Cosmological Signatures of Neutrino Seesaw mechanism

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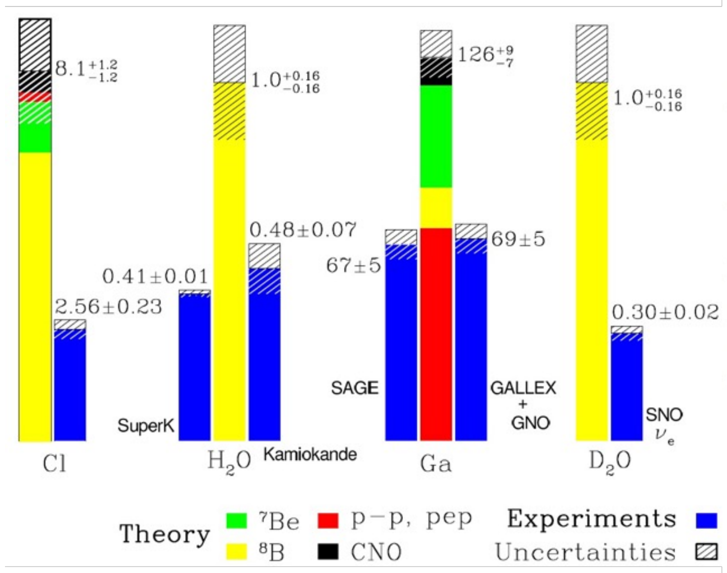
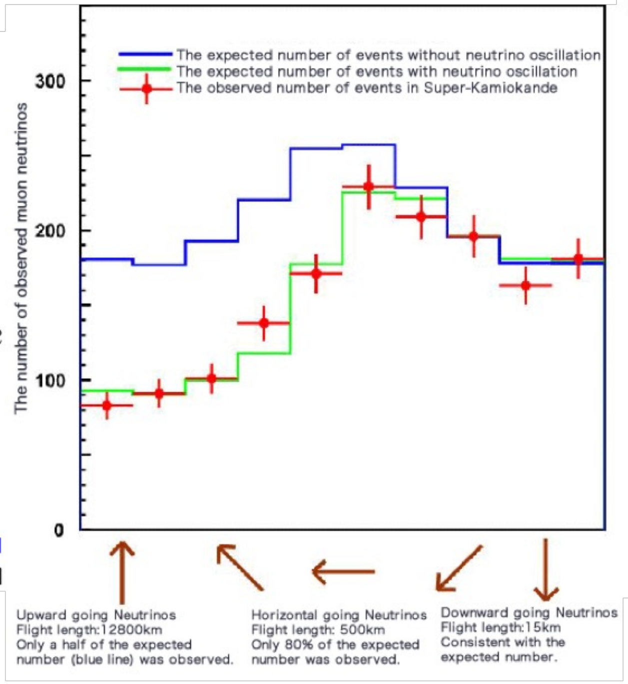
The 2025 Beijing Particle Physics and Cosmology Symposium

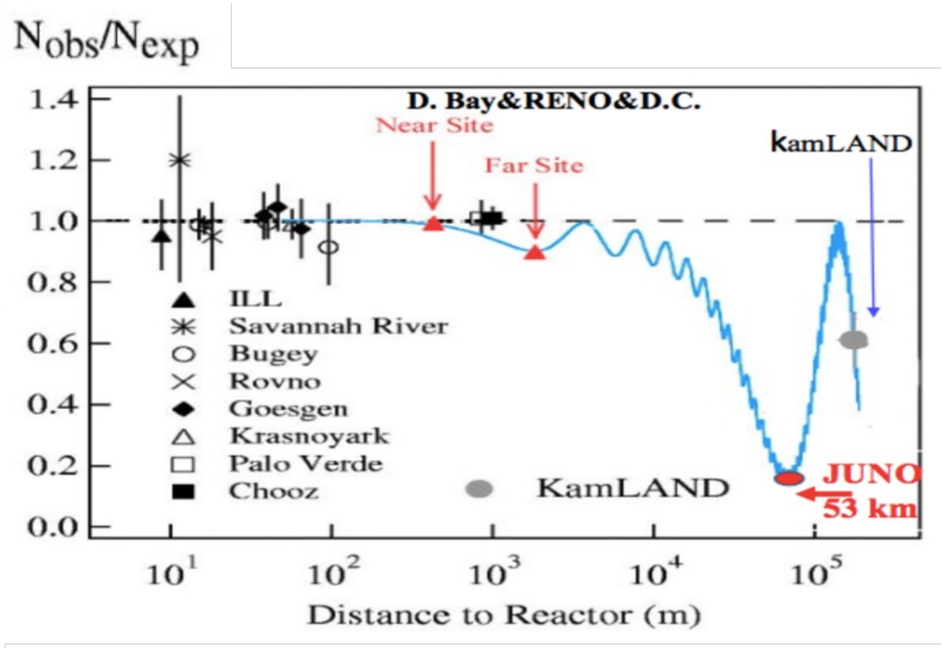
2025.9.27

Neutrino masses

Neutrino oscillation indicates massive neutrinos







Solar Neutrino oscillations

θ_{12}

Atmospheric Neutrino Oscillations

θ_{23}

Reactor Neutrino Oscillations

θ_{13}

$$\Delta m^2_{21} \simeq 7.42 \times 10^{-5}\ \mathrm{eV}^2$$

$$|\Delta m^2_{13}| \approx |\Delta m^2_{23}| \simeq 2.5 \times 10^{-3}\ \mathrm{eV}^2$$

Cosmological limit

Table 26.2: Summary of $\sum m_\nu$ constraints.

PDG

	Model	95% CL (eV)	Ref.
CMB alone			
Pl18[TT+lowE]	ΛCDM+$\sum m_\nu$	< 0.54	[24]
Pl18[TT,TE,EE+lowE]	ΛCDM+$\sum m_\nu$	< 0.26	[24]
CMB + probes of background evolution			
Pl18[TT,TE,EE+lowE] + BAO	ΛCDM+$\sum m_\nu$	< 0.13	[49]
Pl18[TT,TE,EE+lowE] + BAO	ΛCDM+$\sum m_\nu$+5 params.	< 0.515	[25]
CMB + LSS			
Pl18[TT+lowE+lensing]	ΛCDM+$\sum m_\nu$	< 0.44	[24]
Pl18[TT,TE,EE+lowE+lensing]	ΛCDM+$\sum m_\nu$	< 0.24	[24]
Pl18[TT,TE,EE+lowE]+ ACT[lensing]	ΛCDM+$\sum m_\nu$	< 0.12	[50]
CMB + probes of background evolution + LSS			
Pl18[TT,TE,EE+lowE] + BAO + RSD	ΛCDM+$\sum m_\nu$	< 0.10	[49]
Pl18[TT,TE,EE+lowE+lensing] + BAO + RSD + Shape	ΛCDM+$\sum m_\nu$	< 0.082	[51]
Pl18[TT+lowE+lensing] + BAO + Lyman-α	ΛCDM+$\sum m_\nu$	< 0.087	[52]
Pl18[TT,TE,EE+lowE] + BAO + RSD + SN + DES-Y1	ΛCDM+$\sum m_\nu$	< 0.12	[49]
Pl18[TT,TE,EE+lowE] + BAO + RSD + SN + DES-Y3	ΛCDM+$\sum m_\nu$	< 0.13	[53]

The heavy neutrino mass should be around 0.05 eV(IO)-0.06eV(NO)

Seesaw mechanism

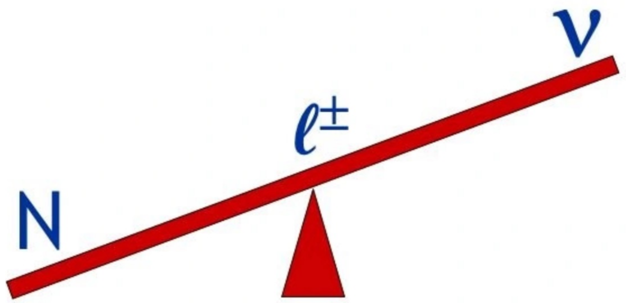
Origin of neutrino masses: seesaw mechanism

$$\mathcal{L} = \mathcal{L}_{\rm SM} + y_\nu \tilde{H} \bar{L} N_R - \frac{1}{2} M_R \bar{N}_R^c N_R + h.c.$$

P. Minkowski ; T. Yanagida; S. L. Glashow;
M. Gell-Mann, P. Ramond and R. Slansky

$$M = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$$

$$m_\nu \sim \frac{m_D^2}{M_R} = \frac{y_\nu^2 \langle h \rangle^2}{2 M_R}$$



- **Natural prediction of small neutrino masses**
- **Explain the baryon asymmetry of the universe: leptogenesis**

Baryogenesis Without Grand Unification, Fukugita and Yanagida, 1986′

Seesaw mechanism

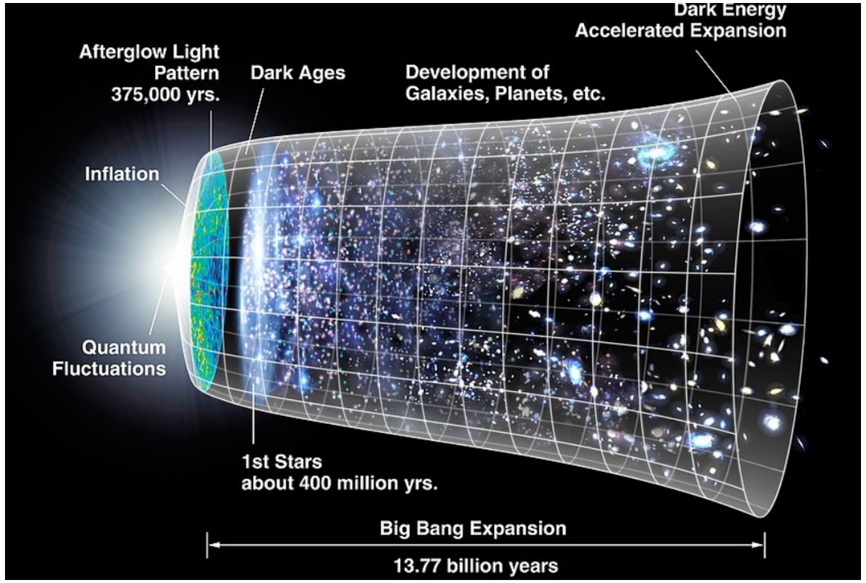
$$m_\nu \sim \frac{m_D^2}{M_R} = \frac{y_\nu^2 \langle h \rangle^2}{2M_R}$$

If the Yukawa coupling is O(1)(as predicted by the GUT), the seesaw scale M_R should be around $10^{13\text{-}14}$ GeV, which is much beyond the reach of particle experiments.

How to test such high scale seesaw?

Inflation

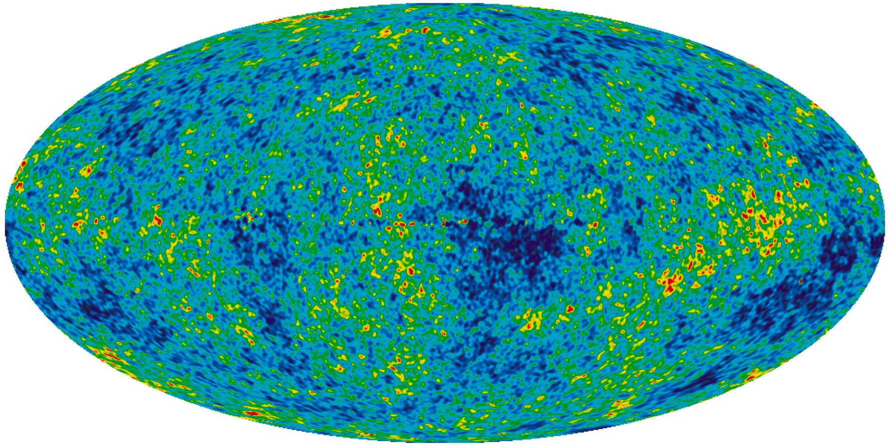
Rapid expansion of the universe in the early time



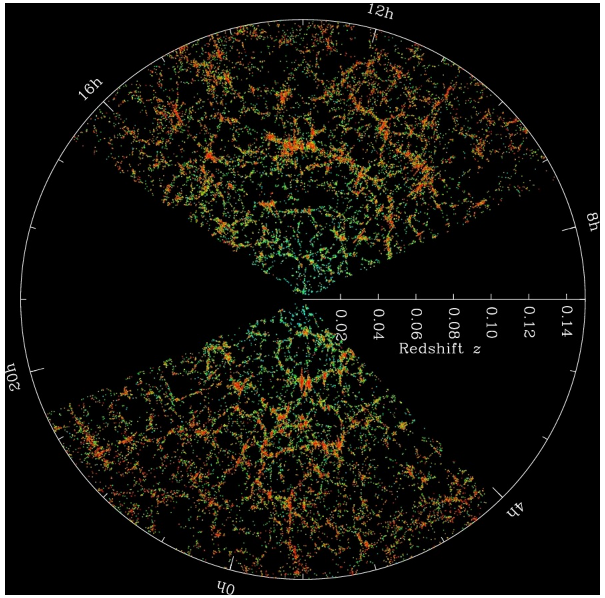
- Flatness problem
- Horizon problem
- Seeding the primordial anisotropies in CMB

Inflation

Stretching quantum fluctuations to large scale



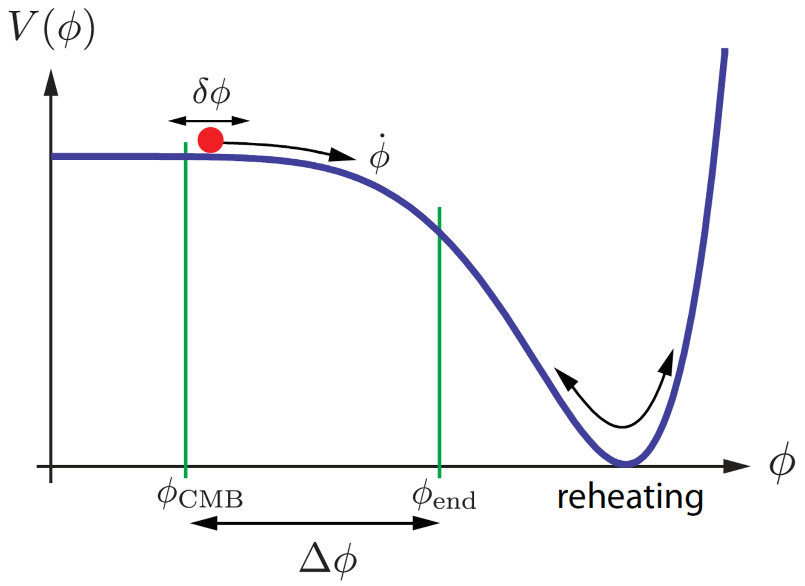
$$\frac{\delta T}{T} \sim 10^{-5}$$



Such small fluctuations finally develops the large structure of our universe

Slow-roll Inflation

Inflation is driven by a scalar field (inflaton)



$$\ddot{\phi} + 3H\dot{\phi} + \frac{\partial V}{\partial \phi} = 0$$

$$H^2 = \frac{1}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right)$$

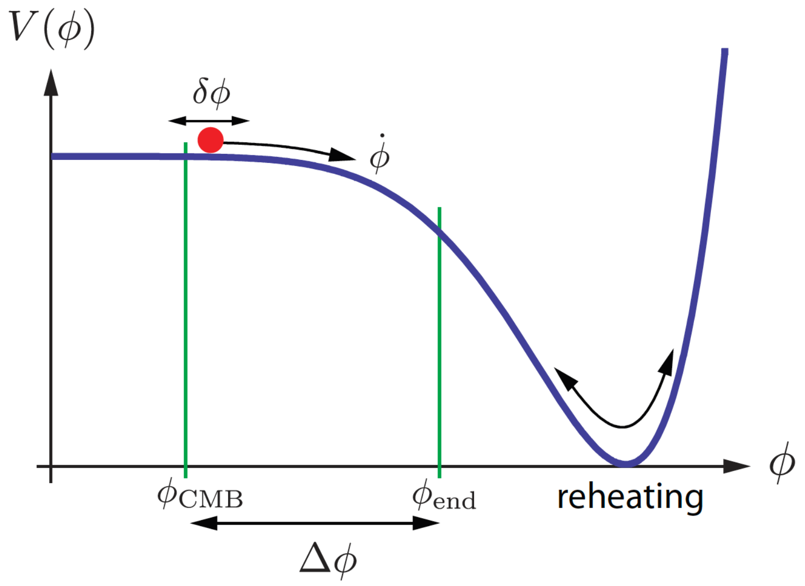
Slow roll condition

$$\dot{\phi}^2 \ll V(\phi) \qquad |\ddot{\phi}| \ll |3H\dot{\phi}|\,,\,|V_{,\phi}|$$

- Hubble parameter is nearly constant(de Sitter universe)
- After inflation, inflaton oscillates at the bottom of the potential and finally decays into SM particles, then reheats the universe(still no clear how it occurs)

Slow-roll Inflation

In a de Sitter universe, scalar fields get quantum fluctuation



$$\delta\phi(x,\tau) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[u_a(\tau,\mathbf{k})b_a(\mathbf{k}) + u_a^*(\tau,-\mathbf{k})b_a^\dagger(-\mathbf{k})\right]e^{i\mathbf{k}\cdot\mathbf{x}}$$

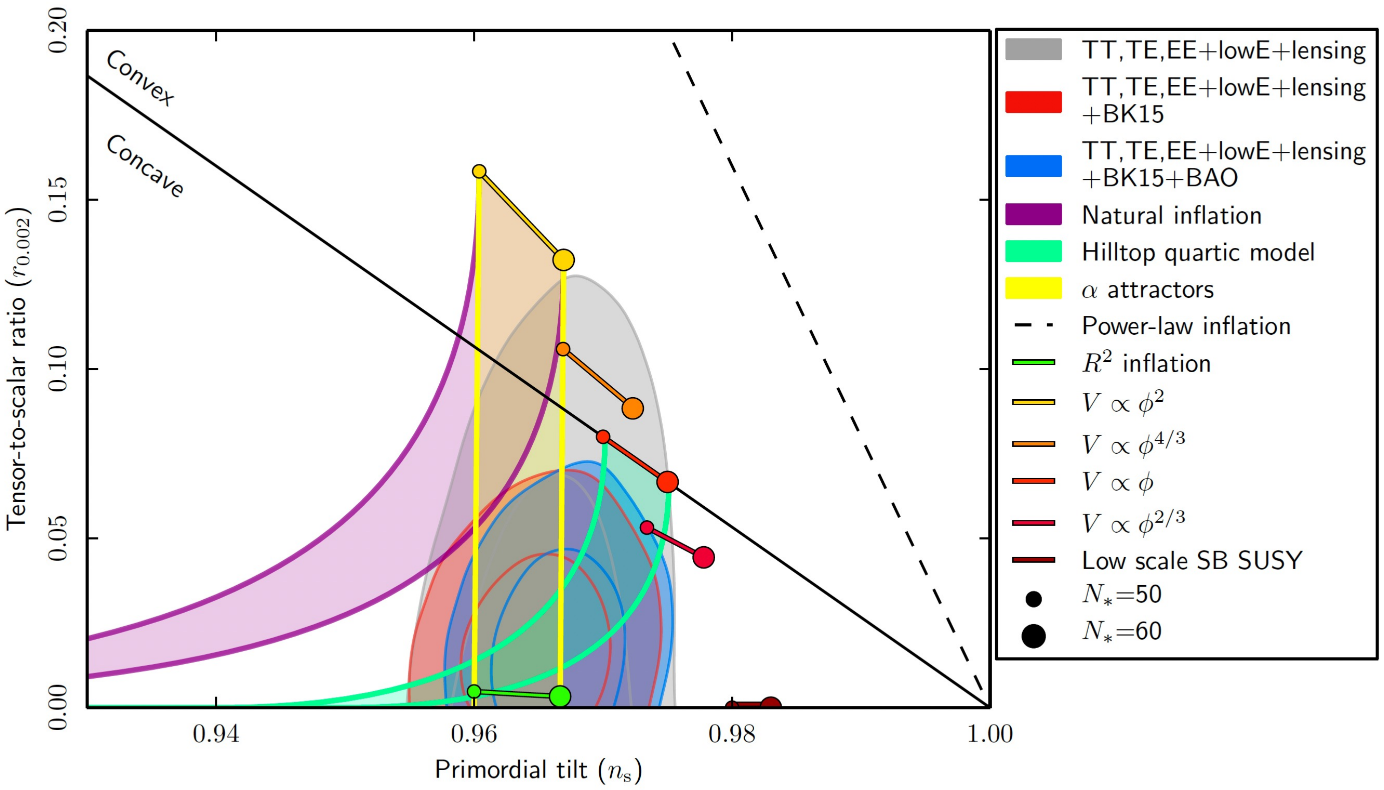
$$u_{\mathbf{k}}(\tau) = \frac{H}{\sqrt{2k^3}}\left[1 + ik\tau\right]e^{-ik\tau}$$

$$\langle \delta\phi_{\mathbf{k}}\,\delta\phi_{\mathbf{k}'}\rangle = (2\pi)^3\,\delta(\mathbf{k}+\mathbf{k}')\,\frac{2\pi^2}{k^3}\left(\frac{H}{2\pi}\right)^2$$

$$\zeta = -\frac{H}{\dot{\phi}}\delta\phi$$

- **Quantum fluctuation of inflaton induces CMB anisotropies(from curvature perturbations)**
- **In the single field inflation, the fluctuations should be nearly gaussian and adiabatic, close to scale invariant**

Inflation



$$\epsilon_V(\phi) \equiv \frac{M_{pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2$$

$$\eta_V(\phi) \equiv M_{pl}^2 \frac{V_{,\phi\phi}}{V}$$

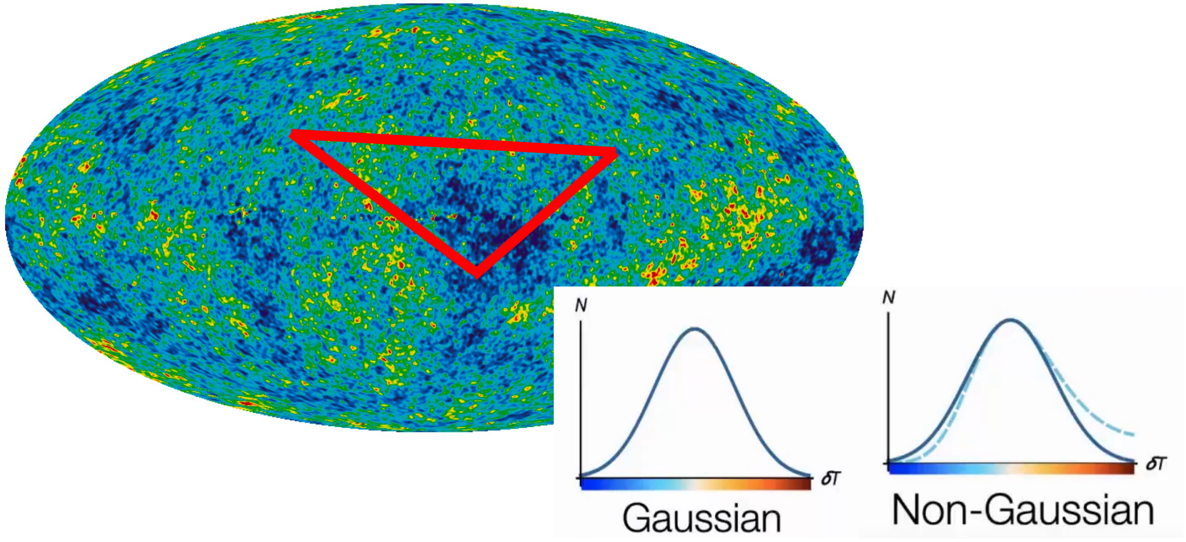
$$\epsilon_V < 0.0097$$

$$\eta_V = -0.010^{+0.007}_{-0.011}$$

$$\frac{H_*}{M_{Pl}} < 2.5 \times 10^{-5}$$

- **Inflaton potential should be flat enough(shift-symmetry?)**
- **Hubble scale could be as high as 6*10^{13} GeV(close to seesaw scale), providing access to the high scale physics**

Non-Gaussianity



Non-Gaussianity is sensitive to new physics

- **New physics could induce large non-Gaussianity : multi-field inflation models, modulated reheating, curvaton scenario…**

- **Current limit from Planck on local type f_{NL}~ O(10), future CMB observations, CMB S4, large scale structure observations DESI O(1), 21 cm tomography O(0.01-0.1)**

- **Non-Gaussianity could provide information to the new particle mass, spin, interactions: cosmological collider signals**

Nima Arkani-Hamed, Juan Maldacena, arXiv:1503.08043

Xingang Chen, Yi Wang, JCAP 04 (2010) 027

A minimal model

Minimal model incorporates inflation and seesaw

$$\Delta\mathcal{L} = \sqrt{-g}\left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi) + \overline{N}_\mathrm{R} i\partial\!\!\!/ N_\mathrm{R} + \frac{1}{\Lambda}\partial_\mu\phi\,\overline{N}_\mathrm{R}\gamma^\mu\gamma^5 N_\mathrm{R} + \left(-\frac{1}{2}M\overline{N^c_\mathrm{R}}N_\mathrm{R} - y_\nu\overline{L}_\mathrm{L}\tilde{\mathbb{H}}N_\mathrm{R} + \mathrm{H.c.}\right)\right]$$

- V(phi) is the potential for inflation is unknown but denominated by the mass term after inflation
- Derivative coupling to keep the flatness of the inflaton potential(shift-symmetry, dim-4 coupling should be suppressed, otherwise the induced phi^4 potential would destroy the flatness of the potential)
- Lambda > 60 Hubble to keep perturbative unitarity
- After inflation, inflaton oscillates at the bottom of the potential until decays into heavy neutrinos (mphi > 2 mN). The heavy neutrinos quickly decay into SM particles and reheat the universe

Seesaw mechanism

Consequence of the seesaw mechanism

$$\mathcal{L} \supset \frac{1}{2}\bar{\psi}_L \mathbf{M}_\nu \psi_R + \text{h.c.}, \qquad \mathbf{M}_\nu = \begin{pmatrix} 0 & \frac{y_\nu h}{\sqrt{2}} \\ \frac{y_\nu h}{\sqrt{2}} & M \end{pmatrix}$$

$$m_\nu \simeq -\frac{y_\nu^2 h^2}{2M}, \qquad M_N \simeq M + \frac{y_\nu^2 h^2}{2M}$$

- **Light neutrino gets a mass**
- **Heavy neutrino mass are get lifted (h dependent)**

Heavy neutrino mass is h dependent, then decay rate of the inflaton is h dependent:

$$\Gamma \simeq \frac{m_\phi M^2}{4\pi\Lambda^2}\left[1 + \frac{1}{4}\left(\frac{y_\nu h}{M}\right)^2\right]$$

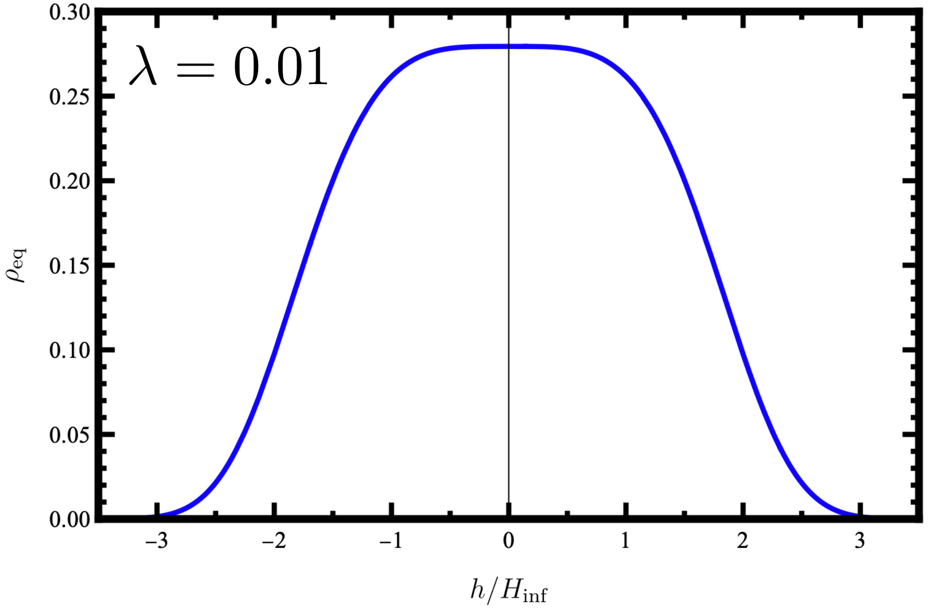
What happens to h in the early universe?

Higgs during inflation

Alexei A. Starobinsky, Jun'ichi Yokoyama,
Phys.Rev.D 50 (1994) 6357-6368

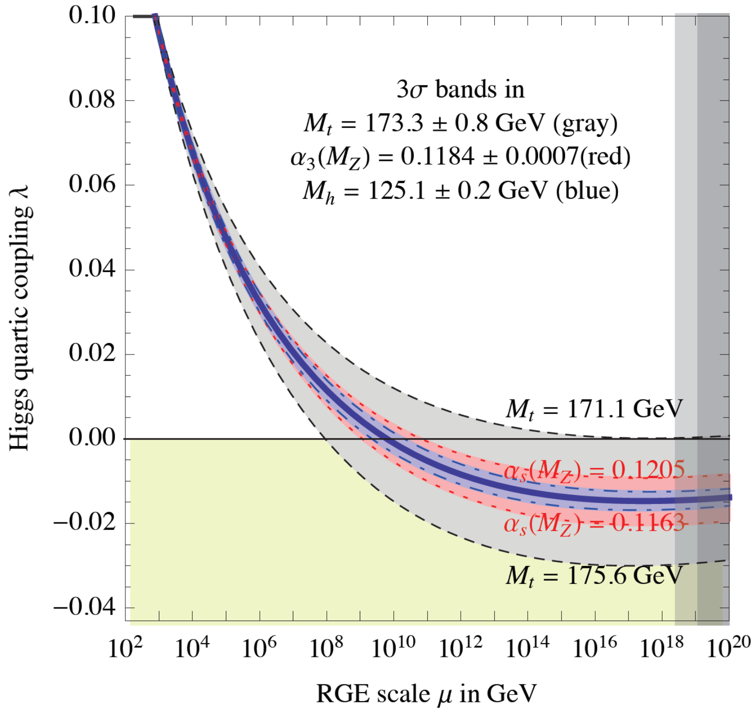
- **During inflation(de-Sitter universe), Higgs also gets quantum fluctuations**
- **The fluctuations reach a equilibrium state(solution of F-P equation)**

$$\rho_{\rm eq}(h) = \frac{2\lambda^{1/4}}{\Gamma(1/4)}\left(\frac{2\pi^2}{3}\right)^{1/4}\exp\left(\frac{-2\pi^2\lambda h^4}{3H_{\rm inf}^4}\right)$$



$$\bar{h} = \sqrt{\langle h^2 \rangle} = \left[\int_{-\infty}^{+\infty} dh\, h^2 \rho_{\rm eq}(h)\right]^{1/2} \simeq 0.363\left(\frac{H_{\rm inf}}{\lambda^{1/4}}\right)$$

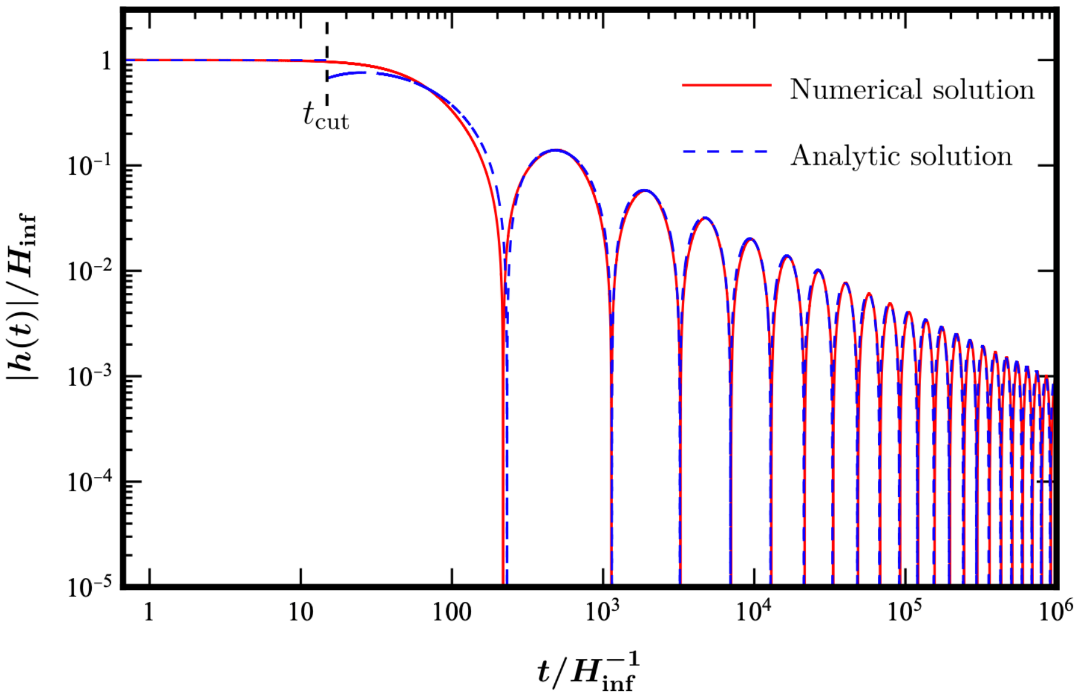
D. Buttazzo, et al arXiv:1307.3536



Higgs after inflation

Inflaton oscillates at the bottom potential. If the inflaton potential is dominated by the mass term, the Universe is matter-dominated

$$\ddot{h}(t) + \frac{2}{t}\dot{h}(t) + \lambda h^3(t) = 0$$



$$h(t) = \begin{cases} h_{\rm inf}\,, & t \leq t_{\rm cut} \\ A H_{\rm inf}\left(\frac{h_{\rm inf}}{H_{\rm inf}\lambda}\right)^{\frac{1}{3}} (H_{\rm inf}\,t)^{-\frac{2}{3}} \cos\left(\lambda^{\frac{1}{6}} |h_{\rm inf}|^{\frac{1}{3}}\, \omega\, t^{\frac{1}{3}} + \theta\right), & t > t_{\rm cut} \end{cases}$$

$$t_{\rm cut} = \frac{\sqrt{2}}{3\sqrt{\lambda}\, h_{\rm inf}}$$

$$\begin{aligned} A &= 2^{1/3} 3^{-2/3} 5^{1/4} \simeq 0.9 \\ \omega &= \frac{\Gamma^2(3/4)}{\sqrt{\pi}} 2^{1/3} 3^{1/3} 5^{1/4} \simeq 2.3 \\ \theta &= -3^{-1/3} 2^{1/6} \omega - \arctan 2 \simeq -2.9 \end{aligned}$$

Higgs value would oscillate and decrease

Higgs modulated reheating

Higgs value varies from different patches of the universe, right-handed neutrino mass also varies as well as the decay rate of the inflaton

$$\Gamma \simeq \frac{m_\phi M^2}{4\pi\Lambda^2}\left[1+\frac{1}{4}\left(\frac{y_\nu h}{M}\right)^2\right]$$

Gia Dvali, Andrei Gruzinov, Matias Zaldarriaga, Phys.Rev. D69 (2004) 023505

- **Different patches of the universe reheat differently (modulated reheating)**
- **The curvature perturbation is generated by Higgs field**
- **Delta N formalism (zeta=delta N~ N- <N>)**

$$\zeta_h(t > t_{\rm reh}, \mathbf{x}) = \delta N(\mathbf{x}) = N(\mathbf{x}) - \langle N(\mathbf{x})\rangle$$
$$= -\frac{1}{6}\left[\ln(\Gamma_{\rm reh}) - \langle \ln(\Gamma_{\rm reh})\rangle\right]$$

$$\zeta = \zeta_\phi + \zeta_h$$

Higgs modulated reheating

Curvature perturbation contains two parts

$$\zeta = \zeta_\phi + \zeta_h \qquad \mathcal{P}_\zeta^{(\phi)} = \left(\frac{H}{\dot\phi}\right)^2 \mathcal{P}_\phi = \left(\frac{H}{\dot\phi}\right)^2 \frac{H^2}{4\pi^2}$$

Taylor expansion of the curvature perturbation

$$\zeta_h(\mathbf{x}) = -\frac{1}{6}\left[\frac{\Gamma_0'}{\Gamma_0}\delta h_{\rm inf}(\mathbf{x}) + \frac{\Gamma_0\Gamma_0'' - \Gamma_0'\Gamma_0'}{2\Gamma_0^2}\delta h_{\rm inf}^2(\mathbf{x})\right] \equiv z_1 \delta h_{\rm inf}(\mathbf{x}) + \frac{1}{2} z_2 \delta h_{\rm inf}^2(\mathbf{x})$$

$$\mathcal{P}_\zeta^{(h)} = z_1^2 \mathcal{P}_{\delta h} = \frac{z_1^2 H^2}{4\pi^2} \qquad R = \left(\frac{\mathcal{P}_\zeta^{(h)}}{\mathcal{P}_\zeta^{(o)}}\right)^{1/2} = |z_1| \left(\frac{\mathcal{P}_{\delta h}}{\mathcal{P}_\zeta^{(o)}}\right)^{1/2}$$

R should be less than 1

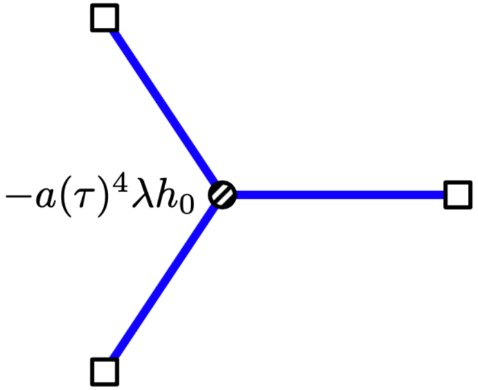
Bispectrum

Considering the three point correlation function

$$\langle \zeta_{\mathbf{k_1}} \zeta_{\mathbf{k_2}} \zeta_{\mathbf{k_3}} \rangle_h = z_1^3 \langle \delta h_{\mathbf{k_1}} \delta h_{\mathbf{k_2}} \delta h_{\mathbf{k_3}} \rangle + z_1^2 z_2 \langle \delta h^4 \rangle_{2\mathrm{nd}} (\mathbf{k_1}, \mathbf{k_2}, \mathbf{k_3})$$

First term is from Higgs self-coupling

$$z_1^3 \langle \delta h_{\mathbf{k_1}} \delta h_{\mathbf{k_2}} \delta h_{\mathbf{k_3}} \rangle$$



Calculated by in-in formalism/Schwinger-Keldysh formalism

Steven Weinberg, Phys.Rev.D 72 (2005) 043514, Phys.Rev.D 74 (2006) 023508
Xingang Chen, Yi Wang, Zhong-Zhi Xianyu, JCAP 1712 (2017) 006

Bispectrum

$$\langle \delta h_{\mathbf{k_1}} \delta h_{\mathbf{k_2}} \delta h_{\mathbf{k_3}} \rangle' = 12\lambda\bar{h}\mathrm{Im}\left(\int_{-\infty}^{\tau_f} a^4 \prod_{i=1}^{3} G_+(\mathbf{k}_i, \tau)\, d\tau\right)$$

$$\mathrm{Im}\left(\int_{-\infty}^{\tau_f} a^4 \prod_{i=1}^{3} G_+(\mathbf{k}_i, \tau)\, d\tau\right)$$

$$N_e = \log(\frac{a_{\mathrm{end}}}{a_k}) = \log(\frac{-\frac{1}{H\tau_f}}{\frac{k_t}{H}}) = -\log(k_t|\tau_f|) \sim 60$$

$$= \mathrm{Im}\int_{-\infty}^{\tau_f} \frac{d\tau}{(H\tau)^4} \cdot \frac{H^6}{8k_1^3k_2^3k_3^3}\left(\prod_{i=1}^{3}(1 - ik_i\tau)\right) e^{i(k_1+k_2+k_3)\tau}$$

$$= \frac{H^2}{24k_1^3k_2^3k_3^3} \cdot \left\{(k_1^3 + k_2^3 + k_3^3)[\log(k_t|\tau_f|) + \gamma - \frac{4}{3}] + k_1k_2k_3 - \sum_{a\neq b} k_a^2k_b\right\}$$

Bispectrum

Second term is from non-linear evolution of the Higgs

$$z_1^2 z_2 \langle \delta h^4 \rangle(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \frac{z_1^2 z_2}{2} \int \frac{\mathrm{d}^3\mathbf{k}_0}{(2\pi)^3} \langle \delta h(\mathbf{k}_1)\delta h(\mathbf{k}_2)\delta h(\mathbf{k}_0)\delta h(\mathbf{k}_3 - \mathbf{k}_0)\rangle + (2\,\mathrm{perm.})$$

$$= \frac{z_1^2 z_2}{2}\left[\int \frac{\mathrm{d}^3\mathbf{k}_0}{(2\pi)^3} \langle \delta h(\mathbf{k}_1)\delta h(\mathbf{k}_0)\rangle \langle \delta h(\mathbf{k}_2)\delta h(\mathbf{k}_3 - \mathbf{k}_0)\rangle + (\mathbf{k}_1 \leftrightarrow \mathbf{k}_2)\right] + (2\,\mathrm{perm.})$$

$$= \frac{z_1^2 z_2}{2}\left[\int \mathrm{d}^3\mathbf{k}_0\, (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_0)\delta^3(\mathbf{k}_2 + \mathbf{k}_3 - \mathbf{k}_0) \frac{H^4}{4k_1^3 k_2^3} + (\mathbf{k}_1 \leftrightarrow \mathbf{k}_2)\right] + (2\,\mathrm{perm.})$$

$$= (2\pi)^3 \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) z_1^2 z_2 \left[\frac{H^4}{4k_1^3 k_2^3} + (2\,\mathrm{perm.})\right].$$

Local type non-gaussianity

The local type non-gaussianity which is defined by Bardeen Potential $\Phi \equiv \frac{3}{5}\zeta$

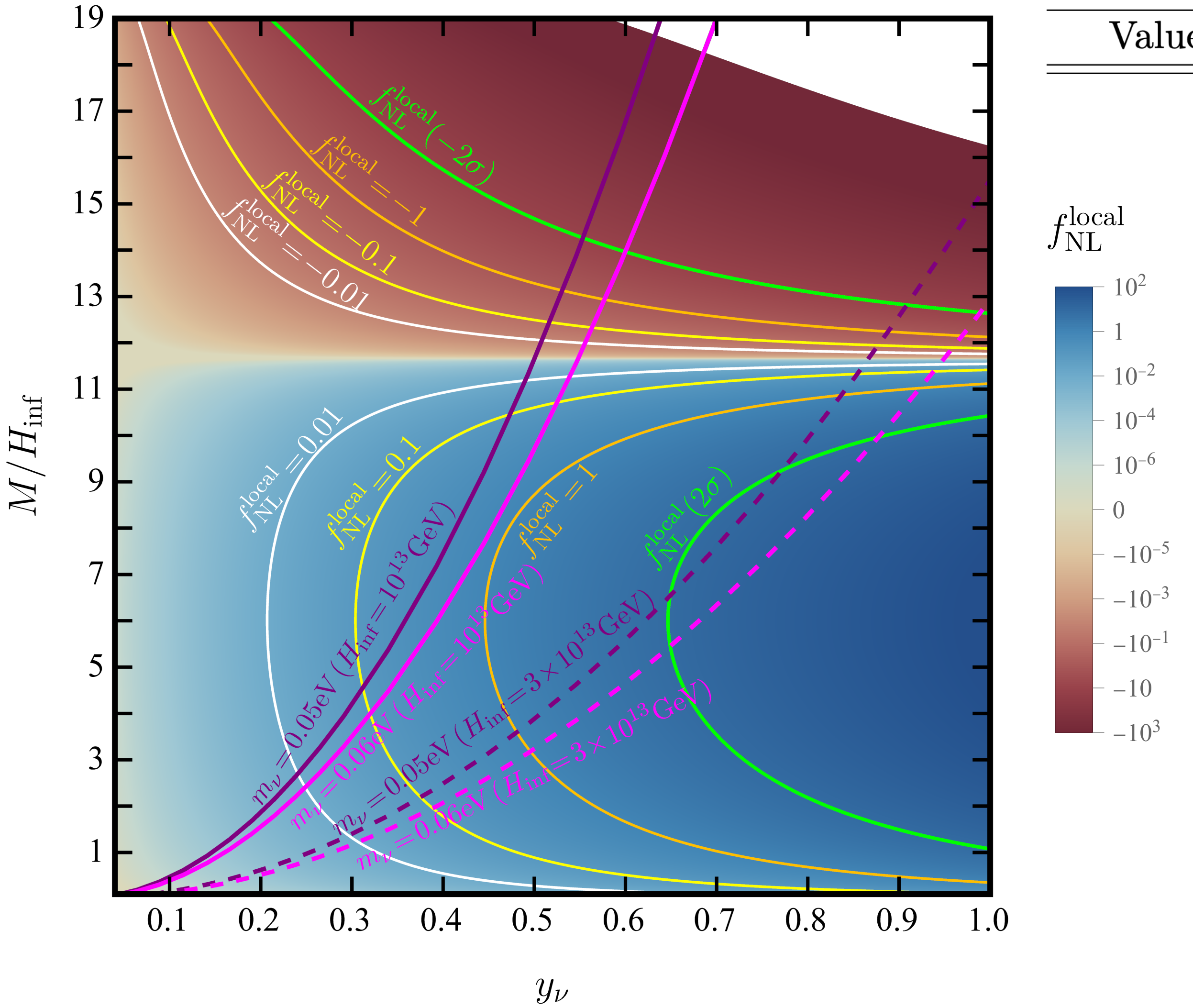
$$\langle \Phi_{\mathbf{k_1}} \Phi_{\mathbf{k_2}} \Phi_{\mathbf{k_3}} \rangle'_{\mathrm{local}} = 2A^2 f_{\mathrm{NL}}^{\mathrm{local}} \left\{ \frac{1}{k_1^3 k_2^3} + \frac{1}{k_2^3 k_3^3} + \frac{1}{k_3^3 k_1^3} \right\}$$

In the limit $k_1 \sim k_2 >> k_3$, we find

$$f_{\mathrm{NL}}^{\mathrm{local}} \simeq -\frac{10}{3} \frac{z_1^3 H^3}{(2\pi)^4 \mathcal{P}_\zeta^2} \left(\frac{\lambda \bar{h}}{2H} N_e - \frac{z_2 H}{4 z_1} \right)$$

$$f_{\mathrm{NL}}^{\mathrm{local}} = -0.9 \pm 5.1 \quad \text{(68\% C.L., Planck 2018)}$$

Local type non-gaussianity



Parameters	$\mathcal{P}_\zeta$	N_e	H_{inf}	m_ϕ	Λ	λ
Values	2.1×10^{-9}	60	$(1,3)\times10^{13}\,\text{GeV}$	$40H_{inf}$	$60H_{inf}$	0.01

- **Colored curves indicating future searches**
- **Parameter space with Yukawa O(1) could be probed by future observations**
- **The contribution from self-interaction and non-linear term are both important**
- **Interplaying with neutrino experiments(JUNO, DUNE for neutrino ordering)**

Summary

- We propose a minimal model incorporating inflation and seesaw
- Seesaw generated non-Gaussianity could be probed in near future CMB or large-scale structure observations
- Cosmological collider signals from fermion loop (in progress)

Thanks!

S-K formalism

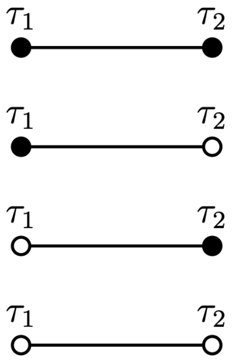
$$Q(\tau) \equiv \varphi^{A_1}(\tau, \mathbf{x}_1) \cdots \varphi^{A_N}(\tau, \mathbf{x}_N)$$

$$\langle Q(\tau) \rangle = \langle \Omega | \bar{F}(\tau, \tau_0) Q_I(\tau) F(\tau, \tau_0) | \Omega \rangle$$

$$F(\tau, \tau_0) = \mathrm{T} \exp \left(-\mathrm{i} \int_{\tau_0}^{\tau} \mathrm{d}\tau_1 \, H_I(\tau_1) \right),$$
$$\bar{F}(\tau, \tau_0) = \bar{\mathrm{T}} \exp \left(\mathrm{i} \int_{\tau_0}^{\tau} \mathrm{d}\tau_1 \, H_I(\tau_1) \right),$$

S-K formalism

$$
\begin{cases}
G_{++}(\mathbf{k};\tau_1,\tau_2) & \equiv G_{>}(\mathbf{k};\tau_1,\tau_2)\,\theta(\tau_1-\tau_2)+G_{<}(\mathbf{k};\tau_1,\tau_2)\,\theta(\tau_2-\tau_1) \\
G_{+-}(\mathbf{k};\tau_1,\tau_2) & \equiv G_{<}(\mathbf{k};\tau_1,\tau_2) \\
G_{-+}(\mathbf{k};\tau_1,\tau_2) & \equiv G_{>}(\mathbf{k};\tau_1,\tau_2) \\
G_{--}(\mathbf{k};\tau_1,\tau_2) & \equiv G_{<}(\mathbf{k};\tau_1,\tau_2)\,\theta(\tau_1-\tau_2)+G_{>}(\mathbf{k};\tau_1,\tau_2)\,\theta(\tau_2-\tau_1)
\end{cases}
$$



$= G_{++}(k;\tau_1,\tau_2)$

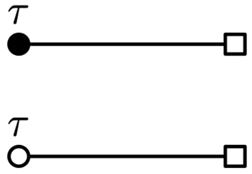
$= G_{+-}(k;\tau_1,\tau_2)$

$= G_{-+}(k;\tau_1,\tau_2)$

$= G_{--}(k;\tau_1,\tau_2)$

$$G_{>}(k;\tau_1,\tau_2)\equiv u(\tau_1,k)u^*(\tau_2,k)$$

$$G_{<}(k;\tau_1,\tau_2)\equiv u^*(\tau_1,k)u(\tau_2,k)$$



$= G_{+}(k;\tau)\equiv G_{++}(k;\tau,\tau_f)$

$= G_{-}(k;\tau)\equiv G_{-+}(k;\tau,\tau_f)$

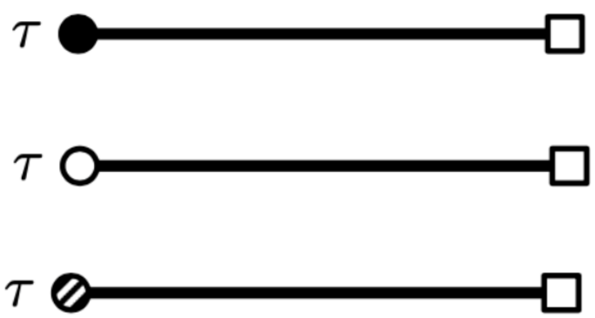
$$\Box u_{\mathbf{k}} = \ddot{u}_{\mathbf{k}} + 3H\dot{u}_{\mathbf{k}} + \frac{\mathbf{k}^2}{a^2(t)}u_{\mathbf{k}} = 0$$

$$u_{\mathbf{k}}(\tau) = \frac{H}{\sqrt{2k^3}}\left[1+ik\tau\right]e^{-ik\tau}$$

S-K formalism

Bulk-to-Boundary propagator

$$G_{\pm}(\mathbf{k},\tau) \equiv G_{\pm +}\left(\mathbf{k};\tau,\tau_f\right)$$



$$= G_{+}(\mathbf{k},\tau)$$

$$= G_{-}(\mathbf{k},\tau)$$

$$= G_{+}(\mathbf{k},\tau) + G_{-}(\mathbf{k},\tau)$$

$$G_{+}(\mathbf{k},\tau) = \frac{H^2}{2k^3}\left[1 - ik(\tau-\tau_f) + k^2\tau\tau_f\right]e^{ik(\tau-\tau_f)} \simeq \frac{H^2}{2k^3}\left[1 - ik\tau\right]e^{ik\tau}$$

$$G_{-}(\mathbf{k},\tau) \simeq \frac{H^2}{2k^3}\left[1 + ik\tau\right]e^{-ik\tau}$$

Higgs during inflation

During inflation(de-Sitter universe), Higgs also gets quantum fluctuations

暴胀期间，希格斯场可以分为长波和短波两部分

Alexei A. Starobinsky, Jun'ichi Yokoyama,
Phys.Rev.D 50 (1994) 6357-6368

$$h(\mathbf{x},t) = h_L(\mathbf{x},t) + \int \frac{d^3k}{(2\pi)^3}\theta(k-\epsilon a(t)H_{\rm inf})\left[a_{\mathbf{k}}h_{\mathbf{k}}(t)e^{-i\mathbf{k}\cdot\mathbf{x}} + a^\dagger_{\mathbf{k}}h^*_{\mathbf{k}}(t)e^{i\mathbf{k}\cdot\mathbf{x}}\right]$$

$$h_{\mathbf{k}} = \frac{H_{\rm inf}}{\sqrt{2k^3}}(1+ik\tau)\,e^{-ik\tau}$$

长波部分可以用郎之万方程描述

$$\dot{h}_L(\mathbf{x},t) = -\frac{1}{3H_{\rm inf}}\frac{\partial V}{\partial h_L} + f(\mathbf{x},t)$$

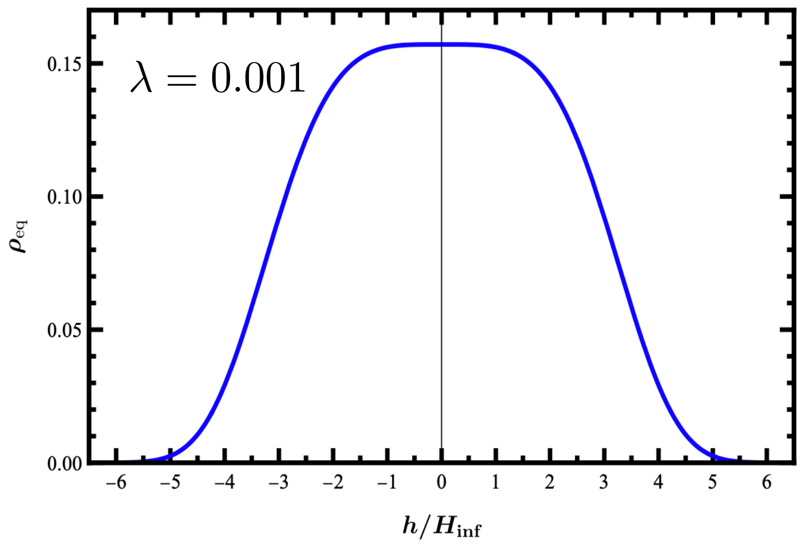
$$\langle f(\mathbf{x}_1,t_1)f(\mathbf{x}_2,t_2)\rangle = \frac{H^3_{\rm inf}}{4\pi^2}\delta(t_1-t_2)\,j_0\left(\epsilon a(t_1)H_{\rm inf}|\mathbf{x}_1-\mathbf{x}_2|\right)$$

带自相互作用的随机游走模型

Higgs during inflation

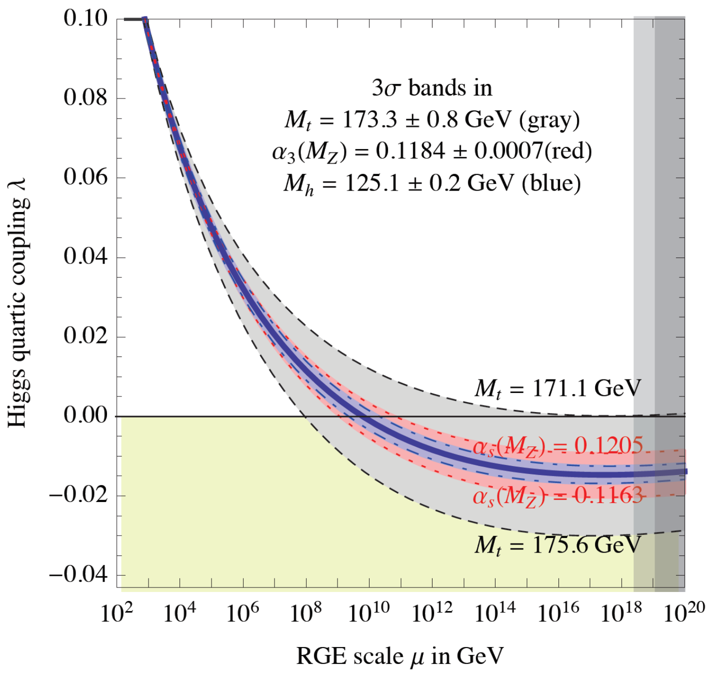
- **If inflation lasts long enough, these fluctuations reach a equilibrium state**
- **Different part of universe Higgs field takes different value**

$$\rho_{\rm eq}(h) = \frac{2\lambda^{1/4}}{\Gamma(1/4)}\left(\frac{2\pi^2}{3}\right)^{1/4} \exp\left(\frac{-2\pi^2\lambda h^4}{3H_{\rm inf}^4}\right)$$



$$\bar{h} = \sqrt{\langle h^2\rangle} = \left[\int_{-\infty}^{+\infty} \mathrm{d}h\, h^2 \rho_{\rm eq}(h)\right]^{1/2} \simeq 0.363\left(\frac{H_{\rm inf}}{\lambda^{1/4}}\right)$$

D. Buttazzo, et al arXiv:1307.3536

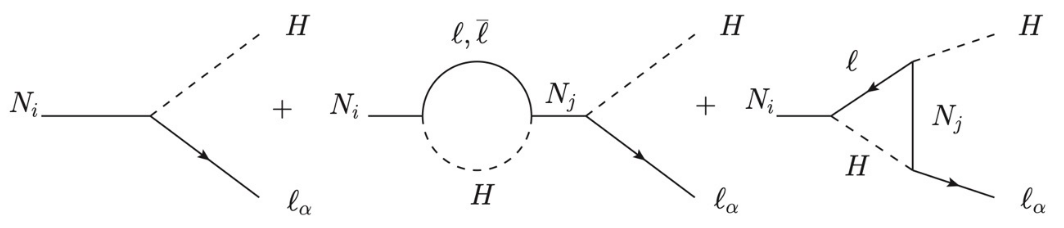


Leptogenesis

Baryogenesis Without Grand Unification, Fukugita and Yanagida, 1986′

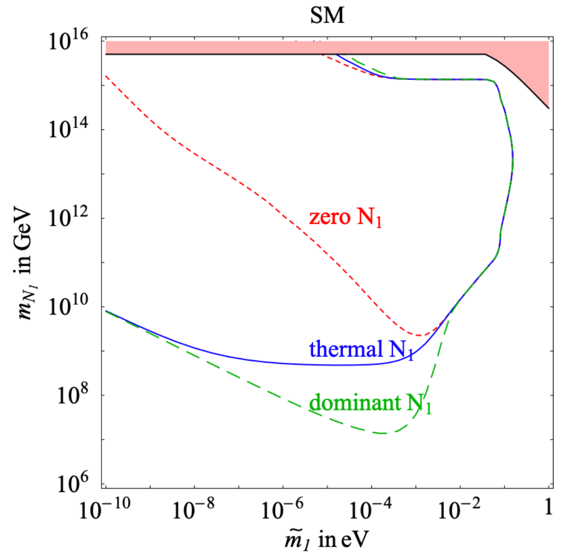
G.F. Giudice, et al,
Nucl.Phys.B 685 (2004) 89-149

$$\mathcal{L}_I = \mathcal{L}_{SM} + i\overline{N_{R_i}}\partial\!\!\!/ N_{R_i} - \left(\frac{1}{2}M_i\overline{N^c_{R_i}}N_{R_i} + \epsilon_{ab}Y_{\alpha i}\overline{N_{R_i}}\ell^a_\alpha H^b + h.c.\right)$$



$$\epsilon_{i\alpha} = \frac{\gamma\left(N_i \to \ell_\alpha H\right) - \gamma\left(N_i \to \overline{\ell_\alpha} H^*\right)}{\sum_\alpha \gamma\left(N_i \to \ell_\alpha H\right) + \gamma\left(N_i \to \overline{\ell_\alpha} H^*\right)}$$

$$n_B = \frac{28}{79}(\mathcal{B} - \mathcal{L})_i$$



Mass of the right-handed neutrino should heavier than 10^9 GeV