The 2025 Beijing Particle Physics and Cosmology Symposium

Sound waves from primordial black hole formations

Zhuan Ning (宁专), UCAS

Based on: arXiv:2504.12243

Co-authors: Xiang-Xi Zeng, Zi-Yan Yuwen, Shao-Jiang Wang, Heling Deng, Rong-Gen Cai

2025.09.29 @ Beijing

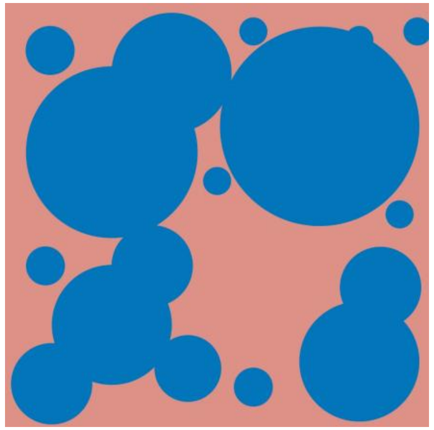
Outline

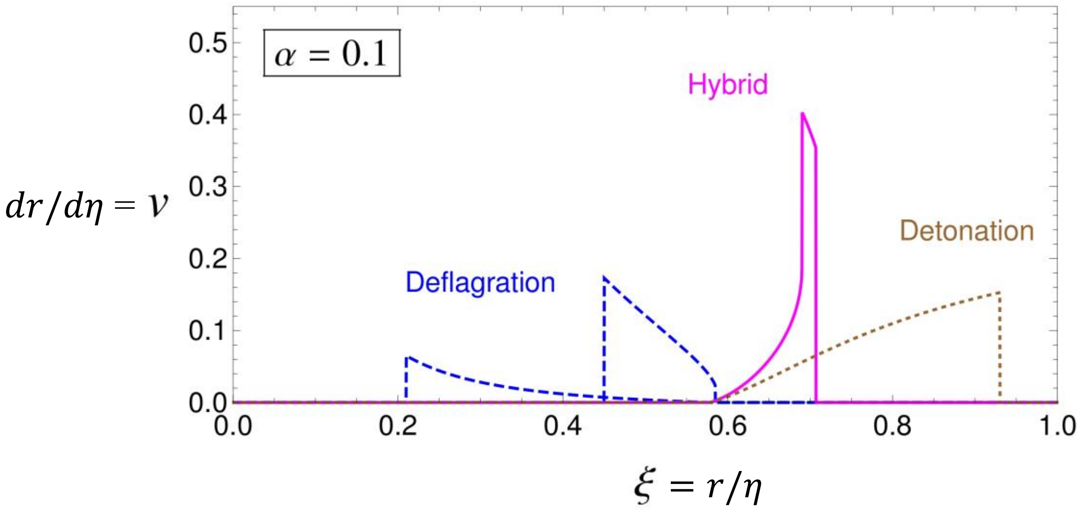
中国科学院大学
University of Chinese Academy of Sciences

01 Introduction

01 | Introduction

- Sound waves can be generated and propagate in the early Universe
- Cosmological first-order phase transitions:
  - Wall-fluid interaction generates sound waves
  - Self-similar velocity profiles
  - Serve as initial conditions for the sound shell model to semi-analytically calculate their gravitational wave spectra [1608.04735,1909.10040,2007.08537]

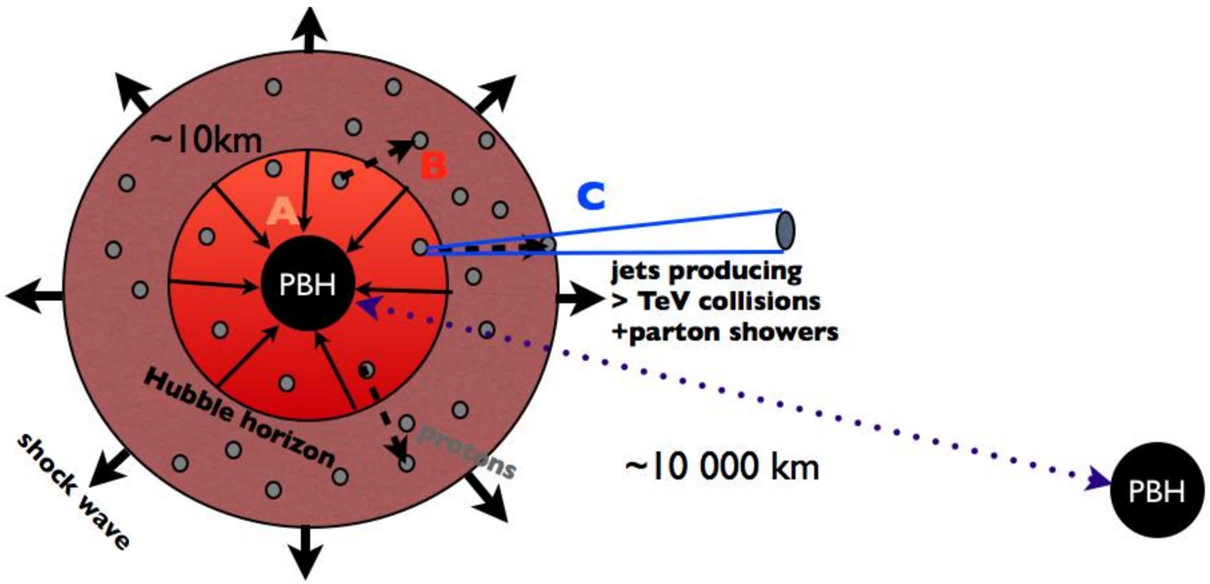




2007.08537 Huai-Ke Guo et.al. Phase Transitions in an Expanding Universe: Stochastic Gravitational Waves in Standard and Non-Standard Histories

01 | Introduction

- Primordial black hole (PBH) formations also generate sound waves!
- Collisions between these sound waves are expected to generate gravitational waves as well
- Profiles of PBH-induced sound waves are governed by nonlinear dynamics of gravitational collapse
- Need nonlinear numerical simulation!



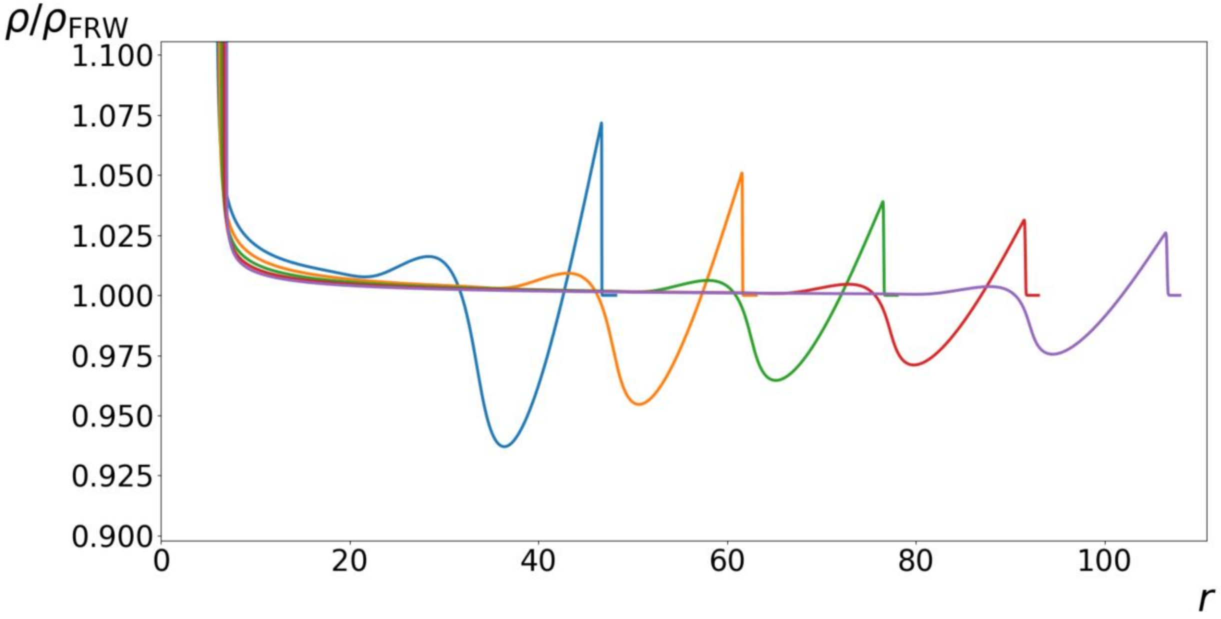
1904.02129 Bernard Carr et.al. Primordial black holes from the QCD epoch: linking dark matter, baryogenesis, and anthropic selection
1904.11482 García-Bellido Juan et.al. Primordial Black Holes and a Common Origin of Baryons and Dark Matter

01 | Further cosmological implications

- Serve as high-density hotspots to facilitate baryogenesis via electroweak sphaleron transitions

$$E_+ \simeq \left(\frac{1}{\gamma} - 1\right) M_{\rm H} = \left(\frac{1-\gamma}{\gamma^2}\right) M_{\rm PBH} \qquad M_{\rm PBH} = \gamma M_{\rm H}$$

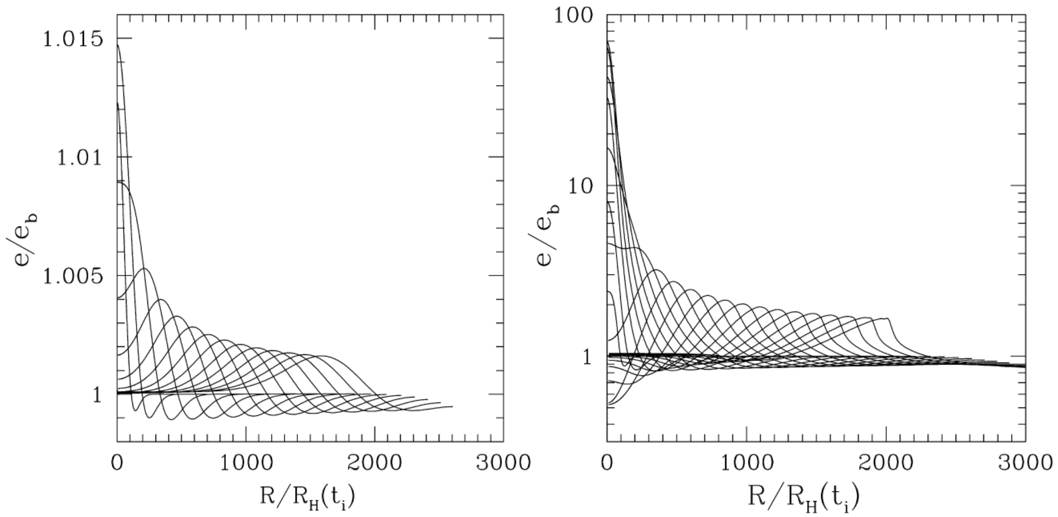
- Dissipation of sound waves due to photon diffusion result in a spectral distortion in the CMB



1710.02865 Heling Deng et.al. Primordial black hole formation by vacuum bubbles
1804.10059 Heling Deng et.al. CMB spectral distortions from black holes formed by vacuum bubbles

01 | PBH formation from curvature perturbations

- A PBH is formed if the amplitude of perturbation exceeds a critical threshold
- For the subcritical perturbations, previous studies observe compression waves



- However, for the supercritical perturbations, their evolution time is insufficient
- Using the Misner-Sharp formalism with an excision technique, our simulations extend to significantly later times than previous work

gr-qc/0605122 Ilia Musco et.al. Curvature profiles as initial conditions for primordial black hole formation

中国科学院大学
University of Chinese Academy of Sciences

02 Simulation Setup

02 | Misner-Sharp formalism

- PBH formation from curvature perturbations
- Metric ansatz: $ds^2 = -A(t,r)^2 \mathrm{d}t^2 + B(t,r)^2 \mathrm{d}r^2 + R(t,r)^2 \mathrm{d}\Omega^2$
- Energy-momentum tensor: $T_{\mu\nu} = (\rho + p) u_\mu u_\nu + p g_{\mu\nu}$
- Equation of state: $p = \omega\rho$

$$
\begin{aligned}
D_r A &= \frac{-A}{\rho + p} D_r p, \\
D_r M &= 4\pi\Gamma\rho R^2, \\
\Gamma &= \sqrt{1 + U^2 - \frac{2M}{R}}, \\
D_t R &= U, \\
D_t U &= -\left[\frac{\Gamma}{(\rho + p)} D_r p + \frac{M}{R^2} + 4\pi R p\right], \\
D_t M &= -4\pi R^2 U p, \\
D_t \rho &= -\frac{(\rho + p)}{\Gamma R^2} D_r (U R^2).
\end{aligned}
$$

- Break down due to the singularity formation
- Need to excise the black hole interior region

02 | Cosmological setup

- To enhance numerical accuracy and stability, we factor out the background evolution from the dynamical variables

$$\xi \equiv \ln(t/t_0)$$
$$\bar{r} \equiv r/R_H$$
$$R_H = H^{-1}(t_0) = t_0/\alpha$$

$$\rho = \rho_b \bar{\rho},$$
$$p = \rho_b \bar{p},$$
$$R = R_b \bar{R} = ar\bar{R},$$
$$U = HR\bar{U},$$
$$M = \frac{4\pi}{3}\rho_b R^3 \bar{M},$$

$$A = \bar{\rho}^{-3\alpha\omega/2}, \tag{11a}$$

$$\bar{\rho} = \bar{M} + \frac{\bar{r}\bar{R}}{3(\bar{r}\bar{R})'}\bar{M}', \tag{11b}$$

$$\bar{\Gamma}^2 \equiv \frac{\Gamma^2}{a^2 H^2 R_H^2} = e^{2(1-\alpha)\xi} + \bar{r}^2\bar{R}^2\left(\bar{U}^2 - \bar{M}\right), \tag{11c}$$

$$\partial_\xi \bar{R} = \alpha\bar{R}\left(\bar{U}A - 1\right), \tag{11d}$$

$$\partial_\xi \bar{U} = \bar{U} - \alpha A\left[\bar{\Gamma}^2 \frac{\bar{p}'}{\bar{r}\bar{R}(\bar{R}+\bar{r}\bar{R}')(\bar{\rho}+\bar{p})} + \frac{1}{2}\left(2\bar{U}^2 + \bar{M} + 3\bar{p}\right)\right], \tag{11e}$$

$$\partial_\xi \bar{M} = 2\bar{M} - 3\alpha\bar{U}A\left(\bar{p} + \bar{M}\right), \tag{11f}$$

$$\partial_\xi \bar{\rho} = -\alpha(\bar{\rho}+\bar{p})A\left(3\bar{U} + \frac{\bar{r}\bar{R}\bar{U}'}{\bar{R}+\bar{r}\bar{R}'}\right) + 3\alpha(1+\omega)\bar{\rho}, \tag{11g}$$

02 | Initial conditions

- Super-horizon initial conditions: long-wavelength approximation $\epsilon(t) \equiv \frac{H^{-1}(t)}{a(t)r_m} = \frac{1}{a(t)H(t)r_m} \ll 1$
- Spatial perturbations are encoded in $K(r)$:

$$ds^2 = -dt^2 + a^2(t)\left(\frac{dr^2}{1-K(r)r^2} + r^2 d\Omega^2\right)$$

- Perturbative expansion

$$R(t,r) = a(t)r\left(1+\epsilon^2(t)\tilde{R}(r)\right),$$
$$U(t,r) = H(t)R(t,r)\left(1+\epsilon^2(t)\tilde{U}(r)\right),$$
$$M(t,r) = \frac{4\pi}{3}\rho_b(t)R(t,r)^3\left(1+\epsilon^2(t)\tilde{M}(r)\right),$$
$$\rho(t,r) = \rho_b(t)\left(1+\epsilon^2(t)\tilde{\rho}(r)\right).$$

$$\tilde{U}(r) = -\frac{1}{5+3\omega}K(r)r_m^2,$$
$$\tilde{M}(r) = -3(1+\omega)\tilde{U}(r),$$
$$\tilde{\rho}(r) = \frac{3(1+\omega)}{5+3\omega}\left[K(r)+\frac{r}{3}K'(r)\right]r_m^2,$$
$$\tilde{R}(r) = -\frac{\omega}{(1+3\omega)(1+\omega)}\tilde{\rho}(r) + \frac{1}{1+3\omega}\tilde{U}(r).$$

02 | Numerical implementation

- Curvature perturbation with a Gaussian profile:

$$K(r) = \mathcal{A}\bar{K}(r) = \mathcal{A}e^{-(r/r_m)^2}$$

- Critical threshold: $\delta_c \approx 0.49774$

$$\mathcal{A} = \frac{\delta_m}{f(\omega)\bar{K}(r_m)r_m^2} = \frac{(5+3\omega)\delta_m}{3(1+\omega)r_m^2}$$

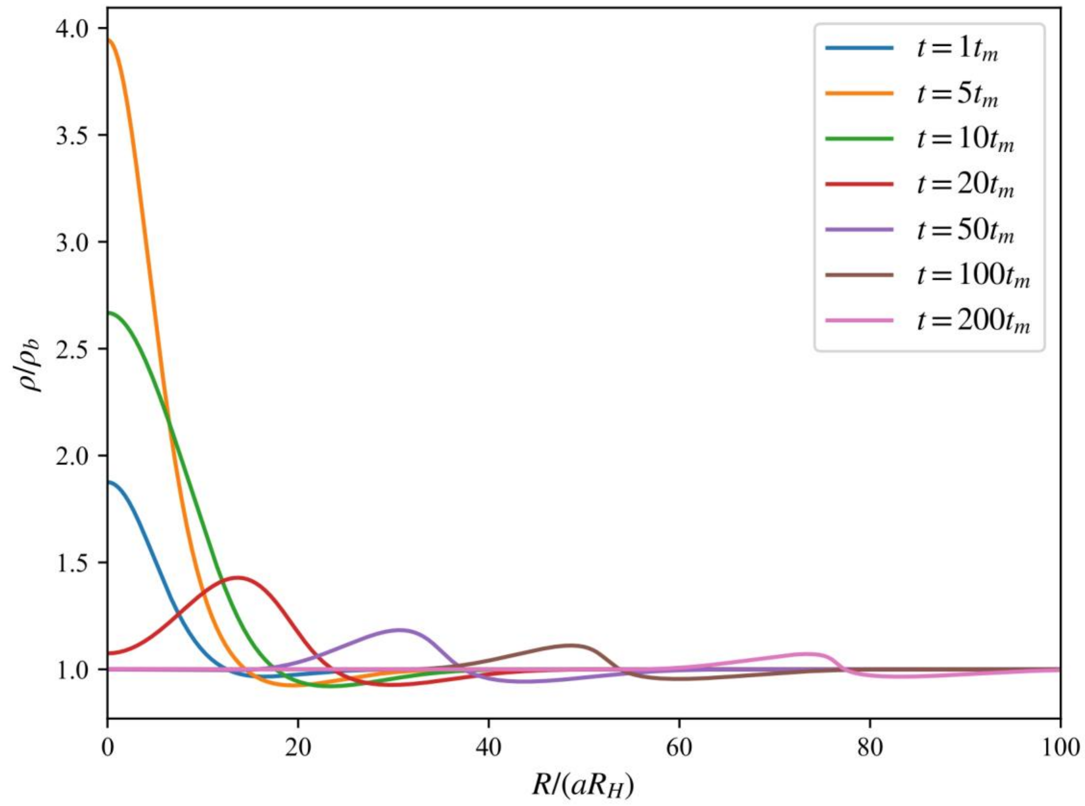
- Radial direction: 4th-order central difference + logarithmic grid
- Time direction: 4th-order Runge-Kutta method

中国科学院大学
University of Chinese Academy of Sciences

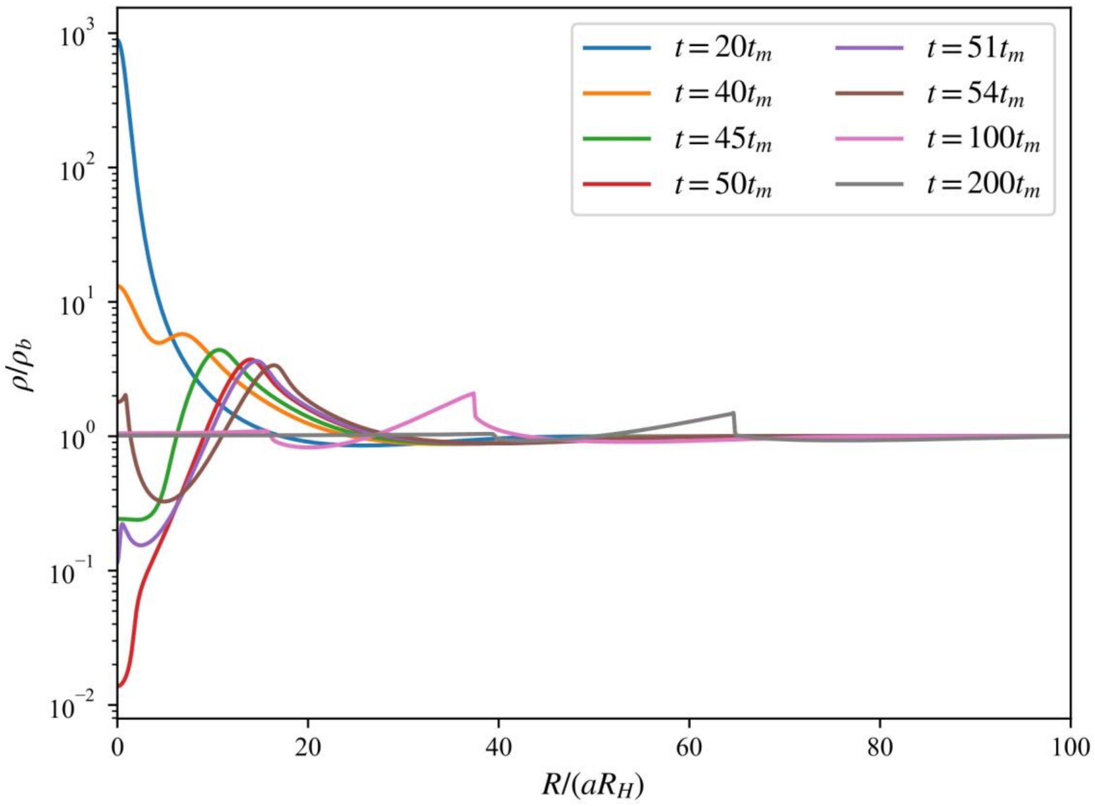
03 Numerical Results

03 | Subcritical cases

- No black hole is formed ($\omega = 1/3$)



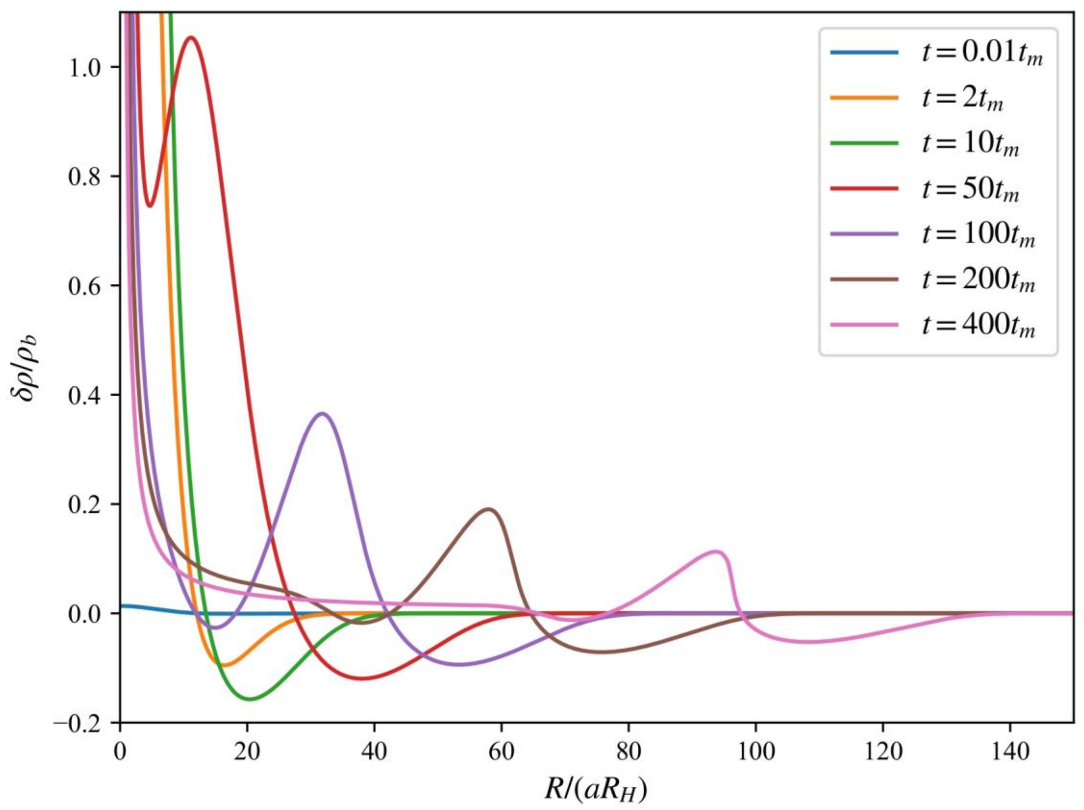
$\delta_m = 0.3 \ll \delta_c$



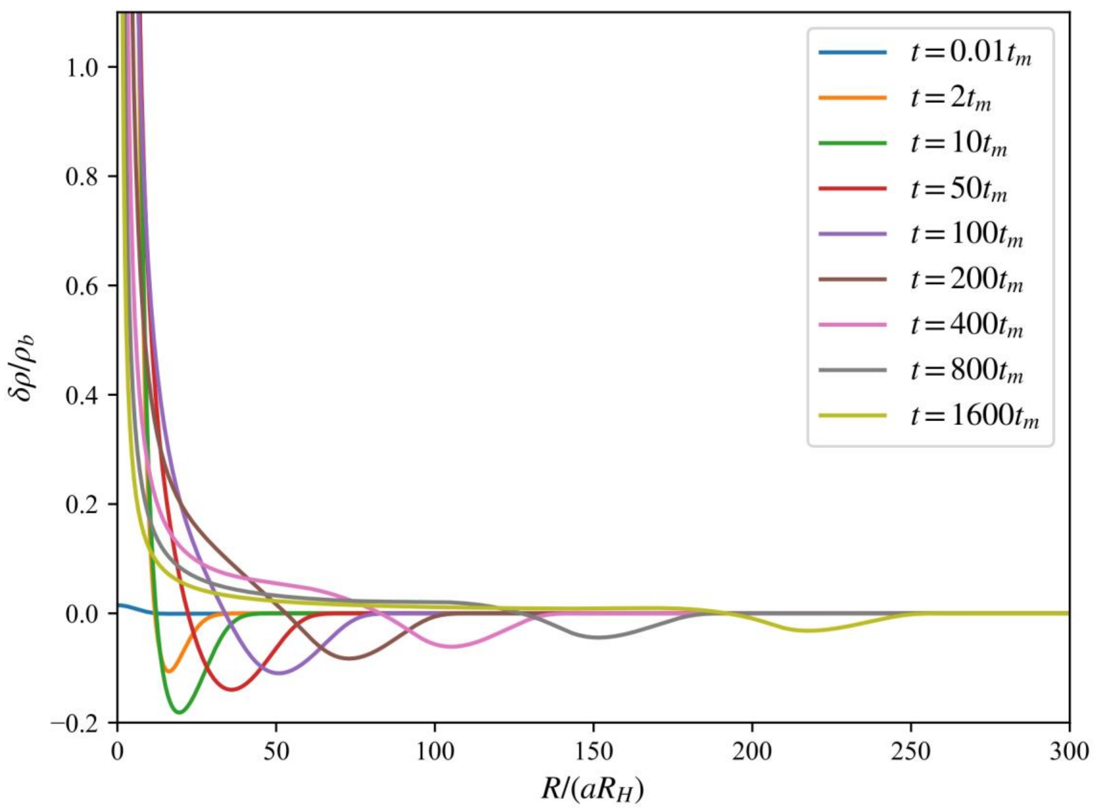
$\delta_m = 0.495 \lesssim \delta_c$

03 | Supercritical cases

- The PBH is formed ($\omega = 1/3$)



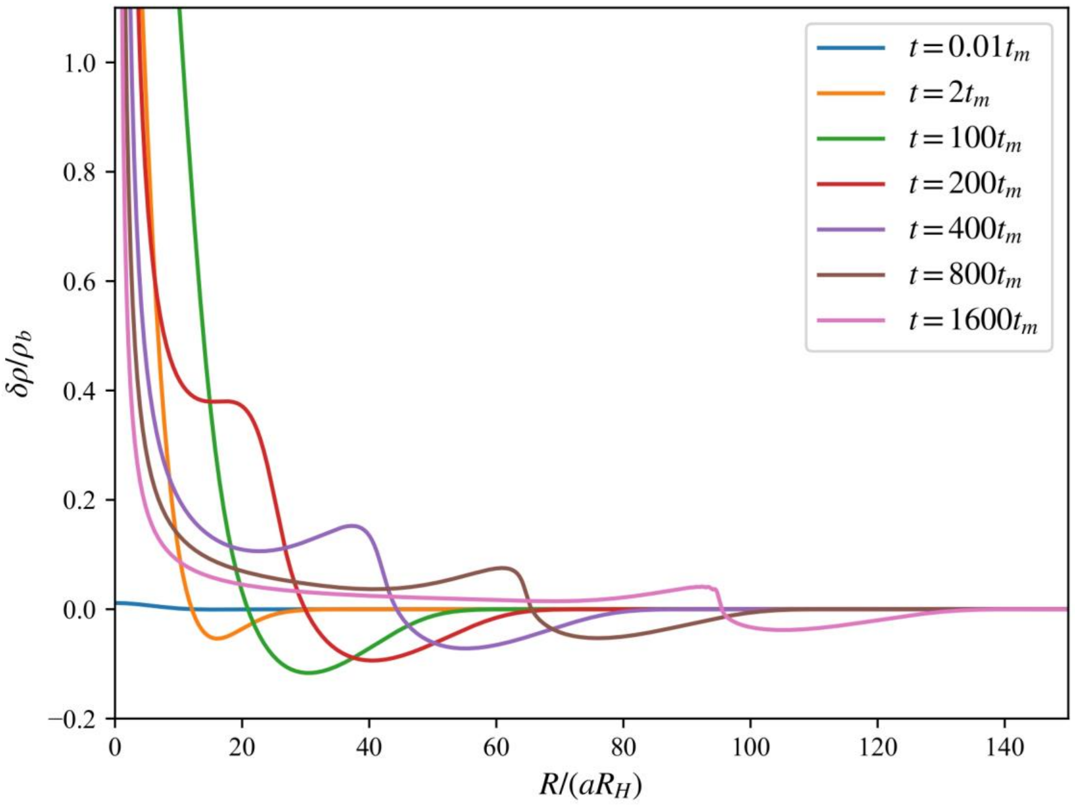
$\delta_m = 0.5 \gtrsim \delta_c$



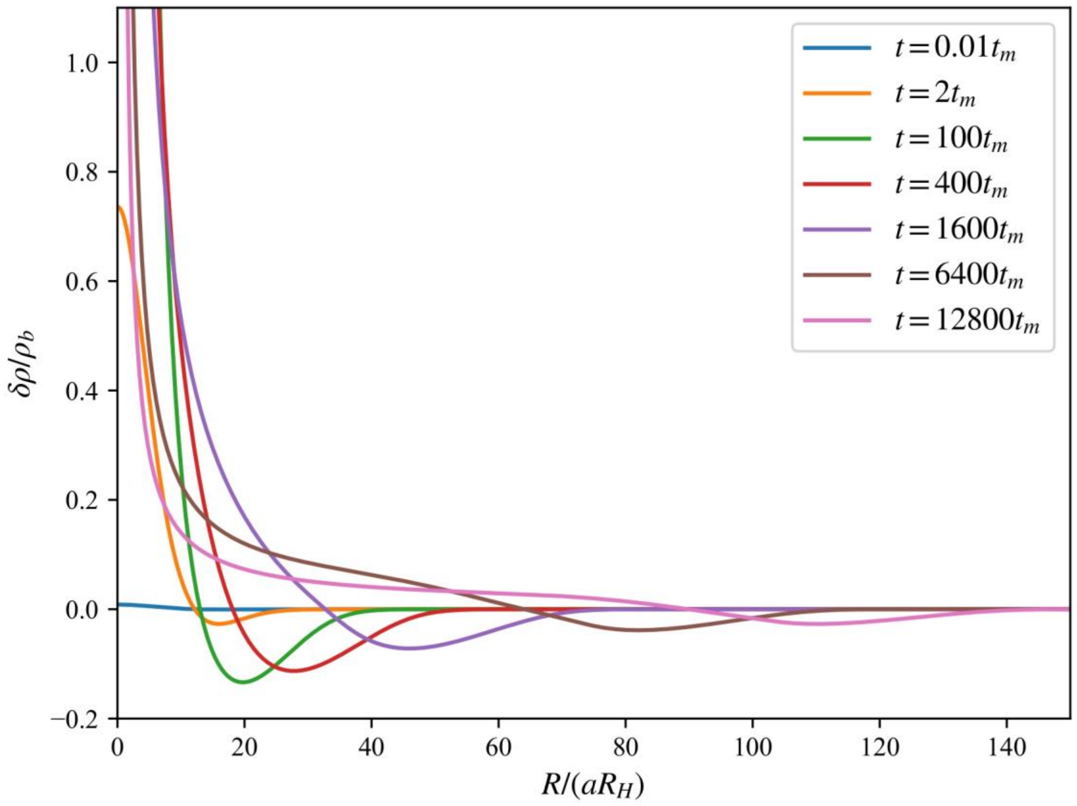
$\delta_m = 0.55 > \delta_c$

03 | Softer equations of state

- $\delta_m - \delta_c \approx 0.00226$



$\omega = 1/5$

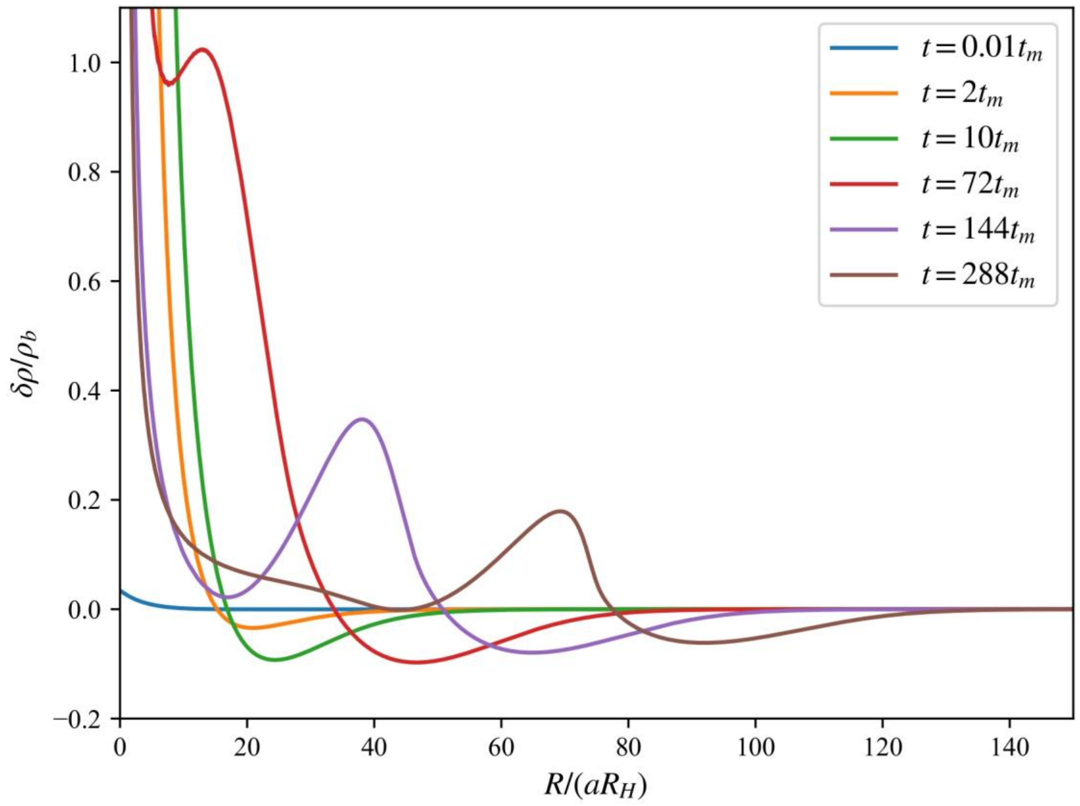


$\omega = 1/10$

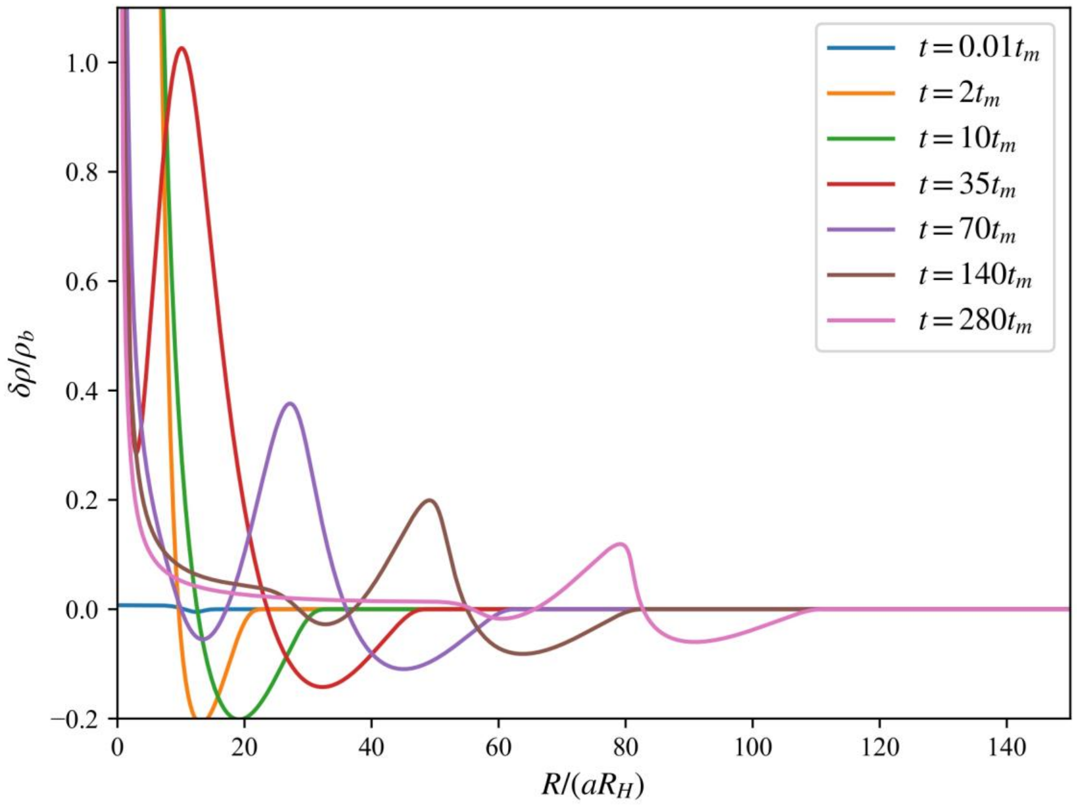
03 | Generalized curvature perturbations

- $\delta_m - \delta_c \approx 0.00226$

$$K(r) = \left(\frac{r}{\Delta}\right)^{2\lambda} \mathcal{A} \exp\left[-\frac{1}{2}\left(\frac{r}{\Delta}\right)^{2\beta}\right]$$



steeper profile $\beta = 0.5$

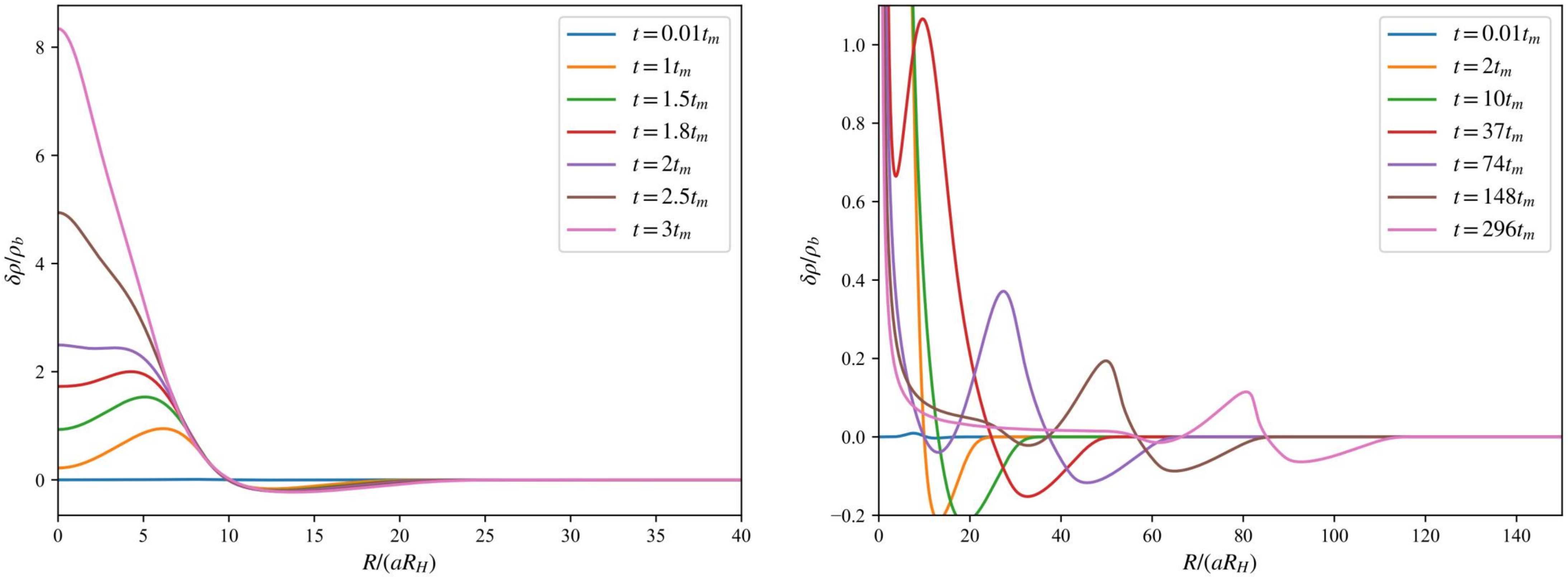


flatter profile $\beta = 4$

03 | Off-centered curvature perturbations

- $\delta_m - \delta_c \approx 0.00226$

$$K(r) = \left(\frac{r}{\Delta}\right)^{2\lambda} \mathcal{A} \exp\left[-\frac{1}{2}\left(\frac{r}{\Delta}\right)^{2\beta}\right]$$



off-centered profile $\lambda = 4$

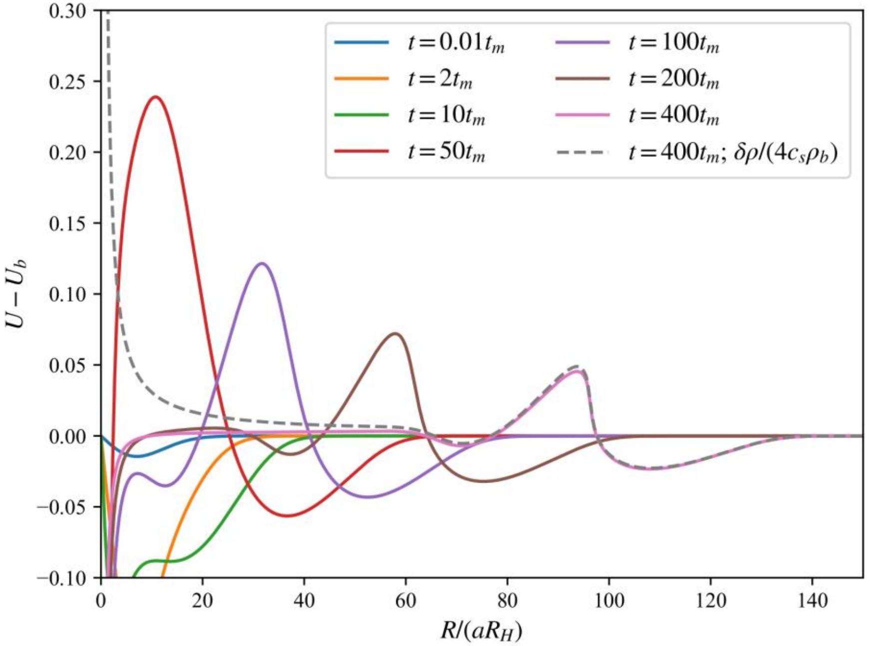
03 | Propagation of sound waves

- Once these waves move sufficiently far from the PBH, they can be treated as linear perturbations on a flat FLRW background, i.e., sound waves

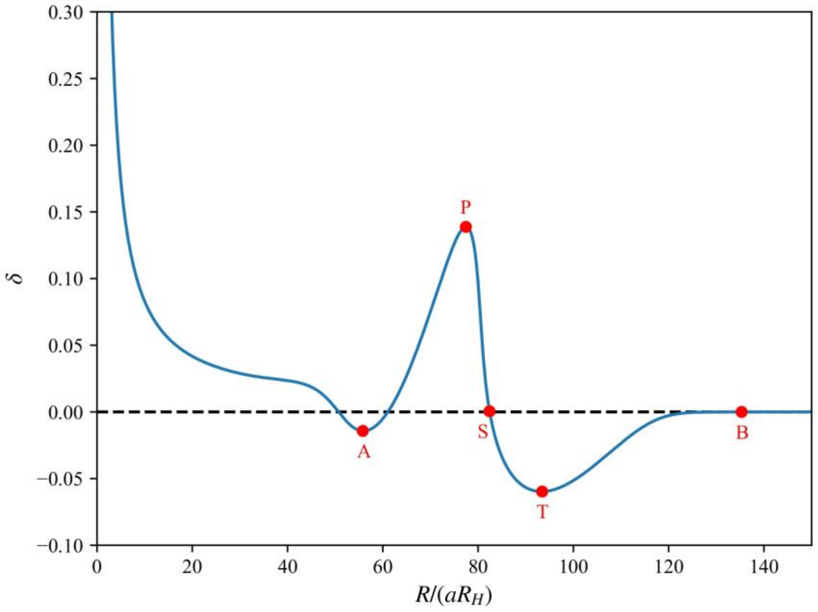
$$
\begin{aligned}
T^{00} &= \rho_b(1+\delta) + (1+\omega)\rho_b(1+\delta)a^2u^iu_i,\\
T^{0i} &= (1+\omega)\rho_b(1+\delta)u^i,\\
T^{ij} &= \omega\frac{h^{ij}}{a^2}\rho_b(1+\delta) + (1+\omega)\rho_b(1+\delta)u^iu^j,
\end{aligned}
$$

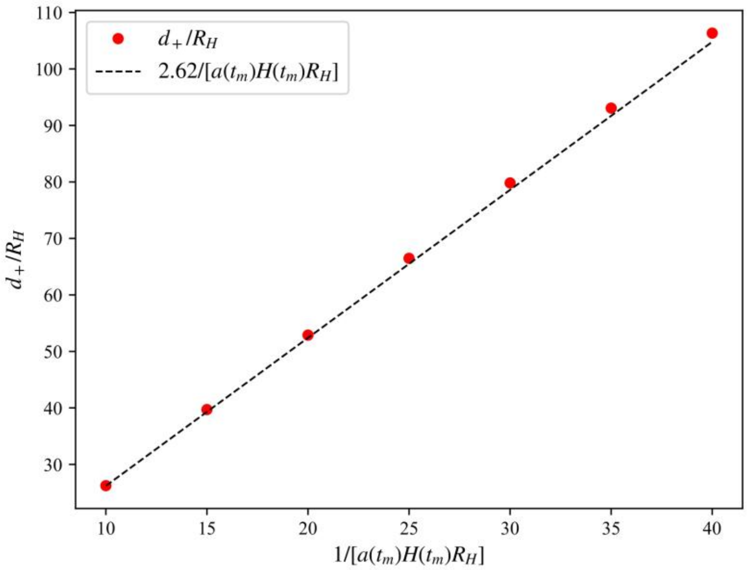
$$
\begin{aligned}
\dot{\delta} + (1+\omega)\left(u' + \frac{2u}{r}\right) &= 0,\\
(1+\omega)\dot{u} + \omega\frac{\delta'}{a^2} + (2-3\omega^2-\omega)\frac{\dot{a}}{a}u &= 0,
\end{aligned}
$$

$$
\begin{aligned}
\delta &= -4a\partial_t(a\phi) = 4c_s\frac{f'(r-c_sR_Ha)}{r},\\
u = \phi' &= \frac{f'(r-c_sR_Ha)}{ar} - \frac{f(r-c_sR_Ha)}{ar^2}\\
&\approx \frac{f'(r-c_sR_Ha)}{ar} = \frac{\delta}{4ac_s},\\
U - U_b &\approx \frac{\delta\rho/\rho_b}{4c_s}
\end{aligned}
$$

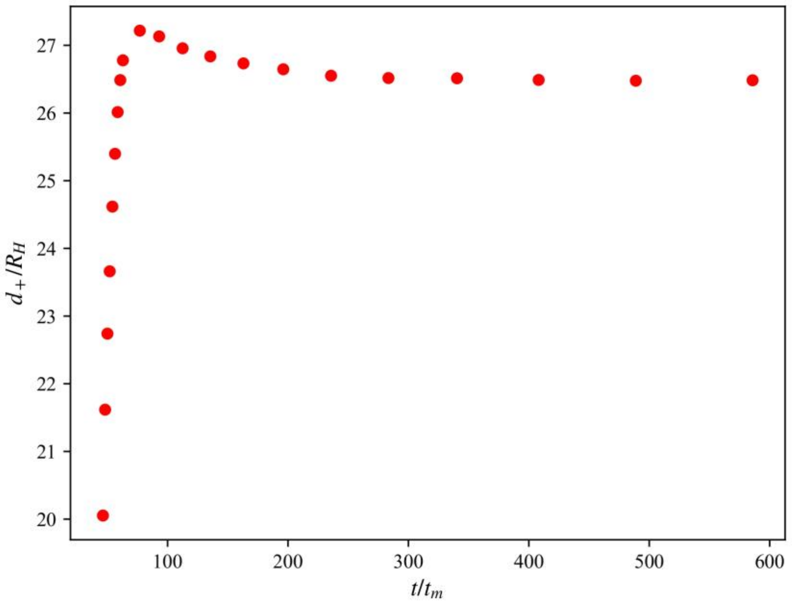


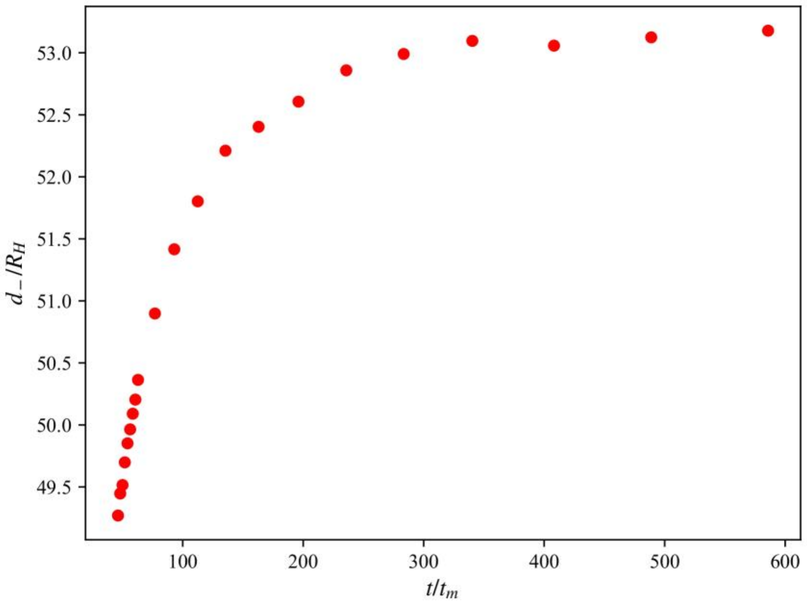
03 | Thickness of sound shells





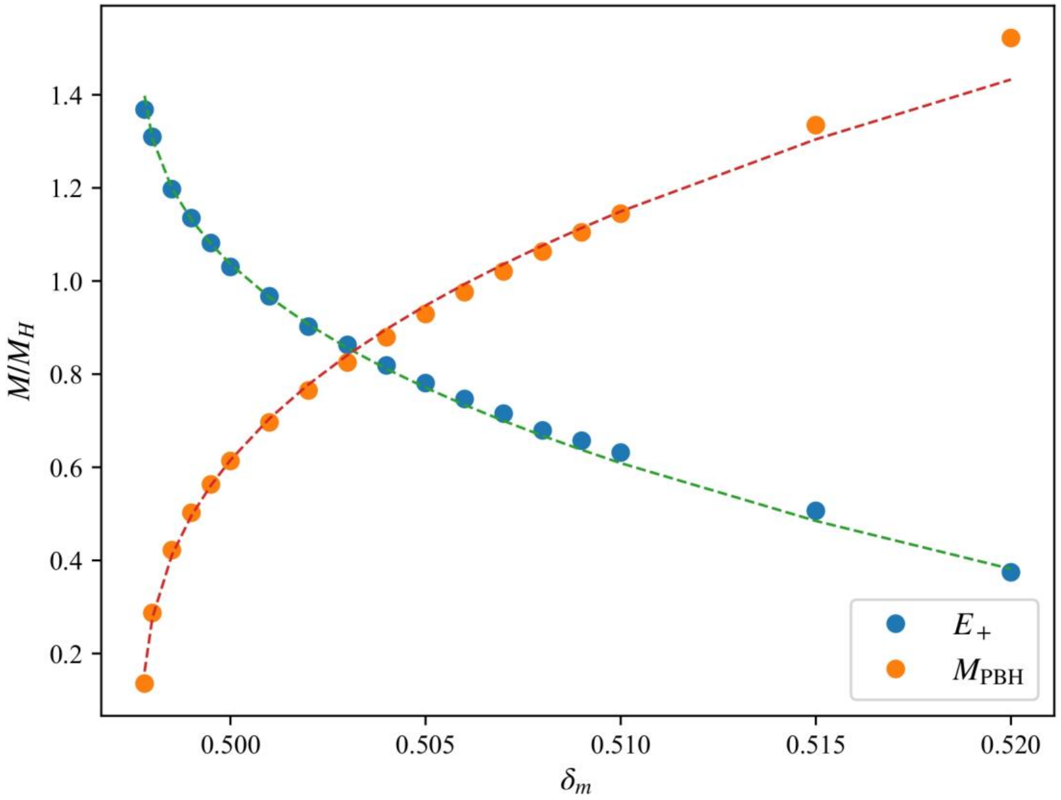
$$d_+ \approx n/a(t_m)H(t_m)$$





03 | Energy of sound shells

- PBH mass follows the scaling law
- We find that the energy of overdense shells follows a similar scaling law



$$M_{\rm PBH} = M_H \mathcal{K}(\delta_m - \delta_c)^{\sigma}$$

$$E_+ = M_H \left[\mathcal{K}_+(\delta_m - \delta_c)^{\sigma_+} + C_+\right]$$

the fit yields $\sigma \approx \sigma_+ \approx 0.37$, $\mathcal{K} \approx 5.84$, $\mathcal{K}_+ \approx -4.70$, and $C_+ \approx 1.52$.

中国科学院大学
University of Chinese Academy of Sciences

04 Conclusions

04 | Conclusions and discussion

- Near-critical perturbations produce a compression wave with both overdense and underdense shells
- Significantly supercritical perturbations yield only an underdense shell
- A softer equation of state suppresses the formation of compression waves
- The comoving thickness of sound shells remains nearly constant during propagation and scales with the comoving Hubble radius at horizon re-entry
- Energy of the overdense shells follows a scaling law similar to that of the PBH mass

- Outlook
  - More complex curvature perturbations, such as multi-peak or oscillatory profiles
  - Non-spherical effects
  - Other PBH formation mechanisms

Thank You!