The Gravitational Wave-Collider Interface

M.J. Ramsey-Musolf

- *T.D. Lee Institute/Shanghai Jiao Tong Univ.*
- *UMass Amherst*
- *Caltech*

About MJRM:

Science

Family

Friends

My pronouns: he/him/his
 # MeToo

Bejing Particle Physics & Cosmology Symposium, September 26, 2025

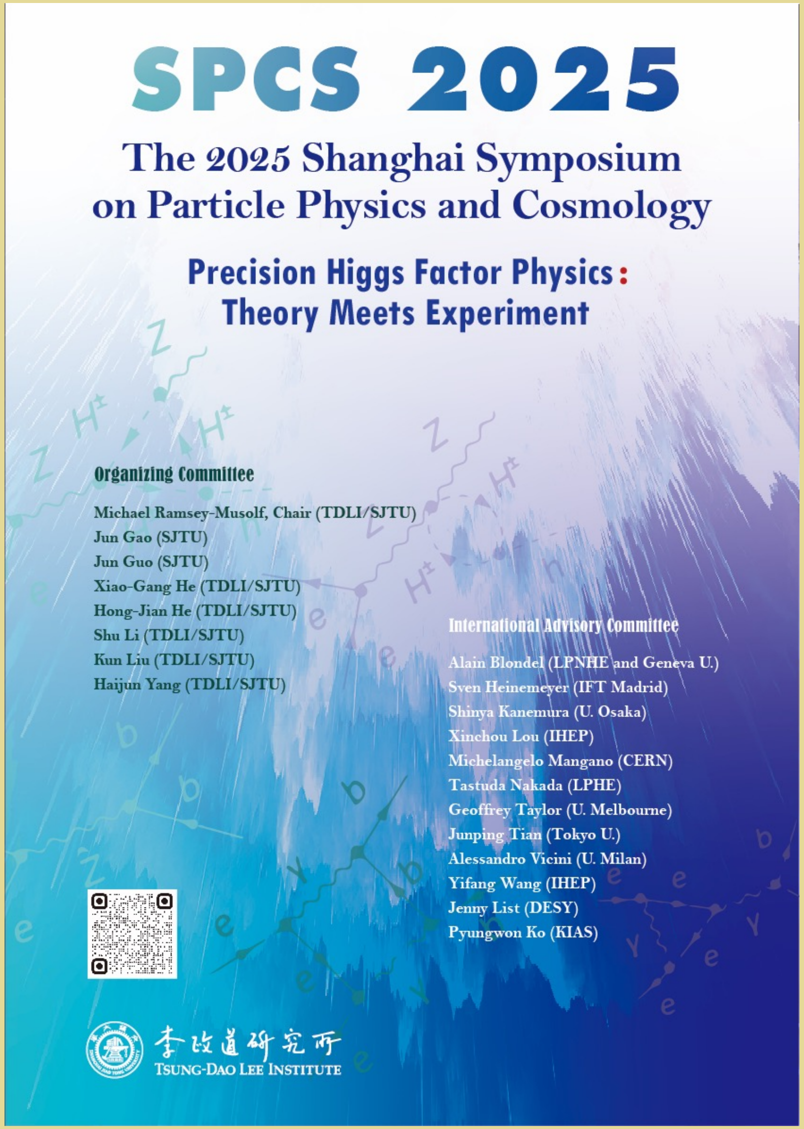
Welcome !

Thanks !

Annual series

- *All participants*
- *Organizing Committee*
- *ITP-CAS Staff*
  - *Qing-Rong Ni*
- *The Executive Chair !*
  - *Shao-Jiang Wang*

A Related Initiative: SPCS 2025



- *Launch a China-based community on precision Higgs Factory Physics*
- *Followed by 4-day intensive school for PhD students & post-docs*

Symposium

https://indico-tdli.sjtu.edu.cn/event/4194/

School

https://indico-tdli.sjtu.edu.cn/event/4428/

Key Themes for This Talk

- *The possibility of primordial gravitational waves generated from various particle physics dynamics has become an exciting area of exploration*
- *There exist many creative ideas for novel phenomena and dynamics that could have generated GW*
- *Realizing which, if any, of these ideas was realized in nature requires input from additional observables and performing the most rigorous theoretical calculations*
- *The electroweak phase transition provides a unique "laboratory" for testing our theoretical methods and ideas, with LHC and next generation collider measurements providing key input*

Outline

I. Context & Questions

II. Electroweak Phase Transition: A Laboratory

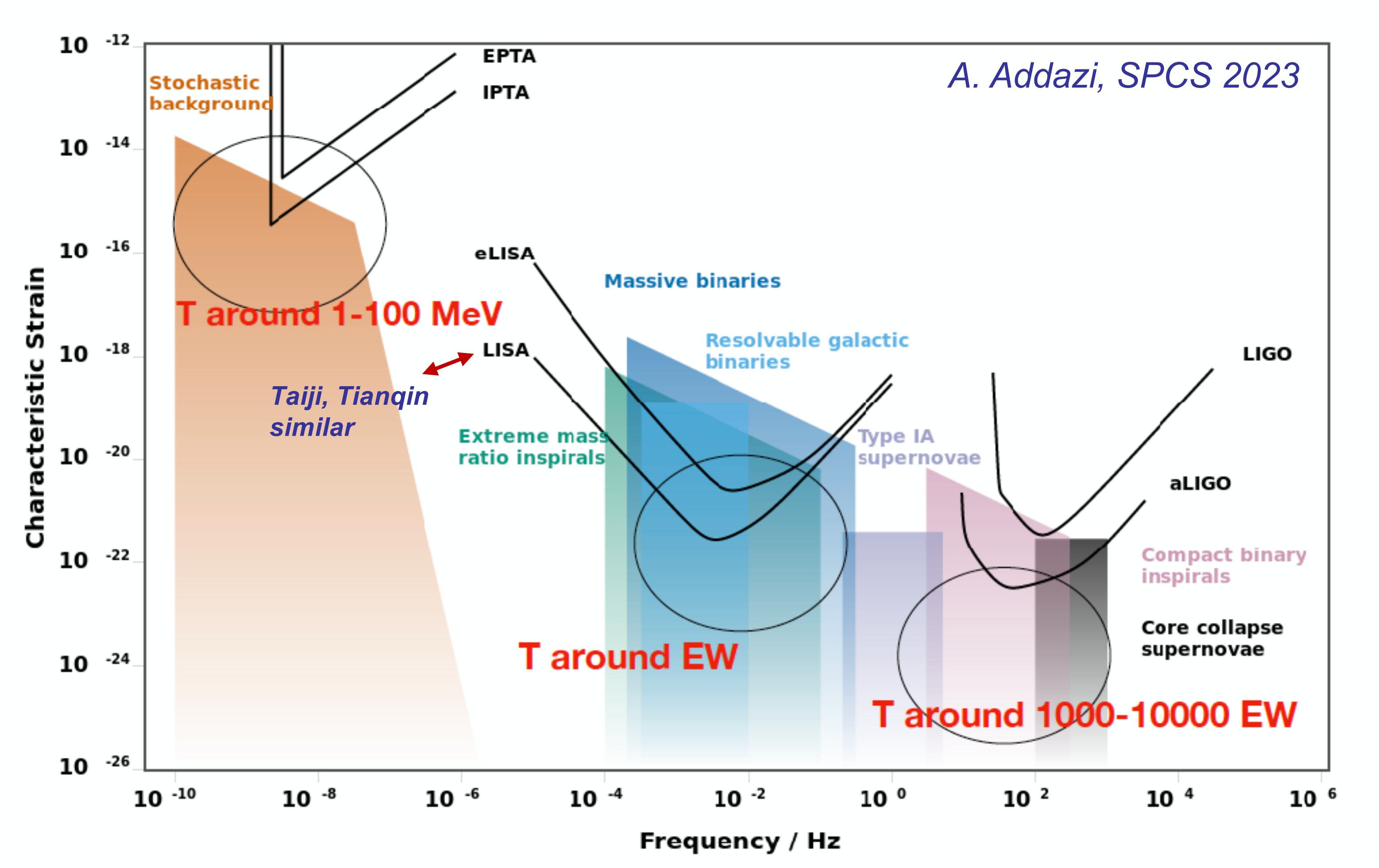
III. Theoretical Robustness & Pheno Interface:

- IR Problem
- Nucleation & gauge invariance
- Wall velocity

IV. Outlook

I. Context & Questions

Gravitational Waves



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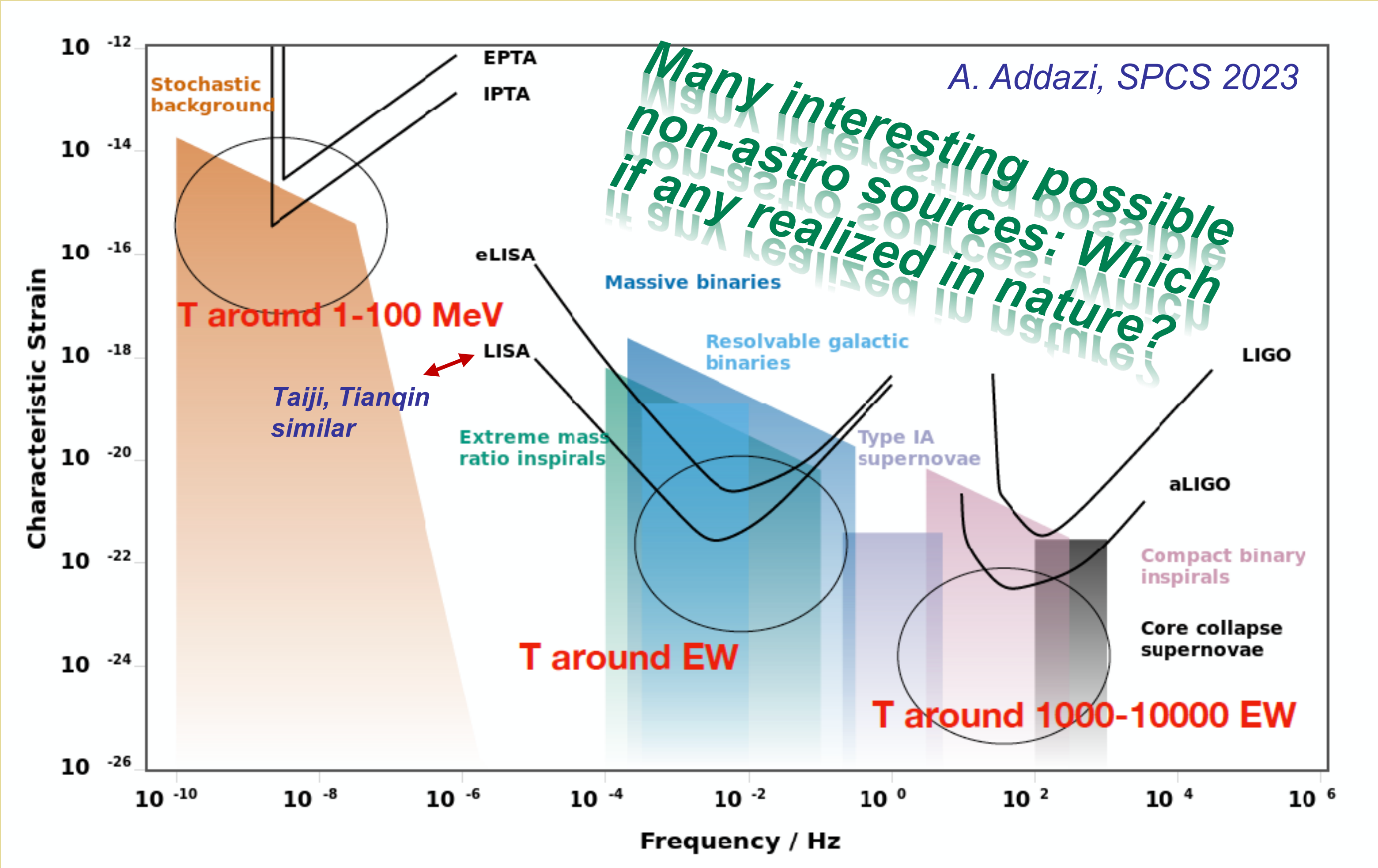

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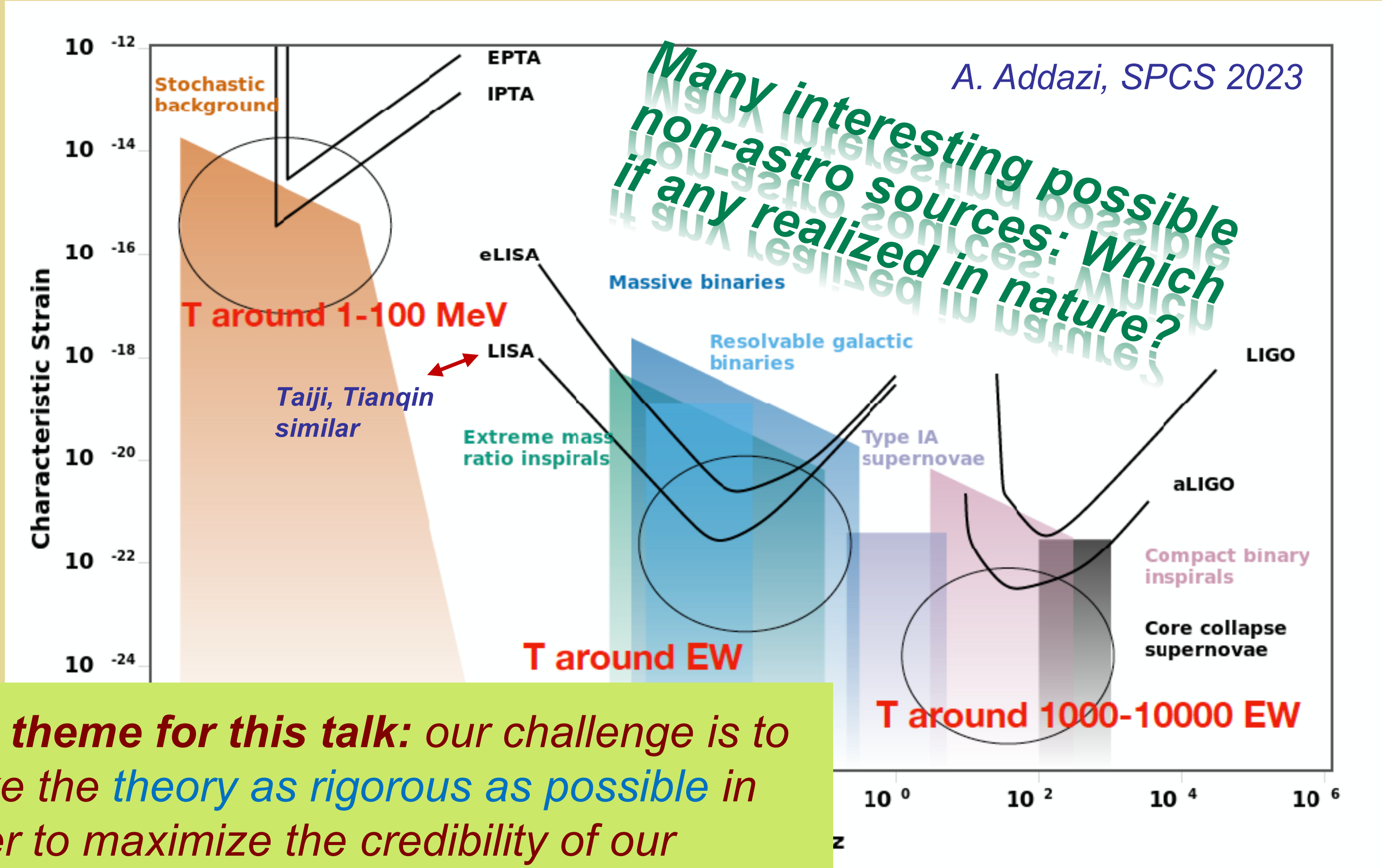
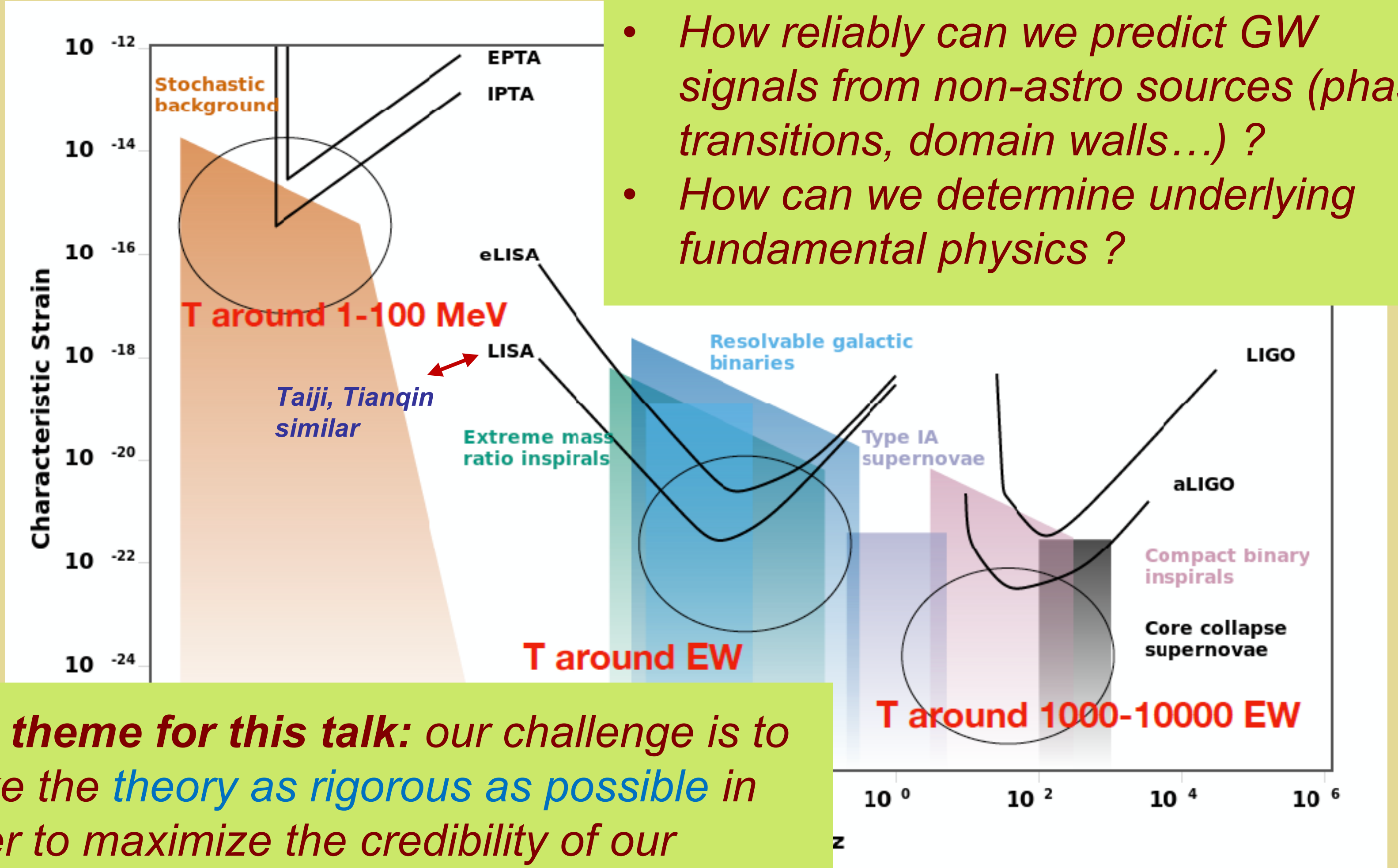


***Key theme for this talk:** our challenge is to make the theory as rigorous as possible in order to maximize the credibility of our creative ideas and enable a robust confrontation with experiment to address this question*

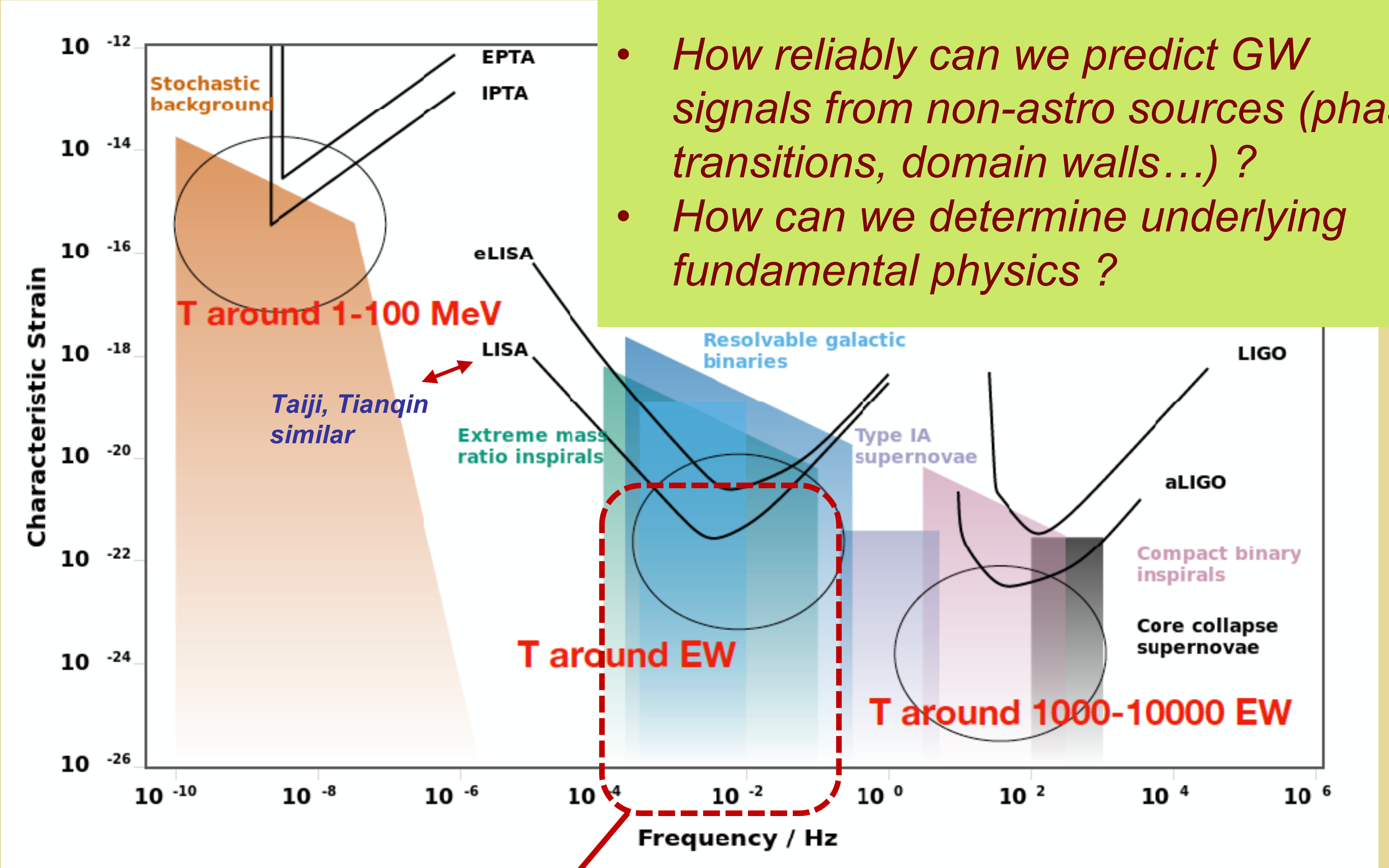
Gravitational Waves



- *How reliably can we predict GW signals from non-astro sources (phase transitions, domain walls…) ?*
- *How can we determine underlying fundamental physics ?*

Key theme for this talk: *our challenge is to make the theory as rigorous as possible in order to maximize the credibility of our creative ideas and enable a robust confrontation with experiment to address this question*

GW: Electroweak Phase Transition



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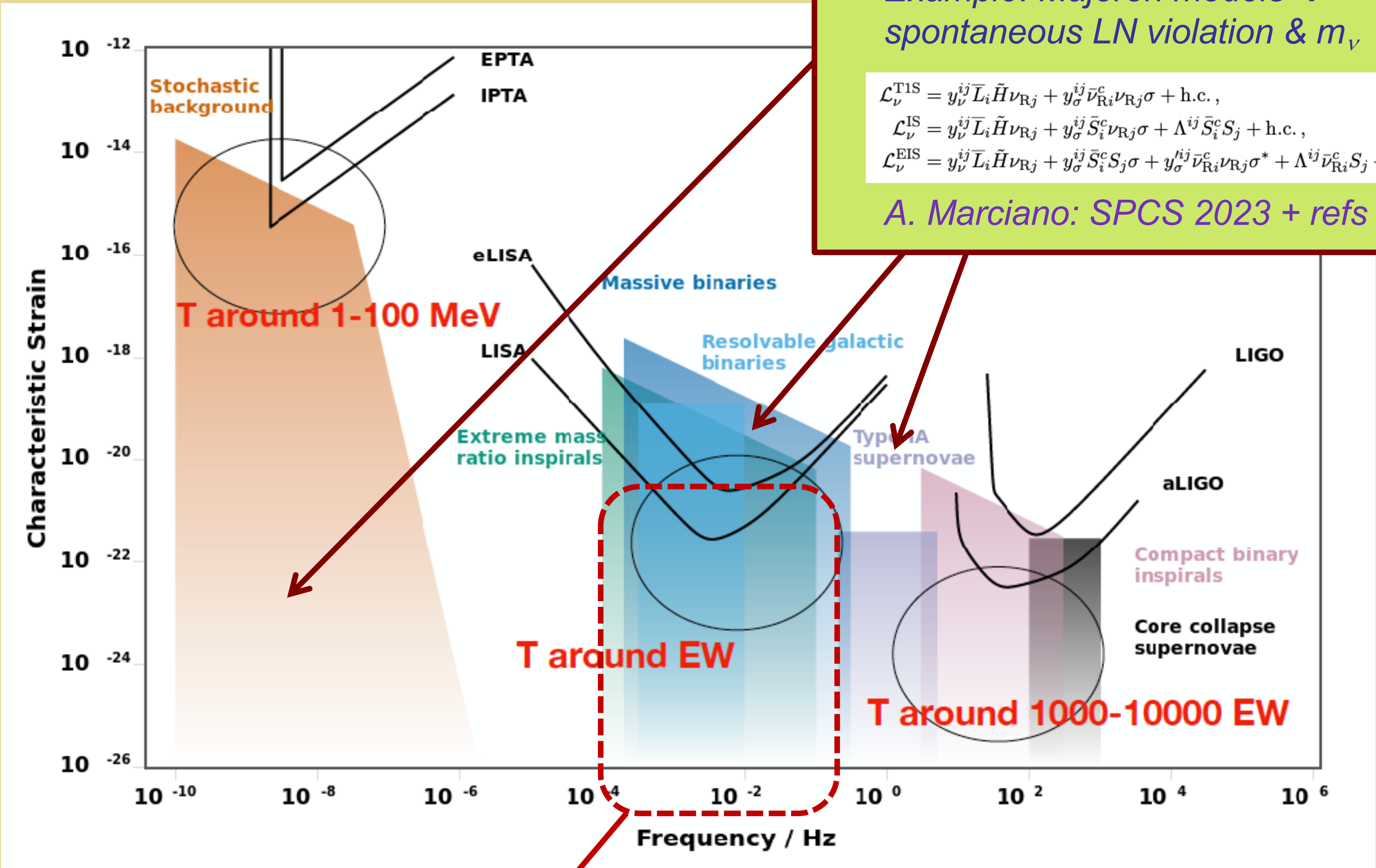
EWPT laboratory for GW micro-physics: *colliders can probe particle physics responsible for non-astro GW sources → test our framework for GW microphysics at other scales*

GW: Electroweak Phase Transition



Example: Majoron models → spontaneous LN violation & m_ν

$$\mathcal{L}_\nu^{\rm T1S} = y_\nu^{ij}\bar{L}_i\tilde{H}\nu_{{\rm R}j} + y_\sigma^{ij}\bar{\nu}^c_{{\rm R}i}\nu_{{\rm R}j}\sigma + {\rm h.c.}\,,$$
$$\mathcal{L}_\nu^{\rm IS} = y_\nu^{ij}\bar{L}_i\tilde{H}\nu_{{\rm R}j} + y_\sigma^{ij}\bar{S}^c_i\nu_{{\rm R}j}\sigma + \Lambda^{ij}\bar{S}^c_i S_j + {\rm h.c.}\,,$$
$$\mathcal{L}_\nu^{\rm EIS} = y_\nu^{ij}\bar{L}_i\tilde{H}\nu_{{\rm R}j} + y_\sigma^{ij}\bar{S}^c_i S_j\sigma + y'^{ij}_\sigma\bar{\nu}^c_{{\rm R}i}\nu_{{\rm R}j}\sigma^* + \Lambda^{ij}\bar{\nu}^c_{{\rm R}i}S_j + {\rm h.c.}$$

A. Marciano: SPCS 2023 + refs

EWPT laboratory for GW micro-physics: *colliders can probe particle physics responsible for non-astro GW sources → test our framework for GW microphysics at other scales*

II. EW Phase Transition

Was There an Electroweak Phase Transition ?

- *Interesting in its own right*
- *Key ingredient for EW baryogenesis*
- *Source of gravitational radiation*
- *Laboratory for testing phase transition theoretical tools*

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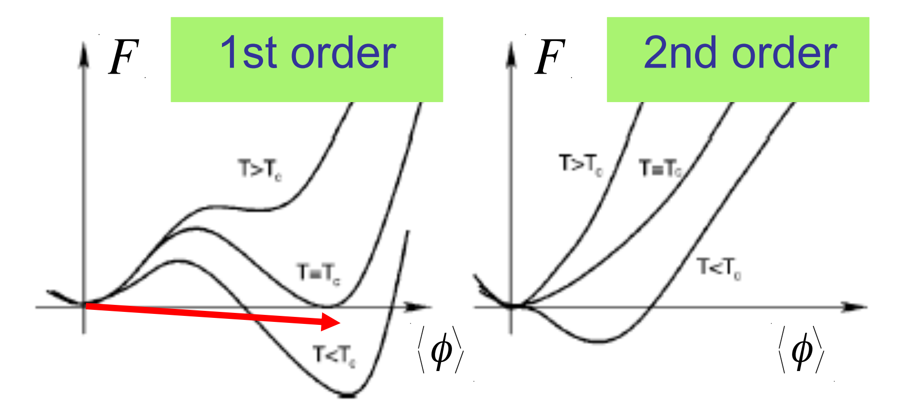
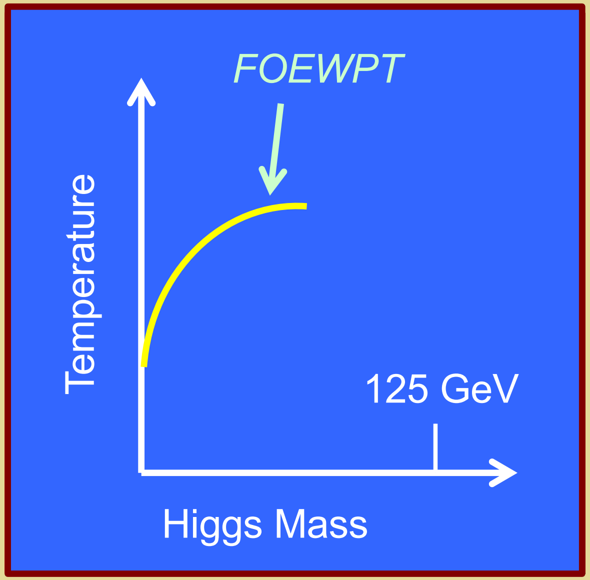
This talk

Was There an EW Phase Transition?



Increasing m_h ⟶



EW Phase Diagram

Lattice	Authors	M_h^C (GeV)
4D Isotropic	[76]	80 ± 7
4D Anisotropic	[74]	72.4 ± 1.7
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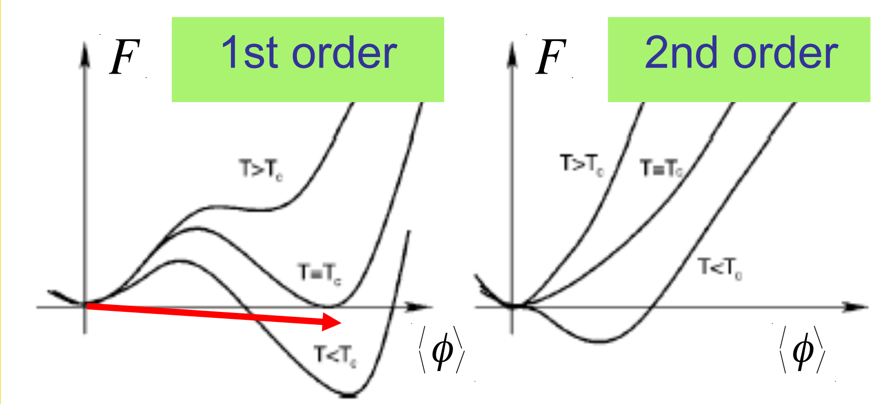
SM EW: Cross over transition

Was There an EW Phase Transition?



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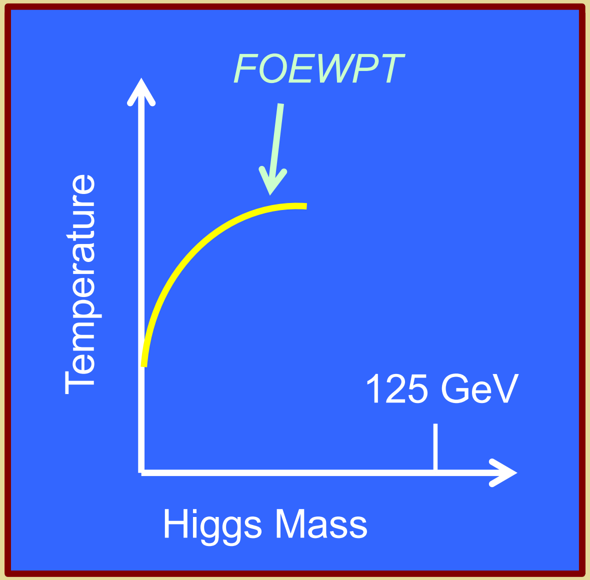
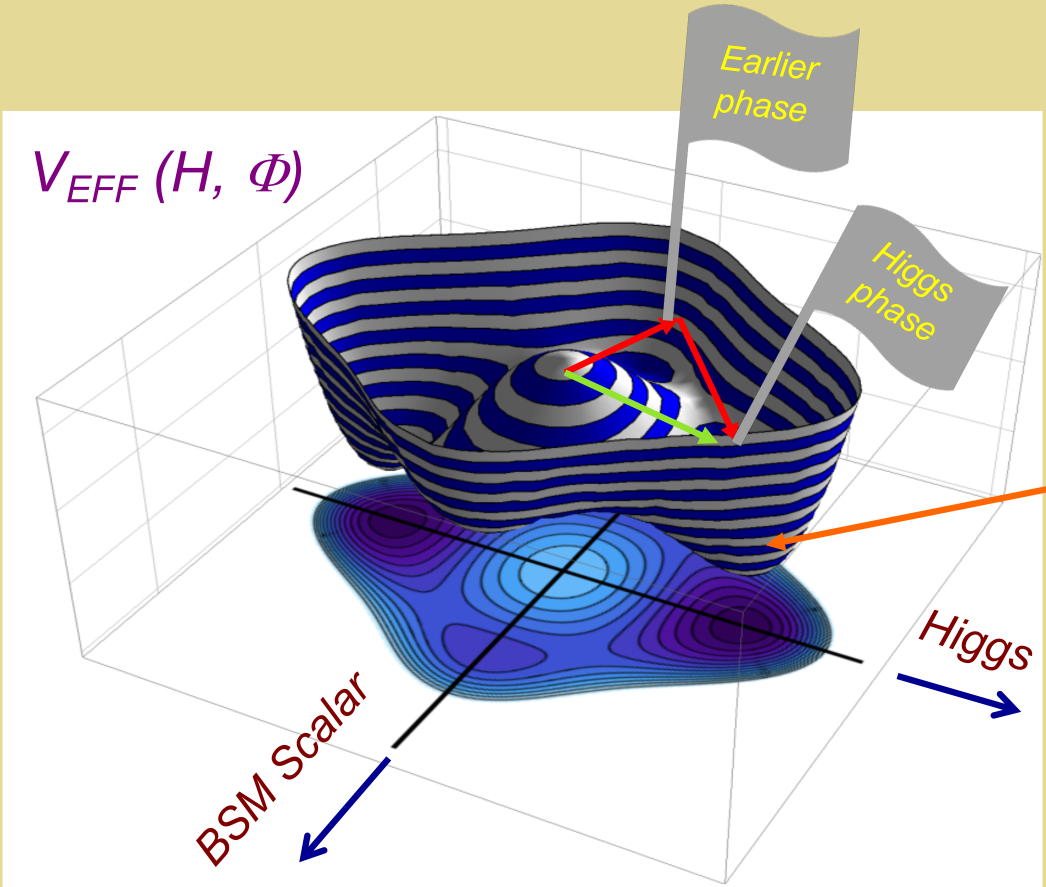


EW Phase Diagram

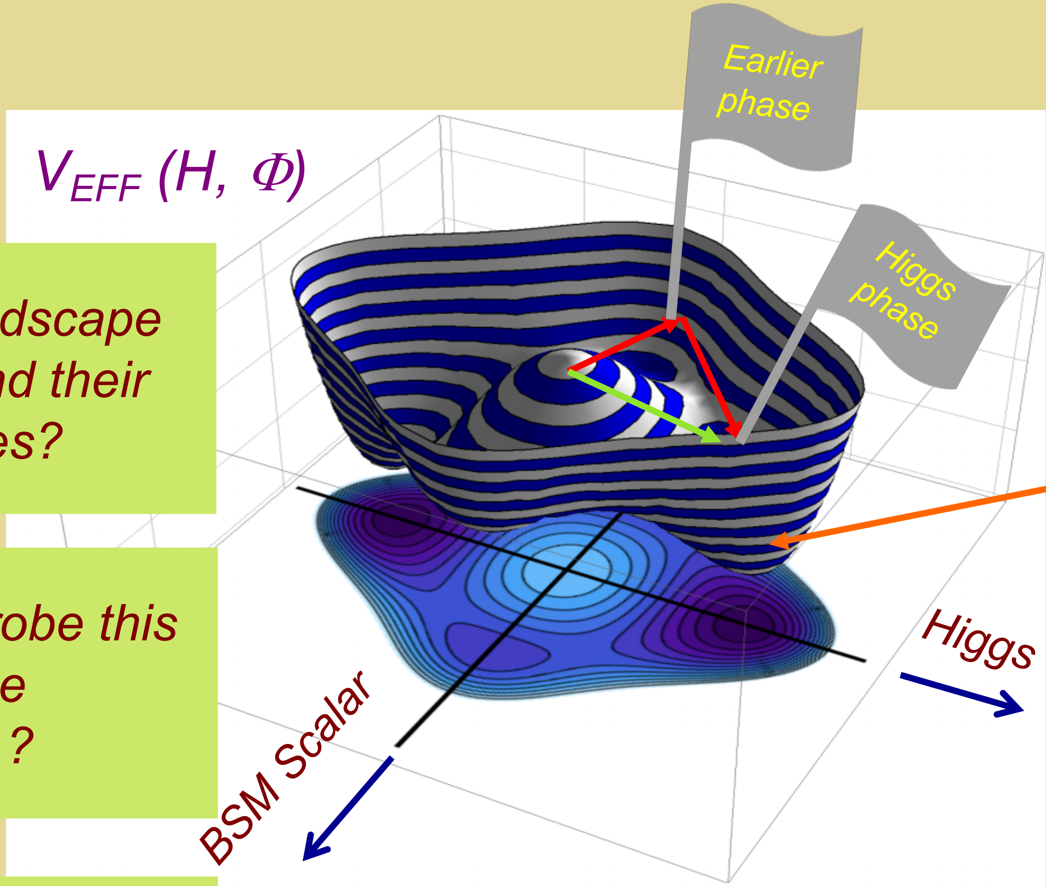
How does this picture change in presence of new TeV scale physics ? What is the phase diagram ? SFOEWPT ?

What Was the EWSB Thermal History ?



Extrema can evolve differently as T evolves →
rich possibilities for symmetry breaking

What Was the EWSB Thermal History ?



- What is the landscape of potentials and their thermal histories?

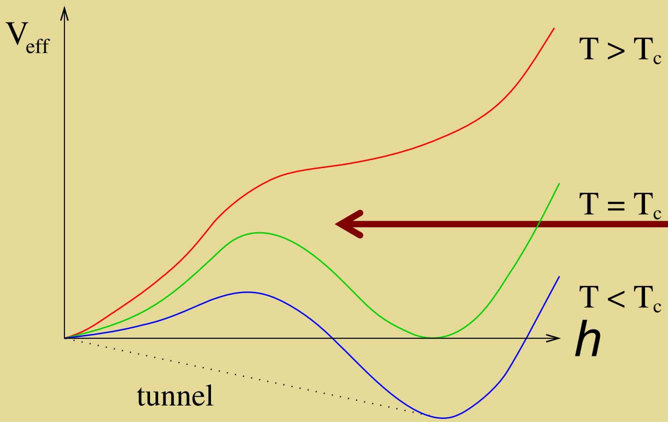
- How can we probe this T > 0 landscape experimentally ?

- How reliably can we compute the thermodynamics ?

How did we end up here ?

n evolve differently as T evolves →
ilities for symmetry breaking

First Order EWPT from BSM Physics



Representative thermal histories → barrier for SFOEWPT

T_{EW} ~ 140 GeV

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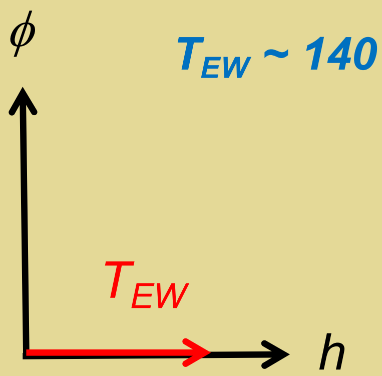
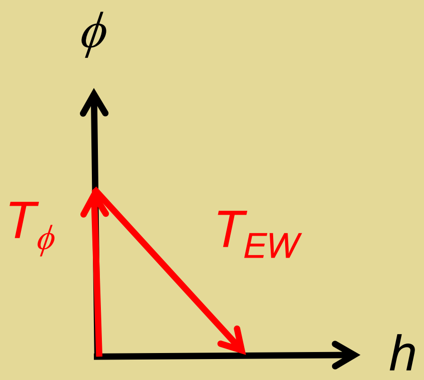
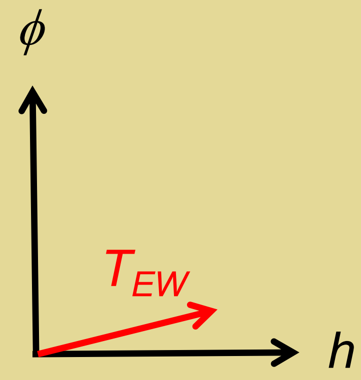


$a_2\, H^2\phi^2$: $T > 0$
loop effect

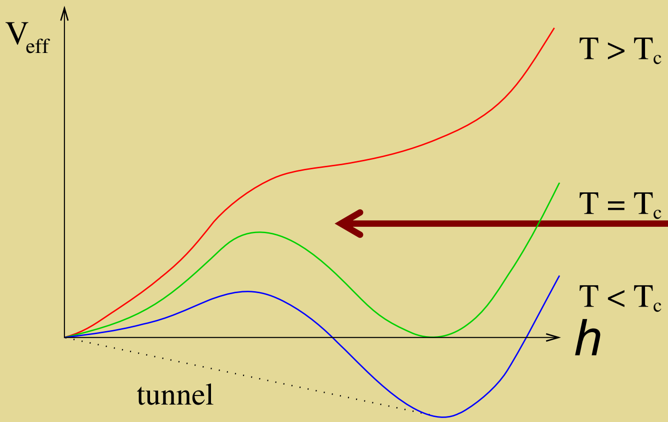


$a_2\, H^2\phi^2$: $T = 0$
tree-level effect



$a_1\, H^2\phi$: $T = 0$
tree-level effect

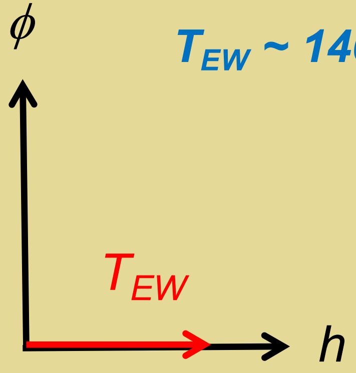
First Order EWPT from BSM Physics



Simple arguments: T_{EW} + first order EWPT → $M_\phi \lesssim 700$ GeV

Collider target

T_{EW} ~ 140 GeV



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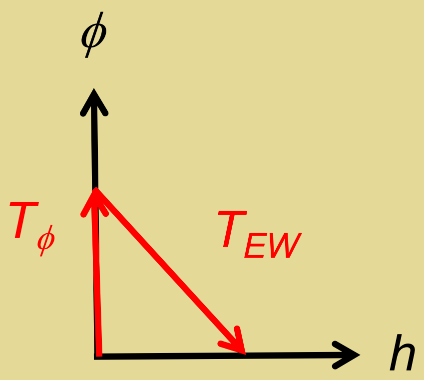

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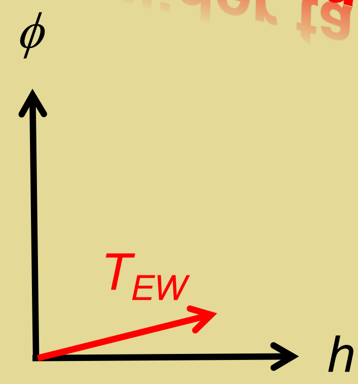

$a_1\, H^2\phi$: $T = 0$
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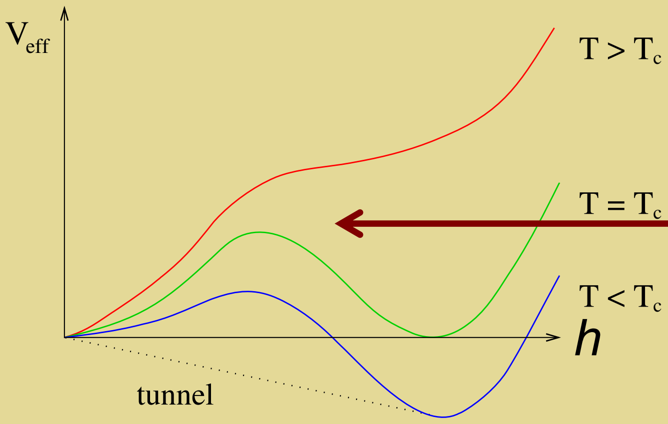
MJRM: 1912.07189

First Order EWPT from BSM Physics

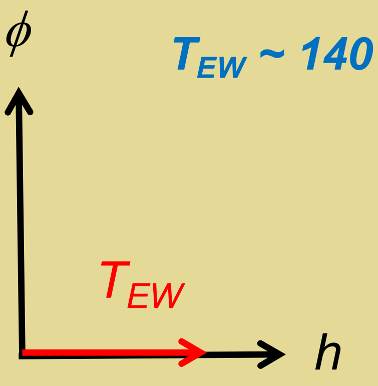


$|\sin\theta| \gtrsim 0.01 \qquad |\Delta\lambda/\lambda| \gtrsim 0.003$

Higgs – ϕ^0 Mixing

Collider target

$T_{EW} \sim 140$ **GeV**



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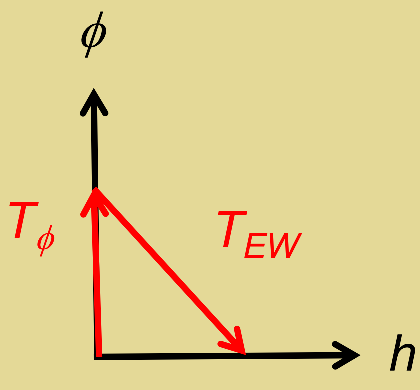

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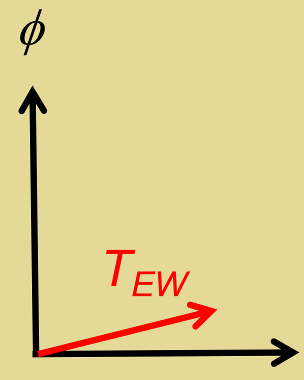

$a_1\, H^2\phi$: $T = 0$
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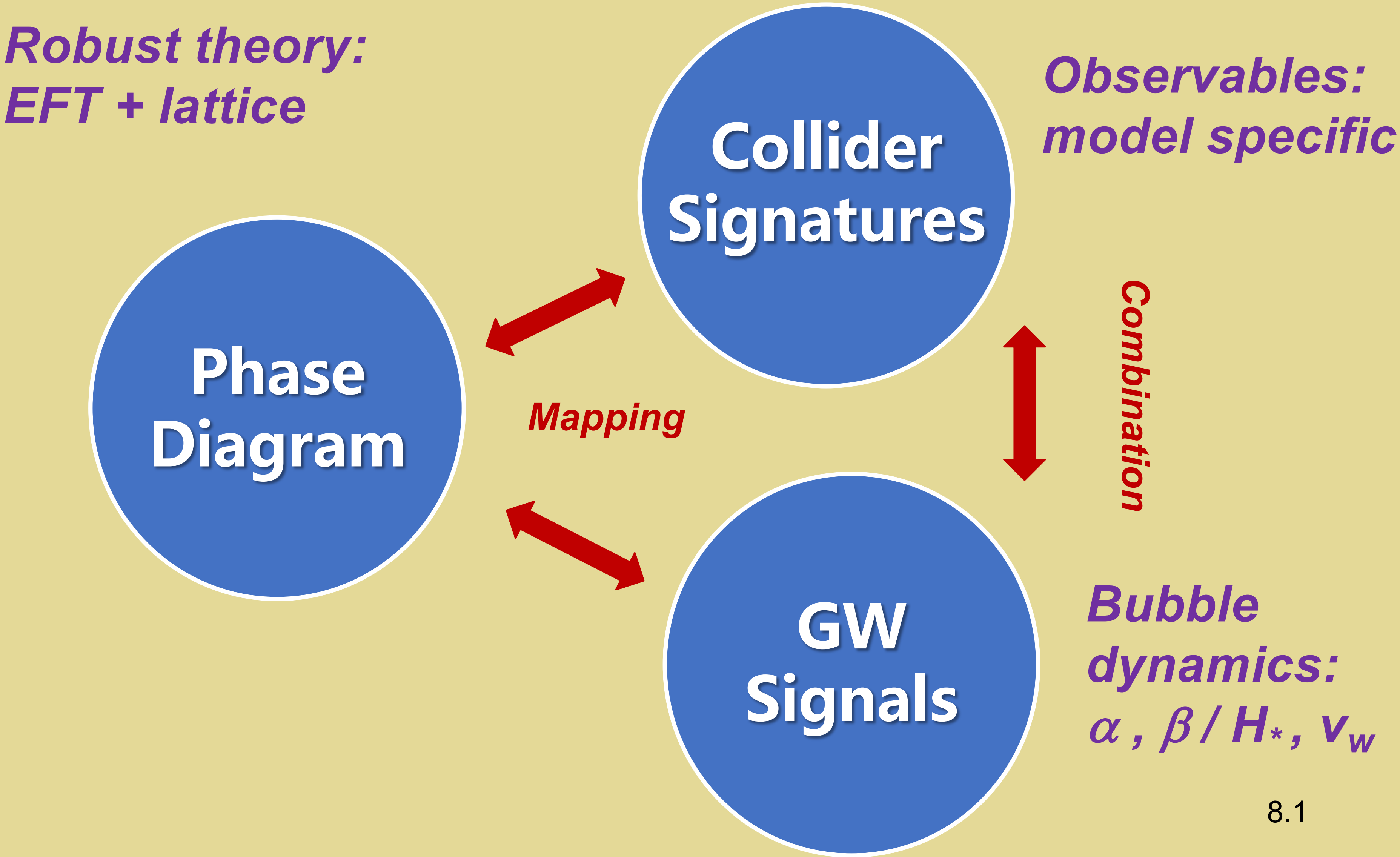
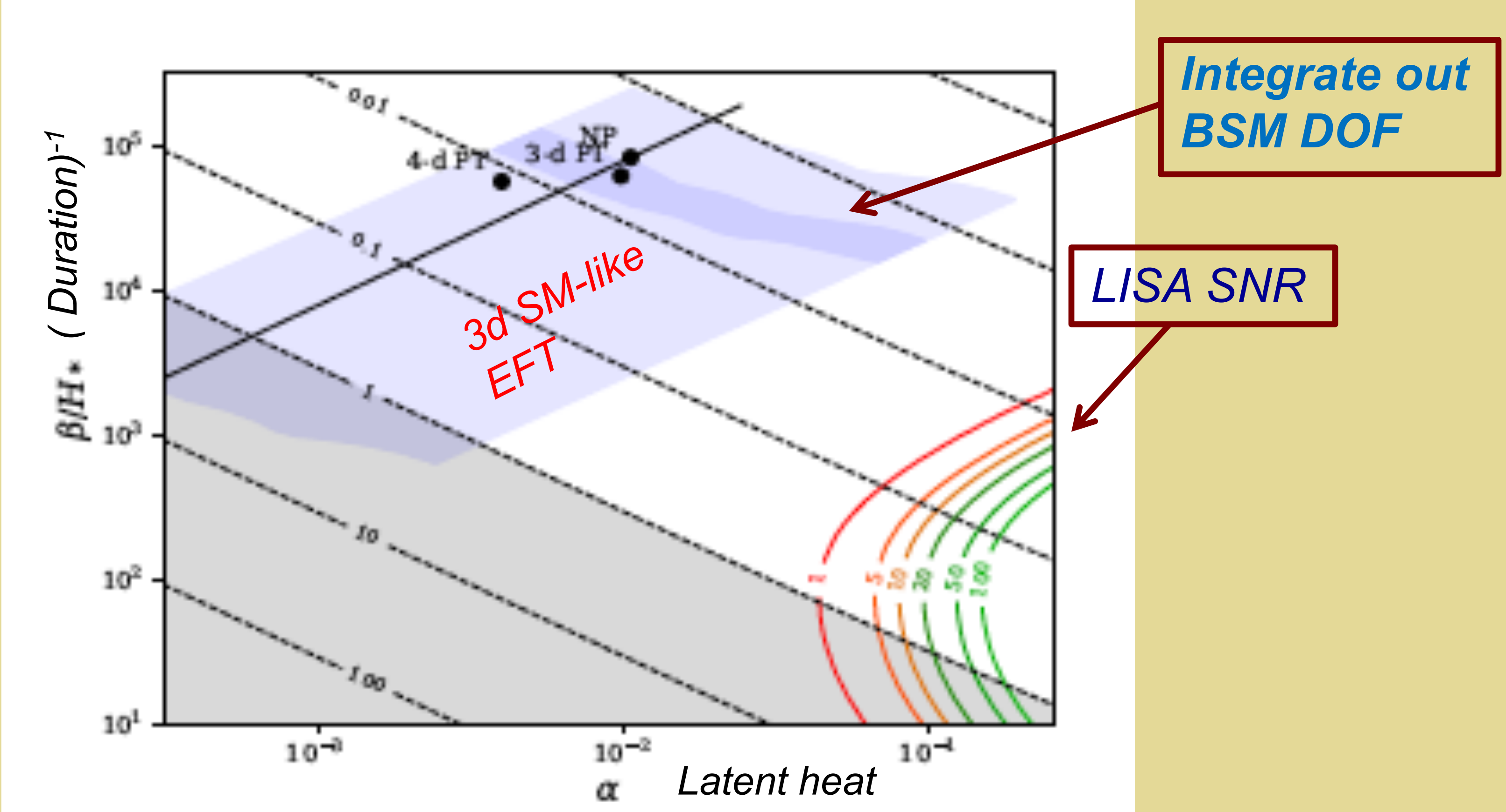
BSM EWPT: Inter-frontier Connections



BSM Scalar: EWPT & GW

High-T dimensional reduction: DR 3d EFT



Gould, Kozaczuk, Niemi, R-M, Tenkanen, Weir 1903.11604

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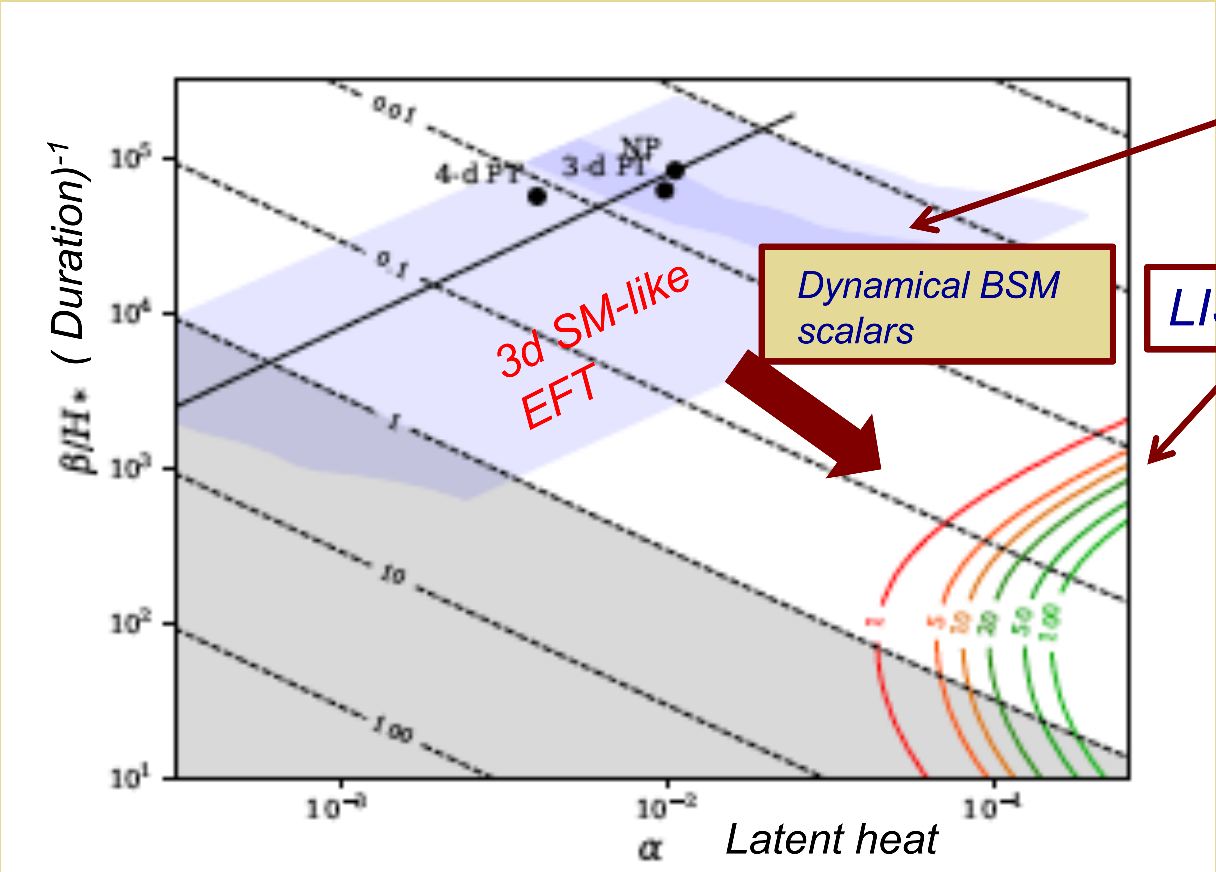

Integrate out BSM DOF

LISA SNR

Gould, Kozaczuk, Niemi, R-M, Tenkanen, Weir 1903.11604

BSM Scalar: EWPT & GW

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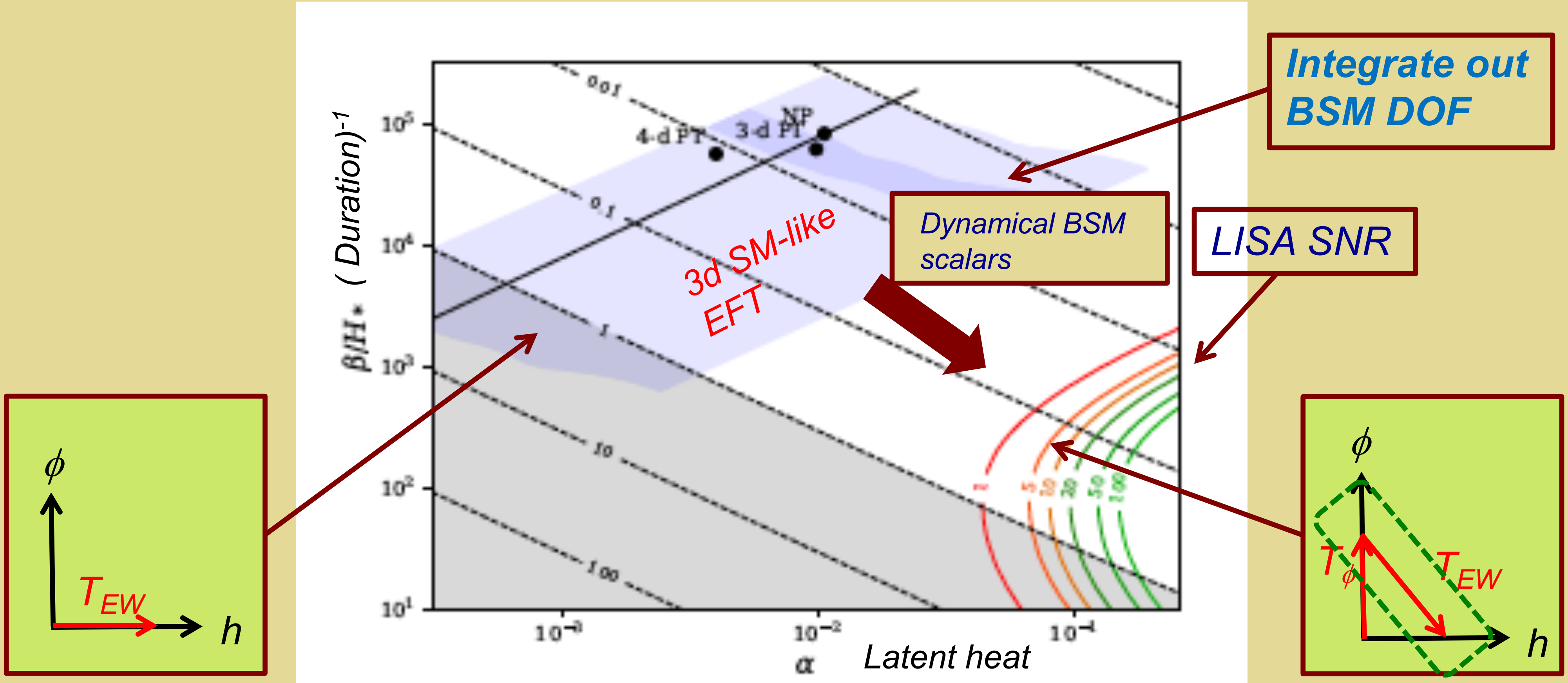

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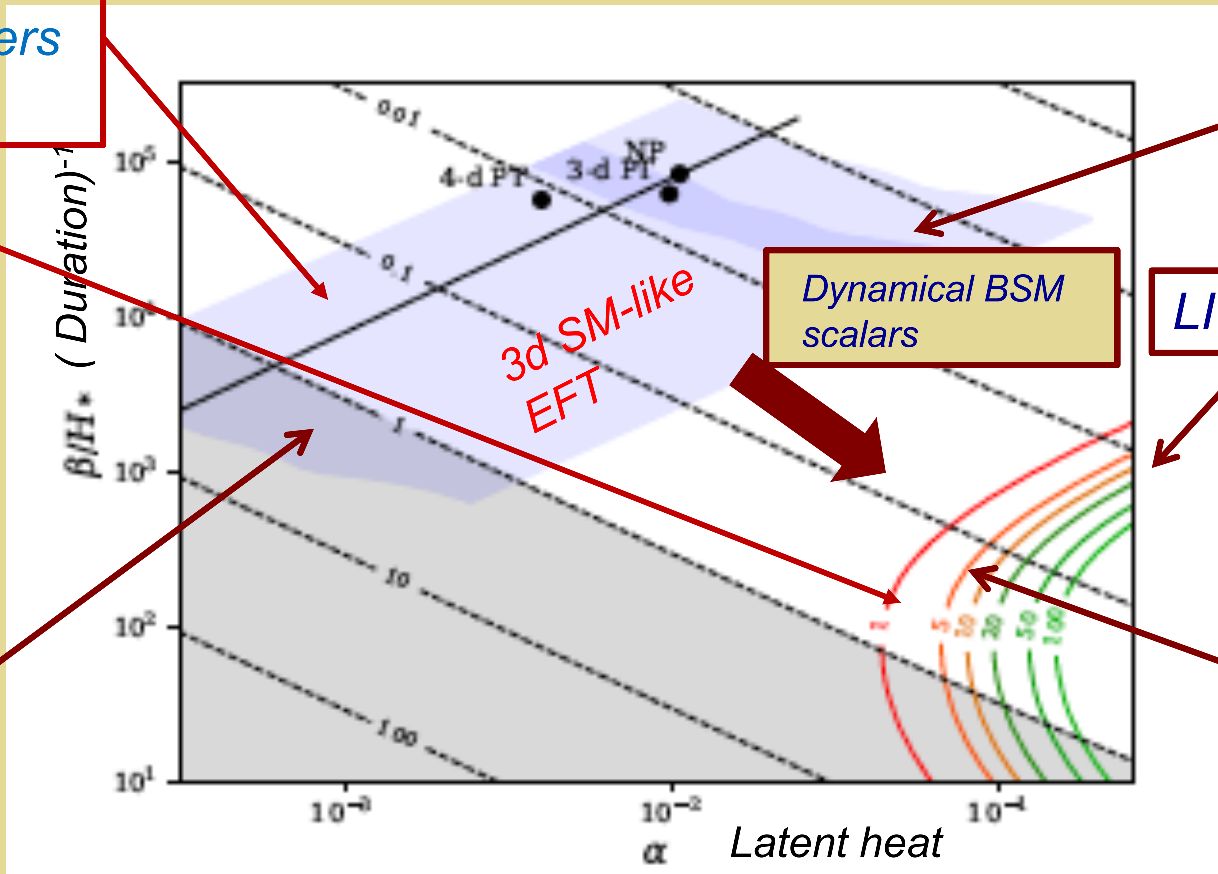
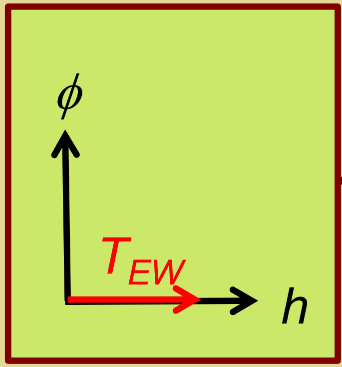
BSM Scalar: EWPT & GW

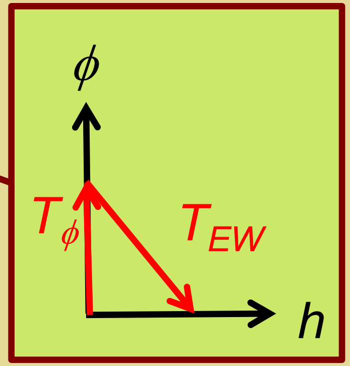
Collider probes of BSM parameters in $\mathcal{L}_{full}$

Integrate out BSM DOF

LISA SNR







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III. Theoretical Robustness

- *IR Problem*
- *Nucleation @ finite T: gauge invariance*
- *Wall velocity*

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Inputs from Thermal QFT

Thermodynamics

- *Phase diagram: first order EWPT?*
- *Latent heat: GW*

Dynamics

- *Nucleation rate: transition occurs? T_N ? Transition duration (GW) ?*
- *EW sphaleron rate: baryon number preserved?*

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How reliable is the theory ?

IR Problem

EWPT & Perturbation Theory: IR Problem

Bosonic loop at T>0

Bose dist fn

$$I(T) = g^2 \int \frac{d^3p}{(2\pi)^3}\, f_B(E,T)\, \frac{1}{(p^2+m^2)^n}$$

Small p regime

$$f_B(E,T) \longrightarrow \frac{T}{m}$$

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Effective expansion parameter

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Field-dependent thermal mass

$$m^2(\varphi, T) \sim C_1\, g^2\varphi^2 + C_2\, g^2 T^2 \equiv m_T^2(\varphi)$$

- *Near phase transition:* $\varphi \sim 0$
- $m_T(\varphi) < g\,T$

Theory Meets Phenomenology

A. Non-perturbative

- *Most reliable determination of character of EWPT & dependence on parameters*
- *Broad survey of scenarios & parameter space not viable*

B. Perturbative

- *Most feasible approach to survey broad ranges of models, analyze parameter space, & predict experimental signatures*
- *Quantitative reliability needs to be verified*

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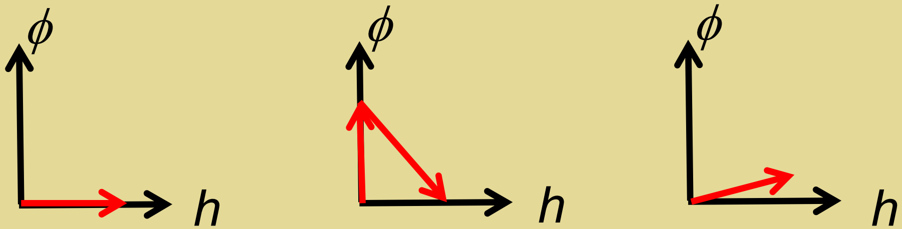
Benchmark pert theory

Theory-Pheno Interface

Simple Higgs portal models:

- *Real gauge singlet (SM + 1)*
- *Real EW triplet (SM + 3)*

$$V \supset a_1 H^2 \phi + a_2 H^2 \phi^2$$



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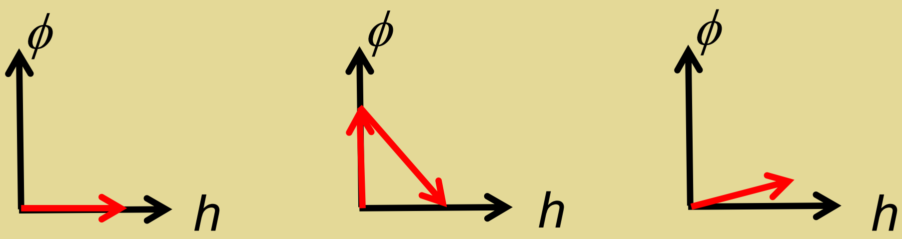

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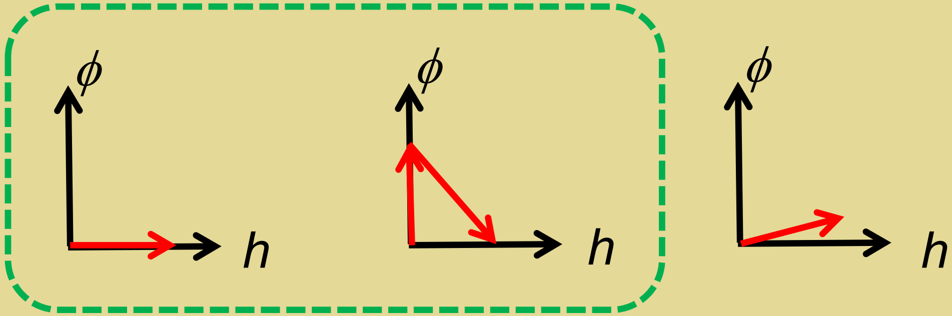

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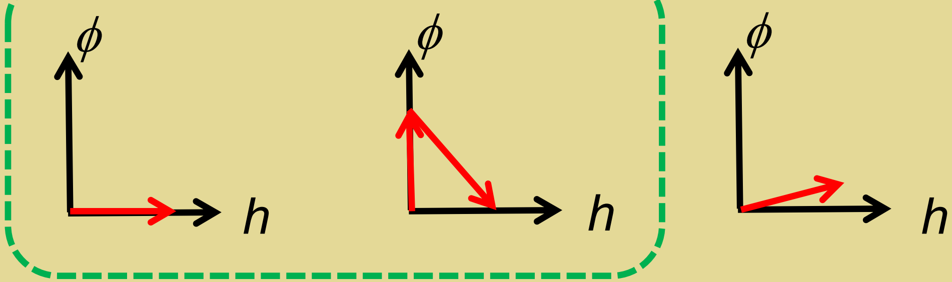
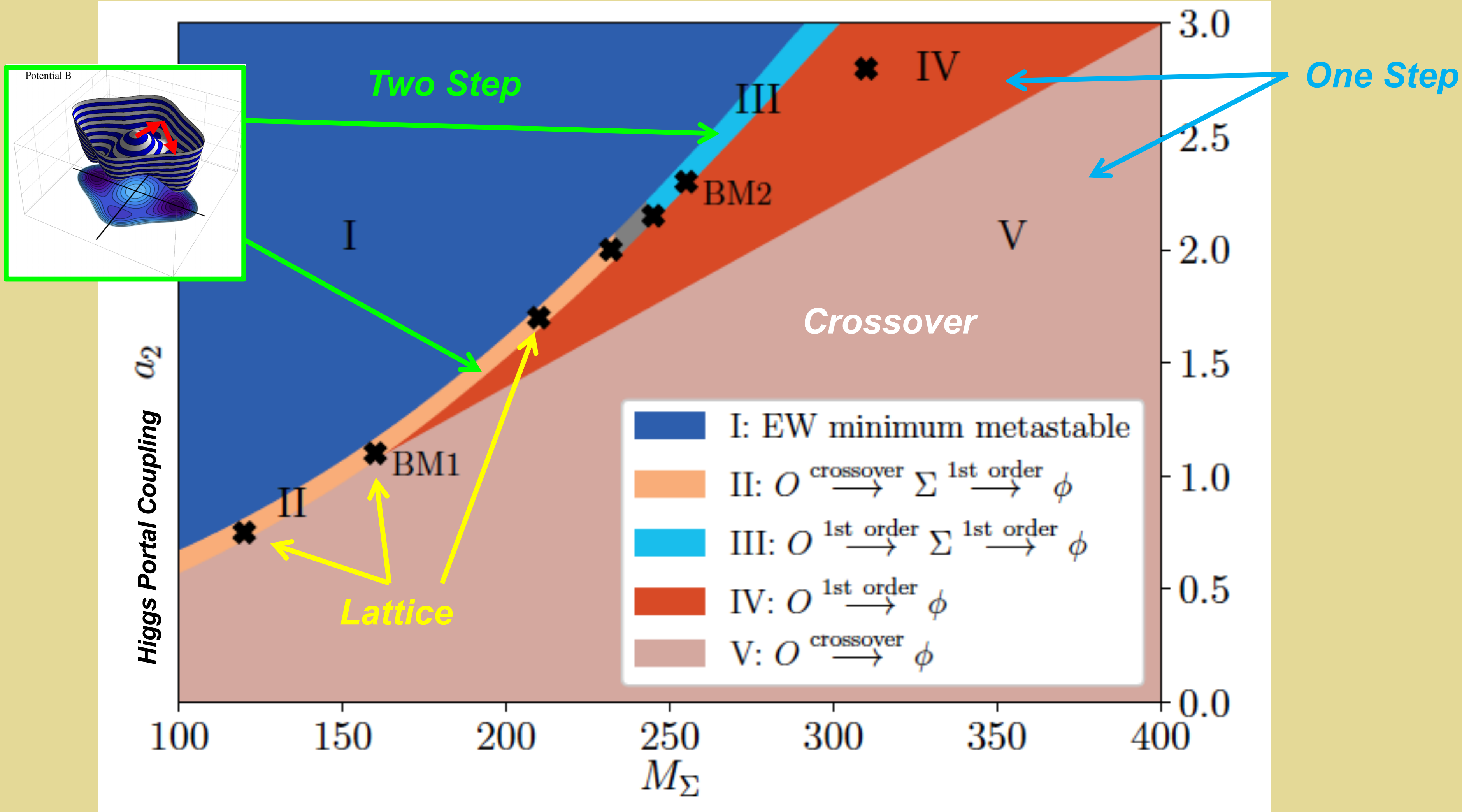


Phenomenology

- *Gravitational waves*
- *Collider: h → $\gamma\gamma$, dis charged track, NLO e^+e^- → Zh…*

Real Triplet & EWPT: Novel EWSB



Niemi, R-M, Tenkanen, Weir 2005.11332

- *1 or 2 step*
- *Non-perturbative*

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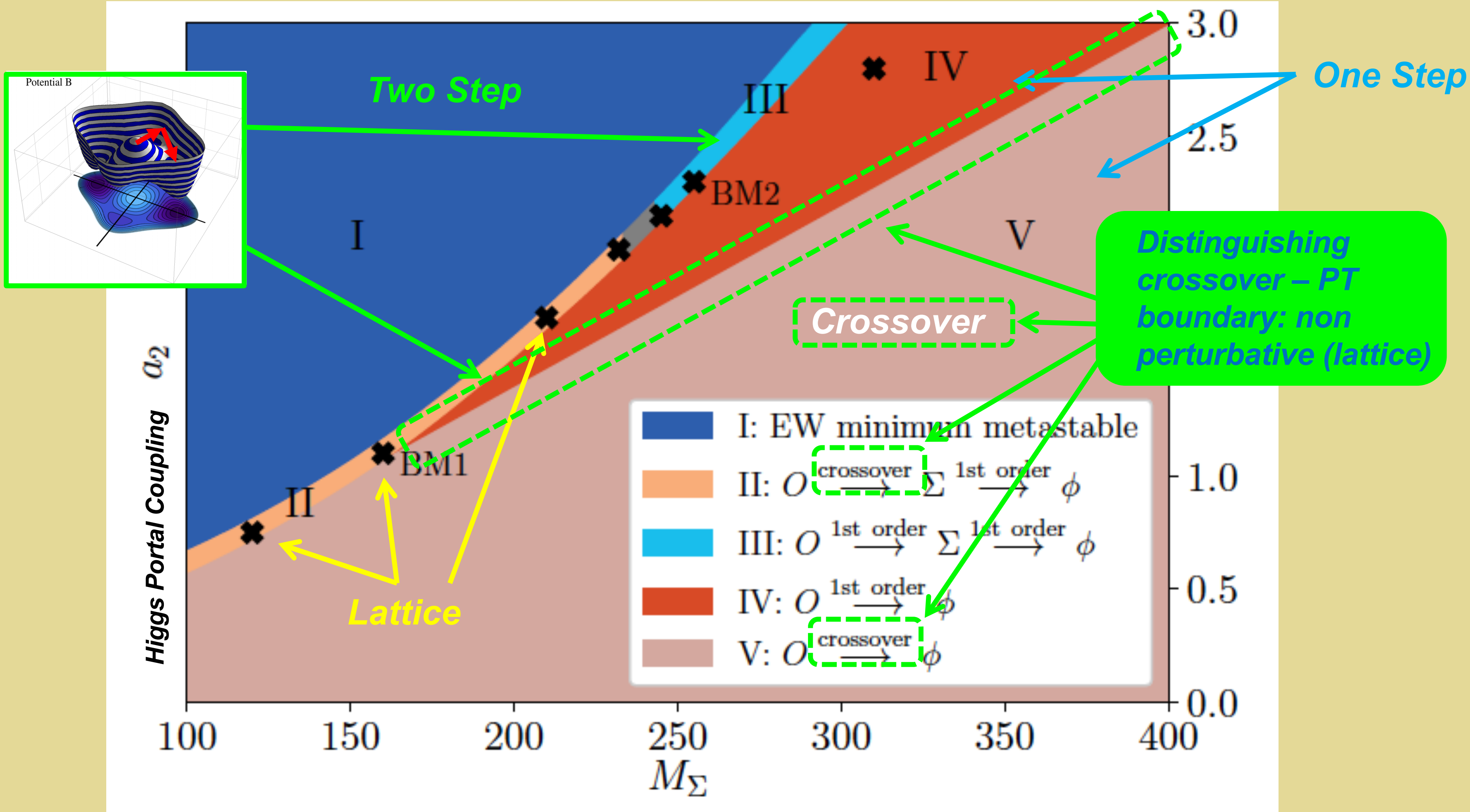

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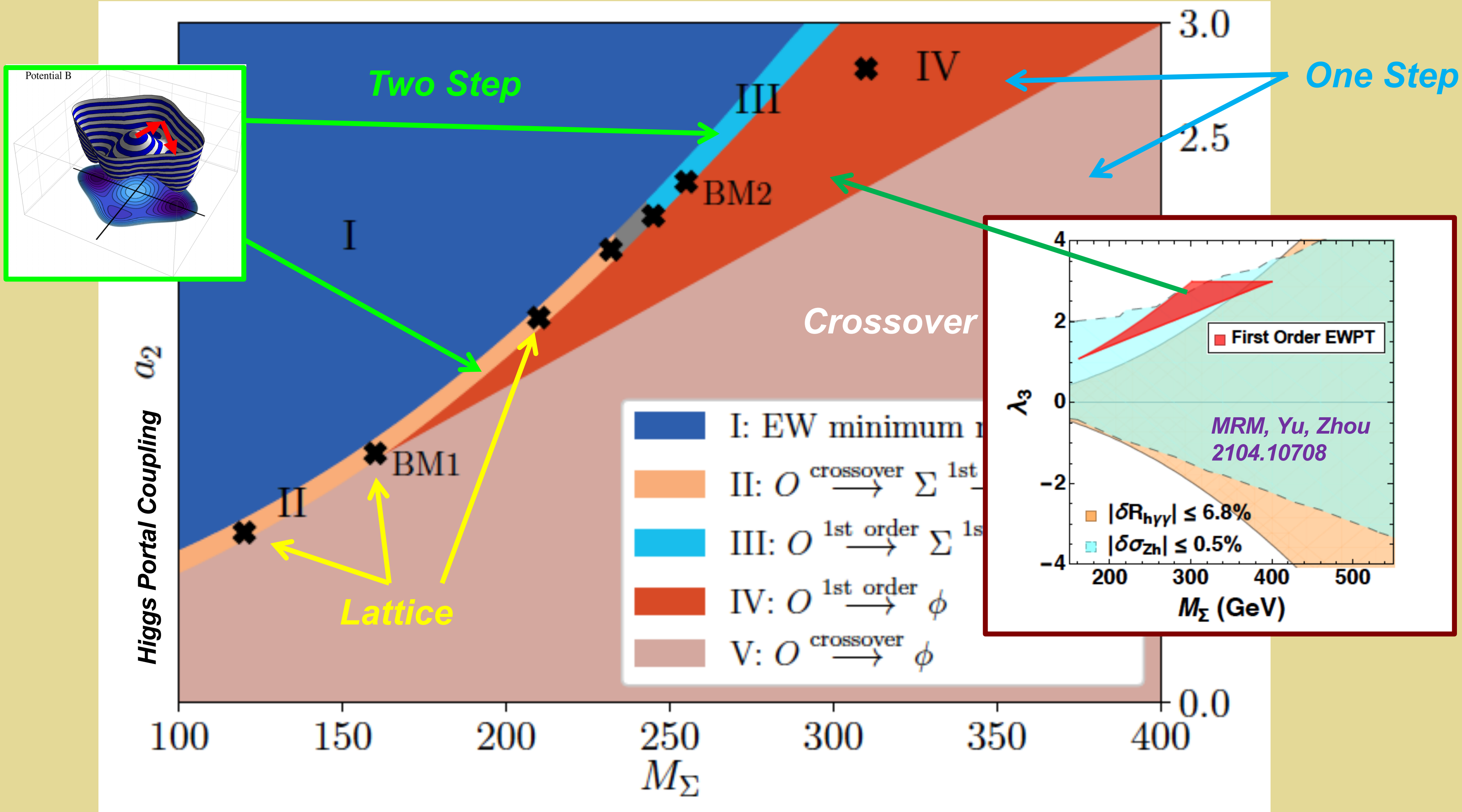

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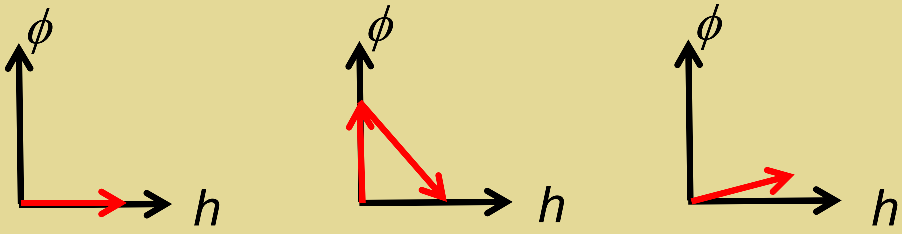

Theory-Pheno Interface

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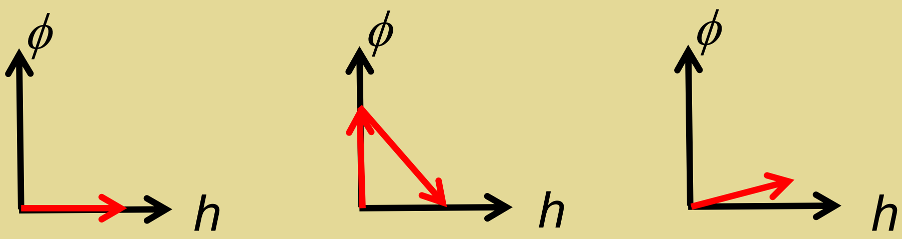
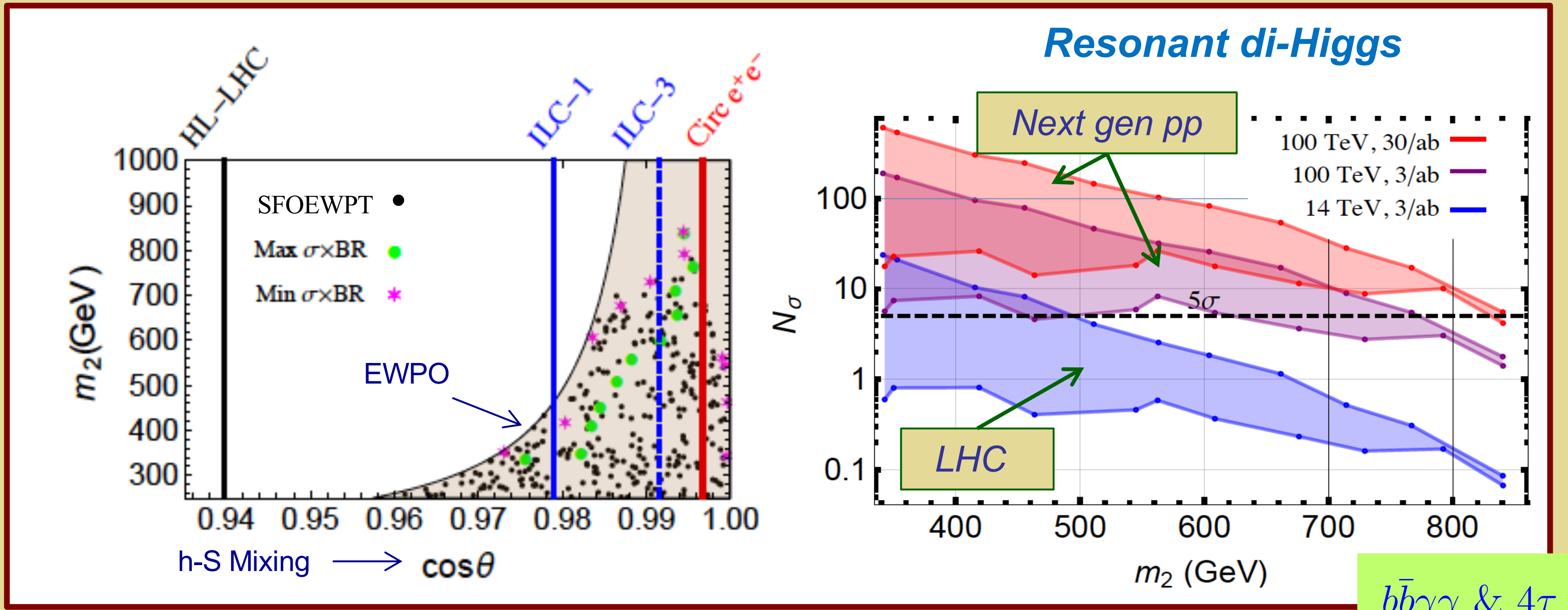
H-ϕ mixing



Singlets: Precision & Res Di-Higgs Prod

*SFOEWPT Benchmarks**: Resonant di-Higgs & precision Higgs studies*



$b\bar{b}\gamma\gamma$ & 4τ

Kotwal, No, R-M, Winslow 1605.06123

** ***Perturbative thermal QFT***

See also: Huang et al, 1701.04442;
Li et al, 1906.05289

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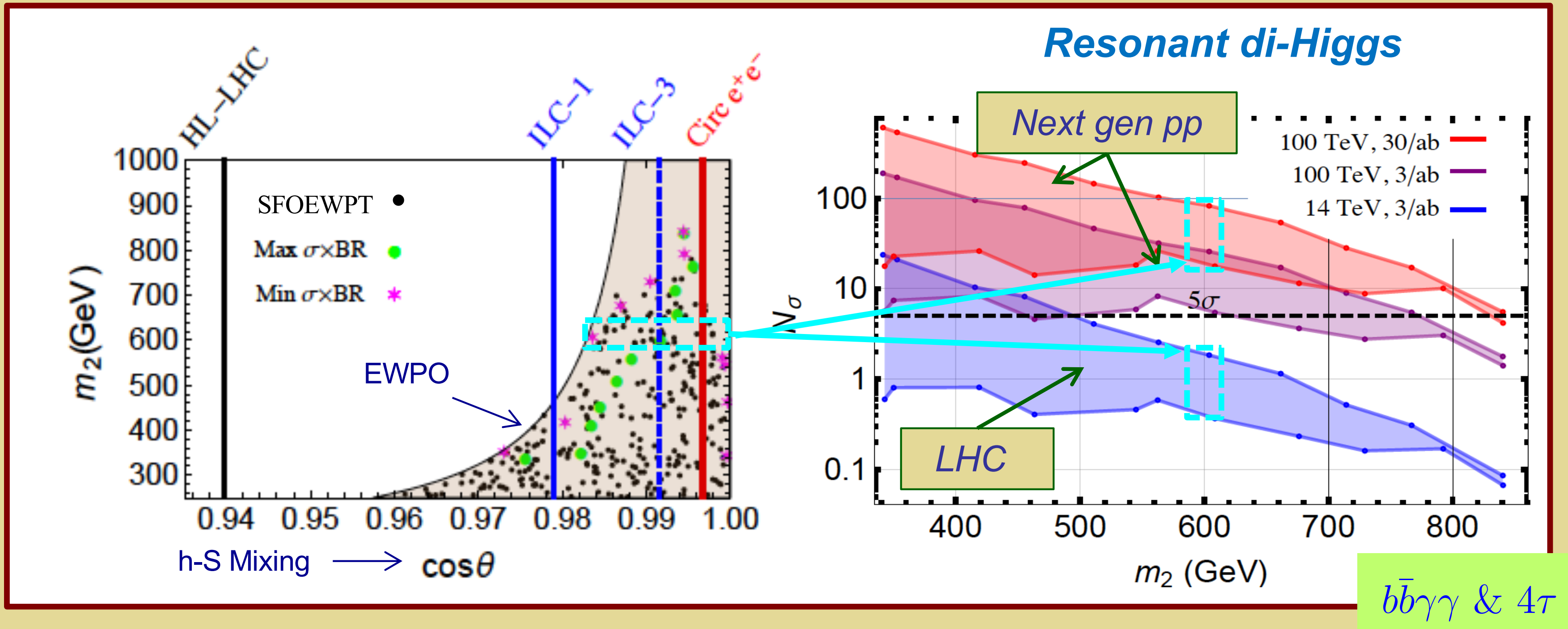

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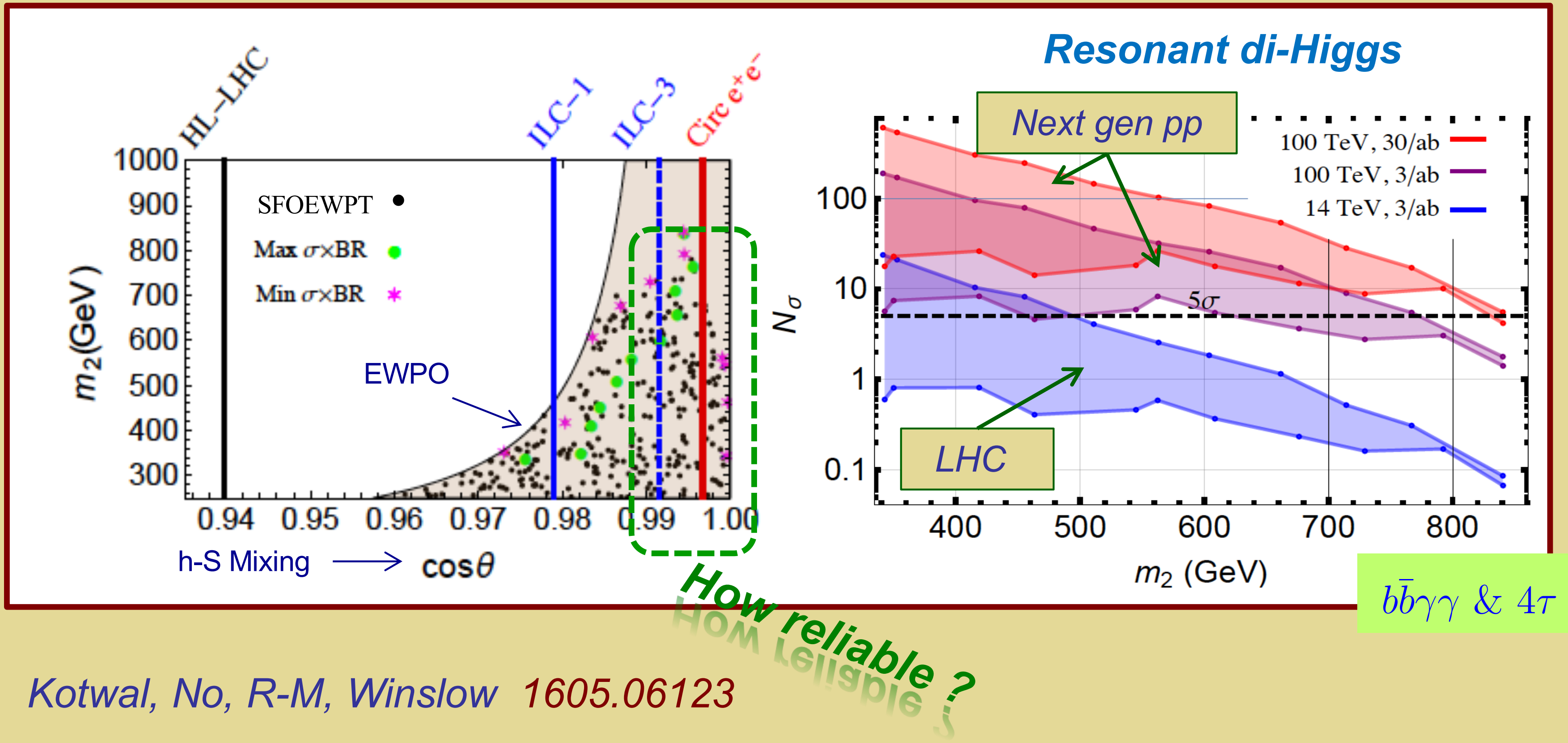

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** **Perturbative thermal QFT**

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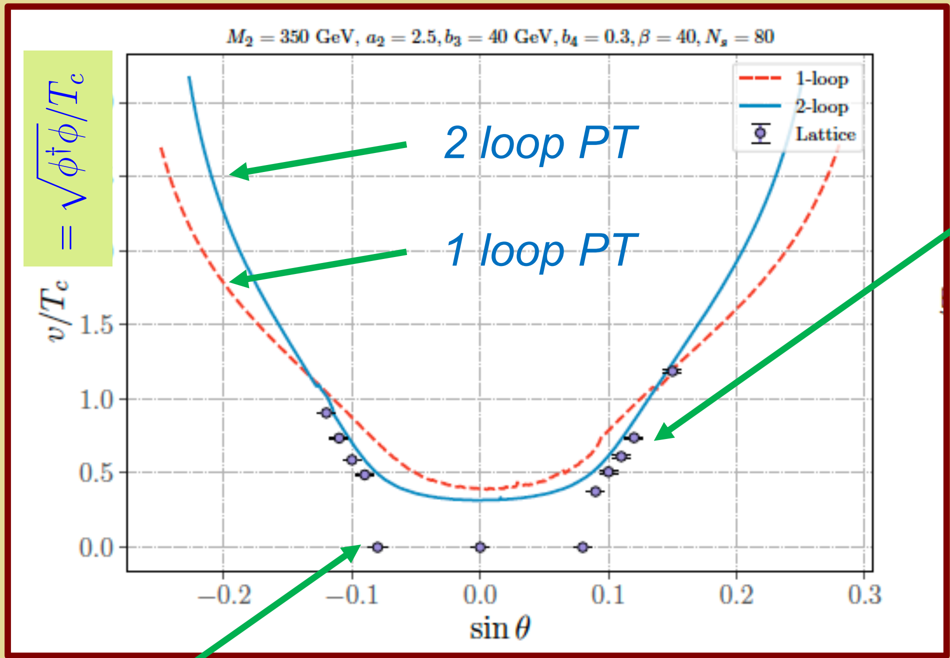
Lauri Niemi, MJRM, Gutao Xia, 2405.01191

Singlets: Lattice vs. Pert Theory

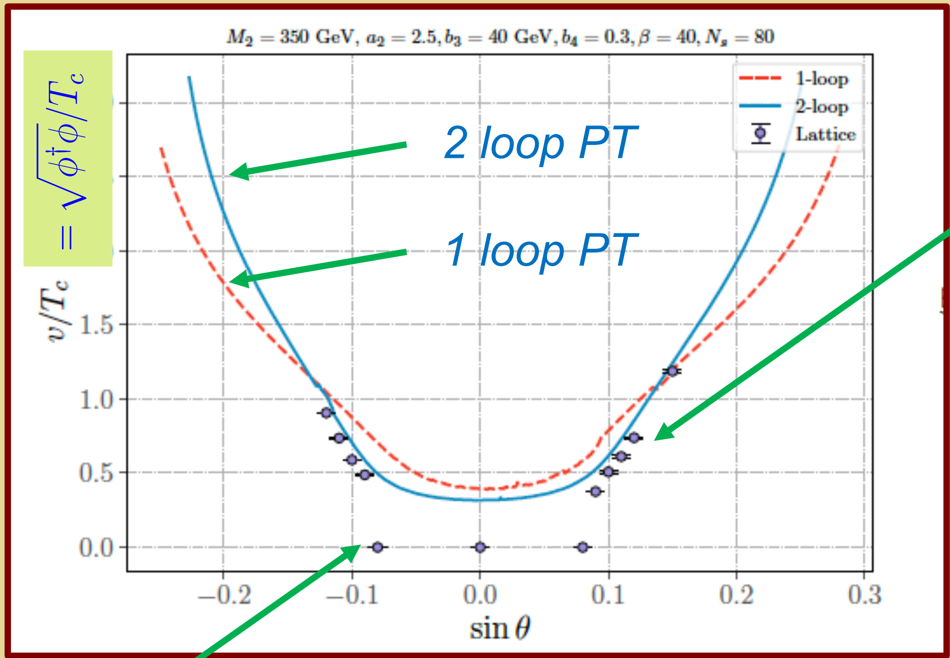


Lattice:
FOEWPT

Lattice:
Crossover

Lauri Niemi, MJRM, Gutao Xia, 2405.01191

Singlets: Lattice vs. Pert Theory



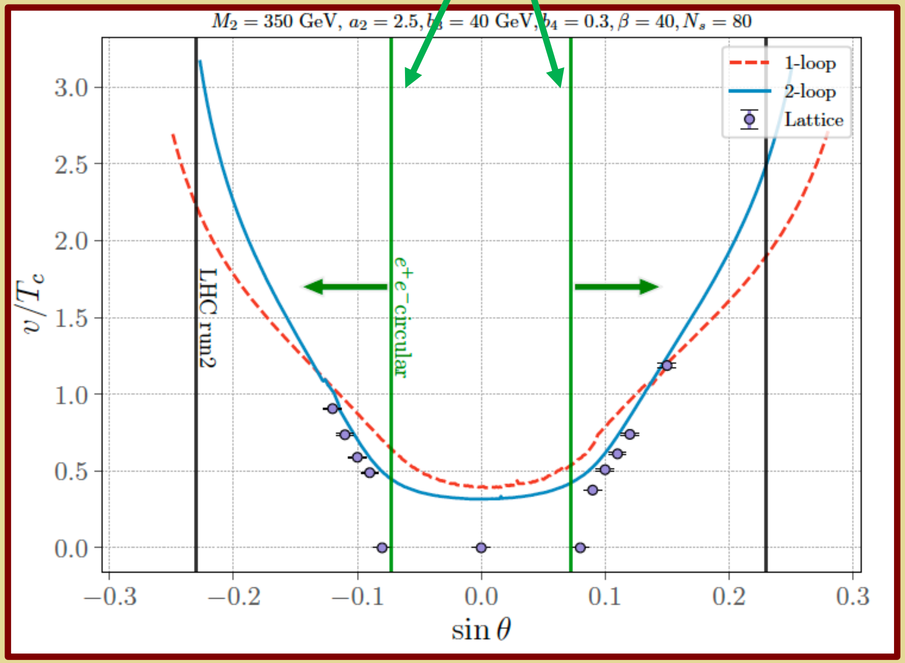
2 loop PT

1 loop PT

Lattice: FOEWPT

Lattice: Crossover

Future e^+e^-



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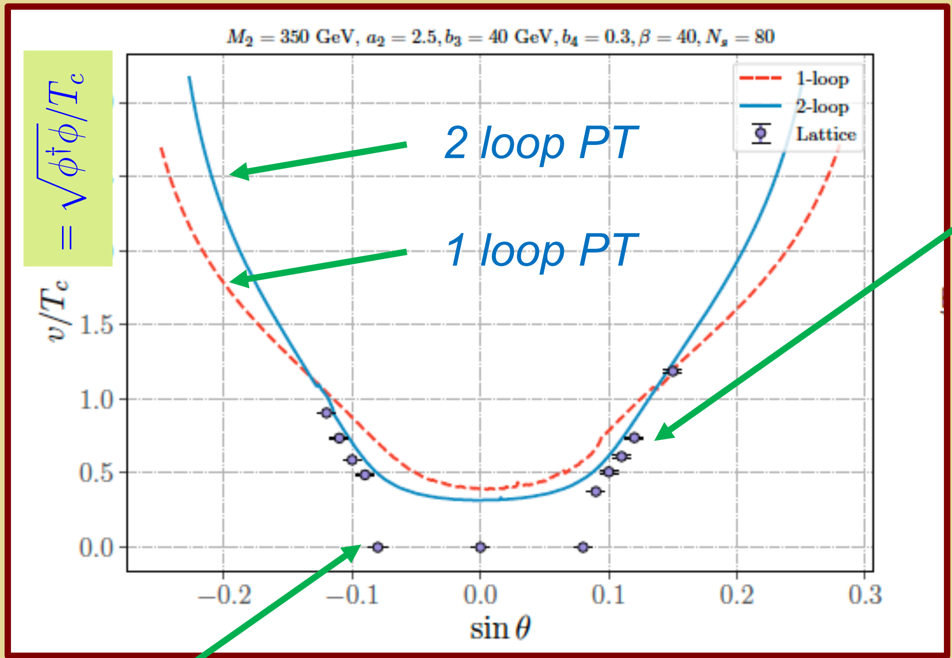
Singlets: Lattice vs. Pert Theory



Lattice: FOEWPT

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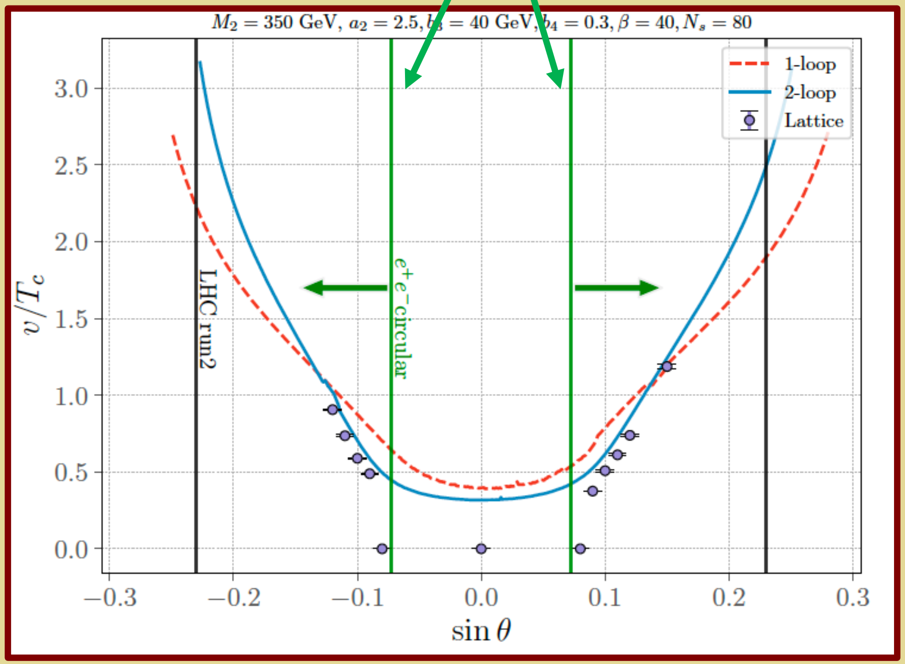


Lattice: Crossover

- *Lattice: crossover-FOEWPT boundary*
- *FOEWPT region: PT-lattice agreement*
- *Pheno: precision Higgs studies may be sensitive to a greater portion of FOEWPT-viable param space than earlier realized*

III. Theoretical Robustness

- *IR Problem*
- *Nucleation @ finite T: gauge invariance*
- *Wall velocity*

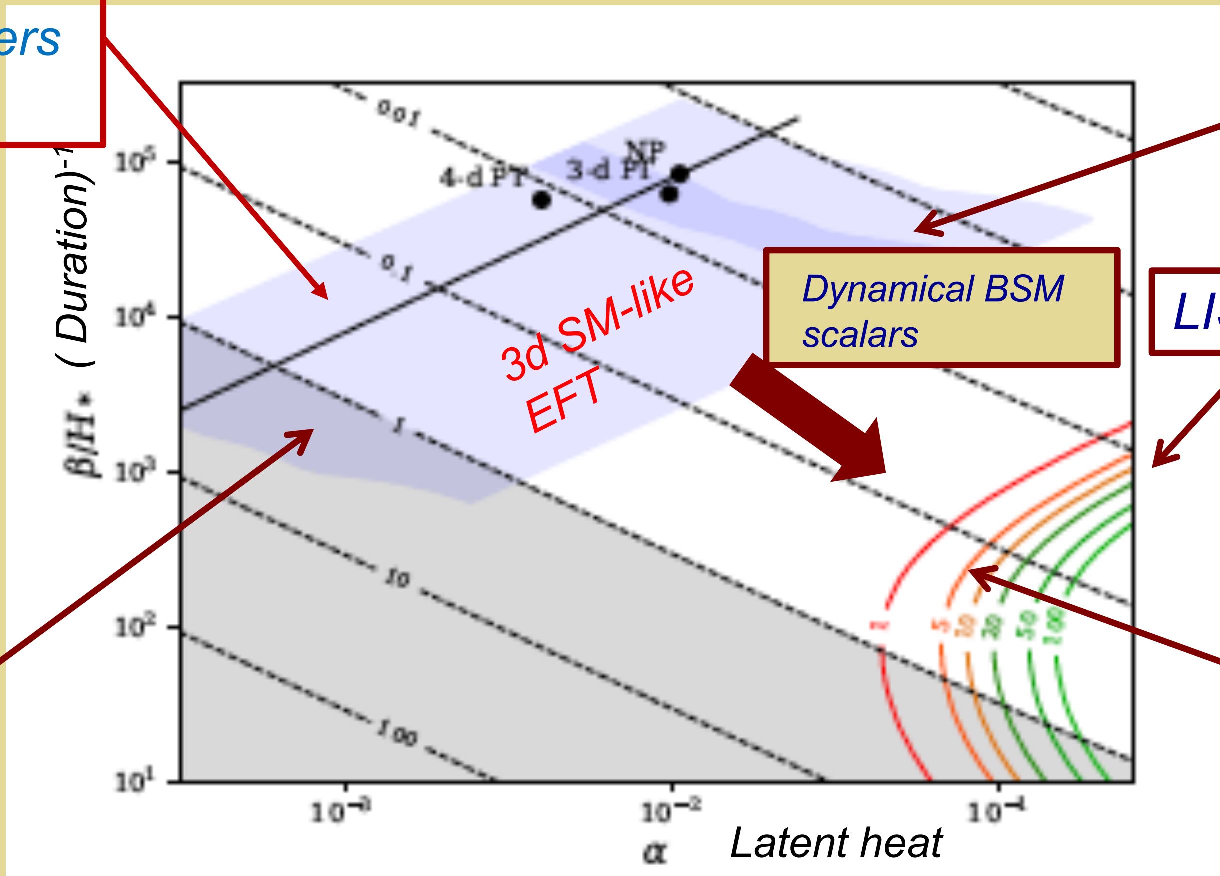
$$\frac{\beta}{H_*} = T\frac{d}{dT}\frac{S_3}{T}$$

+ T_N

BSM Scalar: EWPT & GW

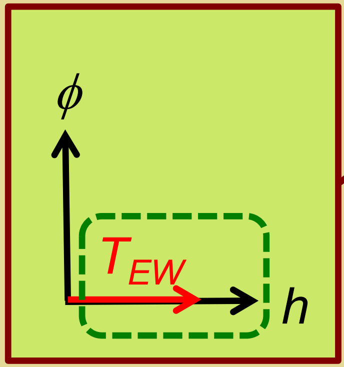
Collider probes of BSM parameters in $\mathcal{L}_{full}$

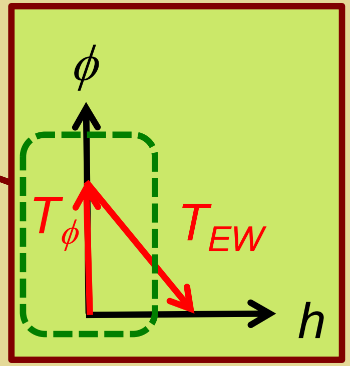
Integrate out BSM DOF



Dynamical BSM scalars

LISA SNR

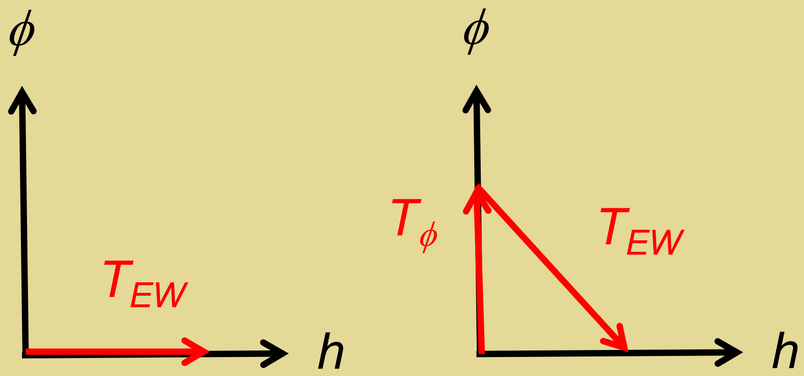




Radiative barrier from gauge sector

Gould, Kozaczuk, Niemi, R-M, Tenkanen, Weir 1903.11604

Tunneling @ T>0

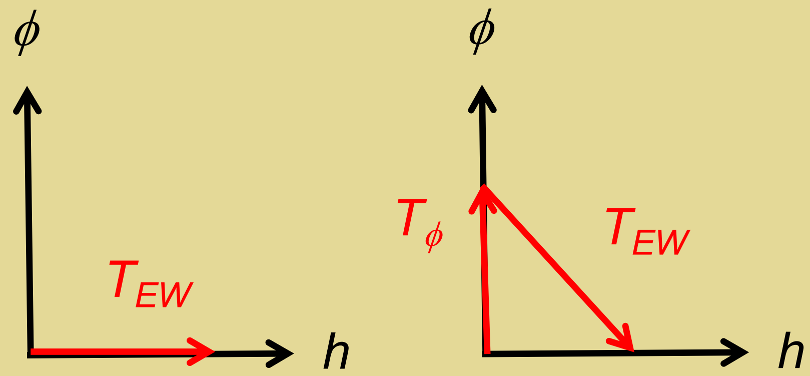


Tunneling rate / unit volume:

$$\Gamma = A e^{-\beta S_3/\hbar}\left[1 + \mathcal{O}(\hbar)\right]$$

$$A \sim \mathcal{O}(1) \times T^4$$

Tunneling @ T>0



Tunneling rate / unit volume:

$$\Gamma = A e^{-\beta S_3/\hbar}\,[1 + \mathcal{O}(\hbar)]$$

$$\frac{d^2\varphi}{dr^2} + \frac{2}{r}\frac{d\varphi}{dr} = V'(\varphi, T)$$

Exponent in Γ

Path: minimize S_E

$$S_3 = \int d^3x \left\{ \frac{1}{2}(\vec{\nabla}\varphi)^2 + V(\varphi, T) \right\}$$

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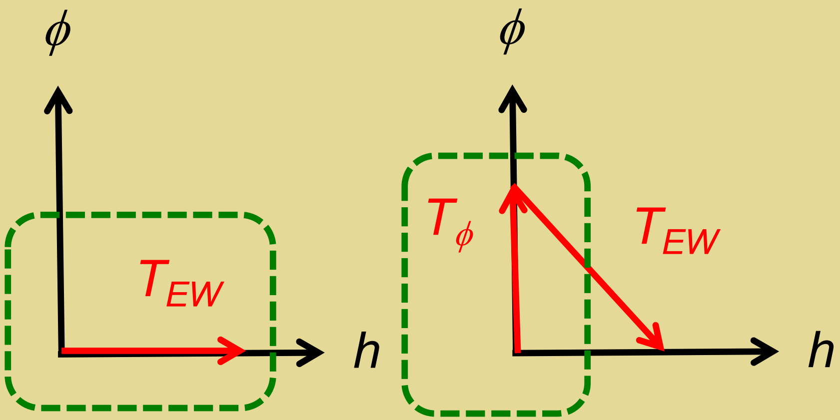
Tunneling @ T>0

Radiative barriers → standard method gauge-dependent Γ



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SSB @ T>0 : Power Counting

Lofgren, MRM, Tenkanen, Schicho 2112.0752 → PRL

$$\mu^2_{\rm eff} \equiv \mu^2 + (4\lambda + 3g^2)\frac{T^2}{12}$$

T=0 parameter < 0

Thermal corrections > 0

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T=0 parameter < 0

Thermal corrections > 0

Near cancellation for $T \sim T_C$

For a range of $T \sim T_{nuc}$: N = 1

$$\mu^2_{\rm eff} \sim \mathcal{O}(g^{2+N} T^2) < \mathcal{O}(g^2 T^2)$$

Power Counting

Lofgren, MRM, Tenkanen, Schicho 2112.0752 → PRL

ϕ	~	T
λ	~	g^3
μ^2	~	$g^2 T^2$
μ_{eff}^2	~	$g^3 T^2$

→

$$V^{\rm eff}_{\rm LO} = \frac{1}{2}\mu^2_{\rm eff}\phi^2 + \frac{1}{4}\lambda\phi^4 - \frac{g^3 T}{12\pi}\left[2\phi^3 + \left(\frac{1}{3}T^2 + \phi^2\right)^{\frac{3}{2}}\right]$$

Radiative barrier:
ξ-independent

(Re) Organize the Perturbative Expansion

Illustrate w/ Abelian Higgs

$$\mathcal{L} = \frac{1}{4}F_{\mu\nu}F_{\mu\nu} + (D_\mu\Phi)^*(D_\mu\Phi) + \mu^2\Phi^*\Phi + \lambda(\Phi^*\Phi)^2 + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}}$$

- *Lofgren, MRM, Tenkanen, Schicho 2112.0752 → PRL*
- *Hirvonen, Lofgren, MRM, Tenkanen, Schicho 2112.08912*

Full 3D effective action

$$S_3 = \int \mathrm{d}^3x \Big[V^{\text{eff}}(\phi,T) + \frac{1}{2}Z(\phi,T)\,(\partial_i\phi)^2 + \dots\Big]$$

Adopt appropriate power-counting in couplings

$$S_3 = a_0 g^{-\frac{3}{2}} + a_1 g^{-\frac{1}{2}} + \Delta$$

(Re) Organize the Perturbative Expansion

Illustrate w/ Abelian Higgs

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Adopt appropriate power-counting in couplings

$$S_3 = a_0 g^{-\frac{3}{2}} + a_1 g^{-\frac{1}{2}} + \Delta$$

G.I. pertubative expansion only valid up to NLO → Δ: higher order contributions only via other methods

G.I. pertubative expansion

Tunneling @ T>0: G.I. & Nielsen Identities

Lofgren, MRM, Tenkanen, Schicho 2112.0752 → PRL

Adopt appropriate power-counting in couplings

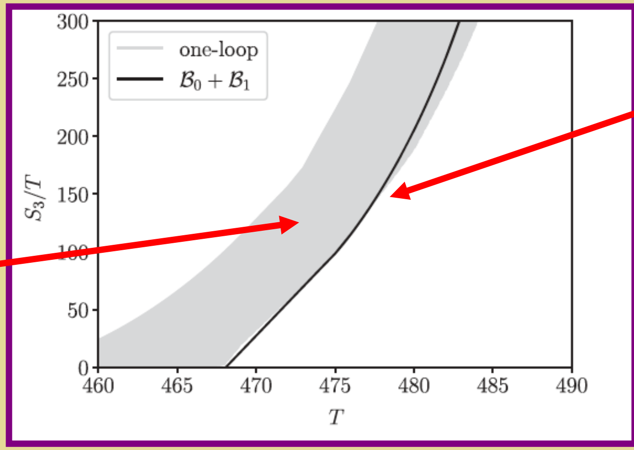
$$S_3 = a_0 g^{-\frac{3}{2}} + a_1 g^{-\frac{1}{2}} + \Delta$$

Order-by-order consistent with Nielsen Identities

$$\xi \frac{\partial S^{\mathrm{eff}}}{\partial \xi} = - \int \mathrm{d}^d \mathbf{x} \frac{\delta S^{\mathrm{eff}}}{\delta \phi(x)} \mathcal{C}(x)$$

$$\mathcal{C}(x) = \frac{ig}{2} \int \mathrm{d}^d \mathbf{y} \Big\langle \chi(x) c(x) \bar{c}(y) \times \Big[\partial_i B_i(y) + \sqrt{2} g \xi \phi \chi(y) \Big] \Big\rangle$$

Numerical comparison with conventional approach



S_3 to $\mathcal{O}(g^{-1/2})$: $0 < \xi < 4$

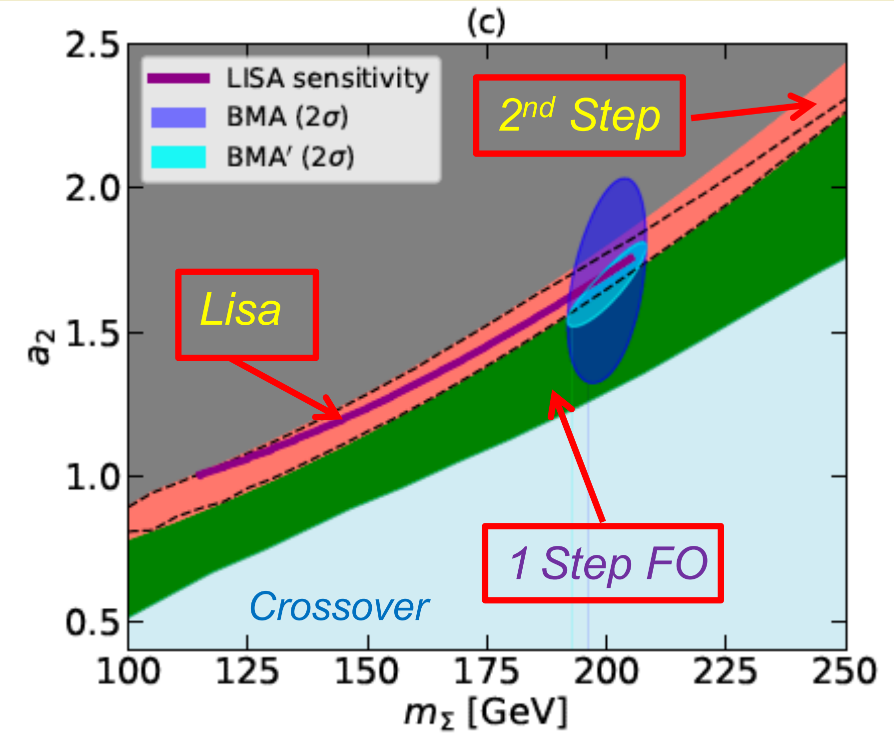
Conventional: $0 < \xi < 4$

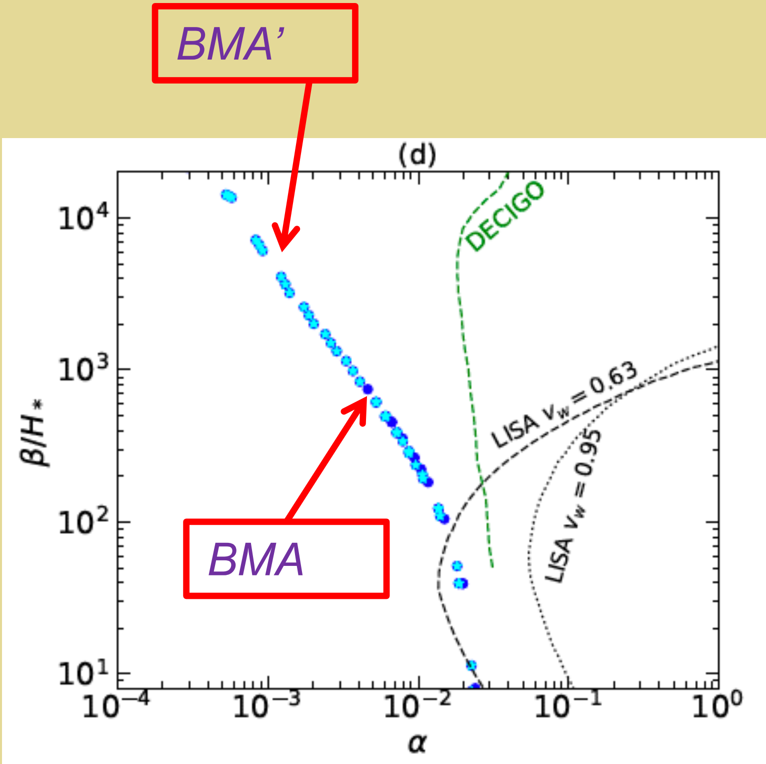
III. Theoretical Robustness

- ***IR Problem***
- ***Nucleation @ finite T: gauge invariance***
- ***Wall velocity***

GW & Collider : EWPT Phase Diagram

Real triplet extension





BMA: m_Σ + h→ $\gamma\gamma$

BMA' : BMA + Σ^0→ ZZ

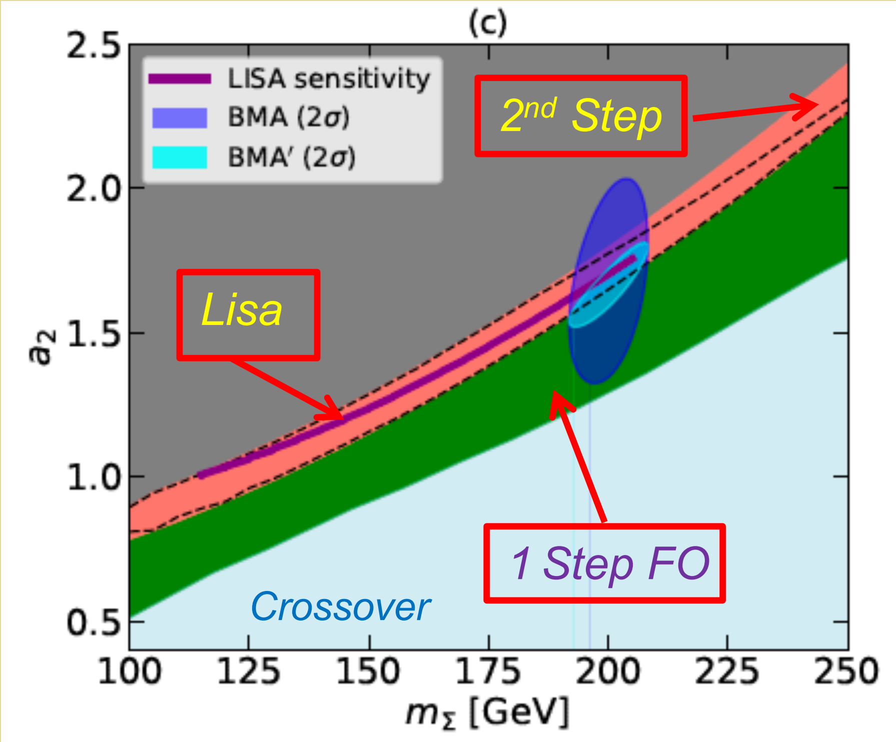
- *Two-step*
- *EFT+ Non-perturbative*

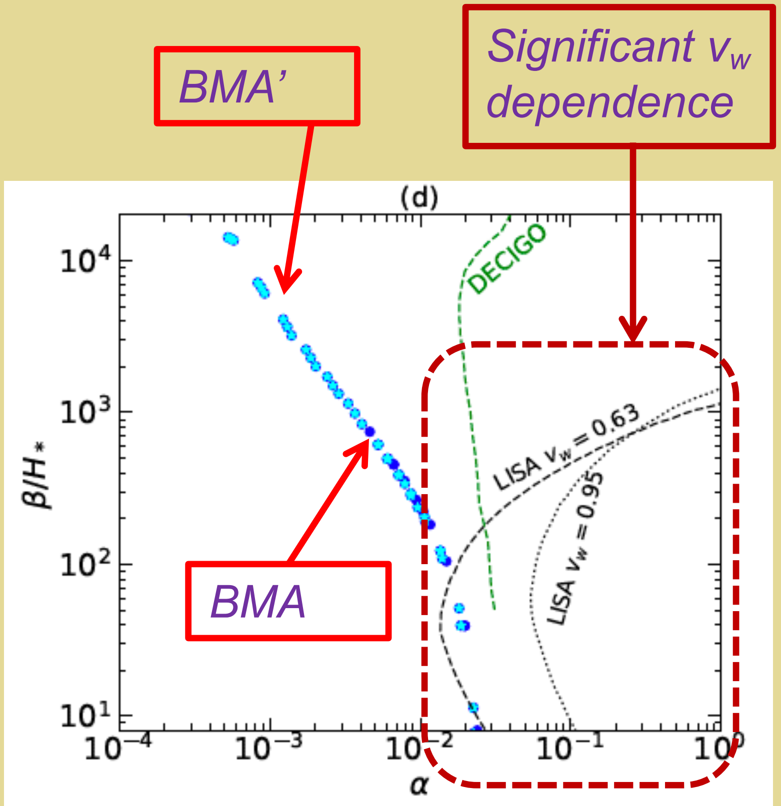
27.1

Friedrich, MJRM, Tenkanen, Tran 2203.05889

GW & Collider : EWPT Phase Diagram

Real triplet extension





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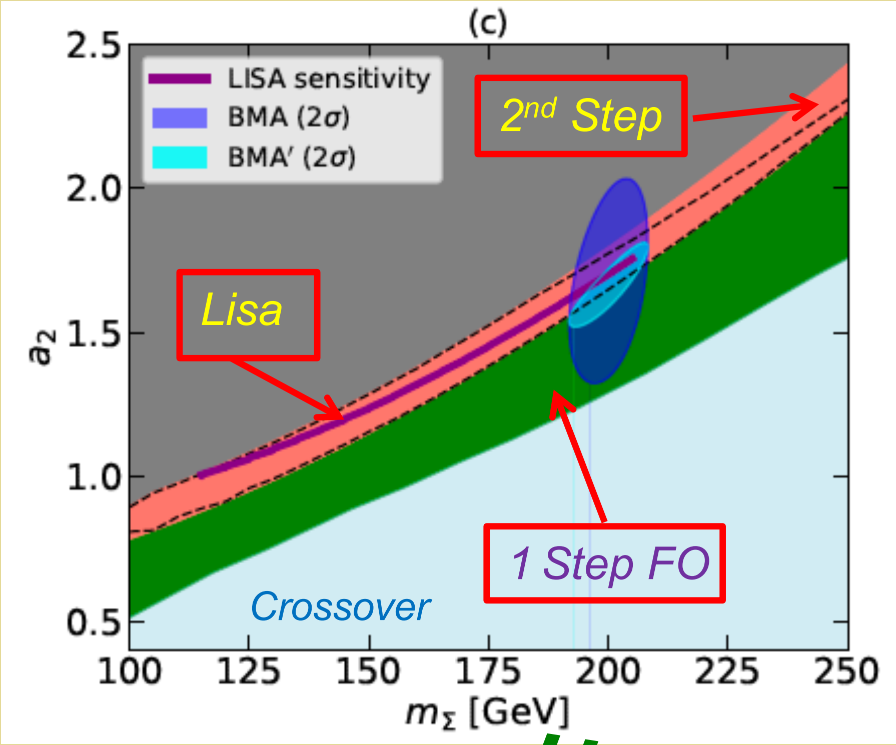

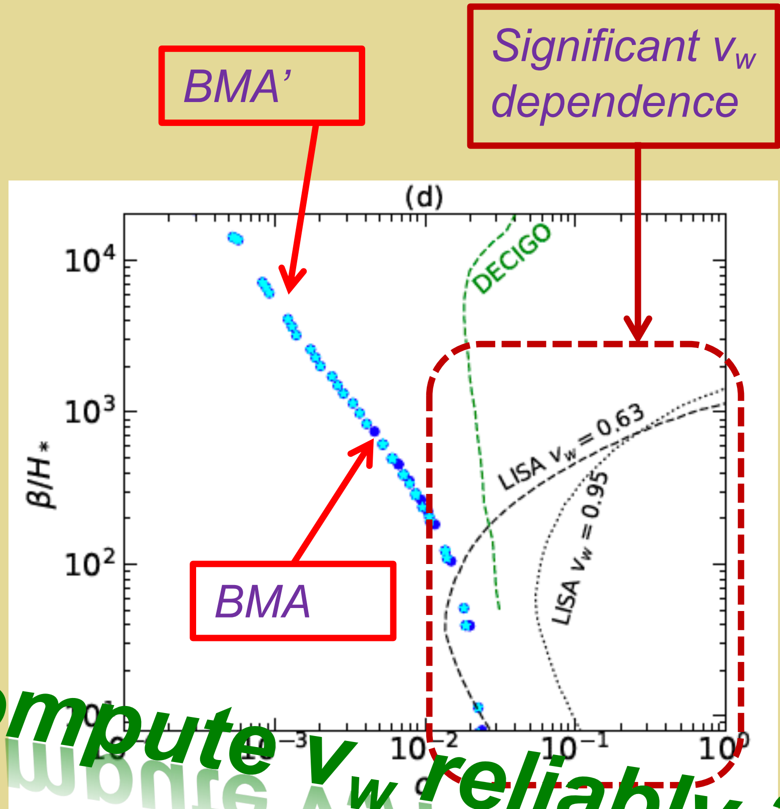

How to compute v_w reliably ?

BMA: m_Σ + h → γγ

BMA' : BMA + Σ^0 → ZZ

- *Two-step*
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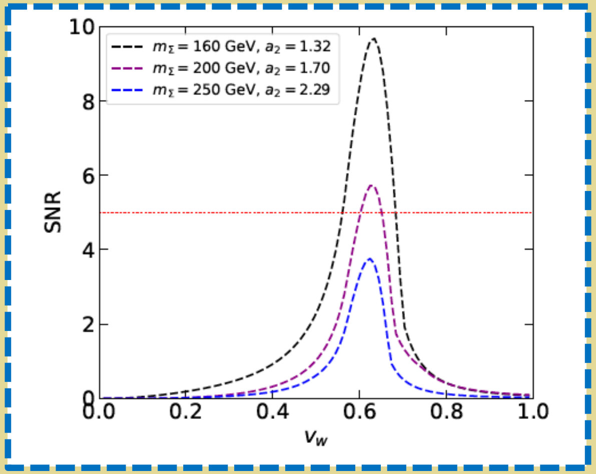
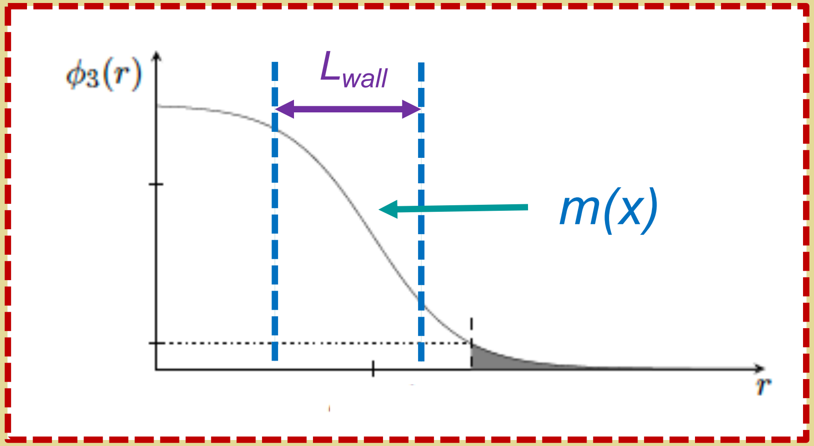
Friedrich, MJRM, Tenkanen, Tran 2203.05889

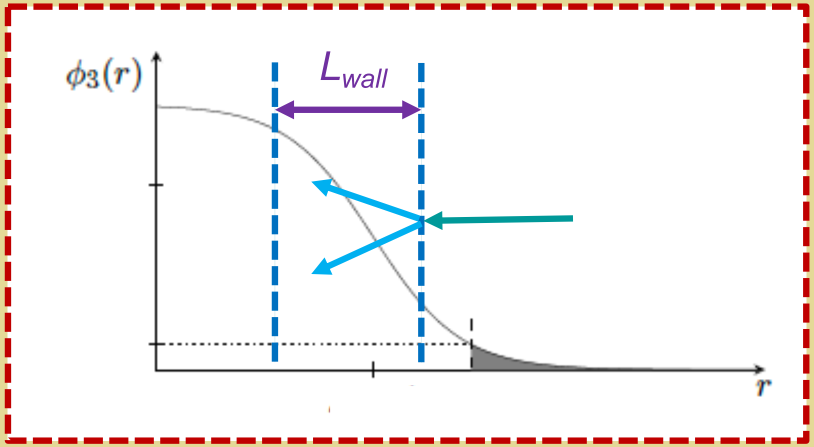
Computing Wall Velocity

LISA SNR

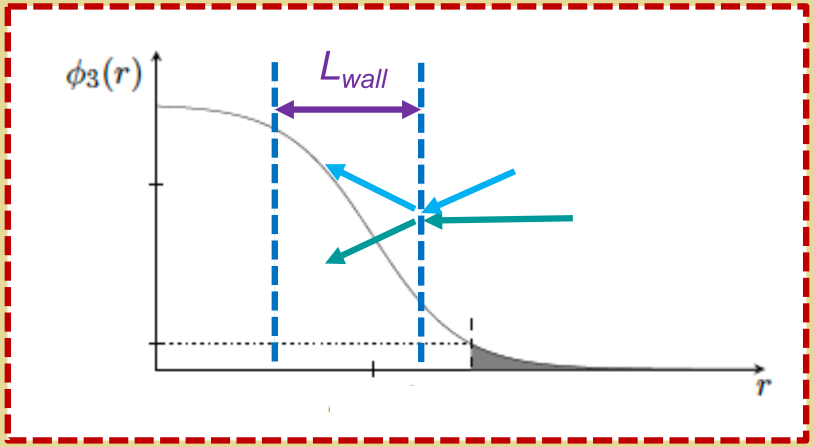


Mass variation





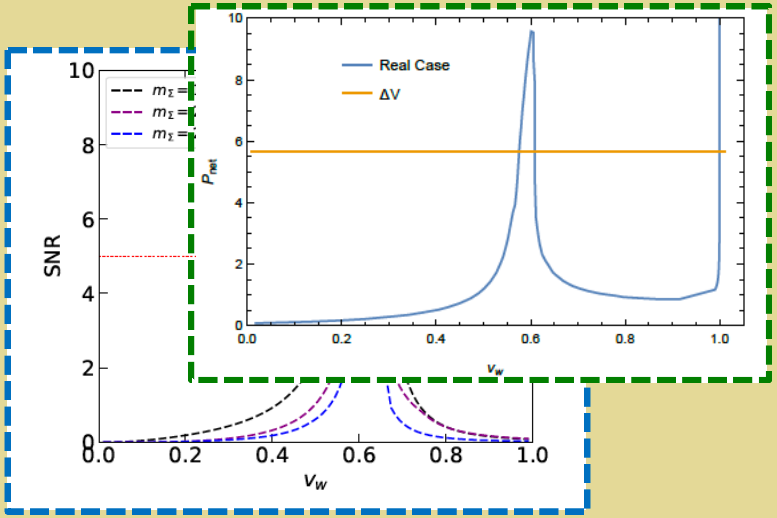
Splitting



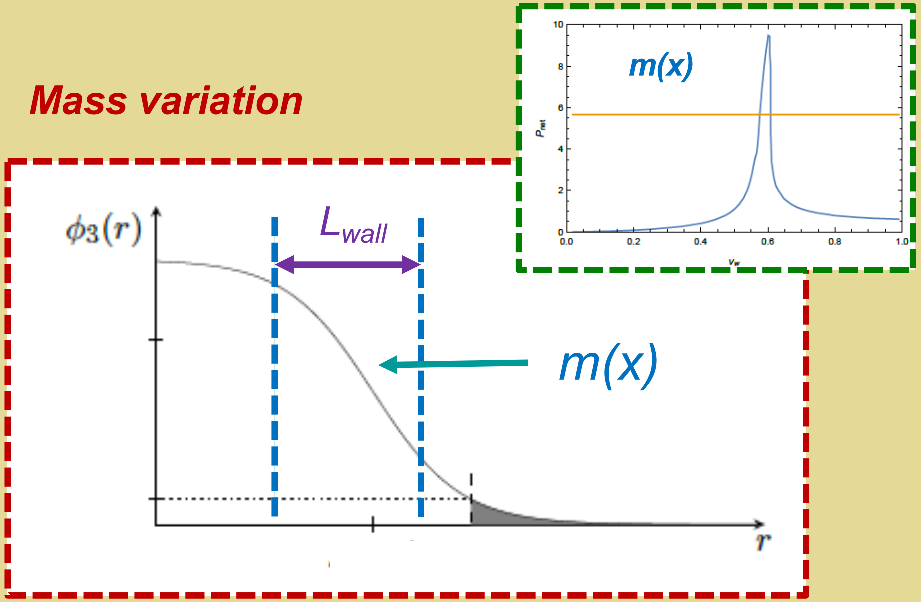
Scattering & annihilation

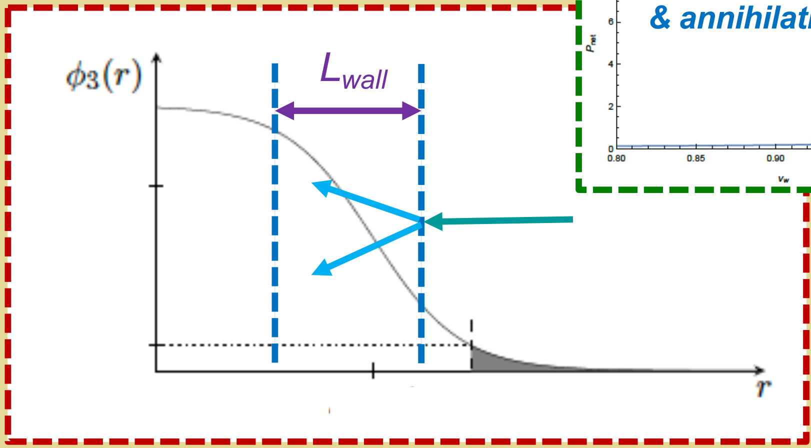
Computing Wall Velocity

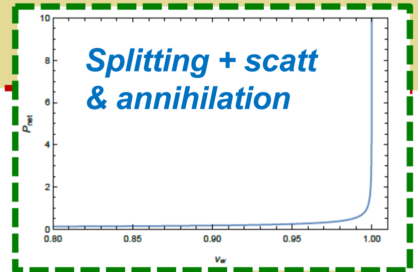
Total friction pressure

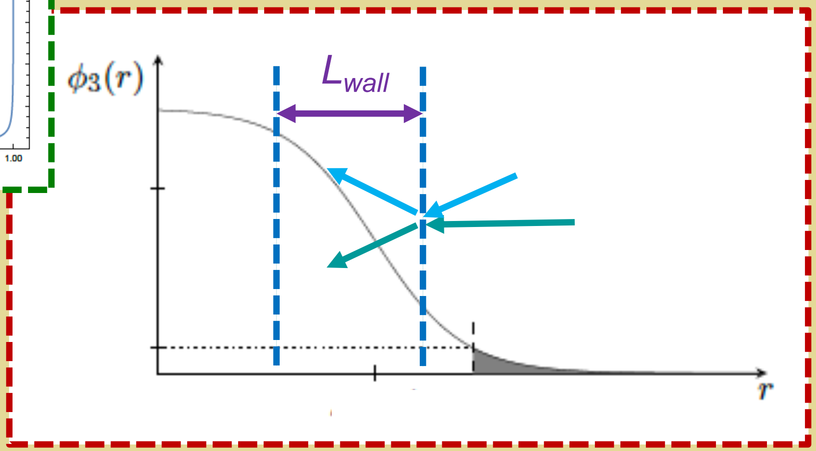


Mass variation





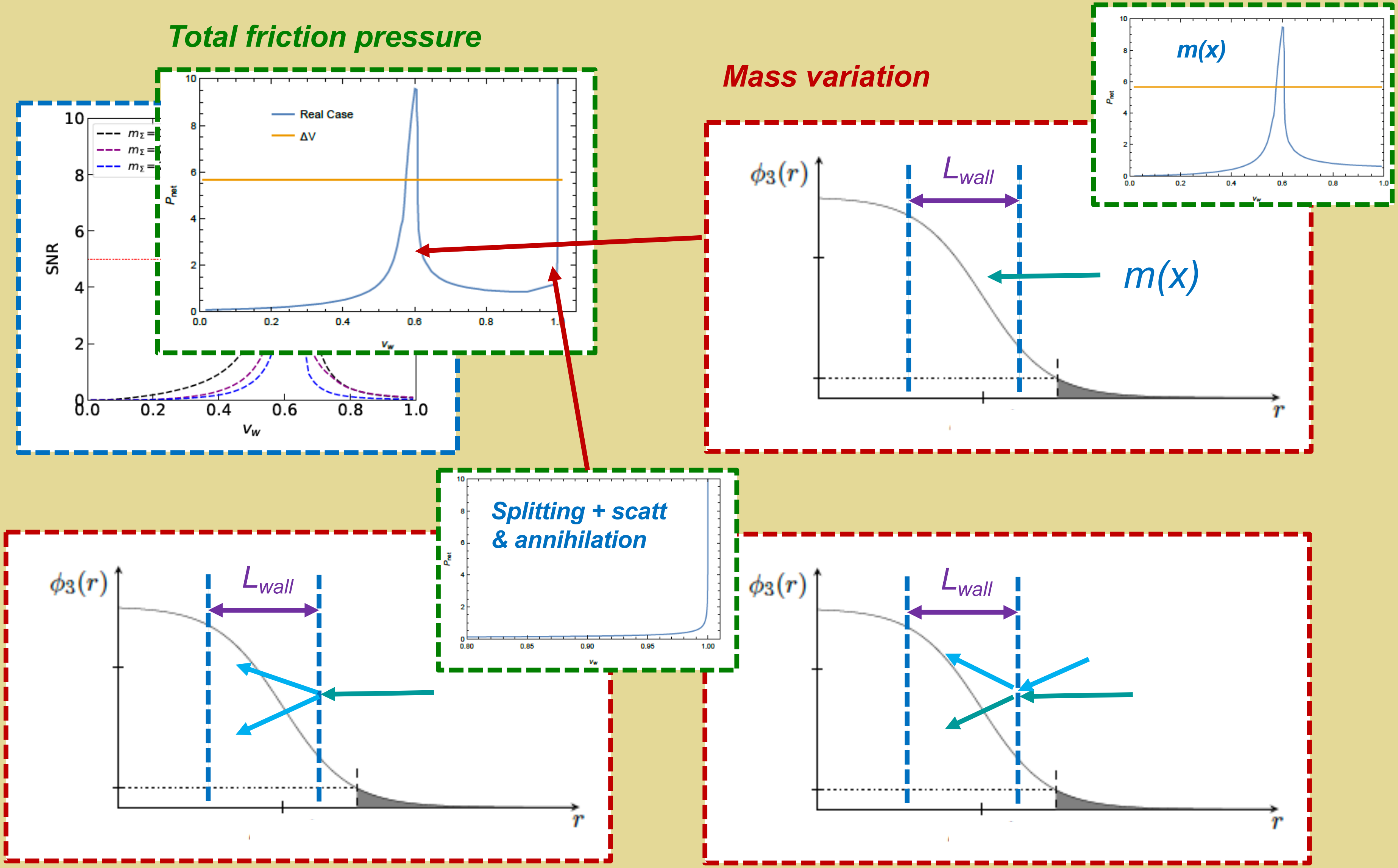




Splitting

Scattering & annihilation

Computing Wall Velocity



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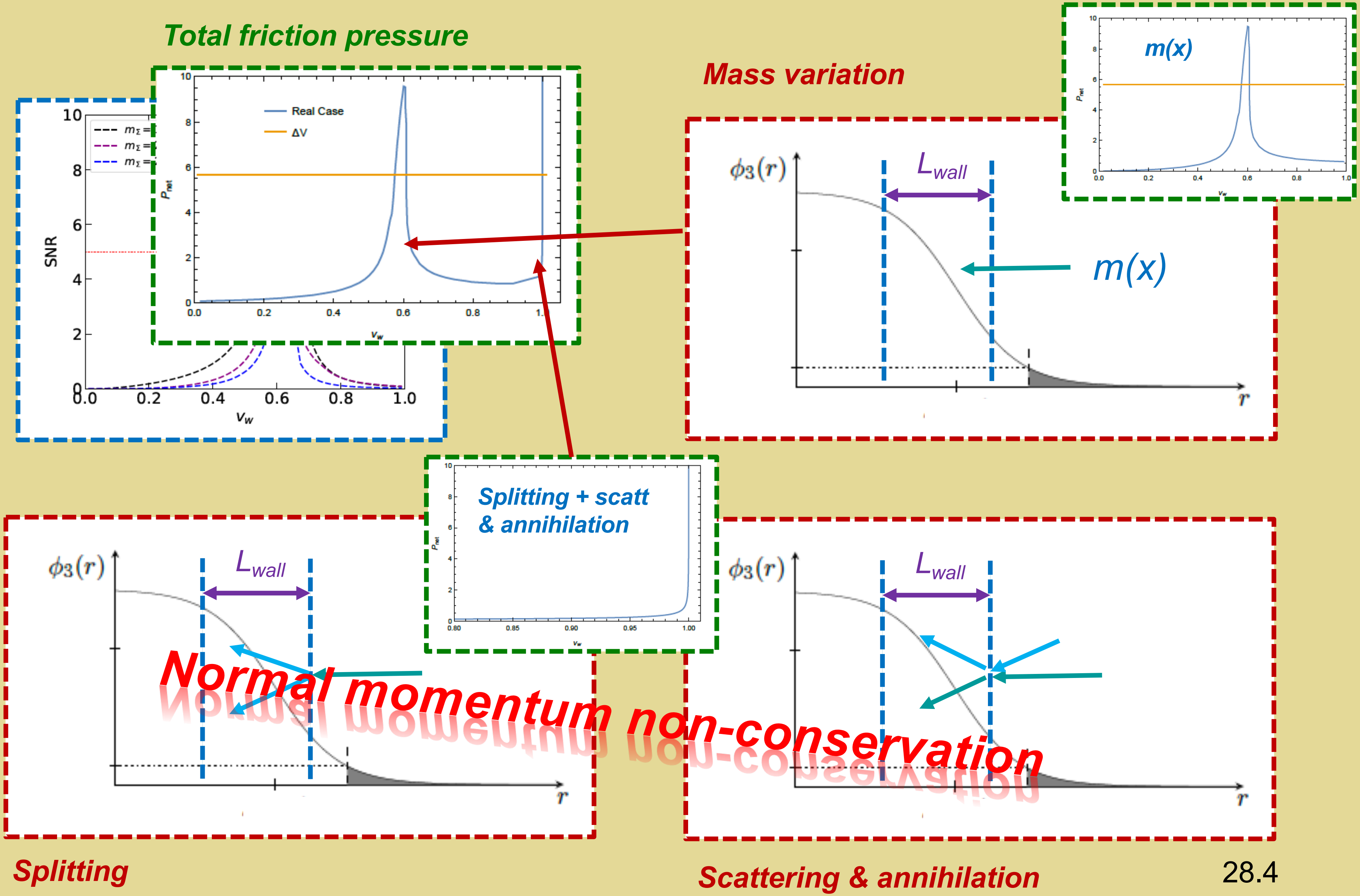

Computing Wall Velocity

$$\partial_\mu T^{\mu\nu}_\phi + \partial_\mu T^{\mu\nu}_{\text{plasma}} = 0$$

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EOM

Kadanoff-Baym → Boltzmann

$$\Delta V(\phi, T)$$

$$L[f_j] = \mathcal{C}[f_j]$$

Computing Wall Velocity

$$\partial_\mu T^{\mu\nu}_\phi + \partial_\mu T^{\mu\nu}_{\rm plasma} = 0$$

EOM

Kadanoff-Baym → Boltzmann

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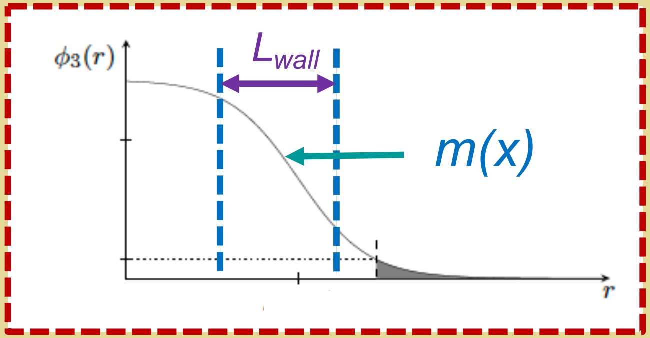

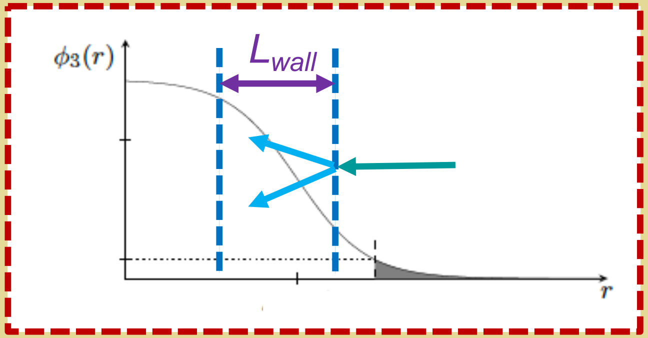

Computing Wall Velocity

Kadanoff-Baym "constraint eq"

$$-2ik\cdot\partial_X G^{\gtrless} + e^{-i\diamond}[m^2, G^{\gtrless}] = -ie^{-i\diamond}([\Pi^h, G^{\gtrless}] + [\Pi^{\gtrless}, G^h] + \frac{1}{2}\{\Pi^>, G^<\} - \frac{1}{2}\{G^>, \Pi^<\}),$$

Project out distribution functions

$$\begin{aligned}\int_0^\infty \frac{dk^0}{2\pi} k_z \frac{d}{dz} G^<(k,z) + \frac{i}{2}\int_0^\infty \frac{dk^0}{2\pi} e^{-i\diamond}[m_a^2(z), G^<(k,z)] \\ + \frac{1}{4}\int_0^\infty \frac{dk^0}{2\pi} e^{-i\diamond}(\{\Pi_a^>, G_a^<\} - \{\Pi_a^<, G_a^>\}) \\ = \frac{1}{2}\int_0^\infty \frac{k^0}{2\pi} e^{-i\diamond}([\Pi_a^h, G_a^<] + [\Pi_a^<, G_a^h]).\end{aligned}$$

Gradient expansion except on δ fns

$$\begin{aligned}&\left[2k_z\frac{\partial}{\partial z} - \frac{dm^2(z)}{dz}\frac{\partial}{\partial k_z}\right]\frac{f_\phi(k,z)}{E_k} \\ &\quad = -\int \frac{d^3\mathbf{p}}{(2\pi)^3}\int \frac{d^3\mathbf{p}'}{(2\pi)^3} F(k,z) \frac{1+f_\Phi(p,z)}{2E_p}\frac{1+f_\Phi(p',z)}{2E_{p'}} \\ &\qquad \times (2\pi)^3 \delta(E_k - E_p - E_{p'})\delta^2(\mathbf{k}_\perp - \mathbf{p}_\perp - \mathbf{p}'_\perp) + (\Delta p_z \leftrightarrow -\Delta p_z) \\ &\quad + \text{InverseProcess},\end{aligned}$$

Computing Wall Velocity

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Wigner transformed Wightman functions

"Diamond operator"

$$\diamond\Big(A(k,x)B(k,x)\Big) = \frac{1}{2}\left(\frac{\partial A}{\partial x^\mu}\frac{\partial B}{\partial k_\mu} - \frac{\partial A}{\partial k_\mu}\frac{\partial B}{\partial x^\mu}\right)$$

Project out distribution functions

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MJRM, J. Zhu 2504.13724

Computing Wall Velocity

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$$+\frac{1}{4}\int_0^\infty \frac{dk^0}{2\pi} e^{-i\diamond}\left(\{\Pi_a^>, G_a^<\} - \{\Pi_a^<, G_a^>\}\right)$$
$$= \frac{1}{2}\int_0^\infty \frac{k^0}{2\pi} e^{-i\diamond}\left([\Pi_a^h, G_a^<] + [\Pi_a^<, G_a^h]\right).$$

Gradient expansion except on δ fns

$$\left[2k_z\frac{\partial}{\partial z} - \frac{dm^2(z)}{dz}\frac{\partial}{\partial k_z}\right]\frac{f_\phi(k,z)}{E_k}$$
$$= -\int \frac{d^3\mathbf{p}}{(2\pi)^3}\int \frac{d^3\mathbf{p}'}{(2\pi)^3} F(k,z)\frac{1+f_\Phi(p,z)}{2E_p}\frac{1+f_\Phi(p',z)}{2E_{p'}}$$
$$\times (2\pi)^3\delta(E_k - E_p - E_{p'})\delta^2(\mathbf{k}_\perp - \mathbf{p}_\perp - \mathbf{p}'_\perp) + (\Delta p_z \leftrightarrow -\Delta p_z)$$
$$+ \text{InverseProcess},$$

mass variation

Collision term: p_z non-cons

$$F(k,z) = \int dz' f_\phi(k,z')Y(z')Y(2z-z')e^{-2i\Delta p_z(z-z')}$$

Computing Wall Velocity

Kadanoff-Baym “constraint eq”

Wigner transformed Wightman functions

$$-2ik\cdot\partial_X G^{\gtrless} + e^{-i\diamond}[m^2, G^{\gtrless}] = -ie^{-i\diamond}\left([\Pi^h, G^{\gtrless}] + [\Pi^{\gtrless}, G^h] + \frac{1}{2}\{\Pi^>, G^<\} - \frac{1}{2}\{G^>, \Pi^<\}\right),$$

“Diamond operator”

$$\diamond\Big(A(k,x)B(k,x)\Big) = \frac{1}{2}\left(\frac{\partial A}{\partial x^\mu}\frac{\partial B}{\partial k_\mu} - \frac{\partial A}{\partial k_\mu}\frac{\partial B}{\partial x^\mu}\right)$$

Project out distribution functions

$$\begin{aligned}\int_0^\infty \frac{dk^0}{2\pi} k_z \frac{d}{dz} G^<(k,z) + \frac{i}{2}\int_0^\infty \frac{dk^0}{2\pi} e^{-i\diamond}[m_a^2(z), G^<(k,z)] \\ + \frac{1}{4}\int_0^\infty \frac{dk^0}{2\pi} e^{-i\diamond}\left(\{\Pi_a^>, G_a^<\} - \{\Pi_a^<, G_a^>\}\right) \\ = \frac{1}{2}\int_0^\infty \frac{k^0}{2\pi} e^{-i\diamond}\left([\Pi_a^h, G_a^<] + [\Pi_a^<, G_a^h]\right).\end{aligned}$$

See Jiang Zhu talk

Gradient expansion except on δ fns

$$\begin{aligned}&\left[2k_z\frac{\partial}{\partial z} - \frac{dm^2(z)}{dz}\frac{\partial}{\partial k_z}\right]\frac{f_\phi(k,z)}{E_k} \\ &= -\int\frac{d^3\mathbf{p}}{(2\pi)^3}\int\frac{d^3\mathbf{p}'}{(2\pi)^3}F(k,z)\frac{1+f_\Phi(p,z)}{2E_p}\frac{1+f_\Phi(p',z)}{2E_{p'}} \\ &\quad\times(2\pi)^3\delta(E_k - E_p - E_{p'})\delta^2(\mathbf{k}_\perp - \mathbf{p}_\perp - \mathbf{p}'_\perp) + (\Delta p_z \leftrightarrow -\Delta p_z) \\ &+ \text{InverseProcess},\end{aligned}$$

mass variation

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IV. Outlook

- *The possibility of primordial gravitational waves generated from various particle physics dynamics has become an exciting area of exploration*
- *There exist many creative ideas for novel phenomena and dynamics that could have generated GW*
- *Realizing which, if any, of these ideas was realized in nature requires input from additional observables and performing the most rigorous theoretical calculations*
- *The electroweak phase transition provides a unique "laboratory" for testing our theoretical methods and ideas, with LHC and next generation collider measurements providing key input*

IV. Outlook

- The possibility of primordial gravitational waves generated from various particle physics dynamics has become an exciting area of exploration
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Recent progress with EFT, lattice, rigorous QFT…more challenges remain

谢谢！

Back Up Slides

EWPT & Perturbation Theory

Expansion parameter

$$g_{\rm eff} \equiv \frac{g^2 T}{\pi m_T(\varphi)}$$

Infrared sensitive near phase trans

SM lattice studies: *g_{eff} ~ 0.8 in vicinity of EWPT for m_H ~ 70 GeV* *

** Kajantie et al, NPB 466 (1996) 189; hep/lat 9510020 [see sec 10.1]*

Tunneling @ T>0: Take Aways

- *For a radiatively-induced barrier, a gauge-invariant perturbative computation of nucleation rate can be performed for S_3 to $\mathcal{O}(g^{-1/2})$ by adopting an appropriate power counting for T in the vicinity of T_{nuc}*

- *Abelian Higgs example generalizes to non-Abelian theories as well as other early universe phase transitions*

- *Remaining contributions to Γ_{nuc} beyond $\mathcal{O}(g^{-1/2})$ in S_3 and including long-distance (nucleation scale) contributions require other methods*

- *Assessing numerical reliability will require benchmarking with non-perturbative computations*

Tunneling @ T>0

Theoretical issues:

- *Radiatively-induced barrier (St'd Model) → gauge dependence*
  - *T = 0 Abelian Higgs: E. Weinberg & D. Metaxas: hep-ph/9507381*
  - *T=0 St'd Model: A. Andreassen, W. Frost, M. Schwartz 1408.0287*
  - *T > 0 Gauge theories:* ***recently solved in 2112.07452 (→ PRL) and 2112.08912***
- *Multi-field problem (still gauge invar issue)*
  - *Cosmotransitions: C. Wainwright 1109.4189*
  - *Espinosa method: J. R. Espinosa 1805.03680*

Challenges for Theory

Perturbation theory

- I.R. problem: poor convergence
- Thermal resummations
- Gauge Invariance (radiative barriers)
- RG invariance at T>0

Non-perturbative (I.R.)

- Computationally and labor intensive

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BSM proposals

Non-perturbative (I.R.)

- *Computationally and labor intensive*

Challenges for Theory

Perturbation theory

- *I.R. problem: poor convergence*
- *Thermal resummations*
- *Gauge Invariance (radiative barriers)*
- *RG invariance at $T>0$*

Non-perturbative (I.R.)

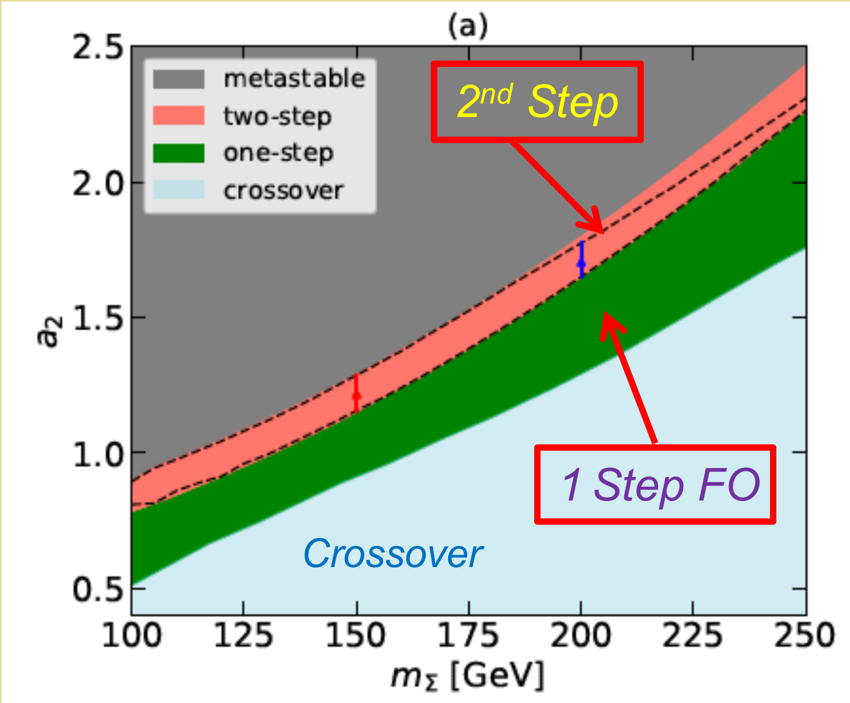
- *Computationally and labor intensive*

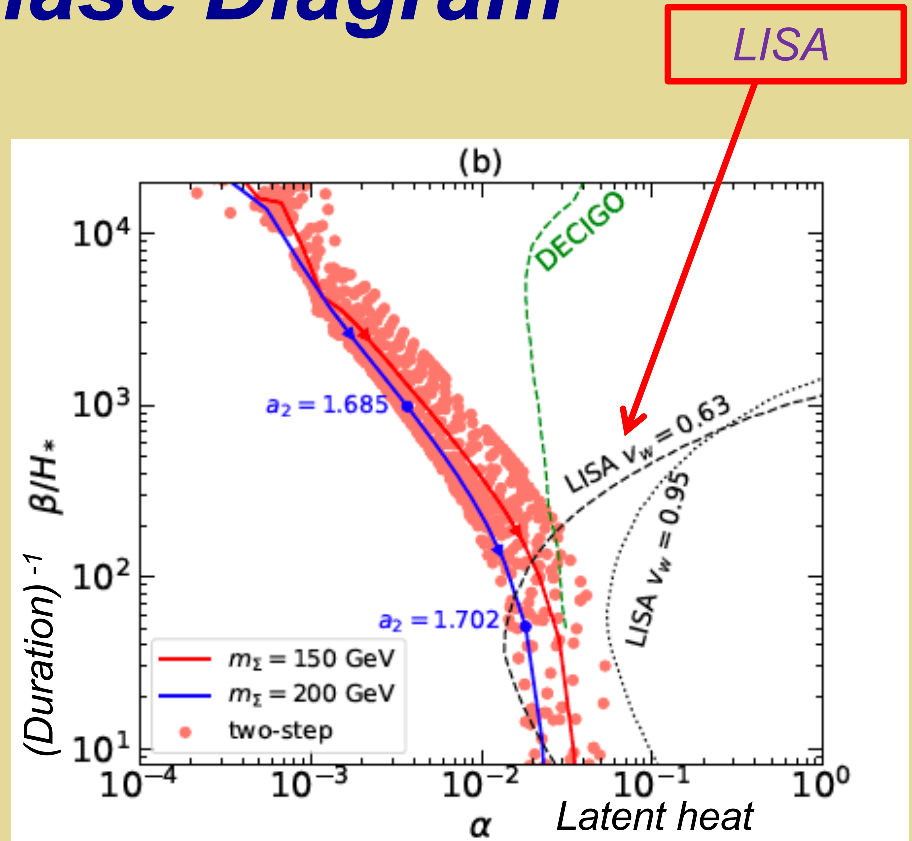
Dimensionally reduced 3D EFT at $T > 0$

BSM proposals

GW & EWPT Phase Diagram

Real triplet extension

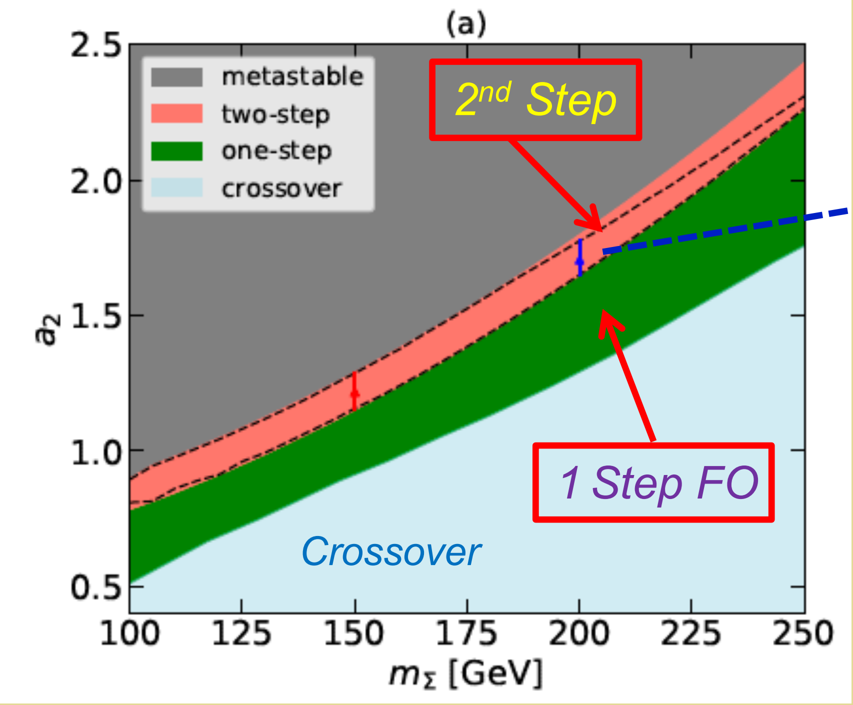


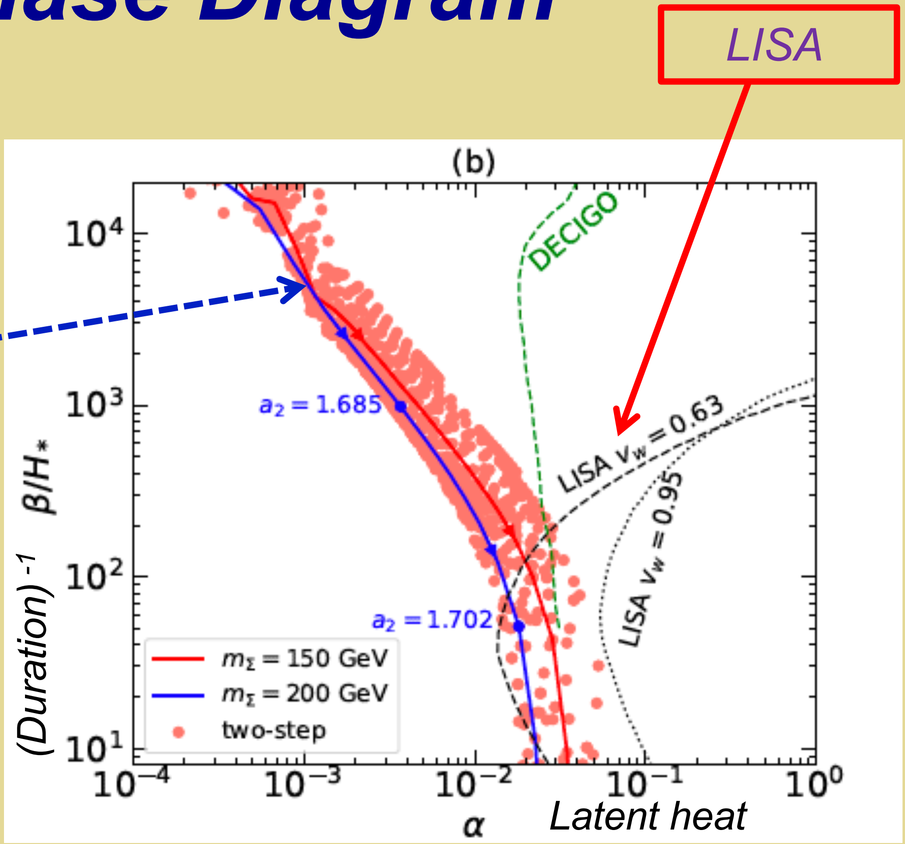


Friedrich, MJRM, Tenkanen, Tran 2203.05889

GW & EWPT Phase Diagram

Real triplet extension





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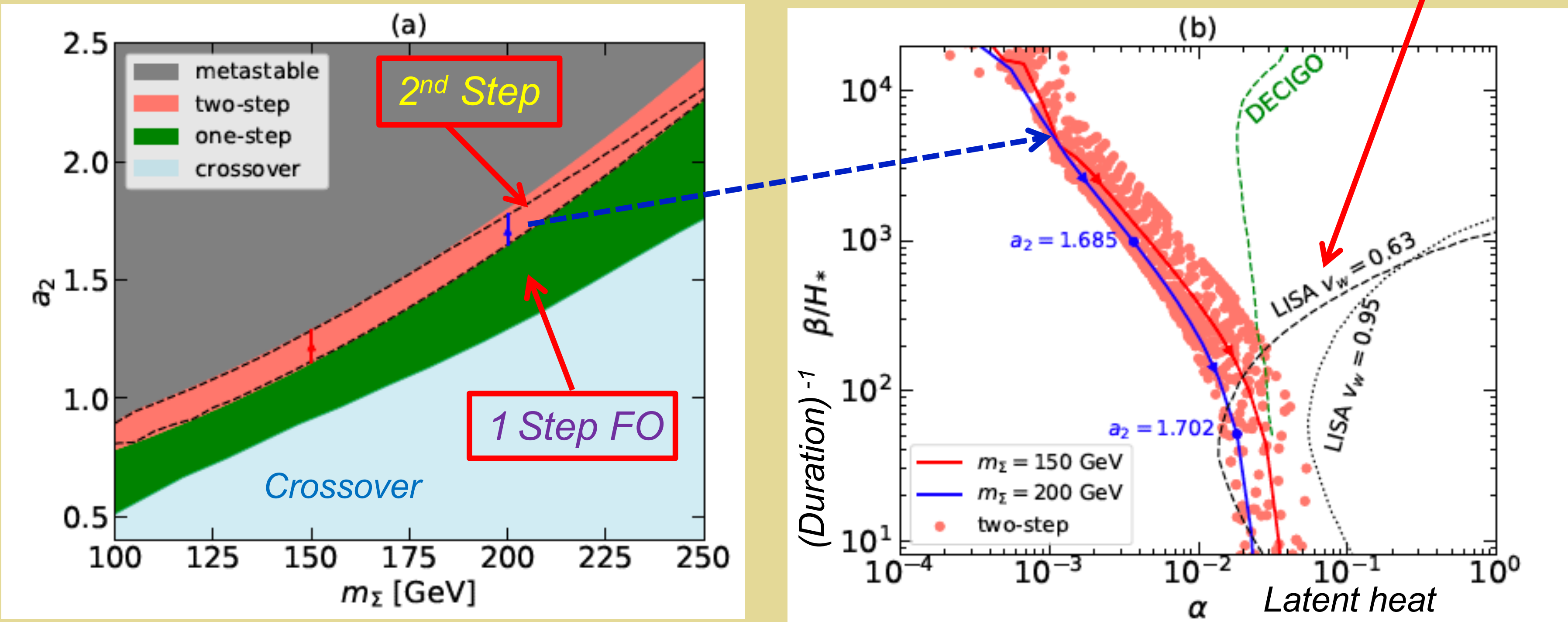
GW & EWPT Phase Diagram

Real triplet extension



- ***Single step transition: GW well outside LISA sensitivity***
- ***Second step of 2-step transition can be observable***
- ***Significant GW sensitivity to portal coupling***

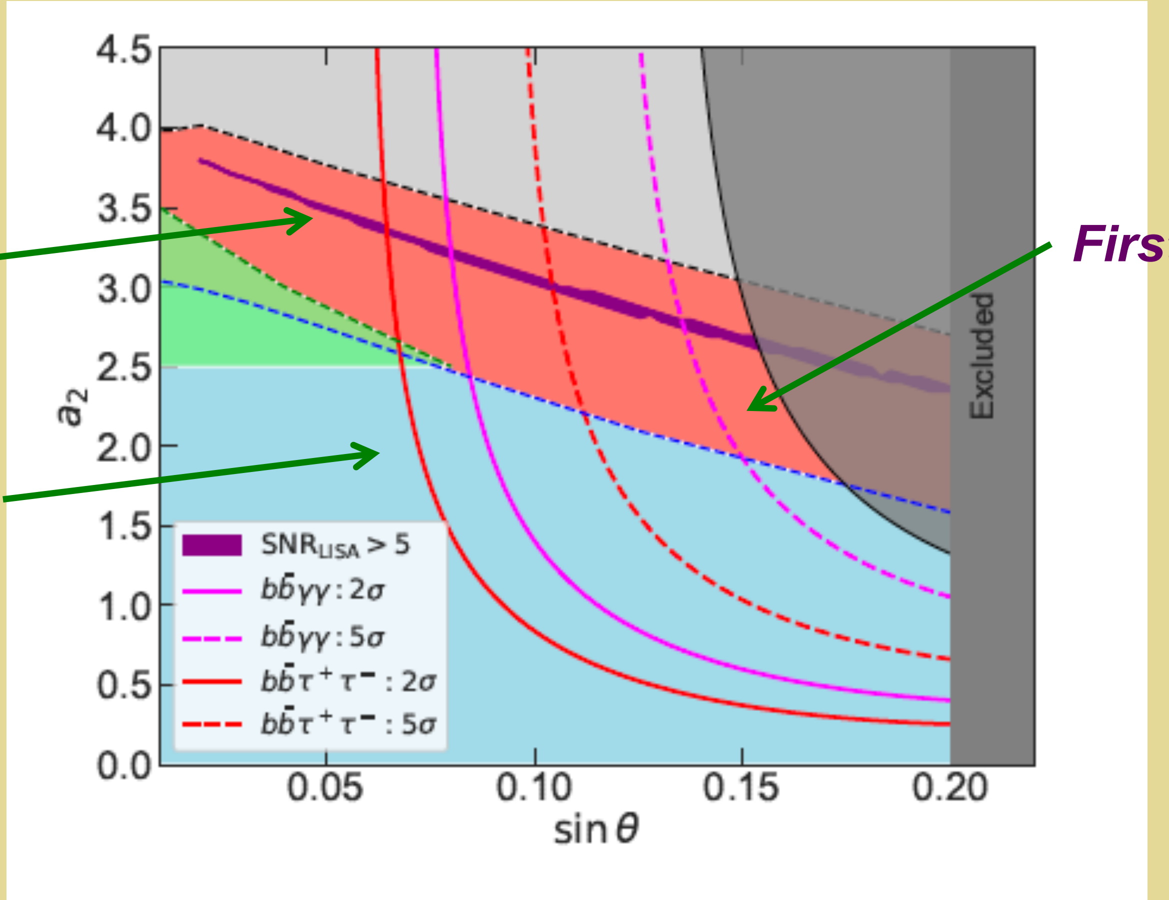
Friedrich, MJRM, Tenkanen, Tran 2203.05889

Gravitational Radiation & Colliders

LISA sensitivity

First order EWPT

HL LHC di-Higgs



MJRM, T.V.I. Tenkanen, V.Q. Tran, 2409.17554

S. Coleman, PRD 15 (1977) 2929

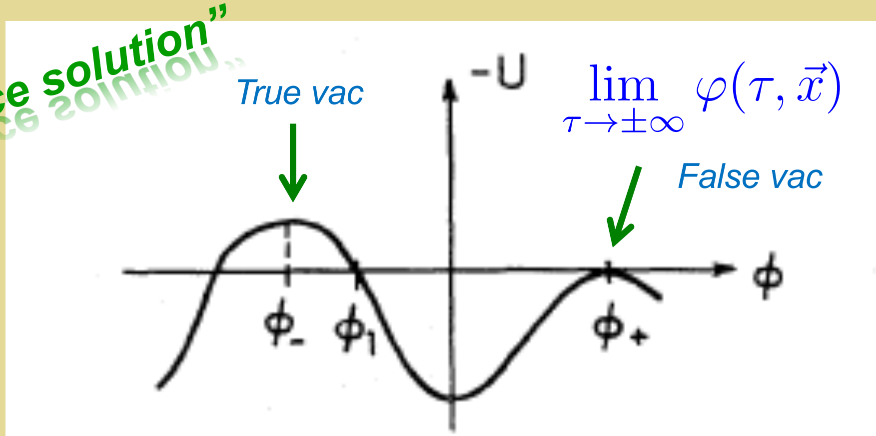
Tunneling @ T=0: Coleman

Scalar Quantum Field Theory

Rotational symmetry

$$\rho^2 \equiv \tau^2 + |\vec{x}|^2$$

"Bounce solution"



$$\frac{d^2\varphi}{d\rho^2} + \frac{3}{\rho}\frac{d\varphi}{d\rho} = U'(\varphi)$$

Ln Γ

Path: minimize S_E

Friction term

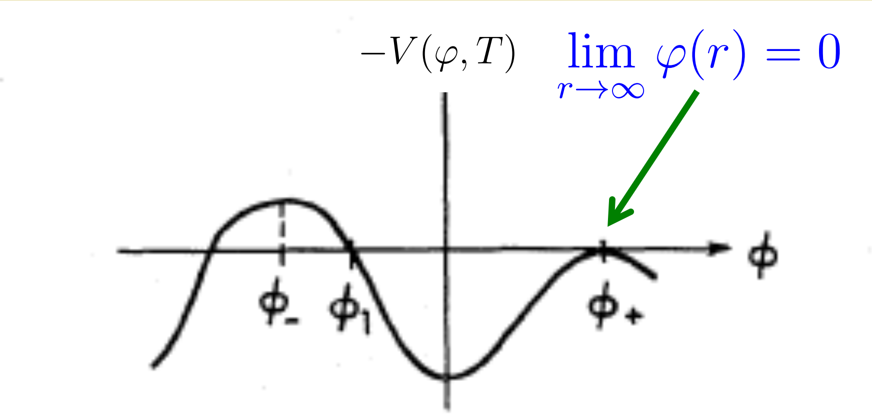
$$S_E = \int d\tau d^3x \left\{ \frac{1}{2}(\partial_\tau \varphi)^2 + \frac{1}{2}(\vec{\nabla}\varphi)^2 + U(\varphi) \right\}$$

Tunneling @ T>0

Scalar Quantum Field Theory

Tunneling rate / unit volume:

$$\Gamma = A e^{-\beta S_3/\hbar}\,[1+\mathcal{O}(\hbar)]$$



$$\frac{d^2\varphi}{dr^2} + \frac{2}{r}\frac{d\varphi}{dr} = V'(\varphi,T)$$

Exponent in Γ

Path: minimize S_E

Friction term

$$S_3 = \int d^3x \left\{ \frac{1}{2}(\vec{\nabla}\varphi)^2 + V(\varphi,T) \right\}$$

$$A \sim \mathcal{O}(1) \times T^4$$

Inputs from Thermal QFT: EFTs

Thermodynamics

- *Phase diagram: first order EWPT?*
- *Latent heat: GW*

EFT 1

Dynamics

EFT 2

- *Nucleation rate: transition occurs? T_N ? Transition duration (GW) ?*

- *EW sphaleron rate: baryon number preserved?*

EFT 3

DR 3dEFT: Scales

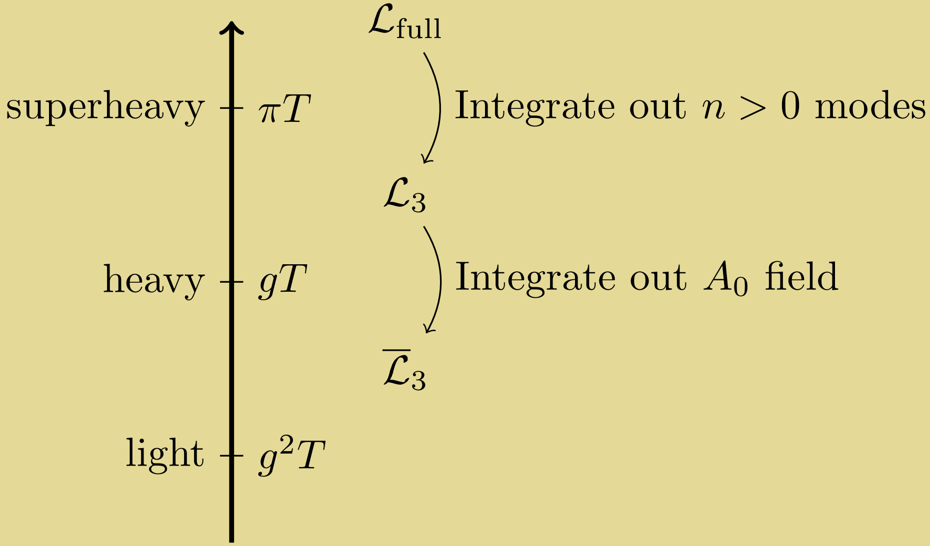
- πT — *Non-zero Matsubara modes*
- M_{BSM} — *BSM mass scale: can be > or < πT*
- $g\,T$ — *Thermal masses*
- M_{NUC} — *Nucleation scale ~ $1/r_{bubble}$*
- $g^2\,T$ — *Light scale*

EFT 2

EFT 1

Thermal Effective Field Theory: EFT 1

Meeting ground: 3-D high-T effective theory

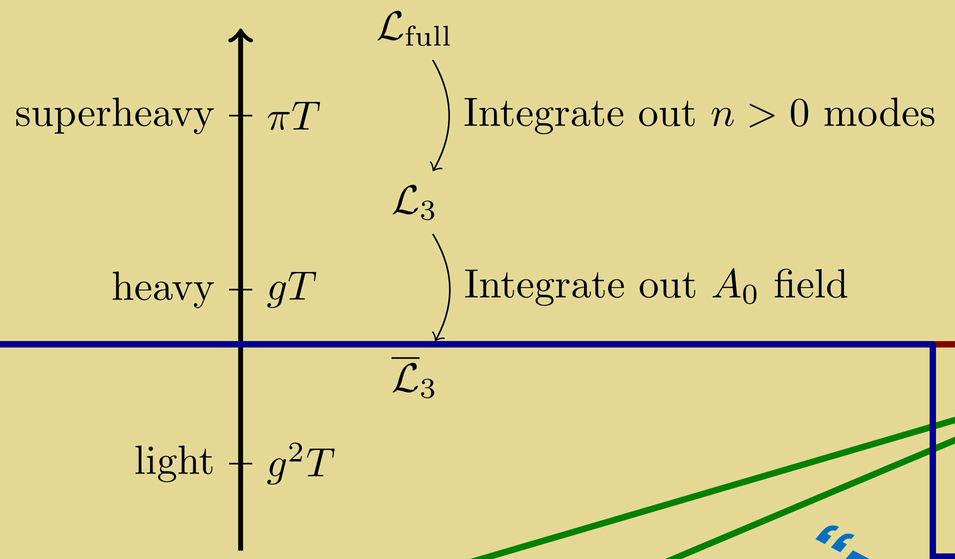


$$V(\phi) = \bar{\mu}^2_{\phi,3}\phi^\dagger\phi + \bar{\lambda}_3(\phi^\dagger\phi)^2 \quad + V(\Phi) + V(\phi, \Phi)_{portal}$$

Non-dynamical BSM scalars — *Dynamical BSM scalars*

EFT 1-A: Integrate Out All BSM Fields

Meeting ground: 3-D high-T effective theory



$$V(\phi) = \bar{\mu}^2_{\phi,3}\phi^\dagger\phi + \bar{\lambda}_3(\phi^\dagger\phi)^2$$

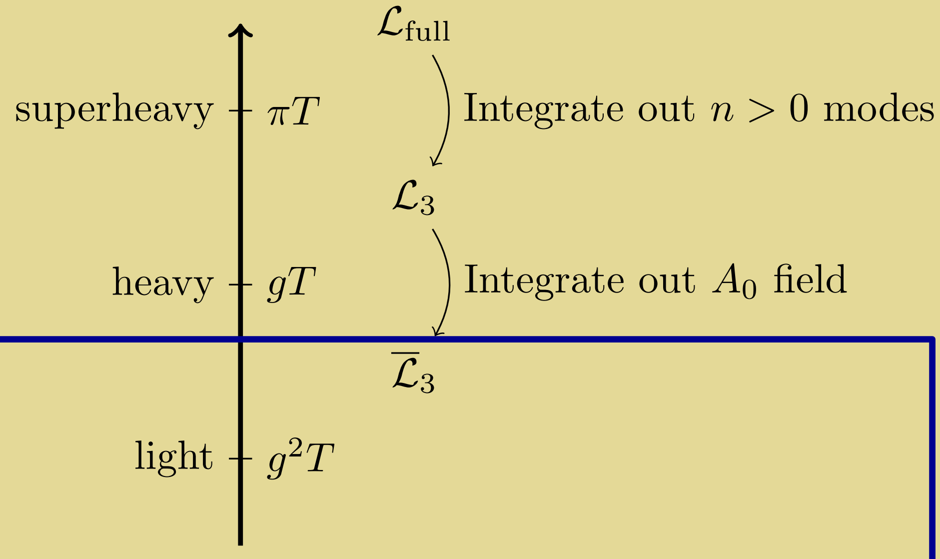
- *Assume BSM fields are “heavy” or “supeheavy” : integrate out*
- *Effective “SM-like” theory parameters are functions of BSM parameters*
- *Use existing lattice computations for SM-like effective theory & matching onto full theory to determine FOEWPT-viable parameter space regions*

“Repurpose” lattice results

Lattice simulations exist (e.g., Kajantie et al ’95)

EFT 1-A: Integrate Out All BSM Fields

Meeting ground: 3-D high-T effective theory



$$V(\phi) = \bar{\mu}^2_{\phi,3}\phi^\dagger\phi + \bar{\lambda}_3(\phi^\dagger\phi)^2$$

When $\mathcal{L}_{full}$ contains BSM interactions, λ_3 and $\mu_{\phi,3}$ can accommodate first order EWPT and m_h =125 GeV

Lattice simulations exist (e.g., Kajantie et al '95)

Tunneling @ T>0: Gravitational Waves

Amplitude & frequency: latent heat & intrinsic time scale

Normalized latent heat

$$\Delta Q = \Delta F + T\Delta S$$
$$S = -\partial F/\partial T$$
$$F \approx V$$

$$\Delta Q \approx \Delta V - T\partial\Delta V/\partial T$$

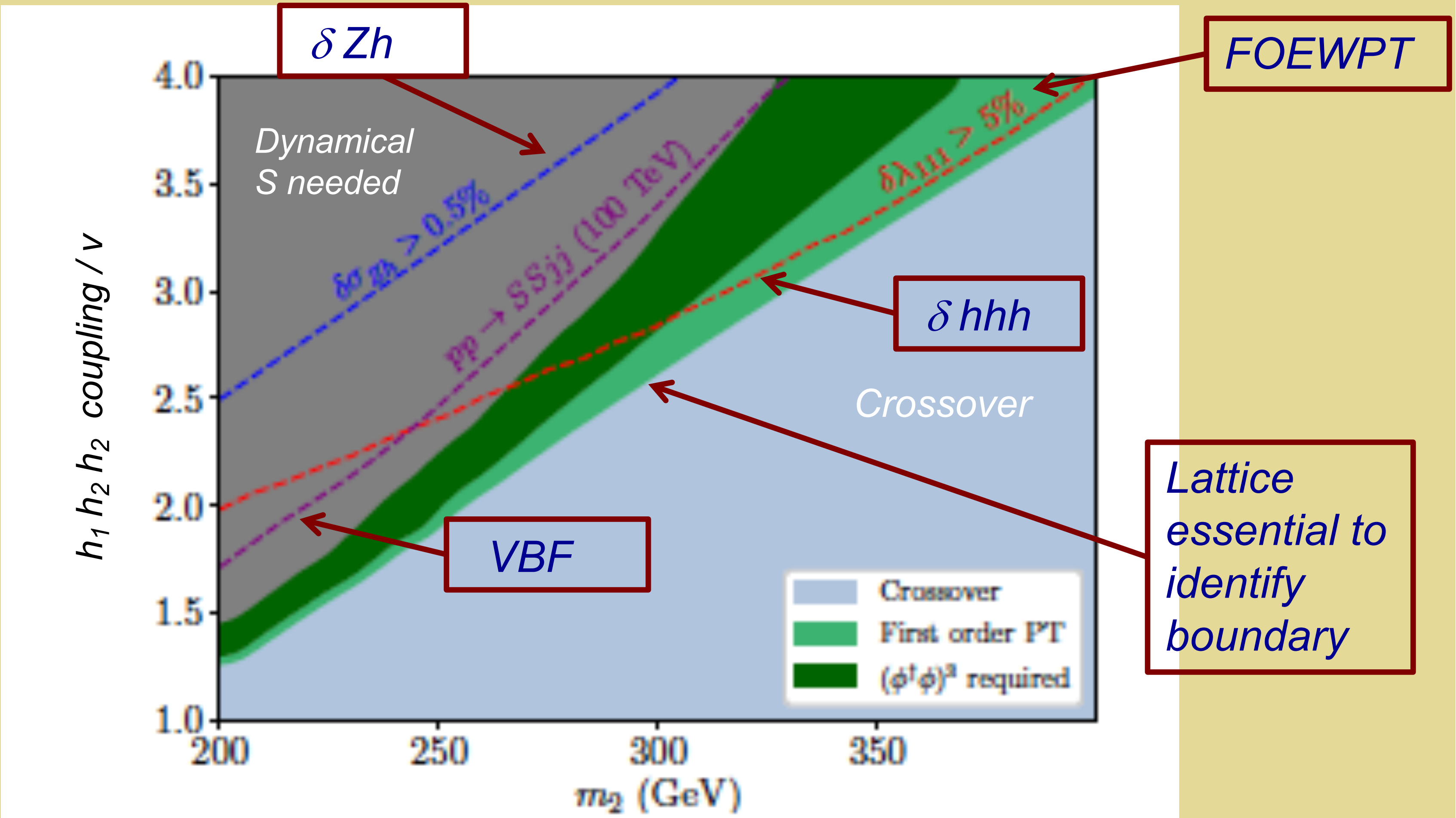
$$\alpha = \frac{30\Delta q}{\pi^2 g_* T^4}$$

Time scale

$$\frac{\beta}{H_*} = T\frac{d}{dT}\frac{S_3}{T}$$

How Reliable?

Non-Dynamcial Real Singlet & EWPT: Probes



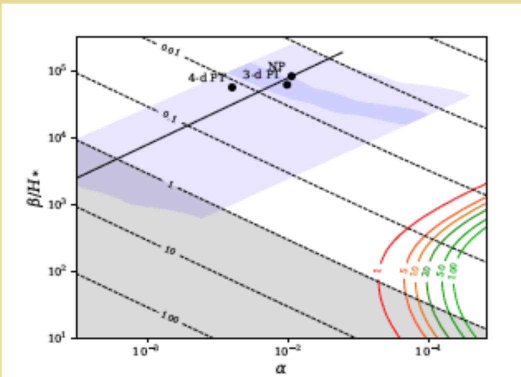
- One-step
- Non-perturbative

Gould, Kozaczuk, Niemi, R-M, Tenkanen, Weir 1903.11604

Non-Dynamical Real Singlet: Lattice vs PT

Benchmark pert theory

	T_c/GeV	T_n/GeV	$\alpha(T_c)$	β/H_*
NP	140.4	140.2	0.011	8.20×10^4
3-d PT	140.4	140.0	0.010	6.11×10^4
4-d PT	131.0	130.7	0.004	5.59×10^4

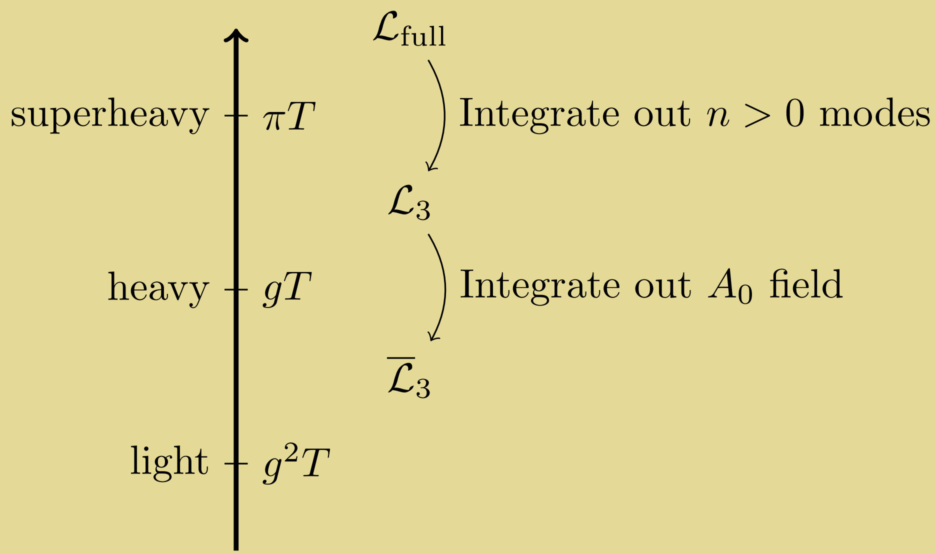


Gould, Kozaczuk, Niemi, R-M, Tenkanen, Weir 1903.11604

- *One-step*
- *Non-perturbative*

Dynamical Real Singlet

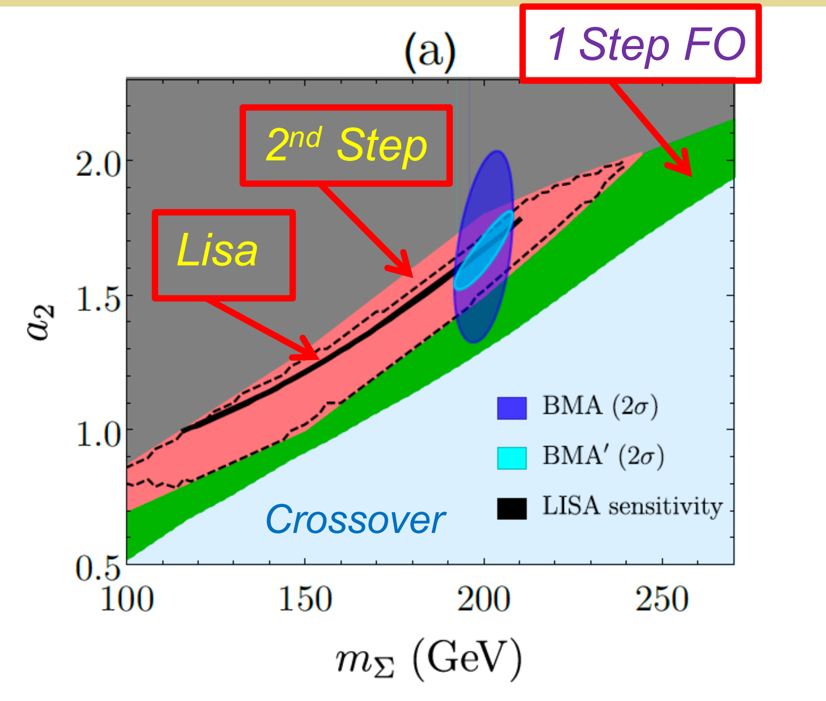
Meeting ground: 3-D high-T effective theory



$$V(\phi) = \bar{\mu}^2_{\phi,3}\phi^\dagger\phi + \bar{\lambda}_3(\phi^\dagger\phi)^2 \qquad + V(\Phi) + V(\phi, \Phi)_{portal}$$

Non-dynamical BSM scalars — *Dynamical BSM scalars*

GW, Collider & EWPT Phase Diagram



How combine sensitivities ?

$$\mathrm{SNR} = \left\{ \mathcal{T} \int_{f_{\min}}^{f_{\max}} df \left[\frac{\Omega_{\mathrm{GW}}(f)}{\Omega_{\mathrm{sens}}(f)} \right]^2 \right\}^{1/2}$$

- *Gaussian significance (N_σ)*

BMA: m_Σ + h → $\gamma\gamma$

BMA' : BMA + Σ^0 → ZZ

Friedrich, MJRM, Tenkanen, Tran 2203.05889

Collider Signatures (Model-Dep)

- ***Thermal Γ (h → $\gamma\gamma$)***
- ***Higgs signal strengths***
- ***$\delta\sigma$ (e+e- → Zh)***
- ***Higgs self-coupling***
- ***Exotic Decays***
- ***Single ϕ production***

MJRM: 1912.07189

T_{EW} → Scale for Colliders & GW probes

High-T SM Effective Potential

$$V(h,T)_{\rm SM} = D(T^2 - T_0^2)\, h^2 + \lambda\, h^4 \;\; + \ldots$$

T_0 ~ 140 GeV **$\equiv$ T_{EW}**