Numerical simulations on vacuum decay

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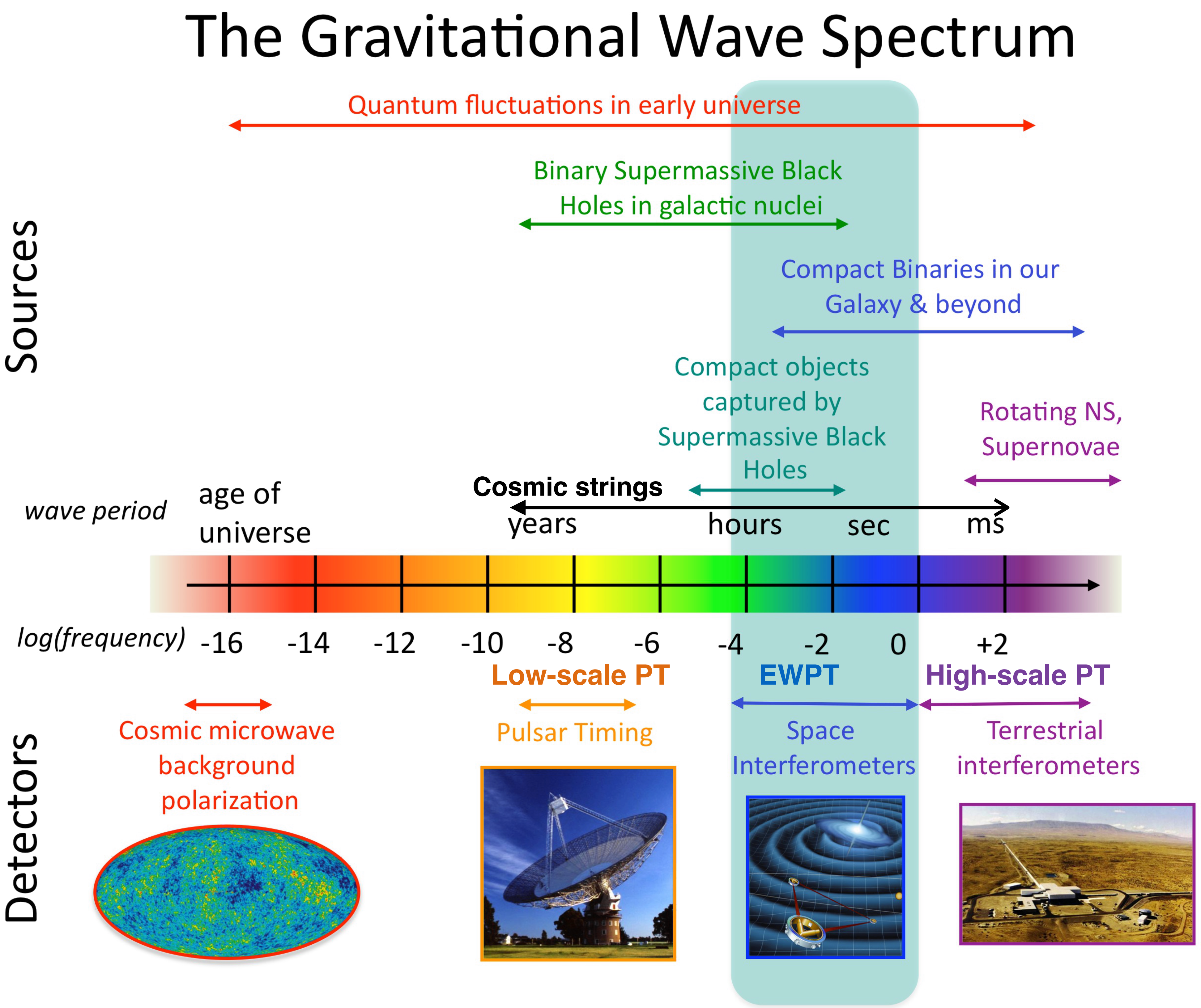
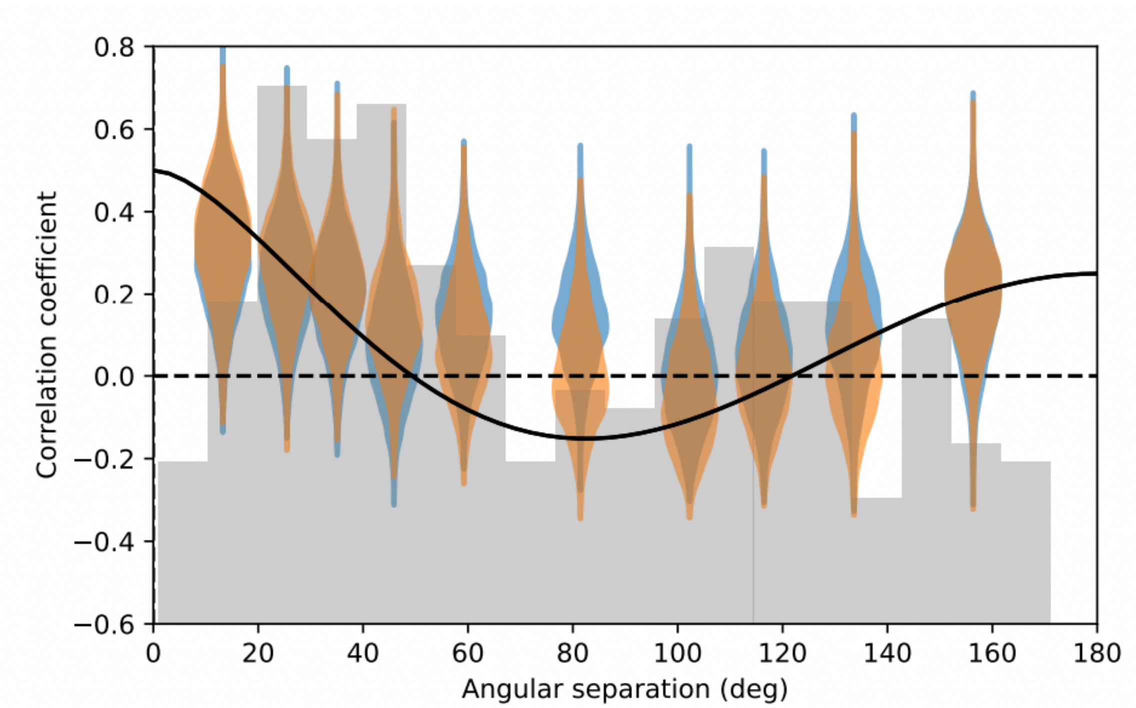
2025年北京粒子物理与宇宙学：早期宇宙、引力波模板、对撞机唯象研讨会（BPCS 2025）
28th Septemper 2025

Contents

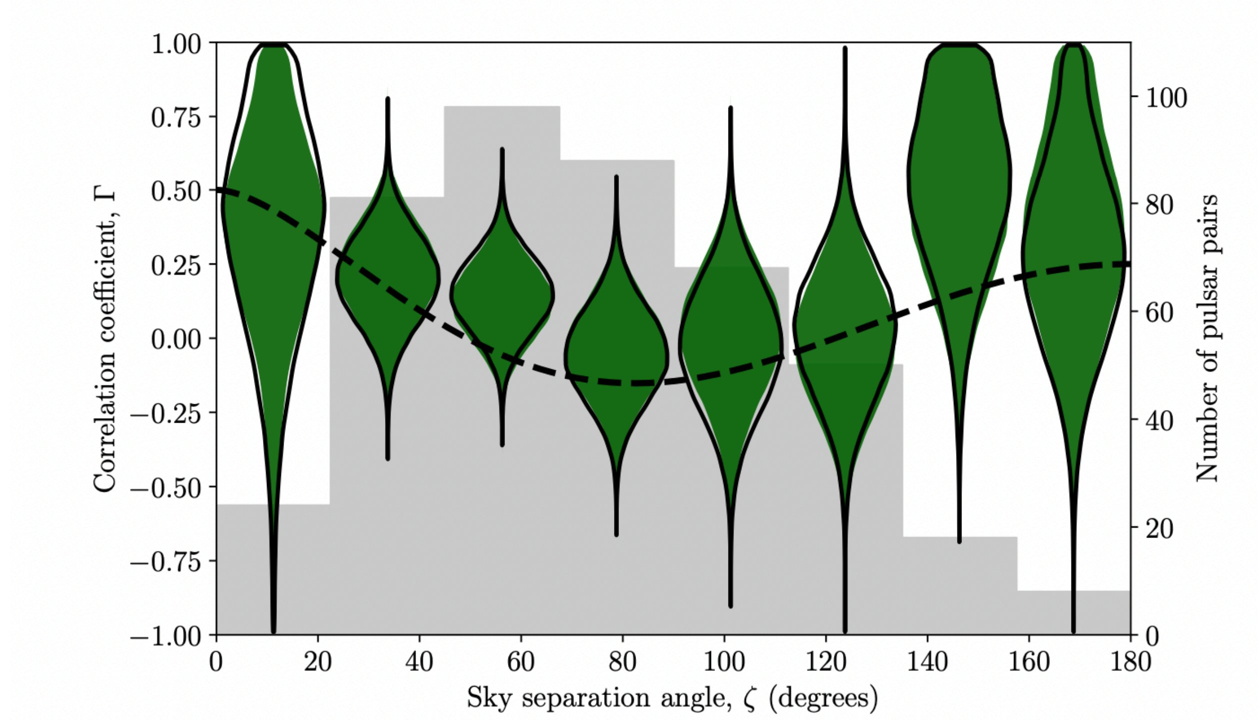
Stochastic gravitational wave background and New physics



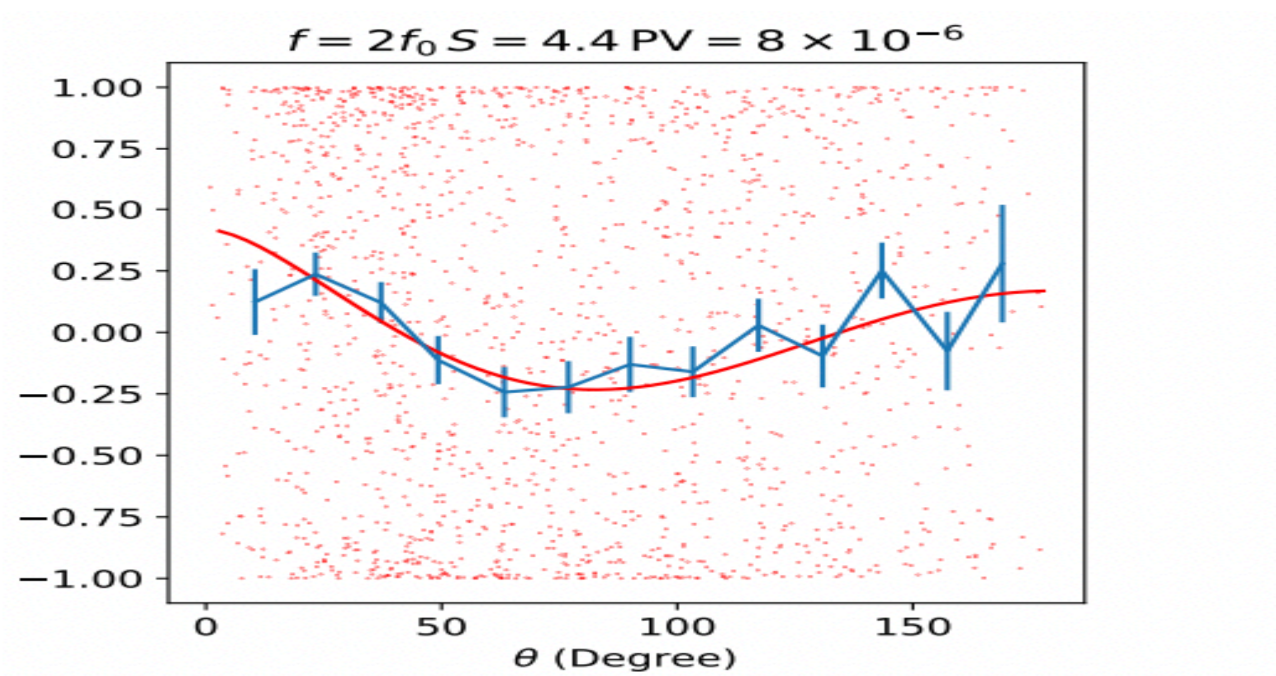
New dataset from PTAs



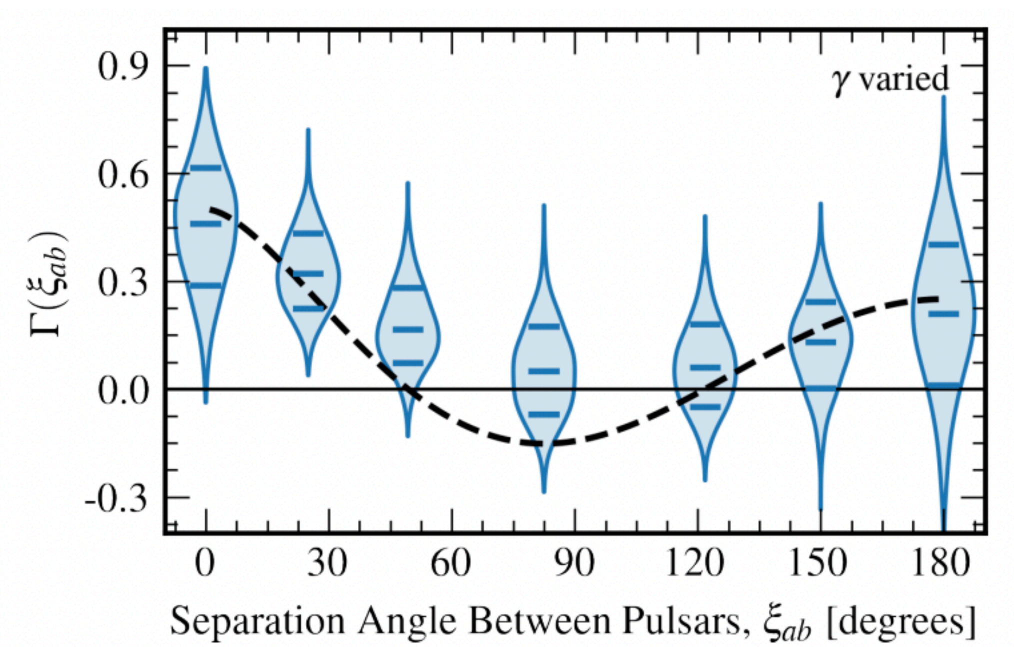
EPTA,2306.16214



PPTA,2306.16215

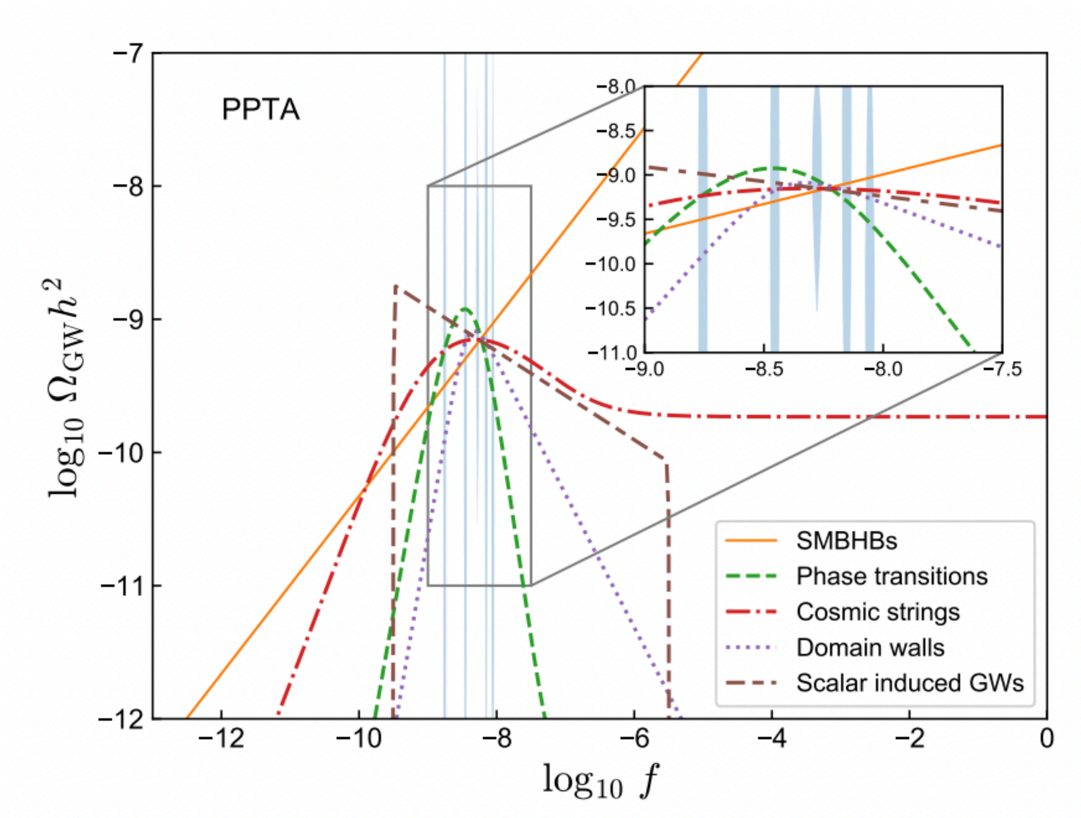


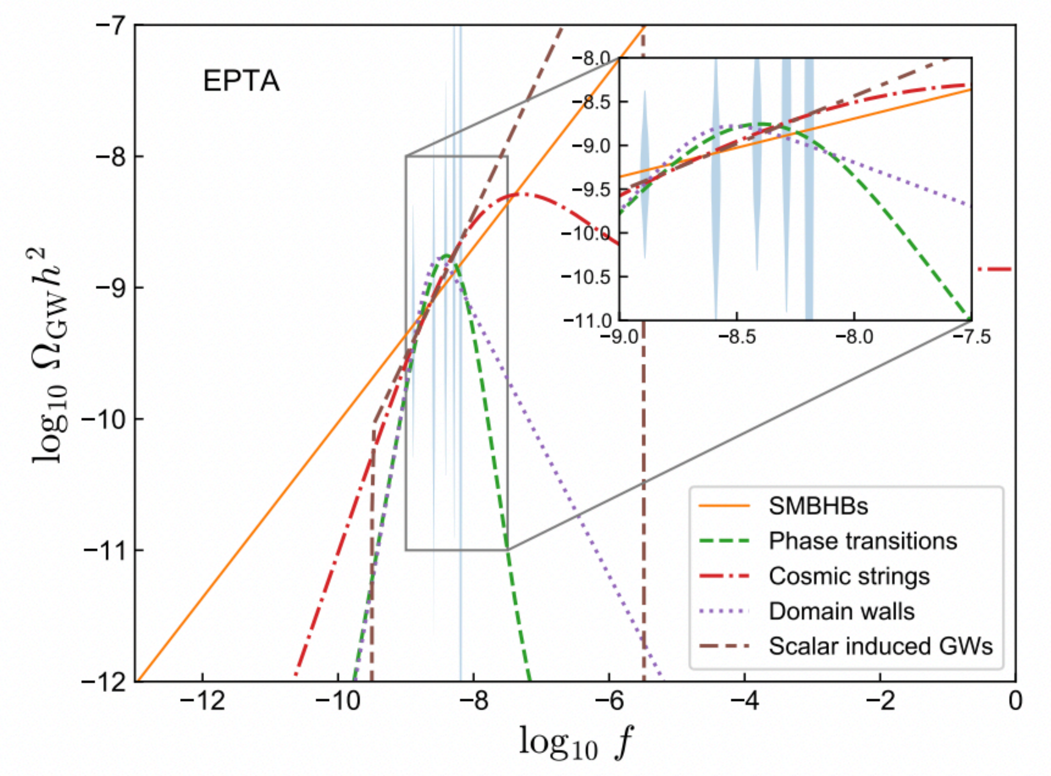
CPTA ,2306.16216

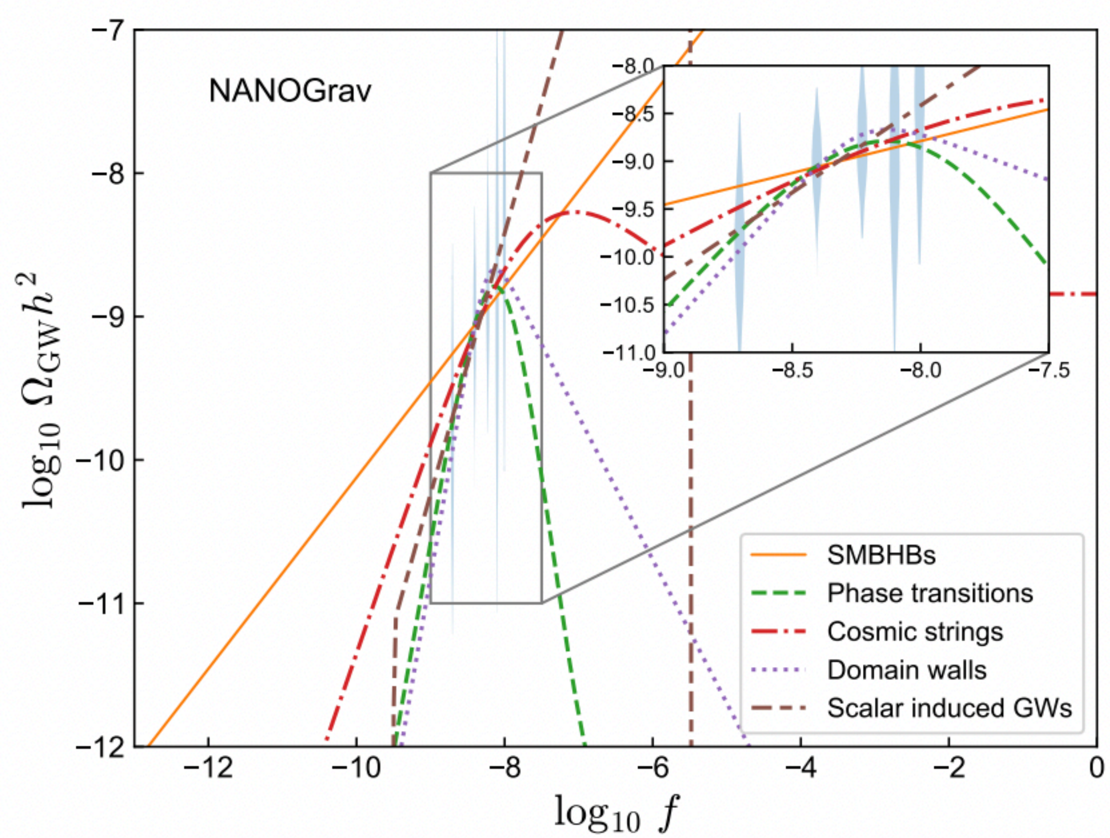


NANOGrav,2306.16213

Gravitational wave sources for Pulsar Timing Arrays

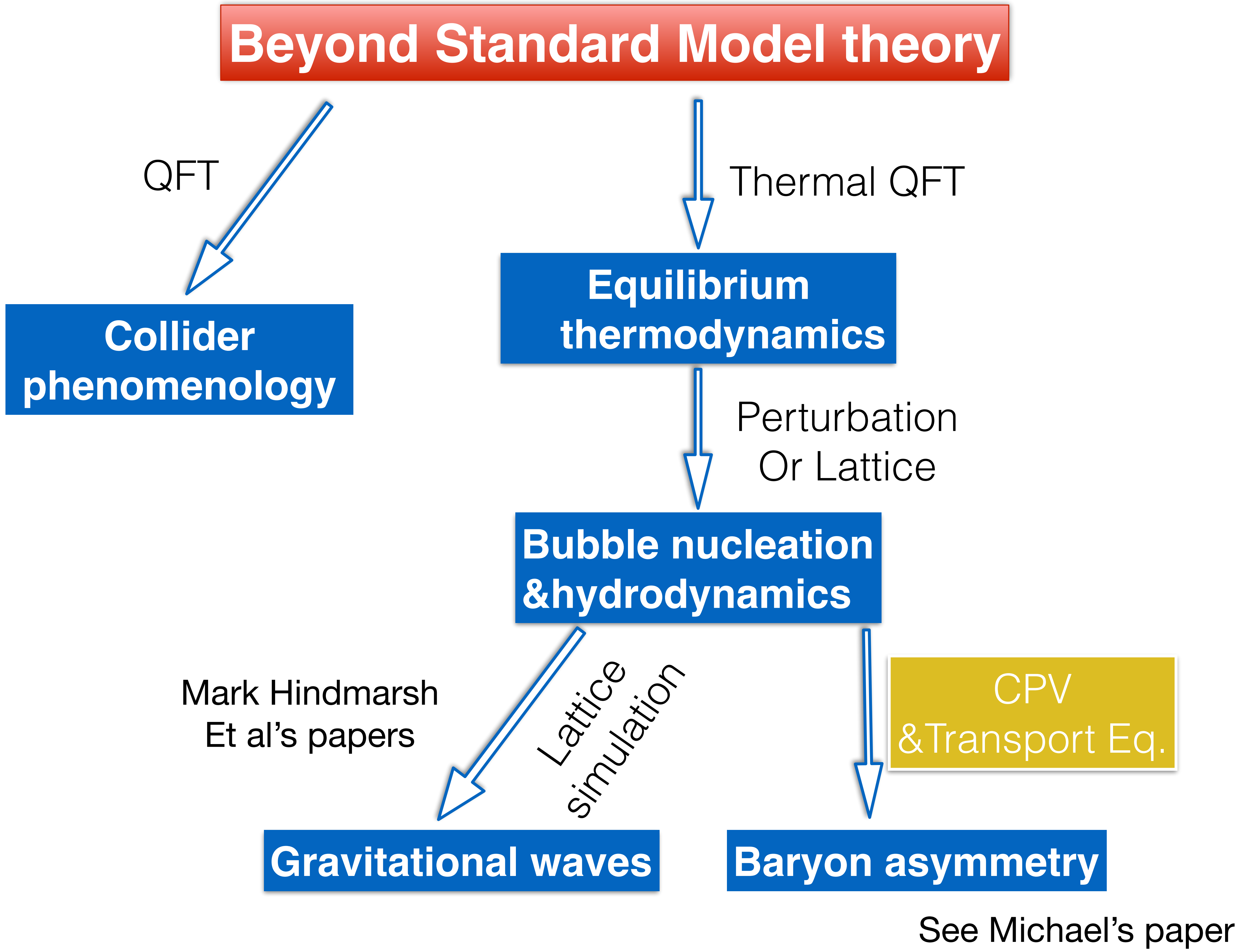






Bian,Ge,Shu,Wang,Yang,Zong,2307.02376

Particle cosmology related with electroweak 1st PT



Methods for PT dynamics study

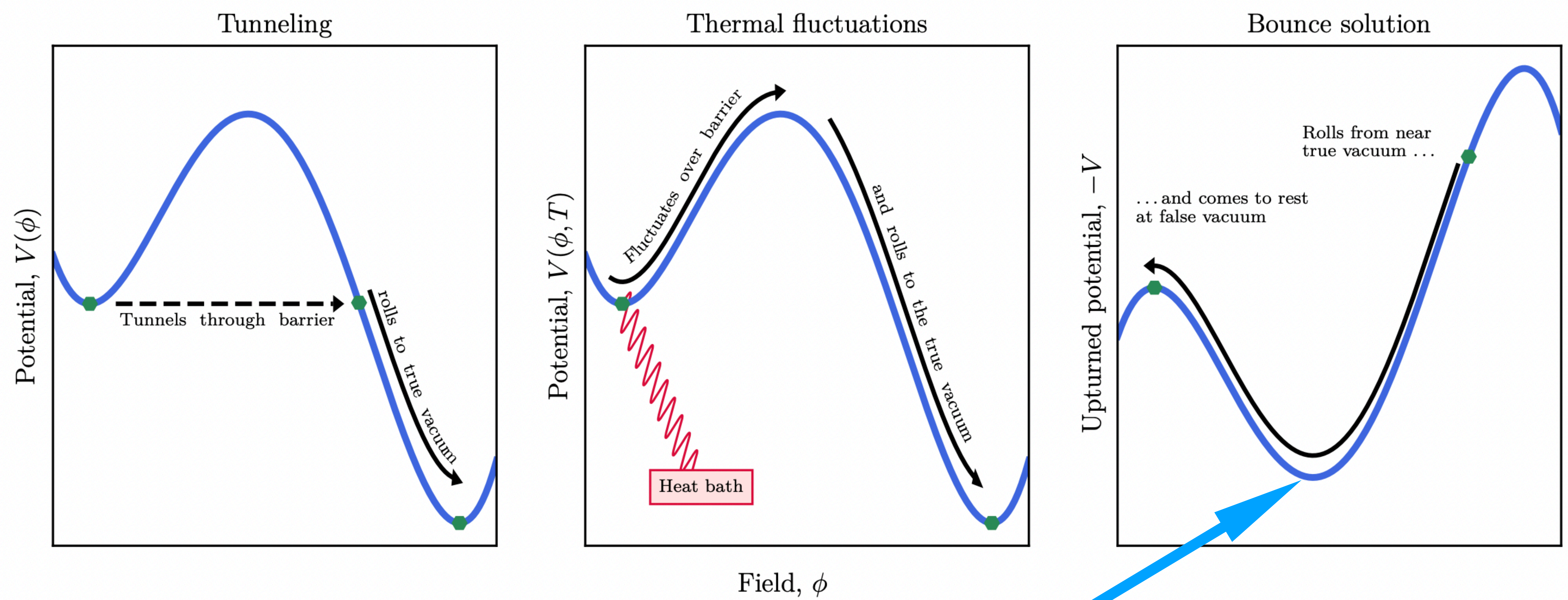
$\mathscr{L}_{\mathbf{4d}} \longrightarrow \mathscr{L}_{\mathbf{3d}} \longrightarrow \mathscr{L}_{3d}^{Lat}$

Perturbative 4d approach	Perturbative 3d approach	Lattice simulation
$\mathscr{L}_{\mathbf{4d}}$ ↓ V_{4d}^{eff} ↓	$\mathscr{L}_{\mathbf{3d}}$ ↓ V_{3d}^{eff} ↓	$\mathscr{L}_{3d}^{Lat}$ ↓ Monte Carlo simulation ↓

Thermodynamics of PT

See Philipp and Michael's talk

Vacuum decay at finite temperature



$$p(t;T) \equiv \Gamma/V = |A(T)|e^{-B(T)/T}$$

$$A(T) \simeq T^4 \left(\frac{S_3[\bar{\phi}(T)]}{2\pi T}\right)^{\frac{3}{2}}$$

Peter Athron , Csaba Balázs, Andrew Fowlie, Lachlan Morris , Lei Wu, 2305.02357

GW parameters and FOPT

Bounce solution

$$S_3(T) = \int 4\pi r^2 dr \left[\frac{1}{2}\left(\frac{d\phi_b}{dr}\right)^2 + V(\phi_b, T)\right]$$

$$\lim_{r\to\infty} \phi_b = 0\ , \qquad \frac{d\phi_b}{dr}|_{r=0} = 0$$

Bubble nucleation

$$\Gamma \approx A(T)e^{-S_3/T} \sim 1$$

PT strength

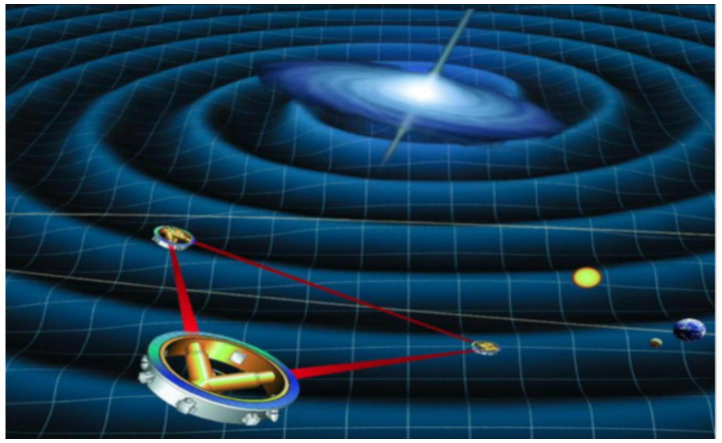
$$\alpha \equiv \frac{1}{\rho_r}\left(\Delta V_{\rm eff}(\phi, T) - \frac{T}{4}\Delta\frac{\partial V_{\rm eff}(\phi, T)}{\partial T}\right)$$

Phase transition inverse duration

$$\frac{\beta}{H_n} = T\frac{d(S_3(T)/T)}{dT}|_{T=T_n}$$

New physics & Gravitation waves

PTA,LIGO,LISA,TianQin,Taiji,⋯



Particle physics model

↓

PT parameters

Effective action $\to \beta,\ H_*$

Energy budget $\to \alpha,\ \kappa(\alpha, v_w)$

Bubble wall dynamics $\to v_w$

→

GW power spectrum

Numerical simulations $\to$

$h^2\Omega_{GW}(f; H_*, \alpha, \beta, v_w)$

→

LISA sensitivity

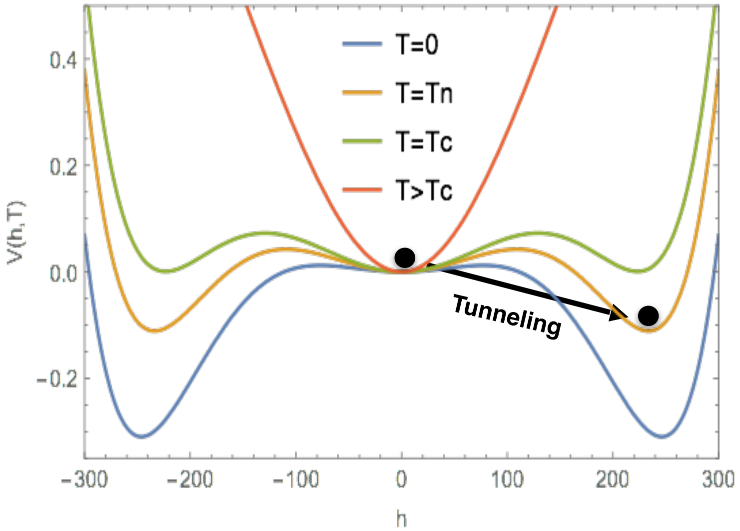
Configuration + noise level $\to$

$h^2\Omega_{sens}(f)$

↓

Signal-to-noise ratio

Finite-T effective potential



Chiara Caprini *et al* JCAP03(2020)024

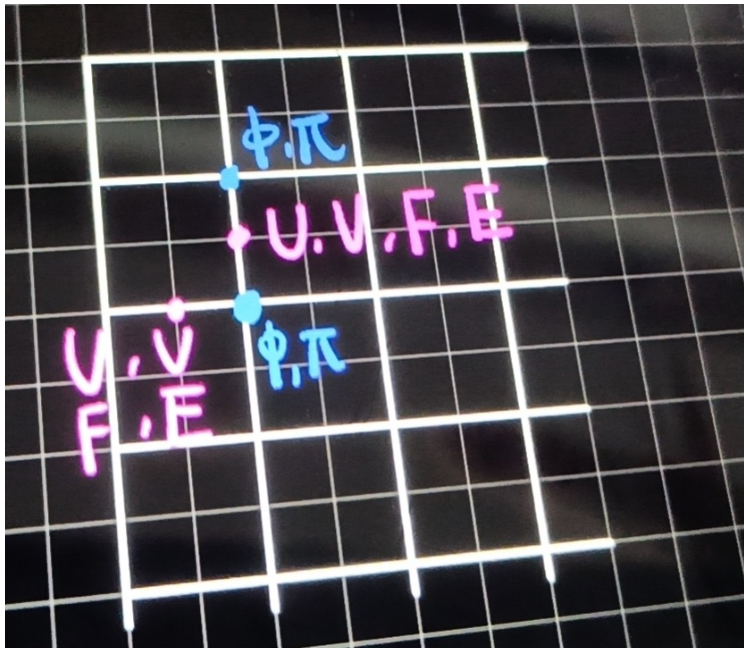
Lattice electroweak theory

$\Phi(t, x)$: Higgs field doublet defined on sites;

$U_i(t, x)$ and $V_i(t, x)$: SU(2) and U(1) link fields, defined on the link between the neighboring sites x and x + i , $\Phi(t, x)$, $U_i(t, x)$ and $V_i(t, x)$ are defined at time steps t + Δt, t + 2Δt, . . .;

Conjugate momentum fields: $\Pi(t+\Delta t/2, x)$, $F(t+\Delta t/2, x)$ and $E(t+\Delta t/2, x)$, are defined at time steps t + Δt/2, t + 3Δt/2.

$$U_i(t,x) = \exp\left(-\frac{i}{2}g\Delta x\sigma^a W_i^a\right)$$
$$U_0(t,x) = \exp\left(-\frac{i}{2}g\Delta t\sigma^a W_0^a\right)$$
$$V_i(t,x) = \exp\left(-\frac{i}{2}g\Delta x B_i\right)$$
$$V_0(t,x) = \exp\left(-\frac{i}{2}g\Delta t B_0\right).$$



$$D_i\Phi = \frac{1}{\Delta x}\left[U_i(t,x)V_i(t,x)\Phi(t,x+i) - \Phi(t,x)\right]$$
$$D_0\Phi = \frac{1}{\Delta t}\left[U_0(t,x)V_0(t,x)\Phi(t+\Delta t,x) - \Phi(t,x)\right].$$

$$\Phi(t+\Delta t,x) = \Phi(t,x) + \Delta t\Pi(t+\Delta t/2,x)$$
$$V_i(t+\Delta t,x) = \frac{1}{2}g'\Delta x\Delta t E_i(t+\Delta t/2,x)V_i(t,x)$$
$$U_i(t+\Delta t,x) = g\Delta x\Delta t F_i(t+\Delta t/2,x)U_i(t,x),$$

Temporal gauge
$U_0(t, x) = I_2$, $V_0(t, x) = 1$

leapfrog

Di, Wang, Zhou, **Bian***, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

1st order EWPT simulation-quantum tunneling

Field basis equation of motion

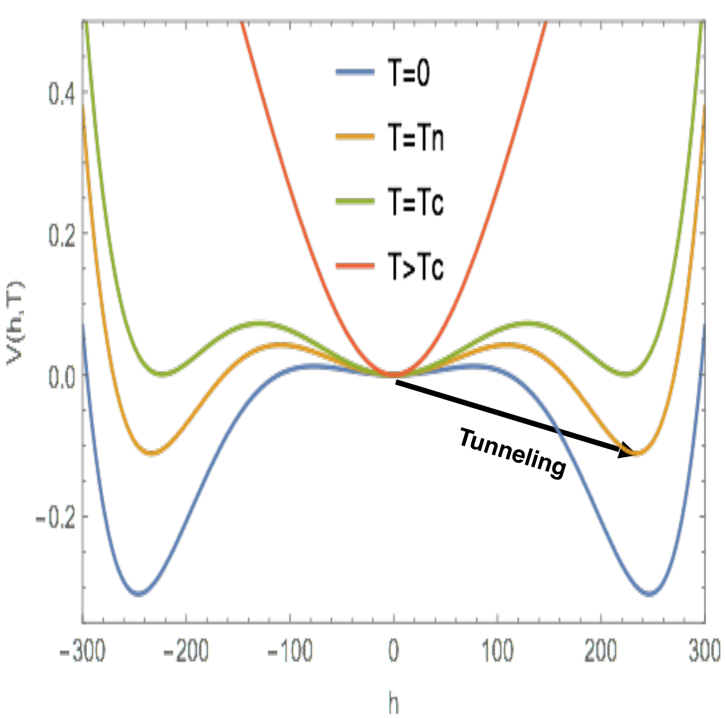
$$
\begin{aligned}
\partial_0^2 \Phi &= D_i D_i \Phi - \frac{dV(\Phi)}{d\Phi}, \\
\partial_0^2 B_i &= -\partial_j B_{ij} + g' \,\mathrm{Im}[\Phi^\dagger D_i \Phi], \\
\partial_0^2 W_i^a &= -\partial_k W_{ik}^a - g\,\epsilon^{abc} W_k^b W_{ik}^c + g\,\mathrm{Im}[\Phi^\dagger \sigma^a D_i \Phi]\,,
\end{aligned}
$$

$$
\begin{aligned}
\partial_0 \partial_j B_j - g' \,\mathrm{Im}[\Phi^\dagger \partial_0 \Phi] &= 0, \\
\partial_0 \partial_j W_j^a + g\,\epsilon^{abc} W_j^b \partial_0 W_j^c - g\,\mathrm{Im}[\Phi^\dagger \sigma^a \partial_0 \Phi] &= 0.
\end{aligned}
$$

Lattice implementation

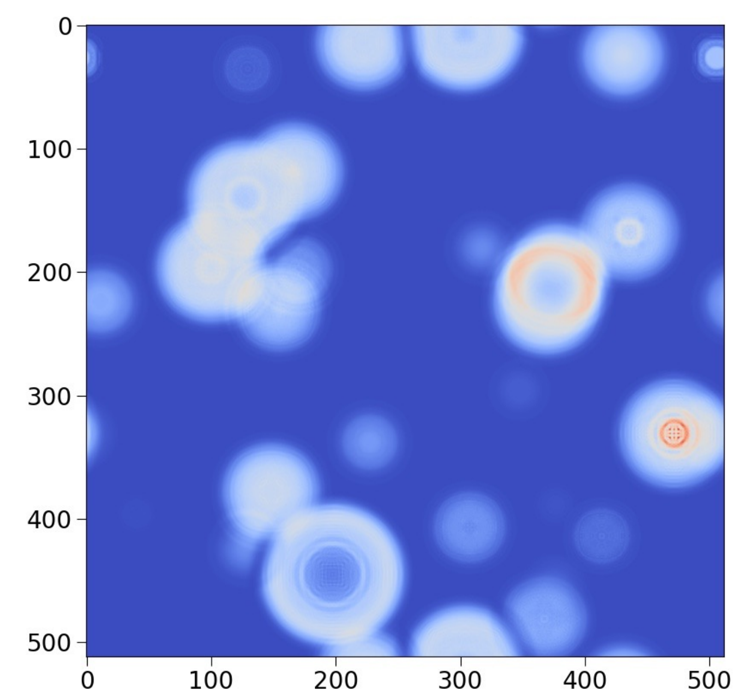
$$
\begin{aligned}
\Pi(t+\Delta t/2, x) =& \Pi(t-\Delta t/2, x) + \Delta t \Big\{ \frac{1}{\Delta x^2} \sum_i \big[U_i(t,x) V_i(t,x) \Phi(t, x+i) \\
& - 2\Phi(t,x) + U_i^\dagger(t, x-i) V_i^\dagger(t, x-i) \Phi(t, x-i) \big] - \frac{\partial U}{\partial \Phi^\dagger} \Big\} \\
\mathrm{Im}[E_k(t+\Delta t/2, x)] =& \mathrm{Im}[E_k(t-\Delta t/2, x)] + \Delta t \Big\{ \frac{g'}{\Delta x} \mathrm{Im}[\Phi^\dagger(t, x+k) U_k^\dagger(t,x) V_k^\dagger(t,x) \Phi(t,x)] \\
& - \frac{2}{g' \Delta x^3} \sum_i \mathrm{Im}[V_k(t,x) V_i(t, x+k) V_k^\dagger(t, x+i) V_i^\dagger(t,x) \\
& + V_i(t, x-i) V_k(t,x) V_i^\dagger(t, x+k-i) V_k^\dagger(t, x-i)] \Big\} \\
\mathrm{Tr}[i\sigma^m F_k(t+\Delta t/2, x)] =& \mathrm{Tr}[i\sigma^m F_k(t-\Delta t/2, x)] + \Delta t \Big\{ \frac{g}{\Delta x} \mathrm{Re}[\Phi^\dagger(t, x+k) U_k^\dagger(t,x) V_k^\dagger(t,x) i\sigma^m \Phi(t,x)] \\
& - \frac{1}{g \Delta x^3} \sum_i \mathrm{Tr}[i\sigma^m U_k(t,x) U_i(t, x+k) U_k^\dagger(t, x+i) U_i^\dagger(t,x) \\
& + i\sigma^m U_k(t,x) U_i^\dagger(t, x+k-i) U_k^\dagger(t, x-i) U_i(t, x-i)] \Big\},
\end{aligned}
$$

Finite-T Veff

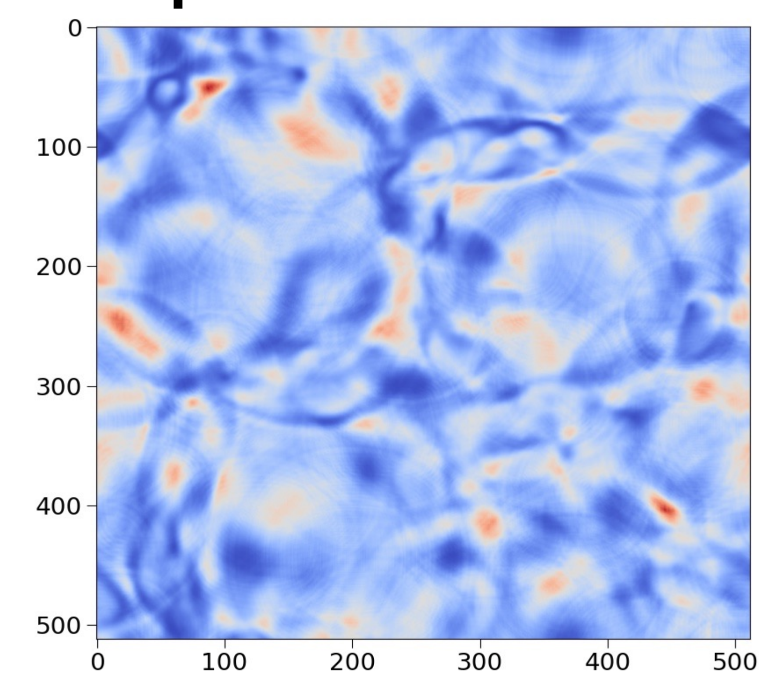


PT temperature（T_*）
duration（β）
strength（α）

Nucleation



Expansion&Percolation



Finite-T calculation

Lattice Simulation

Di, Wang, Zhou, **Bian***, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

See Jinno and Chi's talk for the fluid simulation

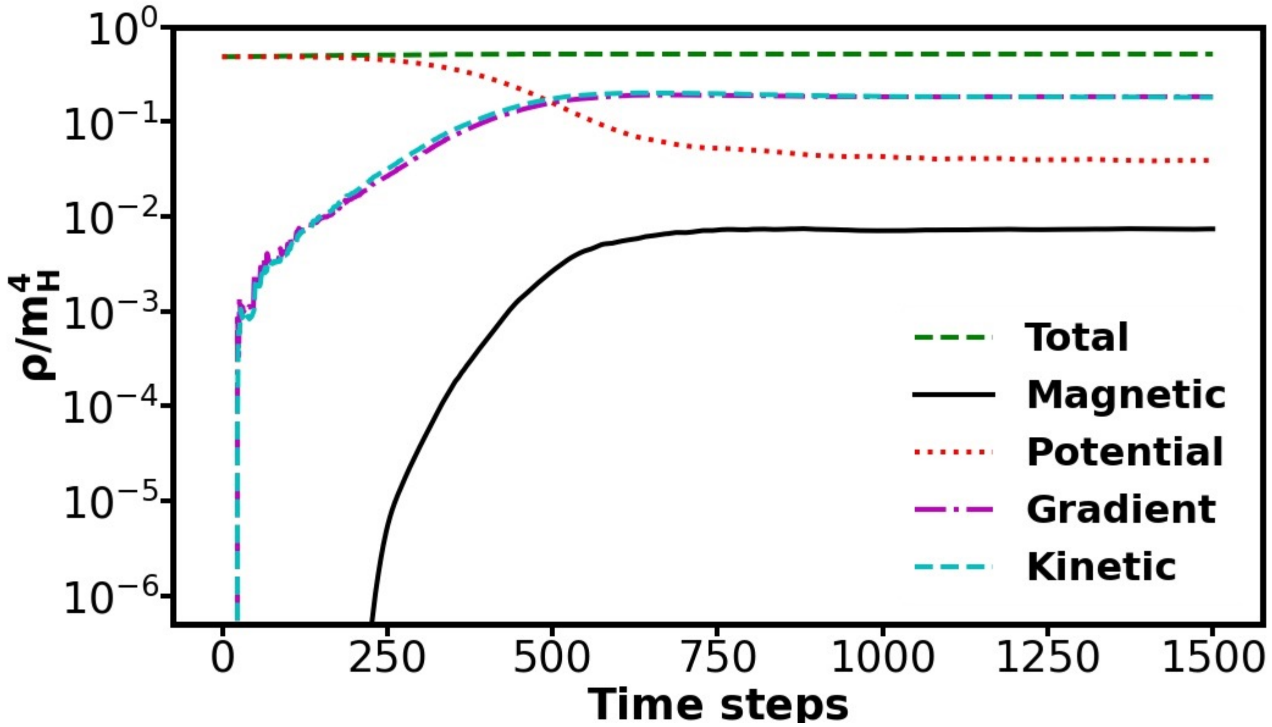
Bubble collision and GWs generation

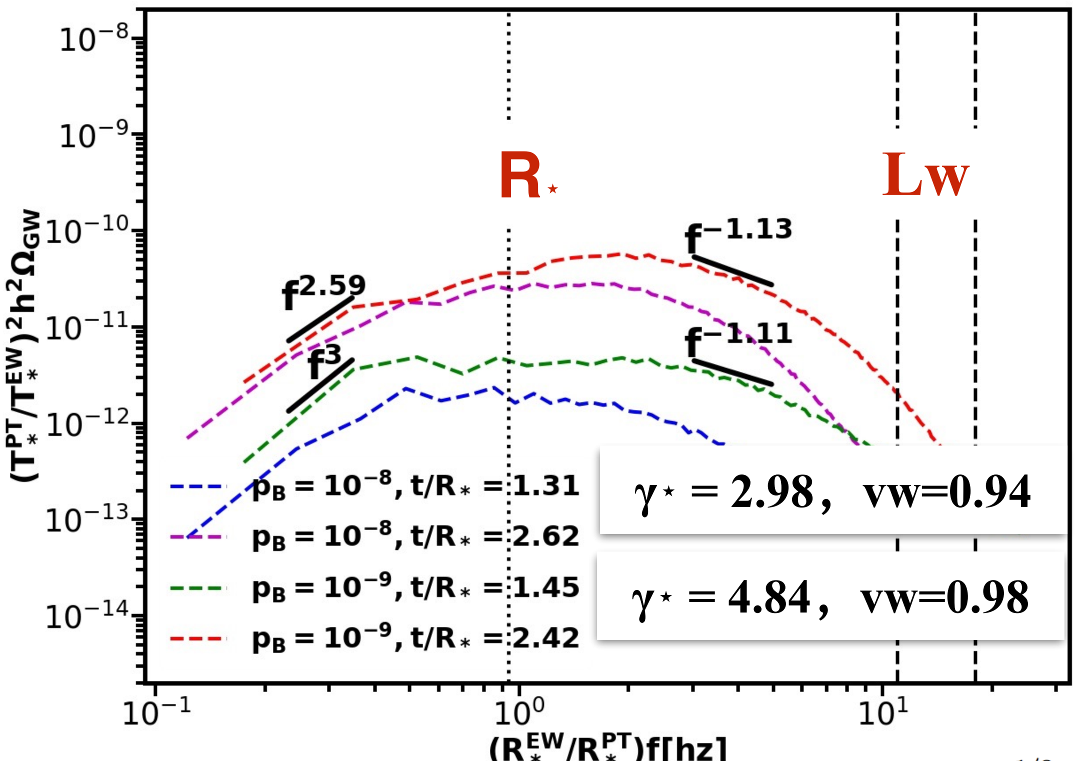
$$\ddot{h}_{ij} - \nabla^2 h_{ij} = 16\pi G T_{ij}^{\rm TT}$$

$$T_{\mu\nu} = \partial_\mu \Phi^\dagger \partial_\nu \Phi - g_{\mu\nu}\frac{1}{2}{\rm Re}[(\partial_i \Phi^\dagger \partial^i \Phi)^2]$$

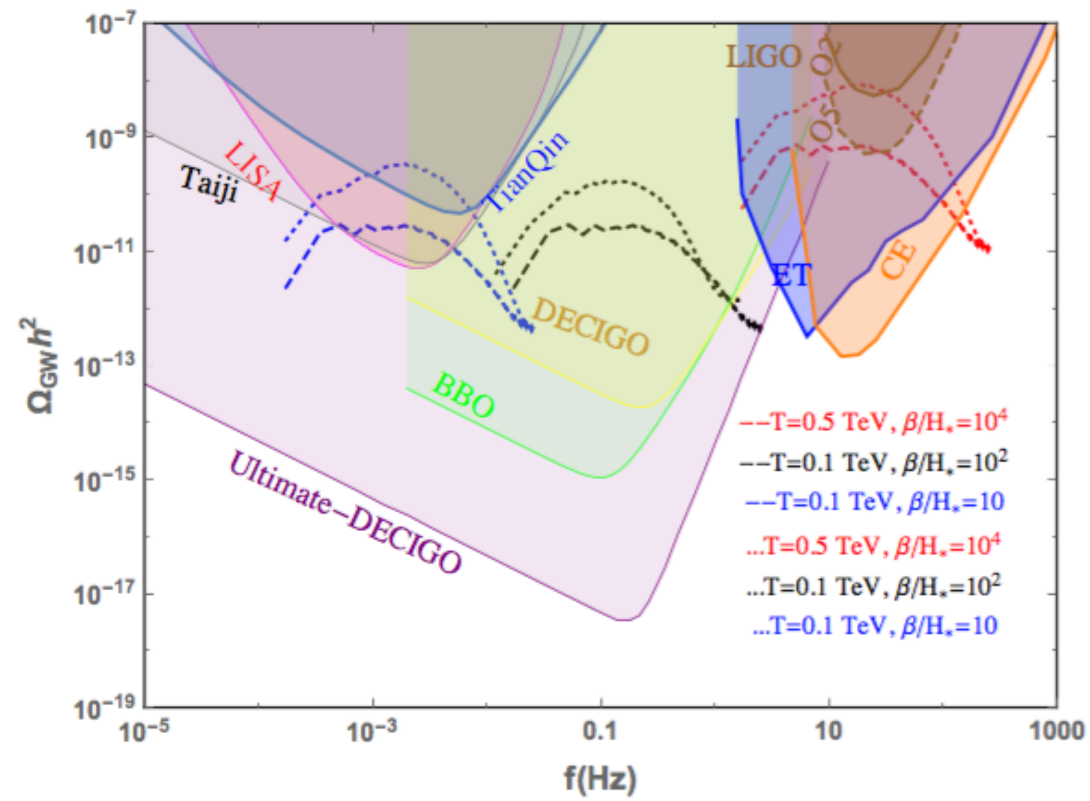
$$\langle \dot{h}_{ij}^{TT}(\mathbf{k},t)\dot{h}_{ij}^{TT}(\mathbf{k}',t)\rangle = P_{\dot{h}}(\mathbf{k},t)(2\pi)^3\delta(\mathbf{k}+\mathbf{k}')$$

$$\frac{d\Omega_{\rm gw}}{d\ln(k)} = \frac{1}{32\pi G\rho_c}\frac{k^3}{2\pi^2}P_{\dot{h}}(\mathbf{k},t)$$





$\gamma_\star = R_\star/(2Rc)$ $\quad R_* = \left(\frac{\mathcal{V}}{N_b}\right)^{1/3}$



Di, Wang, Zhou, **Bian***, Cai*,Liu*, Phys.Rev.Lett. 126 (2021) 251102

See Jiang Zhu's talk for the wall velocity's calculation

Bubble dynamics of FOPT

From QFT $$p(t;T) \equiv \Gamma/V = |A(T)|e^{-B(T)/T}$$

Tunneling

$$A(T) = T\left(\frac{S_3[\bar{\phi}(T)]}{2\pi T}\right)^{\frac{3}{2}}\left(\frac{\det'[-\nabla^2 + V''(\bar{\phi})]}{\det[-\nabla^2 + V''(\phi_f)]}\right)^{-\frac{1}{2}}$$

Fluctuation

$$\begin{aligned} A(T) &= \frac{\sqrt{-\lambda_-}}{2\pi}\left(\frac{S_3[\bar{\phi}(T)]}{2\pi T}\right)^{\frac{3}{2}}\left(\frac{\det'[-\nabla^2 + V''(\bar{\phi})]}{\det[-\nabla^2 + V''(\phi_f)]}\right)^{-\frac{1}{2}} \\ &= \frac{1}{2\pi}\left(\frac{S_3[\bar{\phi}(T)]}{2\pi T}\right)^{\frac{3}{2}}\left(\frac{\det^+[-\nabla^2 + V''(\bar{\phi})]}{\det[-\nabla^2 + V''(\phi_f)]}\right)^{-\frac{1}{2}}, \end{aligned}$$

2305.02357

False vacuum fraction considering bubbles expansion

Bubble Volume

$$h(t) = \exp\left[-\int_{t_c}^{t} dt' \, \frac{4\pi}{3} v_w^3 (t-t')^3 \frac{\Gamma(t')}{\mathcal{V}}\right]$$

$$\frac{\Gamma}{\mathcal{V}} = \frac{\Gamma_f}{\mathcal{V}} e^{\beta(t-t_f)} \qquad \beta \equiv \frac{d}{dt}\log\left(\frac{\Gamma(t)}{\mathcal{V}}\right)\Bigg|_{t=t_f}$$

$$\begin{aligned} -\log h(t) &\simeq \int^{t} dt' \frac{4\pi}{3} v_w^3 (t-t')^3 \frac{\Gamma_f}{\mathcal{V}} e^{\beta(t'-t_f)} \\ &= \frac{4\pi}{3} v_w^3 \frac{\Gamma_f}{\mathcal{V}} \frac{3!}{\beta^4} e^{\beta(t-t_f)} . \end{aligned}$$

h(tf)=1/e

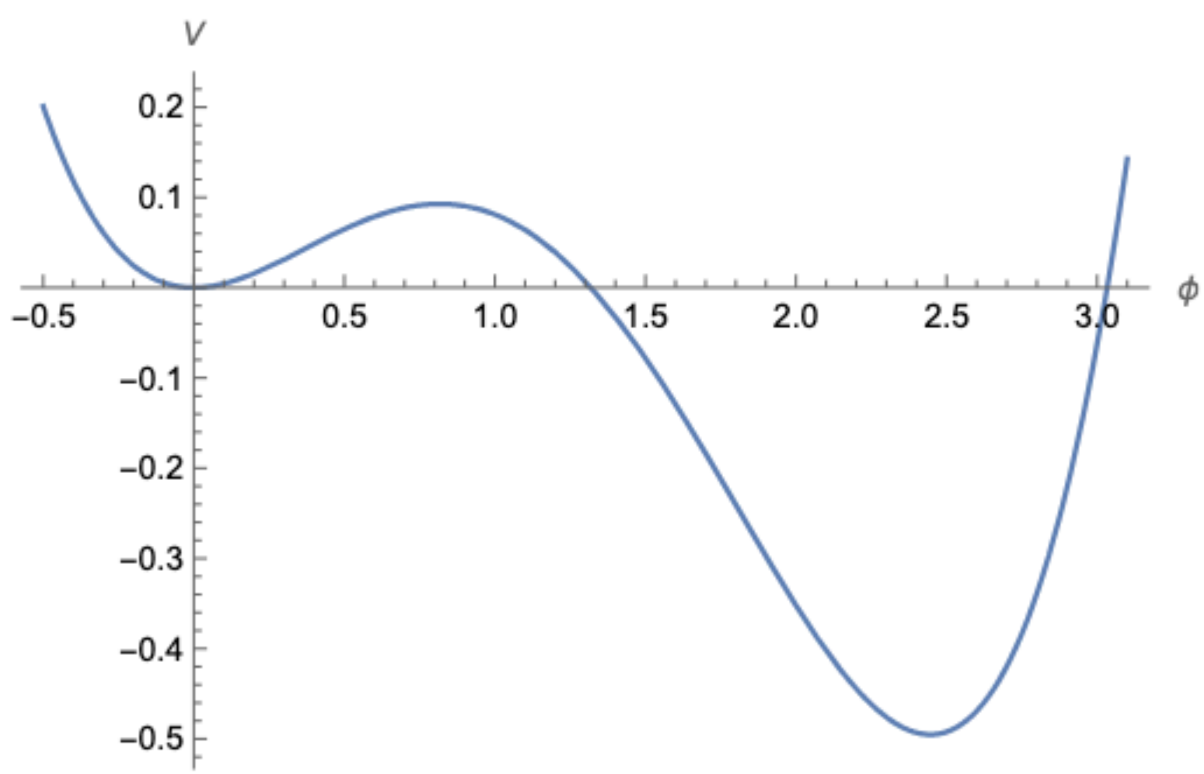
$$8\pi \frac{v_w^3}{\beta^4}\frac{\Gamma_f}{\mathcal{V}} = 1$$

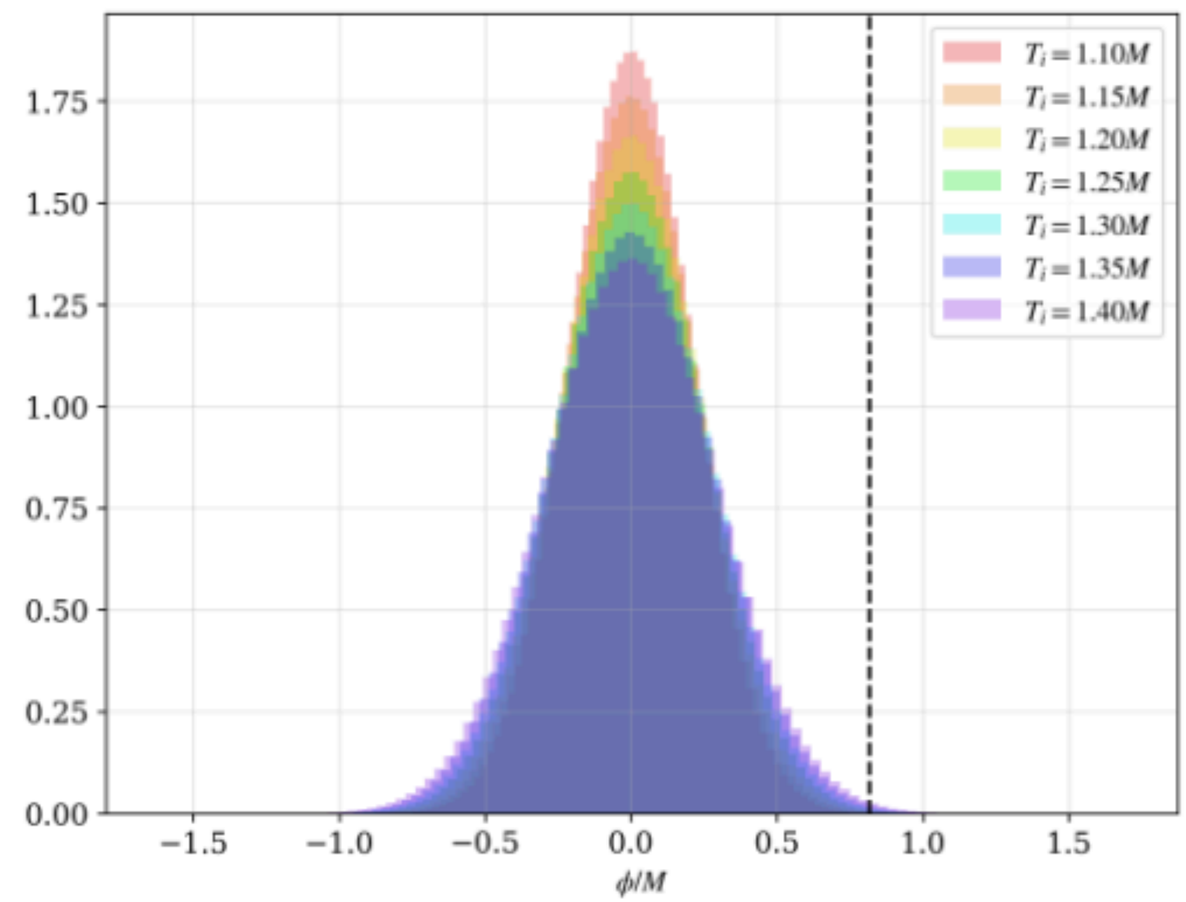
$$h(t) = \exp\left[-e^{\beta(t-t_f)}\right]$$

2008.09136

1st order EWPT simulation-thermal fluctuation

Toy model $V(\phi) = \frac{1}{2}M^2\phi^2 + \frac{1}{3}\delta\phi^3 + \frac{1}{4}\lambda\phi^4$





Bose-Einstein distribution

$$\tilde{\phi} = \phi/f_*, \quad d\tilde{t} = w_* dt, \quad d\tilde{x}^i = w_* dx^i,$$

$$\tilde{S} = \left(\frac{w_*}{f_*}\right)^2 S(f_*\tilde{\phi})$$

$$\ddot{\tilde{\phi}} - \tilde{\nabla}^2\tilde{\phi} + \tilde{V}_{,\tilde{\phi}} = 0$$

Dimensionless EOM

$$\mathcal{P}_\phi(k) = \frac{n_k}{w_k} = \frac{1}{w_k}\frac{1}{e^{w_k/T}-1}, \quad \mathcal{P}_{\dot{\phi}}(k) = n_k w_k = \frac{w_k}{e^{w_k/T}-1},$$

$$\langle\phi(\mathbf{k})\phi(\mathbf{k}')\rangle = (2\pi)^3\mathcal{P}_\phi(k)\delta(\mathbf{k}-\mathbf{k}'), \quad \langle|\phi(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_\phi(k), \quad \langle\phi(\mathbf{k})\rangle = 0,$$

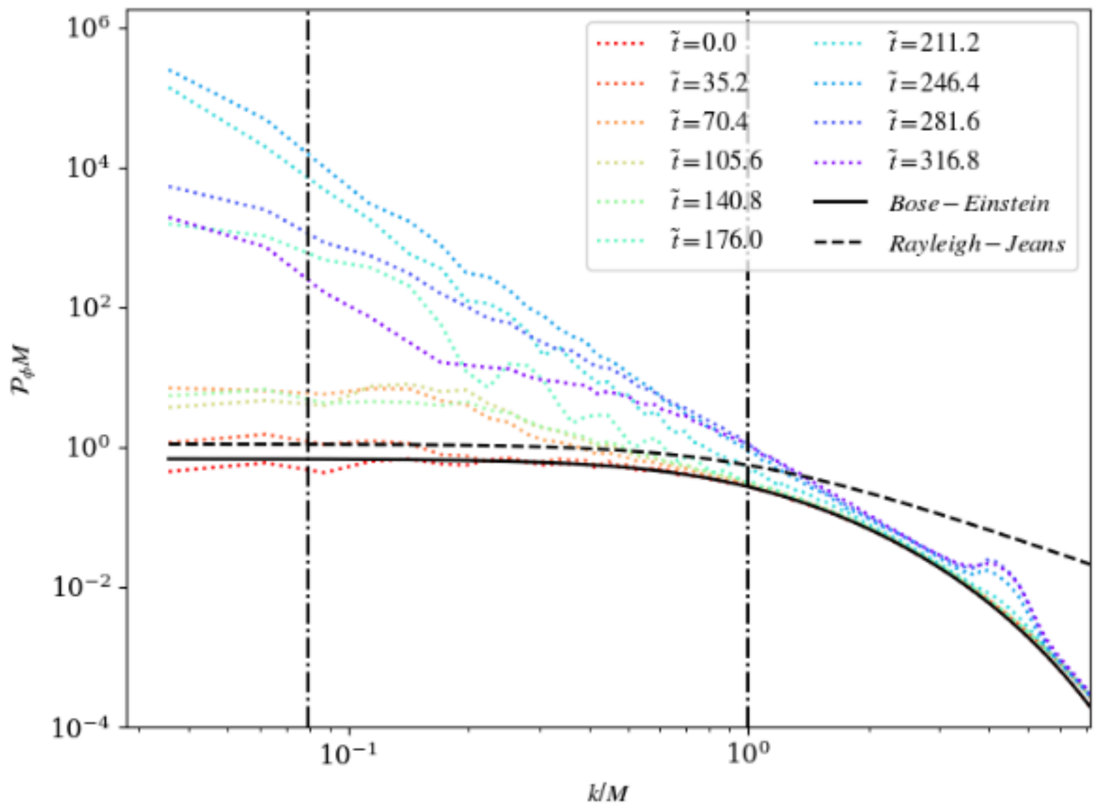
$$\langle\dot{\phi}(\mathbf{k})\dot{\phi}(\mathbf{k}')\rangle = (2\pi)^3\mathcal{P}_{\dot{\phi}}(k)\delta(\mathbf{k}-\mathbf{k}'), \quad \langle|\dot{\phi}(\mathbf{k})|^2\rangle = \left(\frac{N}{\delta x_{\rm phy}}\right)^3\mathcal{P}_{\dot{\phi}}(k), \quad \langle\dot{\phi}(\mathbf{k})\rangle = 0,$$

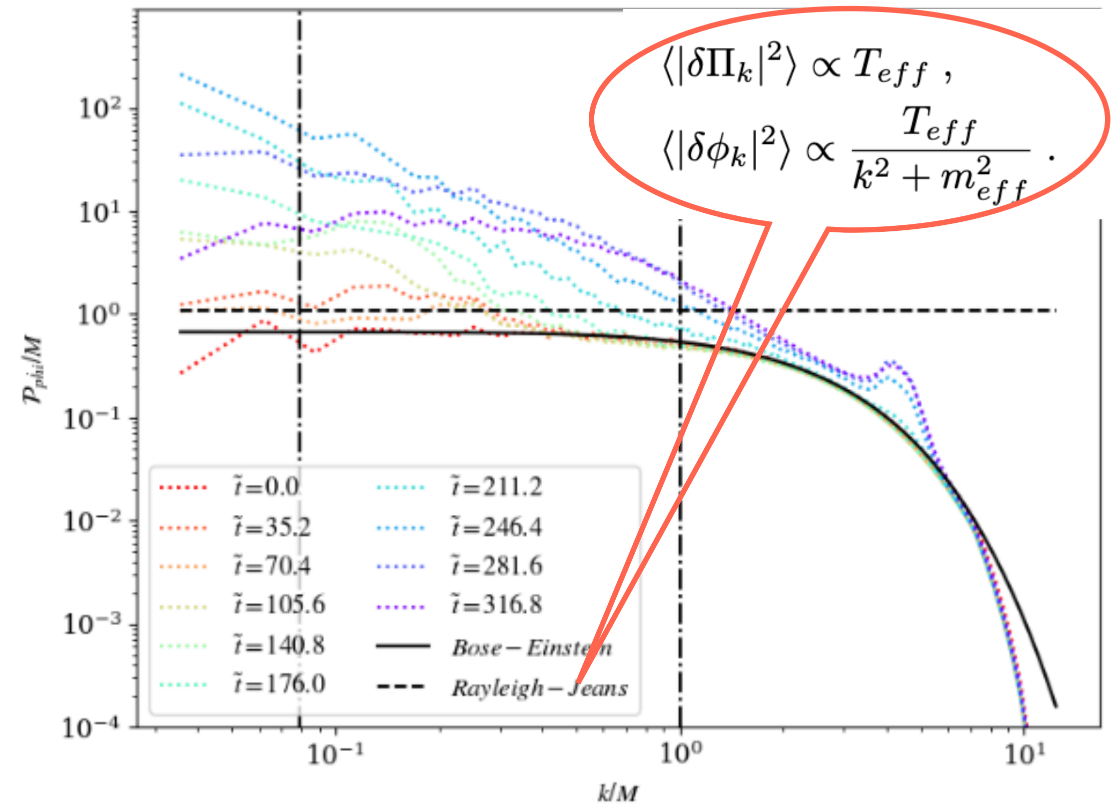
$$\langle\phi(\mathbf{k})\dot{\phi}(\mathbf{k}')\rangle = 0\,.$$

Ligong Bian,Yuefeng Di, Yongtao Jia,Yang Li,Kehao Zeng， arXiv:2505.15360

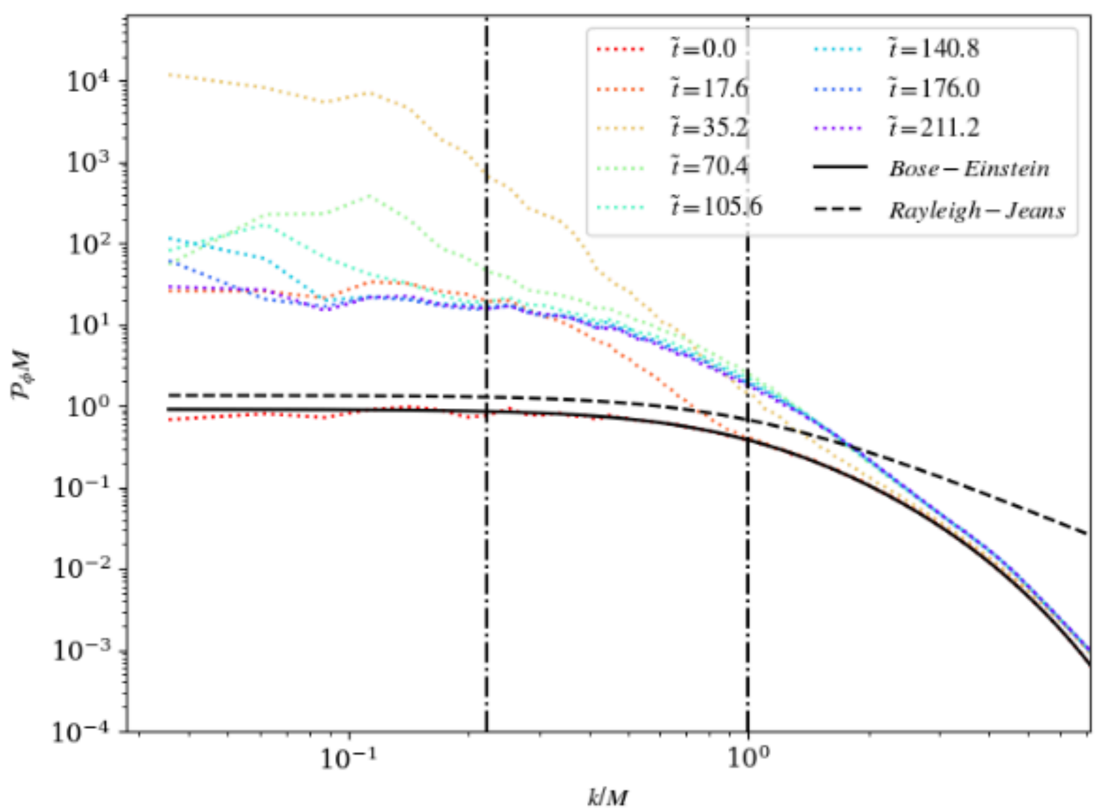
1st order EWPT simulation-the real thermal scenario?

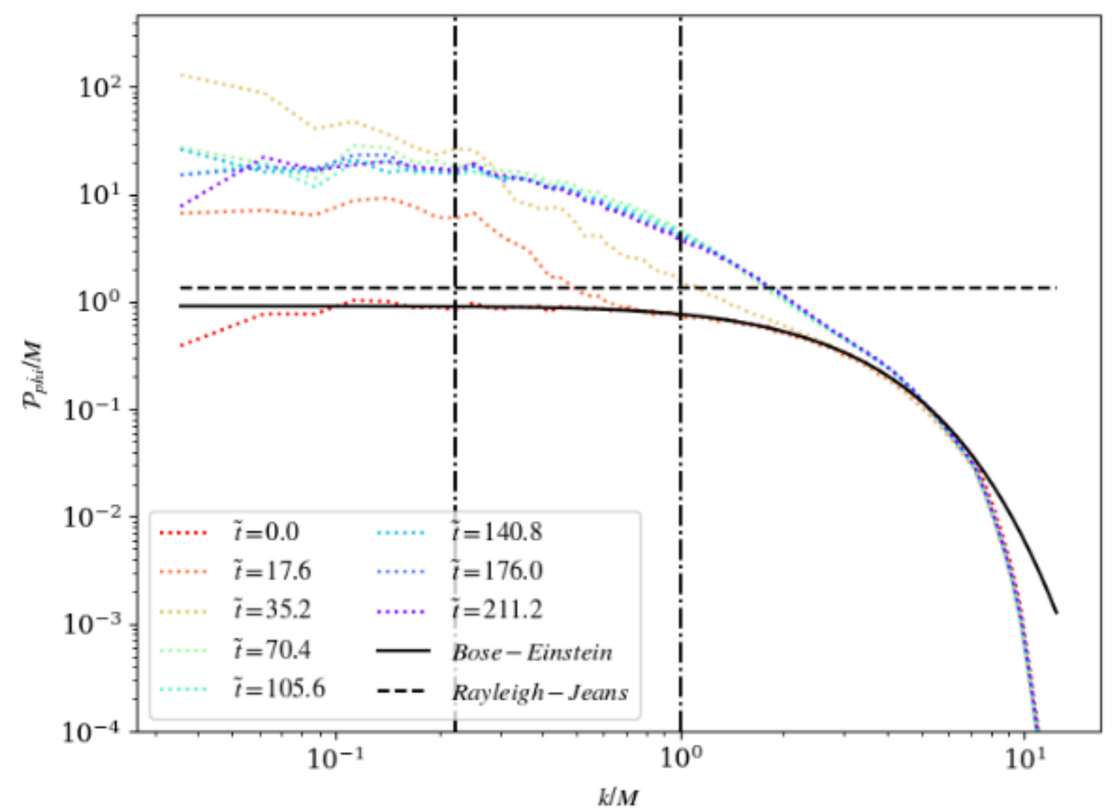
Ti = 1.1M





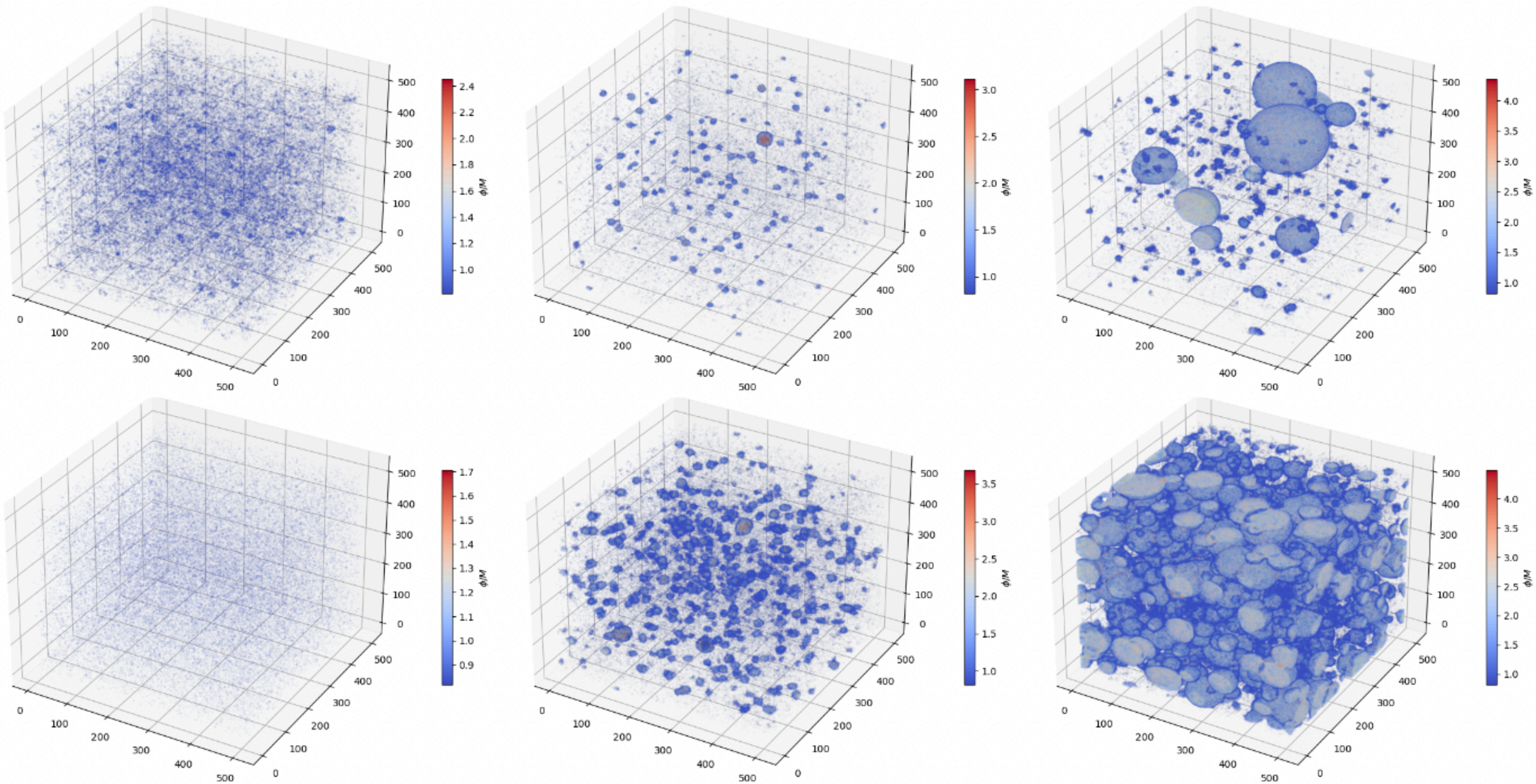
Ti = 1.3 M





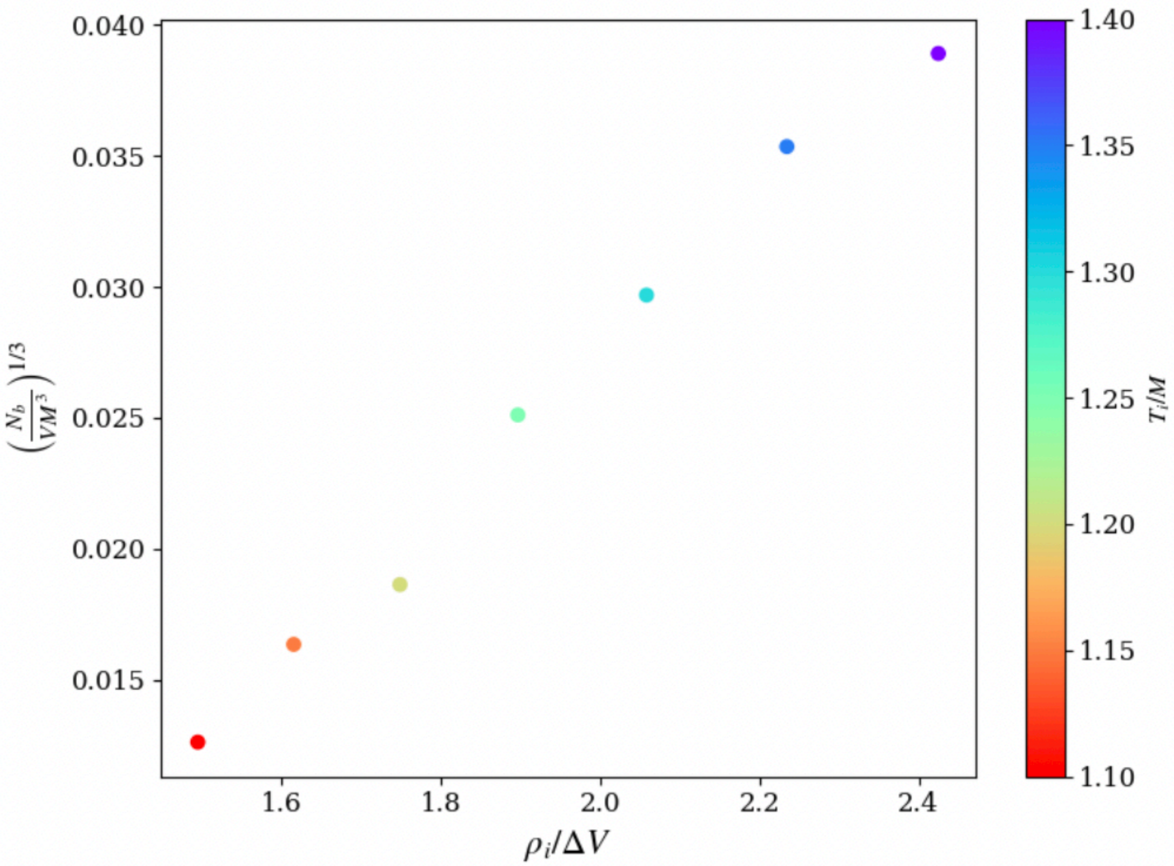
Oscillons-like bubble

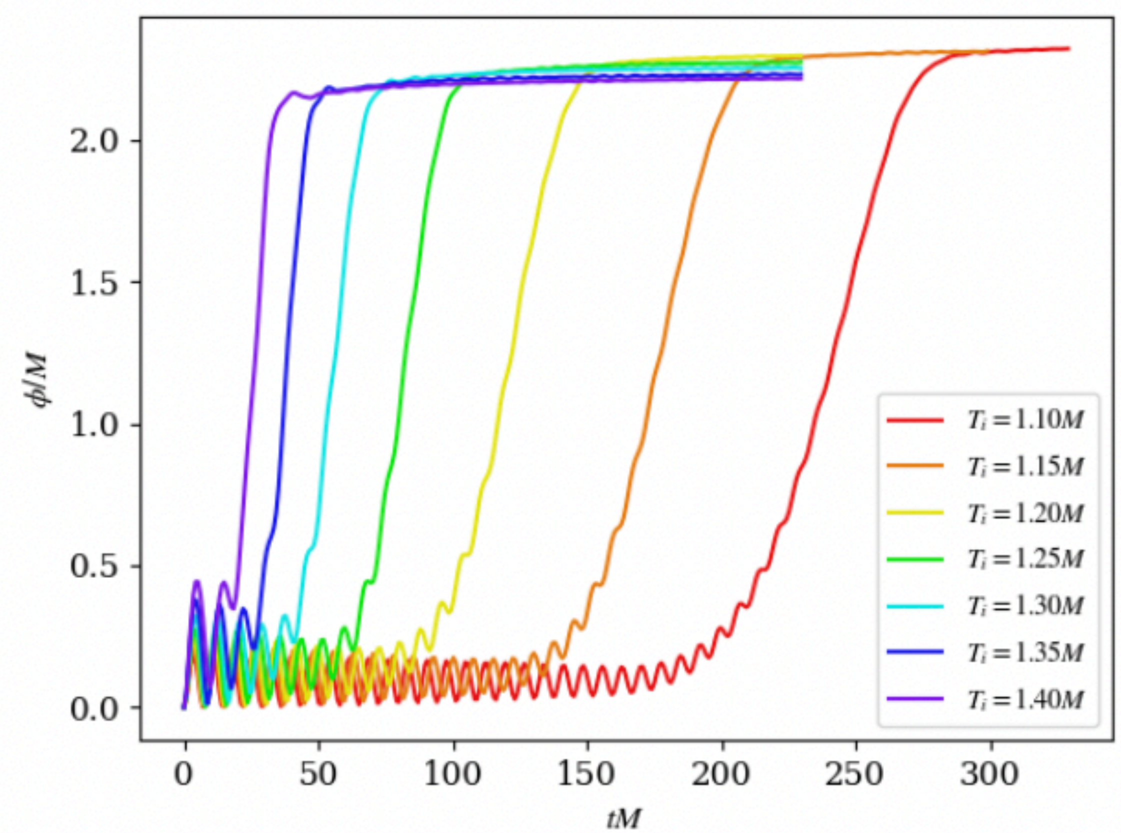
Ti = 1.15 M

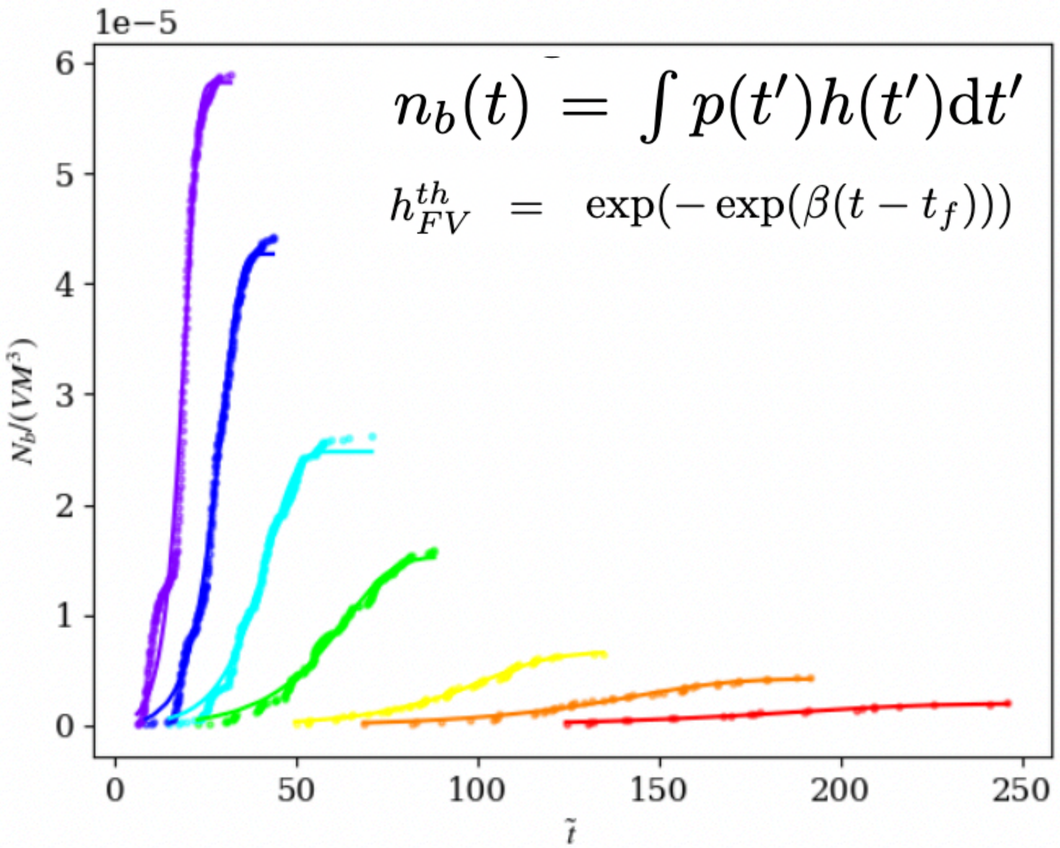


Ti = 1.35 M

Nucleation theory





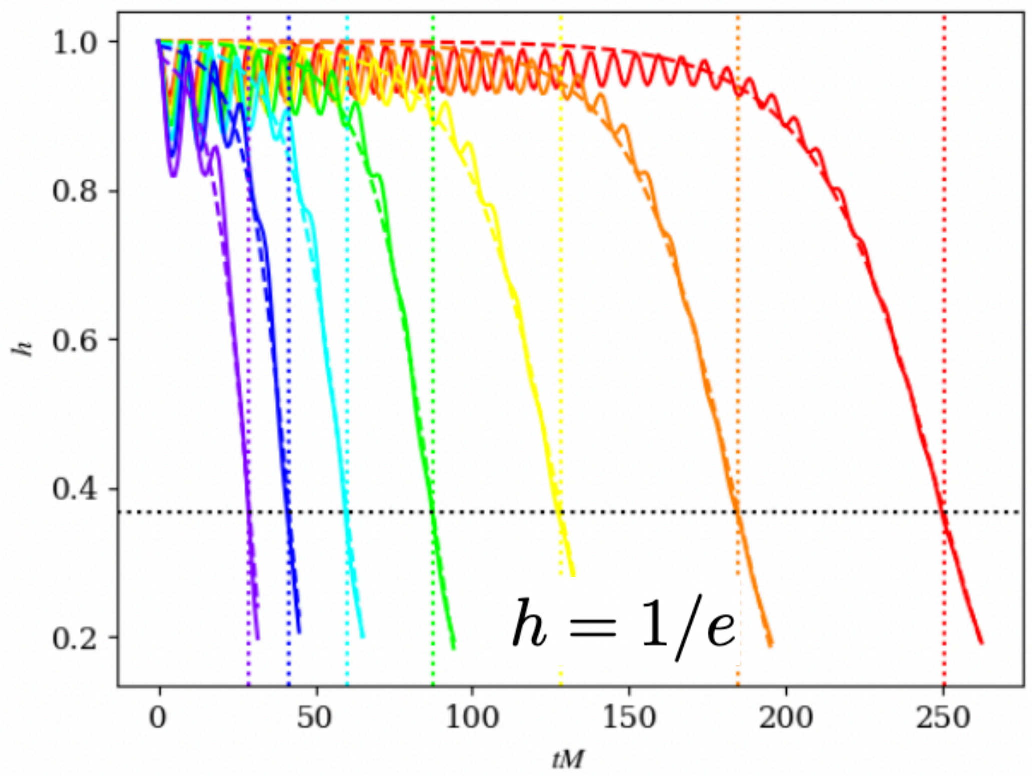


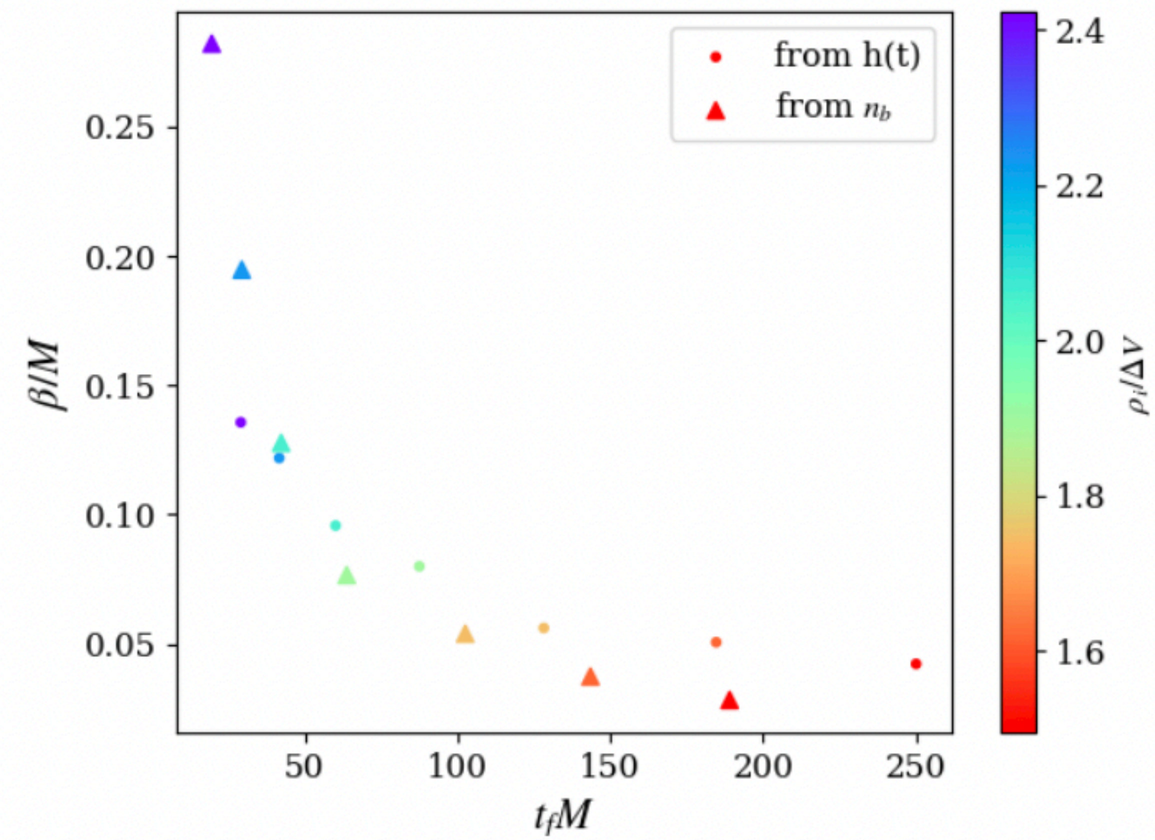
p(t) = pf exp[β(t – tf)]

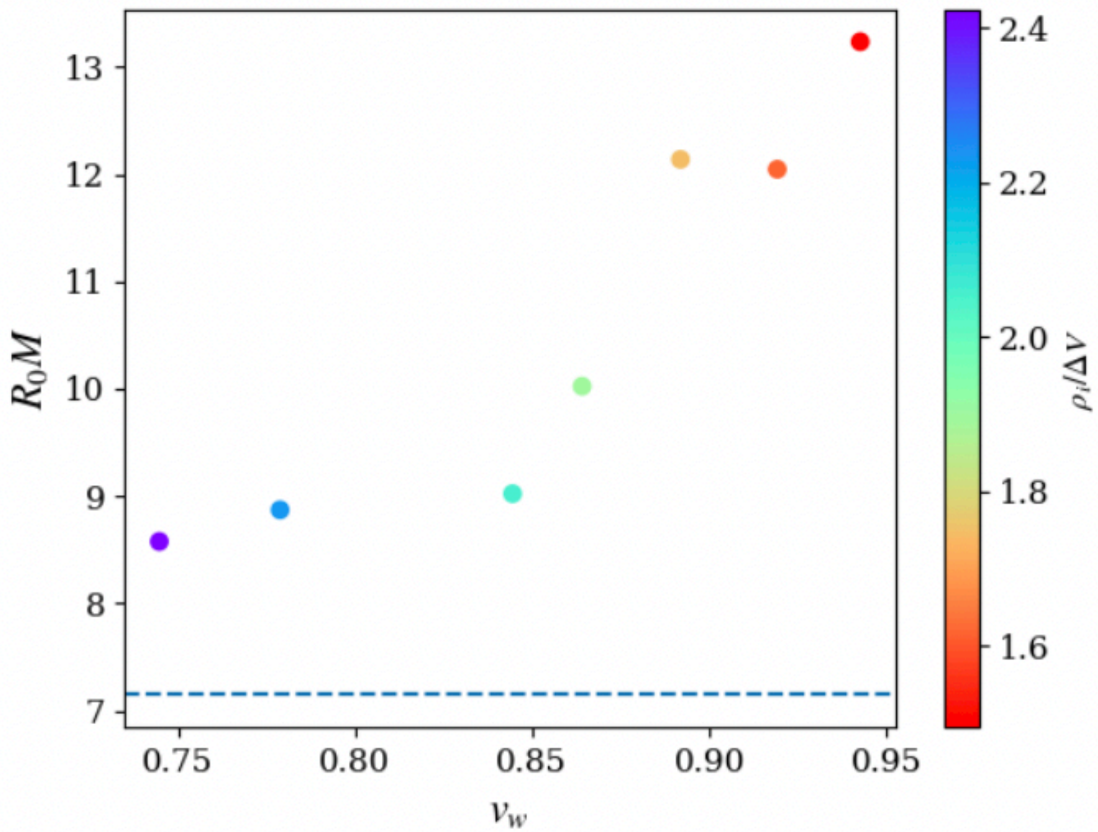
T_i/M	p_f/M^4	β/M	$t_f M$
1.10	5.56×10^{-8}	0.028	189.10
1.15	1.58×10^{-7}	0.037	143.53
1.20	3.57×10^{-7}	0.054	102.54
1.25	1.17×10^{-6}	0.077	63.72
1.30	3.18×10^{-6}	0.128	42.25
1.35	8.34×10^{-6}	0.195	29.35
1.40	1.65×10^{-5}	0.282	19.53

Nucleation theory

$$h_{FV}^{sim} = 1 - \frac{\langle\phi\rangle}{\phi_v} \qquad h_{FV}^{th} = \exp(-\exp(\beta(t - t_f)))$$







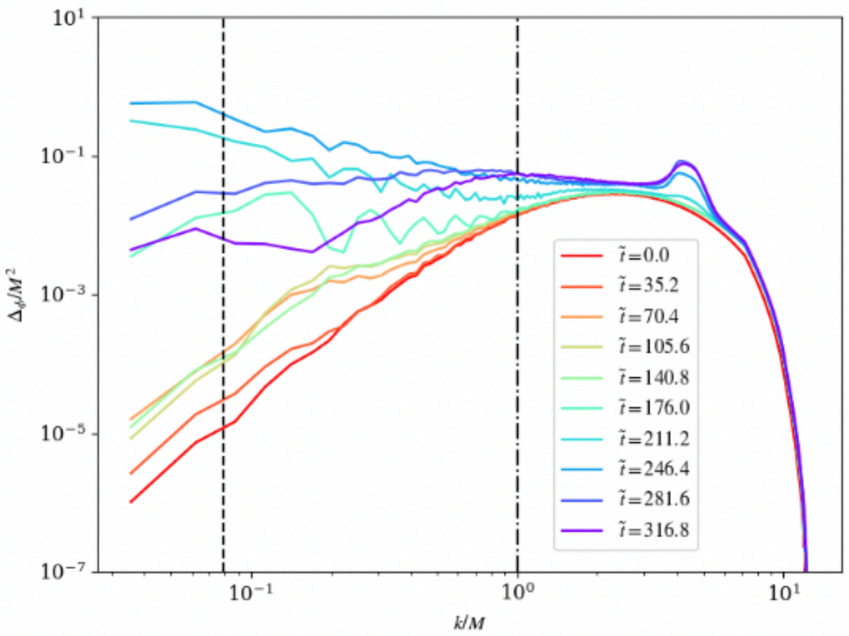
$$v_w \approx \sqrt{\Delta\rho_k / \Delta\rho_g}$$

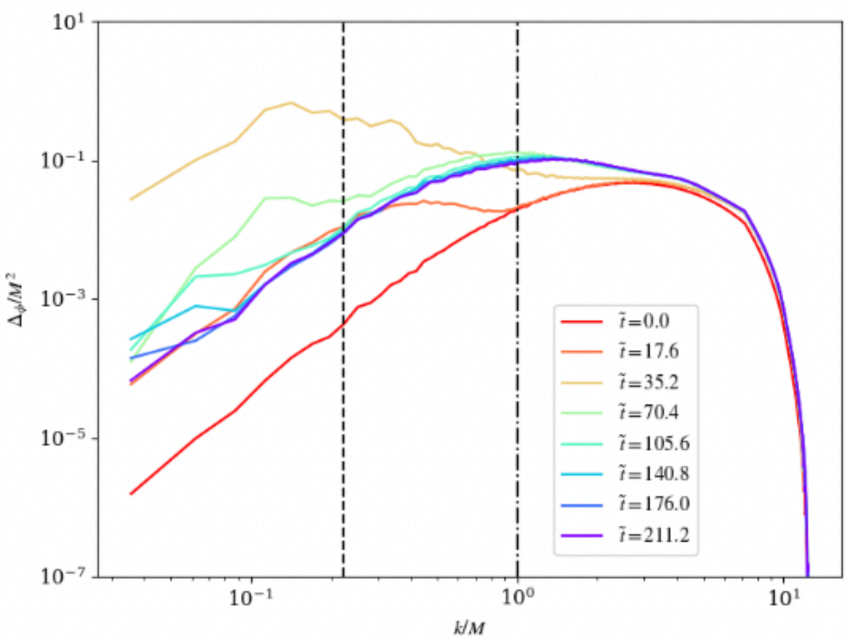
$$\frac{1}{2}\dot{\phi}^2 \qquad \frac{1}{2}(\nabla\phi)^2$$

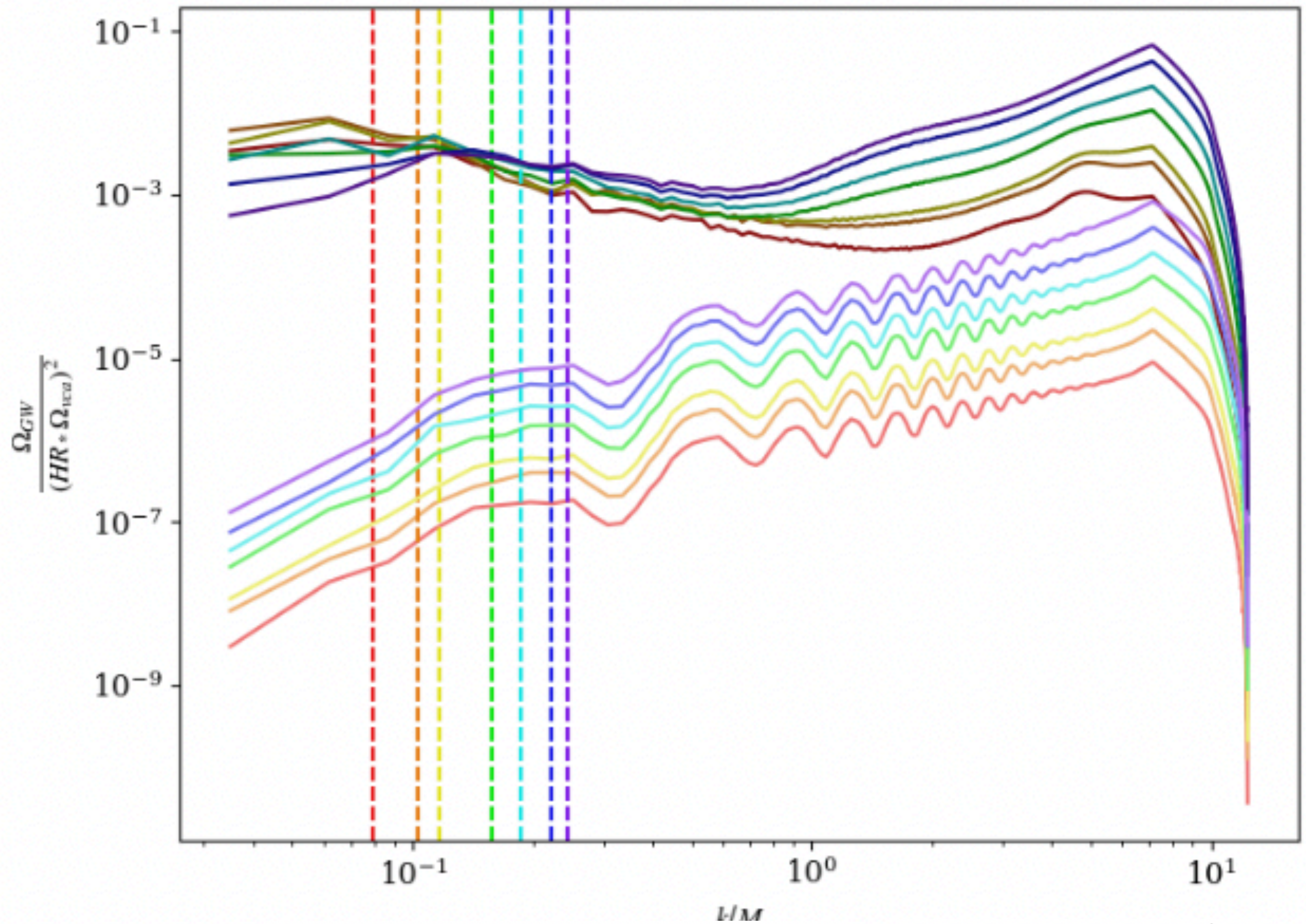
$$R_0 = R_* \sqrt{1 - v_w^2}/2$$

$$R_* = (V/N_b)^{1/3}$$

1st order EWPT simulation-GWs







$\Omega_{GW} \approx \mathcal{O}(10^{-3})(HR_*\Omega_{vac})$ $\Omega_{vac} = \Delta V/\rho_r$

1st order EWPT simulation-thermal fluctuation

Wigner function in field theory

$$P_R(T) \equiv \int_R dx|\psi(x,T)|^2 \quad \Longrightarrow \quad P_R(t) \equiv \int_R \mathcal{D}\phi|\Psi(\phi,t)|^2$$

（量子力学中Wigner函数：(q, p)相空间中准概率密度性质）

$$\Psi(\phi,t) = \langle\phi(x)|\Psi(t)\rangle = \int \mathcal{D}\phi_i \langle\phi|\hat{U}(t|t_0)|\phi_i\rangle\langle\phi_i(\mathbf{x})|\Psi(t_0)\rangle$$

$$W(q,p;t) \sim |\Psi(x,t)|^2$$

$$W(q,p;t) = \int du e^{-\frac{i}{\hbar}pu}\left\langle q+\frac{u}{2}\right|\hat{\rho}(t)\left|q-\frac{u}{2}\right\rangle \Longrightarrow W[\phi(x),\Pi(x);t] = \int \mathcal{D}\varphi(x)\exp\left[-\frac{i}{\hbar}\int dx\Pi(x)\varphi(x)\right]\times\left\langle\phi(x)+\frac{\varphi(x)}{2}\right|\hat{\rho}(t)\left|\phi(x)-\frac{\varphi(x)}{2}\right\rangle$$

Wigner function：$(\phi, \ \Pi)$ phase space Quasi-probability distribution

$$\hat{\rho}(0) = \frac{1}{Z}e^{-\beta\hat{H}} \qquad W[\phi(x),\Pi(x);t] \gtrless 0$$

$$W[\phi(x),\Pi(x);0] = \frac{1}{Z}\int \mathcal{D}\varphi(x)\exp\left[-\frac{i}{\hbar}\int dx\Pi(x)\varphi(x)\right]\times\left\langle\phi(x)+\frac{\varphi(x)}{2}\right|e^{-\beta\hat{H}}\left|\phi(x)-\frac{\varphi(x)}{2}\right\rangle$$

$$\approx \frac{1}{Z}exp\left[-\beta\int dx[\frac{1}{2}\Pi^2(x)+\frac{1}{2}(\nabla\phi(x))^2+V(\phi)]\right]$$

$$Z \equiv Tr\hat{\rho} = \int \mathcal{D}\phi(x)\frac{\mathcal{D}\Pi(x)}{2\pi}W[\phi,\Pi;t]$$

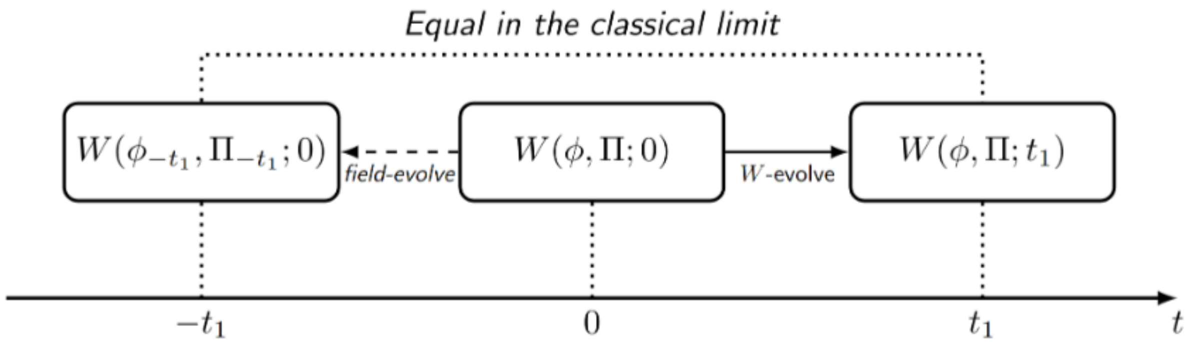
$\phi(x)$ is the real scale field

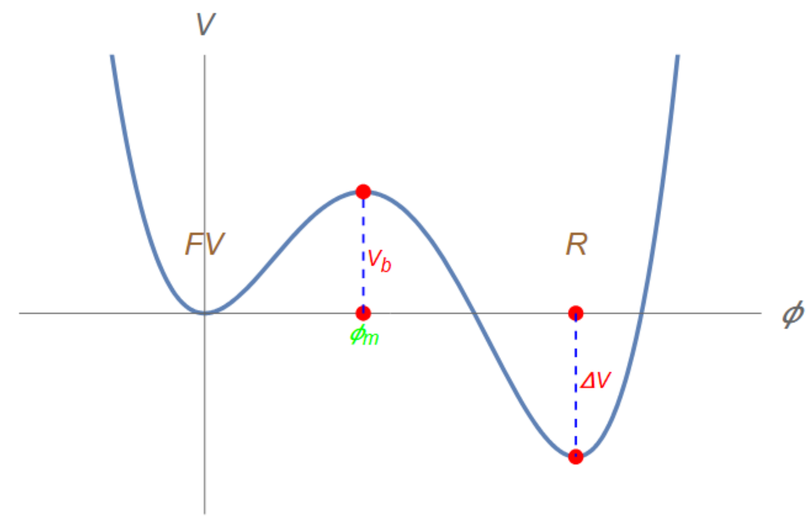
$\Pi(x) \equiv \frac{\delta\mathcal{L}}{\delta\dot{\phi}(x)}$

$\hat{\rho}(t) = |\Psi(t)\rangle\langle\Psi(t)|$

Stanisław Mrówczyński and Berndt Müller, Phys. Rev. D, 50:7542–7552, Dec 1994.

Wigner function's EOM





$$\hat{L} = \int d^3x (\frac{1}{2}\partial^\mu \hat{\phi}\partial_\mu \hat{\phi} - V(\hat{\phi})) \quad \hat{H} = \int d^3x \hat{\mathcal{H}} = \int d^3x [\frac{\hat{\Pi}^2}{2} + \frac{(\nabla\hat{\phi})^2}{2} + V(\hat{\phi})]$$

$$i\hbar\frac{\partial}{\partial t}\hat{\rho}(t) = [\hat{H},\hat{\rho}(t)] \quad \frac{\partial}{\partial t}W[\phi,\pi;t] = -2H\frac{1}{i\hbar}\sin(\frac{i\hbar}{2}\Lambda)W[\phi,\pi;t]$$

$$\Lambda = \frac{\overleftarrow{\delta}}{\delta\Pi}\frac{\overrightarrow{\delta}}{\delta\phi} - \frac{\overleftarrow{\delta}}{\delta\Phi}\frac{\overrightarrow{\delta}}{\delta\Pi}$$ is the Poisson bracket operator

Ignore $O(\hbar^2)$

$$\left[\frac{\partial}{\partial t} + \int d^3x (\frac{\delta H}{\delta\Pi}\frac{\delta}{\delta\phi} - \frac{\delta H}{\delta\phi}\frac{\delta}{\delta\Pi})\right]W[\phi,\Pi;t] = 0$$

$$W[\phi,\Pi;t] = W[\phi_{-t},\Pi_{-t};0]$$

$$\begin{cases} \frac{\delta H}{\delta\Pi} = \frac{d\phi}{dt} = \Pi \\ -\frac{\delta H}{\delta\phi} = \frac{d\Phi}{dt} = \nabla^2\phi - \frac{\delta V(\phi)}{\delta\phi} \end{cases}$$

$$P_{\rm FV}(T) \sim e^{-\Gamma T} \Longrightarrow \Gamma = -\frac{1}{P_{\rm FV}}\frac{d}{dT}P_{\rm FV}$$

$$P_{FV}(t) \equiv \int_{FV}\mathcal{D}\phi|\Psi(\phi,t)|^2 = \frac{1}{Z}\int_{FV}\mathcal{D}\phi\int\frac{\mathcal{D}\Pi}{2\pi}W[\phi,\Pi;t]$$

$$\begin{aligned} P_{FV}(t) &= \frac{1}{Z}\int_{FV}D\phi\frac{D\Pi}{2\pi}W(\phi,\Pi;t) \\ &= \frac{1}{Z}\int_{FV}D\phi\frac{D\Pi}{2\pi}W(\phi_{-t},\Pi_{-t};0) \\ &= \frac{1}{Z}\int_{FV}D\phi_{-t}\frac{D\Pi_{-t}}{2\pi}J(\phi_{-t},\Pi_{-t})W(\phi_{-t},\Pi_{-t};0) \end{aligned}$$

$$J(\phi_{-t},\Pi_{-t}) = \det\begin{vmatrix} \frac{\partial\phi}{\partial\phi_{-t}} & \frac{\partial\phi}{\partial\Pi_{-t}} \\ \frac{\partial\Pi}{\partial\phi_{-t}} & \frac{\partial\Pi}{\partial\Pi_{-t}} \end{vmatrix} = 1$$

Haiyang Wang, Renhui Qin, **Ligong Bian**, arXiv: 2506.18334

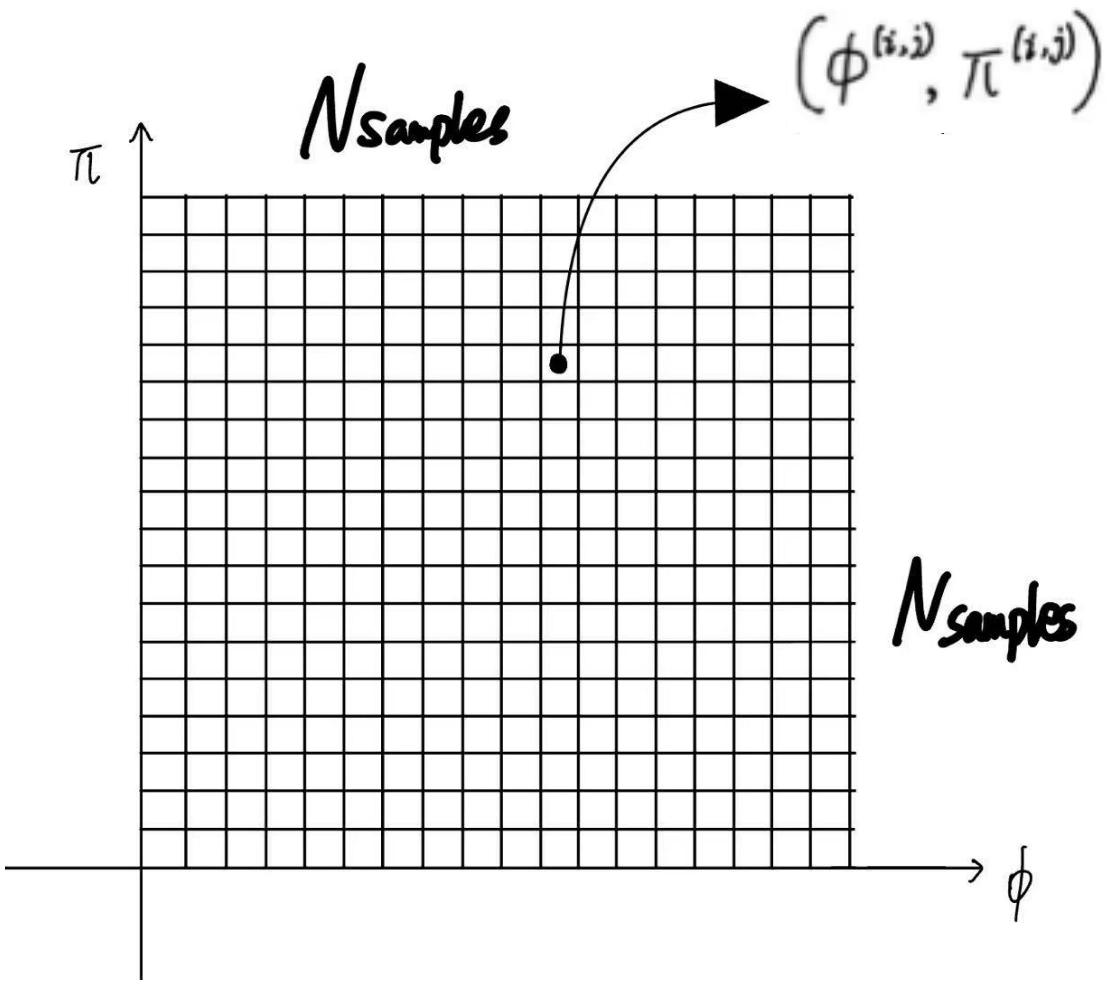
Numerical simulation's setup

$$\hat{L} = \int d^3x(\frac{1}{2}\partial^\mu\hat{\phi}\partial_\mu\hat{\phi} - V(\hat{\phi})) \quad V(\phi) = a\phi^2 + b\phi^3 + c\phi^4$$

Initial condition: $\mathcal{P}_\phi(k) = \frac{n_k}{w_k} = \frac{1}{w_k}\frac{1}{e^{w_k/T}-1}, \mathcal{P}_\Pi(k) = n_k w_k = \frac{w_k}{e^{w_k/T}-1}$

$$w_k = \sqrt{k^2/R^2 + m_{eff}^2}, m_{eff}^2 = V''(\phi_{fv})$$

N_samples*N_samples for（phi， pi）phase space

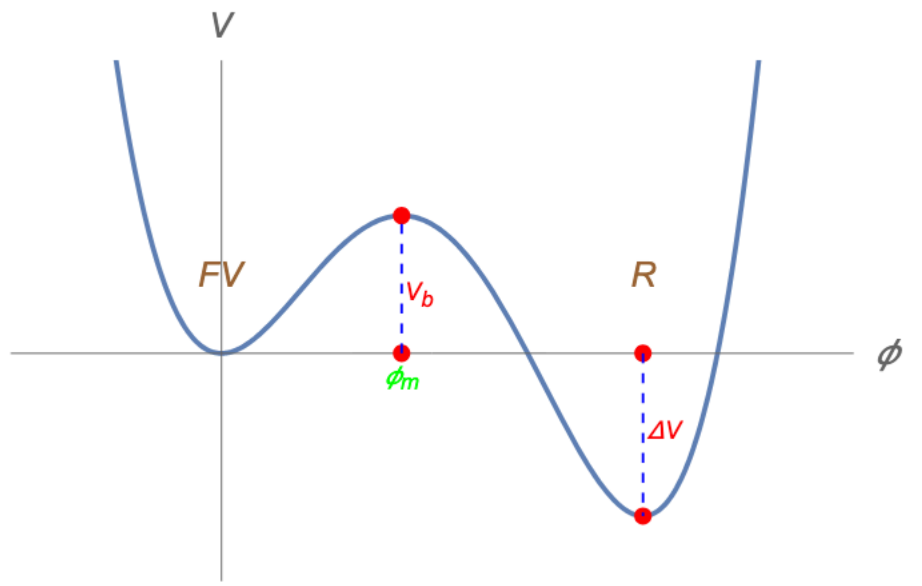


$$P_{FV}(t) \approx \frac{\sum_{ij}\chi_{FV}(\phi_{-t}^{(i,j)}) \cdot W(\phi_{-t}^{(i,j)}, \Pi_{-t}^{(i,j)}; 0)}{\sum_{ij} W(\phi_{-t}^{(i,j)}, \Pi_{-t}^{(i,j)}; 0)}$$

Summation range： $i, j = 1,2,3 \ldots N_{samples}$

$$\chi_{FV}(\phi_{-t}^{(i,j)}) = \frac{N_x(\phi_x^{(i,j)} < \phi_m)}{N_x}$$

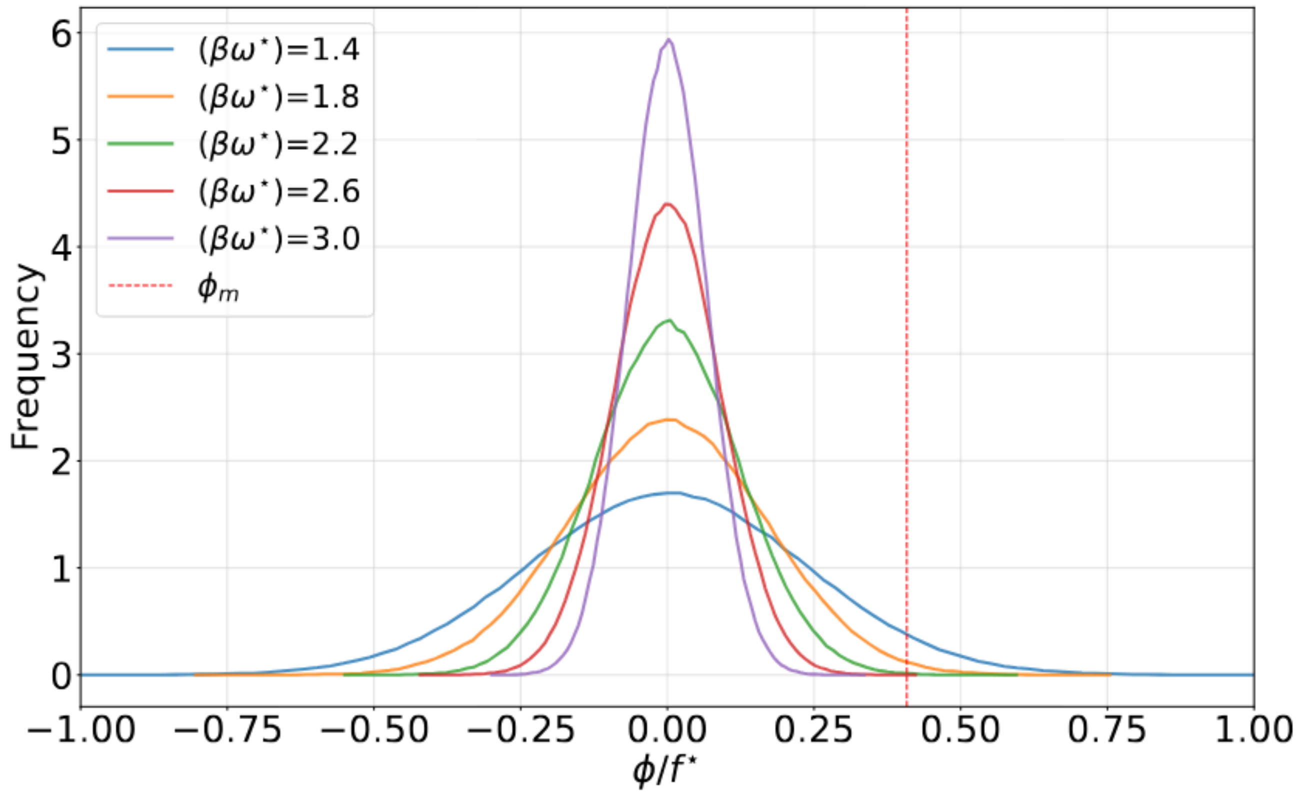
Simulation results

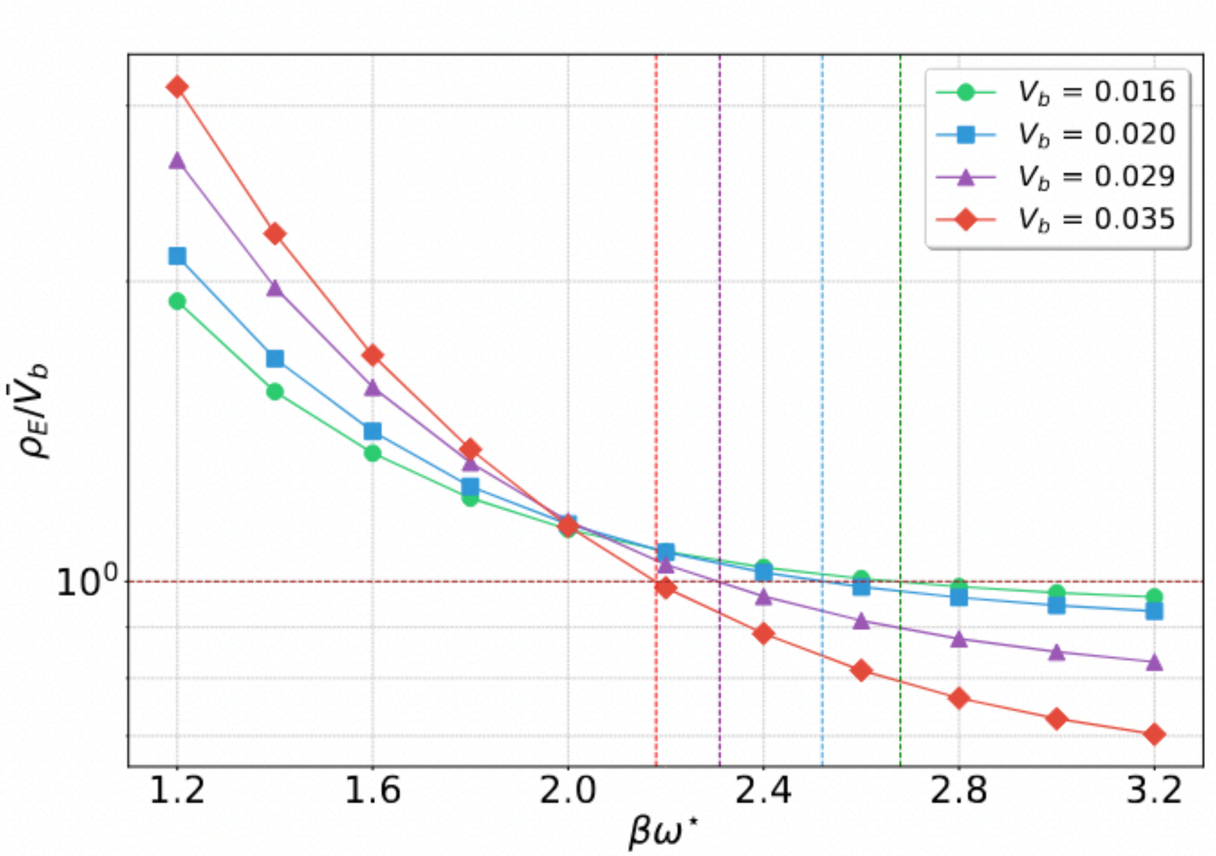


Nsamples = 1000
Δx = 0.25
Δt = 0. 01
Nx = 1024

$$\phi_s = \phi/f^\star \quad t_s = \omega^\star * t \quad x_s = \omega^\star * x$$

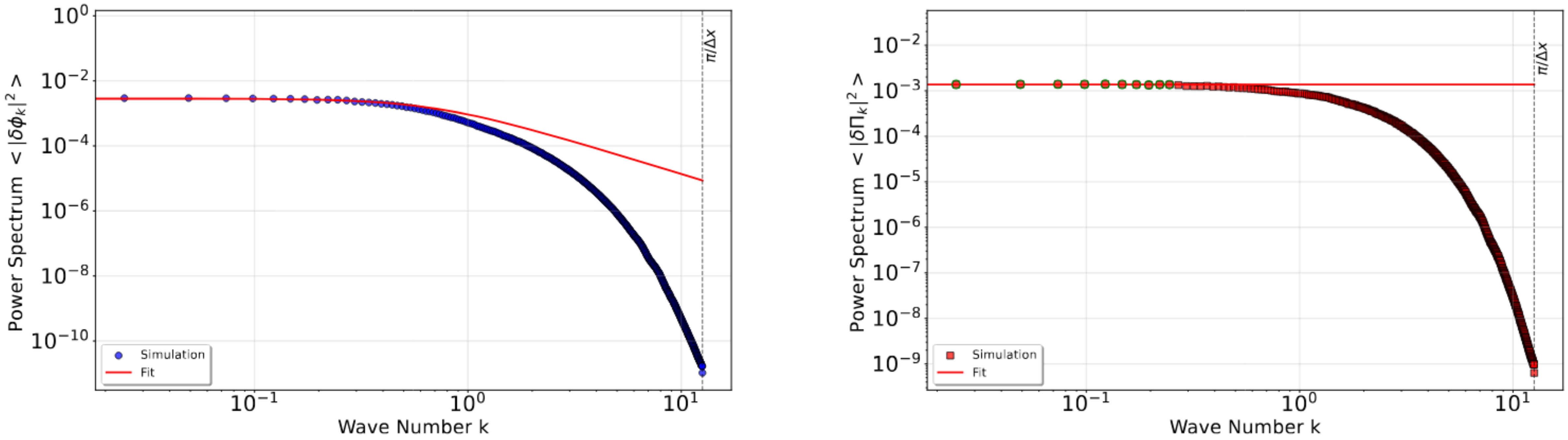
$$\frac{\partial^2 \phi_s}{\partial t_s^2} = \frac{\partial^2 \phi_s}{\partial x_s^2} - (2a\phi_s + 3b\phi_s^2 + 4c\phi_s^3)$$



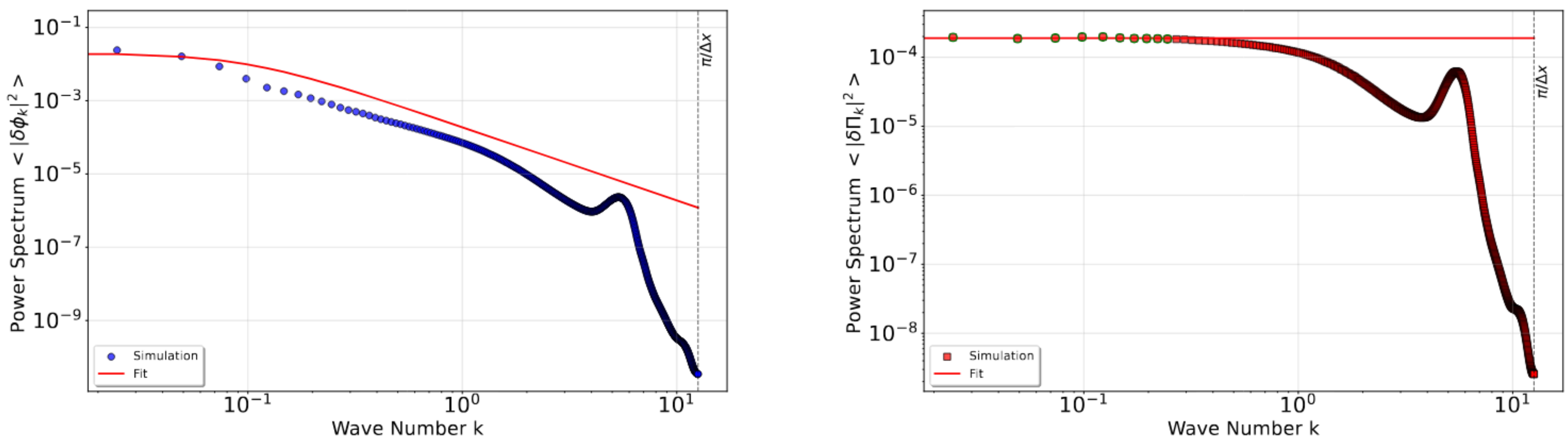


Simulation results-field spectrum

$\beta\omega_\star$ = 1.4



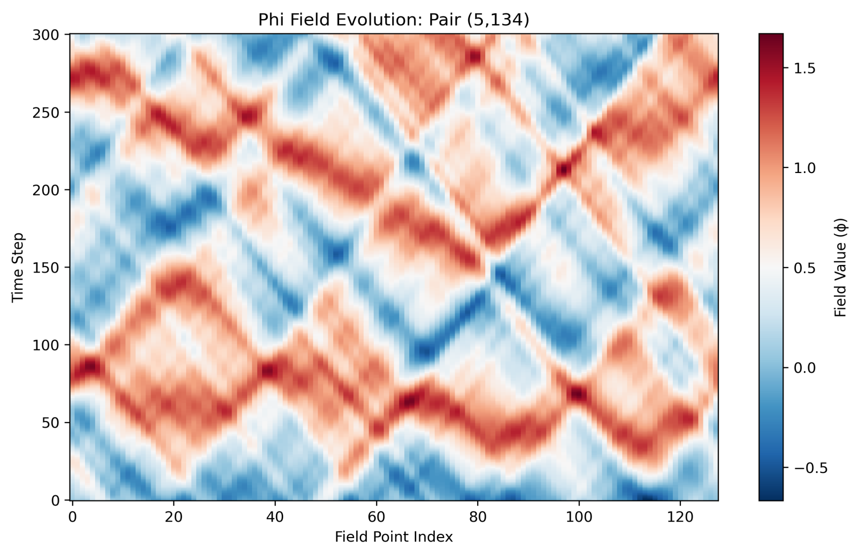
$\beta\omega_\star$ = 2.6



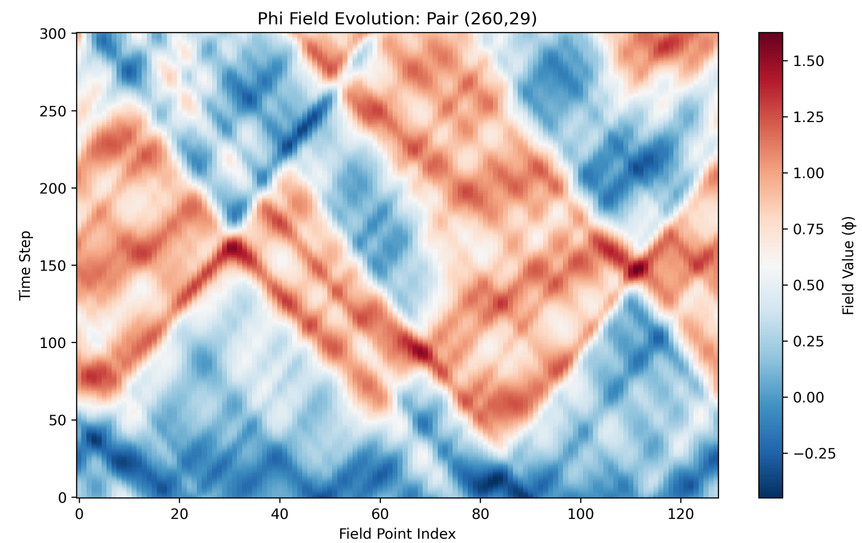
Simulation results

1D field

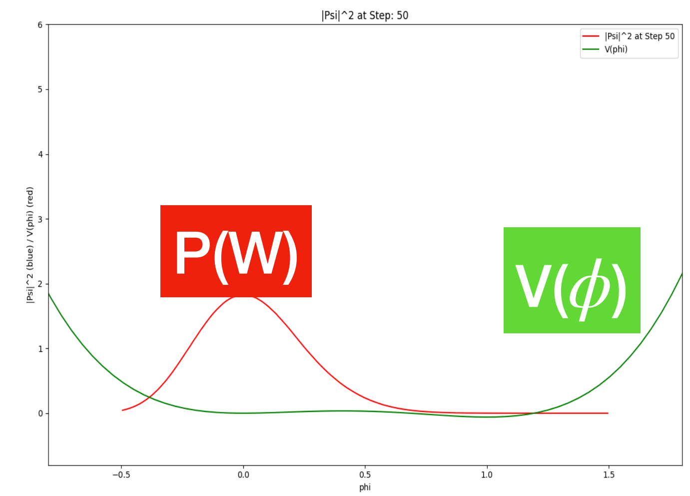
Wave function evolution

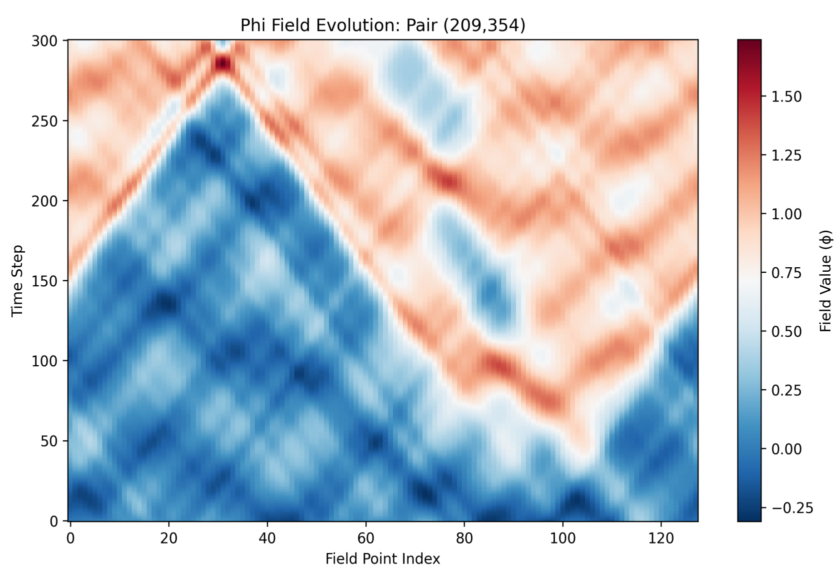


β=0.8

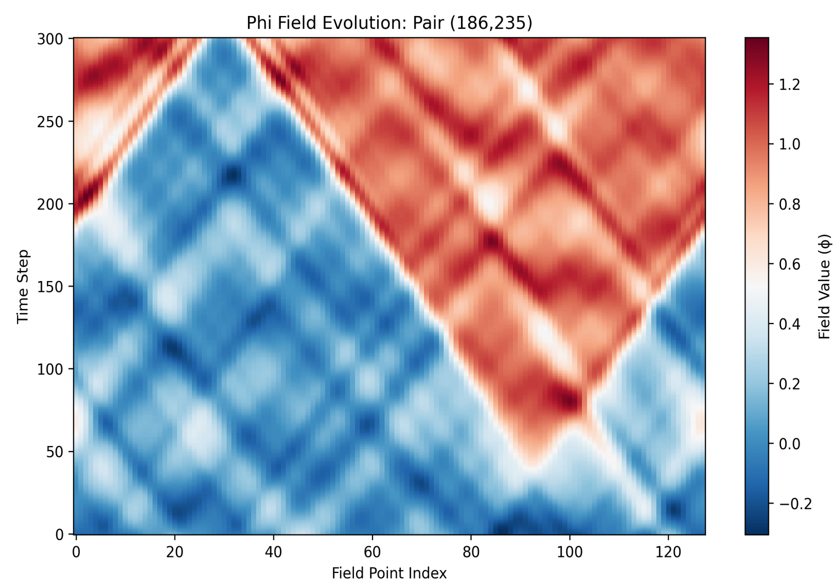


β=1.0

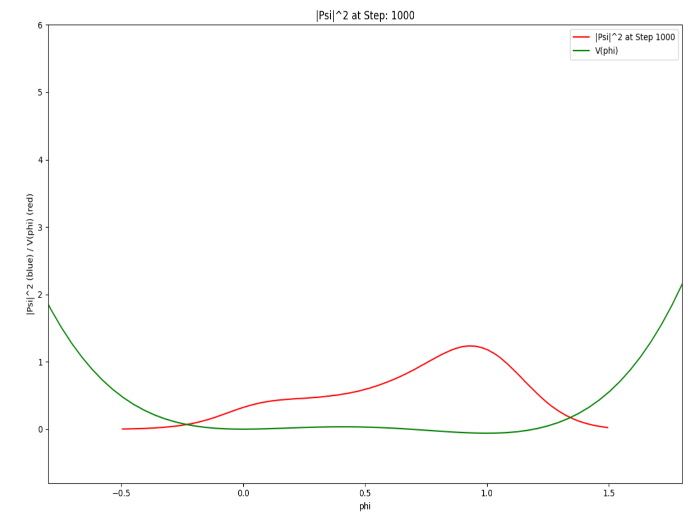




β=1.2

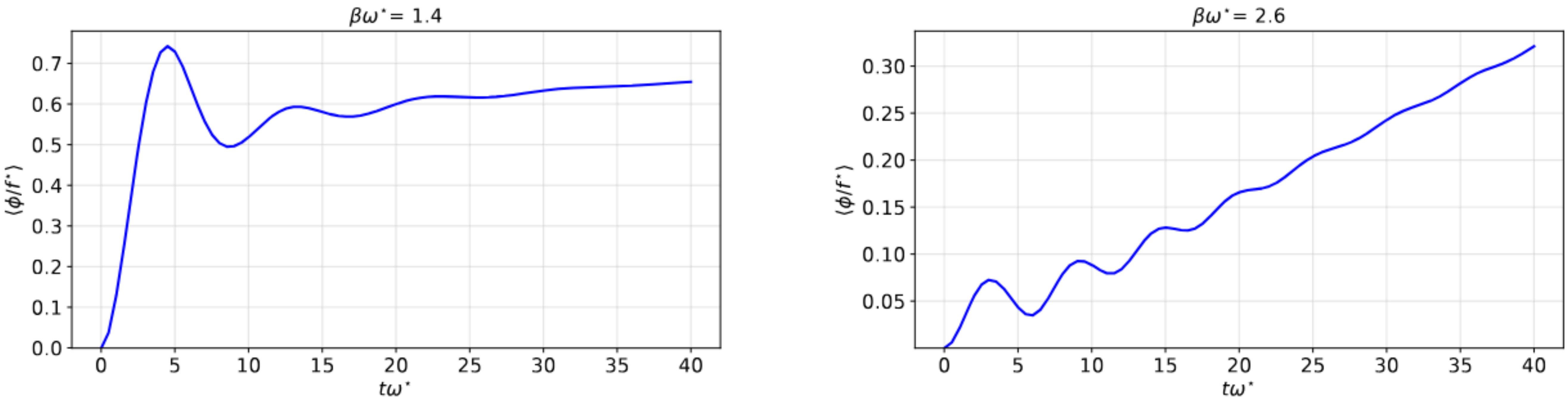


β=1.4

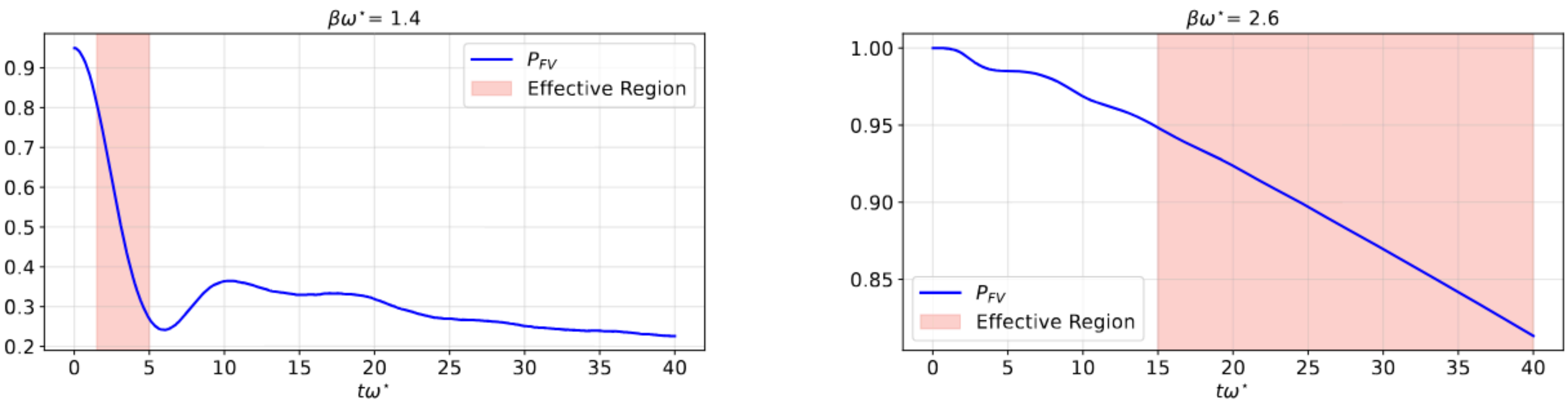


$\beta\omega^* = 1$

Simulation results

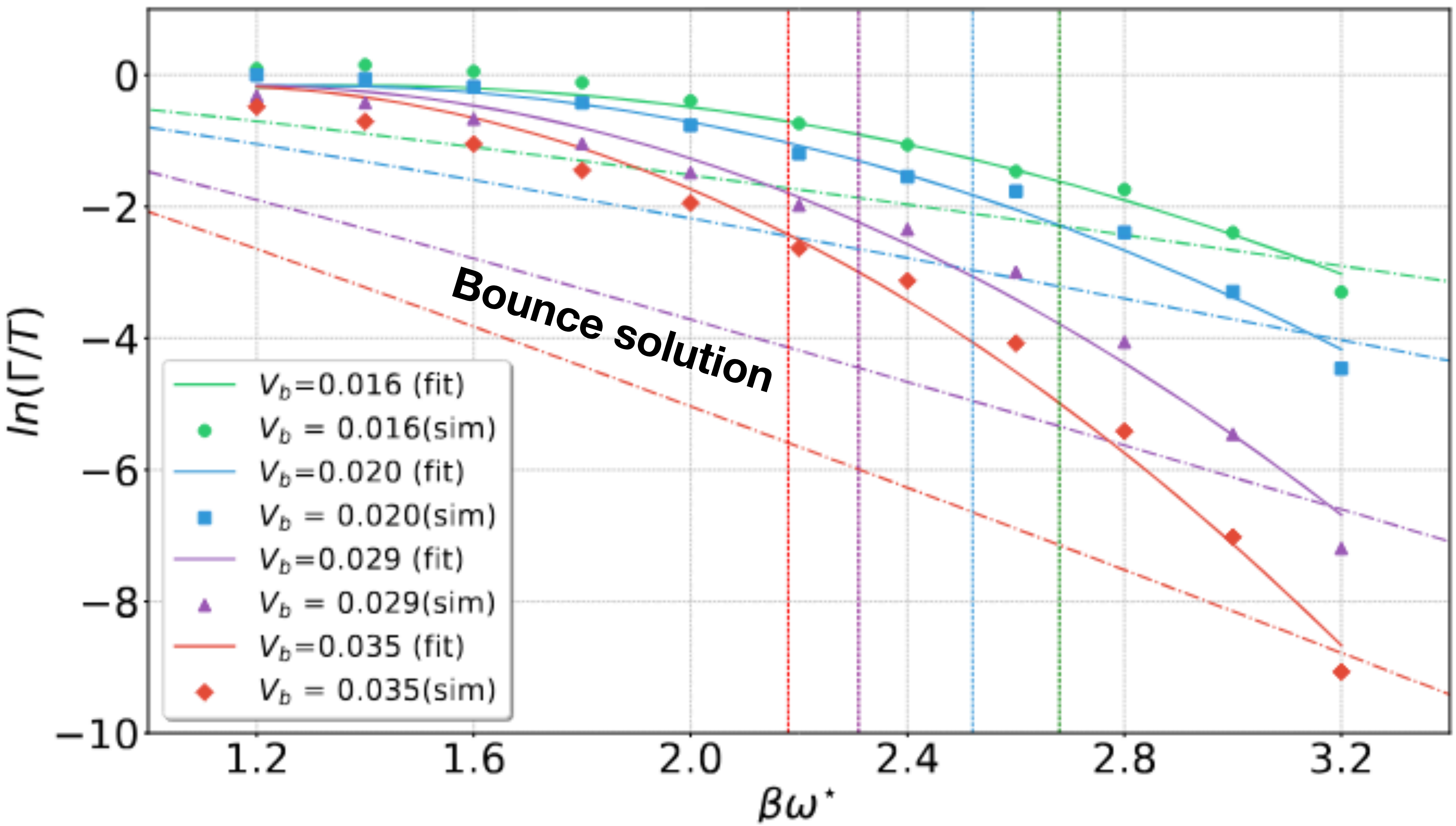


Variations of average field values over time



Variations of false vacuum probabilities over time

Simulation results



▸ Summary

Gravitational waves provide a new window to probe/constrain beyond standard model physics

❖Lattice simulation

- **Nucleation/Sphaleron rate simulations**
- **PT-GW simulation**
- **Topological defects: Magnetic monopoles, cosmic strings, domain walls, string-wall**

❖Pheno

1. Baryon Asymmetry of the Universe and GW from FOPT

- **Sphaleron process, bubble velocity （local equilibrium？）**

2. DM and GW from FOPT

- **DM and high/low-scale PT, DM out-of-equilibrium & FOPT, PBH DM&FOPT**

Thanks!