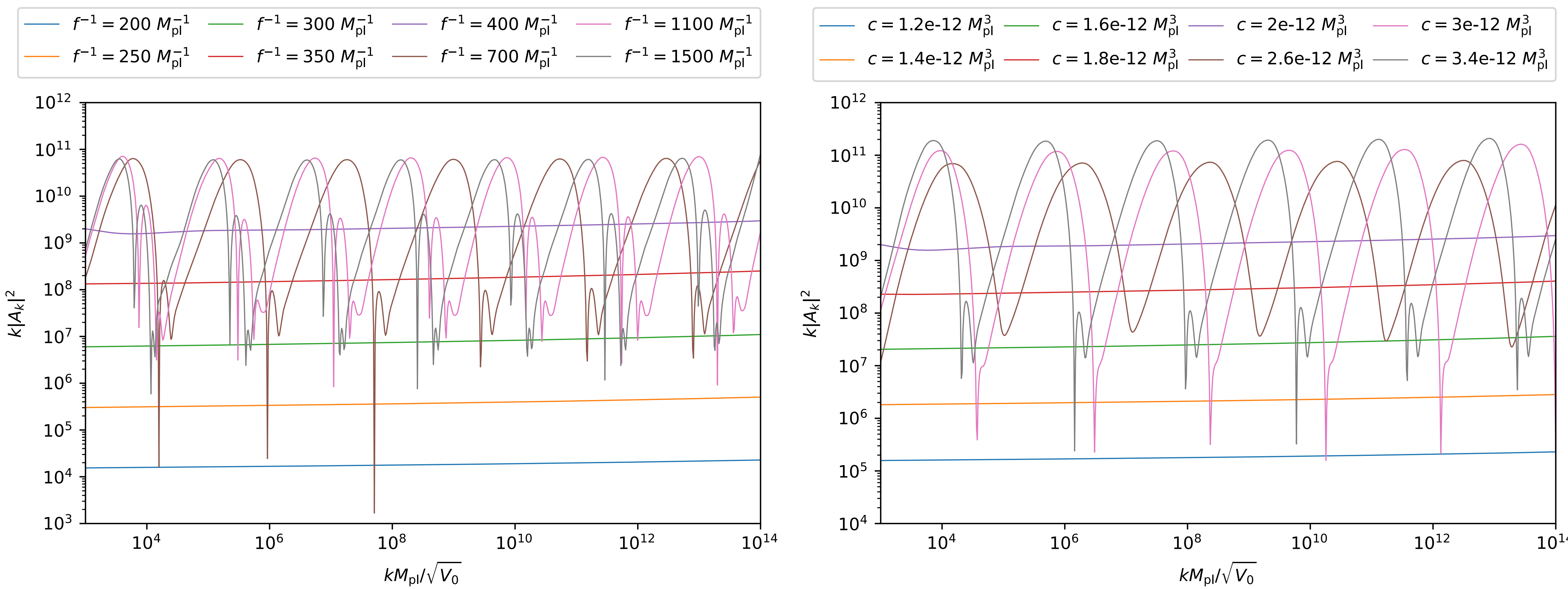


Black dashed line represents $f^{-1}\langle\overrightarrow{E}\cdot\overrightarrow{B}\rangle/U_{,\chi}$ = 0. They show oscillation only happen when $f^{-1}\langle\overrightarrow{E}\cdot\overrightarrow{B}\rangle \sim U_{,\chi}$, which verifies our argument about strong backreaction condition.

2 Model 1: Linear Potential

[2502.13158, Jian-Feng He, Kai-Ge Zhang, Chengjie Fu, Zong-Kuan Guo]

2.3 Spectrum of the Gauge Field



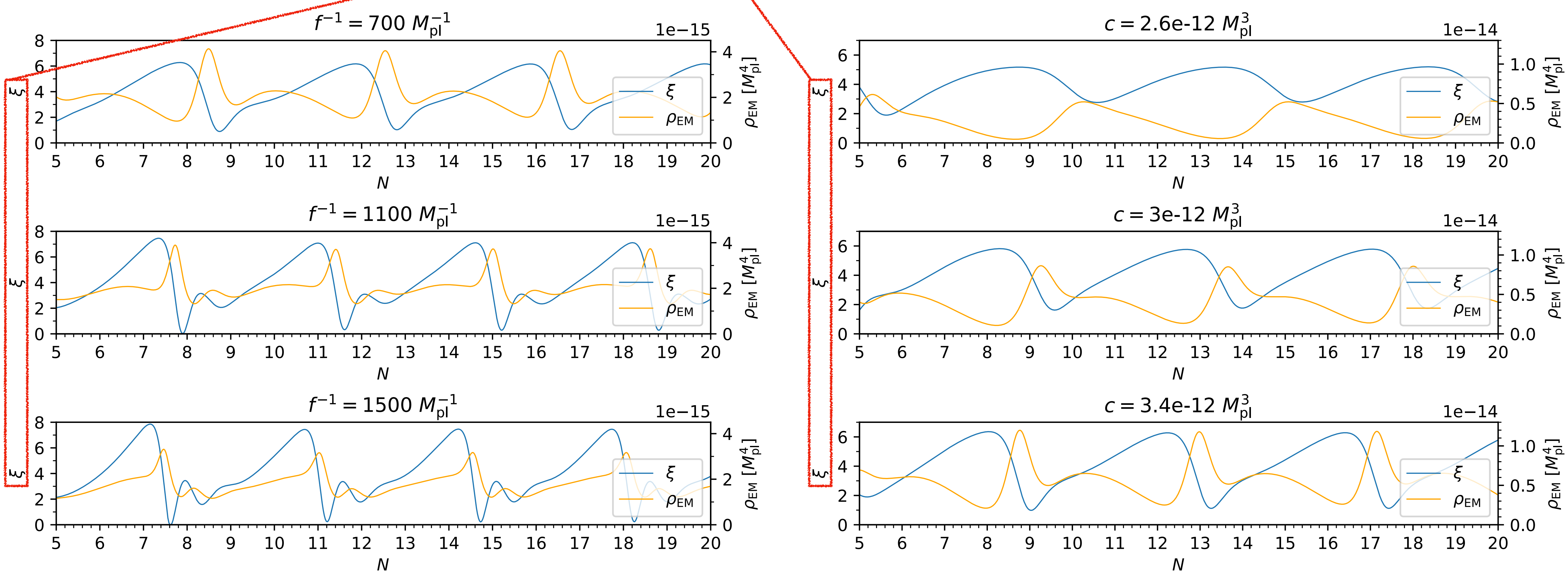
In the strong backreaction region, a larger f^{-1} don't significantly influence $k|A_k|^2$, but a larger $U_{,\chi}$ can improve it.

2 Model 1: Linear Potential

[2502.13158, Jian-Feng He, Kai-Ge Zhang, Chengjie Fu, Zong-Kuan Guo]

2.4 Time Delay between Axion Speed and Gauge Quanta Production

$$\xi \equiv -\frac{\dot{\chi}}{2fH}$$



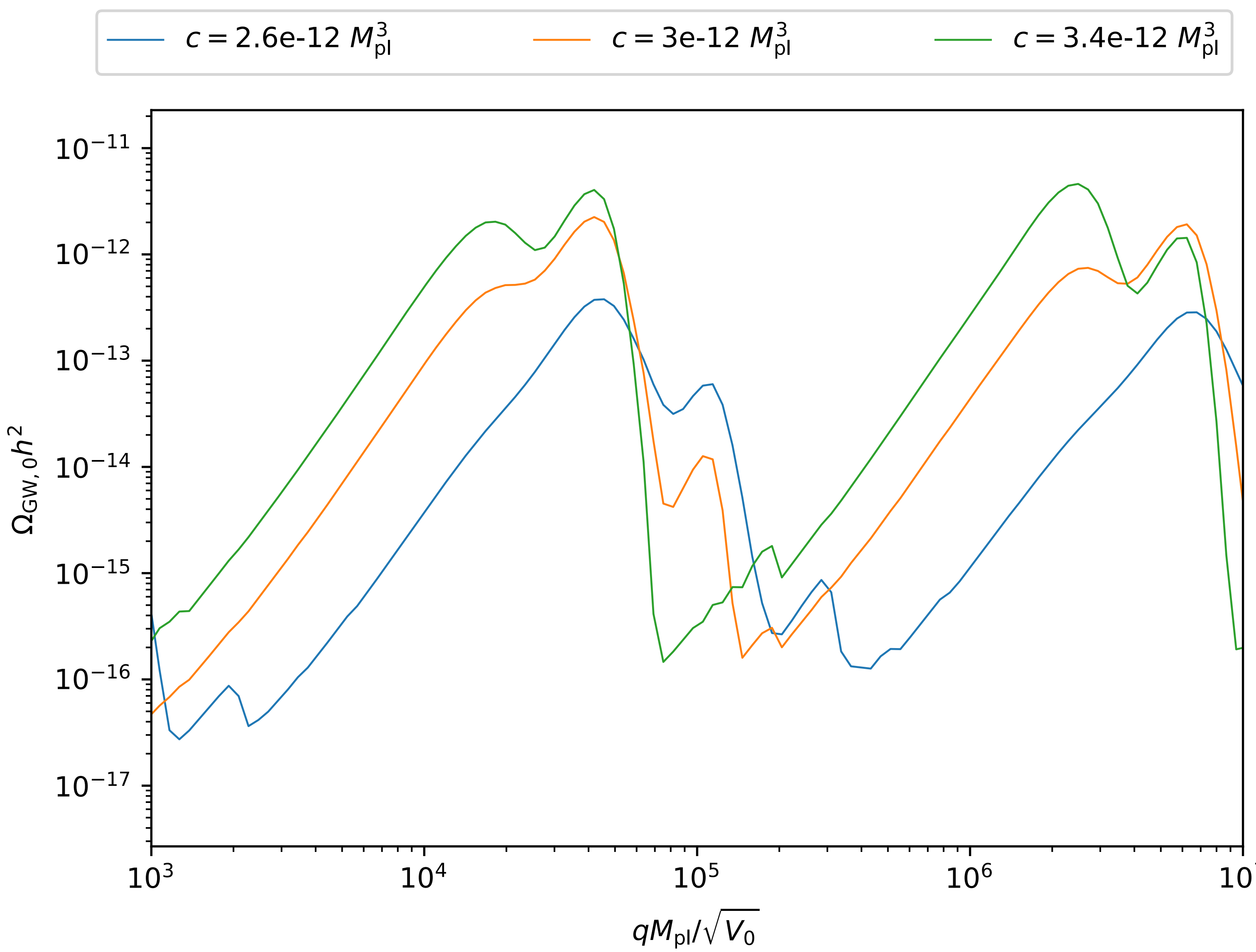
The peak of ξ is always earlier than the peak of $\rho_{\rm EM}$; large f^{-1} can linearly suppress $\rho_{\rm EM}$, which can be explained by strong backreaction condition $f^{-1}\langle \vec{E}\cdot\vec{B}\rangle \sim U_{,\chi}$

early research about the delay: [2002.02952, Domcke etc.] [2209.08131, M. Peloso, L. Sorbo]

2 Model 1: Linear Potential

[2502.13158, Jian-Feng He, Kai-Ge Zhang, Chengjie Fu, Zong-Kuan Guo]

2.5 Energy Spectrum of the GWs



similar oscillating spectrum: [2303.13425, J. Garcia-Bellido et al.]

Since $\Omega_{\rm GW,0}(k)$ is mainly determined by A_k, the oscillation of A_k leads to the oscillation of $\Omega_{\rm GW,0}(k)$

3 Model 2: Cosine Potential

[2507.02611, Kai-Ge Zhang, Jian-Feng He, Chengjie Fu, Zong-Kuan Guo]

3.1 Model

notation of the coefficient now is $\alpha/f_{\rm a}$ instead of $1/f$

$$\mathcal{L} = \frac{M_{\rm pl}^2}{2}R - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - U(\chi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \boxed{\frac{\alpha}{4f_{\rm a}}}\chi\tilde{F}^{\mu\nu}F_{\mu\nu}$$

where $U(\chi)$ is the most commonly used cosine potential

$$U(\chi) = \frac{\Lambda^4}{2}\left[1 + \cos\left(\frac{\chi}{f_{\rm a}}\right)\right]$$

The slope is given by

$$U_{,\chi}(\chi) = -\frac{\Lambda^4}{2f_{\rm a}}\sin\left(\frac{\chi}{f_{\rm a}}\right)$$

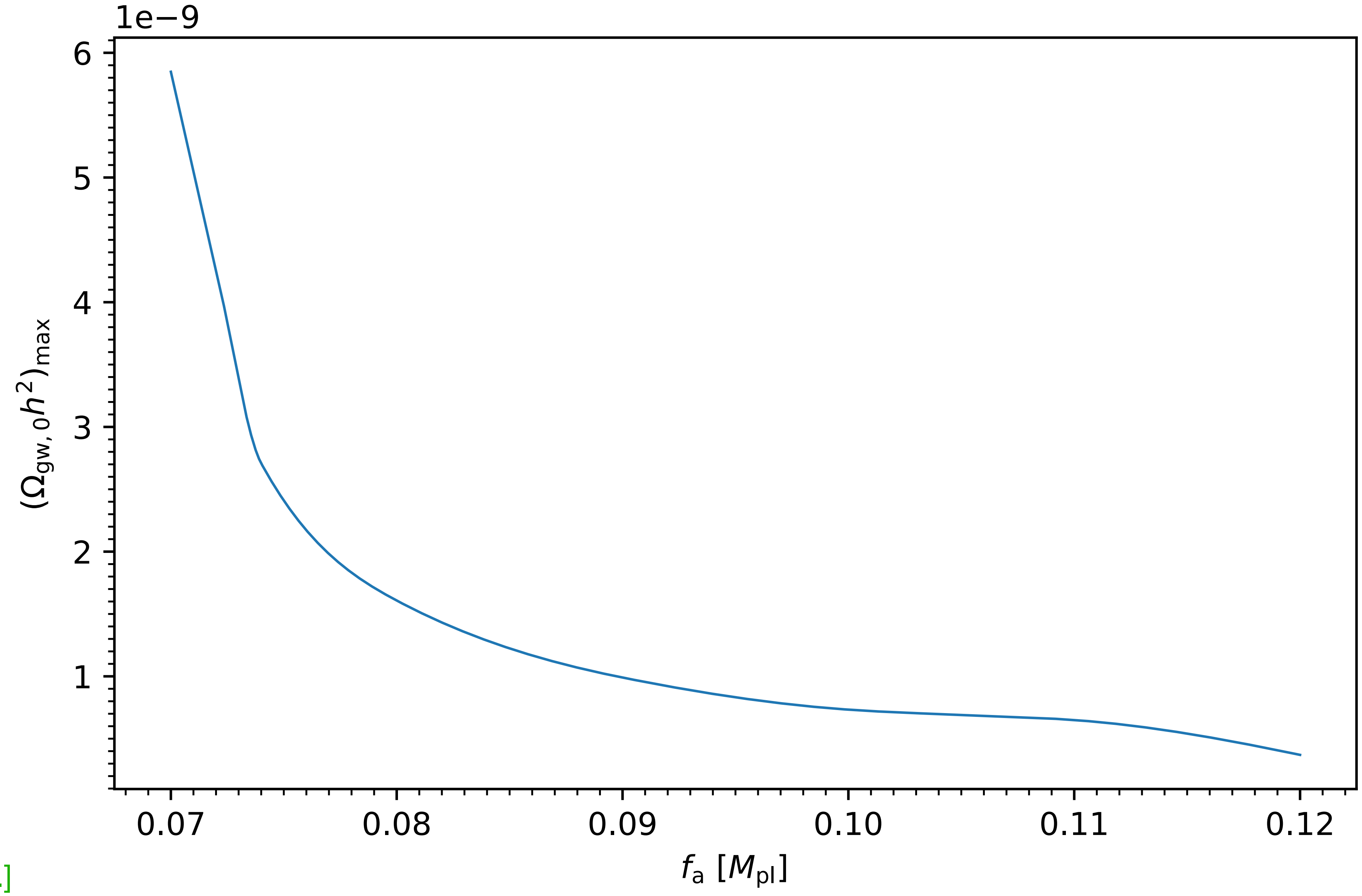
To obtain a stronger particle production, one need a larger $U_{,\chi}$. Since Λ^4 should satisfy $\Lambda^4 \ll V(\phi)$, the only free parameter left is $f_{\rm a}$.

3

Model 2: Cosine Potential

[2507.02611, Kai-Ge Zhang, Jian-Feng He, Chengjie Fu, Zong-Kuan Guo]

3.2 Maximum GWs due to Backreaction



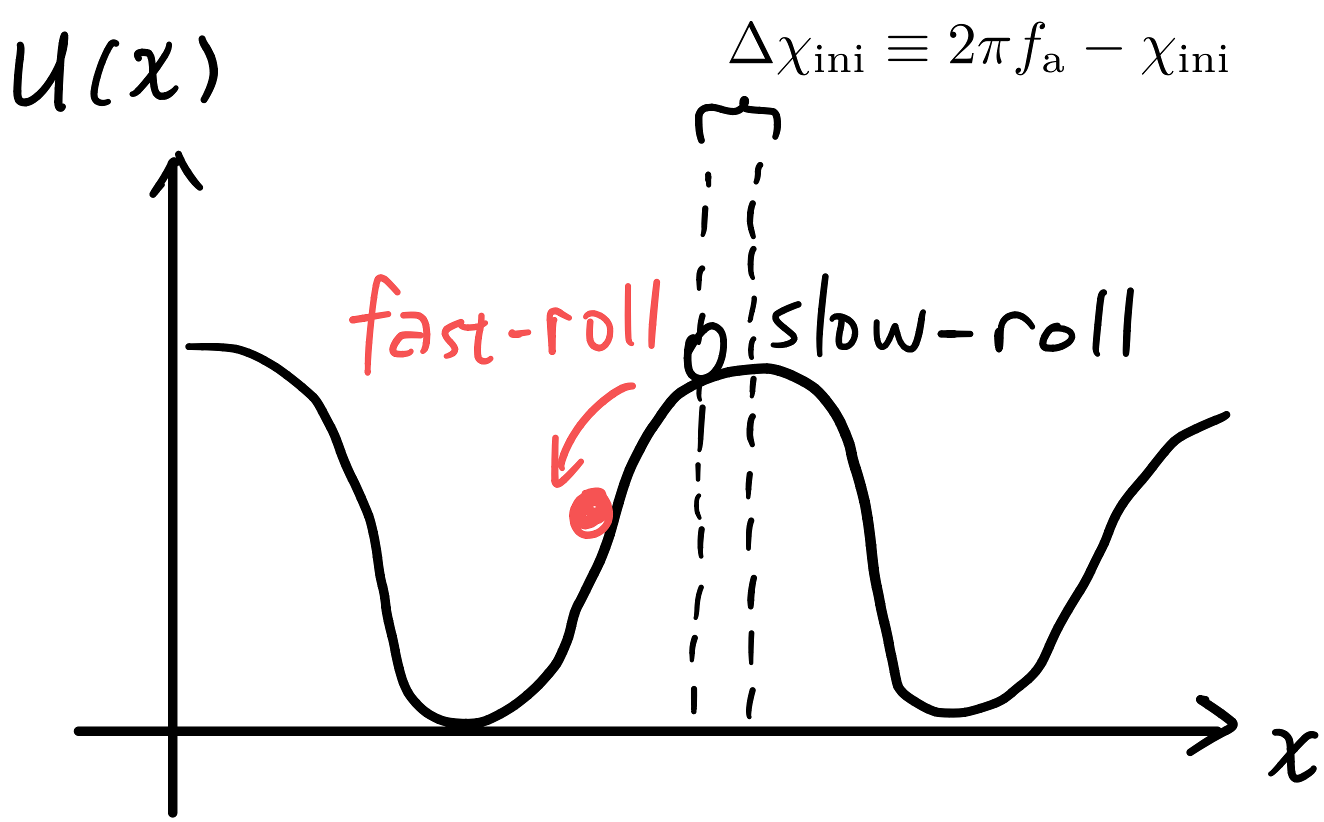
similar order of magnitude: [2303.13425, J. Garcia-Bellido et al.]

Maximum value of $\Omega_{\rm GW,0}(k)h^2$ as a function of $f_{\rm a}$. For each value of $f_{\rm a}$, we scan the parameter space of α to determine the maximum spectrum.

[2507.02611, Kai-Ge Zhang, Jian-Feng He, Chengjie Fu, Zong-Kuan Guo]

3.3 Constraint of f_a

However, f_a cannot be arbitrary small. If f_a is too small, the potential will be too sharp, and the quantum fluctuation will prevent one to put the axion near the peak of $U(\chi)$, which means one cannot obtain a long-enough slow-roll stage.



$$\Delta\chi_\mathrm{ini} > \delta\chi_\mathrm{ini} \sim \sqrt{\mathcal{P}_{\delta\chi}} \sim \frac{H}{2\pi}$$

$$\Longrightarrow 2 f_a\, \mathrm{e}^{-\frac{\Lambda^4}{6H^2 f_\mathrm{a}^2}(N_\mathrm{ini} - N_*)} > \frac{H}{2\pi}$$

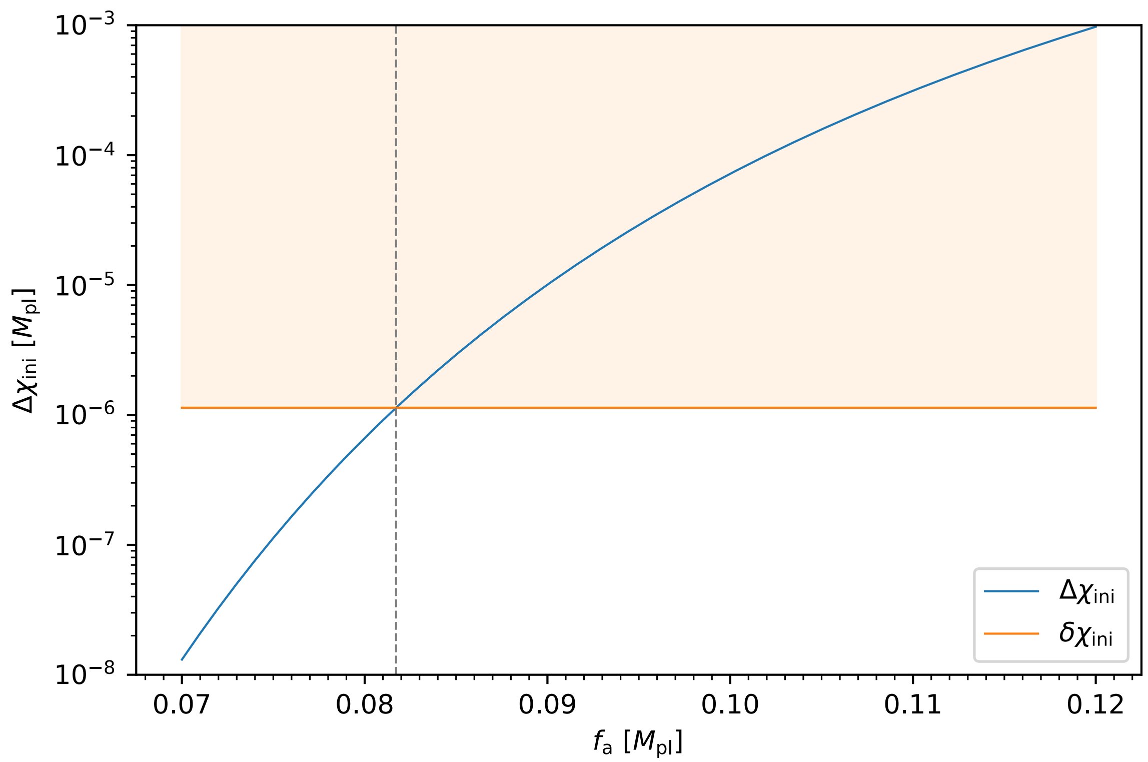
the time where the fast-roll happened

Model 2: Cosine Potential

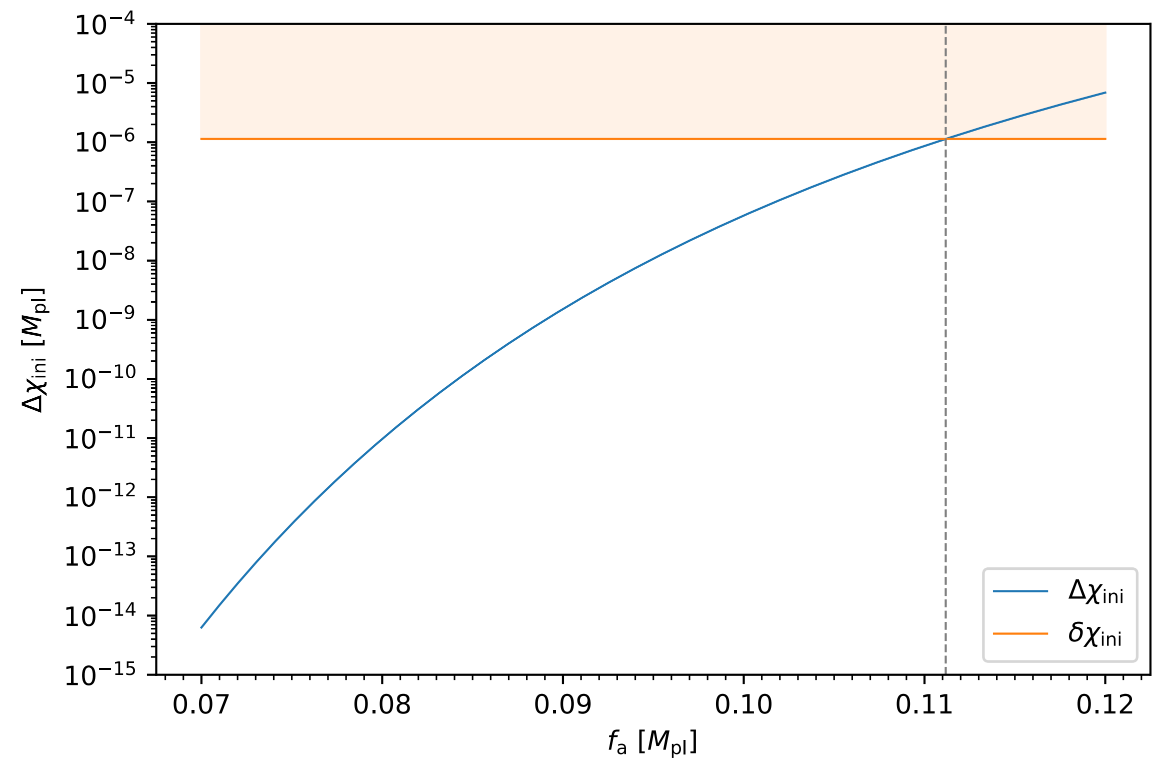
3

[2507.02611, Kai-Ge Zhang, Jian-Feng He, Chengjie Fu, Zong-Kuan Guo]

3.3 Constraint of f_a



Constraint on the parameter f_a for GW spectrum peaking at nHz region



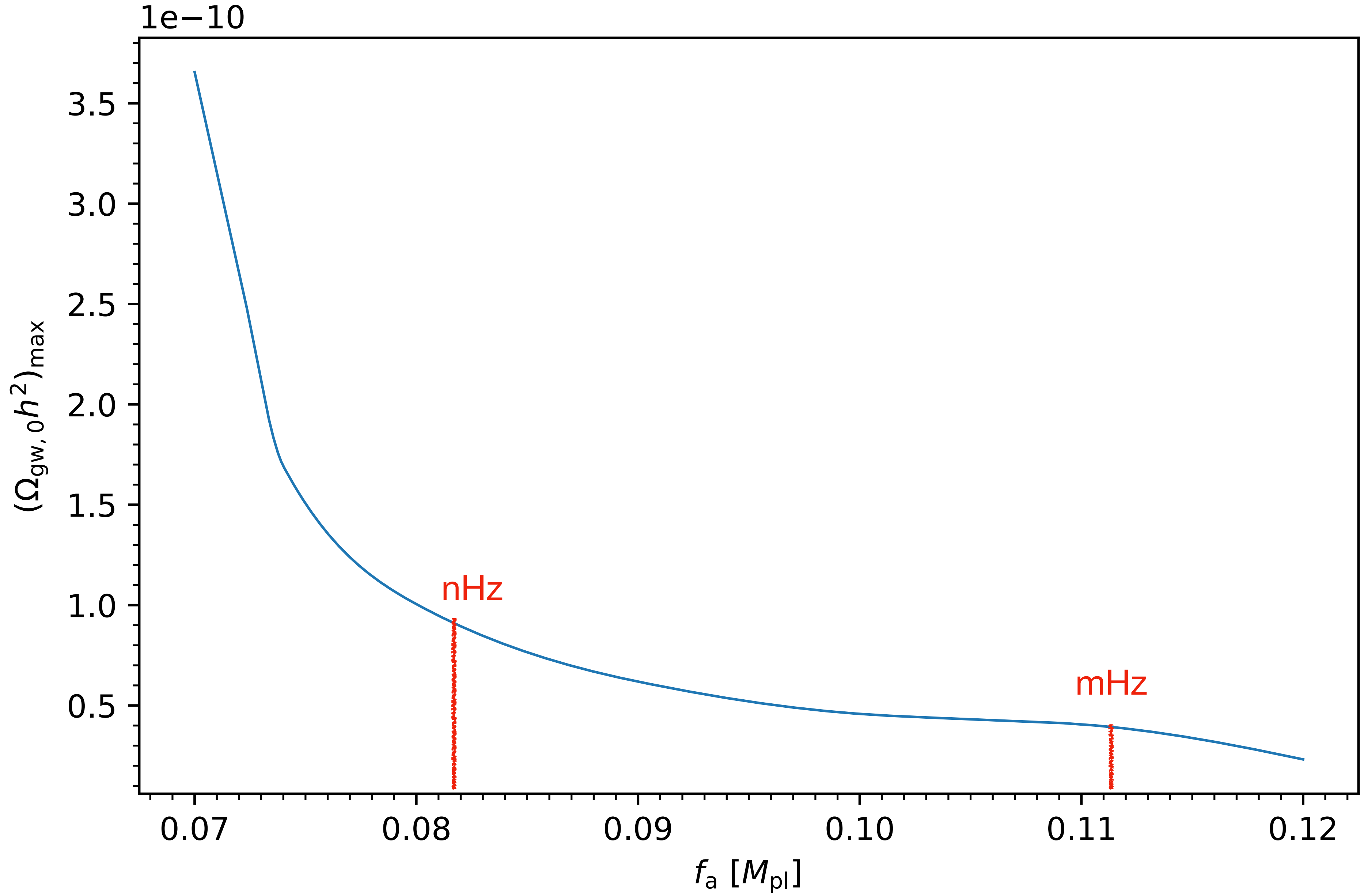
Constraint on the parameter f_a for GW spectrum peaking at mHz region

3

Model 2: Cosine Potential

[2507.02611, Kai-Ge Zhang, Jian-Feng He, Chengjie Fu, Zong-Kuan Guo]

3.3 Constraint of f_a



For both nHz and mHz cases, the maximum $\Omega_{\mathrm{gw},0}h^2$ is both $\sim 10^{-10}$

4 Conclusion

- peak of $\dot{\chi}$ is always prior than ρ_{EM}, and their time-delay leads to the oscillation behavior
- system **enter strong backreaction** when $f^{-1}\langle\vec{E}\cdot\vec{B}\rangle \sim U_{,\chi}$
- in **strong backreaction** region peak value of $k|A_k|^2$ nearly linearly relay on $U_{,\chi}$, but is almost not influenced by f^{-1}
- $\Omega_{\mathrm{GW},0}(k)$ mainly relay on $k|A_k|^2$, thus oscillation of $k|A_k|^2$ leads to oscillation of $\Omega_{\mathrm{GW},0}(k)$
- For cosine potential, the maximum $\Omega_{\mathrm{GW},0}$ are roughly 10^{-10} for both nHz and mHz

Thanks for listening!